Fine-tailored for the Cartel

- Favoritism in Procurement -*

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Abstract

In this paper, we investigate the interaction between two firms, which are involved in a repeated procurement relationship modeled as a multiple criteria auction, and an auctioneer (a government employee) who has discretion in devising the selection criteria.

Our main result is that favoritism substantially facilitates collusion. It increases the gains from collusion and contributes to solving basic implementation problems for a cartel of bidders operating in a stochastically changing environment. A most simple allocation rule where firms take turns in winning, independently of stochastic social preferences and firms' costs, achieves full cartel efficiency (including price, production and design efficiency). In each period the selection criteria is fine-tailored to the in-turn winner: the "environment" adapts to the cartel. This result holds true when the expected punishment is a fixed cost. When the cost varies with the magnitude of the distortion of the selection criteria (compared to the true social preferences), favoritism only partially shelters the cartel from the environment. We thus find that favoritism generally facilitates collusion at a high cost for society. Our analysis suggests some anti-corruption measures that could be effective in curbing favoritism and collusion in public markets. It also suggests that the much-advocated rotation of officials is likely to be counter-productive.

Keywords: auction, collusion, favoritism, procurement

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1 Introduction

Many cartels operate in a stochastically changing environment. This is, in particular, the case for firms involved in public procurement. Public demand for e.g. construction works typically depends on a number of factors that are difficult to predict. These include social needs, the political agendas of elected representatives, internal budget concerns, etc. In addition, firms' technologies change over time. Together, these factors result in significant uncertainty about the profitability of future contracts. In the face of such an uncertain environment, a cartel of firms must devise a mechanism that, while being responsive to changes, does not induce opportunistic behavior. In this paper we claim that favoritism can contribute to solving key problems for a cartel of bidders operating in a stochastically changing environment. A main motivation for the paper is the mounting body of evidence that collusion and corruption often go hand in hand in public procurement.

In France, practitioners and investigators in courts of accounts, competition authorities, and the judiciary have long been aware of the close links between collusion and corruption in public procurement.[1] According to one of the leading Parisian anti-corruption judges, there is hardly any single case of large stake collusion in public procurement in France that is free of corruption.[2] Beside empirical motivations, there are theoretical motivations for investigating the links between favoritism and collusion. In particular, a cartel typically faces tension between the goal of efficiency and the need to provide firms with incentives to reveal private information. A fair amount of attention has been given to the theoretical problems facing a cartel operating in an imperfectly or privately observable environment.[3] Our analysis focuses on a specific mechanism that may sustain collusion between bidders who face both incomplete

[1] The testimony of J. C. Mery provides suggestive evidence of those links (*Le Monde*, September 22 and 23, 2000). J. C. Mery, a City Hall official, admitted that over a period of ten years (1985-94), he organized and arbitrated collusion in the allocation of most construction and maintenance contracts for the Paris City Hall. The system included "measures to restrict competition and favor over-costly solutions" (Le Monde, December 15, 2000). In return, firms paid bribes that were used to finance political parties. A recent judgment in 'Les Yvelines' (Cour d'Appel de Versaille, January 2002) provides a vivid illustration as well. The head of the Conseil General was sentenced to 5 years in prison for organizing collusion between firms.

[2] This judge from the "Pole Financier" was, among other things, involved in the investigation of corruption allegations in the procurement of a 4.3 billion euro program for the reconstruction of Paris high schools (see *Le Monde* April 23 2005). The investigation concerned collusion, corruption and favoritism. (Ordonnance de Justice sur le Marchés d'Ile-de-France 2001). It resulted in a series of convictions (L'Express March 14 2005).

[3] See for instance Green and Porter (1984) and Athey et al. (2004).

information about demand, i.e. social preferences, and asymmetric information about shocks to firms' costs. This approach brings us close to Athey and Bagwell (2001), who study a price cartel operating on a market with a given demand but under asymmetric information about costs. They show that cartel efficiency (including production and price efficiency) is achievable in a scheme where firms are rewarded for truthfully reporting high costs by future market share. Their efficiency result relies on assumptions that ensure the existence (with sufficiently high probability) of states where firms have an identical cost structure.[4] Our main contribution is to show that in an auction context, corruption can solve the cartel's information revelation problem in a situation characterized by both asymmetric information and stochastic government demand. Full cartel efficiency, including production, price and design efficiency (the contract is fine-tailored to the cartel) is achievable in a very simple scheme relying on a non-contingent allocation rule, so that firms take turns to win bids in a pre-determined manner.[5] Favoritism effectively shelters the cartel from random events in the environment. The expected cost of corruption determines the extent of favoritism. We establish this result for the case where the expected punishment cost is independent of the magnitude of the distortion of government preferences. When the expected punishment varies with that magnitude, favoritism only partially shelters the cartel from unpredictable events in the environment. We find that favoritism generally exacerbates the social costs of collusion: the selected specification is socially inefficient and the price paid by the government is higher than in the absence of favoritism.

We model the procurement procedure as a "first score auction". Two firms characterized by a vector of cost parameters compete in scores with offers that include a specification of the project and a price. Social preferences are stochastic. The procedure is administered by an auctioneer who is a government employee. At the beginning of the period, the auctioneer privately observes a signal of social preferences. His duty is to devise and announce a scoring rule that reflects (current) social preferences. In the absence of favoritism, the procedure selects the socially efficient specification of the project.

The presence of asymmetric information between the government and its auctioneer im-

[4]In those states, utility can be transferred at no cost for the cartel. As a consequence, future market shares can be used to provide incentives for disclosing private information without relinquishing production or price efficiency.

[5]A key assumption is that in each period, each firm is at least as efficient as the other firm at producing some specification of the project.

plies that the auctioneer has some discretion in defining the scoring rule. We call favoritism the act of biasing the scoring rule in favor of one of the firms. Corruption is modeled as an auction-like procedure that takes place before the official auction. Firms compete in (menus of) corrupt “deals” including a bribe and a requested scoring rule. We first find that in the one-shot game there exists a major hindrance to favoritism, due to firms' incentives to free-ride in the bribing game.

We then consider a situation where firms meet repeatedly, each period on a new market (the auctioneers are short-run players). We show that favoritism can solve the cartel's problems due to stochastic social preferences and privately observable costs. Provided each firm is efficient at producing some specification of the project, the cartel can earn the maximal income in a scheme that selects the winner independently of true public preferences and firms' costs. The intuition is that with corruption the auctioneer has incentives to fine-tailor the scoring rule to the firm whose turn it is to win. The firms' main concern is to limit competition in bribes. This is achieved by opting for a fixed in-turn allocation rule which makes any defection from the equilibrium strategies immediately observable.

In an extension, we investigate a case where the expected punishment for favoritism is a function of the magnitude of the distortion between the announced scoring rule and true public preferences. We find that the central insights from the fixed punishment case carry over. But with a high cost of punishment, the cartel faces a problem due to imperfect public information. The official auction outcome is bounded away from full cartel efficiency. With a low cost of punishment, the pre-determined in-turn allocation rule is optimal and full cartel efficiency obtains.

The equilibrium allocation patterns emerging from the analysis are consistent with empirical findings. There exists ample evidence, e.g. in developing countries, of problems of maintenance of constructions due to the non-standard design selected in the international procurement procedure (see Rose-Ackerman 1999). Evidence from corruption scandals in France[6] also show that the tender winner is the most efficient firm and that its profits are often larger than the average in the branch.[7] In our analysis, we get a "bang-bang" result: favoritism results in the "environment" fully adapting to the cartel's interests. In reality, we may expect the agent to face some constraints and to attempt to mask favoritism, e.g. by

[6] See footnote 2.

[7] L'Express of 14/03/2005 gave this quotation from the judge: "These rates of return (up to 16.3 %), clearly above those achieved in a competitive environment, testify to the enormous cost to society of these practices"

giving some weight to more than one component. We discuss the interpretation of this result in Section 6.

A first key policy implication emerges from our analysis: collusion and corruption must be investigated conjointly. A second implication is that increasing the severity of punishment can have a real impact on the extent of favoritism. On the other hand, the much advocated anti-corruption policy of reducing the time spent in any particular office, i.e. the rotation of officials, finds no support in our analysis. On the contrary, it makes collusion more profitable.

This paper contributes to a growing literature on corruption in auctions.[8] The auctioneer's abuse of discretion in devising the selection rule has been studied in Che and Burget (2004). But our focus is on the links between collusion and favoritism. So the present article is also related to Compte, Lambert-Mogiliansky and Verdier (2005) and Lambert-Mogiliansky and Sonin (2006), both of which investigate links between corruption and collusion. Those papers focus on the issue of enforcement of the cartel's agreement.[9] In the present paper we bring to light the role of corruption with respect to another central problem of cartels, i.e. how to achieve cartel efficiency in a stochastically changing environment.

The paper is organized as follows. The model is described in Section 2. Section 3 presents an analysis of the one-stage game. In Section 4 we derive our central results. Section 5 proposes an extension to the case with varying punishment costs. Central assumptions are discussed in Section 6. In section 7, we formulate some policy implications for procurement and control agencies.

2 The model

In each time period a project is allocated. A project allows for a multiplicity of specifications. A specification is a vector $\mathbf{q} = (q_1, ...q_k)$, $\mathbf{q} \in \mathbf{R}_+^k$ where q_j represents the level of the (quality) component j. There are two firms indexed i, $i = 1, 2$. Firm i is characterized by its cost function

[8] See for instance Laffont and Tirole (1991), Celentani and Ganunza (2002), Che and Burget (2004), Compte, Lambert-Mogiliansky and Verdier (2005).

[9] In Compte et al. (2005), the auctioneer sells an illegal opportunity to resubmit, which is shown to enable collusion to be sustained in a single-object auction. In Lambert-Mogiliansky and Sonin (2006), the auctioneer abuses a legal right to allow all firms to readjust their offer simultaneously in the context of a multiple-object auction. As a consequence, collusive market-sharing becomes sustainable.

$$c\left(\mathbf{q};\boldsymbol{\theta}_i^t\right)=\sum_{j=1}^{k}\frac{\theta_{ij}^t q_j^2}{2}$$

where $\theta_{ij}^t \in \left\{\underline{\theta},\overline{\theta}\right\}$, $j=1,...,k$ is firm i's cost parameter associated with quality component q_j in period t. The vector of cost parameters $\boldsymbol{\theta}_i^t=\left(\theta_{i1}^t,......\theta_{ik}^t\right)$ is firm i's private information.[10] In each time period there is a new draw of $(\boldsymbol{\theta}_1,\boldsymbol{\theta}_2)$. We assume that no firm has high cost $\overline{\theta}$ on all components in any period, i.e. no firm is ever fully inefficient. This assumption greatly simplifies the presentation of the results. Where it is of interest, we comment on the effect of relaxing it.

The (benevolent) government derives utility from the realization of a project in period t:

$$\begin{aligned} W\left(\mathbf{q}^t,p^t;\boldsymbol{\alpha}^t\right) &= \alpha_1^t q_1^t+...+\alpha_k^t q_k^t-p^t, \\ \text{with } \alpha_j^t &\geq 0,\ \forall j=1...k,\ \sum_{j=1}^{k}\alpha_j^t=1, \end{aligned}$$

where p^t is the price paid to the firm that delivers the project and $\boldsymbol{\alpha}^t=\left(\alpha_1^t,...,\alpha_k^t\right)$ is a vector of parameters representing the true social preference in period t. We assume that the price only takes discrete values with a smallest increment of $\varepsilon>0$.[11] The formulation of the $W(.)$ function implies that the government gives equal weight to price and quality, while the relative weights given to the different quality components vary between projects. A zero value for component j, $\alpha_j^t=0$ is understood as no social value of q_j above a *minimal level* that defines a "basic good". The vector $\boldsymbol{\alpha}^t$ is random with support Δ^{k-1}. The government does not know the true $\boldsymbol{\alpha}^t$. It hires an auctioneer who privately observes a signal of the true $\boldsymbol{\alpha}^t$ at the beginning of each period. For simplicity, we assume that the signal is fully informative.[12] To simplify the exposition, we drop the time index whenever this does not lead to confusion.

The auction rule

At the beginning of each period, the auctioneer announces a selection criterion that is a function of both price p and quality $\mathbf{q}=(q_1,...,q_k)$. We consider a class of selection criteria

[10] As is well-known, the discrete structure of the type space implies that, under asymmetric information, the optimal bidding strategies in the auction are mixed (see e.g. Riley, 1989). This issue only arises in Proposition 1, however. On the other hand, dealing with discrete types facilitates the treatment of favoritism.

[11] Assuming discrete prices enables us to use a general existence result in the stage game, but has no further implication for our results.

[12] This assumption is not crucial to our results.

similar to the government's utility function:

$$S(\mathbf{q}, p, \widehat{\boldsymbol{\alpha}}) = s(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) - p = \sum_{j=1}^{k} \widehat{\alpha}_j q_j - p, \ \sum_{j=1}^{k} \widehat{\alpha}_j = 1,$$

where $\widehat{\boldsymbol{\alpha}}$ is the vector of parameters *announced* by the auctioneer (see *Timing* below). Throughout the paper we refer to $\widehat{\boldsymbol{\alpha}}$ as the "scoring rule". This is a slight abuse of language, since the price also enters into the determination of the score of an offer. At each time period t, the firms simultaneously submit an offer in a sealed envelope, including a project specification $\mathbf{q}_i^t$ and a price p_i, $i = 1, 2$. The contract is awarded to firm i^{*t} whose offer maximizes (from among the offers submitted) the announced selection criterion, subject to a "reservation score" normalized to zero:

$$\begin{aligned} i^{*t} &\in \arg \max_{i \in \{1,2\}} S\left(\mathbf{q}_i^t, p_i^t, \widehat{\boldsymbol{\alpha}}\right) \\ s.t. &: S\left(\mathbf{q}_i^t, p_i^t, \widehat{\boldsymbol{\alpha}}\right) \geq 0. \end{aligned}$$

The winner undertakes to deliver the specification $\mathbf{q}_{i^{*t}}^t$ at price $p_{i^{*t}}^t$. In case of a tie in scores, the project is awarded to the firm with the highest "quality score" (i.e. $s(\mathbf{q}, \widehat{\boldsymbol{\alpha}})$). In case of a tie in both price and quality, the auctioneer randomizes. We refer to this procedure as a First Score Auction (FSA).

The firm i's per period profit-if-win is

$$\pi_i^t = p_i^t - c_i\left(\mathbf{q}_i^t; \boldsymbol{\theta}_i^t\right). \tag{1}$$

Profit-if-lose is zero.

The game is infinitely repeated with the same two firms but with a different auctioneer in each period. The firms discount future gains with a common factor δ. Their payoff for the whole game is the discounted sum of the per-period profits.

Corruption

The auctioneer is opportunistic. He accepts bribes in exchange for announcing a scoring rule, i.e. some $\widehat{\boldsymbol{\alpha}}$. The auctioneer's utility is

$$U = w + b - d[\{\widehat{\boldsymbol{\alpha}} \neq \boldsymbol{\alpha}\}] + D[\{\widehat{\boldsymbol{\alpha}} \neq \boldsymbol{\alpha}\} \cap \{b = 0\}]$$

where w is a wage that we normalize to 0, and $b \in \mathbb{R}^+$ is the bribe paid to the agent. The parameter d ($d \geq 0$) captures the expected punishment cost associated with distorting social preferences[13] and the parameter D (large) captures the moral cost associated with being cheated in corruption, i.e. when the agent grants a favor but receives no bribe.[14] [.] is an indicator function; it takes value 1 under the event in brackets and 0 otherwise. In the basic model, the expected cost for manipulating the scoring rule is a fixed cost. This is consistent with French legislation, for example (Code Penal 432-14, 432-11).[15] In Section 5 we consider a case where the expected cost depends on the magnitude of the distortion of social preferences: $U = b - d\left(\widehat{\alpha}_1 - \alpha_1\right)^2 + D\left[\{\widehat{\boldsymbol{\alpha}} \neq \alpha\} \cap \{b = 0\}\right]$. We discuss these assumptions in Section 6.

Corruption is modeled as a procedure where the firms compete in corrupt "deals". A deal is an offer to pay a bribe in exchange for a specific scoring rule. The two firms simultaneously and secretly submit a menu of deals of the following form: $M_i = \{(\boldsymbol{\alpha}_{il}, b_{il}),\ l = 1, ..., n_i\}$, where $\boldsymbol{\alpha}_{il}$ is a requested scoring rule by firm i, b_{il} is the promised bribe for $\widehat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}_{il}$ and n_i is (finite and) freely chosen by firm i.

The bribe is assumed to be enforceable. This can be justified by a community enforcement argument (Kandori (1992)) similar to the one developed in "Why firms pay occasional bribes" (Lambert-Mogiliansky (2002)).[16] We further assume that the bribe is only paid by the winner of the official auction if the announced scoring rule corresponds to one he requested.[17]

3 The stage game

The stage game is defined by the following *Timing*:

step 0: Firms privately learn their cost parameters $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$;

step 1: The auctioneer learns $\boldsymbol{\alpha}$, the firms submit their menus of deals $M_1 = \{(b_{1l}, \boldsymbol{\alpha}_{1l}),\ l = 1, ..., n_1\}$

[13] The government can conduct a procedure to discover the true social preferences and punish the auctioneer if he distorted them in his announcement.

[14] This happens when the firm that asked for the favor fails to win the auction.

[15] The punishment for favoritism is independent of the social economic loss induced.

[16] The idea is that the corrupt officials interact in a network which allows the transmission of some (limited) information, for instance concerning firms' trustworthiness. Although each agent only meets each firm once, all other agents will refuse to enter into a corrupt deal with an untrustworthy firm, which effectively punishes cheating firms.

[17] An alternative would be to let the firm pay after the requested scoring rule has been announced, i.e. whether the firm wins the auction or not. Such a modeling option would not affect the main results. But it would make the presentation less transparent. This is because when deciding which bribe to offer, the firms must account for new uncertainty including the possibility of ending up with a negative payoff. This happens when a firm pays the bribe but is out-competed in the official auction.

and $M_2 = \{(b_{2l}, \boldsymbol{\alpha}_{2l}), \ l = 1, ..., n_2\}$ respectively;

step 2: The auctioneer makes an announcement $\widehat{\boldsymbol{\alpha}}$, $\widehat{\boldsymbol{\alpha}} \in \Delta^{k-1}$;

step 3: The firms simultaneously submit their offers $(\mathbf{q}_i, p_i), \ i = 1, 2$;

step 4: The auctioneer publicly opens the envelopes and selects the firm (i^*) whose offer maximizes the selection criterion defined in the announced scoring rule. If the announced $\widehat{\boldsymbol{\alpha}}$ is among the scoring rules requested by the winner i^* in a corrupt deal, the winner pays the corresponding bribe. Otherwise no bribe is paid.

We first establish a result applying to the First Score Auction described by the *Timing* above when step 1 is omitted from consideration. Throughout the paper we consider Perfect Bayesian equilibria.

Lemma 1 *In any Perfect Bayesian equilibrium of the FSA where firm i wins the auction with positive probability, firm i's offer is characterized by specification efficiency:* $\mathbf{q}_i^* = \arg\max_{\mathbf{q} \in \mathbf{R}_+^k} s(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) - c(\mathbf{q}; \boldsymbol{\theta}_i)$.

All proofs are presented in the appendix.

The result in Lemma 1, which exploits separability between quality and price in the selection criterion, greatly simplifies the forthcoming analysis.[18] The equilibrium values of the components are the efficient ones corresponding to the announcement. They are

$$q_{ij}^* = \frac{\widehat{\alpha_j}}{\theta_{ij}}, \ i = 1, 2, \ j = 1, ..., k.$$

When the announcement corresponds to the true social preferences, Lemma 1 implies social efficiency in the design of the project.

The free-riding problem

We now investigate the whole game described in the *Timing* above. It turns out that it is plagued by a most serious free-riding problem. With asymmetric information between firms, each firm has an incentive to let the other firm commit to pay for a favor (in a corruption deal) and then to undercut its offer in the official auction to win the favorable contract without paying any bribe. Below, we formulate a Claim that under a slightly modified informational assumption, firms can avoid free-riding and favoritism can occur in equilibrium.[19] But for

[18] A similar result can be found in Che (1993).

[19] It can be shown in a simpler version of the basic model with asymmetric information, that when D is small the stage game does not always have an equilibrium in pure strategies with respect to the decision whether or not to free-ride.

our main result in proposition 2, we wish to retain the (standard) assumption of asymmetric information. So we note that when free-riding occurs the agent ends up with a negative payoff of $d + D$. We assume that the auctioneer's cost of being cheated (D) is large enough to rationalize the following decision rule. He only agrees to bias the scoring rule if the bias is requested by both firms (so he always get paid) and associated with bribe offers whose average is not less than d. Our first result is that:

Proposition 1 *There exists a Perfect Bayes Nash equilibrium of the stage game in which*

- *no firm demands any favor;*
- *the agent announces the true scoring rule;*
- *firms' expected profit is* $E\pi(\widehat{\boldsymbol{\alpha}}) \in \left[0, \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}\right]$.

In the proof of Proposition 1 we derive the value of D that rationalizes the agent's strategy not to grant a favor that is only requested by one firm. It is easy to show that it is then optimal for firms never to ask for favors, since the agent disregards any request made by only one firm. To prove the existence of an equilibrium, we use the fact that the price is discrete and invoke a general existence result. Because of the complex cost structure, it is difficult to derive a closed-form solution. Instead, we characterize a firm's expected profit in terms of an interval. In the following, we take the upper-bound of that interval as an approximation of the stage game payoff and we use it as the equilibrium threat payoff of the repeated game below. We wish to emphasize that this is a very conservative approach: the threat is evaluated in its weakest possible realization.

To show that this paper investigates collusion with favoritism under relatively demanding conditions, let us suppose instead that it is common knowledge that each firm has a comparative advantage on one of the k components (but assuming asymmetric information for the rest of the cost structure). Then, we claim that the one-shot game described in Timing has an equilibrium with very costly favoritism:

Claim 1 *For* $d \leq \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}$, *there exists a Perfect Bayesian equilibrium characterized by* $b_{1i}^* = b_{2j}^* = \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}$ *for some* $\theta_{1i} = \underline{\theta}$ *and* $\theta_{2j} = \underline{\theta}$, $i \neq j$. *Both firms have an expected payoff equal to zero. The agent appropriates all the rents.*

The proof of this claim is very similar to the proof of Proposition 3. In equilibrium, each firm offers a deal on the single component with (known) comparative advantage. The fixed punishment cost d simplifies the reasoning at step 1 as compared to the proof of Proposition 3. Specifically: the profit-if-win associated with the requested $\alpha^j = (0, ..., 1_j, ...0)$ is the same for both firms and equal to $\frac{\overline{\theta}-\underline{\underline{\theta}}}{2\underline{\theta}\overline{\theta}}$. The corruption game boils down to a symmetric information, common-value auction. By a standard argument, the firms submit a bribe equal to the common value $b_1^* = b_2^* = \frac{\overline{\theta}-\underline{\underline{\theta}}}{2\underline{\theta}\overline{\theta}}$.

The claim states that competition in bribes dissipates all the rents, so that the modified informational assumption delivers a threat equilibrium with zero payoff, which makes collusion easier to sustain than under plain asymmetric information.

4 Collusion and Favoritism: A Strategic Complementarity

We now investigate a situation where the two firms interact repeatedly. In each period they meet on a public market administered by a new auctioneer, e.g. by different local governments. In each period there is a new draw of $\left(\boldsymbol{\theta}_1^t, \boldsymbol{\theta}_2^t, \boldsymbol{\alpha}^t\right)$. We are interested in collusion between the two firms under the assumption that transfers between them are precluded.

Information assumptions: At the end of each period, the submitted contract offers are (publicly) observed by the two firms and the current auctioneer. The corrupt deal offers remain the private information of the parties involved. The true value of $\boldsymbol{\alpha}$ is never publicly revealed. Each auctioneer is appointed for one period only and there is no communication between auctioneers from different periods.

We consider a repetition of the game described in the *Timing* above. The firms discount future payoffs with a common discount factor δ.

Below, we characterize an equilibrium of the repeated game that exhibits full cartel efficiency in the official auction. Full cartel efficiency is defined as follows: i) In each period the winner is (one of) the most efficient firms with regard to the announced selection criterion (productive efficiency). ii) The price paid to the winner is the highest price that the government is willing to pay (price efficiency). iii) The selection criterion that applies yields the highest gains to the winning firm from among all possible selection criteria (design efficiency). Note that the third part of our criterion goes beyond the standard definition of cartel efficiency.

Proposition 2 constitutes the central result of this paper.

Proposition 2 *i. There exists $\underline{\delta_1} < 1$ and $\overline{d} > 0$ such that for $\delta \geq \underline{\delta_1}$ and $d \leq \overline{d}$ full cartel* *efficiency is achievable in a Perfect Bayesian equilibrium of the repeated game.*

ii. In the official auction firms take turn in winning independently of the true social preferences and of the firms' costs.

iii. The equilibrium scoring rule is extreme (i.e. $\widehat{\alpha}^* = (0, ..., 1, ...0)$) *and the winning firm* i^* *pays a bribe* $b_{i^*} = d$.

Proposition 2 establishes that with favoritism full cartel efficiency is achievable in spite of incomplete and asymmetric information.[20] The cartel does not need to adapt to the "environment", i.e. to the current cost structure or to current social preferences. Instead, it is the environment that adapts to the cartel: in each period the auctioneer fine-tailors the scoring rule to the in-turn winner, which secures production efficiency. The optimal allocation rule is extremely simple: firms take turn in winning in a pre-determined manner. This allocation rule ensures that there is no competition in bribes and no incentive to free-ride. At the corruption stage both firms offer a menu of deals, each of which requests an extreme scoring rule. We return to this result below. The out-of-turn firm offers a zero bribe while the in-turn firm (the designated winner) offers a bribe that just covers the expected punishment cost d. The out-of-turn firm could consider deviating and secretly offering a bribe of, e.g. $d+\varepsilon$ in exchange for a favorable scoring rule. But this would be immediately detected (because the pre-determined in-turn rule would be violated) and punished by reverting to the equilibrium of Proposition 1. This explains why the bribe can be kept to a minimum of d. In the official auction, the out-of-turn firm submits an offer that scores at most zero, which secures cartel price efficiency. Since contract offers become public information, any defection at that stage is detected after the official opening and punished similarly.

The third point in our cartel efficiency criterion is satisfied because favoritism entails an extreme scoring rule $\widehat{\boldsymbol{\alpha}}^* = \widehat{\boldsymbol{\alpha}}^j = (0, ..., 1_j, ...0)$ for some j.[21] The cartel's profit is maximal when the scoring rule puts all the weight on a single component for which the in-turn winner has a comparative advantage. The extreme scoring rule result follows from our specification of the selection criterion.[22] In particular, if each α_i belongs to $[0, 1]$ without the additional

[20] The agent is a short-run agent. He does not know the history of the interaction between the firms but he learns whether or not the firms are engaged in collusive behavior from the deal offers.

[21] We show in section 5 below (Extensions) that this result is robust to other specifications of the punishment costs.

[22] Note that in contrast, the convexity of firms' cost function tends to make it optimal for the firms to request a smooth scoring rule - to minimize production costs.

constraint that they must sum to 1, the result would be different. But in real life, scoring rules are often expressed in terms of the relative weights given to different dimensions of the project. Our specification captures this feature under the constraint that the weight given to price versus quality is fixed.[23] A consequence of this result is that favoritism induces the selection of "non-standard" projects (see Discussion in Section 6).

We note that quite remarkably, firms' private information about their cost structure is a minor concern in our context. The intuition is that it is incentive-compatible for the corrupt auctioneer to use information to devise a scoring rule that maximizes the winning firm's rents. Therefore firms have an incentive to truthfully disclose their private information, which they can do at step 1 of the game. The submitted menus of corrupt deals inform the agent of the firms' cost structures.[24]

We thus see that favoritism facilitates collusion in several ways. The gains from collusion are higher than with an honest auctioneer: the scoring rule is fine-tailored to maximize the winner's profit. Most importantly, we find that favoritism solves key problems for a repeated cartel operating in a stochastically changing environment. This is because it is incentive-compatible for the auctioneer to effectively shelter the cartel from fluctuations in the profitability of projects due to stochastic social preferences and changing costs. The environment "adapts" to the cartel and ex-post efficiency, i.e. efficiency relative to the *announced* scoring rule, is ensured. But this comes at a cost, the bribe which is equal to the expected cost of punishment d.[25]

We now turn to the impact of relaxing the assumption ensuring that no firm is ever fully inefficient. Not surprisingly, this destroys our full cartel efficiency result. If there is some positive probability for such a draw, the fixed in-turn rule does not achieve production efficiency. On the other hand, all that is required for our result to hold is that in any period each firm has low cost on *at least* one component, which is quite a weak requirement.[26]

[23] It can be shown that if the weight given to the price was a choice variable for the agent, favoritism would always give minimal weight to the price.

[24] In the equilibrium of Proposition 2, firms do truthfully disclose all their private information to the agent. But that is not necessary. We could construct another cartel efficient equilibrium where they truthfully disclose only some of their private information about their cost structure.

[25] When reviewing court cases, it appears quite clear that the expected cost of punishment for favoritism is low. The only instances of conviction for favoritism in France pertain to cases where the auctioneer explicitly required a technology that is proprietary to some firm. (Cour des Graces 2002).

[26] In reality, even simple projects are composed of numerous components, i.e. k is large .

Moreover, our conjecture is that for the case where the probability of fully inefficient firms is not too large, the optimality result with regard to the fixed in-turn rule remains true. It is well-known that under asymmetric information the optimal contingent mechanism might not achieve full cartel efficiency either (see, for example, Athey and Bagwell, 2004). But it is beyond the scope of this paper to investigate the optimal contingent mechanism.

Finally, it follows from our results in Proposition 2 that the social cost of favoritism is twofold, as compared with collusion alone. Firstly, a socially inefficient project specification is selected. Secondly, the price paid by the government is higher than it would be in the absence of favoritism, because fine-tailoring maximizes the winner's rents.

5 Variable expected punishment cost

In this section we extend the analysis by considering the case where the expected punishment for favoritism depends on the magnitude of the distortion of social preferences. The auctioneer's utility is $U = b - d(\widehat{\alpha} - \alpha)^2 + D[\{\widehat{\boldsymbol{\alpha}} \neq \boldsymbol{\alpha}\} \cap \{b = 0\}]$. To simplify the presentation we let $k = 2$ so $\alpha_1 = \alpha$ and $\alpha_2 = (1 - \alpha)$ and α is uniformly distributed on $[0, 1]$. We also assume that firms have anti-symmetric cost structures. This implies that firms have complete information about each other and that there is no issue of free-riding. As a consequence, favoritism is sustainable in the stage game. But that is not the point we want to make here. Instead, we are interested in comparing the equilibrium of the repeated game with the one characterized in Proposition 2, to highlight the implications of variable expected punishment costs. However, we first need to describe the stage game and its equilibrium.

The timing of events in the stage game is as follows:

step 0: The firms privately learn their cost parameters $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$;

step 1: The auctioneer learns α and the firms submit their corruption deals $\left(b_{1j}, \alpha_{1j}\right)$ and (b_{2j}, α_{2j}), $j = 1, 2$;

step 2: The auctioneer makes his announcement of $\widehat{\alpha}$, $\widehat{\alpha} \in [0, 1]$;

Step 3 and *4* of the game are the same as in Section 3.

Proposition 3 characterizes the symmetric Perfect Bayesian equilibria of the stage game described above. We show that for $d \leq \frac{4}{5}\frac{\overline{\theta} - \underline{\theta}}{2\underline{\theta}\overline{\theta}}$:[27]

[27] Since $\frac{\overline{\theta} - \underline{\theta}}{2\underline{\theta}\overline{\theta}}$ is the competitive profit-if-win associated with the most favorable scoring rule, this range covers most interesting cases. Over that range of value for d the profit-if-win is simply $\pi_{i^*} = d$.

Proposition 3 *Any symmetric Perfect Bayesian equilibrium is characterized by*

i. The equilibrium scoring rule is $\widehat{\alpha}^*(d,\alpha) = \begin{cases} 1 \text{ for } \alpha \geq 1/2 \\ 0 \text{ for } \alpha < 1/2 \end{cases}$*;*

ii. The equilibrium bribe offers are $b_1^* = b_2^* = b^*(d) = \frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}} - d$*;*

iii. The contract offers are the competitive equilibrium offers corresponding to the announced selection criterion.

A first result is that the equilibrium scoring rule is extreme, as it is in the case of a fixed punishment case (Proposition 2). The reasoning behind the result is, however, different (see Lemma 2 in the appendix).[28] We also note that, as in Proposition 1, the official auction offers are the (specification efficient) competitive equilibrium offers corresponding to the announced scoring rule.

We wish to emphasize that the occurrence of favoritism in the equilibrium of the stage game is not due to the variable punishment cost. Instead, it is firms' information about each other's costs that solves the free-riding problem. As a consequence the agent can extract rent from firms in favoritism as in Claim 1. Interestingly, the variability of the expected punishment costs introduces an asymmetry between firms: the firm whose requested scoring rule is closer to true government preferences has an advantage over the other firm. Firms' incomplete information about true preferences therefore induces a continuity of the probability to win in the submitted bribe. Therefore, and in contrast to our result in Claim 1, some of the rent stays with the firms. The equilibrium bribe depends negatively on the size of the punishment cost, the smaller d is, the more symmetric the firms and the fiercer the competition.

We now consider a repeated version of the game described above. As in the fixed punishment case, the two firms meet in each period with a new (short-run) auctioneer. At the end of each period, the submitted contract offers are publicly observed by the two firms and the current auctioneer. The corrupt deal offers remain the private information of the parties in-

[28] The intuition is that in the corruption game firms compete in the auctioneer's utility levels, i.e. the deal that achieves the highest utility level for the agent wins. This utility can be separated into bribe and expected punishment cost. We show that for any deal with $\alpha \notin \{0,1\}$ that achieves a given utility to the auctioneer, there exists a deal with $\alpha \in \{0,1\}$ that achieves the same utility level but yields a higher expected profit for the firms.

volved. The true value of α is never revealed. There is no communication between auctioneers from different periods. The condition on d is the same as in Proposition 3: $d \leq \frac{4}{5}\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}$.

Proposition 4 *i. For $\delta \geq \underline{\delta_2} \in (0,1)$, there exists a Public Perfect Equilibrium of the repeated game with collusion in contract offers and in corruption deals.*

ii. For small d a simple pre-determined in-turn allocation rule is optimal, while for large d any optimal collusive scheme entails a contingent allocation rule.

A first important remark is that collusion in official offers and in corrupt deals is achievable in a simple pre-determined in-turn scheme at $b^* = d$. The reasoning is similar to that in proposition 2. However, for d relatively large, the simple scheme implies a significant loss in revenue for the cartel. This is because in such a scheme the bribe always covers the punishment cost associated with the maximal distortion of the scoring rule in relation to true preferences. The bribe cost can be reduced in a contingent scheme, but that may not always be worthwhile because of imperfect public information which induces new inefficiencies.

We first note that once the winner has been designated, collusion in the official auction is sustainable, according to a standard trigger punishment argument. This is because offers become public information with the official opening of the envelopes. As in Propositions 2 and 3, in the scheme of Proposition 4, the agent's announcement results from competition in corrupt deals. A major challenge for the cartel is to sustain collusion in corrupt deals in order to limit the cost of favoritism and make efficient use of the stochastic social preferences as an allocation rule. The problem is that firms do not know the true scoring rule and do not observe the submitted bribe deals. They only observe the announced scoring rule, which is an imperfect public signal of firms' actions at step one of the game. Consequently, firms are sometimes "punished" even when complying (this result is similar to results in Green and Porter (1986) and Radner, Myerson and Maskin (1986)). In the appendix we provide an example showing that collusion is sustainable in a Public Perfect Equilibrium (PPE) with $b^* = \frac{1}{4}d$.[29] Deterrence from defection at the bribing stage is achieved by the threat of competition in the official auction. In the event that a firm wins twice in a row, it is "punished" by the other firm which then submits an offer that scores more than zero. This reduces the cartel's revenue (we have price inefficiency). In our example, we have an

[29] A PPE is a profile of public strategies that, beginning at any date t and given any public history till time t, forms a Nash equilibrium. A strategy for player i is public if, at each time t, the strategy depends only on the public history and not on i's private history.

equilibrium with a contingent scheme that is bounded away from full cartel efficiency but that dominates the fixed in-turn rule scheme for sufficiently large d.

We do not aim to characterize an optimal contingent mechanism, but it can be shown that any contingent mechanism is bounded away from full cartel efficiency. This appears to contradict the results in Fundenberg, Levine and Maskin (1994). One of the main reasons is that our model does not satisfy a property of pairwise identifiability which is necessary for their results. More generally, efficiency often fails when the cartel only has a limited set of instruments to achieve conflicting goals. Athey and Bagwell (2001) encountered a similar problem. But in their model efficiency is achievable, although under some restricted configuration of the parameters (and with direct communication between firms). As mentioned in the Introduction, their efficiency result relies on the assumption that there exist states in which firms are identical. With firms that are anti-symmetric in costs and with social preferences uniformly distributed over $[0,1]$, there is no state where firms are identically efficient. Utility cannot, therefore, be transferred at no cost for the cartel. As a consequence, to achieve information disclosure (or collusion in bribes) one must relinquish price efficiency (see appendix).[30]

The main insight from Proposition 4 is that favoritism facilitates collusion even when the expected punishment cost varies with the magnitude of the distortion. For not-too-large d, favoritism increases the collusive gain and the simple fixed in-turn rule is optimal (when the free-riding problem can be overcome in the stage game, favoritism also reduces the threat payoffs). For larger d, and unlike the case with a fixed punishment cost, favoritism does not fully shelter firms from random fluctuations in public demand (social preferences) and costs. Nevertheless, matters are simplified for colluding firms. With favoritism, the profit-if-win is the same for both firms (by force of the optimality of the extreme scoring rule), which they also know. As a result, favoritism makes it possible to sustain collusion in a reasonably simple contingent collusive scheme.

6 Discussion

The main insights of the analysis can be summarized as follows:

[30] In an earlier version we considered a case where there existed states where firms were identical. Even then, full cartel efficiency including price, production and design efficiency cannot be obtained, because we have a trade-off between design efficiency and bribe minimization.

- Favoritism facilitates collusion because:
  - It induces the disclosure of firms' private information, as this information is used by the corrupt auctioneer to *maximize* the winner's rent;
  - It shelters firms from random fluctuations in government preferences. The selected contract specification reflects the cartel's interests instead of social preferences.
- Favoritism exacerbates the cost of collusion for society. The contract specification is socially inefficient and the price is higher than with collusion alone.

The analysis thus reveals that favoritism fundamentally perverts the auction mechanism, in terms of both the use of firms' private information (about their costs) and the use of the agent's private information about social preferences.

A central intermediary result is that the equilibrium scoring rule is extreme. As a consequence, the project that is selected by the procedure tends to be "non-standard". Often the winning firm is the only one to be efficient in its production (as in Proposition 3). In the repeated setting competition is not a direct concern because of collusion but it may be preferable to select a firm that is clearly more efficient. A possible criticism of this result is that with such a selection rule, favoritism is easy to detect and/or may not even be feasible. We first address the issue of feasibility. Detectability and its links to punishment are addressed in the next section. It is true that most procurement codes include provisions that preclude the use of non-standard (*a fortiori* firm-specific) specifications. They typically encourage generic technical specification and corresponding selection rules. Our view is that the result should be understood as applying within the spectrum of discretion consistent with typical anti-favoritism provisions. It says that within that spectrum, favoritism results in the selection of a project specification that maximizes the winner's rent.

The optimality of the extreme scoring rule obtains from the conjunction of a series of assumptions, most of which are standard and/or reasonable. Two assumptions deserve comment: the separability in costs between components and the separability between bribes and punishment cost. There is a natural way to reinterpret our result for the case where there are complementarities in costs. One should then group together components that are complementary in production into a composite component that is given full weight in a proper manner. Clearly, a more elaborate cost structure would entail more complex operations to compute the scoring rule that maximizes the winner's rent. A conjecture is that the menu of deal offers is a sufficiently rich message language to allow for quite sophisticated information to be revealed, so that the auctioneer can maximize rents as in the basic model. On the other hand, some additional analysis may be required if we want to relax the assumption about separability between bribes and expected punishment, for example if the magnitude of the bribe (significantly) affects the risk of detection.

Our conjecture is thus that the main insights of the analysis do not depend on the fine details of the model but capture central features of the reality of favoritism in procurement as revealed by empirical evidence. Firstly, there is a great deal of anecdotal evidence, from developing countries for example, consistent with our findings. In one case, an Africa country defined its telephone specifications to require "equipment that could survive in freezing climates". Only one telephone company from Scandinavia could satisfy this obviously pointless specification (Rose-Ackerman (1999), p.64). Similarly, problems of maintenance of constructions are often due to the non-standard project specifications selected by the international procurement procedures. Secondly, the allocation pattern emerging from the analysis - a predetermined in-turn rule that allocates the contract to the most efficient firm while generating large profits - is very close to the patterns observed in the Paris City Hall case mentioned in the Introduction. According to Jean Montaldo (2006), new procedures, organizations and enterprises were artificially created to facilitate collusion, e.g. the METP (Marchés Entreprises Travaux Publics)[31] and make it easier for the protagonists to affect the terms of reference of the contracts. Among them the BET (Bureau d'Etude Techniques) and AMO (Assistant à la Maitrise d'Oeuvre) were directly in charge of preparing the technical specifications and other aspects of the tender documentation. Interestingly, people have argued that the fact that the contracts were allocated to the most efficient firms suggest that there was no collusion. The present analysis shows that it is sufficient for each firm to have a comparative advantage in some specification for this outcome to obtain in a collusive equilibrium with favoritism.

[31] The METP procedure amounted to bundling construction, loan financing and exploitation/maintainance. It was later forbidden when its negative impact on competition became widely recognized. See for instance Ministère de l'Economie, des Finances et de l'Industrie 26/04/1999.

7 Policy implications

A central message of the analysis is that the risks of collusion and favoritism are linked and must be addressed simultaneously. Yet the investigation of collusion is often the jurisdiction of Competition Authorities while that of corruption is the jurisdiction of criminal courts. A first recommendation is to develop cooperation to overcome this institutional separation, so as to improve efficiency in the prosecution of cases that involve both favoritism (corruption) and collusion.

The analysis confirms earlier results (see, for example, Laffont and Tirole (1993) and Che and Burget (2004)) that discretion in defining the scoring rule is subject to capture by firms. This seems to suggest that one should eliminate the agent's discretion, i.e. let the agent administer a first price auction. But that would be a very naive conclusion. Scoring rules are used to add design flexibility, which generally increases competitive pressure. In a pure first price auction the object has to be fully defined by the technical specifications. Compte and Lambert-Mogiliansky (2000) show that the decisions relating to technical specifications are even more sensitive to capture than those relating to the scoring rule, because they are linked to higher rents. Only when the first price auction is associated with standardization of the technical specifications can the agent's discretion be truly reduced. When standardization is too costly (or not feasible), the auctioneer's decision should be subjected to close scrutiny. This recommendation is in line with Steven Kelman (1994),[32] who argues in favor of preserved flexibility combined with increased accountability of procurement officials. In practice, this means for instance an obligation for procurement agents to justify their decisions in writing. Another type of measure recognizes that firms often have better information about each other than the government has. They can be in a position to recognize when a scoring rule has been fine-tailored to suit another firm. One recommendation would then be to consider designing a mechanism to reveal this information, e.g. by performing an anonymous consultation before the official submission.

Our results suggest that there is a real role for anti-corruption policy (controls and punishments). In the appendix we show that defection from the equilibrium strategies supporting our result in Proposition 2 is deterred if the incentive constraint: $\frac{\delta}{1-\delta^2}\left[\frac{1}{2\underline{\theta}} - d - \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}\right] \geq \frac{1}{2\underline{\theta}}$ is

[32] Harvard professor Steven Kelman was the director of the Office of Federal Procuremnt Policy during the period 1993-1997.

satisfied.[33] For $d < \frac{1}{2\theta} = \overline{d}$, the expression in brackets is positive and the incentive constraint defines $\underline{\delta_1} < 1$. It is easy to see that $\frac{\partial \delta_1}{\partial d} > 0$, i.e. the larger the expected cost of punishment, the more patient firms must be to be able to sustain collusion with favoritism. The potential efficiency of the repressive tools contrasts with the current legislation in the European Union that makes it very difficult to convict for favoritism. A central reason for this is that favoritism is difficult to prove. In general, any selection criterion is likely to favor some firm(s) at the expense of others. The problem is therefore to compare between selection criteria that favor different firms. The honest auctioneer picks the one that is congruent with public preferences, while the corrupt official selects another. But public preferences are seldom so well-defined that congruence can be measured in a non-controversial manner (which also suggests that a fixed punishment cost model may be the most appropriate). In particular, the occurrence of an extreme scoring rule is by no means sufficient evidence. The analysis shows that collusion and favoritism result in specific allocation and specification patterns over time. It therefore suggests that more attention should be paid to a careful study of those patterns. Unfortunately, courts tend to focus on bribery and few cases of favoritism are prosecuted. We therefore suggest that sophisticated economic expertise be given more power in cases where there is a suspicion of favoritism. While this is the rule in cases of standard collusion, economic expertise is rarely requested in cases involving favoritism.

Finally, in our analysis we have assumed that the agents were short-run players. The idea is that the firms are quite specialized and meet on public markets in different jurisdictions and therefore administered by different agents. The demand for public sport facilities, for example, is not recurrent in any single jurisdiction. But we could also interpret our results in the context of firms who meet on public markets organized by the same administration but with officials who are often moved from one position to another. Such a policy is often advocated to prevent corruption, which is presumed to be easier to sustain within the framework of a long-running relationship. Our result shows that this presumption is not warranted here. On the contrary, the short-run character of the agents enables firms to earn all the rents (above d) from collusion. So our results suggest that a high turnover of officials certainly does not make favoritism more difficult. On the contrary, it makes the use of favoritism more profitable to the cartel.

[33] This inequality is derived after some manipulations, see appendix.

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A Proof of Lemma 1

For any announcement $\widehat{\boldsymbol{\alpha}}$, the efficient specification for firm i is defined: $\tilde{\mathbf{q}}_i = \arg\max_{\mathbf{q}\in\mathbf{R}_+^k} s(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) - c(\mathbf{q}; \boldsymbol{\theta}_i)$. We claim that in the subgame perfect equilibrium of the FSA, both firms offer the efficient specification corresponding to their cost structure. Assume that this was not the case, i.e. that in equilibrium, firm i offers $(\widehat{\mathbf{q}}, \widehat{p})$ with $\widehat{\mathbf{q}} \neq \tilde{\mathbf{q}}_i$. Remember that in Lemma 1 we consider equilibria where $prob\{\text{win}|(\widehat{\mathbf{q}},\widehat{p})\} > 0$.

We now show that offer $(\tilde{\mathbf{q}}_i,p')$ with $p' = \widehat{p} + s(\tilde{\mathbf{q}}_i, \widehat{\boldsymbol{\alpha}}) - s(\widehat{\mathbf{q}}; \widehat{\boldsymbol{\alpha}})$ yields a higher expected payoff to firm i than $(\widehat{\mathbf{q}},\widehat{p})$ does. Note that $S(\tilde{\mathbf{q}}_i,p', \widehat{\boldsymbol{\alpha}}) = S(\widehat{\mathbf{q}},\widehat{p}, \widehat{\boldsymbol{\alpha}})$ by construction, so, in particular, $prob\{\text{win} \mid (\widehat{\mathbf{q}},\widehat{p})\} = prob\{\text{win} \mid (\tilde{\mathbf{q}}_i,p')\}$. Now the expected profit from submitting $(\tilde{\mathbf{q}}_i,p')$ is

$$
\begin{aligned}
\pi_i\left(\tilde{\mathbf{q}}_i,p';\boldsymbol{\theta}_i\right) &= \{\, p' - c(\tilde{\mathbf{q}}_i;\boldsymbol{\theta}_i)\}\, prob\left\{\text{win} \mid \left(\tilde{\mathbf{q}}_i,p'\right)\right\} \\
&= \{\widehat{p} - c(\widehat{\mathbf{q}};\boldsymbol{\theta}_i) + [s(\tilde{\mathbf{q}}_i,\widehat{\boldsymbol{\alpha}}) - c(\tilde{\mathbf{q}}_i;\boldsymbol{\theta}_i)] - [s(\widehat{\mathbf{q}},\widehat{\boldsymbol{\alpha}}) - c(\widehat{\mathbf{q}};\boldsymbol{\theta}_i)]\}\, prob\{\text{win} \mid (\widehat{\mathbf{q}},\widehat{p})\} \\
&> \{\widehat{p} - c(\widehat{\mathbf{q}};\boldsymbol{\theta}_i)\}\, prob\{\text{win} \mid (\widehat{\mathbf{q}},\widehat{p})\} = \pi_i(\widehat{\mathbf{q}},\widehat{p};\boldsymbol{\theta}_i)\,.
\end{aligned}
$$

To get the second equality, we plug in the expression for p' and add and subtract $c(\widehat{\mathbf{q}};\boldsymbol{\theta}_i)$. The last inequality holds because $s(\tilde{\mathbf{q}}_i,\widehat{\boldsymbol{\alpha}}) - c(\tilde{\mathbf{q}}_i;\boldsymbol{\theta}_i) > s(\widehat{\mathbf{q}},\widehat{\boldsymbol{\alpha}}) - c(\widehat{\mathbf{q}};\boldsymbol{\theta}_i)$. *QED*

B Proof of Proposition 1

We consider the following strategies for the players:

Firms:

At *step 1,* no firm submits any deal offers.

At *step 3,* the firms submit the competitive equilibrium offers corresponding to the announced scoring rule.

The auctioneer:

At *step 2,* the auctioneer selects a scoring rule requested by the two firms provided the bribes average at least d. Otherwise he announces the true scoring rule.

At *step 4,* the auctioneer selects the offer that scores highest, and if the firms submitted identical offers he randomizes.

Below, we show that the strategies described above form a Perfect Bayes-Nash equilibrium of the game.

At *step 4,* the agent follows the standard official procedure (he does not have any choice).

By Lemma 1 at *step 3* each firm i chooses the efficient quality specification: $q_{ij}^{*} = \widehat{\alpha_j}/\theta_{ij}$. A firm gets zero payoff for sure if the chosen price is so high that it violates the "reservation score" constraint. Hence, we can restrict our attention to the following range of prices $[0, \bar{p}]$, where $\bar{p} = \max\limits_{\boldsymbol{\alpha},\boldsymbol{\theta}} \boldsymbol{\alpha}\mathbf{q}^{*}(\boldsymbol{\alpha}, \boldsymbol{\theta}) = 1/\underline{\theta}$. Together with a "discretized" set of prices, this implies that the firms choose from a finite set of prices. We appeal to a basic result in game theory to assert that there exists a symmetric equilibrium in mixed strategies over prices. Note that in this equilibrium, no firm sets a price which does not cover costs, otherwise it could increase its payoff by setting the price just above the costs of production. This gives us the range of the expected payoff of a firm $\left[0, \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}\right]$.

At *step 2* the agent distorts $\boldsymbol{\alpha}$ if both firms request the same scoring rule $\hat{\boldsymbol{\alpha}}$ with positive bribes that average no less than d. He believes that each firm will win with equal probability. This strategy ensures that he is never cheated, incurring the cost D, and that he is, in anticipation, compensated for his expected punishment cost d. But could he be tempted to accept an offer made by only one firm? Take the symmetric equilibrium from step 3 and let q, $q < 1$ be the highest probability of winning across all cost types of firms. Assume now that a given firm is the only one to offer a certain bribe b, $b > 0$ in exchange for a distorted scoring rule. Under the distorted scoring rule, the firm will bid less aggressively because the size of the "prize" is reduced by b. Hence the agent's expected payoff is less than $bq - d - (1-q)D$.

Next, the agent knows that any firm would prefer to lose the auction if $b > \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}$. As a result, for sufficiently large D or

$$\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} q - d - (1-q)D < 0$$

it is optimal for the agent to disregard any deal offer made by only one firm. Note that this agent's behavior is optimal under any beliefs about the type of the firm making the deal offer.

At *step 1*, each firm knows that the other is not submitting any deal offer. Since the agent rejects all offers offered by only one firm, it is optimal not to submit any deal offers. *QED*

C Proof of Proposition 2

We show that full cartel efficiency can be achieved in an equilibrium supported by a trigger strategy with a punishment phase corresponding to the play of the equilibrium of Proposition 1. We denote by "in-turn firm" (with subscript *in)* the firm whose turn it is to win and by "out-of-turn" firm (with subscript *out)* the other firm. The cooperative phase is characterized by the following:

Firms' strategy:

At *step 1,* the in-turn-firm submits a menu $M_{in} = \left\{ \left(b^{*}, \boldsymbol{\alpha}^{j} \right) \right\}$ with $\boldsymbol{\alpha}^{j} = (0, ..., 1_{j}, ..., 0)$, $\theta_{in,j} = \underline{\theta}$, $b^{*} = d$. The out-of-turn firm submits $M_{out} = \left\{ \left(0, \boldsymbol{\alpha}^{j} \right) \right\}$ for some $\theta_{out,j}$, $b^{*} = 0$.[34]

At *step 3,* for any $\widehat{\boldsymbol{\alpha}}^{t}$ the in-turn firm submits an offer that scores zero. The out-of-turn firm bids to score strictly less than zero.

The auctioneer's strategy:

At *step 2,* if the submitted deals are the equilibrium deals of the cooperation phase, the auctioneer selects from among the submitted corrupt deals one of the deals associated with the highest bribe. If that bribe covers the costs, he announces the associated scoring rule. Otherwise he acts as in Proposition 1.

Let $H_{t-1} = H^{*}$ denote a public history of the game when it is in a cooperative phase, i.e. in all $t' = 1, ..., t-1$ the play corresponds to the strategies described above and the outcome is characterized by the firms winning alternately, i.e. every second period.

The trigger strategy entails that in any subgame following $H_{t-1} \neq H^{*}$, the firms move to (stay in) the punishment phase. Since it is a Nash equilibrium, it is by construction a best response for all players.

We now consider a subgame following $H_{t-1} = H^{*}$ to show that cooperating according to the strategies defined above is optimal. We proceed by backward induction and check optimality with respect to one-stage deviations.

At *step 3,* whatever $\widehat{\boldsymbol{\alpha}}^{t}$, the in-turn firm expects the out-of-turn firm to bid less than zero. The maximal payoff $\pi^{c} = \frac{1}{2\underline{\theta}}$ obtains when the in-turn firm offers the efficient specification and a price such that its offer scores just zero. So the proposed strategy is optimal. The out-of-turn firm may deviate. The most profitable deviation occurs when the announced scoring rule is extreme, i.e. $\boldsymbol{\alpha}_{1}^{j} = (0, ..., 1_{j}, ..., 0)$, the out-of-turn firm also has low cost on the emphasized component and submits $p = \frac{1}{\underline{\theta}} - \varepsilon$. Its gain is $\pi^{d} = \frac{1}{2\underline{\theta}} - \varepsilon$. However, the in-turn rule is violated and from the next period on the firms revert to the competitive equilibrium of proposition 1. So the out-of-turn firm complies with the collusive strategy whenever the

[34] The deal offer of the out-of-turn firm only serves as a signal to inform the agent that the firms are colluding. Many deal offers are suitable for this purpose, provided the proposed bribe does not exceed d.

following incentive constraint is satisfied

$$IC : \frac{\delta}{1-\delta^2}\left(\frac{1}{2\underline{\theta}} - d\right) \geq \frac{1}{2\underline{\theta}} + \frac{\delta}{1-\delta}\frac{1}{2}\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} \tag{2}$$

where the right-hand side depicts the deviation payoff including the first period gain followed by the highest possible expected competitive profit (each firm has, ex-ante, a 50% chance of winning). Since we use the upper limit on the competitive gain, our constraint is more restrictive than it needs to be. Next, in order to facilitate the comparison, we write $\frac{\delta}{1-\delta^2}\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}$, further increasing the value of the deviation since $\frac{\delta}{1-\delta^2} > \frac{\delta}{1-\delta}\frac{1}{2}$ for $\delta < 1$. Simple algebra shows that $\frac{1}{2\underline{\theta}} - d - \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} > 0$ for $d \leq \frac{1}{2\overline{\theta}}$. Hence, by the usual argument, there exists $\delta \geq \underline{\delta_1}$, $\underline{\delta_1} \in (0,1)$ and $\overline{d} < \frac{1}{2\overline{\theta}}$ such that the IC constraint is satisfied. We note that $\frac{\partial \underline{\delta_1}}{\partial d} > 0$.

At *step 2,* the auctioneer is a short-run player (he does not observe the history). The proposed strategy is optimal under the following beliefs. If he observes the equilibrium deal offers, he believes that the firms are involved in collusion and he knows that he will get paid for $d < \overline{d}$, since the out-of-turn firm has no incentive to deviate. It is then optimal for him to grant the favor to the in-turn firm. If, on the other hand, he observes something different from the equilibrium deals, then he believes that the firms are not colluding, and acting as in the stage game is optimal (for sufficiently large D - see the proof of Proposition 1).

At *step 1,* the firms submit their menu of deals. The scoring rule determines the profit-if-win in the official auction. The expected punishment is fixed, i.e. does not depend on the distortion, so the firm requests the scoring rule that yields the largest profit. With collusion there is no competition, i.e. the government's zero score constraint binds, so price is equal to the quality score. The winning firm's profit is for arbitrary $\boldsymbol{\alpha}$

$$s(\mathbf{q}^*, \boldsymbol{\alpha}) - c(\mathbf{q}^*, \boldsymbol{\alpha}) = \sum_{i=1}^{k} \frac{\alpha_i^2}{\theta_i} - \sum_{i=1}^{k} \frac{\alpha_i^2}{2\theta_i} = \sum_{i=1}^{k} \frac{\alpha_i^2}{2\theta_i}$$

maximizing this expression with respect to the vector $\boldsymbol{\alpha}$ yields an extreme scoring rule $\boldsymbol{\alpha}_1^j = (0, ..., 1_j, ..., 0)$, with $\theta_j = \underline{\theta}$. The firm requests a scoring rule that puts all the weight on a single component, which it can produce at low cost. The in-turn firm expects the out-of-turn firm to offer $b = 0$. It is therefore sufficient to offer $b = d$ to cover the auctioneer's cost so that he announces one of the in-turn firm's preferred scoring rules. The out-of-turn firm can defect and offer $b = d + \varepsilon$ associated with a menu including a most preferred scoring rule $\boldsymbol{\alpha}_{out}$. It knows that the auctioneer would respond by announcing that $\widehat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}_{out}$. But such a deviation brings at most a profit of $\frac{1}{2\underline{\theta}} - m - \varepsilon$. Since we know that such a win triggers a punishment phase defection it is not profitable under (2). Hence for $\delta \geq \underline{\delta_1}$ and $d < \overline{d}$ satisfying (2) the proposed strategies do form a Perfect Bayesian equilibrium of the repeated game. The cartel's gain is maximized. In each period, the scoring rule maximizes the winner's gain, the price is given by the reserve score and the bribe is the lowest possible. *QED*

D Proof of Proposition 3

Firms' strategy:

At *step 1,* each firm submits a deal offer $\left(\boldsymbol{\alpha}^j, b^*\right)$ where j is such that $\theta_{ji} = \underline{\theta}$, $j = 1, 2$, with $\boldsymbol{\alpha}^1 = (1, 0)$ or $\boldsymbol{\alpha}^2 = (0, 1)$;

At *step 3,* the firms submit the competitive equilibrium offers.

The auctioneer:

At *step 2,* the auctioneer selects, from among the submitted corrupt deals, a deal that maximizes his utility given his beliefs that the favored firm wins with probability 1. In other words, he picks the deal that maximizes $b - d\left(\alpha - \widehat{\alpha}\right)^2$. He announces the associated scoring rule. In case of a tie, he randomizes.

Below, we show that the strategies described above form a symmetric Perfect Bayesian equilibrium with favoritism. We develop the proof in terms of firm 1 which has its advantage in the production of q_1. Firm 2 is symmetric with advantage in the production of component 2.

At *Step 3,* the favored firm (1) knows that the opponent firm (2) has high cost. By a standard argument, the opponent is expected to bid the lowest price that just ensures non-negative profit, $p_2^* = c\left(\mathbf{q}_2^*, \boldsymbol{\theta}_2\right) = \frac{1}{2\overline{\theta}}$. Our firm's best response is to bid the highest price that ensures a win: $p_1^* = c\left(q_2^*, \theta_2\right) + s\left(q_1^*\right) - s\left(q_2^*\right)$. Firm 1's gain (including the bribe) is

$$\pi_1\left(\widehat{\alpha^1}\right) = \frac{\overline{\theta} - \underline{\theta}}{2\underline{\theta}\overline{\theta}}. \tag{3}$$

At step 2, the auctioneer's utility function is $b_i - d\left(\alpha_i - \alpha\right)^2 + D\left[\{\alpha_i \neq \alpha\} \cup \{b = 0\}\right]$. If no corrupt deal can ensure a win in the official auction or if $b_{\widehat{i}} < d\left(\alpha_{\widehat{i}} - \alpha\right)^2$ where $\widehat{i} = \arg\max_{\{(b_1,\alpha_1)(b_2,\alpha_2)\}} b_i - d\left(\alpha_i - \alpha\right)^2$, the auctioneer announces the true alpha. Otherwise he selects $\widehat{i}$ and announces $\alpha_{\widehat{i}}$.

At step 1, we start with

Lemma 2 *In a symmetric equilibrium, firms always request the "cartel efficient" scoring rule contingent on their cost structure, i.e.* $\alpha_1^* = 1$ *and correspondingly* $\alpha_2^* = 0$.

For firm 1, the "cartel efficient" scoring rule is defined as $\alpha_1^* = \arg\max_{\widehat{\alpha}_1} \left\{ \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} (2\widehat{\alpha}_1 - 1) - d(\widehat{\alpha}_1 - \alpha)^2 \right\}$. It is the scoring rule that maximizes the cartel's payoff given that there is a cost associated with deviations from the true scoring rule (that the agent must be compensated for). We know that the auctioneer selects the firm whose deal maximizes $U(b_i, \alpha_i) = b_i - d(\alpha_i - \alpha)^2$. Suppose by contradiction that an equilibrium offer is (b_1, α_1) with $\alpha_1 \neq \alpha_1^*$. We now construct an offer (b_1', α_1^*) with $b_1' = b_1 - d(1/2 - \alpha_1)^2 + d(1/2 - \alpha_1^*)^2$. By construction,

$$prob\left(U\left(b_1', \alpha_1^*\right) > U(b_2, \alpha_2)\right) = prob\left(U(b_1, \alpha_1) > U(b_2, \alpha_2)\right) = 1/2.$$

Now

$$\begin{aligned} E\pi\left(b_1', \alpha_1^*\right) &= \left[\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - b_1'\right] prob\left(U\left(b_1', \alpha_1^*\right) > U(b_2, \alpha_2)\right) \\ &= \left[\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - b_1 + d(1/2 - \alpha_1)^2 - d(1/2 - \alpha_1^*)^2\right] \frac{1}{2} \\ &= \left[\left(\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}(2\alpha_1 - 1) - b_1\right) + \left(2\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - d\alpha_1\right)(1 - \alpha_1)\right] \frac{1}{2} \\ &> \left[\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}(2\alpha_1 - 1) - b_1\right] prob\left(U(b_1, \alpha_1) > U(b_2, \alpha_2)\right) \end{aligned}$$

where the inequality holds because $d < \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}$.

Hence $\alpha_1^* = 1$ and similarly for firm 2: $\alpha_2^* = 0$.

We now consider the determination of b_1 and b_2. The expected profit of firm 1

$$\begin{aligned} E\pi_1(b_1; b_2) &= \left(\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - b_1\right) prob\left\{U(b_1, 1) > U(b_2, 0)\right\} \\ &= \left(\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - b_1\right) prob\left\{b_1 - d(1-\alpha)^2 > b_2 - d(\alpha)^2\right\} \\ &= \left(\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - b_1\right) prob\left\{\alpha > \frac{1}{2} + \frac{b_2 - b_1}{2d}\right\} \\ &= \left(\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - b_1\right)\left(\frac{1}{2} - \frac{b_2 - b_1}{2d}\right). \end{aligned}$$

Taking the derivative with respect to b_1 an interior solution satisfies

$$b_1^* = \frac{1}{2} b_2 + \frac{1}{2}\left(\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - d\right).$$

In a symmetric equilibrium we obtain

$$b^* = b_1^* = b_2^* = \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - d. \tag{4}$$

In a symmetric equilibrium the auctioneer never distorts more than by 1/2 so the highest cost for distortion is $d/4$. Firms bid the bribe in (4) which is a best response to the other firm. Hence, for $\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - d > \frac{1}{4}d \Leftrightarrow d < \frac{4}{5}\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}$, the investigated strategies described above including the deal offers $(b^*, 1)$ and $(b^*, 0)$ form a Perfect Bayesian equilibrium of the FSA with favoritism. *QED*

E Proof of proposition 4

In this proof we consider two types of collusion, characterize the conditions for their sustainability and compare them in terms of cartel efficiency.

"In-turn rule" collusion:

This type of collusion is similar to the one in Proposition 2. The strategies are the same as the ones described in the proof of Proposition 2 when putting $k = 2$. Any deviation from those strategies triggers the play of the Perfect Bayesian equilibrium of proposition 3 from the next period on.

As usual we investigate the game by backward induction and focus on incentives to comply in a period following a history of compliance play.

At *step 5,* the same argument as in the proof of Proposition 3 applies, since our agent is short-run.

At *Step 3,* the designated winner, say firm 1, has no incentive to deviate, while the other firm might undercut the offer of firm 1 and get at most $\frac{1}{2\overline{\theta}}$ in the current period and a continuation payoff of $\frac{\delta}{1-\delta}\frac{1}{2}\left\{\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - b^*(d)\right\}$ afterwards, which is the expected payoff of the repeated play of the equilibrium of Proposition 3. Given that $b^*(d) = \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - d$, the incentive constraint is

$$\frac{\delta}{1-\delta^2}\left(\frac{1}{2\underline{\theta}} - d\right) \geq \frac{1}{2\overline{\theta}} + \frac{\delta}{1-\delta}\frac{1}{2}d, \tag{5}$$

where the left-hand side is the compliance payoff and the right-hand side is the deviation payoff. In order to facilitate the comparison and derive the limit value on the discount factor δ satisfying (5) we write $\frac{\delta}{(1-\delta^2)}d$ instead of $\frac{\delta}{1-\delta}\frac{1}{2}d$. Since $\frac{\delta}{1-\delta^2} > \frac{\delta}{1-\delta}\frac{1}{2}$ for $\delta < 1$, we effectively make the value of the deviation more attractive. By assumption, we have $d < \frac{4}{5}\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}$ but to simplify the calculation, we check for $d = \frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}$, when the constraint is more restrictive. The limit value for the discount factor $\underline{\delta_2}$ is then given by

$$\frac{\underline{\delta_2}}{1-\underline{\delta_2}^2} = \frac{\underline{\theta}}{2\underline{\theta}-\overline{\theta}}. \tag{6}$$

At *step 2,* the agent has no incentive to deviate, for reasons similar to those in proposition 3. At *step 1,* firm 2 might make a secret bribe bid of $b \in (0, 2d]$ and request $\hat{\alpha} = 0$. The agent will grant firm 2 the favor of choosing its requested scoring rule when $\alpha \in [0, b/(2d)]$. Firm 2 then gets a payoff of $\frac{1}{2\underline{\theta}} - b$ and a continuation payoff $\frac{\delta}{1-\delta}\frac{1}{2}\left(\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}} - b^*(d)\right)$ because the fixed in-turn rule is violated and the firm revert to the play of the equilibrium of Proposition 3. Since the gain from deviation at *step 1* is lower than at *step 3* (because firm 2 has to pay the bribe in the current period), $\delta \geq \underline{\delta_2}$ defined in (6) also guarantees the satisfaction of its incentive to comply at *step 1.*

Contingent rule

In this subsection, we shall describe an equilibrium where the winner of the auction depends on the true α. In order to discourage defection in the bribing game (which allows to win the official auction) we must introduce some punishment for winning "too often". More precisely we shall distinguish between firm 1(2)'s continuation payoff when winning following a period of win which we denote $w_{1(2)}(YES)$ and the continuation payoff of winning following a period of loss, $w_{1(2)}(NO)$. We focus on a smaller set of strategies including $b \in \left\{\frac{1}{4}d, \frac{5}{4}d\right\}$, so in particular we only consider a defection that ensures a win. To out-compete firm 1 when the true scoring rule is most favorable to 1, e.g. $\alpha = 1$, firm 2 must offer $\frac{5}{4}d$. Restricting the set of possible deviations is not crucial to the result but it simplifies the presentation.

Let $H_{t-1} = H^*$ denote a history of the game where in all periods t', $t' = 0, ..t-1$, we have $\widehat{\boldsymbol{\alpha}}^{t'} \in \{0, 1\}$.

We propose the following strategies for the players:

i. If $H_{t-1} \neq H^*$, the firms and the auctioneer play the equilibrium strategies described in proposition 3.

ii. If $H_{t-1} = H^*$, the firms' strategy:

At *step 1,* firm 1(2) submits a menu of deals, one for each component where cost is low, requesting an extreme scoring rule and offering the same bribe $b = \frac{1}{4}d$.

At *step 3,* if $\widehat{\boldsymbol{\alpha}}^t$ is extreme, we distinguish between two situations. If the firm with the congruent cost structure is not the the previous period's winner it submits an offer including the corresponding efficient specification such that the offer scores zero. Firm 2 with the non congruent cost structure submits an offer that scores at most zero. If the designated winner is to win for the second time in a row, the other firm submits an offer with a positive score and the designated winner overbids that offer to win.

If the announced scoring rule is not extreme, the firms play the Nash equilibrium of the complete information asymmetric auction game.

The auctioneer's strategy:

At *step 2*, the auctioneer selects a corruption deal that maximizes his utility, provided the bribe covers expected costs, and he announces the corresponding scoring rule. In case of tie he randomizes.

Below, we show that these strategies form a Perfect Public Equilibrium of the repeated game with the stage game as described in section 5. Collusive bidding at *step 3* is sustainable, relying on an argument similar to the one developed in Proposition 2 when setting $k = 2$. This is because defection is immediately detected. The non-favored firm's (say 2) incentives to comply with the collusive strategy is satisfied for $\delta > \underline{\delta}_2$ where $\underline{\delta}_2$ is defined by the following equality

$$\underline{\delta}_2 \frac{1}{2} w_2 (NO) = \frac{1}{2\overline{\theta}} + \frac{\underline{\delta}_2}{1 - \underline{\delta}_2} \frac{1}{2} E\pi^f$$

where the left-hand-side is the compliance payoff when losing in the current period. The right-hand-side if the payoff when winning today and reverting to the equilibrium of Proposition 3 from the next period on with $E\pi^f = d$ (see Proof of Prop. 3 above).

At *step 2,* the proposed strategy is optimal for the auctioneer, according to the same argument as in proposition 3. At *step 1,* the firms may consider defection and offer a deal with a bribe equal to $\frac{5}{4}d$. The current period defection payoff is at most $\pi^d = \pi^c - \frac{5}{4}d$ (where $\pi^c = \frac{1}{2\underline{\theta}}$ has been defined in the proof of Proposition 2) while the expected compliance payoff is $\frac{1}{2}\left[\pi^c - \frac{1}{4}d\right]$. We first note that for $d \geq \frac{4}{9}\pi^c$ there is no incentive to defect. But for $d < \frac{4}{9}\pi^c$ (recall that we only consider the interval $d < \frac{4}{5}\frac{\overline{\theta}-\underline{\theta}}{\overline{\theta}\underline{\theta}}$ implying $d < \frac{4}{9}\pi^c$), the following incentive constraint applies in any period t preceded by a loss:

$$\frac{1}{2}\left[\pi^c - \frac{1}{4}d\right] + \delta\left[\frac{1}{2}w_1 (YES) + \frac{1}{2}w_1 (NO)\right] > \left(\pi^c - \frac{5}{4}d\right) + \delta w_1 (YES)$$

where the left-hand-side is the compliance payoff. In the current period the firm has 50% chance to win implying that in the next period it faces either continuation payoffs with equal chance. On the right-hand-side we give the payoff when the firm defects and wins in the current period so it faces that continuation payoff following a win with probability 1. Simplifying the expression above we get

$$\delta\left[\frac{1}{2}w_1 (NO) - \frac{1}{2}w_1 (YES)\right] > \left(\frac{1}{2}\pi^c - \frac{9}{8}d\right)$$

$$\delta\left[w_1(NO) - w_1(YES)\right] \geq \left(\pi^c - \frac{9}{4}d\right) \tag{7}$$

So as $\delta \longrightarrow 1,\ w_1(NO) - w_1(YES) \geq \left(\pi^c - \frac{9}{4}d\right)$ the continuation payoff of firm 1 following an announcement of $\widehat{\alpha} = 1$ must be lower that the one following $\widehat{\alpha} = 0$, which means that we need an in-equilibrium punishment to deter deviations. This payoff is achieved by letting firm 2 submit an offer in the official auction that induces a lower profit for firm 1. For $d = \frac{4}{9}\pi^c$ (the highest d for which there is an incentive to deviate), the right-hand side is equal to 0. A possible implementation entails that the full punishment is taken in the period $t+1$ and "the clock is reset", i.e. the next following payoffs in $t+2$ are determined as if no win ever occurred. Note that when (7) holds, incentives to comply in a period following an a gain instead of a loss (as in (7) are also satisfied). This is because the gains from defection in the current period are then lower since the firm can only win the lower profit associated with winning twice in a row. Hence, for $\delta \geq \underline{\delta}_2$ and $w_1(YES) = w_2(YES)$ satisfying (7), the proposed strategies form a Perfect Public Equilibrium of the repeated game.

As $d \rightarrow \frac{4}{9}\pi^c$ the equilibrium average expected payoff of the contingent scheme tends toward $\frac{1}{2}\left(\pi^c - \frac{d}{4}\right) = \frac{4}{9}\pi^c$ while the average payoff of the fixed in-turn rule is $\frac{\frac{5}{9}\pi^c}{1+\delta}$. So for any $\delta > \frac{5}{4} - 1$, the contingent scheme yields a higher expected payoff. We note that the condition $d \geq \frac{4}{9}\pi^c$ is consistent with the condition in proposition 4 $d \leq \frac{4}{5}\pi_{ne}$ when $\frac{4}{9}\frac{1}{2\underline{\theta}} \leq \frac{4}{5}\frac{\overline{\theta}-\underline{\theta}}{2\underline{\theta}\overline{\theta}}$ which requires $\underline{\theta} \leq \frac{4}{9}\overline{\theta}$.

On the other hand, when $d \rightarrow 0$ the right-hand side of (7) tends to π^c implying $\pi_1^{t+1}(1) \rightarrow 0$ corresponding to an average equilibrium expected payoff in the contingent scheme of $\frac{1}{4}\pi^c$ which is strictly smaller than the average payoff in the in-turn scheme $\frac{\pi^c}{1+\delta}$. *QED*