Math Refresher
Module 1, 2017/2018

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Course description

The main goals of the course are the following:
- to refresh basic facts from mathematical analysis and linear algebra;
- to learn the bases of probability theory and mathematical statistics;
- to set the theoretical tools for studying econometrics;
- to introduce the principles of statistical modeling.

The course consists of five parts:
1. Mathematical analysis: short refresher on the theory of single and multiple variable functions, including conditional optimization and ordinary differential equations theory basics;
2. Linear algebra: short refresher of the linear systems, linear operators and quadratic forms theory;
3. The foundations of probability theory. Random variables and random vectors. Limit theorems.
4. The foundations of mathematical statistics. Population and sample. Point and interval parameters estimations. Hypothesis testing.
5. Introduction to econometrics. Simple and multiple regression. Ordinary least squares (OLS)/ Properties of OLS estimator.

Course requirements, grading, and attendance policies

Course prerequisites: none. Because of introductory nature of the course there are no control tasks and exams.

Course contents

The main topics of the course:
1. Single variable functions, their properties.

2. Derivative, integral, differentiation of integrals.

3. Number sequences and series. Taylor series.

4. Multiple variables functions. Implicit functions.

5. Unconditional optimization, necessary and sufficient extremum conditions. Conditional optimization.

6. Differential equations, systems of linear differential equations with constant coefficients.

7. Vectors, linear space, linear dependency. Matrices, matrix operations.
8. Linear equation systems.
9. Eigenvalues and eigenvectors.
10. Classification of matrices and linear operators. Symmetric matrices.
11. Quadratic forms. A criterion to determine whether a symmetric matrix is positive-definite.

12. The foundations of probability theory. Experiment with uncertain outcomes, probability space, random events. Properties of probability. Discrete and continuous probability schemes.
13. Conditional probability. Bayes formula. Independence.
14. Random variables and random vectors.
15. Limit theorems: law of large numbers, central limit theorem.
16. The foundations of mathematical statistics. Population and sample. Point and interval parameters estimations.
17. Hypothesis testing. Statistical test. Significance and power of a test.
18. Introduction to econometrics. Simple and multiple regression. Ordinary least squares (OLS)/ Properties of OLS estimator.

Description of course methodology

The class teaching combines lectures, solutions of the problems and considerations of typical applied examples. There are weekly home tasks. The outline of the lectures are provided.

Selected home task problems

Problem 1. Two dice, black die and red die, are rolled. Find is the probabilities of the following events:
(a) to get at least one "6";
(b) to get and odd number on the black die and an even number on the red die;
(c) to get the different number on the dice.

Problem 2. The box contains 3 black and 5 white balls. Two balls are randomly taken in series. What is the probability that the first ball is white and the second ball is black if balls are taken
(a) with replacement;
without replacement?

Problem 3. The probability that a person with certain symptoms has hepatitis is 0.8. A blood test used to confirm this diagnosis gives positive results for 90% of those who have the disease, and 5% of those who do not have the disease. What is the probability that a person who reacts positively to the test actually has hepatitis?

Problem 4. A company prices its hurricane insurance using the following assumptions:
(i) In any calendar year, there can be at most one hurricane.
(ii) In any calendar year, the probability of a hurricane is 0.05 .
(iii) The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.
Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

Problem 5. Scores on a standardized exam are normally distributed with mean 1000 and standard deviation 160.
(a) What proportion of students score under 850 on the exam?
(b) They wish to calibrate the exam so that 1400 represents the 98th percentile. What should they set the mean to? (without changing the standard deviation)

Problem 6. In a simple random sample of 25 subway cars during rush hour, the average number of riders per car was 91.5 with standard deviation of 6.4.

    (a) Find a 90% confidence interval estimate for the average number of riders per car during rush hour.

    (b) Assuming the same standard deviation of 6.4 how large a sample of cars would be necessary to determine the average number of riders to within ± 2 at 90% confidence level?

Problem 7. Find all extrema for the function $f(x, y) = xy + \dfrac{50}{x} + \dfrac{20}{y}$ and $x > 0$, $y > 0$.

Problem 8. Find all extrema for the function $f(x, y, z) = x - 2y + 2z$ given the following condition: $x^2 + y^2 + z^2 = 1$.

Problem 9. Does the following linear system have a solution?

$$2x - 3y + z = 2$$
$$3x - 5y + 5z = 3.$$
$$5x - 8y + 6z = 5$$

Problem 10. Find the inverse matrix for $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$.

Problem 11. Is the following quadratic form positive-definite? $f(x, y, z) = x^2 + 2y^2 + 4z^2 - 2xy + 3xz + yz$.

Course materials

Required textbooks and materials

    1. Simon and Blume, Mathematics for Economists, Norton, 1994.
    2. Sh. Ross. A First Course in Probability, Pearson, Prentice Hall, 2009.
    3. V.Chistyakov. The course on Probability Theory (5th edition), Moscow, "Agar", 2000 (in Russian: Чистяков В.П. Курс теории вероятностей (5-е издание). М., «Агар», 2000).
    4. V.Gmurman. Probability Theory and Mathematical Statistics. Moscow, "Higher School", 2003 (in Russian: Гмурман В.Е. Теория вероятностей и математическая статистика, М., "Высшая школа", 2003).

Additional materials

    1. A.Shiryaev. Probability, MCNMO, 2011 (in Russian:Ширяев А.Н.. Вероятность, МЦНМО, 2011).
    2. S.Aivazian, V.Mkhitarian. Applied Statistics and Basic Econometrics (2nd edition). Vol. 1: Probability Theory and Applied Statistics, Moscow, UNITY, 2001 (in Russian: Айвазян С.А., Мхитарян В.С. Прикладная статистика и основы эконометрики. (2-е издание). Том 1: Теория вероятностей и прикладная статистика. — М.: ЮНИТИ, 2001).

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.