# Spatial-economic-ecological model for the assessment of sustainability policies of the Russian Federation

## Project 213091

## D3
## Description of the economic model, mathematical formulation and derivations

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# 1. Introduction

The objective of the SUST-RUS project is to develop and implement for Russia an integrated spatio-economic-ecological modelling approach, which represents the state-of-the-art in different areas of economic, transport, resource-use and environmental modeling.

The SUST-RUS modelling approach will provide Russian and international community with the sound scientific support for formulating sustainability policies. The use of the SUST-RUS approach will assist the implementation of the EU strategy for sustainable development in Russia as well as an efficient incorporation of the sustainability goals into the existing Russian policy tools at regional and federal levels. The SUST-RUS modelling approach represents the state-of-the-art in many different areas of knowledge and, hence, will be superior to other models available for Russia.

Sustainability means that the needs of the present generation should be met without compromising the ability of future generations to meet their own needs. The EU sets the following key objectives concerning sustainability[1]:

1. Environmental protection
2. Social equity and cohesion
3. Economic prosperity
4. Meeting international responsibilities

The SUST-RUS modelling approach is characterized by a balanced integration of social, economic and environmental policy objectives. Therefore, in this deliverable we will uncover the appropriate modelling techniques along these dimensions. The deliverable presents a full mathematical formulation of the model. The model is formulated as a system of simultaneous nonlinear equations, which represent the solutions to utility maximization and producer costs minimization problems as well as the market equilibrium conditions.

The major problem of sustainable development is the rational use of spatially distributed natural resources such as minerals, water, land and ecosystem services. The use of most of these resources depends upon the allocation of production and consumption activities. By incorporating the representation of geographically distributed consumption and production patterns into the SUST-RUS modelling framework, we will be able to account for the use of natural resources in the economy as well as to assess the effects of sustainability policies on different Russian regions.

The SUST-RUS model, among others, allows incorporating the following features:

- region-specific factor endowments of capital and labour
- regional production and consumption
- intermediate inputs of the sectors (total output is produced using not only capital and labour but also inputs of various services and goods)
- interregional trade
- representation of government finances (taxes, subsidies and transfers) and multi-level governance system
- emissions related to production and energy inputs of the sectors
- negative effect of emissions on the households' welfare
- investment decisions of households and firms
- representation of agglomeration mechanism in some sectors via Dixit-Stiglitz framework with monopolistic competition (optional)

[1] EU-SDS: EU sustainable development strategy

Chapter 2 gives an overview of the main structure of the model and discusses its main components and the underlying theory. In the subsequent chapters the model will be introduced in mathematical details including a full set of model parameters and variables and a full description of all model equations and their economic interpretation. Most of the model equations are the results of utility maximization or costs minimization problems. We do not present the full derivation of these formulas but just indicate, which optimization problem has to be solved to get them. In case of investment decisions of households and firms, we give a full derivation of the formulas in inter-temporal dynamic framework.

# 2. Main assumptions and structure of the model

SUST-RUS belongs to the group of special computable general equilibrium (SCGE) models, which apply a mix of conventional modelling techniques used in standard computable general equilibrium models at regional level. Typically, SCGE models are comparative static equilibrium models of interregional trade based on microeconomic theory. In particular, SCGE models depend on utility and production functions with substitution between inputs. Firms can operate under economies of scale in markets with monopolistic competition of the Dixit-Stiglitz (1977) type or under perfect competition. Interesting theoretical simulations with a SCGE model with a land market are found in Fan et al. (1998). These models are part of the new economic geography school (Krugman, 1991, Fujita, Krugman and Venables, 1999) and have been around for less than two decades.

The present SCGE models have a sophisticated theoretical foundation and rather complex, non-linear mathematics. The latter is precisely the reason why SCGE models are able to model (dis)economies of scale, external economies of spatial clusters of activity, continuous substitution between capital, labour, energy and material inputs in production, and between different goods in consumption.

The model represents a real economy with no inflation or banking sector. Consequently, there is no monetary authority in the model. All prices are relative and calculated in terms of a numeraire, with the GDP deflator used as the numeraire. Since there is no intertemporal optimization in the model and economic agents do not have the possibility to borrow money, the interest rate is fixed exogenously; so is the marginal propensity to save by consumers.

The model utilizes the notion of a representative economic agent. In particular, the behavior of a whole population group or industrial sector is modeled as the behavior of one single agent. It is further assumed that the behavior of each agent is driven by certain optimization criteria such as maximization of utility or minimization of costs. The model is neo-classical and assumes average cost pricing and no profits in equilibrium. Positive profits are normally due to the existence of monopoly or oligopoly on the market. Accounting distributed firm profits (dividends) are subsumed in the total return to capital (interest payments) to households who own all capital goods in the economy.

The modeling of interregional trade flows is an essential part of the interregional linkage. However, the only available data are the data on the total origin-destination flows of commodities between regions by type of commodity without specification of intermediate and final goods. There is no information about interregional trade in services, nor is there information about differences in the geographical mix of goods used in each region by different industrial sectors and households. Accordingly, the model adopts a simplified structure with no trade in services between regions and no difference in the geographical mix of commodities bought by various sectors in a region.Given this assumption, it is possible to represent the decisions of both sectors and households about buying commodities from a particular region as the decision of a representative agent called “wholesaler”. There is one wholesaler per region and per commodity type, who decides upon the geographical mix of commodities. Regional households and sectors further use the composite commodity, which is produced by the wholesaler. In this way both production sectors and households use the same geographical mix.

The model includes the representation of the micro-economic behavior of the following economic agents:

- At the regional level – one household type by region, production sectors differentiated by NACE95 classification categories; regional governments; wholesalers differentiated by NACE95 classification categories;
- At the national level - investment banks; federal governments and external trade sector.

**Time dynamics**

The model is dynamic, recursive over time, involving dynamics of physical capital accumulation. Recursive dynamics is a structure composed of a sequence of several temporary equilibria. The first equilibrium in the sequence is given by the benchmark year. In each time period, the model is solved for an equilibrium given the exogenous conditions assumed for that particular period. The equilibria are connected to each other through physical and human capital accumulation as well as through changes in migration flows and the number of operating firms. Thus, the endogenous determination of investment behavior of households and firms is essential for the dynamic part of the model.

Because of the elaborate regional dimension of the model, it is quite difficult to implement full dynamics. This would drastically increase the number of equations in the model (the number of equations of the static model should be multiplied by the number of time periods) and make it non-manageable. Instead, we use the recursive-dynamic framework which allows  the model size to be manageable.

**Households**

The behavior of households is based on the utility-maximization principle. Household's utility is associated with the level and structure of its consumption and the level of emissions.  The household cannot influence the level of emission and takes this as exogenous variables. It is assumed that the utility of household is separable in consumption and emissions.

The household spends its consumption budget on services and goods in order to maximize its satisfaction from the chosen consumption bundle. The household is able to substitute between different consumption commodities. In the model the substitution is captured by the Stone-Geary utility function, which corresponds to the Linear Expenditure System (LES) of demands. According to the Stone-Geary utility function a household derives its utility only from the amount of consumption higher than the minimum subsistence amount and the elasticity of substitution between commodities is equal to one. In the special case when all subsistence amounts are equal to zero, the Stone-Geary utility function reduces to the Cobb-Douglas utility function.

The household maximizes utility given the budget constraint, which states that the household's consumption spending is equal to its income minus income tax and the household's savings. The household's  income is a sum of wage, capital rents, unemployment benefits and other transfers (pensions and other social transfers) from the federal government.

Capital rents are the dividends and interest (return to capital) paid to the households by the firms. It is assumed that households own all firms in the domestic economy. In reality each household receives its capital rents from various regions and sectors. However, the present data availability does not allow for such formulation of the model. There is no data on the flows of investments and corresponding capital rent flows between regions of the country.

The level of unemployment benefits received by the household depends upon the level of unemployment of individuals within the household. The unemployment is modeled according to a simplified wage curve, where the household reduces or increases its labor participation depending on the real market wage. The Russian labor market is known for (1) high participation rates of both sexes and (2) a high wage flexibility. Adjustment to negative labor market shocks mostly goes through wages.

**Firms**

The behavior of a production sector is based on the profit-maximization principle and is captured by the behavior of a representative firm. The return to capital in the sector depends on technology and the structure of intermediate and factor inputs. The sector's technology is summarized by its technological constraint. Intermediate inputs include energy, various commodities and services. Factor inputs include sector-specific infrastructure, physical capital and labor.

At each time period, the instantaneous behavior of a production sector is based on the minimization of production costs for a given output level under the sector's technological constraint. The level of the sector's output is equal to the aggregate demand for its production so that the market clears. Production costs of each sector in the model include labor costs, capital costs and the costs of intermediate inputs. The sector's technological constraint describes the production technology of each sector. It provides information on how many different units of labor, capital, sector-specific infrastructure and commodities are necessary for the production of one unit of the sectoral output.

The production technology of the sector is represented by the nested Constant Elasticity of Substitution (CES) functions. Nested CES function is quite flexible and allows for different assumptions about the degree of substitutability between production inputs. Inputs which can easier be substituted for one another are put into the same nest. Inputs which are more difficult to substitute in the production process are put into different nests. The degree of substitutability is the lowest on top of the nested CES function and the highest at the bottom of it. All production inputs in the CES tree have a certain degree of substitutability, which depends on their relative position in the tree. In accordance with their production technology, sectors have substitution possibilities between different intermediate inputs and production factors.

At the top level of the CES function the firms can substitute between intermediate inputs and the aggregate capital-labour-energy bundle. At the second nest they can substitute between capital-labour and energy. At the lowest nests they can substitute between the use of different energy types, capital and labour.

The model adopts perfectly competitive industrial structure, which leads to the average cost pricing for firms inside each sector. The assumption of perfect competition greatly simplifies modelling and allows an easy interpretation of the model results. Simulations within the perfect competition framework are also an important benchmark if a modeler would wish to deviate from the assumption of perfect competition.

One such type of deviation from the perfect competition rule will be implemented as an optional part of the model, through incorporation of the Dixit-Stiglitz framework. Monopolistic competition framework assumes that each sector consists of a number of symmetric firms, each producing a unique specification of a particular commodity. The type of the commodity, produced by an individual firm, is slightly different from the types of commodity produced by other firms inside the sector.

These differences in the commodity specification then give individual firms a certain monopolistic power over the consumers. Certain consumers prefer a certain specification of the commodity and, hence, they are prepared to pay a bit more for it. The monopolistic power of the individual firms results in the deviation from the marginal costs pricing rule of perfect competition. The producer prices are now equal to the sector's average production costs and depend upon the number of the individual firms, which operate on the market. The sectoral variable costs are equal to the marginal costs multiplied by the sectoral output level. The sectoral fixed costs depend upon the number of the individual operating firms and are equal to the number of firms inside a sector multiplied by the fixed costs per firm.

At this moment the Dixit-Stiglitz monopolistic competition is not implemented. While we have prepared the mathematical and theoretical basis, it is not clear if we will have sufficiently detailed data to apply Dixit-Stiglitz type of competition modelling in a consistent way.

**Sales**

Domestic regional sales of services are modeled as the production of a service sector in the region. We assume that services are not traded between regions and countries. This is a restrictive assumption and is justified by the absence of data on inter-regional trade in services.

Domestic regional sales of each type of commodities are composed of commodities and services produced by domestic sectors, those imported from other regions and those imported from the rest of the world. According to the Armington assumption, the same type of commodity produced by domestic sectors, imported from other regions or imported from the rest of the world, all have different specifications and, hence, cannot be treated as a homogenous good. Domestic consumers have different preferences for these specifications and can substitute between them in case their relative prices change. The substitution possibilities between these commodities' specifications are captured by a CES function that takes the commodity by different origins as its arguments. As a result, the proportions in which the commodity is bought from domestic producers, from other regions and from the rest of the world are determined by the relative producer prices of the commodity, along with transport and trade costs.

All regional households and firms purchase the same geographical mix of commodities, which is produced by the commodity-specific wholesaler in each region. This mix consists of commodities bought from different regions and from commodities bought from different producers in the domestic region. The assumption that all economic agents in the region consume the same geographical mix of commodities is not well-grounded in reality, but is imposed notwithstanding due to the lack of data on trade flows between regions.

The equilibrium prices of all commodities and services are defined by the market equilibrium conditions. Under the market equilibrium the sum of demands for a particular commodity and service is equal to the sum of its supply.

**Savings**

The model incorporates the representation of investment and savings decisions of economic agents. Savings in the economy are made by households, government and the rest of the world. The total savings accumulated at each period of time are invested into the sector-specific physical capital, which is not mobile between the sectors.

The total investment into the sector-specific capital stock is spent on buying different types of capital goods, such as machinery, equipment and buildings. A particular mixture of different capital goods used for physical investment is determined by maximization of the investment agent's utility. The investment agent is an artificial national economic agent responsible for buying capital goods for physical investment in all domestic sectors.

**Governments**

The model incorporates the representation of the federal and regional governments. The governmental sector collects taxes, pays subsidies and makes transfers to households, subsidies to production sectors and transfers to the rest of the world. Tax revenues are shared by the federal and regional governments according to certain rates determined from the base year data. The federal and regional governments consume a number of commodities and services, and the optimal governmental demand is determined according to maximization of the governmental consumption utility function. We use a Cobb-Douglas utility function for this purpose, so the expenditure shares of different commodities and services' purchases by the government stay constant over time. The governmental budget constraint implies that the total governmental tax revenues are spent on subsidies, transfers, governmental savings and consumption. There are also transfers between the regional and national governments.

Finally, the model includes the trade balance constraint, according to which the value of the country's exports plus the governmental transfers to the rest of the world are equal to the value of the country's imports.

# 3. Symbols and notation used

**Table 1: Federal regions of Russian Federation**

| Region number | Federal Region of Russian Federation |
|---|---|
| Reg1 | Central region |
| Reg2 | North-West |
| Reg3 | South |
| Reg4 | Volga area |
| Reg5 | Urals |
| Reg6 | Siberia |
| Reg7 | Far East |

**Table 2: Subscripts used in mathematical formulation**

| | Subscript |
|---|---|
| Sectors/products (each sector produces only one product) | *I* |
| Intermediate inputs (products ii, sectors i) | *ii,i* |
| Regions (Federal regions of Russia) | R |
| Rest of the world regions | *RoW* |
| Flows of goods (from region r to region rr) | *r,rr* |
| Superscript 0 is used to indicate the initial (previous period) level of variable | *0* |

**Table 3: Overview of variables of SUST-RUS economic module**

| VARIABLE | DESCRIPTION |
|---|---|
| **Prices** | |
| $P_{i,r}$ | domestic sales prices of commodities |
| $PD_{i,r}$ | domestic producer prices of commodities |
| $PDDT_{i,r}$ | composite domestic producer prices of domestic commodities |
| $PDD_{i,r}$ | price level of domestic good, delivered to domestic market |
| ER | exchange rate |
| $INDEX_{r}$ | consumer price index |
| PI | price of investments private |
| $PMROW_{i}$ | import price of imports form ROW in local currency |
| $PL_{r}$ | domestic price of labour |
| $PKLEM_{i,r}$ | price of capital-labour-energy-materials bundle |
| $PKLE_{i,r}$ | price of composite capital-labour-energy bundle |
| $PMAT_{i,r}$ | composite price of materials |
| $PKL_{i,r}$ | price of composite labour-capital bundle |
| $PENER_{i,r}$ | energy price |
| $PNONELEC_{i,r}$ | non electricity price |
| $PELEC_{i,r}$ | electricity price |
| $PGASOIL_{i,r}$ | price of oil-gas bundle |
| $RK_{i,r}$ | return to capital |
| **Production and inputs** | |
| $KS_{r}$ | capital endowment (exogenous) |
| $LS_{r}$ | labor supply (exogenous) |
| $X_{i,r}$ | domestic sales (domestic+foreign origin) |
| $XD_{i,r}$ | gross domestic output |
| $XDDE_{i,r,rr}$ | domestic production delivered to domestic market |
| $XDD_{i,r}$ | gross domestic output bought from domestic market |
| $XXD_{i,r}$ | gross domestic output delivered to domestic market |
| $TMX_{i,r}$ | Commodity consumed for prod of transp and trade margins |
| $EROW_{ii,i,r}$ | exports to RoW |
| $MROW_{i,r}$ | imports from RoW |
| ET | total exports |
| MT | total imports |
| IT | Total investments private |
| $K_{i,r}$ | capital input |

| | |
|---|---|
| $L_{i,r}$ | labor input |
| $KL_{i,r}$ | capital-energy bundle |
| $ENER_{i,r}$ | energy input |
| $ELEC_{i,r}$ | electricity input |
| $NONELEC_{i,r}$ | non-electricity input |
| $GASOIL_{i,r}$ | Oil-gas inputs |
| $GAS_{i,r}$ | Fuels (bottom-nest) oil, gas and coal |
| $COAL_{i,r}$ | Coal and coal derivates as input to the production process |
| $OIL_{i,r}$ | Oil as input to the production process |
| $IOE_{i,ii,r}$ | Intermediary energy inputs |
| **Consumption of households and government** | |
| $C_{i,r}$ | demand for consumption goods |
| $CBUD_{r}$ | consumer expenditure on consumption |
| $Y_{r}$ | household income |
| SH | household savings |
| $SG_{gov}$ | Government savings |
| SROW | savings of or from RoW (exogenous) |
| S | national savings |
| $I_{i,r}$ | demand for investment goods private |
| $CG_{i,r}$ | Intermediate public demand for goods |
| $CGR_{r,gov}$ | public spendings at regional level |
| $CGG_{i,r,gov}$ | Intermediate public demand regional governments |
| TAXR | tax revenues |
| SUBS | Total subsidies |
| $TAXRG_{gov}$ | total tax revenue of regional government |
| $SUBSG_{gov}$ | total subsidies of regional government |
| $TRF_{r}$ | total transfers of government to households (exogenous) |
| $TRFF_{r,gov}$ | total transfers of regional government to households |
| GDP | Gross domestic product (real) |
| GDPC | Gross domestic product (nominal) |
| GDPDEF | GDP deflator (exogenous-numeraire) |
| $GDPR_{r}$ | regional gross domestic product (real) |
| $GDPRC_{r}$ | regional gross domestic product (nominal) |
| INDEXE | price index for exports |
| INDEXM | price index for imports |
| PTM | composite price of trade and transport margin |
| $PEV_{r}$ | equivalent variation price index |
| $EV_{r}$ | welfare change as a percentage of households income |

| | |
|---|---|
| $U_r$ | regional utility level |
| **Labour market** | |
| $UNEMP_r$ | regional unemployment level |
| $UNRATE_r$ | regional unemployment rate |
| $UNEMPB_{r,gov}$ | unemployment benefits |
| $trmV_{r,rr,i}$ c | freight transport costs |
| **Regional governments** | |
| TRFG | total intra-government transfers |
| $TRFGE_{gov}$ | outgoing transfers from government |
| $TRFGY_{gov,govv}$ | incoming transfers from government |
| $TRFGG_{gov,govv}$ | Intra government transfers gov to govv |
| PB | total public budget |
| $CBUD\_GOV_{gov}$ | regional consumption budget of government |
| **Monopolistic competition (optional)** | |
| $PDC_{i,ii,r}$ | Monopolistic competition price of domestic good |
| $NF_{i,r}$ | equilibrium number of monopolistic firms |
| $AUXV_{i,r}$ | auxiliary variable |
| $PROFITS_{i,r}$ | profits of the sectors |
| $Kv_{i,r}$ | variable capital input |
| $Lv_{i,r}$ | variable labour input |

**Table 4: Parameters associated with the model**

| Parameters associated with taxation and government consumption | |
|---|---|
| $aTRFGOV_{gov,govv}$ | coefficient for initial intra-government transfers |
| $shareTRFGE_{gov}$ | share of the government income going to transfers |
| $aTRFGE_{gov,govv}$ | division of transfers between subgovernments |
| $aG_{i,r,gov}$ | Cobb-Douglas parameter for government spending on regional level |
| $\alpha G_{r,gov}$ | Cobb-Douglas power in government utility function (goods |
| $sp\_gov_{r,gov}$ | share of subsidies on production subgovernment |
| $sc\_gov_{r,gov}$ | share subsidies on products subgovernment |
| $tc\_gov_{r,gov}$ | share of tax products subgovernment |
| $tk\_gov_{r,gov}$ | share of corporate tax rate subgovernment |
| $tl\_gov_{r,gov}$ | share of labour tax |
| $txd\_gov_{r,gov}$ | share of production tax subgovernment |
| $ty\_gov_{r,govv}$ | income tax |
| $sp_i$ | subsidies rate on production |

| | |
|---|---|
| $sc_i$ | subsidies rate on products |
| $tc_i$ | tax rate on products |
| $txc_i$ | tax rate on intermediates |
| $tcg_i$ | tax rate on government consumption |
| $ti_i$ | tax rate on investment goods |
| $tk_i$ | corporate tax rate |
| $tl_i$ | tax rate on labor |
| $txd_i$ | tax rate on production |
| ty | tax rate on income |
| **Parameters of the labour market** | |
| $trep_r$ | replacement rate of unemployed |
| **Technical coefficients of production and input-output** | |
| $trm_{r,rr,i}$ | trade and transport margins |
| $io_{i,ii,reg}$ | Technical coefficients intermediate inputs |
| $iop_{i,ii,reg}$ | technical coefficients outputs |
| $iops_{i,ii,reg}$ | technical coefficients outputs (production share in demand) |
| $ioKLE_{i,r}$ | Technical coefficients for BDLKLE bundle (labour-capital –energy) |
| $\sigma KLE_{i,r}$ | CES elasticity of subsitutiton between energy and capital-labor bundle |
| $\gamma KLE_{i,r}$ | CES share parameter for labor-capital bundle |
| $aKLE_{i,r}$ | scaling parameter of the CES function |
| $\sigma KLE_{i,r}$ | CES elasticity of subsitutiton between capital, labor and energy |
| $\gamma KL_{i,r}$ | CES share parameter for capital and labour bundle |
| $\gamma E_{i,r}$ | CES share parameter for energy inputs |
| $\gamma GASOIL_{i,r}$ | CES share parameter for gas-oil bundle |
| $\gamma COAL_{i,r}$ | CES share parameter for coal |
| $\gamma OIL_{i,r}$ | CES share parameter for oil |
| $\gamma GAS_{i,r}$ | CES share parameter for gas |
| $aKLE_{i,r}$ | scaling parameter of the CES function |
| $aECNEC_{i,r}$ | scaling parameter of CES function of energy |
| $aGASOIL_{i,r}$ | scaling parameter of CES function of fuels |
| $\sigma E_{i,r}$ | CES elasticity of subsitutiton between electricity and non-electricity |
| $\sigma NE_{i,r}$ | CES elasticity of substitution between fuels (non electricity) |
| $\sigma OIL_{i,r}$ | CES elasticity of substitution between oil and gas |
| $\gamma K_{i,r}$ | CES share parameter for capital and labour bundle |
| $\gamma E_{i,r}$ | CES share parameter for energy |

| | |
|---|---|
| $\gamma$ EC $_{i,r}$ | CES share parameter for Electricity |
| $\gamma$ NEC $_{i,r}$ | CES share parameter for non-electricity |
| aKL $_{i,r}$ | scaling parameter of the CES function |
| $\sigma$ KE $_{i,r}$ | CES elasticity of subsitutiton between capital and labor |
| $\gamma$ L $_{i,r}$ | CES share parameter for labor |
| delta $_{i,r}$ | Depreciation rate |
| **Parameters associated with international and interregional trade** | |
| $\sigma$ A $_{i,r}$ | Armington elasticity of substitution between domestic prod and imports |
| $\sigma$ A1 $_{i,r}$ | Armington elasticity of substitution between domestic prod from diff regions |
| $\gamma$ A2 $_{i,r}$ | CES share parameter of ARMINGTON function for imports from ROW |
| $\gamma$ A3 $_{i,r}$ | CES share parameter of ARMINGTON function for domestic goods |
| $\gamma$ A4 $_{i,r}$ | CES share parameter of ARMINGTON function for XDDE $_i$ sec |
| aA $_{i,r}$ | scale parameter of ARMINGTON function of sector $_i$ |
| aA1 $_{i,r}$ | scale parameter of ARMINGTON function of sector $_i$ |
| **Parameters of household consumption and investment** | |
| mps $_r$ | marginal propensity to save of households |
| $\alpha$ H $_r$ | power in nested-LES household utility on good $_i$ |
| muH $_{i,r}$ | subsistence household consumption quantity of good $_i$ |
| $\alpha$ I $_{i,r}$ | Cobb-Douglas power in investment production function |
| atm $_{i,r}$ | share of commodity for prod of transp and trade margins |

# 4. Main elements of the model

## 4.1 Households

### 4.1.1 Households' income, savings and consumption budget

The model utilizes the notion of the representative economic agent. It represents the behavior of the whole population group as the behavior of one single agent.

Each household owns a certain amount of physical capital in the economy. This gives additional income to the households in the form of return to capital as a dividend ($K \cdot RK$). In reality households in one region can own physical capital in other regions, which creates a bi-regional flow of capital incomes. However, since we do not have statistical information about the origin of the capital incomes of the households we need to make simplifying assumptions, which implies that the regional household receives all capital rents from its region of origin.

Besides capital each household is also endowed with a certain amount of labour. These labor endowments represent efficient units of time and can be used by the households for work and being unemployed. The shares of household's labour endowment spent on work and being unemployed add up to one, that is why the share of the labour endowment used for work can be calculated as one minus the share used on being unemployed (unemployment rate) (i.e., *1-UNRATE*).

The total income of each household is calculated as the sum of its regional labour income and capital income. Households' capital income includes income from capital investments in the production sectors. The labour income comes from work in the home region.

$$Y_{th,r} = (LS_r - UNEMP_r) \cdot PL_r + \sum_i K_{i,r} \cdot RK_{i,r} \quad (1)$$

The labour income of the regional household (*Y*) is calculated as the total endowment of labour in the region supplied for work minus unemployment (*UNEMP*) times the price of labour (*PL*). It is assumed that the return to capital used by all regional sectors (capital input (*K*) times the return to capital (*RK*)) is received by the regional household. The capital income of the regional household is calculated as the sum over all regional sectors of their capital inputs multiplied by the sector-specific rate of return to capital.

The households' income received from labour and physical capital is spent on savings and consumption. We made a distinction between the households' income and households' budget available for consumption of goods and services. The difference between the two is not only the households' savings but also social transfers (*TRF*) such as pensions and child care money and unemployment benefits received by the households. Unemployment benefits received by the household are calculated as the sum of the labour endowment of the household used for being unemployed times the wage it would get if employed (*PL*) time the share of the this wage which is paid to the household through unemployment benefits. This share of wage is called replacement rate of unemployment (*trep*). The disposable net income of the household is calculated as the income (*Y*) times one minus the income tax rate (*1-ty*).

The total consumption budget of the households' (*CBUD*) is calculated as the sum of after-tax income (net income) plus the social transfers of national and regional governments (*TRF* and *TRFR*) minus the households' savings (*SH*) plus the unemployment benefits received by the household (calculated as the unemployment level (*UNEMP)* times the price of labour times the replacement rate of unemployment (*trep*) minus the investments of households' into education:

$$CBUD_r = Y_r \cdot (1 - ty_r) + TRF_r \cdot GDPDEF + TRFR_r \cdot GDPDEF - SH_r \\ + UNEMP_r \cdot PL_r \cdot trep \quad (2)$$

Where *ty* is the income tax rate and *GDPDEF* is the GDP deflator. Governmental transfers are indexed in the model with the GDP deflator. If the overall price level in the economy goes up so will the transfers.

The savings of the regional household are calculated as a fixed proportion of its total disposable income that consists of the household's net income plus the social transfers and unemployment benefits. This fixed proportion (marginal propensity to save (*mps*)) is different for each region and household. Marginal propensity to save can also be negative, reflecting persistent debts.

$$SH_r = mps \cdot ((Y_i \cdot (1 - ty_r) + TRF_r \cdot GDPDEF + TRFR_r \cdot GDPDEF \\ + UNEMP_r \cdot PL_r \cdot trep \quad (3)$$

### 4.1.2 Household utility'

The amounts of the goods and services bought by the regional household types are determined from a utility-maximization problem, where the household maximizes the following utility function. This is a utility function based on the LES or Stone-Geary function. The LES function is a variation on the Cobb-Douglas utility function, where we subtract a fix part of the consumption of goods which is defined as 'basic' or 'subsistence' consumption ($\mu_i$) from the total consumption of a good (C).The utility from consumption is associated only with the amount of good and service which is higher than its subsistence consumption level. The regional household defines its consumption levels in order to maximize the LES utility function under the budget constraint that the total expenditures of the household are equal to its consumption budget.

$$U_r = \prod_i (C_i - \mu_i)^{\alpha_i} \tag{5}$$

Utility of the household is maximized under the budget constraint, where the household's consumption spending is equal to its income minus income tax and the household's savings. Households in the model receive their income in the form of wages, capital rents, unemployment benefits and other transfers (pensions and other social transfers) from the federal government.

Utility is maximized subject to this budget constraint:

$$CBUD_r = \sum_i (P_{i,r} \cdot C_{i,r} \cdot (1 - sc_{i,r} + tc_{i,r})) \tag{6}$$

In order to solve this maximization problem we need to form the Lagrangian:

$$\Lambda(C_{i,r}, \lambda_r) = \prod_i (C_i - \mu_i)^{\alpha_i} + \lambda_{i,r} \cdot \left[ CBUD_r - \sum_i (P_{i,r} \cdot (1 - sc_{i,r} + tc_{i,r})) \cdot C_{i,r} \right] \tag{7}$$

The First Order Conditions (FOC) are derived by differentiating the Lagrangian with respect to decision variables and the Lagrangian multiplier:

$$\frac{\partial \Lambda(C_{i,r}, \lambda_r)}{\partial C_{i,r}} = \alpha \frac{\overline{U}}{(C_i - \mu_i)} + \lambda_{i,r} \cdot \sum_i (P_{i,r} \cdot (1 - sc_{i,r} + tc_{i,r})) = 0 \tag{8}$$

$$\frac{\partial \Lambda(C_{i,r}, \lambda_r)}{\partial \lambda_{i,r}} = CBUD_r - \sum_i (P_{i,r} \cdot (1 - sc_{i,r} + tc_{i,r})) = 0 \tag{9}$$

Eliminating *U* and $\lambda$ by dividing the FOC of *i* with the FOC of *j* gives:

$$\frac{\alpha_i}{\alpha_j} \frac{(C_j - \mu_j)}{(C_i - \mu_i)} = \frac{P_{i,r} \cdot (1 - sc_{i,r} + tc_{i,r})}{P_{j,r} \cdot (1 - sc_{j,r} + tc_{j,r})} \tag{10}$$

$$C_j = \mu_j + \frac{\alpha_j}{\alpha_i} \cdot \frac{P_{i,r} \cdot (1 - sc_{i,r} + tc_{i,r})}{P_{j,r} \cdot (1 - sc_{j,r} + tc_{j,r})} \cdot (C_i - \mu_i) \tag{11}$$

Substituting $C$ in the budget constraint gives solution for the demand function, leading eventually to the following solution

$$P_{i,r} \cdot (1 - sc_i + tc_i) \cdot C_{th,i,r} = P_{i,r} \cdot (1 - sc_i + tc_i) \cdot \mu H_{i,r} + \alpha H_{i,r} \cdot \left( CBUD_{i,r} - \sum_i \mu H_{i,r} \cdot P_{i,r} \cdot (1 - sc_i + tc_i) \right) \tag{13}$$

### 4.1.3 Household welfare

The welfare of a representative regional household is calculated as the change in equivalent variation of the representative regional household. The equivalent variation is defined as monetized change in utility, based on the LES utility function.

$$EV = \frac{1}{scalU} [U^A - U^0] \cdot \frac{1}{PEV_r} \tag{14}$$

The calculation of the equivalent variation measure according to this formula is based on the price of equivalent variation and on the level of utility. The superscript '0' refers to the initial baseline values of the utility price and the budget. The price index of utility obtained by the household is derived according to the following equation. This price depends on the after-tax prices of goods and services as well as the utility shares ($\alpha_{i,r}$)

$$PEV_r = \prod_{i=products} \left( \frac{P_{i,r} \cdot (1 - sc_i + tc_i)}{\alpha_{i,r}} \right)^{\alpha_{i,r}} \tag{15}$$

## 4.2 Firms

### 4.2.1 Firms' profit maximization problem

The behavior of the firms is based on the minimization of the production costs for a given output level under the firm's technological constraint. Production costs of each sector in the model include labor costs, energy costs, capital costs, and the costs of intermediate inputs. By capital we mean physical capital of the sector, which includes machinery, equipment and buildings. The sector's technological constraint describes the production technology of the sector. It provides information on how many units of labor, energy, capital and commodities are necessary for the production of one unit of the sectoral output.

Production sectors are assumed to operate under constant return to scale and perfect competition. The output prices are equal to marginal production costs, which are in turn equal to the average production costs.

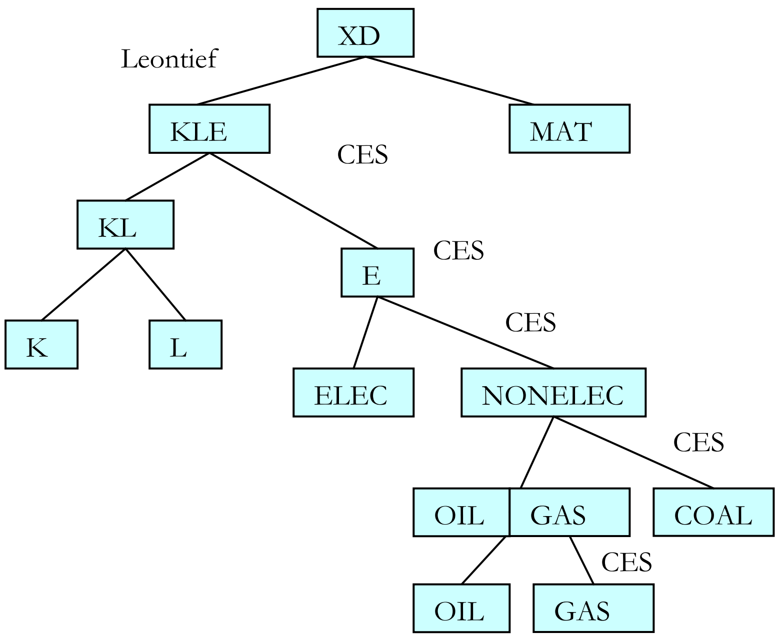
The production technology of the firm is represented by the nested Constant Elasticity of Substitution (CES) functions. The nested CES function is quite flexible and allows for different assumptions about the degree of substitutability between the production inputs. Inputs which are easier to substitute one with another are put into the same nest. Inputs which are more difficult to substitute in the production process are put into different nests. All production inputs in the CES tree have a certain degree of substitutability between each other and it depends on their relative position in the tree. The degree of substitutability is the lowest on top of the nested CES function and the highest at the bottom of it. In accordance with their production technology, sectors have substitution possibilities between different intermediate inputs and production factors.

At the top level of the CES function firms can substitute between intermediate inputs and the aggregate capital-labour-energy bundle. At the second top level they can substitute between capital-labour and energy. At the lowest nests they can substitute between capital and labour and among different types of energy inputs.

The nested CES function follows certain assumptions about how easy it is to substitute between different production inputs. In reality various firms can have different nested production structure reflecting different production technology of the firms. Moreover several different production trees could be tested econometrically in order to identify which of them fits given sector the most. Estimation of the production functions is a part of the SUST-RUS project. This will be done after the finalization of the model database.

We assume that one cannot substitute materials used in production, sector-special infrastructure and aggregated capital-labour-energy among each other. They should be used in fixed proportions to the overall output of the sector according to the Leontief production technology. Those three inputs have the lowest elasticity of substitution (zero) and are put at the top level of the CES tree. Alternative assumption would be to assume that they have a very low (close to zero) elasticity of substitution. It is also important to ensure that each production input enters to one and only one group in order to avoid double counting of their effect.

**Figure 1: CES production tree**



### 4.2.2 The production function

In order to estimate demands for the intermediate inputs corresponding to the CES production function let us start from the general CES production function:

$$Y = A \cdot \left( \sum_i a_i \cdot X_i^{\rho} \right)^{1/\rho} \tag{16}$$

where $Y$ is the production output, A is the scaling parameter of the production function, $X_i$ the intermediate inputs, $a_i$ the share parameters of the CES function and $\rho = (\sigma - 1)/\sigma$ ,where $\sigma$ is the elasticity of substitution between production inputs. Let us solve the problem of minimization of production costs under the technological constraint represented by the CES production function. The Lagrangian of the problem can be written as:

$$L = \sum_i P_i \cdot X_i + \lambda \cdot \left( Y - A \cdot \left( \sum_i a_i \cdot X_i^{\rho} \right)^{1/\rho} \right)$$

(17)

The FOC of the maximization problem are:

$$\frac{\partial L}{\partial X_i} = P_i - \lambda \cdot \frac{1/\rho \cdot \left( \sum_j a_j \cdot X_j^{\rho} \right)^{1/\rho}}{\sum_j a_j \cdot X_j^{\rho}} \cdot a_i \cdot \rho \cdot \frac{X_i^{\rho}}{X_i} = 0 \tag{18}$$

$$\frac{\partial L}{\partial \lambda} = Y - A \cdot \left( \sum_i a_i \cdot X_i^{\rho} \right)^{1/\rho} = 0 \tag{19}$$

The first FOC can be simplified in the following way:

$$P_i = \lambda \cdot \frac{Y}{X_i} \cdot \frac{a_i \cdot X_i^{\rho}}{\sum_j a_j \cdot X_j^{\rho}} \tag{20}$$

Let us divide $\frac{\partial L}{\partial X_i}$ by $\frac{\partial L}{\partial X_j}$, this gives us the formula:

$$\frac{P_i \cdot X_i}{P_j \cdot X_j} = \frac{a_i \cdot X_i^{\rho}}{a_j \cdot X_j^{\rho}} \Leftrightarrow a_i \cdot X_i^{\rho} = \frac{P_i \cdot X_i}{P_j \cdot X_j} \cdot a_j \cdot X_j^{\rho} \tag{21}$$

Let us substitute the last expression into the CES production function:

$$Y = A \cdot \left( \sum_i \frac{P_i \cdot X_i}{P_j \cdot X_j} \cdot a_j \cdot X_j^{\rho} \right)^{1/\rho}$$

$$= A \cdot \left( \frac{a_j \cdot X_j^{\rho}}{P_j \cdot X_j} \cdot \sum_i P_i \cdot X_i \right)^{1/\rho} \tag{22}$$

$$= A \cdot \left( \frac{a_j \cdot X_j^{\rho}}{P_j \cdot X_j} \cdot PY \cdot Y \right)^{1/\rho}$$

In the last part of the derivations we have used the assumption of the zero profits of the sector. According to this assumption the firms always charge their average production costs, that is:

$$PY \cdot Y = \sum_i P_i \cdot X_i \tag{23}$$

where *PY* is the price of the output of the sector.

Let us modify the last expression further in the following way:

$$Y^{\rho} = A^{\rho} \cdot \frac{a_j \cdot X_j^{\rho}}{P_j \cdot X_j} \cdot PY \cdot Y$$

$$\Leftrightarrow X_j^{1-\rho} = Y^{1-\rho} \cdot \frac{a_j}{P_j} \cdot A^{\rho} \cdot PY \tag{24}$$

$$\Leftrightarrow X_j = Y \cdot \left( \frac{a_j}{P_j} \right)^{1/(1-\rho)} \cdot A^{\rho/(1-\rho)} \cdot PY^{1/(1-\rho)}.$$

From the relationship between $\rho$ and $\sigma$ we know that $1/(1-\rho) = \sigma$ and $\rho/(1-\rho) = \sigma - 1$. If we use these two expressions in the formula above we get the final expression for the CES demands of the production sectors:

$$X_j = Y \cdot \left( \frac{a_j}{P_j} \right)^{\sigma} \cdot PY^{\sigma} \cdot A^{\sigma-1}. \tag{25}$$

### 4.2.3 Demand functions of production sectors

The following equation derives the value of the top CES bundle (*KLE*) which is equal to the total domestic production (*XD*) multiplied by a Leontief coefficient.

$$KLE_{i,r} = ioKLE_{i,r} \cdot XD_{i,r}, \tag{26}$$

where *KLE* is the composite labour and capital bundle and *io* are technical coefficients. *PD* is the domestic producer price of commodities. The composite price of this bundle is equal to the price of capital-energy-labour bundle (*KLE*).

$$PKLE_{i,r} \cdot KLE_{i,r} = PKL_{i,r} \cdot KL_{i,r} + PENER_{i,r} \cdot ENER_{i,r} \tag{27}$$

The value of the capital-labour-energy bundle is calculated according to the CES demand function and depends upon the value of the top CES bundle (*KLE*), the composite price of the capital-labour-energy bundle (*PKLE*), the composite price of the top CES bundle (*PKLE*) and the CES technological coefficients

$$KL_{i,r} = KLE_{i,r} \cdot \left(\frac{\gamma KLE_{i,r}}{PKL_{i,r}}\right)^{\sigma KLE_{i,r}} \cdot PKLE_{i,r}^{\sigma KLE_{i,r}} \cdot aKLE_{i,r}^{\sigma KLE_{i,r}-1} \tag{28}$$

Likewise, the composite price of this bundle is equal to the weighted average of the prices of energy (*ENER*) and capital-labour (*KL*) bundle.

$$PKLE_{i,r} \cdot KLE_{i,r} = PKL_{i,r} \cdot KL_{i,r} + ENER_{i,r} \cdot PENER_{i,r} \tag{29}$$

The value of the capital-labour bundle is calculated according to the CES demand function and depends upon the value of the top CES bundle (*KLE*), the composite price of the capital-labour bundle (*PKL*), the composite price of the top CES bundle (*PKLE*) and CES technological coefficients ($\sigma$ here is the elasticity of substitution between capital and labour).

$$KL_{i,r} = KLE_{i,r} \cdot \left(\frac{\gamma KL_{i,r}}{PKL_{i,r}}\right)^{\sigma KLE_{i,r}} \cdot PKLE_{i,r}^{\sigma KLE_{i,r}} \cdot aKLE_{i,r}^{\sigma KLE_{i,r}-1}$$

(30)

The composite price of this bundle is equal to the weighted average of the prices of capital (*K*) and labour input (*LT*).

$$PKL_{i,r} \cdot KL_{i,r} = (RK_{i,r}(1+tk_{i,r}) + \delta_{i,r} \cdot PI_c) \cdot K_{i,r} + PLT_{i,r} \cdot LT_{i,r}$$

(31)

Where *tk* is the corporate tax rate; $\delta$ the depreciation rate, *PI* the price of private investments and *PLT* the price of the labour.

### 4.2.4 Energy inputs

SUST-RUS distinguishes between 4 aggregate energy inputs: electricity, gas, oil and coal. The demand for energy is derived from a standard nested-CES tree as used throughout the entire model.

Aggregate energy inputs (gas-oil, coal and electricity) are derived from the capital-labour-energy bundle by the following formula.

$$ENER_{i,r} = KLE_{i,r} \cdot \left(\frac{\gamma ENER_{i,r}}{PENER_{i,r}}\right)^{\sigma KLE_{i,r}} \cdot PENER_{i,r}^{\sigma KLE_{i,r}} \cdot aKLE_{i,r}^{\sigma KLE_{i,r}-1} \tag{32}$$

The price of the composite energy bundle PENER is equal to the weighted price of the electricity and non-electricity inputs. This is defined by the equation below.

$$PENER_{i,r} \cdot ENER_{i,r} = PNONELEC_{i,r} \cdot NONELEC_{i,r} + P_{ii=electricity,r} \cdot ELEC_{i,r} \tag{33}$$

The demands for electricity and non-electricity inputs are given by the following equations. These are essentially at a lower nest of the energy inputs.

$$NONELEC_{i,r} = ENER_{i,r} \cdot \left(\frac{\gamma NEC_{ii,i,reg}}{PNONELEC_{i,r}}\right)^{\sigma E_{i,r}} \cdot PENER_{i,r}^{\sigma E_{i,r}} \cdot aECNEC_{i,r}^{\sigma E_{i,r}-1} \tag{34}$$

$$ELEC_{i,r} = ENER_{i,r} \cdot \left(\frac{\gamma EC_{ii,i,reg}}{PELEC_{i,r}}\right)^{\sigma E_{i,r}} \cdot PENER_{i,r}^{\sigma E_{i,r}} \cdot aECNEC_{i,r}^{\sigma E_{i,r}-1} \tag{35}$$

The demand for each type of fossil fuel is again a sub nest of the NONELEC bundle, given by the next equation. We distinguish 3 types of fuels: oil, coal and gas bundles. Oil and gas act as a separate bundle, different from coal (as suggested in Figure 4 in section 5: construction of the database of model parameters).

$$GASOIL_{i,reg} = NONELEC_{i,r} \cdot \left( \frac{\gamma GASOIL_{ii,i,reg}}{PGASOIL_{ii,reg}} \right)^{\sigma NE_{i,r}} \cdot PNONELEC_{i,r}{}^{\sigma NE_{i,r}} \cdot aFUEL_{i,r}{}^{\sigma NE_{i,r}-1} \quad (36)$$

$$COAL_{i,reg} = NONELEC_{i,r} \cdot \left( \frac{\gamma COAL_{ii,i,reg}}{P_{ii=coal,reg}} \right)^{\sigma NE_{i,r}} \cdot PNONELEC_{i,r}{}^{\sigma NE_{i,r}} \cdot aFUEL_{i,r}{}^{\sigma NE_{i,r}-1} \quad (37)$$

The demands for gas and oil are derived at the bottom nest.

$$GAS_{i,reg} = GASOIL_{i,r} \cdot \left( \frac{\gamma GAS_{ii,i,reg}}{P_{ii=gas,reg}} \right)^{\sigma OIL_{i,r}} \cdot PGASOIL_{i,r}{}^{\sigma OIL_{i,r}} \cdot aOIL_{i,r}{}^{\sigma OIL_{i,r}-1} \quad (37)$$

$$OIL_{i,reg} = GASOIL_{i,r} \cdot \left( \frac{\gamma OIL_{ii,i,reg}}{P_{ii=oil,reg}} \right)^{\sigma OIL_{i,r}} \cdot PGASOIL_{i,r}{}^{\sigma OIL_{i,r}} \cdot aOIL_{i,r}{}^{\sigma OIL_{i,r}-1} \quad (38)$$

### 4.2.5 Capital stock

For cost minimizing (and profit maximizing) firms operating under constant return to scale, expenditures on capital (*K*) are derived as a sub-nest from the capital-labour bundle, as a solution of the cost minimization problem.

$$K_{i,r} = KL_{i,r} \cdot \left( \frac{\gamma K_{i,r}}{(1+tk_{i,c}) \cdot RK_{i,r} + \delta_{i,r} \cdot PI_c} \right)^{\sigma KL_{i,r}} \cdot PKL_{i,r}{}^{\sigma KL_{i,r}} \cdot aKL_{i,r}{}^{\sigma KL_{i,r}-1} \quad (39)$$

### 4.2.6 Dixit-Stiglitz varieties and monopolistic competition (optional)

This mathematical description of the model includes a set of equations that deviate from the assumption of perfect competition. We will allow turning on monopolistic competition as an option to the modeler. Under the monopolistic competition framework, it is assumed that each sector consists of a number of symmetric firms, each producing a unique specification of a particular commodity. The type of the commodity, produced by an individual firm, is slightly different from the types of commodity, produced by other firms inside the sector. These differences in the commodity specifications give individual firms a certain monopolistic power over the consumers.

Each new production firm under monopolistic competition faces initial fixed costs of establishing itself in the market. The fixed production costs of an individual firm are related to its initial establishment in the industry and include both labour and capital costs. Each new firm produces one particular type of the product type/variety. The firms charge prices higher than their marginal costs in order to be able to cover their fixed costs. Since consumers have widely differentiated preferences with respect to the types/varieties of goods and services produced by the firms, they purchase outputs from all firms in the sector. The functional form of the consumer utility function associated with this assumptions is represented by the CES function, which positively depends on the number of firms (varieties) in a region. This setup is generally called the Dixit-Stiglitz form of monopolistic competition.

The sector variable cost is equal to the marginal output cost multiplied by the sectoral output level. The sector fixed cost depends upon the number of the individual operating firms and are equal to the number of firms inside a sector multiplied by the fixed costs per firm. Given that there are no statistical data that describe the production process of each firm in the industry, all firms are assumed to be symmetric and have the same production technology, the same output size and the same fixed production costs.

The monopolistic competition framework, allows to model agglomeration and dispersion forces. Agglomeration forces in this set-up follow the following logic: when the number of the operating firms in the region increases, the variety of differentiated goods available in the region will increase. This means that the cost of obtaining a certain set of differentiated goods will decrease. For a given nominal wage, this decrease in the price index will increase the real wage of regional workers in relative terms. This leads to in-migration. The new migration reinforces the agglomeration because migrants expand the consumption market in the region, again increasing the offered variety, reducing the price index and increasing real wages in a cumulative process.

Given that the entry into the industry  is assumed to be free, the number of the firms in each sector (NF) is determined by the condition that the total costs of the firms are equal to their total revenues (zero profit condition). Once the firms in the industry start making profits, new firms enter the market and drive total profits down to zero again. The fixed capital and labour costs for each firm are assumed to be constant, making the total number of the firms operating in a sector endogenous, defined by the zero profit condition for the sector as a whole:

$$NF_{i,r} \cdot elas\,\mathrm{Re}\,g_{i,r} \cdot fcL_{i,r} + fcK_{i,r} \cdot INDEX_r = XD_{i,r} \cdot PD_{i,r} \qquad (40)$$

Where $elas\,\mathrm{Re}\,g_{i,r}$ is the demand elasticity for imperfectly competitive sectors in regions and $fcK_{i,r}$ the total labour fixed costs. Just as in equation (n2) of the standard NEG model the price of the goods or services produced by a monopolistically competitive sector (PDC) depend negatively on both the number of the operating firms and on the elasticity of substitution between the varieties of a good or a service produced by each firm. However, this is made operational by using a simple auxiliary variable. Under the assumption that the firms operating in a sector are identical, the price of a monopolistically competitive sector is derived according to the following formula:

$$PDC_{i,r} = PD_{i,r} \cdot AUXV_{i,r} \qquad (41)$$

This price is higher than the marginal production costs. Which is the domestic production price (PD), multiplied by the auxiliary variable (AUXV)

$$AUXV_{i,r} = (NF_{i,r})^{\frac{1}{1-elas\,\mathrm{Re}\,g_{i,r}}} \qquad (42)$$

Firms charge prices higher than their marginal costs, which results in obtaining the profits. The profits made by the monopolistic firms are identical to the sum of their fixed labour and capital costs. This equality determines the total number of operating firms in each sector.

$$PROFITS_{ir} = NF_{i,r} \cdot (fcL_{i,r} + fcK_{i,r}) \cdot INDEX_{i,r} \qquad (43)$$

If a sector does not include spatially bound inputs agglomeration in a small set of regions is possible. If spatially bound inputs are needed, the price of this input will act as a spreading force, since the input cannot migrate. Agglomeration is still possible, but given the countervailing force, it will occur in a larger set of regions and is less likely to be catastrophic. Simulations will be needed to assess the sensitivity of results.

For the firms operating under increasing returns to scale, the variable expenditures on capital (Kv) is derived as a sub-nest from the capital-labour bundle, as a solution of the cost minimization problem. The total expenditures on capital are a sum of the variable capital inputs and the fixed capital costs. These are the fixed cost of capital per firm (fcK), multiplied by the amount of firms (NF) in the sector.

$$K_{i,r} = KL_{i,r} \cdot \left( \frac{\gamma K_{i,r}}{(1+tk_{i,c}) \cdot RK_{i,r} + \delta_{i,r} \cdot PI_c} \right)^{\sigma KL_{i,r}} \quad i \in monopolistic \qquad (45)$$
$$\cdot PKL_{i,r}^{\sigma KL_{i,r}} \cdot aKL_{i,r}^{\sigma KL_{i,r}-1} + (NF_{i,r} \cdot fcK_{i,r})$$

## 4.3 Government

The Russian government is modeled at two levels, a regional and a country level governments. The parameters of the governments in SUST-RUS model are related o the type and share of tax income and subsidy, monetary transfers between governments and government consumption.

### 4.3.1 Government tax income and subsidies

Each government gets two types of income: tax revenues from the economic agents within the regions under its jurisdiction and income due to inter government transfers.

The tax revenues within each region (TAXRG) are calculated as the sum of the labour taxes, profit taxes of the firms (tk), taxes on production (txd) and taxes on the total consumption (tc). The taxes on consumption are subdivided in: final tax on consumption of households, tax on investment, tax on government consumption and export taxes. They are all modelled as a fixed percentage of the value of a good. Regional governments get a different fixed share of the total tax revenues from each tax subtype. The total tax income for each government is equal to the sum of its tax revenues within each region.

$$TAXRG_{gov} = \sum_r \begin{bmatrix} PL_r \cdot L_{i,r} \cdot (tl1_i \cdot tl1\_gov_{r,gov}) \\ + tk_i \cdot tk\_gov_{r,gov} \cdot K_{i,r} \cdot RK + txd_i \cdot txd\_gov_{r,gov} \cdot XD_{i,r} \cdot TFP \cdot PD_{i,r} \\ \left( \begin{matrix} + \sum_i (tc_i \cdot tc\_gov_{r,gov}) \cdot P_{i,r} \cdot \\ \sum_{th,i} C_{th,i,r} \\ + I_{i,r} + CG_{i,r} \end{matrix} \right) \\ + Y_{th,r} \cdot ty_{th} \cdot ty\_gov_{r,gov} \end{bmatrix} \qquad (46)$$

The total subsidies of each government consist of subsidies on production and consumption. Subsidies are treated similarly as tax revenues. The national rates are fixed and are the same in each province, but the share of the total subsidies paid by each government is region specific. The relevant equations can be found in the mathematical appendix.

### 4.3.2 Government transfers

The governments transfer income to the households and to the other governments. For the transfers to the households a distinction is made between unemployment benefits and 'other transfers'. Transfers to the households are partially fixed; the 'other transfers' are assumed to be constant, but the unemployment benefits depend on the wage level and on unemployment within each region. Unemployment benefits only partially compensate the loss in real wage (PW); the degree of compensation depends on the exogeneously fixed parameters *trep* (wage replacement rate).

$$UNEMPB_{r,gov} = \left(UNEMP_r \cdot trep_r \cdot PW_r\right) \cdot indic\_UNEMPB_{gov} \tag{47}$$

Transfers from government to government are endogenous and are calculated in the following way.

First, we assume that a fixed share of the total government income (tax revenues and income from transfers) is transferred.

$$TRFGE_{gov} = shareTRFGE_{gov} \cdot (TAXRG_{gov} + TRFGY_{gov}) \tag{48}$$

Next, we assume that each government gets a fixed share of the government transfer expenditures

$$TRFGG_{gov,govv} = aTRFGE_{gov,govv} \cdot TRFGE_{gov} \tag{49}$$

The income from transfers is assumed to be the sum of the total transfers from each government

$$TRFGY_{govv} = \sum_{gov} TRFGG_{gov,govv} \tag{50}$$

### 4.3.3 Government consumption

The consumption budget of each government (CBUD_GOV) consists of the total tax revenues (TAXRG) minus total subsidies (SUBSG), minus the unemployment benefits, minus the transfers to the households (TRFF), plus the income from intergovernmental transfers (TRFGY) minus the expenditures on intergovernmental transfers (TRFGE) and savings.

$$\begin{aligned} & CBUD\_GOV_{gov} = \left(TAXRG_{gov} - SUBSG_{gov}\right) \\ & \sum_{th,r} \cdot TRFF_{th,r,gov} \cdot GDPDEF - \sum_{th,r} UNEMPB_{th,r,gov} + (TRFGY_{gov} - TRFGE_{gov}) \cdot GDPDEF \\ & - SG_{gov} \cdot GDPDEF \end{aligned} \tag{51}$$

There are several possible closures of the government budget, each with a distinct effect on model results. The first possibility is a closure via government savings. In this case, a change in the government revenues is added or subtracted from the public budget surplus or deficit, keeping government consumption constant.

A second possibility is that extra revenues are redistributed via the government consumption and have a direct effect on the economy. (However, note that this can lead to rather large price and consumption effects on education, government services and health provision). Another possibility is that government tries to balance budget through an increase or decrease of lump sum transfers to households or by increasing taxation of other goods.

We model the government expenditures on commodities in two stages. In the first stage we assume that each region gets a fixed part of the government spendings on commodities.

$$CGR_{r,gov} = \alpha G_{r,gov} \cdot CBUD\_GOV_{gov} \quad (52)$$

In the next stage, we assume that the consumption budget within each regions is distributed on the basis of government's maximization of a Cobb-Douglas welfare utility function, which depends upon its consumption of goods and services under its budget constraint. This broadly corresponds to one of the theoretical models of governments, where the Government "knows best" while maximizing economic welfare (this model is referred to as the despotic benevolent model; Bailey, 1995, 1999). The result is the following demand function for regional goods by the national Government:

$$P_{i,r} \cdot (1 + tcg_i) \cdot CGG_{i,r,gov} = aG_{i,r,gov} \cdot CGR_{r,gov} \quad (53)$$

## 4.4 Trade

### 4.4.1 International trade

The formulation of the trade part of the model is based on the theory for a small open economy. Domestic sales in each region are a composite commodity of domestically produced goods and imports from all foreign countries (Rest Of the World). The domestically produced goods are a CES-composite of goods produced in the region itself and imports from the other regions.

In the first nest, representing international trade, the region chooses to buy domestically produced or imported goods (from the ROW). This part of the model is based on the Armington assumption of heterogeneity between the goods and services produced abroad and domestically. Goods and services produced abroad cannot be perfectly substituted with the domestically produced ones. The substitution possibilities between domestic and foreign commodities are described by the CES production function, according to which domestic and foreign commodities are used in a certain proportion in order to produce a composite commodity used in consumption by the domestic firms and households.

The equation below defines imports from the ROW.

$$MROW_{i,r} = X_{i,r} \cdot \left( \frac{\gamma A2_{i,r}}{PMROW_i} \right)^{\sigma A_{i,r}} \cdot \left(P_{i,r}\right)^{\sigma A_{i,r}} \cdot \left(aA_{i,r}\right)^{\sigma A_{i,r} - 1} \quad (54)$$

The prices of the commodities imported to the country from the rest of the world in foreign currency are exogenously fixed in the model and their prices in the domestic currency are calculated according to the following formulas, where the subscript '0' refers to the commodity prices in foreign currency:

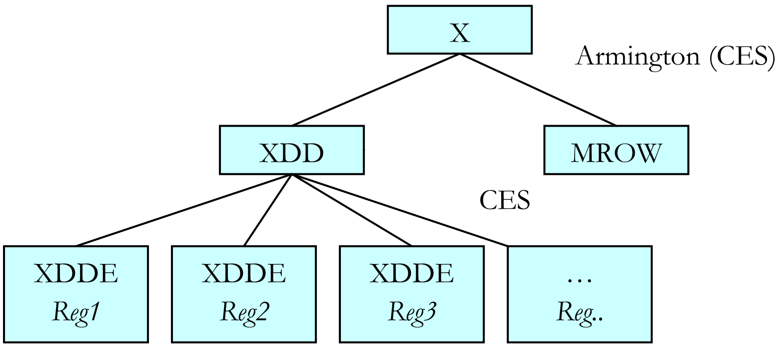
$$PMROW_i = PWMROW_i^0 \cdot ER \quad (55)$$

Domestic sectors have the possibility to export their production to the rest of the world. Exports are determined through a similar function as the Armington CES function in the case of imports. This function is mathematically equivalent and is commonly referred to as the CET function or the constant elasticity of transformation. Note that in this case, X (sales) are replaced by XD (production) and P (sales price) is replaced by PD (producers price):

$$EROW_{i,r} = XD_{i,r} \cdot \left( \frac{\gamma T2_{i,r}}{PROW_i \cdot (1 - t\exp_i)} \right) \cdot \left(PD_{i,r}\right)^{\sigma T_{i,r}} \cdot \left(aT_{i,r}\right)^{\sigma T_{i,r} - 1} \quad (56)$$

### 4.4.2 Interregional trade

**Figure 2: CES-tree for the international and interregional trade**




In the second nest, representing interregional trade, the region allocates the domestic consumption of commodities over the different regions within the country. The composite domestic commodity consists of the goods and services produced in all the regions of the country. Similar to the international trade part, we assume heterogeneity between the goods and services produced in different domestic regions. The substitution possibilities between the commodities produced in different regions are again described by the CES production function.

The demand for the composite domestic commodity is determined in the first CES nest

$$XDD_{i,r} = X_{i,r} \cdot \left( \frac{\gamma A3_{i,r}}{PDDT_{i,r}} \right) \cdot \left( P_{i,r} \right)^{\sigma A_{i,r}} \cdot \left( aA_{i,r} \right)^{\sigma A_{i,r} - 1} \qquad (57)$$

The price of the composite domestic goods and services is derived as the weighted average of the prices of the commodities bought from all domestic regions. This weighted price includes the price of domestically produced goods (PDD) in each region, plus the relative transport costs.

$$PDDT_{i,r} \cdot XDD_{i,r} = \sum_{rr} XDDE_{i,rr,r} \cdot \left( PDD_{i,r} + PTM \cdot trm_{rr,r,i} \right) \qquad (58)$$

The regional demand for domestic commodities is given by the next equation

$$XDDE_{i,rr,r} = XDD_{i,r} \cdot \left( \frac{\gamma A4_{i,r}}{PDD_{i,rr} + PTM \cdot trmV_{rr,r,i}} \right)^{\sigma A1_{i,r}} \cdot PDDT_{i,r}^{\ \sigma A1_{i,r}} \cdot aA1_{i,r}^{\ \sigma A1_{i,r} - 1} \qquad (59)$$

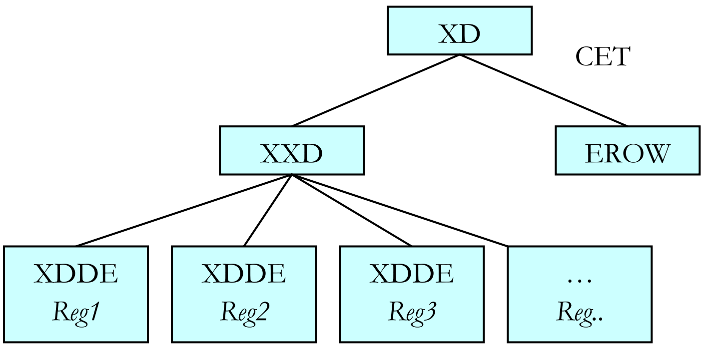
The calculation of the transport costs of commodities deserves some additional explanations. Instead of using the commonly applied iceberg transportation costs, the model bases transport costs on the relative production and consumption of transport margins. The countrywide (!) price of trade margins (PTM) is a weighted sum of the **production cost** of transport margins **relative to the sales price of some sectors**. The sectors producing transport margins are the trade and retail sector and the transport sector. The shares (atm) are exogenously fixed.

$$PTM = \sum_{i} \sum_{r} \left( atm_{i,r} \cdot P_{i,r} \right) \qquad (60)$$

The **consumption of trade margins** is calculated **relative to the value of the specific commodity transported.** Like the relative production shares, the relative transport margin consumption (trm) is fixed exogenously. However, not all products consume transport margins, only the commodities that use freight transport. While service sectors use inputs that consume freight transport, the products of service sectors have zero transport margins.

We have now determined the interregional trade equations from the consumption side, however we did not do this for the producer side of the model. The figure below resumes the CET function as a mirror image of the CES function.

**Figure 3: CET tree of international and interregional trade**




While comparing equation 55 and the equation below, we see that in this case producers are choosing between delivery to the domestic market, based on the production price (PD) and the price on the domestic market (PDD). This makes the main assumptions behind the interregional trade market clear. Producers sell at a price PDD on the domestic market, which is the so called 'mill price' of the good. A competitive transport agent is responsible for moving the good and demands a total value equal to the transport and trade margin.

$$XXD_{i,r} = XD_{i,r} \cdot \left( \frac{\gamma T3_{i,r}}{PDD_{i,r}} \right) \cdot \left( PD_{i,r} \right)^{\sigma A_{i,r}} \cdot \left( aT_{i,r} \right)^{\sigma A_{i,r}-1} \quad (61)$$

The next equations close the interregional trade market. The first one is an obvious restriction, but probably one of the most important ones, related to interregional trade. This equation states that all production of a region, delivered to the domestic market, has to be equal to the total domestic demand for goods from that region.

$$XXD_{i,r} = \sum_{rr} XDDE_{i,r,rr} \quad (62)$$

The second and last equation is related to the **production** of transport and trade margins. The production of trade margins is made by the transport and trade sectors and is determined by a fixed share (comparable to the Leontief configuration). This equation relates production of trade margins to the consumption of transport and trade.

$$TMX_{i,r} = \text{atm}_{i,r} \sum_{i,rr,rrr} trm_{i,rr,rrr} \cdot XDDE_{i,rr,rrr} \quad (63)$$

## 4.5 Investments

The total domestic savings consists of the savings made by all regional households, government and the regional production sectors. The savings of the regional sectors are assumed to be equal to their depreciation costs. The total domestic savings are calculated according to the following formula:

$$S = \sum_{th,r} SH_{th,r} + \sum_{gov} SG_{gov} + \sum_{i,r} \partial_{i,r} \cdot K_{i,r} \cdot PI \tag{64}$$

The total investments in the economy consist of domestic savings and the savings/investments received from the rest of the world minus the total changes in stocks:

$$IT = S + SROW \cdot ER - \sum_{i} \sum_{r} \left( SV_{i,r} \cdot P_{i,r} \right) \tag{65}$$

The total investments are spent on buying physical investments goods from various domestic regions, where the demand for them is determined according to the Cobb-Douglas demand function:

$$I_{i,r} \cdot P_{i,r} \cdot \left(1 + ti_{i,r}\right) = \left(\alpha_{i,r}\right) \cdot IT \tag{66}$$

The nominal rate of return in the economy is calculated as the average return to capital of all domestic sectors:

$$RGD_r = \frac{\sum_i \left( RK_{i,r} \cdot K_{i,r} \right)}{\sum_i K_{i,r}} \tag{67}$$

The price of additional unit of the composite physical investment good is calculated in accordance to the Cobb-Douglas demand function and has the following form:

$$PI = \prod_i \prod_r \left( \frac{P_{i,r} \cdot \left(1 + ti_{i,r}\right)}{\alpha_i} \right)^{\alpha_{i,r}} \tag{68}$$

## 4.6 Labour market

The labour market was chosen deliberately to be very simple. This specification is justified by the high labour participation rate in Russia, the weak position of labour unions and subsequently high bargaining power of firms, limited enforcement of labour regulations and relatively low labour mobility between regions.

The price of labour is determined from the labour market clearing condition indicated below. This basic equation simply indicates that all labour is either employed or unemployed. There is no leisure in the utility function of households and no involuntary unemployment. The labour supply of the region is fixed on a yearly basis.

$$\sum_i L_i = LS_r - UNEMP_r \tag{69}$$

Unemployment is determined from the so-called Philips curve. This curve provides a very basic link between real wage (PL/INDEX) and unemployment rate (UNRATE)..

$$\left( \frac{PL}{PL^0} \cdot \frac{INDEX^0}{INDEX} - 1 \right) = philips \cdot \left( \frac{UNRATE}{UNRATE^0} - 1 \right) \tag{70}$$

## 4.7 Market equilibrium conditions

Markets for goods and services are in equilibrium in each region of the country. According to the market clearing condition the total supply of a certain commodity in each region is equal to the sum of the demand of the regional households, region-specific demands of the governments, region-specific demand for physical investment goods, changes in stocks, region-specific demand for commodities used for production of freight trade and transport margins, intermediate demands of the regional production sectors both of materials as energy inputs.

$$X_{i,r} = \sum_{th} C_{th,i,r} + CG_{i,r} + I_{i,r} + SV_{i,r} + TMX_{i,r} + \sum_{ii} io_{i,ii,r} \cdot IO_{ii,r} + \sum_{ii} IOE_{ii,i,r} \qquad (71)$$

The corresponding sales price is determined from the internal and external market equilibrium from goods of the local market and imported goods.

$$P_{i,r} \cdot X_{i,r} = PDDT_{i,r} \cdot XDD_{i,r} + PMROW_i \cdot (1 + tm_{i,r}) \cdot MROW_{i,r} \qquad (72)$$

## 4.8 Closure and exogenously fixed variables

The formal introduction of the concept of closure rule can be traced back to Sen (1963). Sen (1963), showed that the necessary ex-post equality between savings and investment cannot be fulfilled when all the following conditions are satisfied: the factors are paid at their marginal productivity, household consumption is a function of real income, real investment is fixed and the factors are fully employed. The equilibrium is achieved only by relaxing one of these constrains. The choice of the constraint to be dropped, represents in fact the choice of the closure rule. In mathematical terms, the model should consist of an equal number of independent equations and endogenous variables. The closure rule reflects the choice of the model builder on which variables are exogenous and which variables are endogenous, so as to achieve ex-post equality. The following variables are exogenously fixed and define the closure:

- Sector-specific capital endowments in each region
- Governmental transfers to households and savings (optional)
- Transfers from abroad
- Price of labor in the rest of the world
- Labour supply in each region
- Transport margins
- Public savings / Government consumption (one of these has to be fixed, government consumption is fixed by default)
- Exchange rate / foreign savings (exchange rate is fixed by default)
- Fixed numeraire

## 4.9 Calculation of GDP and the Walras law

Regional real GDP (*GDPR*) is calculated according to the value added approach and is equal to the sum of output values minus intermediates inputs, where the prices are fixed at their initial levels:

$$GDPR_r = \sum_i (XD_{ir} \cdot PD_{i,r}{}^0) - \sum_{ii,i} (IO_{ii,i,r} \cdot P_{ii,r}{}^0)$$
$$- \sum_i (ENER_{i,r} \cdot \sum_{ii} P_{ii,r}{}^0) + \sum_i (tc_{i,c} - sc_{i,c}) \cdot P_{i,r}{}^0 \cdot C_{th,i,r}$$
$$+ \sum_i t_{i,r} \cdot I_{i,r} P_{i,r}{}^0 + \sum_i tcg_{i,r} \cdot CG_{i,r} P_{i,r}{}^0 + \sum_i tcg_{i,r} \cdot CGR_{i,r} P_{i,r}{}^0$$

(73)

Regional nominal GDP (*GDPCR*) is calculated according to the value added approach and is equal to the sum of output values minus intermediates inputs, all calculated in current prices:

$$GDPR_r = \sum_i (XD_{ir} \cdot PD_{i,r}) - \sum_{ii,i} (IO_{ii,i,r} \cdot P_{ii,r})$$
$$- \sum_i (ENER_{i,r} \cdot \sum_{ii} P_{ii,r}) + \sum_i (tc_{i,c} - sc_{i,c}) \cdot P_{i,r} \cdot C_{th,i,r}$$
$$+ \sum_i t_{i,r} \cdot I_{i,r} P_{i,r} + \sum_i tcg_{i,r} \cdot CG_{i,r} P_{i,r} + \sum_i tcg_{i,r} \cdot CGR_{i,r} P_{i,r}$$

(74)

Country-level GDP (real and nominal) is calculated as the sum of the regional-level GDPs:

$$GDP_c = \sum_r GDPR_r$$

(75)

$$GDPC_c = \sum_r GDPCR_r$$

(76)

Country-level GDP deflator is used as a numeraire of the model. All prices in the model are calculated relative (in terms of) to GDP deflator. GDP deflator is calculated as the ratio between nominal GDP of the country divided by the real GDP of the country.

$$GDPDEF = \frac{\sum_c GDPC_c}{\sum_c GDP_c} \qquad (77)$$

General equilibrium model represents a system of non-linear equations, where the number of variables is equal to the number of equations. Given that the functional forms of the production and utility functions are well-behaved (continuous and concave), this ensures that the model has a unique solution. All prices in the model are relative prices and calculated in terms of the numeraire, in our case it is the GDP deflator. Numeraire is exogenously fixed in the model. Once we have fixed one of the variables of the nonlinear system of equations (numeraire) it is necessary to remove one of the equations from the system in order to keep the equality between the number of equations and the number of variables. In case of our model the following trade balance equation has been dropped:

$$PMROW_{i,r} \cdot MROW_{i,r} \cdot ER + \sum_r LROW_r \cdot PLROW_{ed} \cdot ER =$$
$$SROW_r + TROW_r \cdot ER \qquad (78)$$

Since our system of equations describes a closed economic system where all monetary flows have origin and destination, the trade balance equation will be satisfied even if it is dropped from the system of nonlinear equations describing the model. This property is called Walras law which states that if N-1 market is in equilibrium than the Nth market will also be in equilibrium even if it is not a part of the general equilibrium problem. In the case of trade balance, it represents the market clearing condition for the exchange rate.

## 4.10 Recursive dynamics

SUST-RUS has a recursive dynamic structure composed of a sequence of several temporary equilibria. The first equilibrium in the sequence is given by the benchmark year 2006. In each time period, the model is solved for an equilibrium given the exogenous conditions assumed for that particular period. The equilibria are connected to each other through capital accumulation. In the benchmark case, we assume that the economy is on a steady-state growth path, where all the quantity variables grow at the same rate and all relative prices remain unchanged. When a policy measure is implemented the economy enters on a transition path, until, after some time it has reached a new steady-state growth path (Ballard, Fullerton, Shoven and Walley, 1985). We are interested in the transition path induced by the policy measure and the characteristics of the new growth path.

The endogenous determination of investment behavior is essential for the dynamic part of the model. Investment and capital accumulation in year t depend on expected rates of return for year t+1, which are determined by actual returns on capital in year t. This approach involves adaptive expectations. In the dynamic economic processes a composite investment commodity is allocated between sectors according to the actual (year t) returns on capital in sector sec (subscript i). The equilibrium expected rate of return $RK_{i,t}$ in the sector in the year t, is specified as an inverse logistic function (see Figure 1) of the proportionate growth in sector's capital stock (Dixon and Rimmer, 2002):

$$
\begin{aligned}
RK_{i,r,t} &= RK^{0}_{i,r,t} + (1/B_{i,r})\cdot[(\ln(Kg_{i,r,t} - Kg\min_{i,r})) \\
&- \ln(Kg\max_{i,r} - Kg_{i,r,t}) - \ln(Ktrend_{i,r} - Kg\min_{i,r}) \\
&+ \ln(Kg\max_{i,r} - Ktrend_{i,r})]
\end{aligned}
\qquad (79)
$$

where $RK^{0}_{i,r,t}$ is the sector's historically normal rate of return, $Kg_{i,r,t}$ is the actual capital growth rate in the sector, $Kg\min_{i,r}$ and $Kg\max_{i,r}$ are the minimum and the maximum possible growth rates of capital in the sector, $Ktrend_{i,r}$ is the sector's historically normal growth rate and $B_{i,r}$ is a positive parameter. The minimum possible growth rate is set at the negative of the rate of depreciation in the sector, while the maximum rate is set at $Ktrend_{i,r}$ plus 0.062 in order to avoid unrealistically large simulated growth rate (Dixon and Rimmer, 2002).

Parameter $B_{i,r}$ reflects the sensitivity of capital growth in sector sec to variations in its equilibrium expected rate of return. It is derived by differentiating the above equation with respect to $Kg_{i,r,t}$:

---

2 The specification of the maximum possible growth rate implies that if the historically normal rate in a sector is 4 per cent, the upper limit in any year t would not exceed 10 per cent.

$$B_{i,r} = SEA \cdot \left( \frac{Kg\max_{i,r} - Kg\min_{i,r}}{(Kg\max_{i,r} - Ktrend_{i,r}) \cdot (Ktrend_{i,r} - Kg\min_{i,r})} \right) \quad (80)$$

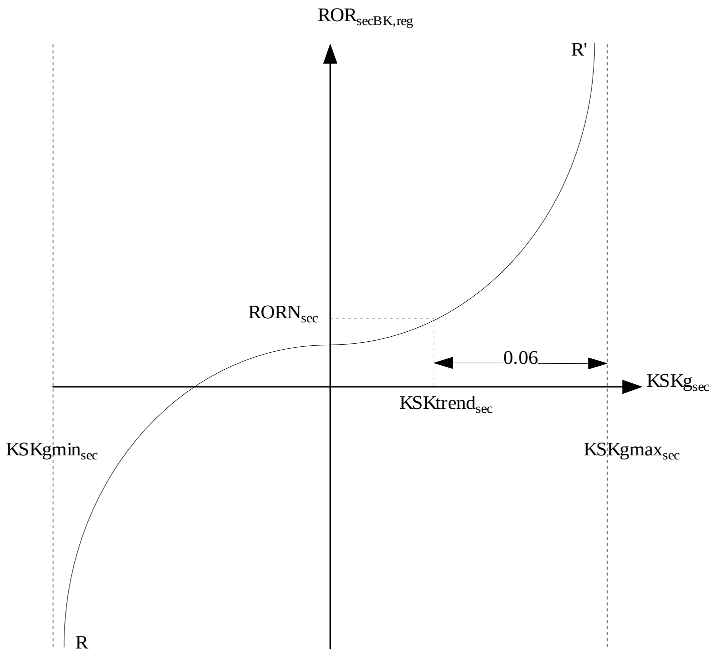
where:

$$SEA = \left( \frac{\partial RK_{i,r,t}}{\partial Kg_{i,r,t}} \right)^{-1} \quad (81)$$

The present value of purchasing a unit of capital to be used in the sector in year t is defined as:

$$PVK_{i,r,t} = -PI_t + [RRK_{i,r,t+1} + PI_{t+1} \cdot (1 - delta_{i,r})]/[1 + RINT_t]$$

where $PI_t$ is the cost of buying a unit of capital (the price of composite investment commodity) in year t, $RRK_{i,r,t}$ is the rental rate on sector's sec capital, $delta_i$ is the depreciation rate of the sector and $RINT_t$ is the interest rate in year t (Dixon and Rimmer, 2002). The purchase of one unit of capital in year t by sector involves an immediate expenditure, followed by two benefits in year t+1 which are discounted by $(1 + RINT_t)$: the rental value of an extra unit of capital in year t+1 and the value at which the depreciated unit of capital can be sold in year t+1. The actual rate of return on capital in sector sec in year t is further given by dividing both sides of by $PI_t$, and reflects the present value of an investment of one euro.




The expected rate of return $ROR_{i,t}$ under adaptive expectations is derived as:

$$ROR_{i,r,t} = -1 + [RK_{i,r,t} / PI_t + (1 - delta_{i,r})]/[1 + RINT_t / GDPDEF_t] \quad (82)$$

where we assume that investors expect no change in the price of composite investment commodity and rental rates. The real rate of return in year t is given by:

$$RGD_{r,t} = \frac{\sum_i (RK_{i,r,t} \cdot K_{i,r,t})}{\sum_{ii} (K_{ii,r,t})} \quad (83)$$

and $[1 + RINT_t / GDPDEF_t]$ reflects the adaptive expectation of the real interest rate.
The capital stock of the sector in the next period (year t+1) is given by:

$$K_{i,r,t+1} = (1 - delta_i) \cdot K_{i,r,t} + INV_{i,r,t} \tag{84}$$

The actual capital growth rate in sector sec can be derived from the above equation as:

$$\begin{aligned} Kg_{i,r,t} &= [\alpha RK_{i,t} \cdot Kg\max_i \cdot (Ktrend_i - Kg\min_i) \\ &+ Kg\min_i \cdot (Kg\max_i - Ktrend_i)] / \\ &[\alpha RK_{i,t} \cdot (Ktrend_i - Kg\min_i) + (Kg\max_i - Ktrend_i)] \end{aligned} \tag{85}$$

where:

$$\alpha RK_{i,r,t} = e^{B_{i,t} \cdot (RK_{i,r,t} - RK^0_{i,r,t})} \tag{86}$$

and the capital growth rate in terms of capital stock in year t+1 and the capital stock in year t is given by:

$$Kg_{i,r,t} = K_{i,r,t+1} / K_{i,r,t} - 1 \tag{87}$$

From the previous equations we derive the investment carried out in the sector in year t:

$$\begin{aligned} INV_{i,r,t} &= K_{i,r,t} \cdot ([\alpha RK_{i,r,t} \cdot Kg\max_i \cdot (Ktrend_i - Kg\min_i) \\ &+ Kg\min_{i,r} \cdot (Kg\max_{i,r} - Ktrend_{i,r})] / \\ &[\alpha RK_{i,r,t} \cdot (Ktrend_{i,r} - Kg\min_{i,r}) + (Kg\max_{i,r} - Ktrend_{i,r})] + 1) \\ &- K_{i,r,t} \cdot (1 - delta_i) \end{aligned} \tag{88}$$

The model is solved dynamically with annual steps. The simulation horizon of the model has been set up until 2020 but it can easily be extended. In between periods, some other variables like the transfers between firms, government and the rest of the world, and the balance of payment balance (foreign savings) are updated exogenously.

# 5. Construction of the database of model parameters

## 5.1 Introduction

To construct the database of exogenous model parameters, we performed a review of applied general equilibrium models with respect to the non-calibrated parameters. These parameters can subsequently be introduced into the modelling framework. Hereafter, we in particular focus on the sector-specific elasticities of substitution between different input factors in production and the Armington elasticities.

Our literature review encompasses three single-country CGE studies for Russia (Rutherford and Paltsev (1999), Alekseev et al. (2004), Lokhov and Welsch (2008)). However, we also review CGE and econometric studies with a multi-regional focus (Capros et al. (1998), Burniaux and Troung (2002), Kemfert and Welsch (2000), Bchir et al. (2002), Kemfert (2002), Liu et al. (2003), Böhringer and Löschel (2004), Saito (2004), Paltsev et al. (2005), Van der Werf (2007), Nemeth et al. (2008), Okagawa and Ban (2008), Welsch (2008)).

In section 5.2 we give an overview of the literature on elasticities of substitution between input factors; in section 5.3 we present the values for Armington elasticities. Section 5.3 concludes.

## 5.2 Substitution elasticities between production inputs

Elasticities of substitution between various inputs to production are essential for CGE models, since the specific degree of substitutability determines the cost of a simulated policy. All simulation studies reviewed use nested CES functions to model the production process. Elasticities of substitution therefore are defined for every nest-layer, between e.g. the factor inputs capital, labour, materials and various forms of energy, as well as for the disaggregated sectors included in the model. An overview of the literature review on substitution elasticities between various input factors to production is depicted in Table 5. A short discussion on the different models follows.

**CGE models with a regional focus on Russia (single country studies)**

All in all, we observe a wide range of values in the estimation of the different elasticities of substitution, summarized in the last column. This wide range can, for the most part, be explained by the different foci of the studies with respect to regional and sectoral aggregation. Studies focussing on elasticities of substitution specifically for Russia include the ones by Rutherford and Paltsev (1999) as well as by Lokhov and Welsch (2008).

The more comprehensive study, Lokhov and Welsch (2008), uses a CGE model to analyze emissions trading between Russia and the EU-15, disaggregated into ten sectors, including oil, gas, coal and electricity. Their estimates for the elasticities of substitution are based on literature reviews and recent econometric evidence for Russia and are depicted in Annex 1. Most of the elasticities are assumed to be uniform across both regions, possibly reflecting a lack of information. The energy-capital elasticity, however, is assumed to be higher for Russia (0.7) than for the EU-15 (0.5). This is due to the fact that Russia still has a much higher potential for energy saving. Furthermore, they assume that oil and gas are more easily substituted with each other than with coal as well as that elasticity of substitution between coal and oil/gas is greater than the substitutability of the fuel composite with electricity.

Lokhov and Welsch (2008) do not explicitly display the substitutability between capital and labour, while Rutherford and Paltsev (1999), presenting a static general equilibrium model for Russia, assume a Cobb-Douglas function for the factor-input relationship between capital and labour with a value of one. Other elasticities are not explicitly stated in this paper.

**Table 5 Overview of the literature review on elasticities of substitution**

| | Lokhov& Welsch (2008) | Rutherford & Paltsev (1999) | PACE (Böhringer et al., forthcom.) | EPPA (Paltsev et al., 2005) | WIAGEM (Kemfert, 2002) | GEM-E3 (Capros et al., 1997) | GTAP -E | GREEN | RAEM | Van der Werf (2007) | Okagawa and Ban (2008) | Kemfert& Welsch (2000) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| type of study | CGE model | CGE model | CGE model | CGE model | CGE model | CGE model | CGE model | CGE model | CGE model | econometric analysis | econometric analysis | econometric analysis | |
| Regional scope | Russia and EU-15 | Russia | global | global | global | global | global | global | 40 dutch regions | OECD | OECD | Germany | |
| Substitution | | | | | | | | | | | | | Overall Ranges |
| K/L | | 1 | 0.02 – 0.46 | 1 | | | | | 0.09 - 1 | 0.22 - 0.60 | 0.02 – 0.46 | 0.17 - 0.98 | 0.02 - 1 |
| KL/ E | | | 0 - 0.78 | 0.4-0.5 | | | | | | 0.16 - 0.62 | 0 - 0.78 | 0.64 - 0.98 | 0 - 0.98 |
| (KL)E/ M | | | 0 - 1.26 | | | | | | | | 0 - 1.26 | | 0 - 1.26 |
| K/E | 0.5 - 0.7 | | 0.03 - 0.65 | | | | 0 - 0.5 | 0 - 0.8 | | 0.10 - 0.99 | 0.03 - 0.65 | 0.17 - 0.98 | 0 - 0.98 |
| KE/L | 0.6 | | 0 – 0.94 | | | | | | | 0.81 - 1.04 | 0 – 0.94 | 0.58 - 0.94 | 0 - 1.04 |
| (KE)L /M | | | 0.35 - 1.15 | | | | | | | | 0.35 - 1.15 | | 0.42 - 1.15 |
| E/L | | | | | | | | | | 0.52 - 0.86 | | 0.07 - 0.97 | 0.07 - 0.97 |
| EL/K | | | | | | | | | | 0.22 - 0.60 | | 0.32 - 0.95 | 0.22 - 0.95 |
| KLE/ intermediates | 0 | | | | | | | | | | | | 0 |
| Electricty/ fuels | 0.6 | | | 0.5 | | | 0 - 1.0 | | | | | | 0 - 1 |
| Coal/gas-oil | 1 | | 0.5 - 2 | | | | 0 - 0.5 | | | | | | 0 - 2 |
| Gas/ oil | 2 | | 0.75 - 2 | | 2 | | 0 - 1.0 | | | | | | 0 - 2 |
| Coal/gas/oil | | | | 1 | 1 | | | | | | | | 1 |
| L/M/fuels | | | | | | 0.1 - 0.5 | | | | | | | 0.1 - 0.5 |
| L-M-fuels/ Electricty | | | | | | 0.5 | | | | | | | 0.5 |
| L-E-M/ capital | | | | | | 0.3 - 0.4 | | | | | | | 0.3 - 0.4 |
| KLM/Energy | | | | | 0.5 | | | | | | | | 0.5 |

With *K*= capital, *L*= labour, *E* = energy, *M*= materials.

**Multi-country CGE models**

The newest version of the PACE model (see Böhringer et al., forthcoming) employs substitution elasticities reported by Okagawa and Ban (2008)[3]. This study provides substitution elasticities for nested CES production functions for 19 sectors using panel data for OECD countries. The authors find that for substitution elasticities closely related to energy inputs, higher values in energy intensive industries and lower values in other industries are obtained, compared to the assumed parameters in existing CGE models.

It must be emphasized that drawing on this study, Böhringer et al. (forthcoming) use two possible nesting structures to model the substitution possibilities between the most relevant input factors to production. The estimates for the first version (KL, KL_E, M, KLE_M, Figure 1) are depicted in Table 6 and the estimates for the second version (KE, KE_L, M, KLE_M) are shown in Table 7.

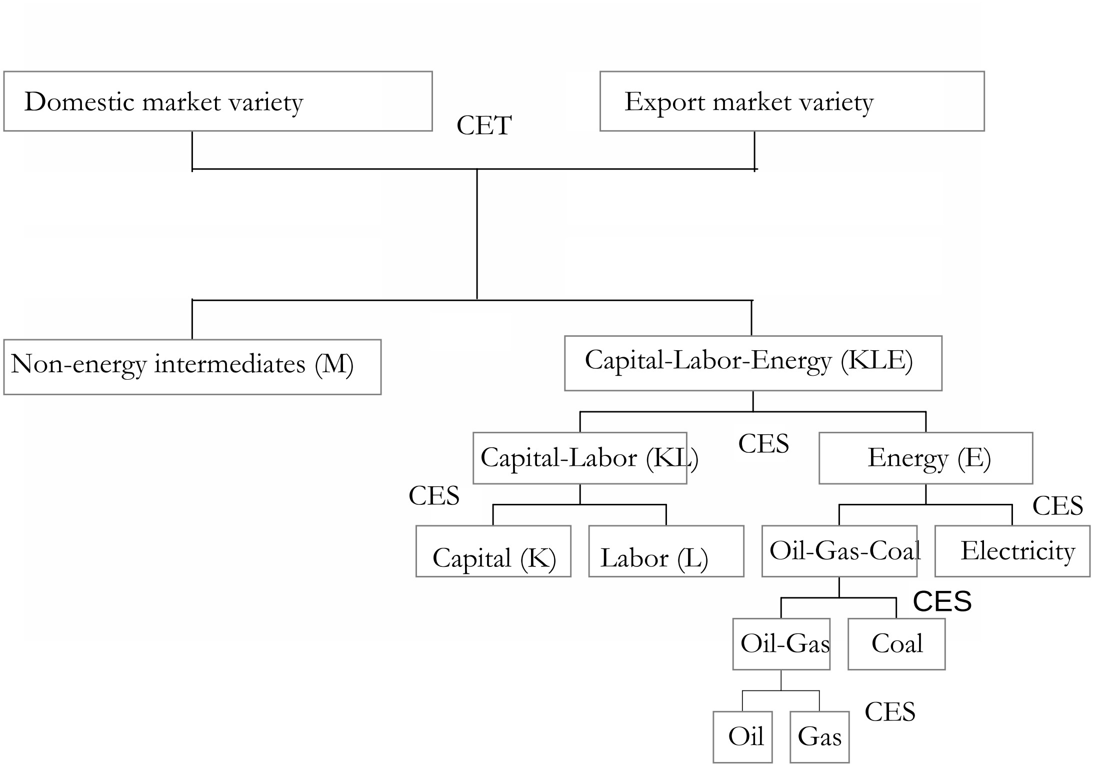
**Table 6 Substitution elasticities for the nesting structure version one employed in PACE**

| Sector/ Substitution[4] | KL | KLE | M | KLEM |
|---|---|---|---|---|
| Agriculture | 0.023 | 0.516 | 0 | 0.392 |
| Mining | 0.139 | 0.553 | 0.309 | 0.729 |
| Food | 0.382 | 0.395 | 0 | 0.329 |
| Textiles | 0.161 | 0.637 | 0.597 | 0.722 |
| Wood | 0.087 | 0.456 | 0.115 | 0.695 |
| Pulp&Paper | 0.381 | 0.211 | 0 | 0.187 |
| Chemical | 0.334 | 0 | 0.082 | 0.848 |
| Other Non-Metallic Mineral | 0.358 | 0.411 | 0.191 | 0.306 |
| Basic Metal | 0.220 | 0.644 | 0.253 | 1.173 |
| Machinery | 0.295 | 0.292 | 0.459 | 0.130 |
| Electrical Equipment | 0.163 | 0.524 | 0.359 | 0.876 |
| Transport Equipment | 0.144 | 0.519 | 1.087 | 0.548 |
| Manufacturing | 0.046 | 0.529 | 0.309 | 0.406 |
| Electricity, Gas and Water | 0.460 | 0.256 | 0.391 | 0 |
| Construction | 0.065 | 0.529 | 0 | 1.264 |
| Transport Equipment | 0.310 | 0.281 | 0.331 | 0.352 |
| Post and Telecommunications | 0.370 | 0.518 | 0.711 | 0.654 |
| Financial& Business Services | 0.264 | 0.320 | 0 | 0.492 |
| Personal Services | 0.316 | 0.784 | 0.132 | 0.90 |

[3] Where the econometric estimate produces a negative value, this is set to zero.

[4] KL = substitution within value added bundle (capital and labour), KLE = substitution within value added and energy bundle, M= materials bundle, KLEM = substitution between KLE and Materials bundle

**Figure 4: Nesting in non-fossil fuel production**




We implement the nesting figure for the SUST-RUS model presented in Figure 4 (cfr. section 4.2.2 and 4.2.3).

Please note that Lokhov and Welsch (2008) use the second nesting structure for their model. Therefore, it seems worthwhile to compare the estimates for this nesting structure between the two studies:

The value for the substitution elasticity between Capital-Energy with Labour (KE/ L) given by Lokhov and Welsch (2008) is within the range of values given by Okagawa and Ban (2008) for various sectors and as employed in the newest version of the PACE model. This suggests that the disaggregated values given in Tables 6 and 7 could be used for the SUST-RUS model. Regarding the substitution elasticity between capital and energy (K/E), Lokhov and Welsch's (2008) values for the EU and Russia are very high compared to Okagawa and Ban's (2008) estimates. Since Lokhov and Welsch (2008) make a sensible argument why Russia's substitution elasticity should be higher (see above), one might contemplate also readjusting the figures employed in PACE when applying these to a CGE model for Russia.

With regard to the elasticities of substitution within the energy aggregate, the values used in the PACE model are lower in general and higher or equal for the electricity sector as compared to the aggregated values given by Lokhov and Welsch (2008). We therefore suggest to follow Lokhov and Welsch's (2008) higher values in general and the specific values provided in PACE for the electricity sector. Other important feature of the PACE model should also be mentioned: First, the possibility to optimally scale the elasticities of substitution for short-run analysis exists.

**Table 7 elasticities for the nesting structure version two employed in PACE**

| Sector/ Substitution[5] | KE | KE_L | M | KEL_M |
|---|---|---|---|---|
| Agriculture | 0.029 | 0.547 | 0 | 0.998 |
| Mining | 0.535 | 0.341 | 0.309 | 0.349 |
| Food | 0.391 | 0.286 | 0 | 0.681 |
| Textiles | 0.170 | 0.467 | 0.597 | 1.023 |
| Wood | 0.052 | 0 | 0.115 | 0.944 |
| Pulp&Paper | 0.372 | 0.163 | 0 | 0.831 |
| Chemical | 0.038 | 0.344 | 0.082 | 0.808 |
| Other Non-Metallic Mineral | 0.350 | 0.207 | 0.191 | 0.987 |
| Basic Metal | 0.290 | 0 | 0.253 | 1.050 |
| Machinery | 0.118 | 0.082 | 0.459 | 1.149 |
| Electrical Equipment | 0.246 | 0.331 | 0.359 | 0.745 |
| Transport Equipment | 0.091 | 0.431 | 1.087 | 1.037 |
| Manufacturing | 0.102 | 0.251 | 0.309 | 1.046 |
| Electricity, Gas and Water | 0.396 | 0.375 | 0.391 | 0.418 |
| Construction | 0.105 | 0.938 | 0 | 0.974 |
| Transport Equipment | 0.449 | 0.466 | 0.331 | 1.045 |
| Post and Telecommunications | 0.288 | 0.345 | 0.711 | 0.439 |
| Financial& Business Services | 0.271 | 0.370 | 0 | 0.854 |
| Personal Services | 0.654 | 0.793 | 0.132 | 1.029 |

A further example of the multi-regional, multi-sector CGE models reviewed is the MIT Emissions Prediction and Policy Analysis (EPPA) model, which is built on the GTAP dataset and additional data for the greenhouse gas and urban gas emissions (version 4: Paltsev et al. 2005). The production technologies are modelled using nested CES functions. The key elasticities of substitution are based on a review of the literature and expert elicitation and are depicted in Annex 4. The substitution elasticity between electricity and fuels of the EPPA model is close to Lokhov and Welsch's (2008) values and the substitutability among the different fuels, which is not obtainable from the econometric studies presented above, is assumed to be one, a value also supported by Kemfert (2002).

Another relevant multiregional and multisector CGE model is the GEM – E3 Model, which includes 14 EU countries and the rest of the world, covering major industrialised and developing countries (Capros et al., 1998). The nested CES structure of the model and the elasticities of substitution are based on an econometric analysis of the Belgian economy, disaggregated into ten industrial sectors. The detailed nesting structure and the respective elasticities of substitution can be found in Annex 5. This model, using a different nested-structure, adds to the previously surveyed elasticities by including those between labour, materials and fuels (values: 0.1- 0.5), labour-materials-fuels and electricity (0.5) and labour-electricity-materials and capital (0.3- 0.4). Burniaux and Troung (2002) develop an energy-environmental version of the Global Trade Analysis Project (GTAP) Model, called GTAP – E. The standard GTAP model is a multiregional, multisector CGE model, with perfect competition and constant returns to scale, with the GTAP-E extending it considering energy substitution, carbon

[5] KE = substitution within capital and energy bundle, KEL = substitution between capital-energy and labour, M = substitution within materials bundle, KELM = substitution between KEL and Materials bundle

emissions and a specific mechanism for international carbon emission trading. The elasticities of substitution employed in this model include a range of values for different elasticities and sectors which are below those in Lokhov and Welsch (2008).

**Further econometric studies**

Van der Werf (2007) employs industry-level data from 12 OECD countries – Belgium, Canada, Denmark, Finland, France, United Kingdom, Italy, the Netherlands, Norway, Sweden, USA and West-Germany – to estimate the substitution elasticities between capital, labour and energy inputs to production, using two level nested CES production functions for the estimation procedure.

**Table 8: Van der Werf's (2007) elasticities of substitution**

| Sector | K/E | KE/L | K/L | KL/E | L/E | EL/K |
|---|---|---|---|---|---|---|
| Basic metals | 0.88 | 0.82 | 0.60 | 0.62 | 0.86 | 0.60 |
| Construction | 0.99 | 0.95 | 0.22 | 0.29 | 0.52 | 0.22 |
| Food&Tob. | 0.99 | 0.92 | 0.46 | 0.40 | 0.85 | 0.46 |
| Transport eq. | 0.10 | 0.98 | 0.44 | 0.16 | 0.80 | 0.37 |
| Non Metal Minerals | 0.10 | 0.94 | 0.45 | 0.25 | 0.82 | 0.39 |
| Paper etc. | 0.96 | 0.81 | 0.38 | 0.39 | 0.77 | 0.32 |
| Textiles etc. | 0.10 | 1.04 | 0.27 | 0.29 | 0.79 | 0,23 |

K= capital, L= labour, E = energy

## 5.3 Review of Armington elasticities

Armington elasticities, which date back to the seminal work by Armington (1969), describe the substitutability between goods produced in the domestic and in the foreign market. This substitutability is assumed to be limited, meaning that domestic and foreign commodities are less than perfect substitutes. CGE models usually incorporate this international trade component by employing a two-level nested CES function structure. Two distinct Armington elasticities are therefore needed to specify the model: The elasticity, which we will call 'Nest1' Armington, describes the substitutability between domestic and foreign goods in the demand for the composite good. The 'Nest2' Armington elasticity then describes the substitutability between imports coming from different regions, the intra-import substitutability. The overview of Armington elasticities employed in different models and estimated using econometric analysis is depicted in Table 8. A short discussion of the various studies follows.

**Table 8: Review of the literature review on Armington elasticities**

| | Alekseev et al. (2004) | Lokhov& Welsch (2008) | Rutherford& Paltsev (1999) | EPPA | GEM-E3 | GTAP | GREEN | MIRAGE | PACE | WIAGEM | Welsch (2008) | Liu et al. (2003) | Saito (2004) | Nemeth et al. (2008) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| type of study | CGE model | CGE model | CGE model | CGE model | CGE model | CGE model | CGE model | CGE model | CGE model | CGE model | Econometric analysis | Econometric analysis | Econometric analysis | Econometric analysis | |
| Regional scope | Russia | Russia and EU-15 | Russia | global | global | global | global | global | global | global | OECD | global | OECD | EU-25 | |
| Armington type and sector | | | | | | | | | | | | | | | Overall Ranges |
| Domestic - import (NEST 1) | 0.6 - 0.94 | 0.1 - 4 | 4 | 2.0 - 3.0 | 0.6 - 1.5 | 1.9 - 10 | 0.3 - ∞ | 1.9 - 4.55 | 1 | 4 | -0.931 - 0.99 | 1.05 - 3.76 | 0.94 - 3.53 | 0.72 - 3.6 | -0.931 - ∞ |
| electricity | 0.75 | | | 0.3 | 0.6 | 2.8 | 0.3 | | | | | | | 1.66 - 1.85 | |
| coal | 0.75 | | | | 0.6 | 2.8 | 4 | | | | | | | 0.72 - 1.09 | |
| crude oil | 0.75 | | | | 0.6 | 10.0 | ∞ | | | | | | | 0.72 - 1.09 | |
| gas | 0.75 | | | | 0.6 | 2.8 | 4 | | | | | | | 0.72 - 1.09 | |
| petroleum, coal products | | | | | | 1.9 | 4 | | | | | | | | |
| ferrous metals | 0.81 | | | | 1.5 | 2.8 | 2 | | | | 1.267 | | 2.60 | | |
| chemical, rubber, plastic products | 0.83 | | | | 1.5 | 1.9 | 2 | 3.95 | | | 0.97 - 1.6 | 1.98 | 2.30 | | |
| other manufacturing: trade, transport | | | | | | 2.59 | 2 | | | | | | | | |
| agriculture, forestry, fishery | 0.60 | | | | 1.2 | 2.47 | 3 | 2.30 | | | 0.575 | 1.05 | 3.53 | 2.0 - 3.6 | |
| commercial, public services, dwellings | | | | | | 1.91 | 2 | | | | | | | | |
| processed food | 0.80 | | | | | | | 2.30 | | | 0.460 | 3.76 | 0.94 | | |
| fuels | | | | | | | | | | | 0.005 | 1.08 | | | |
| minerals | | | | | | | | | | | -0.931 | 1.08 | 1.14 | | |
| clothing and textiles | | | | | | | | 4.54 | | | 0.853 | 2.54 | 1.05 | | |
| other light manufacturing | | | | | | | | | | | | 2.23 | | | |
| machinery equipment | 0.94 | | | | | | | | | | 1.625 | 3.66 | 2.55 | | |
| basic manufacturers | | | | | | | | | | | | 2.24 | | | |
| nonferrous metallurgy | 0.81 | | | | 1.5 | | | | | | 1.267 | | 2.60 | | |
| light industry | 0.79 | | | | | | | | | | | | | | |
| other industries | 0.61 | | | | | | | | | | | | | | |
| construction services | 0.60 | | | | | | | | | | | | | | |
| communication | 0.60 | | | | 0.6 | | | | | | | | | | |
| other services | 0.60 | | | | | | | 1.90 | | | | | | | |
| finance, banking and insurance services | 0.60 | | | | 0.6 | | | | | | | | | | |
| raw materials | | | | | | | | 4.06 | | | | | | | |
| other manuf. products | | | | | | | | 3.27 | | | | | 0.94 | | |
| motors and vehicles | | | | | | | | 4.09 | | | | | | | |
| equipment | | | | | 1.5 | | | 4.19 | | | | | | | |
| houses | | | | | | | | 4.55 | | | | | | | |
| transports | 0.60 | | | | 1.2 | | | 4.35 | | | 1.699 | | | | |
| electrical goods | | | | | 1.5 | | | | | | 0.337 | | | | |
| paper and printing products | | | | | | | | | | | 0.342 | | 2.01 | | |
| mining | | | | | | | | | | | | | 2.07 | | |
| Among imports from different regions (NEST2) | | 0.1 - 12 | | 0.5 - 6 | 0.8 - 2.8 | 3.8 - 20.0 | 3 - ∞ | | 2 | | | | 0.24 - 1.39 | 0.23 - 5.24 | 0.1 - ∞ |
| within the EU-12+Russia | | 0.1-8 | | | | | | | | | | | | | |
| non energy goods | | | | 5 | | | | | | | | | | | |
| gas,coal | | | | 4 | 0.8 | 5.6 | 5 | | | | | | | 0.23 - 2.18 | |
| electricity | | | | 0.5 | 0.8 | 5.6 | 0.5 | | | | | | | 1.73 - 2.06 | |
| ROIL (refined oil) | | | | 6 | 0.8 | | 3 | | | | | | | 0.23 - 2.18 | |
| crude oil | | | | | | 20.0 | ∞ | | | | | | | | |
| petroleum, coal products | | | | | | 3.8 | 3 | | | | | | | | |
| ferrous metal | | | | | | 5.6 | 3 | | | | | | 0.71 | | |
| chemical, rubber, plastic products | | | | | | 3.8 | 3 | | | | | | 1.16 | | |
| other manufacturing | | | | | 2.4 | 6.04 | 3 | | | | | | 1.15 | 2.34 - 3 | |
| agriculture, forestry | | | | | 1.6 | 4.62 | 4 | | | | | | 0.24 | 5.24 - 4.21 | |
| commercial, public services, dwellings | | | | | | 3.8 | 3 | | | | | | | | |
| consumer goods | | | | | 2.8 | | | | | | | | | 1.94 - 2.33 | |
| mining | | | | | | | | | | | | | 1.04 | | |
| paper and printing products | | | | | | | | | | | | | 0.72 | | |
| processed food | | | | | | | | | | | | | 0.92 | | |
| textiles and clothing | | | | | | | | | | | | | 1.39 | | |

Alekseev et al. (2004) present Russia-specific NEST1 Armington elasticities for 15 sectors, based on the work of Zemnitsky (2002). Their values range between 0.6 and 0.94 and are thus much lower than e.g. the value of 4.0 employed by Rutherford and Paltsev (1999) in their CGE model for Russia or the GTAP values which range from 1.9 to 10. Such low values are, however, supported by newer econometric analysis: Welsch (2008) estimated Armington elasticities in four European countries using a common data set with 15 commodity groups and, as a rule of thumb, finds Armington elasticities for European industries to have values between 0 and 1 for consumption goods and between 1 and 2 for investment goods. Although Welsch's (2008) estimation results are only applicable to Europe, they suggest that Alekseev et al.'s (2004) values are not unreasonably low.

Lokhov and Welsch (2008) also use regionally and sectorally differentiated Armington elasticities with values ranging between 0.1 and 4 (between 0.1 and 12 for NEST2 Armington elasticities respectively), where high end figures are default values and lower end values reflect goods with little worldwide trade such as electricity.

Kemfert (2002) specifies an aggregate NEST1 Armington elasticity for the WIAGEM model and Böhringer and Löschel (2004) give aggregate NEST1 and NEST2 figures for the model PACE. Paltsev et al. (2005) provide a range of 2 to 3 for the NEST1 elasticity of the EPPA model without further disaggregating this range with the exception of the case of energy, which receives a low value of 0.3. NEST2 values range from 0.5 to 6, disaggregated for the major energy sources.

Capros et al. (1998) specify NEST1 and NEST2 Armington elasticities for the 18 sectors covered in the GEM-E3 model. Elasticities differ among sectors, but values for each sector are assumed to be identical for all EU countries. Values range between 0.6 and 1.5 for NEST1 and between 0.8 and 2.8 for NEST2 Armington elasticities.

The Armington elasticities for the GTAP and GREEN models, both NEST1 and NEST2, are presented in Burniaux and Troung (2002). The elasticities are sectorally disaggregated and have, on average, very high values. The MIRAGE values for Armington elasticities are drawn from the GTAP database (Bchir et al., 2002).

Liu et al. (2003) estimate NEST1 Armington elasticities using an approximate likelihood function for a variant of the widely used GTAP model of global trade and arrive at values ranging from 1.05 to 3.76.

Saito (2004) finds that the Armington elasticities obtained from multilateral trade data tend to be higher than those obtained from bilateral trade data in the intermediate inputs sector and provides sectorally disaggregated values for NEST1 and NEST2 Armington elasticities ranging between 0.94 - 3.53 and 0.24 - 1.39 respectively.

Nemeth et al. (2008) estimate short and long run Armington elasticities for eight sectors for most countries of Europe for both NEST1 and NEST2 elasticities.

## 5.4 Conclusion

In constructing a database with the parameters for the SUST-RUS model (and given the fact that Russia-specific econometric estimates are fragmentary), we have to rely on values from studies with a different regional focus. To fulfil the objective of the given task, the most recent literature on applied general equilibrium and econometric models, and specifically on the elasticities of substitution between different factor inputs of production as well as on Armington elasticities, has been reviewed. The literature survey encompasses CGE studies for Russia, global CGE models as well as econometric estimations.

Building on this literature, tentative conclusions can be drawn with regard to the selection of values of these non-calibrated parameters for the SUST-RUS model.

First, the study by Lokhov and Welsch (2008) provide values for the elasticities of substitution that can be used as guideposts. For the specific determination of elasticities of substitution disaggregated into various sectors, it seems appropriate to rely on the econometric studies focussing on OECD countries, most notably the newest study by Okagawa and Ban (2008) as employed in the most recent version of the PACE model (Böhringer et al., forthcoming). As explained above, this furthermore opens the possibility for choosing between two different nesting structures. Regarding the more specific suggestions with respect to the values for substitution elasticities, it is suggested to use Okagawa and Ban's (2008) estimates in general but possibly to adjust them upwards for the substitution elasticity between Capital and Energy, as Lokhov and Welsch (2008) provide a higher figure based on the argument that Russia still has a much higher potential for energy saving. For the intra-energy elasticities of substitution (Coal-Oil and Gas; Oil and Gas), we can rely on Lokhov and Welsch's (2008) values.

**Table 9: Proposed exogenous parameter of input substitution**

| Production Technologies | KLEM | M | KLE | KL | ELEC | COAL | OIL/GAS |
|---|---|---|---|---|---|---|---|
| Agriculture, ea | 0.392 | 0 | 0.516 | 0.023 | 0.6 | 0.5 | 0.75 |
| Fishing | 0.392 | 0 | 0.516 | 0.023 | 0.6 | 0.5 | 0.75 |
| Coal | 0.729 | 0 | 0.553 | 0.139 | 0.6 | 0.5 | 0.75 |
| Gas | 0.729 | 0 | 0.553 | 0.139 | 0.6 | 0.5 | 0.75 |
| Oil | 0.729 | 0 | 0.553 | 0.139 | 0.6 | 0.5 | 0.75 |
| Mining (non-energy) | 0.729 | 0 | 0.553 | 0.139 | 0.6 | 0.5 | 0.75 |
| Food, beverage and tobacco | 0.729 | 0 | 0.553 | 0.139 | 0.6 | 0.5 | 0.75 |
| Textiles | 0.329 | 0 | 0.395 | 0.382 | 0.6 | 0.5 | 0.75 |
| Leather | 0.722 | 0 | 0.637 | 0.161 | 0.6 | 0.5 | 0.75 |
| Wood | 0.695 | 0 | 0.456 | 0.087 | 0.6 | 0.5 | 0.75 |
| Pulp&Paper | 0.187 | 0 | 0.211 | 0.381 | 0.6 | 0.5 | 0.75 |
| Coke, refineries | 0.848 | 0 | 0.529 | 0.334 | 0.6 | 0.5 | 0.75 |
| Chemicals | 0.848 | 0 | 0.529 | 0.334 | 0.6 | 0.5 | 0.75 |
| Rubber and plastics | 0.306 | 0 | 0.411 | 0.358 | 0.6 | 0.5 | 0.75 |
| Non-metallic products | 0.306 | 0 | 0.411 | 0.358 | 0.6 | 0.5 | 0.75 |
| Basic metals | 1.173 | 0 | 0.644 | 0.22 | 0.6 | 0.5 | 0.75 |
| Machinery | 0.13 | 0 | 0.292 | 0.295 | 0.6 | 0.5 | 0.75 |
| Electric and optics | 0.876 | 0 | 0.524 | 0.163 | 0.6 | 0.5 | 0.75 |
| Transport Eq. | 0.548 | 0 | 0.519 | 0.144 | 0.6 | 0.5 | 0.75 |
| Other manufacturing | 0.406 | 0 | 0.529 | 0.046 | 0.6 | 0.5 | 0.75 |
| Electricity, gas and water (distribution) | 0 | 0 | 0.256 | 0.46 | 0.6 | 0.5 | 0.75 |
| Electricity | 0 | 0 | 0.256 | 0.46 | 0.6 | 0.5 | 0.75 |
| Construction | 1.264 | 0 | 0.529 | 0.065 | 0.6 | 0.5 | 0.75 |
| Wholesale trade | 0.9 | 0 | 0.784 | 0.316 | 0.6 | 0.5 | 0.75 |
| Hotels and restaurants | 0.9 | 0 | 0.784 | 0.316 | 0.6 | 0.5 | 0.75 |
| Communication | 0.654 | 0 | 0.518 | 0.37 | 0.6 | 0.5 | 0.75 |
| Transport | 0.352 | 0 | 0.281 | 0.31 | 0.6 | 0.5 | 0.75 |
| Financial intermediation | 0.492 | 0 | 0.32 | 0.264 | 0.6 | 0.5 | 0.75 |
| Government service and defence | 0.9 | 0 | 0.784 | 0.316 | 0.6 | 0.5 | 0.75 |
| Real estate, renting and business activities | 0.492 | 0 | 0.32 | 0.264 | 0.6 | 0.5 | 0.75 |
| Education | 0.9 | 0 | 0.784 | 0.316 | 0.6 | 0.5 | 0.75 |
| Health and social work | 0.9 | 0 | 0.784 | 0.316 | 0.6 | 0.5 | 0.75 |

Second, the literature review of Armington elasticities encompasses the types NEST1 (substitutability between domestic and imported goods) and NEST2 (substitutability among imports from different regions), sectorally disaggregated, short and long-term as well as Russia-specific estimations. Alekseev et al. (2004) present a comprehensive database for Russia-specific NEST1 Armington elasticities for 15 sectors, based on the econometric analysis conducted by Zemnitsky (2002), with values ranging between 0.6 (amongst others: agriculture) and 0.94 (machinery equipment). As highlighted above, these values seem very low when compared to the common practice values of Armington elasticities as employed in global CGE models; they are, however, supported by most recent econometric analysis carried out by Welsch (2008). For NEST2 Armington values, we cannot draw on Russia-specific econometric estimates, only on econometric studies for the OECD (Saito, 2004) and the EU-25 (Nemeth et al., 2008). The wide range given by Lokhov and Welsch (2008) might be used for the sensitivity analysis.

**Table 10: Proposed Armington elasticticities for SUST-RUS model**

| Production Technologies | Armington (Alekseev et al.) |
|---|---|
| Agriculture, ea | 0.6 |
| Fishing | 0.6 |
| Coal | 0.75 |
| Gas | 0.75 |
| Oil | 0.75 |
| Mining (non-energy) | 0.75 |
| Food, beverage and tobacco | 0.6 |
| Textiles | 0.79 |
| Leather | 0.79 |
| Wood | 0.79 |
| Pulp&Paper | 0.79 |
| Coke, refineries | 0.83 |
| Chemicals | 0.83 |
| Rubber and plastics | 0.83 |
| Non-metallic products | 0.83 |
| Basic metals | 0.81 |
| Machinery | 0.94 |
| Electric and optics | 0.75 |
| Transport Eq. | 0.75 |
| Other manufacturing | 0.61 |
| Electricity, gas and water (distribution) | 0.75 |
| Electricity | 0.75 |
| Construction | 0.6 |
| Wholesale trade | 0.6 |
| Hotels and restaurants | 0.6 |
| Communication | 0.6 |
| Transport | 0.6 |
| Financial intermediation | 0.6 |
| Government service and defence | 0.6 |
| Real estate, renting and business activities | 0.6 |
| Education | 0.6 |
| Health and social work | 0.6 |

# 6. Main outputs of the model

## 6.1 National level

A set of the model variables, which are elements of the SAM are computed at the national level:

- national GDP
- exports and imports by commodity
- revenues from national level taxes
- spending on subsidies paid at the national level

## 6.2 Regional level

The main variables of the model follow the structure of a SAM. This means that during a simulation run the model calculates new values of all elements in the initial SAM. These include the following variables

- sectoral outputs and intermediate inputs
- sectoral employment and investment
- gross value added by sector
- households consumption by type of commodity
- households' and firms' savings
- inter-regional trade
- unemployment and unemployment rate
- employment by sectors
- regional GDP
- revenues from regional-level taxes
- subsidies paid at the regional level
- unemployment benefits paid to households
- governmental transfers to households
- amount of labour (human capital)

Note that different types of public spending are considered as exogenous variables of the model.

# 7. Model implementation in GAMS

## 7.1 About GAMS

The SUST-RUS model is implemented using the General Algebraic Modeling System (GAMS). This software is widely used for general equilibrium modeling and has proved to be able to efficiently handle large scale economic models. More information about this software is available from www.gams.com.

A manual to work with GAMS is freely available, as well as a test version of the model. However, SUST-RUS requires the use of a fully functional GAMS program, including a solver able to handle "Mixed Complementarity Problems" (MCP). The price of this software packet and the solver is considerable, but allows complex modelling on a large scale.

## 7.2 Main structure of the code

The main structure of the GAMS code consists of the following elements:

- o BASIC MODEL SETS
- o IMPORTING DATA
- o DECLARATION AND INTIALIZATION OF PARAMETERS
- o CALIBRATION OF MODEL PARAMETERS
- o MODEL VARIABLES AND EQUATIONS
- o INITIALIZE VARIABLES
- o MODEL CLOSURE AND NUMERAIRE
- o SIMULATION SET-UP (optional)
- o SET SOLVER OPTIONS
- o MODEL REPORTING
- o RECURSIVE DYNAMICS (loop model over different time periods)

## 7.3 General rules when editing the code

When adding lines of code to the model, you should make sure that the general template of the model is still followed. When introducing a new parameter, the parameter has to be defined and an initial value for the parameter has to be set in the model calibration. It is important to follow the notational conventions that we have set, to avoid creating an unintelligible and therefore unusable code.

**Basic conventions**

- New sets and aliases are introduced in the beginning of the code, before declaring initial parameters.
- Scalars, parameters and data are in lower case. For example Scal (scaling variable), sigma A1,..
- VARIABLES (and their initial levels) and EQUATION names are in CAPITAL letters are named in the shortest and most logical way possible.
- INDICATORS are called after their respective variables and should be named as short and clearly as possible
- EQUATION names always begin with EQ and are named after the variable that results from the calculation, for example: EQP is the equation that calculates the price of a good; EQXD is the equation that calculates domestic production for each sector, in each region.
- Initial values of variables and parameters are indicated with Z added to their names, for example: PZ, XDZ, CZ. Respectively initial price level, intial domestic production and initial consumption.
- VARIABLES are initialized to 1 and are multiplied with their initial values in the model equations.

Within the code numerous comments are added on what happens within the model and why. GAMS forces the researcher to declare each parameter explicitly, together with the sets that each parameter uses. This can be very time demanding, but increases the readability of the code and makes it easier to debug. A comment is introduced by the $ontext/$offtext command or by putting an asterisk in the margin.

## 7.4 Model numeraire

A common assumption for a CGE model, which is also adopted here, is that the economy is initially in equilibrium with the quantities normalized in such a way that the prices are equal to unity. Due to the homogeneity of degree zero in prices the model can only determine relative prices. A particular price has been selected to provide the numeraire against which all the prices in the model will be measured. In the model, the GDP deflator is chosen as the numeraire and exogenously fixed in the model. The ER equation is used to check the Walras law.

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