Notes for a Model of Private Protection of Property Rights *

by
Avinash Dixit
Princeton University

First draft March 2003,
this version May 28, 2007

The purpose of these notes is to endogenize occupation choice between production, professional protection, and predation in a society where the government (if one exists) does not protect property rights. Previous work is limited or differs in different respects; for example: [1] Hirshleifer (1995) considers only self-protection; Dixit (2004, chapter 5) generalizes this to allow each of ex ante identical people to split their time into the different activities of production, protection, and predation. [2] Grossman (2002) compares self-protection and that provided by a government which maximizes either social welfare or the utility of a ruling elite. [3] Anderson and Bandiera (2002) separate the population exogenously into property-'owners' who may choose self-protection or hire a professional, the professional protectors, and bandits. But in reality, protectors and bandits come from the same population, and many of the attributes needed for success in the two occupations are common. And there is mobility between these occupations and production. Therefore it is of interest to examine the outcomes when people can choose freely among these occupations.

Here the players are labeled by $t \in [0, 1]$. Actually each player occupies a tiny interval of length Δt in this space, so there are $1/\Delta t$ players in all. Having done the calculations on this basis, many results are then better stated and interpreted in the limit as $\Delta t \to 0$.

The idea is that each person has two types of skill – farming and fighting (for predation or protection). To keep the problem one-dimensional, it is supposed that the two are perfectly negatively related. Player t has toughness t in fighting and productivity $(1 - t)$ in farming.

Occupation choices:

Each player chooses from among the following four occupations:

[1] Full-time farmer. Produces output $1 - t$; hires a professional protector at wage w (which is endogenous, and must be determined as a part of the equilibrium).

*This is a highly preliminary version of some incomplete ideas; please do not cite but feel free to use and develop. I thank Oriana Bandiera for comments on a previous even more incomplete version, and the National Science Foundation for research support.

[2] Self-protecting farmer. Additionally chooses what fraction y of his own time to devote to protection. Then produces output $(1-t)(1-y)$, and has effective toughness $t\,y$.

[3] Protector. Gets wage w. Type t is private information – potential employer cannot observe this.

[4] Bandit. Chooses at random a farm to hit; cannot observe whether it has professional protection and the type of the farmer or protector guarding it.

Predation probabilities:

I assume that if a farm protected with toughness t' is hit by a single bandit with toughness t, the probability that the protector defeats the bandit and the output remains available to the farmer is

$$\tfrac{1}{2} + k\,(t'-t)\,.$$

To ensure that the probabilities stay between 0 and 1 for all types, I assume $k \leq \frac{1}{2}$. If $k = \frac{1}{2}$, then the best protector meeting the worst bandit is sure to prevail and the worst protector meeting the best bandit is sure to lose. If $k < \frac{1}{2}$, there is positive probability that even the best bandit attacking a farm protected with zero toughness (tantamount to no protection) fails; there may be low cost devices like locks so this is not implausible, and in any case it is not a big issue for the results.

I assume that each full-time farmer hires exactly one protector, and that if two or more bandits hit the same farm (whether self-protected or professionally protected), they will overcome the protection with probability 1, take all the output, and share it equally among themselves.

I take the professional protectors' honesty for granted, but comment on this in light of the results.

Some comments on this structure

[1] The assumption that multiple bandits hitting a farm simultaneously "cooperate" to get all its output and share it equally is convenient because each bandit's payoff in case of simultaneous attack is independent of his type. But it may be realistic to change this, e.g. multiple bandits may fight each other and share proportional to t or have win probabilities proportional to t or the toughest may take all. Or it may be a matter of timing: the first bandit to reach the farm is the only possible taker.

[2] Better information may be available, e.g. is a farm protected, and what is the toughness of the protector? Protectors – and bandits – can build reputation for toughness, and the fact that a farm is protected may be publicized (think of the decals on home and business doors). Then a bandit's decision of which farm to hit may be purposeful rather than random. Then the negative externality identified by Bandiera and others comes into play: if I protect my farm, that raises the probability that other unprotected farms are hit, so reducing the payoffs of those farmers.

[3] Fights may destroy fraction of output.

Numerous other variants are conceivable and are left to interested readers to develop.

Candidate equilibrium:

Intuitively, equilibrium should take the following form:

[1] Full-time farmers: $0 \le t < t_1$
[2] Self-protecting farmers: $t_1 \le t < t_2$
[3] Protectors: $t_2 \le t < t_3$
[4] Bandits: $t_3 \le t \le 1$

Thus there are $t_2/\Delta t$ farmers (of the two types combined) and $(1 - t_3)/\Delta t$ bandits. The t_i and w are the endogenous variables. They are to be determined from

(i) indifference conditions for the adjoining types at t_i,

(ii) balance between full-time farmers and professional protectors, that is, $t_1 = t_3 - t_2$.

Also, it needs to be checked that the inframarginal people have clear preference for the stipulated occupation.

Lemma:

If there are F farmers and B bandits, both $>> 1$, then for any given farmer

$$\text{Prob[No bandit]} \approx e^{-B/F}\,.$$

Proof: The probability that all B bandits choose from the remaining $F - 1$ farms is

$$\left(\frac{F-1}{F}\right)^{B} = \left[\,\left(1 - \frac{1}{F}\right)^{-F}\,\right]^{-B/F} \approx e^{-B/F}\,.$$

Also

$$\text{Prob[One specified bandit]} \approx \frac{1}{F}\;e^{-(B-1)/F}\,.$$

Proof: The probability that one specified bandit hits your farm and the other $(B - 1)$ hit the other $(F - 1)$ farms is

$$\frac{1}{F}\;\left(\frac{F-1}{F}\right)^{B-1} = \frac{1}{F}\;\left[\,\left(1 - \frac{1}{F}\right)^{-F}\,\right]^{-(B-1)/F} \approx \frac{1}{F}\;e^{-(B-1)/F}\,.$$

Income of full-time farmer:

Consider a player of type t in the role of a full-time farmer. In equilibrium we want $0 \le t < t_1$, but we can find the income in this occupation as a function of t for the whole range $0 \le t \le 1$, assuming that the others are choosing as specified in the candidate equilibrium. (The same procedure will apply to the other occupations also.)

This farmer's output is $(1 - t)$. The probability that he is not hit by any bandit is

$$\exp\left[\frac{-(1-t_3)/\Delta t}{t_2/\Delta t}\right] = e^{-(1-t_3)/t_2}\,.$$

The probability that he is hit by precisely the bandit at $x_3 \in [t_3, 1]$ is

$$\frac{1}{t_2/\Delta t}\;\exp\left[\frac{-(1-t_3)/\Delta t - 1}{t_2/\Delta t}\right] = \frac{\Delta t}{t_2}\;e^{-(1-t_3)/t_2}\,,$$

where in the argument of the exponential I have let $\Delta t \to 0$.

The probability that he is hiring the protector $x_2 \in [t_2, t_3)$ is $\Delta t/(t_3 - t_2)$. Conditional on this and on being hit by exactly the x_3-bandit, the probability that he keeps his output is

$$\tfrac{1}{2} + k\,(x_2 - x_3)\,.$$

Integrating (adding) over x_2 and x_3, the probability of his keeping his output when hiring a random protector and being hit by a random bandit is

$$\begin{aligned}
& \frac{1}{t_2}\, e^{-(1-t_3)/t_2}\, \frac{1}{t_3 - t_2} \int_{x_3=t_3}^{1} \int_{x_2=t_2}^{t_3} \left[\tfrac{1}{2} + k\,(x_2 - x_3)\right] dx_2\, dx_3 \\
= \;& \frac{1}{t_2}\, e^{-(1-t_3)/t_2}\, \frac{1}{t_3 - t_2} \int_{x_3=t_3}^{1} \left[\, (\tfrac{1}{2} - k\, x_3)\,(t_3 - t_2) + \tfrac{1}{2}\, k\, \{\, (t_3)^2 - (t_2)^2 \,\}\,\right] dx_3 \\
= \;& \frac{1}{t_2}\, e^{-(1-t_3)/t_2} \int_{x_3=t_3}^{1} \left[\, (\tfrac{1}{2} - k\, x_3)\; + \tfrac{1}{2}\, k\,(t_3 + t_2)\,\right] dx_3 \\
= \;& \frac{1}{t_2}\, e^{-(1-t_3)/t_2} \left\{\, \tfrac{1}{2}\,[1 + k\,(t_3 + t_2)]\,(1 - t_3) - \tfrac{1}{2}\, k\,[\,1 - (t_3)^2\,]\, \right\} \\
= \;& \frac{1 - t_3}{2\, t_2}\, e^{-(1-t_3)/t_2}\, [\,1 + k\,(t_3 + t_2) - k\,(1 + t_3)\,]\,.
\end{aligned}$$

The last line can be simplified one step further but it turns out better not to do that.

Combining with the case where the farmer t is not hit by any bandit, his gross income (G) when hiring a protector (subscript h) is

$$G_h(t) = (1 - t)\; e^{-(1-t_3)/t_2}\; \left\{\, 1 + \frac{1 - t_3}{2\, t_2}\, [\,1 + k\,(t_3 + t_2) - k\,(1 + t_3)\,]\, \right\}, \tag{1}$$

and his corresponding net income (I) is

$$I_h(t) = (1 - t)\; e^{-(1-t_3)/t_2}\; \left\{\, 1 + \frac{1 - t_3}{2\, t_2}\, [\,1 + k\,(t_3 + t_2) - k\,(1 + t_3)\,]\, \right\} - w\,. \tag{2}$$

For given candidate-equilibrium magnitudes t_1, t_2, t_3 and w, $G_h(t)$ is just a constant multiple of $(1 - t)$, and $I_h(t)$ is a vertically parallel downward shift of $G_h(t)$.

Self-Protecting Farmer

Suppose the farmer of type t tries self-protection, dividing his time into fraction y spent on protection and $(1 - y)$ on production. His output is $(1 - t)(1 - y)$. The probabilities of being hit by zero or one bandit are the same as those in the above calculations for full-time farmers. If hit by the bandit of type x_3, the probability that he defeats the bandit is

$$\tfrac{1}{2} + k\,(t\, y - x_3)\,.$$

Integrating (adding) over x_3, the probability of his keeping his output when hit by a random bandit is

$$\begin{aligned}
& \frac{1}{t_2}\, e^{-(1-t_3)/t_2} \int_{x_3=t_3}^{1} \left[\tfrac{1}{2} + k\,(t\, y - x_3)\right] dx_3 \\
= \;& \frac{1}{t_2}\, e^{-(1-t_3)/t_2} \left\{\, [\tfrac{1}{2} + k\, t\, y]\,(1 - t_3) - \tfrac{1}{2}\, k\,[\,1 - (t_3)^2\,]\, \right\} \\
= \;& \frac{1 - t_3}{2\, t_2}\, e^{-(1-t_3)/t_2}\, [\,1 + 2\, k\, t\, y - k\,(1 + t_3)\,]\,.
\end{aligned}$$

Combining with the case where the farmer is not hit by any bandit, his net (equals gross) expected income (I) when self- protecting (subscript s) with fraction y of his time is

$$I_s(t,y) = (1-t)\,(1-y)\; e^{-(1-t_3)/t_2} \;\left\{\, 1 + \frac{1-t_3}{2\,t_2}\; [\, 1 + 2\,k\,t\,y - k\,(1+t_3)\,] \,\right\} . \tag{3}$$

He chooses y to maximize this. Abbreviate the expression in the brackets on the right hand side as $A + B\,y$, where

$$A = 1 + \frac{1-t_3}{2\,t_2}\; [\, 1 - k\,(1+t_3)\,] \quad \text{and} \quad B = \frac{1-t_3}{2\,t_2}\; 2\,k\,t\,.$$

B is obviously positive, and $k \leq \frac{1}{2}$ ensures $A > 0$. The farmer wants to maximize

$$\phi(t,y) \equiv (1-y)\,(A + B\,y) = A + (B-A)\,y - B\,y^2\,.$$

This is a concave function, and $\phi(t,1) = 0$ so an optimum at $y = 1$ cannot occur. This leaves two possibilities. If $B \leq A$, which happens if $t \leq \widehat{t}$ where

$$\widehat{t} \equiv \frac{t_2}{k\,(1-t_3)} + \frac{1 - k\,(1+t_3)}{2\,k}\,, \tag{4}$$

then $\phi_y(t,0) \leq 0$ so we have a corner optimum at $y = 0$. If $B > A$, that is, $t > \widehat{t}$, then the optimum y is positive and is defined by the first-order condition $\phi_y(t,y) = (B-A) - 2B\,y = 0$.

Thus, when not using a professional protector, farmers with low t (high productivity in farming but low toughness) may choose to forgo protection and take their chances of escaping predators. Of course whether this happens in equilibrium will depend on the other parameters and functional forms used in the specification.

Use the notation

$$y^*(t) = \arg\max_y \;\phi(t,y), \qquad \phi^*(t) = \max_y \;\phi(t,y)\,.$$

Then

$$y^*(t) = \begin{cases} 0 & \text{if } t \leq \widehat{t} \\ (B-A)/(2B) & \text{if } t > \widehat{t}\,, \end{cases} \tag{5}$$

and

$$\phi^*(t) = \begin{cases} A & \text{if } t \leq \widehat{t} \\ (A+B)^2/(4B) & \text{if } t > \widehat{t}\,, \end{cases} \tag{6}$$

When $y^*(t) > 0$, the envelope theorem gives

$$\phi^{*\prime}(t) = \phi_t(t, y^*(t)) = [1 - y^*(t)]\,y^*(t)\,\frac{\partial B}{\partial t} = [1 - y^*(t)]\,y^*(t)\,k\,\frac{1-t_3}{t_2}\,.$$

The derivative is positive, and attains its maximum magnitude when $y^*(t) = \frac{1}{2}$. But (5) shows that $y^*(t)$ is an increasing function of t and is always less than $\frac{1}{2}$, so $\phi^{*\prime}(t)$ is positive and an increasing function of t, and always less than $k\,(1-t_3)/(4\,t_2)$.

Using this information, we can find the incomes of self-protecting farmers and compare them to those they would get if they hired professional protectors. For $t < \widehat{t}$ so $y^*(t) = 0$, the self-protecting farmer's income is

$$I_s^*(t) \equiv I_s(t,0) = (1-t)\, e^{-(1-t_3)/t_2} \left\{ 1 + \frac{1-t_3}{2\,t_2} \left[1 - k\,(1+t_3) \right] \right\}. \tag{7}$$

Comparing (1) and (7), we see that for $t < \widehat{t}$,

$$I_s^*(t) \,/\, G_h(t) = \text{constant} < 1\,.$$

For $t > \widehat{t}$, so $y^*(t) > 0$, the self-protecting farmer's income is

$$I_s^*(t) \equiv I_s(t, y^*(t)) = (1-t)\, e^{-(1-t_3)/t_2}\, [1-y^*(t)] \left\{ 1 + \frac{1-t_3}{2\,t_2} \left[1 + 2\,k\,t\,y^*(t) - k\,(1+t_3) \right] \right\}. \tag{8}$$

Then, comparing (8) and (1), we have for $t > \widehat{t}$:

$$\frac{I_s^*(t)}{G_h(t)} = [1-y^*(t)] \frac{1 + \frac{1-t_3}{2\,t_2} \left[1 + 2\,k\,t\,y^*(t) - k\,(1+t_3) \right]}{1 + \frac{1-t_3}{2\,t_2} \left[1 + k\,(t_3+t_2) - k\,(1+t_3) \right]}\,.$$

The right hand side is less than 1 when $t < t_2$ because then $2t < t_2 + t_3$. Also, when $t > \widehat{t}$, $I_s^*(t) > I_s(t,0)$ because zero is not the optimum choice of y in this range of t. Therefore in the range $\widehat{t} < t < t_2$ (assuming for the moment that this is a meaningful range; this will be justified in equilibrium), $I_s^*(t)$ is between the expressions (1) and (7).

Putting together all this information, the various incomes as functions of t are as shown Figure 1.

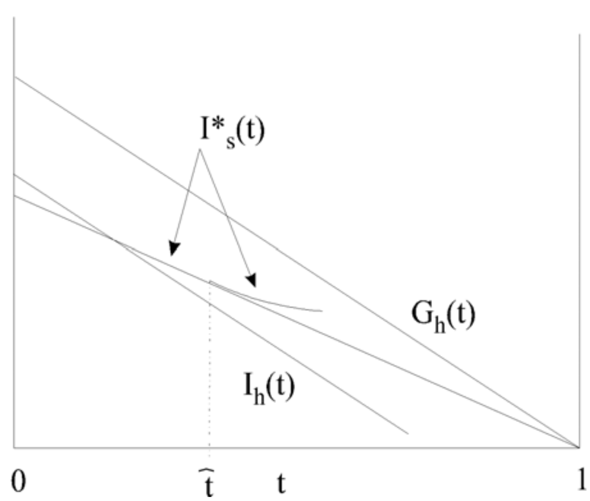

Figure 1: Farmers' Incomes with Different Choices of Protection

Professional protectors:

These earn the constant wage w. I assume that they serve their employers honestly, that is, they do not rob their employers either directly or in collusion with any bandits. I leave in the background any repeated games and reputation motives that explain this behavior. However, I will point out a specific problem about sustaining protectors' honesty in this model.

Bandits:

Consider a bandit with label t. He chooses a farm at random, and the probability that he picks any one farm of size Δt from the total interval $[0, t_2)$ is $\Delta t / t_2$. The probability that he is alone in this, that is, that the remaining $(1 - t_3 - \Delta t)/\Delta t$ bandits pick from the remaining $(t_2 - \Delta t)/\Delta t$ farms, is

$$\left(1 - \frac{\Delta t}{t_2}\right)^{(1-t_3-\Delta t)/\Delta t} \approx e^{-(1-t_3)/t_2} \, .$$

If the farmer is of type $x_0 \in [0, t_1)$, he will have hired a protector of type x_2 chosen randomly from $[t_2, t_3)$, with probability $\Delta t/(t_3 - t_2)$. In this event, the probability that the bandit gets the output is

$$\tfrac{1}{2} + k\,(t - x_2)\, .$$

If the farmer is of type $x_1 \in [t_1, t_2)$, he will a self-protector, devoting a fraction of his time $y^*(x_1)$ to protection. In this case the probability that the bandit gets his output is

$$\tfrac{1}{2} + k\,(t - x_1\, y^*(x_1)\,)\, .$$

If one or more of the other bandits hit the same farm as our t, they will overcome any protection and get and share the output. The expected income of t in these events is a complex expression, but independent of his own type. Call it K. Then, summing over all the possibilities, the bandit's overall income is

$$\begin{aligned} B(t) \;\; &= \;\; K + \int_{x_0=0}^{t_1} \frac{1}{t_2}\; e^{-(1-t_3)/t_2}\; (1 - x_0)\; \frac{1}{t_3 - t_2} \int_{x_2=t_2}^{t_3} \left[\tfrac{1}{2} + k\,(t - x_2)\right] dx_2\; dx_0 \\ &\quad + \int_{x_1=t_1}^{t_2} \frac{1}{t_2}\; e^{-(1-t_3)/t_2}\; (1 - x_1)\; \left[\tfrac{1}{2} + k\,(t - x_1\, y^*(x_1))\,\right] dx_1 \end{aligned} \tag{9}$$

This can be evaluated without much difficulty, but even without explicit evaluation it is clearly an increasing function of t. In equilibrium, K can be found more easily from an identity that puts the total expected income lost by all farmers equal to the expected income of all bandits.

Equilibrium:

The above calculations show that the slopes of the various types' incomes as functions of t are such that they must fit together as shown in Figure 2, justifying the structure of the candidate equilibrium. If we choose different initial values of t_1, t_2, t_3 and w, the curves will shift. Equilibrium is a fixed point in (t_1, t_2, t_3) and a w that achieves $t_1 = t_3 - t_2$. And K needs to be calculated from and income identity as explained above.

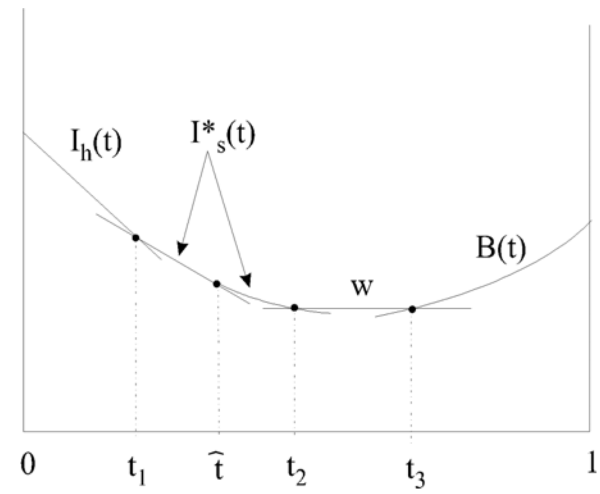

Figure 2: Occupation Choice and Equilibrium

This has the following implications. Some of them confirm or extend our intuition; others point to limitations of the model and lead to suggestions for modification or extension.

[1] The figure shows only the most general possibility. Some of the segments may be missing in equilibrium. For example, we may have $t_1 = \hat{t}$, when there are no farmers who take their chances and do without protection. Or we may have $t_1 = 0$ and $t_2 = t_3$, so there are no professional protectors. Alas, the case that cannot occur is an equilibrium without any bandits. If there were such an equilibrium, then there would be no professional protectors and no effort on self-protection either. But then the person close to $t = 1$, who would have near-zero output as a farmer, can do much better as a bandit, thus upsetting the purported equilibrium.

[2] The choice of productive occupations is governed in an obvious way by comparative advantage; the people with the lowest toughness t and highest agricultural productivity $(1 - t)$ concentrate on farming and hire professional protectors. Those in the next interval do without any protection, and the next ones divide their time into production and self-protection.

[3] It is also obvious that those with the highest t should choose banditry or the profession of providing protection against it. But the split between these two activities is not so obvious. In this model we find that the toughest become the bandits, and the guards are the next toughest lot. The reason is that any individual guard has no way of credibly signaling his toughness in this model. Therefore all guards must get the same wage, whereas a bandit's income increases with his t. If credible screening or reputation mechanisms for guards became available, this could change.

[4] This ordering of types also has the consequence that the professional protectors have lower incomes than either the people whose property they guard or the people against whom they guard these properties. Alas, this may broadly conform to the situation in the real world. The assumption that individual toughness is private information is crucial in generating this result. But it raises serious questions about my other assumption that guards act honestly.

Their low income must create a temptation for them to collude with bandits in robbing the farmers. Some repeated game mechanism, with efficiency wages as rewards for honesty or some device to punish dishonesty, may solve this problem. And repeated interaction should also ease the problem of the unobservability of individual toughness.

[5] While bandits have higher incomes than guards, and some bandits have higher incomes than some farmers, the richest bandit ($t = 1$) cannot be as rich as the richest farmer ($t = 0$), because the most a bandit can have is the certainty of robbing the richest farmer. This can change if one robber can hit more than one farm.

References

Anderson, James E. and Oriana Bandiera. "Private enforcement and social efficiency." Working paper, Boston College and London School of Economics, 2002.

Dixit, Avinash. *Lawlessness and Economics.* Princeton University Press.

Herschel I. Grossman. "Make us a king: Anarchy, predation, and the state." *European Journal of Political Economy,* 18, 2002, 31–46.

Hirshleifer, Jack. "Anarchy and its Breakdown." *Journal of Politcal Economy,* 105(1), February 1995, 26–52.