ECONOMIC GOVERNANCE 3 – FORMAL MODELS CONTD.

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4. Graduated Punishments

Based on work in progress, "Self-Enforcing Cooperation with Graduated Punishments"
by Dilip Abreu, Douglas Bernheim, and Avinash Dixit.
Available at `http://www.princeton.edu/~dixitak/home/wrkps.html`

Motivation

Theory and evidence on repeated games agree in most respects:
roles of patience, small stable group, accuracy of information, ...
One discrepancy – size of punishments
Most applied theory based on grim trigger punishments
General idea – harshest subgame-perfect punishment best
for sustaining tacit cooperation – Abreu, Fudenberg-Tirole
Where punishments vary through time, harsh phase first
mild phase / rewards later, for subgame perfectness

Empirical observations – (1) punishments are initially mild,
(2) are graduated in response to experience, and
(3) breaking off relationship is last resort, not first
Examples: Ostrom, Ellickson, "three strikes" laws

Imperfect monitoring can explain milder punishments
(Green-Porter, Abreu-Pearce-Stachetti), time-path (Rubinstein)
But [1] no learning, updating, [2] no actual cheating
Therefore better theory also needs asymmetric information
about incentives to cheat (type of the other player)

So combination of moral hazard and adverse selection.
Produces some complex, non-intuitive outcomes
Will present only two examples that illustrate the issues
Paper has a few more results in special cases, conjectures

Basic structure of our model

One-sided prisoner's dilemma game (Williamson, Greif).
Typical example – Each period, A can invest in B's enterprise;
then B can behave opportunistically

Stage game specification

1. Two players, A and B. A decides whether to invest.
   If he doesn't, outside payoffs a for A, 0 for B.
2. If A invests, B has choice between Generous/Honest and Selfish/Opportunistic.
   B's expected extra payoff from Selfish is his "temptation" y.
3. A's payoff can be $c > a$ or $0 < a$. That's all A observes.

$$\begin{aligned} \mu &= \text{Prob}[\text{A gets low payoff} \mid \text{B takes selfish action}], \\ \lambda &= \text{Prob}[\text{A gets low payoff} \mid \text{B takes generous action}], \end{aligned}$$

where

$$0 < \lambda < \mu \leq 1$$

4. A commits to a punishment rule: If he gets 0 payoff,
   that will trigger costly punishment reducing both payoffs: A's by $\gamma\, y$
   and B's by y (so this is the maximum temptation A can deter).
   (Commitment is assumed; we know how to endogenize it so leave out)

5. Player B's types parametrized by θ.
   A has prior distribution on θ; does Bayesian updating across plays.
6. Two cases:
   Fixed temptation: $y \equiv \theta$; CDF of prior over θ's is $F(\theta)$
   Stochastic temptation: y random; CDF $F(y, \theta)$ over $[0, \overline{y}]$
     B observes realization of y before choosing action
     $F_\theta < 0$ for all $y \in (0, \overline{y})$ (larger $\theta \Rightarrow$ worse type)
     (Of course $F(0, \theta) = 0$ and $F(\overline{y}, \theta) = 1$ for all θ.)
     And common-knowledge prior over types θ.
7. Drastic simplification here: assume that both players have
   perfect memory but infinite impatience ($\delta = 0$).
   So focus on updating of prior about types.

   (Can't have $\delta = 0$ when commitment is endogenized,
   so will need a more complex model. Also other complexities
   when better dynamic model; this is work in progress.)

Fixed temptation case

Given A's choice of y, we have
Prob[B chooses generous] $=$ Prob[$\theta < y$] $= F(y)$
Write $L(y) =$ Prob[A gets low payoff] $= \lambda\, F(y) + \mu\, [1 - F(y)]$

Therefore A chooses y to maximize

$$\begin{aligned} V(y) &= c\,(1 - L(y)\,) + (-\gamma\, y)\; L(y) \\ &= c - (c + \gamma\, y)\; L(y) \\ &= c - (c + \gamma\, y)\; \{\; \lambda\; F(y) + \mu\, [1 - F(y)]\; \} \end{aligned}$$

Next period, A will revise the prior and pick new y
First intuition: revise y upward after bad experience
and downward after good experience. But this is not correct!

Correct result:

y is not revised at all (downward or upward) after good outcome;
after a bad outcome it may be revised upward or downward.

Correct intuition:

Suppose y^* is optimal in period 1, and outcome is good.

Then Bayes' formula shows that in the posterior distribution,

likelihood of all $y < y^*$ shifts up, multiplied by constant factor $k > 1$,

likelihood of all $y > y^*$ shifts down, multiplied by constant factor $h < 1$.

Next period, if punishment is reduced to $y' < y^*$, three effects on A's payoffs:

$E_1 > 0$ because unnecessary punishment not inflicted on B's $\in (0, y')$

$E_2 < 0$ because fails to deter B's with temptations between y' and y^*

(so higher probability and cost of punishing in this range)

$E_3 > 0$ because ineffective punishments on B's beyond y^* cost less

For the prior, y^* was optimal, so $E_1 + E_2 + E_3 < 0$

In the posterior, effect of reducing punishment from y^* to y' is

$$k\ E_1 + k\ E_2 + h\ E_3 = k\ (E_1 + E_2 + E_3) - (k - h)\ E_3 < 0$$

so can't do better by reducing y^*. Similar argument for increase.

If previous optimum was regular, now a kink at y^*

Reversing this argument, if A gets the low payoff, then next period desirability of both increasing and decreasing the punishment goes up, and the balance is unclear, so the optimal y^* can go up or down.

Example where punishment goes down after a bad experience:

Parameters $c = 12$, $\gamma = 1$, $\mu = 1$, $\lambda = 0.5$
Types (temptations) $\theta = 1$, 6, 12; Prior probs 0.25, 0.50, 0.25

First-period decision:

A's choices of y	1	6	12
$1 - L =$ Prob[high payoff to A]	0.125	0.375	0.500
$L =$ Prob[low payoff to A]	0.875	0.625	0.500
$V = c - (c + \gamma\, y)\, L$	0.625	0.750	0.000

The optimum is $y = 6$, which deters the two lower types.
(Breaking B's indifference in A's favor.)

Suppose A does this and experiences the low payoff.
Ex ante, this would happen with probabilities
$0.5 \times 0.25 = 0.125$, $0.5 \times 0.50 = 0.25$, and $1 \times 0.25 = 0.25$
from the three types. Total probability of this $= 0.625$

So posterior probabilities are

0.125/0.625 = 0.2, 0.250/0.625 = 0.4, 0.250/0.625 = 0.4

Comparing posterior and prior, observe

$(0.2, 0.4) = 0.8 * (0.25, 0.5)$, and $0.4 > 0.25$.

Second-period decision:

A's choices of y	1	6	12
$1 - L =$ Prob[high payoff to A]	0.100	0.300	0.500
$L =$ Prob[low payoff to A]	0.900	0.700	0.500
$V = c - (c + \gamma\, y)\, L$	0.300	-0.600	0.000

The optimum is $y = 1$, which deters only the lowest types.

and $y = 6$ has become the worst choice

(if $a > 0.300$, terminate relationship)

New intuition: If bad experience raises prob of very bad type,

it may cause A to accept more cheating (here, deterring 12 is too costly),

or in worst-case situation, to give up on this game.

Stochastic temptation case

A's optimal choice of y is to maximize $V(y) = c - (c + \gamma\, y)\; L(y)$
where $L(y)$ is now an expectation over B's types etc.

Problem: Multiple local optima, discontinuous comparative statics.
Example: Distribution of temptation y conditional on θ is normal
Two types; means 4 and 6, standard deviations 0.14 each
Other parameters $c = 10$, $m = 0.5$, $\mu = 1$, $\lambda = 0.5$, so $\gamma = 1$
Probability of bad type π varies
Table shows global maximizers y^* and maximized values $V(y^*)$
plus one further local but not global max in ( ) on each side

π	Lower y^*	$V(y^*)$	Upper y^*	$V(y^*)$
0.050	4.380	2.426		
0.100	4.374	2.068		
0.130	4.371	1.853	(6.267)	(1.836)
0.135	(4.370)	(1.818)	6.270	1.834
0.150			6.278	1.831
0.200			6.298	1.822

[1] Discontinuity and bifurcation: $y^*(\pi)$ is
decreasing along lower branch, increasing along upper branch

[2] Both optima are 2 or more standard deviations above
the corresponding mean: deterrence almost complete when attempted

[3] Best value $V(y^*)$ is decreasing convex function of π
This can be proved rigorously and with more generality

[4] Therefore outside opportunity taken when π goes
above a critical value: breakdown is last resort, not first

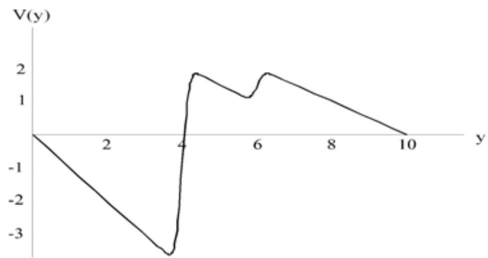


Figure 1: $V(y)$ when $\pi = 0.133$, bifurcation point

Evaluation – Achievements

Explains some basic facts:
Begins to make sense of empirical findings in a rich context:
[1] allows heterogeneous types and some cheating in equilibrium,
[2] strategy includes concept of testing for cause of bad outcome,
[3] clear result that breaking off relationship should be last resort,
[4] finds new possibility and intuition for it.

Shortcomings and Research Opportunities

Examples rather than general theory. Should generalize to allow testing over multiple periods, finite impatience etc.

Other Related Literature

Susan Athey and Kyle Bagwell. 2001. "Optimal Collusion with Private Information." *Rand Journal of Economics* 32:428-465.

A. Mitchell Polinsky and Steven Shavell. 1998. "On Offense History and the Theory of Deterrence." *International Review of Law and Economics* 18: 305-324.

5. Private Protection of Property Rights

Motivation

Literature has models where occupations are exogenous, or else
people are ex ante identical and divide time between production,
protection of one's own property, and predation on others.
In reality, heterogeneous population, different comparative advantages
in the three activities; so endogenous specialization, occupation choice.
For example, Bandiera finds the origins of the Sicilian Mafia when
landlords began to hire the toughest bandits as guards.
Outlining my preliminary, defective model to spur ideas, improvements.

Assumptions of the Model

Continuum of population indexed by $t \in [0, 1]$, uniform.
Type t has fighting toughness t, farm productivity $(1 - t)$
Occupation choices: [1] Full-time farmer: Output $(1 - t)$. Hires guard at wage w
[2] Self-Protecting farmer: Time fraction y to guard; output $(1 - t)(1 - y)$
[3] Professional protector: Gets wage w. Type t private information
[4] Bandit: Chooses farm to hit at random, not knowing its protection status

Fighting technology:
If farm protected with toughness t' is hit by single bandit t,
Prob(Bandit is defeated) $= \frac{1}{2} + k\,(t' - t)$
If multiple bandits hit, they get all output and share equally.

Comments: [0] All this is *very* incomplete, tentative.
[1] Assumption is convenient because bandit's payoff in case of simultaneous attack is independent of his type. But can vary, e.g. bandits fight and share proportional to t or toughest takes all, or the first bandit to reach the farm is the only possible taker.
[2] Better information may be available, e.g. is a farm protected, and what is the toughness of the protector? (Protectors – and bandits – can build reputation for toughness, announce protection using decals.)
[3] Fights may destroy fraction of output. And many other variants.

Lemma: When F farmers and B bandits, both $>> 1$, for any one farmer

$$\text{Prob[No bandit]} \approx e^{-B/F}, \qquad \text{Prob[One specific bandit]} \approx \frac{1}{F}\; e^{-(B-1)/F}$$

Then can calculated expected incomes from various strategies. See paper.

Candidate equilibrium in most general case:

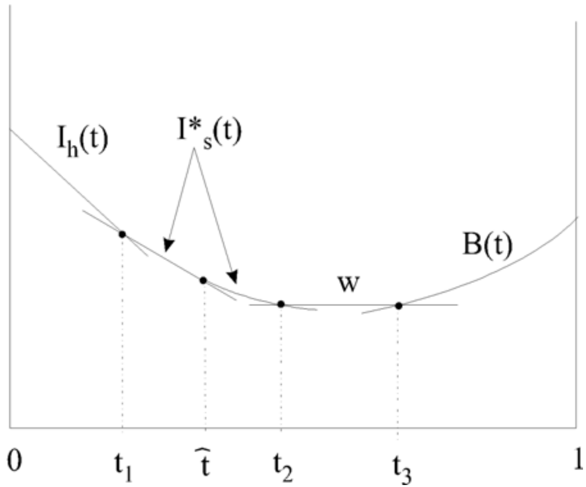
Full-time farmers: $0 \le t < t_1$

Unprotected farmers: $t_1 \le t < \widehat{t}$ ("self-protecting" with $y = 0$)

Self-protecting farmers: $\widehat{t} \le t < t_2$

Professional protectors: $t_2 \le t < t_3$

Bandits: $t_3 \le t \le 1$



Intuition: [1] In first two intervals, farmer's retained output
$= (1-t)$ * Probability of retention; Prob. higher in first interval.
[2] When self-protected, Prob. of retention rises with t;
that is why the income graph is convex.
[3] Protector's type unobservable so w independent of t;
this is crucial for the way the occupational choices split up.

Problems: [1] Only the "second-toughest" bandits become guards.
[2] Professional protectors have lowest incomes; this may alas be realistic,
But then their assumed honesty becomes questionable.

Other Related Literature

Barzel, Yoram. 1989. *Economic analysis of property rights.* Cambridge University Press.

Grossman, Herschel I. and Minseong Kim. 1995. "Swords or Ploughshares? A Theory of Security of the Claims to Property." *Journal of Political Economy* 103: 1275-1288.

Garfinkel, Michelle and Stergios Skaperdas. 2006. "Economics of Conflict: An Overview." University of California-Irvine, Department of Economics, Working Paper 050623.