MASTER THESIS

Endogenous Job Search in Heterogeneous Households Model

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Abstract

The paper examines incomplete-market heterogeneous households model with capital accumulation (Aiyagari, 1994) and labor market frictions. Namely, firms and workers are matched bilaterally, the labor market has Diamond–Mortensen–Pissarides structure and unemployed individuals seek for open vacancies choosing search effort endogenously. Since individual risks are not insured, search effort and unemployment duration crucially depends on a history of individual shocks, that differs between groups of consumers, so the model allows to investigate non-trivial distributional, wage and employment effects of different unemployment insurance programs and fiscal policy in general. Numerical evidence suggests the substantial impact of search variation on the degree of the wealth inequality and the overall importance of the search elasticity rather than the scale of search costs for the shape of the aggregate wealth distribution.

Contents

1 Introduction

The work develops heterogeneous agents general equilibrium framework with labor market frictions, accounting for differential search effort, and, as a consequence, varying unemployment duration. The detailed solution strategy is developed and tested here. However heterogeneous agents models with labor market frictions are currently emerging, and I base my study on one of them presented in [3], the modeling of differential search intensity has not been conducted yet in published papers. The only attempt was taken recently in unpublished work [6], but now it contains only some preliminary analysis of the highly simplified framework and does not present complete dynamic model or solution procedures. Although several important results are presened here, I see the main contribution of the work as methodological: it opens several directions for the further research, allowing to analyze non-trivial consequences of labor market policies, design of unemployment benefits and taxation system.

The computational evidence suggests a number of regularities, added to the basic framework by accounting for search effort. First, the welfare inequality characteristics are shown to be rather elastic with respect to search effort variability, that may serve as an indirect evidence of substantial impact of search variety on aggregate distribution shape. When search becomes more differential, the left tail of welfare distribution becomes heavier, that means an increase of the share of low-welfare agents, and, additionally, the inequality measures like Gini coefficients tend to increase. The plausible mechanism for this effect is described here.

Second, there is an additional impact of the search variability onto wage inequality and aggregate earnings distribution. However, the direction of the effect depends on the methodology of comparison: increase in search variety per se tends to make wages distribution more flat, however, small change of search elasticity induces large shift in vacancy-unemployment ratio. Adjusting vacancy costs to match target labor market outcomes, as the literature suggests, leads to the result that an increase in the search variability increases wage inequality. This issue worth additional consideration to justify the right direction of the relation, since the correct methodology of comparative statics for heterogeneous agents models is a non-trivial issue.

Third, the work suggests that the elasticity rather then the value of search costs matters for aggregate distribution, the scale effects are rather small if they even exist given the numerical approximation errors. That justifies consideration of varying search effort in the model framework rather than simple accounting for the search costs.

However, there is a negative result of the work: given the setting present here, it cannot accommodate all observable unemployment heterogenity. According to, for example, [4], the average unemployment duration interquartile variation is about 5-8 times, and to capture that in the current model, we need extremely persistent shocks of job status, that is not observed in the reality. Being more precise, the model explains the search variation by income and wealth variability, but simple changes in job stauts cannot provide enough of that: we need additional sources of workers' heterogenity. However, incorporating individual productivity shock together with labor market frictions is expected to resolve this contradiction, although it complicates the model, it could, in principle, account for them. Taking into account fact of the limited heterogenity here, I do not rely on quantitative predictions of the model and focus on general qualitative trends, however, further investigation is needed to justify whether these trends are important in models with enough shocks.

The structure of the rest of the work is the following. Section 2 contains full description of the theoretical setting I use here, section 3 discusses the solution strategy and contains the description of the algorithm I develop here, section 4 described calibrated parameter values, section 5 describes the how current model could replicate the literature result with fixed search efforts, section 6 shows the impact of search effort variation in several alternative comparative statics approaches and section 7 concludes. Appendix provides several additional details of theoretical framework.

2 Model

The current model combines several theoretical building blocks. As a baseline model I use Krussel–Mukoyama–Sahin work [3], that refers to classic bilateral matching scheme described by [7]. I changed some of notations and assets timing for my own convenience. Simplified part of the model uses [5] concept of union bargaining. The closest to mine is an unpublished work [6], the author also analyze varying job search intensity impact in the presence of borrowing constraint. However, there are several crucial differences of my work for it. First, Obiols-Homs, the author provides theoretical analysis of two-period model instead of computational analysis of infinite horizon model I present here. Second, the author uses different search cost concept assuming them to be monetary, and I use search costs in terms of utility, that allows for more flexible calibration of the labor market parameters. Third, the author uses flat wage scale, and I also include varying wage for comparison. In this way this work is partially complimentary to the existing studies.

2.1 Outline

The structure of the model is conventional for the search and match general equilibrium framework. There is a unit measure of ex-ante identical consumers and two individual states: employed and unemployed. Employed consumer inelastically supplies one unit of labor to the firm at some wage, unemployed agent receives unemployment benefits from the government and searches for the job, both agents also save their funds in the risk-free assets and cannot borrow up to the certain limit. Markets are incomplete: there does not exist state-contingent assets and agents cannot insure their employment risks.

The firm sector is rather simple: there is some measure of identical firms, each firm can either have a worker or a vacancy that could be filled, holding vacancies are costly so the job search frictions determine the number of firms with vacancies. Unlike [3] I assume firms to pay lump-sum taxes, that allows to get rid of non-trivial taxation effects for consumers that are constrained in borrowing.

The labor market randomly matches unemployed workers to the firms with vacancies, the probability of being matched is proportional to worker's search intensity. Each period each firm-worker pair (both newly matched or already existent) negotiate on the wage amount. That creates an individual risk for the firm: workers with higher assets ask for larger wages and decrease their expected profits. For methodological simplicity I assume that firms are able to pool their risks, or, equivalently, workers are randomly mixed among firms each period, so after in dynamics firm's expected future profits depend on average future wage but not on individual wage determined in the current period.

Finally, government has the balanced budget: the amount of lump-sum taxes collected from operating firms is equal to the sum of benefits to unemployed agents. Therefore, we do not need government securities, and there are two types of riskless assets in the economy: firm's stocks and physical capital, that are assumed to be the perfect substitutes, so we care only about the total assets holdings of each consumer.

The following subsection describes the full theoretical structure of the model.

2.2 Definitions

Let i denotes the identifier of the consumer and denote by $\Omega = [0;1]$ the set of all households, let μ be usual additive measure such that $\mu(\Omega) = 1$. Let Ω_u Ω_e be sets of unemployed and employed agents, $\mu(\Omega_u) = u$ and $\mu(\Omega_e) = 1 - u$, respectively.

Formally, each of heterogeneous consumers is characterized by the pair $\{a_i, o_i\}$, where $a_i \in \mathcal{A} = [-B; +\infty)$ is assets and $o_i \in \{e, u\}$ is individual state of being employed and unemployed. For convenience I will try to avoid integration by the number of agent i, since neither a_i nor o_i is a continuous functions, so I will order the agents in each states by their assets and introduce $f_e(a)$ and $f_u(a)$, that are shares (densities) of agents with each asset holding in each state. As above, now $\int_{\mathcal{A}} f_e(a)da = 1 - u$, $\int_{\mathcal{A}} f_u(a)da = u$, we also define $f(a) = f_e(a) + f_u(a)$, that integrates to one. Note within their assets level a and state o agents are assumed to be homogeneous, therefore pair $\{a_i, o_i\}$ defines the set of i's. Corresponding marginal densities I denote as $\frac{f_e(a)}{1-u}$ and $\frac{f_u(a)}{u}$, both of them integrate to one.

2.3 Matching and labor market

Each period v firms hold vacancies and u unemployed workers seek for a job with different intensity $s(a)$, consider, for example, that each agent posts s_i job ads per period. The number of worker-firm matches m, as in [7], is a continuous function of the total number of ads $S = \frac{1}{u}\int_{\mathcal{A}} s(a) f_u(a) da$ and the number of vacancies v. Denote by $\bar{s} = \frac{S}{u}$ average search intensity, so we have

$$m = m(\bar{s}u, v),$$

as usual, I assume m to be concave and with constant returns to scale. To avoid trivial cases, we assume that the functional forms are chosen such that given each calibration

$$s \in [0;1], \quad 0 \leq m(\bar{s}u, v) \leq \min\{u, v\}.$$

I introduce the following notation in the further sections. Let $\theta = \frac{v}{\bar{s}u}$ be a market tightness parameter, that is a ratio of the number of vacancies to the number of (effective) unemployed. Given CRS nature of matching function, the probability to fill the vacancy $\frac{m}{v}$ is

$$\lambda_f(\theta) \overset{\text{def}}{=} \frac{m}{v} = \frac{m(\bar{s}u, v)}{v} = m\left(1/\theta, 1\right), \quad \frac{\partial \lambda_f(\theta)}{\partial \theta} < 0,$$

and the probability to find a job for each ad is

$$\lambda_w(\theta) \overset{\text{def}}{=} \frac{m}{\bar{s}u} = \frac{v}{\bar{s}u} \cdot \frac{m}{v} = \theta \lambda_f(\theta), \quad \frac{\partial \lambda_w(\theta)}{\partial \theta} > 0,$$

and the probability to find a job for a worker with s_i is $s_i \lambda_w(\theta)$.

Also, let σ denotes the probability to lose the job in each period. Given the measures described about, for each period the law of motion for the stock of unemployed agents is

$$u' = u + \sigma(1-u) - \bar{s}\lambda_w(\theta)u,$$

note that since the matching is random the average search intensity of matched workers is assumed to be equal to the average search intensity of all workers. Under this assumption, there is a steady-state unemployment level where $u' = u$ and $\bar{s}$ is constant, that is

$$u = \frac{\sigma}{\sigma + \bar{s}\lambda_w(\theta)}. \tag{1}$$

2.4 Hoseholds

Again we assume a unit continuum of ex-ante identical households. Each employed consumer earns individual wage w_i, holds a amount of risk-free asset and spends c on consumption and leaves a' assets for the next period. Each period she losses the job with a probability of σ, becoming unemployed in the consequent period. Unemployed consumers receive fixed government benefit of h, and seeks for a job with intensity of s. Search are costly in terms of utility, there are convex search costs $g(s)$. Given the search intensity, the agent becomes employed in the consequent period with probability $s\lambda_w(\theta)$.

When worker is employed, all matched agents with assets are are identical, so their bargaining outcomes w_i are identical, I denote corresponding wages as $w(a)$. Let β be a constant discount factor, and $W(a)$ and $U(a)$ are value functions, corresponding to employed and unemployed agents respectively.

The employed consumer value function is given by

$$\begin{cases} W(a) = \max\limits_{c,a'} \{u(c) + \beta\left[(1-\sigma)W(a') + \sigma U(a'),\right]\} \\ c + a' = (1 + r - \delta)a + w(a), \\ c \geq 0, \quad a' \geq -B, \end{cases} \tag{2}$$

where B is the borrowing limit. Note that r here is the gross capital return rate, and given capital depreciation, $r - \delta$ is the net return that asset holders get.

Now the value function of the unemployed worker is

$$\begin{cases} U(a) = \max_{c,a'} \left\{ u(c) - g(s) + \beta \left[s\lambda_w(\theta) W(a') + (1 - s\lambda_w(\theta)) U(a') \right] \right\}, \\ c + a' = (1 + r - \delta)a + h, \\ c \geq 0, \quad a' \geq -B, \end{cases} \tag{3}$$

the functional form assumptions are $u(c) = \frac{c^{1-\psi}}{1-\psi}$ for $\psi \neq 1$, and $u(c) = \log c$ for $\psi = 1$, so ψ is a risk aversion measure, and $g(s) = \xi \frac{s^{1+1/\phi}}{1+1/\phi}$, so ξ is a scale parameter of importance of the search for the consumer and ϕ is (inverse) elasticiy of marginal search effort, that accounts for search variation among the consumers. See A.1 in Appendix for additional notes on value function forms. Problem 2 results policy functions $(c_e(a), a'_e(a))$, problem 3 results $(c_u(a), a'_u(a), s(a))$.

2.5 Firms

Here I describe the complete framework of firms' problems, I also applied two simplifications to the framework reducing numerical complexity of the model: I did not find them affecting qualitative results. See Appendix A.2 for discussion of them.

Firms with workers

The firm can have two states: either have a worker or a vacancy. Denote by V value of the firm with vacancy and by $J(a)$ value of the firm hired a worker with on a wage a. Each firm has one worker, so the number of working firms is $1 - u$. Firms use common monetary discount factor of $q = \frac{1}{1+r-\delta}$.

The given the wage function $w(a)$, and consumer's decision rule full value function of the firm with the worker should be

$$J(a) = \max_k \left\{ f(k) - rk - w(a) - t + q\left((1-\sigma)J(a'_e(a)) + \sigma V\right) \right\}, \tag{4}$$

so, the firm produces $f(k)$, pays lump-sum tax of t and can lose the worker with probability σ. First order conditions by capital is just

$$f'(k) = r, \tag{5}$$

Firms with vacancies

The problem is simple: firm holding a vacancy has a real costs of ζ and does not make any other decision. The volume of such firms is v. The value function for them is

$$V = -\zeta + q\left((1-\lambda_f(\theta))V + \lambda_f(\theta)\frac{1}{u}\int_{\Omega_u} J(a'_u(a_i))di\right), \tag{6}$$

where the $\frac{1}{u}\int_{\mathcal{A}} J(a'_u(a))f_u(a)da$ expression stands for average next-period assets of new matched worker. When $V \neq 0$, new vacancies are either created or destroyed, so in equilibrium we use free-entry condition $V = 0$, this way

$$0 = -\zeta + q\lambda_f(\theta)\frac{1}{u}\int_{\mathcal{A}} J(a'_u(a))da \tag{7}$$

must hold in the equilibrium.

2.6 Wage setting

To have an incentive to create vacancies we employ Nash Bargaining at the stage of wage determination. Bargaining takes place every time-period, so wage level does not persist. For numerical simplicity we abstract from intertemporal nature of firm employment decisions, the numerical evidence suggests that it does not change crucial features of results (see A.2 for discussion). Therefore, we assume that the firm cares about current profit $\pi = f(k) - rk - t - w$.

In contrast, since we analyze the persistence of the unemployment, we have to care about the intertemporal nature of household's decision: current wage setting affects current earnings, thus it changes savings and future assets position (so we have to compute again value and policy functions for each of suggested wages). Given optimal value functions from 3 and 2 (derived for optimal wage scale $w(a)$) household bargaining value function is

$$\tilde{W}(w,a) = \max_{a'} \left\{ u((1+r-\delta)a + w - a') + \beta \left[\sigma U(a') + (1-\sigma) W(a') \right] \right\},$$

note that continuation value does not depend on current bargaining decision (since renegotiation happens every period). In case of disagreement, firm is left with the vacancy corresponding to one-period profit of $-\zeta$ and the worker is left unemployed with the optimal value function $U(a)$. Taking into account these details, I define optimal wage function $w(a)$ as

$$w(a) = \operatorname*{Argmax}_{w} \left[f(k) - kf'(k) - t - w + \zeta \right]^{1-\gamma} \left[\tilde{W}(w,a) - U(a) \right]^{\gamma}, \qquad (8)$$

where γ is the bargaining power of each worker, note that since $W(a)$ and $U(a)$ depend on $w(a)$, this is non-trivial functional fixed-point problem, however, iterative optimization resolves it quite fast.

Additionally, for the specification with flat wage scale I assumed the following uniform wage determination (repeating the logic of [5]): there is a "labor union", behaving as an agent who holds average amount of assets in the economy $\bar{a} = \int_{\mathcal{A}} a(f_e(a) + f_u(a))\, da$, $w(a) = w\ \forall a$ and

$$w = \underset{w}{\text{Argmax}}\, [f(k) - kf'(k) - t - w + \zeta]^{1-\gamma} \left[\tilde{W}(w, \bar{a}) - U(\bar{a})\right]^{\gamma} \tag{9}$$

2.7 Government

In contrast with [3], I assume that h is not a result of home production process, but fully covered by lump-sum taxes from operating firms. This way, number of firms working $1 - u$ pays taxes t to cover number u of unemployment benefits, I assume that government does not have an access to asset markets (since Ricardian equivalence does not hold here it may create additional distortions), so balanced budget condition is

$$uh = (1 - u)t, \tag{10}$$

I assume that tax level adjusts every period to match target level of h, so $t = \frac{u}{1-u}h$ holds.

2.8 Assets and balances

The asset structure is the following: agent i can hold either physical capital k_i or any share in the firm x_i, the total number of shares is normalized to be one: $\int_{\Omega} x_i di = 1$, and share price is p (this is a price after dividends are paid). Here p is a total share price of all firms in the economy, including firms with vacancies. Individual assets balance is, in this case

$$a_i = k_i + x_i p,$$

integration (by agents) implies (the equivalent integration by assets is used)

$$\int_{\mathcal{A}} a(f_e(a) + f_u(a))da = (1-u)k + p, \tag{11}$$

note that k is an amount of capital per firm and $1-u$ is the number of firms, so total $\int_{\Omega} k_i di = (1-u)k$.

Since p is fair stock price, no arbitrage conditions will hold, recall that $q = \frac{1}{1+r-\delta}$ is a monetary discount factor, so $p = \sum\limits_{t=1}^{\infty} q^t d$, thus

$$p = \frac{1}{r-\delta} d, \tag{12}$$

see that $r > \delta$ is crucial. Assuming equal dividends are paid every period, total dividends must be equal to total period after-tax profits of all firms (firms with vacancies have losses, I also take them into account), so

$$d = \int_{\mathcal{A}} \left[f(k) - rk - t - w(a) \right] f_e(a)da + v(-\zeta),$$

also denote average wage as

$$\bar{w} = \frac{1}{1-u} \int\limits_{\mathcal{A}} w(a) f_e a da, \tag{13}$$

and rewrite it as

$$p = \frac{1-u}{r-\delta}\left[f(k) - rk - t - \bar{w}\right] - \frac{v}{r-\delta}\zeta, \tag{14}$$

in the manner of online-appendix B[1] of [3] it can also be established that, in my notation, in equilibrium with stationary unemployment rate and stationary assets distribution condition

$$p + d = \int_{\mathcal{A}} J(a) f_e(a) da,$$

holds, however, this equation is never used in computations, so I did not replicate the proof here — it is only of methodological interest.

To close the model, the product balance in stationary (with fixed k) equilibrium is the following:

$$(1-u)f(k) = c + \delta(1-u)k + v\zeta, \tag{15}$$

where we used a notation

$$c = \int\limits_{\mathcal{A}} c_e(a) f_e(a) da + \int\limits_{\mathcal{A}} c_u(a) f_u(a) da, \tag{16}$$

that is the total production amount $y = (1-u)f(k)$ is spent on total consumption, investment (equal to depreciation in steady state) and vacancies. Appendix B verifies that suggested balances are consistent with individual and governmental budget constraints.

[1]See http://www.ny.frb.org/research/economists/sahin/KMS_finalappx.pdf

2.9 Definition of equilibrium

I use the concept of recursive stationary equilibrium, common in heterogeneous agents framework for the models with physical capital starting from [1].

We define an equilibrium as a set of value functions $\{W(a), U(a), J(a), V\}$, decision rules $\{c_e(a), a'_e(a), c_u(a), a'_u(a), s(a)\}$, prices $\{w(a), r, p\}$, capital stock k, market tightness θ, unemployment u, taxes t and the distribution μ, that is a pair of densities $\{f_e(a), f_u(a)\}$ assigning density of agents of each employment status to each asset level a.

There are several requirements for these objects to be an equilibrium:

1. $\{c_e(a), a'_e(a), c_u(a), a'_u(a), s(a)\}$ resolve consumer problems 2 and 3 given the distribution of μ, prices $\{w(a), r, p\}$ and market tightness θ for each a, and $W(a)$ and $U(a)$ are corresponding value functions.

2. k resolves producer problem 4 given the prices $\{w(a), r, p\}$, taxes t and market tightness θ, $J(a)$ is corresponding value function.

3. Given θ, u and value function $J(a)$ the number of vacancies $v = \theta \bar{s} u$ is such that free-entry condition holds: $V = 0$.

4. The budget is balanced, that is 10 holds.

5. $w(a)$ resolves the bargaining problem 8.

6. Stock price is determined through 14 (that is dividends are paid according to total one-period profits).

7. Given the decision rules $\{a'_e(a), a'_u(a), s(a)\}$ pair $\{f_e(a), f_u(a)\}$ defines stationary distribution density of ergodic Markov chain on $\mathcal{A} \times \{e, u\}$. In particular, this

implies that

$$\int_{\mathcal{A}} a'_e(a)f_e(a)da + \int_{\mathcal{A}} a'_u(a)f_u(a)da = \int_{\mathcal{A}} a(f_e(a) + f_u(a))da, \tag{17}$$

that means that total amount of assets is unchanged, and determines u directly from 1 given $\bar{s}$ defined above.

8. The product balance

$$(1-u)f(k) = \int\limits_{\mathcal{A}} c_e(a)f_e(a)da + \int\limits_{\mathcal{A}} c_u(a)f_u(a)da + \delta(1-u)k + v\zeta,$$

is satisfied by $\{c_e(a), c_u(a)\}$ and k, this fact together with the invariant distribution implies that aggregate capital stock (and, consequently, stock price and dividends) does not change over time.

9. Invariant distribution, u, k and p satisfies asset market equilibrium condition:

$$\int_{\mathcal{A}} a(f_e(a) + f_u(a))da = (1-u)k + p,$$

3 Solution strategy

Here I describe the solution strategy I applied to find an equilibrium. When the simple models like [1] are solved, the only aggregate price parameter of interest rate r was chosen to balance the assets market. In our case, it is not implementable: we have three dimensions of prices $\{w(a), r, p\}$, one of them is function and price p is determined through the wide set of variables we have to choose an initial values, so the direct search for prices is not the best way here.

Therefore, I used more convenient for search combination $\{w(a), k, \theta\}$, given that combination, value functions and decision rules could be determined, given the decision rules, invariant distribution and u could be found, then aggregate things like p and u could be computed. Although [3] assert that solution is quite fast, there is at least three dimensions of the search and one of them is a function, do the rate of convergence is rather slow, moreover, the direction is non-trivial: when initial guess is far from stationary distribution, divergence in some of three dimensions may happen.

The solution strategy is a mix of conventional computational macroeconomics techniques with several problem-specific adjustment. MATLAB codes was used to obtain the solution, codes are available on demand. The search is based on the uniform grid of G elements for assets, $a \in \mathcal{A} = [-B; \overline{a}]$, I took $B = 0$ and $\overline{a} = 600$ (KMS took 1000, but I found that even assets more than 300 never appear if the system is close to the stationary distribution). Given the grid size, $w(a)$ is finite-dimensional $G \times 1$ vector.

The following basic algorithm was used:

1. Guess initial values of $x_0 = (w(a), k, \theta)$. Make initial $w(a_i) = \gamma\left(f(k) - kf'(k)\right) \forall a_i$ (see 8) to be sure there is non-negative profit of the firm initially. Take $\theta = 1$ (corresponding to [3] calibration) and $k = 1.1 \cdot k_{ss}$, where $k_{ss} = \left(\frac{\alpha}{\beta^{-1}-1+\delta}\right)^{\frac{1}{1-\alpha}}$ is a complete-markets steady-state capital stock (because of market incompleteness precautionary savings occur and capital is over-accumulated, see e.g. [2] for discussion of this fact).

2. Given k we compute r as $r = f'(k)$ necessary to sustain this k in the individual firm's optimum. Given r, $w(a)$ and θ, I use Value Function Iteration algorithm to

compute $W(a)$ and $U(a)$ from 2 and 3, limit assets selection to grid points, get grid value functions $\tilde{W}(a)$, $\tilde{U}(a)$.

3. Given the grid value functions, refine them: solve 2 and 3 with linearly interpolated $\tilde{W}(a)$ and $\tilde{U}(a)$ using numerical optimization, obtain refined $W(a)$ and $U(a)$ and policy functions $a'_e(a)$, $a'_u(a)$ and $s(a)$ with arbitrary assets level. Note, that $a'(a)$ will not be a grid point under this method. To get continuous policy functions, $a'(a)$ and $s(a)$ were also linearly interpolated.

4. Given policy functions $\{a'_e(a), a'_u(a), s(a)\}$ and individual transition probabilities matrix for states $\{e, u\}$:

$$P(s,\theta) = \begin{bmatrix} 1-\sigma & \sigma \\ s\lambda_w(\theta) & 1-s\lambda_w(\theta) \end{bmatrix},$$

note that the model is calibrated to be sure $s\lambda_w(\theta) < 1$, we have two-dimensional (ergodic under basic setting) Markov chain on assets and state. When $a'(a)$ was obtained on grid points, the eigenvalues decomposition was used to derive stationary distribution, however, it is rather complex numerically, therefore, an alternative Monte Carlo approximation was obtained in the following way.

I generate number N of agents, each of them is initially located at random point on assets grid and has random employment status. Given the policy functions and probability matrix, we compute next period assets and random employment status for large number of periods T. We keep last k of each agent's assets-state records, so we have Nk assets-state pairs for each period of time (due to ergodic nature of the chain, it is equivalent to keeping only one record for Nk agents or Nk records for one agent, but I found the mixture to be the fastest approach). To clarify, denoting employment status as $o \in \{e, u\}$ each after k period we have $Nk \times 2$ matrix, where the first index is an agent and the second is a time moment:

$$A = \begin{bmatrix} a_{1,t} & o_{1,t} \\ \dots & \dots \\ a_{1,t-k+1} & o_{1,t-k+1} \\ a_{2,t} & o_{2,t} \\ \dots & \dots \\ a_{2,t-k+1} & o_{2,t-k+1} \\ \dots & \dots \\ a_{N,t} & o_{N,t} \\ \dots & \dots \\ a_{N,t-k+1} & o_{N,t-k+1} \end{bmatrix}$$

I continue iterations until the convergence criterion is met. I choose the following stopping rule: I track five-element vector of mean, median, 0.95 and 0.05 quantiles of the first column (assets) and share of employed of the second column, and I stop the iterations when maximum change (relative to previous value) of the vector component was less then the given precision (I took 10^{-5}). I found that this method is nearly as fast as eigenvalue computation for the model with grid policy functions and the most feasible for my interpolated policy function.

All in all, in this step we get distribution matrix A and we could approximate integrals above by conditional sample means. We also can derive unemployment level u from the second column, and then compute equilibrium number of vacancies $v = \theta u$.

5. Given grid policy function $\tilde{a}'_e(a)$, wages $w(a)$ and r and tax burden $t = \frac{u}{1-u}h$, I solve 4. Since choice of k is determined explicitly, VFI on grid is equivalent to the solution of the linear system of equations and very fast. The drawback of this approach is the fact I use $\tilde{a}'_e(a)$ instead of $a'_e(a)$ that is used for invariant distribution, but the practical evidence is that two functions are rather close, therefore I expect the discrepancy to be negligible. We have $J(a)$ as a result of this step, again I linearly interpolate it between grid points.

6. Given $W(a)$ and $U(a)$ I compute on grid $\tilde{W}(a, w)$ and $\tilde{a}'_e(a, w)$ (functions for arbitrary w). We do not need iterations here (since I assumed that renegotiation takes time every period, so w affects only the current utility, but it anyway affects the decision of next-period assets). Given $\tilde{W}(a, w)$ find new wage scale $w^*(a)$ that resolves 8. To speed up the algorithm, I computed $w^*(a)$ only on more rare grid and then linearly interpolated it to original assets grid.

7. We now have a full set of value functions and decision rules. Compute stock price p from 14 and then new level of k from 9.

8. Now I compute the value of vacancy V from 6. Note that in equilibrium free-entry condition $V = 0$ must hold. I get new value of θ as a solution of equation

$$0 = -\zeta + q\lambda_f(\theta^*)\frac{1}{u}\int_{\mathcal{A}} J(a'_u(a))f_u(a)da,$$

that ignores the fact that $J(a)$ and u also depend on θ, however, it ad hoc gives the right direction for θ to change.

9. We have $x^* = (w^*(a), k^*, \theta^*)$. Let g be a small number, indicating a speed of adjustment (I took $g = 0.01$ when the system is far from steady state and increased it to 0.1 as it moves closer, faster adjustment usually led to divergence due to ad hoc ways of obtaining w^* and θ^*: it seems they are too volatile here). The update rule is the following: $x_i = (1-g)x_{i-1} + gx^*$. I also tried randomization of g and unequal rates of adjustment for each of components (adjusted k faster then w and θ due to its better behavior), but I finally stopped on the small constant rate for each component.

   As an alternative adjustment procedure, I preserved θ in initial value (usually $\theta = 1$), but on the final step I gradually adjusted vacancy cost ζ using ζ^* obtained from 7.

10. The convergence criterion is the following: I track the vector $T = \left(\frac{k^*-k}{k}, \frac{\bar{w}^*-\bar{w}}{\bar{w}}, V\right)$. If maximum by absolute value element of T is less then given precision (I used 10^{-3} here), we stop. If not, try again from 2 with the new value of x_i.

4 Calibration

The calibration of the model is non-trivial issue, since it contains several parameters, that cannot easily be established from micro-data or complete-market models. Therefore I rely on the default calibration of [3], and adjust it to take endogenous search into account.

I applied the following functional form assumptions:

Function	Assumption
Utility of consumption	$u(c) = \frac{c^{1-\psi}-1}{1-\psi}$
Job search utility costs	$g(s) = \xi\frac{s^{1+1/\phi}}{1+1/\phi}$
Production function	$f(k) = k^{\alpha}$
Matching function	$m(u, v) = \chi u^{\eta} v^{1-\eta}$

The following table presents basic parameter values that I did not change during my experiments. Except χ, they are the same as in [3]:

Parameter	Value	Meaning	Comments
ψ	2	Risk aversion	
β	0.995	Discount factor	Quarterly data
δ	0.01	Depreciation rate	
α	0.3	Capital share of output	
χ	0.7	Matching scale parameter	[3] used 0.6, see below
σ	0.05	Job destruction rate	
η	0.72	Matching function u elasticity	
h	1	Unemployment benefits	$\sim$ 40% of average wage
γ	0.72	Bargaining power of the worker	

The rest is to calibrate three unobservable things: ζ — cost of vacancy, ξ — weight of job search costs in worker's utility and ϕ, that is, according to 19, is the search elasticity with respect to utility difference between being employed or not. I discuss the rest of the calibration in the following sections.

5 KMS-replicating calibration

For this experiment, I adjusted $\chi = 0.6$ and $\psi = 2$ to fully match KMS framework. To have a model comparable with the setting with fixed search effort, I need to properly "switch off" the search variety. See 19: when $\phi \to 0$, $s(a) \to 1$, the choice of ξ therefore does not matter. I took $\xi = 8$ and $\phi = 0.001$ (that is equal to very elastic $g(s) = 8\frac{s^{1001}}{1001}$), and adjusted $\zeta = 0.975$ to target $\theta = 1$, so the job-finding probability on average is $\bar{s}\chi \approx 0.6$, as targeted by KMS, and standard deviation of search effort is less than 10^{-4}.

The stationary distribution results fully replicate the KMS framework. Let $p_w(a) = s(a)\lambda_w(\theta)$ be the probability to find a job, bars denote means. The following table present stationary equilibrium parameters. To study the local sensetivity, I also adjusted ϕ four times, the results did not change a lot that suggests that $\phi = 0.001$ is enough to get inelastic search effort:

Model	$r-\delta$	θ	$\bar{p}_w$	p	d	$\bar{w}$	$\sigma(w)/\bar{w}$	Gini a	Gini w
$\phi = 0.001$	0.34%	1.0000	0.5980	1.5855	0.0054	2.4826	0.0035	0.0412	0.0019
$\phi = 0.004$	0.35%	1.0000	0.5921	1.2614	0.0043	2.4767	0.0035	0.0432	0.0020

Parameter ζ was adjusted every time. The resulting distribution is hump-shaped with heavy right tail, see 1 for kernel-smoothed graph. Skewness of the distribution is negative of -0.31, kurtosis is near-normal of 3.09. The overall contribution of unemployment shocks to the distribution seems to be minor here: the unemployment is not really persistent (with probability 0.4 on average the agent continues to be unemployed), and the number of agents that a close to liquidity constraint of zero assets are near zero. So although the model does not pretend to explain all the range of wealth inequality, it still allows to capture distributional effects of job search in isolation.

Note that as KMS assert, the effects of wage-bargaining on the inequality are rather small: wage scale is rather flat for individuals far from liquidity constraint (those who are insured enough), see 2 for stationary $w(a)$.

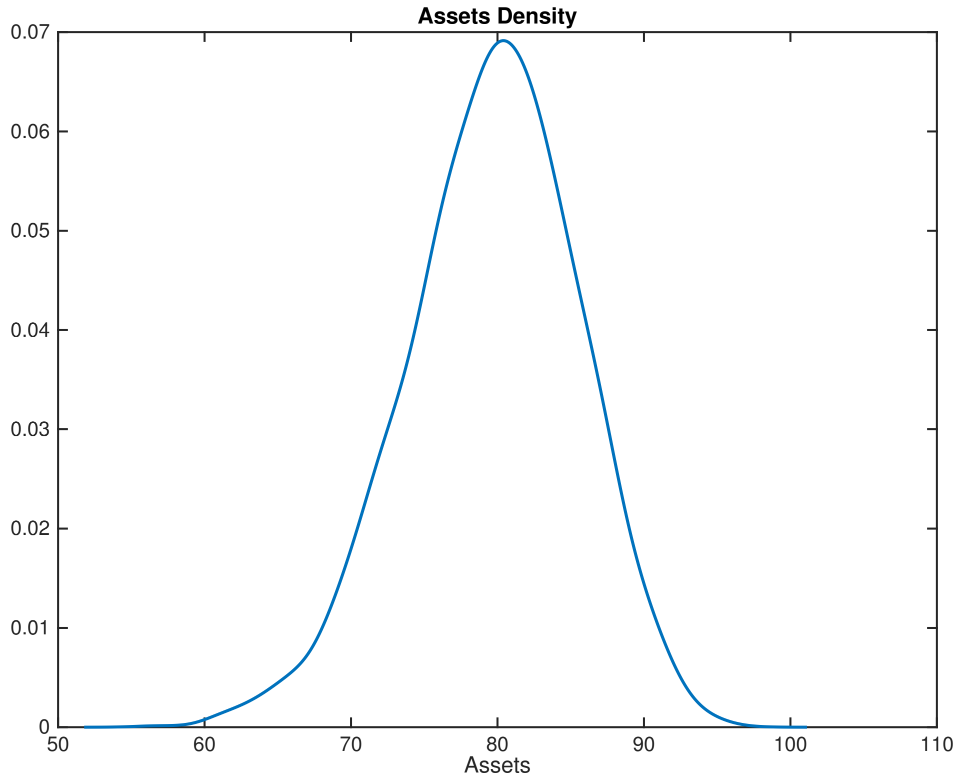

Figure 1: Assets Density in KMS-replicating model

6 Varying search effort calibration

After the basic KMS results were replicated, I am able to study the effect of search effort variety on distribution, namely: how change of search elasticity ϕ affects key model results. The issue here is that we cannot change costs in isolation: when ϕ increases, together with increase in variety of costs, the costs of the same level of effort increase, and obviously

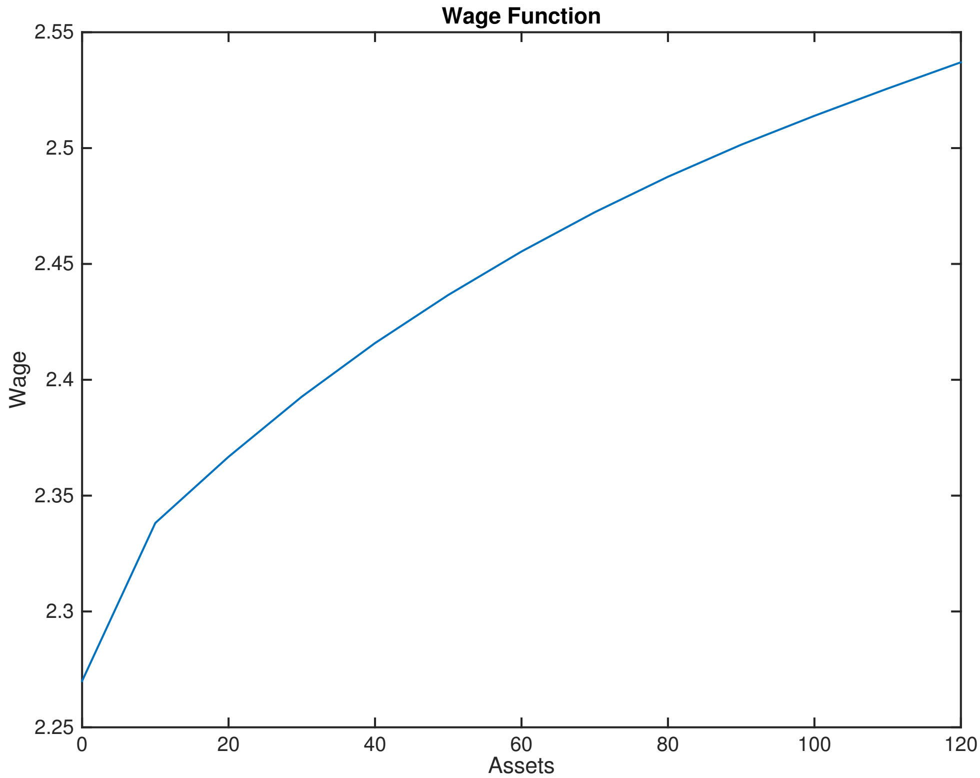

Figure 2: $w(a)$ function for reasonable assets values

everybody is more reluctant to search for job. So since both value and curvature of search costs affects individual's decision, it is not correct to vary only one parameter. However, there is no the only straightforward way to apply comparative statics here, so different approaches may, in principle, lead to different conclusion. Here I present my comparative static idea together with several alternatives.

6.1 Job finding probability targeting

Low search elasticity

The most credible idea of comparative statics I see is to adjust unobservable parameters to match some clear observable targets. To preserve comparability, I decided to adjust vacancy costs ζ and utility weight of search ξ to match the target value of ϕ and the target KMS-calibrated values of $\theta = 1$ and average (equal to everyone in KMS) probability to find a job of 0.6. To keep $s < 1$, I adjusted $\xi = 0.7$, therefore we have to keep $\bar{s} = \frac{6}{7} \approx 0.83$.

I started with calibrations close to KMS and begin to decrease search difficulty by increasing ϕ. I began with calibration with $\phi = 0.04$, presented above, however, results are not fully compatible, because in this section I set $\chi = 0.7$. The following table presents required adjustment of ζ and ξ to preserve average job-finding probability being 0.6 and market tightness being 1:

$\phi = 0.04$	$\zeta = 1.0148$	$\xi = 39.8000$
$\phi = 0.06$	$\zeta = 1.1616$	$\xi = 24.6000$

See table below for resulting aggregate equilibrium parameters:

Model	$r - \delta$	$\bar{w}$	$\sigma(w)/\bar{w}$	Gini w ($\times 100$)	Gini a ($\times 100$)	$s_{0.95}/s_{0.05}$
$\phi = 0.001^*$	0.34%	2.4826	0.0035	0.1932	4.1212	1.0001
$\phi = 0.004^*$	0.34%	2.4767	0.0035	0.1955	4.3242	1.0003
$\phi = 0.04$	0.32%	2.5084	0.0039	0.2195	4.4503	1.0045
$\phi = 0.06$	0.30%	2.5057	0.0045	0.2532	4.8420	1.0069
* with $\chi = 0.6$ as in Section 5						

The columns, respectively, are aggregate net (of depreciation) interest rate, average wage of employed agents, standard deviation of wage as a share of average wage, Gini coefficient on wages (that are both earnings and income of employed individuals, since taxes are paid by firms), Gini coefficient of assets among all individuals (measure of wealth inequality), and ratio of 0.95 and 0.05 quantiles of the search intensity (that is equivalent to the ratio of quantiles of unemployment durations and measures variability of the search). See Figure 3 for the evolution of assets density corresponding to the table.

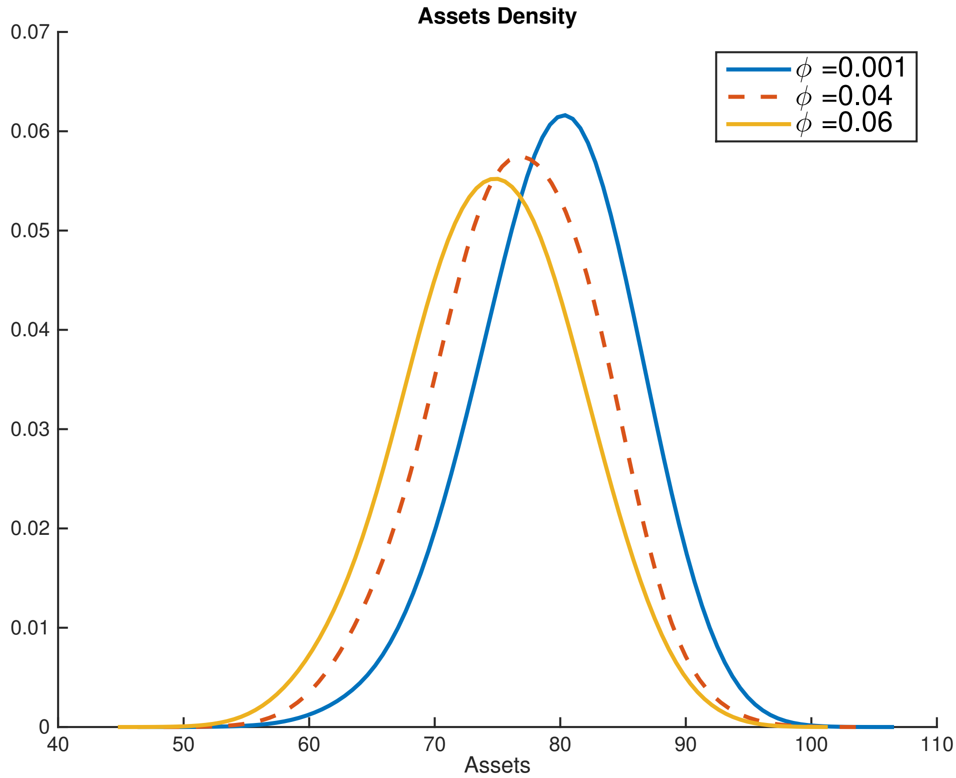

Figure 3: Asset Distribution for different ϕ, $\psi = 2$, $\bar{p}_w = 0.6$

Higher search elasticity, lower risk aversion

I also tried higher values of ϕ, but due to numerical stability issues in this section I had to set $\psi = 1$ (so, less precautionary savings and higher interest rates here are expected, and results are incomparable with previous). I compared three calibrations with $\phi = 0.2$, $\phi = 0.25$ and $\phi = 0.3$. Here is required adjustment to preserve $\bar{p}_w = 0.6$:

$\psi = 1,\ \phi = 0.2$	$\zeta = 1.0569$	$\xi = 2.6$
$\psi = 1,\ \phi = 0.25$	$\zeta = 1.1120$	$\xi = 2.2$
$\psi = 1,\ \phi = 0.3$	$\zeta = 1.1429$	$\xi = 2.0$

The resulting dynamics of the shape of the distribution is presented on 4. The regularity is an increase in the left tail: the distribution becomes more asymmetric, the share of less insured agents increases.

The following table presents results corresponding to these three calibartions:

Model	$r - \delta$	$\bar{w}$	$\sigma(w)/\bar{w}$	Gini w ($\times 100$)	Gini a ($\times 100$)	$s_{0.95}/s_{0.05}$
$\psi = 1,\ \phi = 0.2$	0.41%	2.4299	0.0015	0.0852	3.7328	1.0094
$\psi = 1,\ \phi = 0.25$	0.41%	2.4233	0.0016	0.0872	4.0121	1.0130
$\psi = 1,\ \phi = 0.3$	0.42%	2.4183	0.0018	0.0954	4.5031	1.0163

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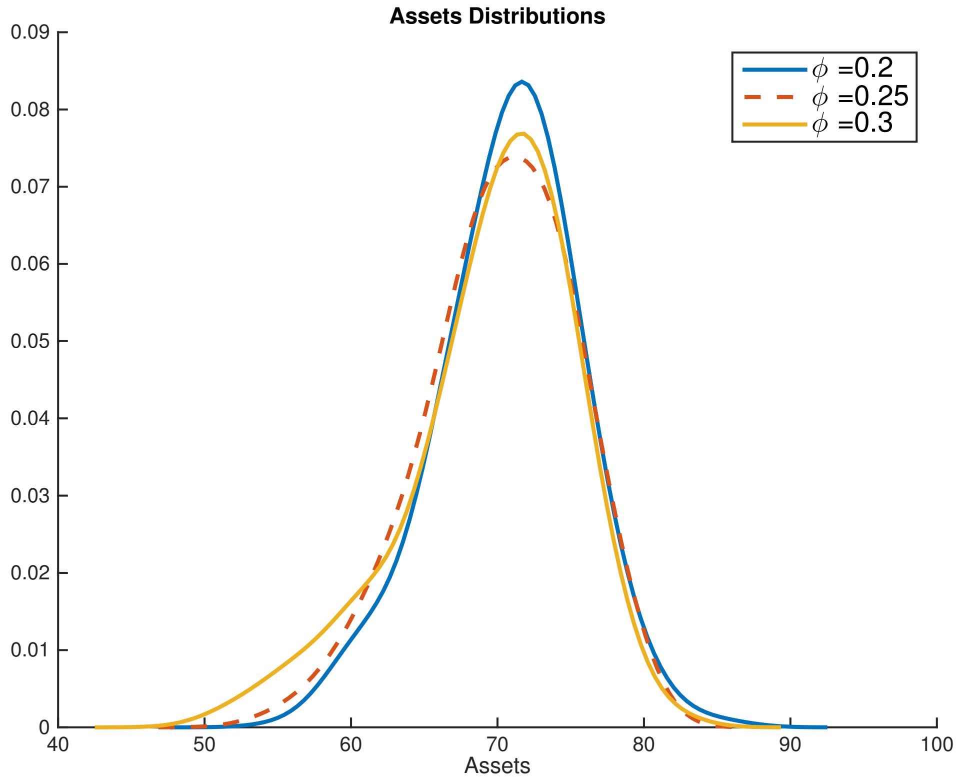

Figure 4: Asset Distribution for different ϕ, $\psi = 1$, $\bar{p}_w = 0.6$

6.2 Alternative comparative statics approach

The approach used above does not perfectly match the usual sense of comparative statics, since I varied three parameters together instead of one. The alternative may be to study the effects of parameter in isolation, taking into account that labor market targets (market tightness of 1 and probability to find a job of 0.6) cannot be reached in that case. The table below presents the results for the model in the calibration with $\chi = 0.6$, $\psi = 1$ (the rest is the same as described above), and set fixed vacancy costs $\zeta = 1$. See how the outcome varies with isolated change of ϕ in table 1. See also Figure 5 for the shifts of

the assets distribution induced by isolated change in ϕ. It looks like the change in the

Model	θ	v	$\bar{p}_w$	$r-\delta$	$\bar{w}$	$\sigma(w)/\bar{w}$	Gini w	Gini a	$s_{0.95}/s_{0.05}$
$\phi=0.05$	0.9320	0.0723	0.5254	0.38%	2.4383	0.0024	0.1295	3.9835	1.0028
$\phi=0.25$	3.1558	0.1741	0.5258	0.43%	2.2795	0.0015	0.0821	4.2546	1.0125
$\phi=0.3$	3.7620	0.1973	0.5166	0.44%	2.2354	0.0012	0.0681	4.9589	1.0162

Table 1: Isolated change of search elasticity, equilibrium values

shape of the distribution is nearly the same, depsit of different directions on aggregate equilibrium parameters presented in the table.

The main contradiction between two approaches is the effects on wage inequality and variation: adjustment of (ϕ, ζ, ξ) together increases both $\sigma(w)/\bar{w}$ and isolated change of ϕ always decreases them. .

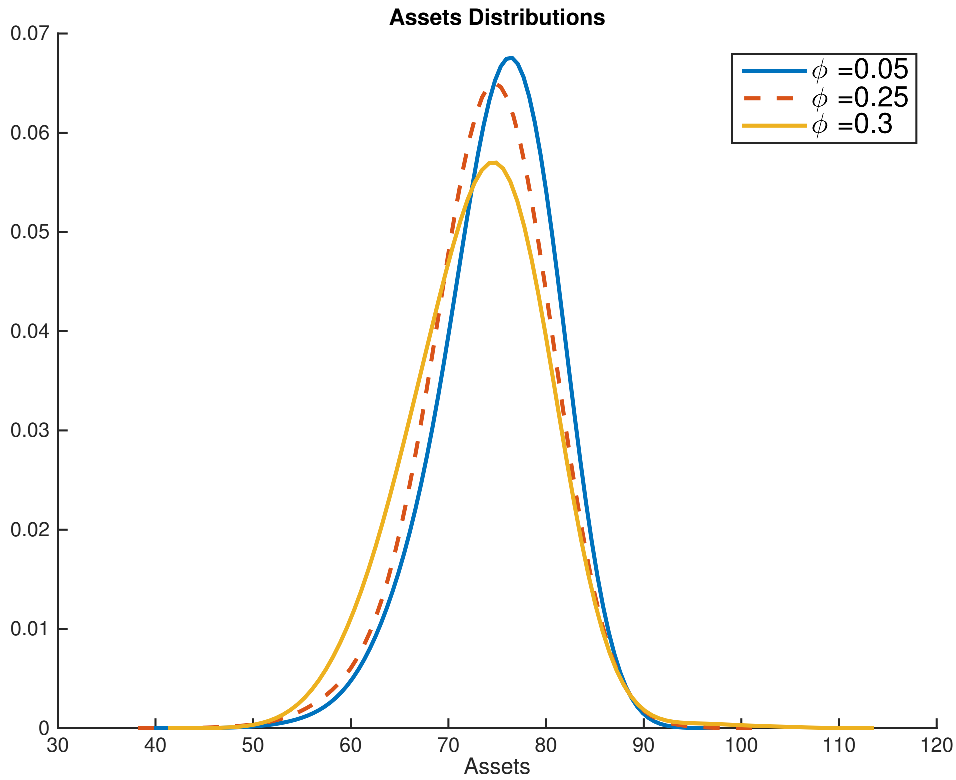

Figure 5: Asset Distribution for different ϕ, alternative approach

To resolve the contradiction, I ran the model with only ξ varying. Recall that an increase in ξ increases search costs regardless of search effort, that may help to isolate the shape and the scale effects of changes of the search function. See Table 2 for the equilibrium aggregate variables.

Model	θ	$\bar{p}_w$	$r-\delta$	$\bar{w}$	$\sigma(w)/\bar{w}$	Gini w	Gini a	$s_{0.95}/s_{0.05}$
$\xi=7.5$	0.9827	0.5350	0.39%	2.4302	0.0022	0.1216	3.9393	1.0026
$\xi=4.5$	0.9290	0.5396	0.39%	2.4325	0.0021	0.1159	3.9011	1.0027
$\xi=3.5$	0.8954	0.5406	0.40%	2.4330	0.0020	0.1135	3.9109	1.0027
$\xi=2.5$	0.8452	0.5405	0.40%	2.4370	0.0020	0.1142	3.9711	1.0028

Table 2: Isolated change of search costs magnitude ξ, equilibrium values

That reveals general direction of the scale effect (change in overall costs of search preserving its elasticity, see the last column that ξ does not change duration variability that much).

7 Conclusions

Although the overall evidence are mixed and there is a large methodological concern of how to conduct comparative statics correctly, there are few regularities that deserve further analysis and consideration.

1. Differential search effort seems to have some clear impact on the aggregate wealth distribution and inequality. Relatively small changes in search effort variability induce large fluctuations of assets inequality: see quantile ratios and corresponding Gini coefficient changes. Shape of the graphs also confirms the effects of varying search intensity: left tail of the distribution becomes more fat, the ratio of less insured agents increases, arguably it tends to slightly decrease the amount of precautionary savings.

The evidence from simulations gives plausible explanations for that: allowing the effort to be more flexible, we allow the poor unemployment agents to quickly escape their unemployed state by searching for a job more intensively. Consequently, this possibility increases "labor force mobility" and decreases the need for precautionary savings for lower assets groups.

2. The effects of search variation are caused mostly by elasticity of the search costs with respect to search effort rather then by the magnitude of serach costs itself. See Table 2: there is no substantial impact of search costs magnitude onto stationary equilibrium.

3. The effects of search intensity variation onto wage inequality is a non-trivial issue and depend on the approaches to analyse comparative statics. Effects of elasticity increase per se seems to decrease wage variation. The plausible explanation is the shift of the disagreement point of individuals: although low-assets agents are less insured, they are able to find the next job faster by increasing of the search intensity, therefore, they have more bargaining power in wage negotiations and get higher wage. See 6 for an evidence of that: wage scale of higher search elasticity becomes flatter, that means more uniform distribution and amplifies the result of KMS about low importance of bargaining wage variation to wealth inequality.

   In contrast, if we apply the first comparative statics approach, account for general equilibrium effects and adjust vacancy costs ζ to keep (hopefully observable) vacancy/unemployment ratio $\theta = 1$, we get steeper wage scale and more wage inequality. This approach, based on observable characteristics rather than on fixed parameter values, seems to be more credible, and under it wage inequality increases as a result of change of the search elasticity. Technical explaination could be that together with change of disagreement point for consumers, adjustment of ζ makes worse the disagreement point for firm, and this effect overlaps the previous, so technically cost of vacancy ζ affects wage scale more than search elasticity, and that is a large concern for the model: unobservable fixed vacancy costs are difficult to calibrate explicitly but reveal their large impact on wage inequality. That suggests trying another bargaining approach or trying endogenous vacancy costs for making correct predictions, this issue cannot be resolved directly in the current setting.

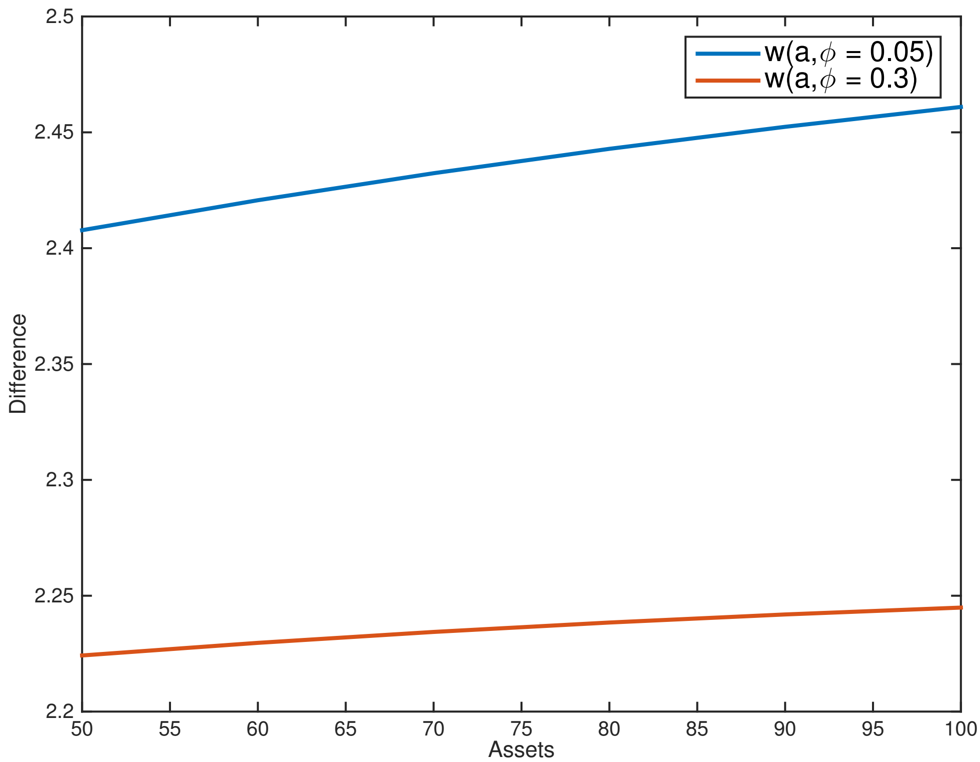

Figure 6: Wage functions for different ϕ, isolated change

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4. The negative result is that the overall scale of the quantitative contribution of the

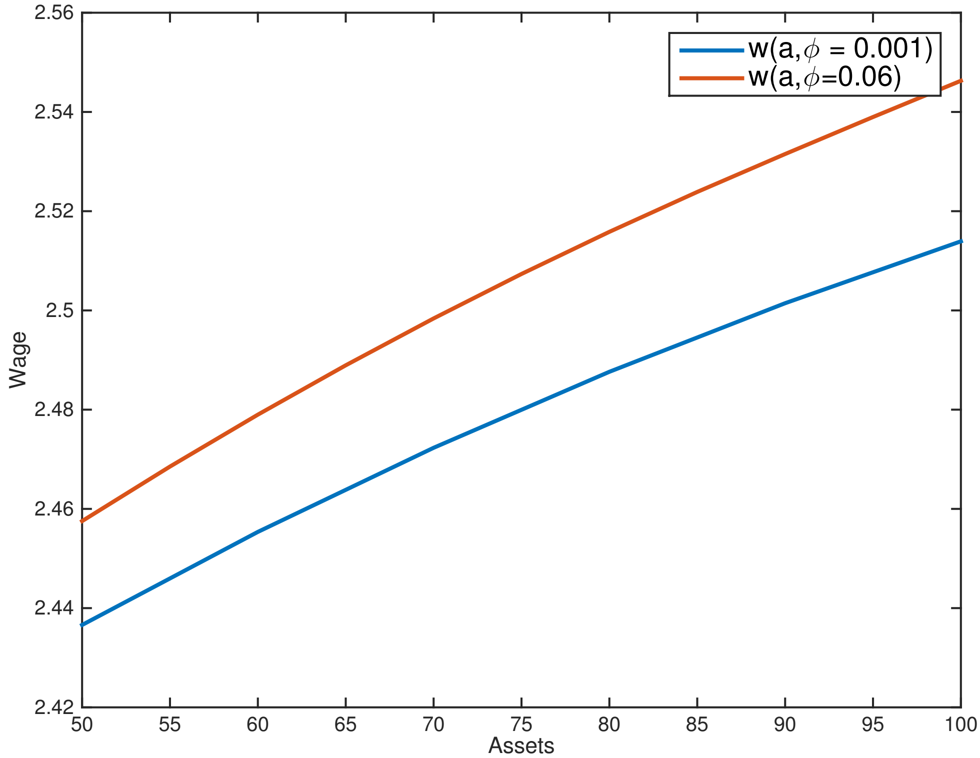

Figure 7: Wage functions for different ϕ, adjustment of ζ to keep $\theta = 1$

search to inequality cannot be evaluated in the current setting. The reason for that is fairly low level of heterogenity: asset level varies very little in comparison to real economies and things like Gini coefficients are really small. Therefore, although the direction of impact of the search and the changes of KMS results may be revealed, the model cannot describe enough of observable heterogenity.

The main technical reason for that is that income shocks are caused by only changes in employment status, and unemployment is not really persistent here: the workers are homogeneous in their productivity and lose their jobs only by some exogenous reasons, and find them at different rates according to their savings level. Since it is quite rare event, we do not see a lot of variation in assets. The alternative approach could be adding productivity shocks in combination with employment shocks, that allows the incomes to vary within the employment status. However it complicates the matching: it will be natural to assume that the firms will be reluctant to hire workers when their productivity is low, and will more likely to fire workers with low productivity. So, combination of employment status with persistent productivity differences may be a good way to accommodate the observable search and unemployment duration variation. It could be implemented in the model, however, it requires additional work and changes in the setting. This may be the major direction for my further work now.

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A Model in details

A.1 Unconstrained value functions

Note that for computations consumer Bellman functions should be transformed to the proper form. The employed value function transformation of 2 is straightforward:

$$W(a) = \max_{a' \in \Gamma_e(a)} \left\{ u\left((1+r-\delta)a + w(a)\right) + \beta\left[(1-\sigma)W(a') + \sigma U(a'),\right]\right\},$$

$$\Gamma_e(a) = [-B; (1+r-\delta)a + w(a)], \quad (18)$$

as to the unemployed agent 3, searching for the job, it requires some analytical work. Note that given functional form assumption, the first order condition for internal solution of s implies that

$$g'(s) = \beta\lambda_w(\theta)\left(W(a') - U(a')\right) \Rightarrow s = \left[\frac{\beta}{\xi}\lambda_w(\theta)\left(W(a') - U(a')\right)\right]^{\phi}, \quad (19)$$

Thus ϕ can be interpreted as elasticity of search effort with respect to the difference of value functions. When ϕ approaches zero, search intensity becomes fully inelastic. After some degree of manipulations, it could be shown that

$$U(a) = \max_{a' \in \Gamma_u(a)} \left\{ u((1+r-\delta)a + h - a') + \right.$$
$$\left. +\beta U(a') + \frac{1}{(1+\phi)\xi^{\phi}}\left[\beta\lambda_w(\theta)\left(W(a') - U(a')\right)^{1+\phi}\right]\right\},$$

$$\Gamma_e(a) = [-B; (1+r-\delta)a + h], \quad (20)$$

and given decision rule $a'(a)$, the $s(a)$ function can be obtained from 19.

A.2 Simplifying firms' problems

For numerical simplicity, I apply a couple of simplifications for firms' problems. Together with the original framework, when the employed worker stays in the firm until it is destroyed, I tried two assumptions. The first is mixing of workers among firms every period (as a result, firm's profit depends on the assets of employed worker only in the current period). The second is fully flat wage scale, in case of which firm does not care about assets of the employee.

Workers mixing

Let the workers are mixed among firms each period, such that the assets of the current worker affect only current period profits: in the next period the firm can be matched to any of workers in the economy, so its value function is equal to average value function $\int_\Omega J(a_i)di$, thus in the modified framework

$$J(a) = \max_k \left\{ f(k) - rk - w(a) - t + q\left((1-\sigma)\int_{\Omega_e} J(a_i) + \sigma V\right)\right\},$$

first-order condition implies that $f'(k) = r$, assuming Cobb–Douglas production function $f(k) = k^\alpha$ we have

$$J(a) = (1-\alpha)k^\alpha - w(a) - t + q\left((1-\sigma)\int_{\Omega_e} J(a_i)di + \sigma V\right),$$

where $k = \left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}}$. Integrating it over the pool of employed workers implies

$$\int_{\Omega_e} J(a_i)di = \frac{1-u}{1-q(1-\sigma)}\left((1-\alpha)k^\alpha - \bar{w} - t + q\sigma V\right), \tag{21}$$

note that $\int_{\Omega_e} di = 1-u$, where $\bar{w}$ is from 13.

Flat wage scale

Assume further that $w(a) = w$, so the wage is uniform across the economy. In this case, 21 reduces to simple

$$J = \frac{1}{1-q(1-\sigma)}\left((1-\alpha)k^\alpha - w - t + q\sigma V\right).$$

B Consistency of balances

Here I check that balancing conditions I apply here are consistent with each other, the most important part here is the relation between individual budget constraints and assets market, because they are directly use in computations.

Given arbitrary consumer decision functions $\{c_e(a), a'_e(a), c_u(a), a'_u(a)\}$ and price combination $\{w(a), r\}$ and unemployment level u, and stationary (in sense described in equilibrium definition) distribution, consistent with corresponding decision rules μ the following consumer budget constraints will hold:

$$c_e(a_i) + a'_e(a_i) = (1+r-\delta)a_i + w(a_i), i \in \Omega_e, \quad c_u(a_i) + a'_u(a_i) = (1+r-\delta)a_i + h, i \in \Omega_u,$$

then we integrate both equalities by corresponding sets and sum them, use 16 here to mark the total consumption, get

$$c + \int_{\Omega_e} a'_e(a_i)di + \int_{\Omega_u} a'_u(a_i)di = (1 + r - \delta) \int_{\Omega} a_i di + \int_{\Omega_e} w(a_i)di + \int_{\Omega_u} hdi,$$

then we use 17 and mean wage definition 13 and get

$$c = (r - \delta) \int_{\Omega} a_i di + (1 - u)\bar{w} + uh,$$

then we use asset balance 9

$$c = (r - \delta)(1 - u)k + (r - \delta)p + (1 - u)\bar{w} + uh,$$

plug stock price 14:

$$c = (r - \delta)(1 - u)k + (1 - u)\left[f(k) - rk - t - \bar{w}\right] - v\zeta + (1 - u)\bar{w} + uh,$$

input balanced budget condition $t = \frac{u}{1-u}h$, rearrange and finally get

$$c + (1 - u)\delta k + v\zeta = (1 - u)f(k).$$