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UEFA Financial Fair Play: A Model of Financial Regulation*

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Abstract

A dramatic surge of revenues from TV broadcasting and brand-selling forced modern football clubs, involved simultaneously in domestic and European competitions, to operate in a new environment. In response, the European Union of Football Associations (UEFA) introduced Financial Fair Play, a set of financial regulation that affects all major European clubs. To assess the new regulation's impact on default risk for individual clubs and competitive balance, we build a game-theoretic model with clubs making decisions on the amount they borrow and spend on the squad. We show that the impact of FFP on "systematic risk" is positive: in equilibrium clubs have lower default probability when financial restrictions are in place; also, we've found a positive effect on the competitive balance. Finally, we examine how new regulations influence football clubs' investors decisions.

Keywords: Financial Fair Play, financial regulation, competitive balance, investment tournaments

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1 Introduction

Football is a game, in which two teams of 11 players try to score a ball into the opposing team's goal. The team that scores more goals wins. According to estimates by football's governing body, the Federation Internationale de Football Association (FIFA), it is played in more than 200 countries by 250 million players, and its worldwide fan base is more than 1.3 billion people, which makes football the most popular sport in the world.

Trivially, main objective of a football club is to win any tournament that it gets into, and gifted players is the main instrument to achieve this goal. Club managers heavily use leverage to magnify purchasing power so that they can attract the best and brightest talent. Their competitors are affected indirectly by such strategy as their likelihood of winning falls, so they are forced into financial "arms race". Clearly, clubs operate in "winner takes it all" world, therefore, it's not surprising that many of them have reported repeated losses in recent years. Novel measures, such as financial regulations, were introduced to improve "financial health" of European football clubs a while ago, and its effects are still unclear. In our model we have found how these regulations shift alignment of forces in European football; moreover, we analyze its impact on clubs' credit risk and entrance of new investors on football market.

The club tournaments with largest TV audience - the UEFA Champions League and the Europa League - are held in Europe and administered by the Union of European Football Associations (UEFA). Approximately 360 million people watched the final of UEFA Champions League 2012-2013 between Borussia Dortmund and Bayern Munich. It comes as no surprise that these competitions attract multinational corporations as sponsors (UEFA Champions League official sponsors are Heineken, UniCredit, Ford, MasterCard, Sony Computer Entertainment Europe and Gazprom); also UEFA receives hefty fees from broadcasters. Later it shares these revenues with participating clubs - according to the official website[1] "A total of €904.6m was distributed to teams competing in the UEFA Champions League last season". Moreover, "All 32 participants were entitled to a minimum €8.6m in accordance with the distribution system. Additionally, performance bonuses were paid in the group stage: teams received €1m for every win and €500,000 for every draw..." and clubs get additional bonuses for their advance to the next stage of the tournament. To sum up, clubs' payoffs from participation in such competitions are estimated at tens of millions of euros, and they might constitute a decent share of clubs' revenues (especially for a team with relatively small budget).

[1] `http://www.uefa.org/management/finance/news/newsid=1975196.html`

Generous payoffs from participation and progression in UEFA competitions prompted club managers to take on huge loans to buy new players with an initial plan to place at national championship (in order to qualify for European tournaments) and successfully perform at these competitions later. However, we are familiar with a number of cases, when things got wrong after implementing such strategy. UEFA reported in 2010 that half of Europe's top-division clubs were losing money. The prime example is Rangers - football club from Glasgow, Scotland. In the end of the 80's new owners of Rangers adopted a novel strategy - instead of relying on a local talent they started borrowing heavily to buy international stars, such as Gascoigne, Amoruso and Laudrup. These and many others purchases were intended to help in conquering European cups, but Rangers never succeeded. In 2011 they were knocked out from early qualifying stages, and Rangers FC was placed into administration in 2012 due to increasing financial problems. Later that year club failed to reach an agreement with its creditors and entered the process of liquidation. Scottish Premier League (SPL) clubs voted to relegate Rangers, Scottish champion a record 54 times, to the fourth tier, the country's lowest professional league. Also club was banned from European competitions for three years and it lost its strongest players. Without Rangers SPL lost its most famous rivalry - Glasgow derby between Celtic and Rangers - and a significant part of its commercial appeal to broadcasters and sponsors. Another remarkable examples are Parma FC from Parma, Italy and Leeds United from Leeds, England, which had experienced similar problems a decade ago.

The professional team sports has specific features: while the objective of any team is to win the tournament and therefore it can be considered as a win-maximizer (in contrast with "ordinary"firms which are usually studied in academic literature as profit-maximizers), it also needs opponents (and preferably stronger ones) in order to improve its economic performance. This observation comes into conflict with standard models of economic competition, where any firm will benefit from driving competitors out of business and staying alone on the market.

As we've noted before, every team prefers to play with stronger opponents. In other words, a weak team imposes a negative externality on its stronger competitors, and attractiveness of sports league increases in its competitive balance. In the famous paper Rottenberg [1956] studied US baseball league and came up with a similar observation that "in baseball no team can be successful unless its competitors also survive and prosper sufficiently so that the differences in the quality of play among teams are not too great". Likewise, Neale [1964] noted that "the closer the standings, and within any range of standings the more frequently the standings change, the larger will be the gate receipts". He also presented "Louis-Schmelling Paradox": heavy-weight champion of the world needs a strong contender to earn more money, though uncertainty of the outcome will rise and chances to defend a title will fall. Many methods were employed to measure competitive balance of sports leagues, including the dispersion of winning probabilities within league (Scully [1989]), Hirfindahl-Hirschman Index (HHI) of the concentration of championship titles and Gini coefficient (Fort and Quirk [1995]).

The concept of Financial Fair Play was adopted by UEFA recently, and there are not many research papers on FFP and specifically about its effect on competitive balance. Peeters and Szymanski [2012] in their working paper model FFP by hardening budget constraint. They argue that the break-even rule operates analogously to a salary cap and this "may restrict competition in exactly the same way as a horizontal agreement between competing firms". Franck and Lang [2012] showed that injecting money by owners induced clubs to pursue riskier strategies; consequently, FFP would limit such opportunities. Under some conditions on parameters of the model they demonstrated that clubs would implement optimal strategies from the social welfare perspective. They've also noticed that FFP rules might decrease bankruptcy risk of football clubs and improve competitive balance. However, neither empirical evidence nor theoretical proof of these claims were presented.

In some aspects FFP is similar to the measures of regulations in the banking sector. Banks do not conform with standard models of economic competition as well, because failure of one bank will have disastrous consequences for others, i.e. it may ignite banking panic. For instance, Lehman Brothers collapse in September 2008 escalated financial crisis and significantly worsened credit conditions all over the world. As a result, its competitors, including Goldman Sachs and Morgan Stanley, were put on a verge of bankruptcy and were saved only after a variety of government actions.

Freixas and Rochet [2008] noticed that "a bank failure may spread to other banks (interbank loans account for a significant proportion of banks' balance sheets) and similarly endanger the solvency of nonfinancial firms". Next, they wrote "...when depositors are insured ..., moral hazard appears. Bankers have incentives to take too much risk and to keep operating (at the expense of the deposit insurance fund) in situations in which liquidation would be efficient". Similarly, football club managers have incentives to gamble on success - borrow money, buy stars and in case of losing and bankruptcy rely on the owner's financial aid.

Next, new set of banking regulations, Basel III, was introduced in response to the late-2000s financial crisis. These measures (including capital requirements and leverage constraint) aim to "improve the banking sector's ability to absorb shocks arising from financial and economic stress, whatever the source" and "improve risk management and governance"[3]. Admati et al. [2013] examine "claims that high capital requirements are costly and would affect credit markets adversely", and they find that "better capitalized banks suffer fewer distortions in lending decisions and would perform better". Similarly, FFP imposes a maximum loss requirement, and club owners may cover these losses only by equity injections. Critics argue that it would harm football industry by forgoing benefits from "external" investments, and therefore it would be interesting to study the impact of borrowing restriction on clubs.

Finally, Allen and Gale [2000] studied interconnectedness of banking system and possibility of contagion. They suggested that regulation can play an essential role: "by intervening appropriately, the central bank can ensure that the inefficient allocation associated with contagion can be avoided". Morris and Shin [2008] considered maximum leverage constraint that "has the potential to prevent the buildup in leverage that leaves the system vulnerable to a sudden reversal". In what follows we want to study the effect of similar restriction - borrowing constraint - on the behavior of football clubs and overall level of risk in European football.

2 UEFA Financial Fair Play basics

To fight adverse effects of excessive risk-taking in European football, UEFA's Executive Committee approved the concept of Financial Fair Play (FFP) in September 2009[2], "with its principal objectives being:

- to introduce more discipline and rationality in club football finances;
- to decrease pressure on salaries and transfer fees and limit inflationary effect;
- to encourage clubs to compete with(in) their revenues;
- to encourage long-term investments in the youth sector and infrastructure;
- to protect the long-term viability of European club football;
- to ensure clubs settle their liabilities on a timely basis."

[3] `http://www.bis.org/bcbs/basel3.htm`

[2] `http://www.uefa.org/footballfirst/protectingthegame/financialfairplay/`

The FFP rules require clubs entering UEFA competitions to pay their obligations in a timely manner ("no overdue payable rule") and live within their revenues ("break-even requirement"). Second rule established maximum deficit amount for a monitoring period (including another limit for a case when losses are covered by contributions from equity participants); relevant expenses do not include some costs - e.g. investments in infrastructure or youth academy. Sanctions that can be taken against a club may be: "a warning, a reprimand, a fine, a deduction of points, the withholding of revenues from a UEFA competition, the prohibition on registering new players in UEFA competitions, the restriction on the number of players that a club may register for participation in UEFA competitions, the disqualification from competitions in progress and/or the exclusion from future competitions and the withdrawal of a title or award". However, these regulations have already encountered significant critique, and the main points of criticism are:

- The effect of FFP on competitive balance is uncertain
- The FFP will ossify current hierarchy, i.e. relatively small clubs won't be able to compete with bigger clubs, if clubs can only spend within their own means;
- Clubs will sign questionable sponsorship contracts to bypass FFP regulations
- Different tax rates across Europe create unequal conditions for clubs from various leagues

The goal of this paper is to investigate the effect of FFP regulations on competitive balance and at the same time observe how individual and overall borrowing change with the introduction of FFP. The rest of the paper is structured as follows: section three introduces formal model setup, section four describes equilibrium without and with FFP regulations, section five studies impact of FFP on investor's decision and section six concludes.

3 Model Setup

We describe the following simultaneous game: there are n clubs in tournament, competing for the prize Π. Each team $i \in 1, \ldots, n$ is endowed with the budget W_i (without the loss of generality: $W_1 < W_2 < \ldots < W_{n-1} < W_n$) and it chooses $D_i \geq 0$ - the amount of borrowing at risk-adjusted interest rate. We consider football club to be not a profit maximizer, but a win maximizer. Probability of winning the tournament and the prize Π (and, therefore, payoff) by ith club depends on investments of all clubs - similar to the rent-seeking game from a seminal paper by Tullock [1980]:

$$P_i(m_1, \dots, m_n) = \frac{m_i}{\sum_{j=1}^{n} m_j}$$

where investment of ith team m_i (in wage bill, which is known to be highly correlated with performance - e.g. Smith and Szymanski [1997]) equals the sum of budget W_i and borrowings D_i. Each club could be in two states of the world: bankrupt or not; bankruptcy is an exogenous event, it's independent of other teams' bankruptcies and the team's default is uniform random variable which depends only on its own level of borrowing; $D_{max} = const > 0$ is high enough:

$$P(bankruptcy|D_i) = \begin{cases} 0 & , \quad D_i < 0, \\ \frac{D_i}{D_{max}} & , \quad 0 \leq D_i < D_{max} \\ 1 & , \quad D_i \geq D_{max} \end{cases}$$

First we consider a case, where each team can borrow without a constraint. Every club cares only about winning: football club maximizes its own probability of winning, although it is a subject to the budget constraint - revenues must be greater or equal than costs in any state of the world.

Costs include

1. payments on debt $(1 + r(D_i))D_i$, where $r(D_i) = r \exp\left(\lambda \frac{D_i}{W_i}\right)$ is risk-adjusted interest rate. We assume that the team with larger budget gets lower rate and borrowing is relatively costly as interest rate grows exponentially.

2. penalties in cases of bankruptcy of all teams (including our own) $c_i(D_1, \dots, D_n) = \sum_{j=1}^{n} cD_i \cdot \mathbb{I}\,[\text{team j is bankrupt}]$ (we assume that bankruptcy of other team has an adverse effect on our debt - i.e. in case of refinancing we'll get a higher interest rate - and our own bankruptcy has larger effect)

Revenues include

1. Value of prize $\Pi \cdot \mathbb{I}\,[\text{team i is a winner}]$ (in case of victory in the tournament)

2. ticket sales $\phi_i(m_1, \dots, m_n)$, where function $\phi_i(m_1, \dots, m_n) = t \cdot m_i \sum_{j=1}^{n} m_j$, $t = const > 0$ shows how these revenues depend on the strength of teams (and investment m_i is a good proxy for a strength) - we assume that matches are more interesting, when teams are

more equal and they spent more on players. This functional form implies that ticket sales increase more in the investment of our team than in the investments of other teams. Also ticket sales are higher when equal and “middle”-sized teams play compared to the case with unequal “rich” and “poor” teams.

Each team solves the general problem:

$$\begin{aligned}
P_i(m_1,\ldots,m_n) &\rightarrow \max_{m_i} \\
\textbf{s.t. } \Pi\cdot\mathbb{I}\,[\text{team i is a winner}] + t\phi_i(m_1,\ldots,m_n) &\geq \sum_{j=1}^{n} cD_i\cdot\mathbb{I}\,[\text{team j is bankrupt}] + (1+r(D_i))D_i \\
\text{in} & \quad \forall \text{ state of the world} \\
m_i &= W_i + D_i
\end{aligned}$$

We must solve for Radner equilibrium in incomplete markets framework. However, if we assume the existence of market for any contingent commodity, then we can search for Arrow-Debreu equilibrium, which is identical with Radner equilibrium[1]. The problem in our setting is following

$$\begin{aligned}
\frac{m_i}{\sum_{j=1}^{n} m_j} &\longrightarrow \max_{m_i} \\
\textbf{s.t. } \Pi\cdot\frac{m_i}{\sum_{j=1}^{n} m_j} + tm_i\sum_{j=1}^{n} m_j &\geq cD_i\frac{\sum_{j=1}^{n} D_j}{D_{max}} + (1+r\exp\left(\lambda\frac{D_i}{W_i}\right))D_i \\
m_i &= W_i + D_i
\end{aligned}$$

4 Analysis

Constraint must be binding in equilibrium or otherwise team can deviate and increase its investment a bit (as long as budget constraint permits) and therefore increase its own probability of winning. Problem of n clubs transforms into the non-linear system of n equations:

$$\begin{aligned}
\Pi\cdot\frac{W_i+D_i}{\sum_{j=1}^{n}(W_j+D_j)} + t(W_i+D_i)\sum_{j=1}^{n}(W_j+D_j) &= cD_i\frac{\sum_{j=1}^{n} D_j}{D_{max}} + (1+r\exp\left(\lambda\frac{D_i}{W_i}\right))D_i \\
& \quad \forall\ i \in 1,\ldots,n
\end{aligned}$$

[1]Andreu Mas-Colell, Michael D. Whinston and Jerry R. Green, Microeconomic Theory, 1995, pp. 696-697

This system has solution $D_i = k \cdot W_i$, where $k = f(W, \Pi, D_{max}, c, t, r, \lambda)$, i.e. each team borrows the fixed proportion of its budget, or, in other words, leverage is the same across all teams. Therefore, probabilities of winning in unconstrained model $P_i(W_1, \dots, W_n) = \frac{W_i}{\sum_{j=1}^n W_j}$ increase in clubs' budget.

Now, in line with FFP regulations, we impose constraint on the amount of debt: $D_i < D_{FFP}, \forall i \in 1, \dots, n$. Let's denote equilibrium debt levels of unconstrained teams $(D_i^*)_{i=1}^n$. There are three cases possible:

1. The constraint is weak: $D_i^* < D_{FFP}, \forall i$, this case is trivial, as nothing changes.

2. The constraint is strong: $D_i^* > D_{FFP}, \forall i$; each team will borrow exactly D_{FFP} and new probability of winning is $\frac{W_i + D_{FFP}}{\sum_{j=1}^n W_j + nD_{FFP}}$; the change in probabilities is $\frac{W_i + D_{FFP}}{\sum_{j=1}^n W_j + nD_{FFP}} - \frac{W_i}{\sum_{j=1}^n W_j} = \frac{nD_{FFP}(\overline{W} - W_i)}{(\sum_{j=1}^n W_j + nD_{FFP}) \sum_{j=1}^n W_j}$, where $\overline{W}$ is the average budget. Thus, teams with budget lower than the average will benefit from such constraint.

   Obviously, the overall debt load decreases, as $nD_{FFP} < \sum_{i=1}^n D_i^*$

3. First $n_0 < n$ teams are unconstrained, other $n - n_0$ teams are not and they borrow D_{FFP}. Note, that the first teams to become constrained are the "rich" ones as they borrow the most in the unconstrained model. However, nothing fundamentally changes as first n_0 teams still borrow the fixed proportion of their budget: $D_i = \widetilde{k} W_i, \forall i \in 1, \dots, n_0$, but the constant is different from original k : $\widetilde{k} = g(W, \Pi, D_{max}, D_{FFP}, c, t, r, \lambda)$. The equation to determine $\widetilde{k}$ is

$$\Pi \cdot \frac{\widetilde{k} + 1}{\sum_{j=1}^n W_j + \widetilde{k} \sum_{j=1}^{n_0} W_j + (n - n_0) D_{FFP}} + t(\widetilde{k} + 1)(\sum_{j=1}^n W_j + \widetilde{k} \sum_{j=1}^{n_0} W_j + \\ + (n - n_0) D_{FFP}) = c\widetilde{k} \cdot \frac{\widetilde{k} \sum_{j=1}^{n_0} W_j + (n - n_0) D_{FFP}}{D_{max}} + \Big(1 + r \exp(\lambda \widetilde{k})\Big) \widetilde{k} \qquad (1)$$

Lemma 1. *The equation 1 has the unique solution* $\widetilde{k}^* > 0$

The proof of Lemma 1 can be found in Appendix. We still haven't ruled out various solutions for specific level of D_{FFP} - though from Lemma 1 we know that $\widetilde{k}$ is unique for the fixed number n_0 of unconstrained teams, this number itself might be non-unique. To deal with this ambiguity we will choose the equilibrium with such number of unconstrained teams n_0, where $\widetilde{k}$ along with overall debt level is at maximum. We could find this number by brute force or with more intelligent algorithm (we provide in appendix the proof of the fact that $\widetilde{k}$ decreases in n_0, as well as debt level) - the steps are following:

1. Set $n_0 := 1$
2. Solve system with n_0 unconstrained teams: $D_i = \widetilde{k}W_i, \forall i \in 1,\ldots,n_0$; $D_i = D_{FFP}, \forall i \in n_0+1,\ldots,n$
3. If $D_i \leq D_{FFP}, \forall i \in 1,\ldots,n_0$ then we have found the solution $n_0, \widetilde{k}$.
4. If for some $i \in 1,\ldots,n_0 : D_i > D_{FFP}$ then we set $n_0 := n_0 + 1$ and go back to the second step of algorithm (if $n_0 > n$, then the solution is $n_0 = 0, D_i = D_{FFP}\, \forall i = 1,\ldots,n$)

Now we want to examine change in a debt level and winning probabilities for this case.

4.1 Debt level

Let's denote debt level D^z, where z is the number of teams that stayed unconstrained. The proof of next result is provided in appendix

Lemma 2. *Compared to unconstrained case, for* $t < \dfrac{\Pi}{4(\sum_{j=1}^{n} W_j)^2}$ *overall debt level falls:* $D^{n_0} < D^n$, *where* $n_0 < n$

The above-mentioned constraint implies that fans do not react too rapidly to the buying spree of clubs and, consequently, do not rush for the tickets after astonishing transfers.

However, change in the *individual* level of debt is not clear for unconstrained teams - for constrained it obviously falls. Next, in case of two equal teams if we're going to limit their borrowing level simultaneously, then their debt level will trivially fall (with sufficiently strict constraint). Suppose we had two different teams - we're going to construct the numerical example, where the debt level of unconstrained team rises:

Example. If we set $W_1 = 1, W_2 = 9, \Pi = 5, D_{max} = 10, r = 0.05, t = 0.1, c = 5, \lambda = 0.1$, then the equations for unconstrained and constrained cases will look like:

$$\frac{1}{2} + (k+1)^2 = 5k^2 + k\left(1 + 0.05e^{0.1k}\right)$$

$$\frac{5(\widetilde{k}+1)}{10+\widetilde{k}+D_{FFP}} + 0.1(\widetilde{k} + 1)(10+\widetilde{k}+D_{FFP}) = = 5\widetilde{k}\frac{\widetilde{k}+D_{FFP}}{10} + \widetilde{k}\left(1 + 0.05e^{0.1\widetilde{k}}\right)$$

The solution for unconstrained case $k \approx 0.741957 \Rightarrow D_2 \approx 6.68$.
For different D_{FFP} we received the following results:

D_{FFP}	2	3	4	5	6
$\widetilde{k}$	1.55	1.26	1.05	0.91	0.80
$D_1 + D_2$	3.55	4.26	5.05	5.91	6.8

Thus, we observe that $\widetilde{k}$ and therefore the debt of first team decreases in D_{FFP} and overall debt level increases in D_{FFP} .

4.2 Winning Probabilities

In this section we'll examine the impact of FFP on individual winning probabilities: probability of winning was $\frac{W_i}{\sum_{j=1}^{n} W_j}$ for all teams, now it is $\frac{W_i + D_{FFP}}{\sum_{j=1}^{n} W_j + \widetilde{k} \sum_{j=1}^{n_0} W_j + (n - n_0) D_{FFP}}$ for constrained teams and $\frac{W_i + \widetilde{k} W_i}{\sum_{j=1}^{n} W_j + \widetilde{k} \sum_{j=1}^{n_0} W_j + (n - n_0) D_{FFP}}$ for unconstrained. The following lemma states results about changes in individual probabilities, and its proof is provided in appendix

Lemma 3. *Change in winning probabilities is:*
1) > 0 *for all unconstrained teams and*
2) < 0 *for constrained teams if* $W_i > \overline{W(n_0 + 1, n)}$

We can combine the results of two previous lemmas in the following proposition:

Proposition 1.

1. *Overall debt load decreases and, therefore, total level of credit risk is lower than previously, as teams will declare bankruptcy less frequently.*

2. *"Poor" teams benefit from new regulations as their probability of winning (which is the sole objective of football club) increases, while probability of winning of relatively "rich" teams (which budget is higher than the average in the subset of constrained teams) decreases compared to the case without any regulation.*

First of all, decreased debt load will reduce the number of bankruptcies and, therefore, improve long-term viability of European club football, which is one of the objectives set by UEFA.

Although "poor" clubs will win more often than previously, it's not clear how new regulations will affect "middle" teams and, consequently, the overall distribution of winning probabilities. We'll provide general result in next section.

4.3 Lorenz curve

It's time to observe how the distribution of winning probabilities is going to be affected by FFP. We can strengthen the second result in terms of Lorenz curve and Lorenz dominance.

Definition. *Lorenz curve is a graphical representation of the distribution of winning probabilities, where for any proportion p between zero and one, the ordinate of the corresponding point on the Lorenz curve for a given distribution is the proportion of teams that accrues to the first* $100p$ *percent of clubs, which are sorted in order of increasing winning probabilities*

Definition. *If for any point p Lorenz curve of distribution 1 lies higher than Lorenz curve of distribution 2, then distribution 1 is said to Lorenz dominate distribution 2*

The proof of next result is provided in appendix

Theorem 1. *Distribution of winning probabilities under FFP Lorenz dominates distribution of winning probabilities without FFP*

From this result we get that teams become more equal in their probabilities of winning. The quantitative measure of inequality is Gini coefficient, where 0 means perfect equality, while Gini coefficient of 1 implies complete inequality. The straight line would represent Lorenz curve for the distribution of teams which are equally likely to win the tournament - in other words, it's the line of "perfect equality". Gini coefficient is the ratio of the area between Lorenz curve and the line of perfect equality over the area under the line of perfect equality. It's easy to see that Lorenz dominance of distribution 1 over distribution 2 implies $Gini_1 < Gini_2$. Many papers in sports economics (e.g. Fort and Quirk [1995]) measure competitive balance of various sports leagues by comparing Gini coefficients of corresponding distributions of winning probabilities - where lower Gini coefficient corresponds to increased competitive balance, so we can state the main result:

Proposition 2. *UEFA Financial Fair Play improves competitive balance of European football*

Suppose we have two distributions F, G with finite support $[a; b]$

Definition. *Distribution F is said to second order stochastically dominate distribution G if and only if* $\forall t \geq a : \int_a^t F(x)dx \leq \int_a^t G(x)dx$

Second-order stochastic dominance is equivalent to generalized Lorenz curve dominance (e.g. Thistle [1989]), where generalized Lorenz curve is just the usual Lorenz curve scaled by the mean of distribution, and its dominance can be defined in similar way with Lorenz dominance. Noting that the mean for all distributions of winning probabilities equal to $\frac{1}{n}$ and using Theorem 1, we have proved the next important result

Corollary 1. *Distribution of winning probabilities under FFP second-order stochastically dominates distribution of winning probabilities without FFP*

From social welfare perspective any planner with increasing and concave utility would prefer distribution F over distribution G if F second-order stochastically dominates distribution G. That is to say, new distribution of winning probabilities under FFP is socially preferable too.

5 Market Entry

In this section we're going to study the effects of new regulations on individuals or organizations considering a purchase of a football club. We decompose investor's problem in two-stage game: at first stage he chooses in which club he'll acquire stake; at second stage he'll make decision about the level of borrowing (behaviour of the investor at this stage is completely described in previous chapter). At first stage we have only one player - the investor, which set of actions consists of choosing a team $i \in 1, \ldots, n$. As clubs in our model have zero expected profits, it's natural to assume that investor wants to buy a club with maximal winning probability P_i, which is going to be investor's "value of project"; we assume that his payoff $\mu(P_i)$ is weakly convex in $P_i : \mu(\cdot)' > 0$, $\mu(\cdot)'' > 0$. We can model this game in similar way with "investment tournaments" of Schwarz and Severinov [2009]. They showed that when decision-maker's payoff is weakly convex in the value of project, then the optimal strategy is "to invest all resources in each time period into one favorite alternative". In our game investor is constrained (he has funds I) and wants to acquire at least $\alpha \geq \alpha_0$ stake in club. We assume that clubs have valuations $f(W_i)$, where f is increasing function - in other words clubs with higher budgets are more valuable, as they are expected to win more frequently. We could write down the investor's problem:

$$\begin{aligned} P_i(m_1, \ldots, m_n) &\rightarrow \max_{i \in 1,\ldots,n} \\ \textbf{s.t. } \alpha f(W_i) &\leq I \\ \alpha &\geq \alpha_0 \end{aligned}$$

In our model: $W_1 < \ldots < W_n$, and corresponding winning probabilities $P_1 < \ldots < P_n$ before and after FFP (as we've found in Theorem 1). Also FFP wouldn't affect investor's budget constraint, so investor still would choose the same team $i = \max_{j \in 1,\ldots,n} \left[j : W_j \leq f^{-1}\left(\frac{I}{\alpha_0}\right)\right]$. Though FFP doesn't impact the ranking of teams in terms of winning probabilities, their values are influenced by new regulations. According to Lemma 3, as a result of FFP, winning probabilities of unconstrained teams have increased, while effect on constrained and relatively richer teams is opposite. This lemma can be summarized in the following picture (where j is such that $W_j < \overline{W(n_0+1, n)}$, but $W_{j+1} > \overline{W(n_0+1, n)}$). Though the effect on "middle" teams is

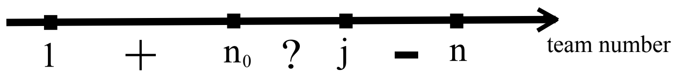

Figure 1: FFP impact on winning probabilities

uncertain, it's clear that investor's utility from investing in teams $\in 1, \ldots, n_0 \cup M$ increased while utility from investing in teams $\in n_0+1, \ldots, n \setminus M$ decreased, where $M = \{l \in n_0+1, \ldots, j : P_l \text{ increased after FFP}\}$. All in all, as our teams sorted in budgets, we can state the following result:

Proposition 2. *FFP regulations increase incentives for investors in teams with lesser budgets ("poor" teams)*

This result reflects the fact that "attractiveness" of rich teams will fall and "attractiveness" of poor teams will rise for sure. Additionally, if we assume that investors have their own reservation utilities (as they usually do), then the change of winning probabilities due to FFP might throw some investors in "rich" clubs out of the game, while attract new investors in "poor" teams, which will even further level playing field in European football.

6 Conclusion

In this paper we've built game theoretic model to illustrate how capital structure of a football club is affected by its own and peers' investment decisions. On the one hand, increasing debt by one team implies increased winning probability for this team and increased ticket sales for all teams - i.e. more investments into the line-up leads to more strong team, and fans prefer to watch matches with such clubs. From the other hand, debt growth leads to higher interest payment (as the interest rate increases in the amount of debt) and, more importantly, to increased probability of bankruptcy, which in turn magnifies expected bankruptcy penalties for all teams. We've showed that in the equilibrium each team equalizes its expected revenues and costs, and borrows a fixed proportion of its budget; that's leverage is the same for all clubs.

Next we introduced FFP regulation by imposing the constraint on the amount of debt. In the resulting equilibrium constrained teams borrow maximum allowed amount, while unconstrained teams still borrow fixed ratio of their budget; though the leverage is different from the one in the equilibrium without FFP. We showed that for unconstrained team the winning probability is greater than it was before. Under reasonable assumption (ticket sales do not react too drastically to the investments of clubs) overall debt load falls, and therefore systemic risk in European football decreases. We proved the main result that competitive balance in UEFA competitions improves after FFP introduction. Finally, we showed that FFP will attract new investors in football clubs with lesser budgets and will further level playing field.

In our further research we plan to study the impact of FFP regulations on players' wages, especially high-earning superstars. Debate about football players' wages inflation is of a great interest at the moment. While UEFA authorities claimed that overall losses declined by €600m after the introduction of FFP,[4] players' wages continue to grow (though at slower pace than clubs' revenues). Meanwhile, Real Madrid superstar Cristiano Ronaldo signed staggering five-year contract with annual after tax salary of €17m. Therefore, the issue of growing wages appears to be a very interesting problem for a further study.

[4] http://www.uefa.org/footballfirst/protectingthegame/financialfairplay/news/newsid=1988099.html

Appendix

Lemma 1. *The equation 1 has the unique solution* $\widetilde{k}^* > 0$

Proof. Derivative of left hand side with respect to $\widetilde{k}$ is positive for $\widetilde{k} \geq 0$:

$$\Pi \cdot \frac{\sum_{j=n_0+1}^{n} W_j + (n-n_0)D_{FFP}}{(\sum_{j=1}^{n} W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n-n_0)D_{FFP})^2} + t(\widetilde{k}+1)\sum_{j=1}^{n_0} W_j +$$

$$+t(\sum_{j=1}^{n} W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n-n_0)D_{FFP}) > 0$$

Derivative of right hand side with respect to $\widetilde{k}$ is also positive for $\widetilde{k} \geq 0$:

$$c\frac{\widetilde{k}\sum_{j=1}^{n_0} W_j + (n-n_0)D_{FFP}}{D_{max}} + c\widetilde{k}\frac{\sum_{j=1}^{n_0} W_j}{D_{max}} +$$

$$+\Big(1 + r\exp(\lambda\widetilde{k})\Big) + \lambda r\exp(\lambda\widetilde{k})\widetilde{k} > 0$$

Right hand side equals zero for $\widetilde{k} = 0$, while left hand side:

$$\frac{\Pi}{\sum_{j=1}^{n} W_j + (n-n_0)D_{FFP}} + t(\sum_{j=1}^{n} W_j + (n-n_0)D_{FFP}) > 0$$

Left and right hand sides both increase in $\widetilde{k} > 0$, but right hand side grows "faster" because of exponent ⇒ two curves must intersect once, and unique solution $\widetilde{k}^* > 0$ of the equation exists. □

Lemma 1.1. *Suppose that for one level of* D_{FFP} *there are two solutions:* $\widetilde{k}, \widehat{k}$ *with number of unconstrained teams* $n_{01} < n_{02}$ *correspondingly. Then it must be* $\widetilde{k} > \widehat{k}$ *or, in other words, leverage decreases in number of unconstrained teams. Additionally,* $D^{n_{01}} > D^{n_{02}}$*.*

Proof. Let's look at teams $i \in n_{01+1}, \ldots, n_{02}$ - they stay unconstrained in the second case, but become constrained in the first case. Consequently, following inequalities must hold $\forall i \in n_{01+1}, \ldots, n_{02}$:

$$\begin{aligned} \widetilde{k}W_i &> D_{FFP} \\ \widehat{k}W_i &< D_{FFP} \end{aligned}$$

It immediately follows that $\widetilde{k} > \widehat{k}$. Turning to debt levels:

$$\begin{aligned} D^{n_{01}} &= \widetilde{k}\sum_{i=1}^{n_{01}} W_i + (n - n_{01})D_{FFP} \\ D^{n_{02}} &= \widehat{k}\sum_{i=1}^{n_{02}} W_i + (n - n_{02})D_{FFP} \end{aligned} \tag{2}$$

Thus, the difference is $D^{n_{01}} - D^{n_{02}} = (\widetilde{k} - \widehat{k})\sum_{i=1}^{n_{01}} W_i + (n_{02} - n_{01})D_{FFP} - \widehat{k}\sum_{i=n_{01}+1}^{n_{02}} W_i > 0$, as $\widetilde{k} > \widehat{k}$ and $\widehat{k}W_i < D_{FFP}\,\forall\, i \in n_{01+1}, \ldots, n_{02}$ (these teams are unconstrained). □

Lemma 2. *Compared to unconstrained case, for* $t < \dfrac{\Pi}{4(\sum_{j=1}^{n} W_j)^2}$ *overall debt level falls:* $D^{n_0} < D^n$*, where* $n_0 < n$

Proof. Let's see how overall debt level changes compared to the case with n teams. Debt levels:

$$D^{n_0} = \widetilde{k}\sum_{j=1}^{n_0} W_j + (n - n_0)D_{FFP}$$

$$D^n = k\sum_{j=1}^{n} W_j$$

Change in debt

$$D^{n_0} - D^n = \left((n - n_0)D_{FFP} - k\sum_{j=n_0+1}^{n} W_j\right) + (\widetilde{k} - k)\sum_{j=1}^{n_0} W_j$$

If $\widetilde{k} < k$, then the first term is negative as if for some team $i \in n_0+1, \ldots, n : \widetilde{k}W_i < kW_i < D_{FFP}$ then we would have that more than n_0 teams were unconstrained because $W_1 < \ldots < W_n \Rightarrow$ for $\widetilde{k} < k$ it follows that $D^{n_0} - D^n < 0$.

If $\widetilde{k} > k$ then we'll prove by contradiction. Let's assume that $D^{n_0} > D^n$. The equations to find k and $\widetilde{k}$ are:

$$\begin{aligned} \Pi \cdot \frac{\widetilde{k} + 1}{\sum_{j=1}^{n} W_j + D^{n_0}} + t(\widetilde{k} + 1)(\sum_{j=1}^{n} W_j + D^{n_0}) &= c\widetilde{k}\frac{D^{n_0}}{D_{max}} + \left(1 + r\exp[\lambda\widetilde{k}]\right)\widetilde{k} \\ \Pi \cdot \frac{k + 1}{\sum_{j=1}^{n} W_j + D^n} + t(k + 1)(\sum_{j=1}^{n} W_j + D^n) &= ck\frac{D^n}{D_{max}} + (1 + r\exp[\lambda k])\,k \end{aligned}$$

which we can rewrite as

$$1 + \frac{1}{\widetilde{k}} = \frac{c\dfrac{D^{n_0}}{D_{max}} + 1 + r\exp[\lambda \widetilde{k}]}{\dfrac{\Pi}{\sum_{j=1}^{n} W_j + D^{n_0}} + t(\sum_{j=1}^{n} W_j + D^{n_0})}$$

$$1 + \frac{1}{k} = \frac{c\dfrac{D^{n}}{D_{max}} + 1 + r\exp[\lambda k]}{\dfrac{\Pi}{\sum_{j=1}^{n} W_j + D^{n}} + t(\sum_{j=1}^{n} W_j + D^{n})}$$

If $\widetilde{k} > k > 0 \Rightarrow 1 + \frac{1}{\widetilde{k}} < 1 + \frac{1}{k} \Rightarrow$

$$\frac{c\dfrac{D^{n_0}}{D_{max}} + 1 + r\exp[\lambda \widetilde{k}]}{\dfrac{\Pi}{\sum_{j=1}^{n} W_j + D^{n_0}} + t(\sum_{j=1}^{n} W_j + D^{n_0})} < \frac{c\dfrac{D^{n}}{D_{max}} + 1 + r\exp[\lambda k]}{\dfrac{\Pi}{\sum_{j=1}^{n} W_j + D^{n}} + t(\sum_{j=1}^{n} W_j + D^{n})}$$

As

$$c\frac{D^{n_0}}{D_{max}} + 1 + r\exp[\lambda \widetilde{k}] > c\frac{D^{n}}{D_{max}} + 1 + r\exp[\lambda k]$$

then it must be

$$\frac{\Pi}{\sum_{j=1}^{n} W_j + D^{n_0}} + t(\sum_{j=1}^{n} W_j + D^{n_0}) > \frac{\Pi}{\sum_{j=1}^{n} W_j + D^{n}} + t(\sum_{j=1}^{n} W_j + D^{n})$$

But if function $\frac{\Pi}{\sum_{j=1}^{n} W_j + D} + tD$ decreases in D ($\Leftrightarrow t < \frac{\Pi}{(\sum_{j=1}^{n} W_j + D)^2}, \forall D$) then the above inequality is wrong. We are free to choose our parameters to calibrate the model - in real life overall debt level is smaller than the sum of all budgets $\Rightarrow$ our condition becomes $t < \frac{\Pi}{4(\sum_{j=1}^{n} W_j)^2}$. The proof is completed. □

Lemma 3. *Change in winning probabilities is:*
1) > 0 *for all unconstrained teams and*
2) < 0 *for constrained teams if* $W_i > \overline{W(n_0+1, n)}$

Proof.

1. the difference of probabilities for unconstrained team is

$$\frac{(\widetilde{k}\sum_{j=n_0+1}^{n} W_j - (n-n_0)D_{FFP})}{\sum_{j=1}^{n} W_j(\sum_{j=1}^{n} W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n-n_0)D_{FFP})}$$

As $D_{FFP} - \widetilde{k}W_j < 0, \forall j = n_0+1, \ldots, n$ (they are all constrained) - we get that this difference is positive or, in other words, that unconstrained (relatively "poor") teams benefit from the introduction of such constraint.

2. the difference of probabilities for constrained team is

$$\frac{(D_{FFP} - \widetilde{k}W_i)\sum_{j=1}^{n_0} W_j + D_{FFP}(\sum_{j=n_0+1}^{n} W_j - (n-n_0)W_i)}{\sum_{j=1}^{n} W_j(\sum_{j=1}^{n} W_j + \widetilde{k}\sum_{j=1}^{n_0} W_j + (n-n_0)D_{FFP})}$$

As $D_{FFP} - \widetilde{k}W_i < 0$ (i-th team is constrained), we see that if $\sum_{j=n_0+1}^{n} W_j - (n-n_0)W_i < 0$ or $W_i > \overline{W(n_0+1, n)}$ - the average budget among constrained teams, then probability of winning decreases. In other words, it decreases for relatively "rich" teams; but for other teams among constrained it's unclear how the probability of winning will change.

□

Theorem 1. *Distribution of winning probabilities under FFP Lorenz dominates distribution of winning probabilities without FFP.*

Proof. Total investments before FFP - $\sum_{i=1}^{n}(W_i + D_i^*) = (k+1)\sum_{i=1}^{n} W_i$
Total investments after FFP (assuming first n_0 clubs are unconstrained) - $\sum_{i=1}^{n} W_i + \widetilde{k}\sum_{i=1}^{n_0} W_i + (n-n_0)D_{FFP}$
Winning probabilities before FFP increase in budget so we can sort teams in this order. For the same probabilities after FFP we can sort teams in the same order as probability of winning of unconstrained team - $\frac{\widetilde{k}W_i}{\sum_{i=1}^{n} m_i}$ - trivially increases in budget (we denote m_i an investment of ith team under FFP regulations); the same is true for constrained team as the probability equals to $\frac{W_i + D_{FFP}}{\sum_{i=1}^{n} m_i}$. We need to show that $\frac{(\widetilde{k}+1)W_{n_0}}{\sum_{i=1}^{n} m_i} < \frac{W_{n_0+1} + D_{FFP}}{\sum_{i=1}^{n} m_i}$ to justify our ordering for all teams: this inequality holds as $\frac{W_{n_0+1} - W_{n_0} + D_{FFP} - \widetilde{k}W_{n_0}}{\sum_{i=1}^{n} m_i} > 0$ because $W_{n_0+1} > W_{n_0}$ and $D_{FFP} - \widetilde{k}W_{n_0} > 0$ (team n_0 was unconstrained)
Before FFP proportions G_k of winning teams (as we study discrete distribution) were

$$\frac{W_1}{\sum_{i=1}^{n} W_i}, \frac{W_1 + W_2}{\sum_{i=1}^{n} W_i}, \ldots, \frac{\sum_{i=1}^{n-1} W_i}{\sum_{i=1}^{n} W_i}, 1$$

Let's denote corresponding proportions after FFP as

$$F_i = \frac{\sum_{j=1}^{i} m_j}{\sum_{j=1}^{n} m_j}, \forall i = 1, \ldots, n$$

1. $\forall i = 1, \ldots, n_0$ obviously $F_i > \frac{\sum_{j=1}^{i} W_j}{\sum_{j=1}^{n} W_j} = \frac{W_1}{\sum_{j=1}^{n} W_j} + \ldots + \frac{W_i}{\sum_{j=1}^{n} W_j}$, as we have proved that the difference of probabilities of winning for unconstrained team is positive or that $\frac{m_k}{\sum_{j=1}^{n} m_j} > \frac{W_k}{\sum_{i=1}^{n} W_i}, \forall k = 1, \ldots, n_0$

2. For $m \in n_0+1,\dots,n$ let's compute the difference of F_m and G_m:

$$F_m - G_m = \frac{\sum_{i=1}^{m} W_i + \widetilde{k}\sum_{i=1}^{n_0} W_i + (m-n_0)D_{FFP}}{\sum_{i=1}^{n} W_i + \widetilde{k}\sum_{i=1}^{n_0} W_i + (n-n_0)D_{FFP}} - \frac{\sum_{i=1}^{m} W_i}{\sum_{i=1}^{n} W_i} =$$

$$= \frac{\widetilde{k}\sum_{i=1}^{n_0} W_i \sum_{i=m+1}^{n} W_i + D_{FFP}((m-n_0)\sum_{i=1}^{n} W_i - (n-n_0)\sum_{i=1}^{m} W_i)}{(\sum_{i=1}^{n} W_i + \widetilde{k}\sum_{i=1}^{n_0} W_i + (n-n_0)D_{FFP})\sum_{i=1}^{n} W_i}$$

Now we consider the numerator:

$$\widetilde{k}\sum_{i=1}^{n_0} W_i \sum_{i=m+1}^{n} W_i + D_{FFP}\left((m-n_0)\sum_{i=1}^{n} W_i - (n-n_0)\sum_{i=1}^{m} W_i\right) =$$

$$= \sum_{i=1}^{n_0} W_i\left(\widetilde{k}\sum_{i=m+1}^{n} W_i - (n-m)D_{FFP}\right) + (n-m)D_{FFP}\sum_{i=1}^{n_0} W_i +$$

$$+ D_{FFP}\left((m-n_0)\sum_{i=1}^{n} W_i - (n-n_0)\sum_{i=1}^{m} W_i\right) = \sum_{i=1}^{n_0} W_i\left(\widetilde{k}\sum_{i=m+1}^{n} W_i - (n-m)D_{FFP}\right) +$$

$$+ D_{FFP}\left((n-m)\sum_{i=1}^{n_0} W_i + (m-n_0)\sum_{i=1}^{n} W_i - (n-n_0)\sum_{i=1}^{m} W_i\right) =$$

$$= |\text{sums up to } n_0 \text{ cancel out}| = \sum_{i=1}^{n_0} W_i\left(\widetilde{k}\sum_{i=m+1}^{n} W_i - (n-m)D_{FFP}\right) +$$

$$+ D_{FFP}\left((m-n_0)\sum_{i=n_0+1}^{n} W_i - (n-n_0)\sum_{i=n_0+1}^{m} W_i\right)$$

The first term is positive as teams $i \in m+1,\dots,n$ are constrained, i.e. $\widetilde{k}W_i > D_{FFP}$. The second term is positive only if $\frac{\sum_{i=n_0+1}^{m} W_i}{m-n_0} < \frac{\sum_{i=n_0+1}^{n} W_i}{n-n_0}$ or if $\overline{W(n_0+1,m)} < \overline{W(n_0+1,n)}$. But the budget set is ordered: $W_1 < W_2 < \dots < W_{n-1} < W_n \Rightarrow$ this inequality for average budgets holds as long as $m < n$. Finally, we get that $F_m > G_m, \forall m \in n_0+1,\dots n-1$. For $m = n:\ F_n = G_n = 1$.

Therefore we get the result that "new" distribution F *Lorenz dominates* "old" distribution G.

□

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