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Title: Capital budgeting and managerial compensation: the effect of division managers' preferences over the set of investment projects

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Abstract

In this paper I study the effect of a division manager's preferences over the set of investment projects on the optimal capital budgeting mechanism within a company. This paper belongs to the stream of literature, that uses mechanism design approach to study capital budgeting procedures. It is common in this literature to consider a division manager, who has access to only one project. However, this setup does not allow to consider manager's preferences over projects. Thus, in order to study the effect of these preferences on capital budgeting and managerial compensation, I consider a division manager, who has information about two investment projects in an environment, where headquarters uses compensation scheme as an incentives devise for the manager to make her report truthfully. In addition, I assume that the headquarters is uninformed and has enough capital to finance only one project. The preferences of the manager are represented by the fact that she can extract private benefits from the projects and the amount of this private benefits differs across these projects. Therefore, such preferences create a conflict of interests between the manager and the headquarters. In this setup I derive the general form of the contract, consisting of managerial compensation and project selection rule, and investigate its properties. In addition, I provide an explicit analytical solution to the mechanism under the assumption that the projects' qualities are uniformly distributed. For this case, I derive implications of the optimal

mechanism for the expected compensation of the manager.

1 Introduction

Investment decisions of companies are important determinants of their future performance. Since typically division managers of a firm are better informed about the prospects of investment projects than the headquarters, the value of company depends on how this information is used during the process of decision making. Therefore a problem of the headquarters is to design a mechanism of capital allocation within the company, which would address this asymmetry of information. This paper belongs to the stream of literature, that uses mechanism design approach to study capital budgeting procedures. While it is common in this literature to consider a case when each division has access to only one project, I consider a manager, who has access to two projects, in order to study the effect of manager's preferences.

In particular, I consider a single-division company and a division manager, who has access to two investment projects and has some preferences over them, which described by the fact that manager can extract private benefits from these projects. Moreover, I assume that the greater the quality of the given project, the larger the amount of private benefits, manager can extract from it, and that this relationship has a linear functional form. Given this assumption, it is useful to define a marginal private benefit as an additional amount of private benefits, that manager receives if project quality increases by one unit. In this case, if qualities of the projects are equal, manager would prefer project with greater marginal private benefits. These preferences of the manager create a conflict of interest between her and the headquarters. The headquarters is uninformed and has to rely on a manager's report about the projects' qualities to make a financing decision. I also assume that the headquarters can finance only one project. In this setup, I look for optimal contract, consisting of a project selection rule and a managerial compensation scheme.

I derive several properties of the optimal mechanism in a general case. I show that optimal project selection rule must have the following form: report of the manager about the first project is compared with some linear function of the manager's report about the second project. The slope of this linear function, is determined by marginal private benefits of the projects; and the intercept is optimally chosen by the headquarters. In addition, I show that, if one of the projects, let's say first one, has greater marginal private benefits than the other (second one), optimal project selection rule has bias towards the second project, when reported project qualities are low, and it has bias to the first project, when reported project qualities are high. Moreover, the optimal managerial compensation contract must be represented by two payments in this case: zero if the first project is selected and some positive wage if the second project is financed.

I also prove that optimal contract should ignore reports of the manager about projects, from which she can not extract private benefits. The result is driven by the fact, that in this case utility of the manager does not depend on true quality of the projects. Thus, if a contract is sensitive to the report of the manager about these projects, headquarters can not provide incentives for the manager to make her report truthfully.

In addition, I provide an analytical solution for the optimal mechanism under the assumption that the projects' qualities are independent and drawn from a uniform distribution. In this case I show that difference in the manager's preferences over the projects, i.e. a bias of the manager to some project, leads to optimal ex-ante bias of headquarters to select this project and higher expected payment to the manager.

This paper is most closely related to the literature on capital budgeting, that uses mechanism design approach to describe capital budgeting procedures of companies as optimal responses to asymmetry of information and conflicts of interest between headquarters and division managers. Harris and Raviv (1996, 1998) study the role of audit in capital budgeting. Harris et al (1982) and Antle and Eppen (1985) focus on transfer prices; Bernardo et al (2001, 2004, 2006) similar to my paper consider managerial compensation as incentive devise.

Harris et al (1982) considers a company that has one "resource producing" division, that uses capital to produce some resource. This resource is used by other divisions in the company to produce intermediate goods. Final output is manufactured from these intermediate goods. Each division is run by a division manager, who privately observes productivity of her division. Conflict of interests between manager and headquarters arises, because manager receives disutility from exerting effort. Hence, given that headquarters demand division to produce some level of output, manager has incentives to understate productivity of capital in her division to obtain more capital and work less, since effort and capital are substitutes in this model. In this setup the authors look for direct revelation mechanism, consisting of resource allocation scheme, amount of output, that headquarters demand division to produce, and reward, that division manager receives. The authors also claim that optimal direct revelation mechanism, that they found, is equivalent to a mechanism, where each division is allocated with initial budget and headquarters announces a set of transfer prices, from which divisions can choose; in addition resource allocation rule, that depends on the divisions' choice of transfer prices, is announced. Given that division has chosen some transfer price, it buys amount of resource, allocated to them, for this price. All remaining budget goes to a division manager as a reward.

Antle and Eppen (1985) consider a somewhat similar model to Harris et al (1982).

Their model has a single-division firm, where productivity of capital in the division is private knowledge of the manager. In this setup Antle and Eppen look for optimal mechanism consisting of capital allocation and required return. Manager has opportunity to invest in the project only a part of allocated capital, if it is sufficient to provide required return. The rest of the budget the manager keeps for herself. This opportunity creates incentives for the manager to understate productivity of capital in the division, in order to increase amount of capital that she able to divert to herself. The authors find that optimal mechanism represent a hurdle rate policy with required rate of return above firm's cost of capital, i.e. it is optimal for headquarter to offer a contract that require this hurdle rate if manager choose to ask for positive amount of capital. They also reinterpret their mechanism as a scheme, where division manager can borrow capital from headquarters at optimal hurdle rate, which serves as a transfer price.

Harris and Raviv (1996) consider a single-division firm with manager, who privately observe quality (productivity) of investment project. A conflict of interests between division manager and headquarter arise due to "empire-building" preferences of the manager. This preferences represent the fact that the manager may extract private benefits from the project she runs. Therefore, the manager prefer more capital to less, all other things equal. In this setup the authors look for optimal contract, consisting of capital allocation rule and probability to audit the project. They show that optimal capital budgeting mechanism should include initial spending account, allocated to the manager, and opportunity of the manager to request additional capital above this budget. In the latter case the project is audited with some probability, and in case of audit optimal amount of capital is allocated[1], otherwise some compromise level is chosen. Harris and Raviv (1998) extend this analysis to the case when manager has access to two investment projets. In this setup they show that, in general, optimal contract has quite similar properties in term of existence of initial spending account and opportunity of manager to ask headquarters for additional amount of capital.

Bernardo, Cai, Luo (2001) also consider single-division firm, with manager that privately observes project quality. Conflict of interests, similarly to Harris and Raviv, is created by "empire-building" preferences of the manager. However, Bernardo at al consider compensation contract of the manager, rather than audit, as an incentives devise. In addition, the authors assume that the division manager can exert costly unverifiable effort to increase project cash flow. Due to this feature, optimal compensation scheme includes profit sharing in division's cash flow. Bernardo, Cai, Luo (2004) extends this analysis to a case of two-division firm (in this case company

[1]if audit reveals that manager lied about project's quality no capital is allocated, however, it never happens in equilibrium

has two division managers and each manager privately observes only quality of the project in her own division)

Bernardo, Luo, Wang (2006) consider a model with a two-division firm, which is similar to Bernardo, Cai, Luo (2004). However, the divisions of the company are described as weak and strong in term of distributions from which project qualities are drawn. Additional assumption is that headquarters has enough capital to finance only one of this two divisions. In this setup, the authors show that optimal contract, consisting of project selection rule and managerial compensation scheme, implies optimal bias towards weak division and compensation contract with profit sharing in the division that manager runs.

My paper contributes to the literature, by considering the role of managerial compensation contract in capital budgeting procedures, when division manager has access to two projects. Note, that usually in the literature it is assumed that each division has only one project (for example Bernardo et al (2001,2004,2006), Harris et al (1982), Antle and Eppen (1985)), thus these setups do not allow to study effect of division managers' preferences over projects. A case when division manager has access to two project is considered in Harris and Raviv (1998), however they focus on the role of auditing and headquarters do not use managerial compensation as incentive devise.

Finally, like all papers mentioned above, my model considers a single-period capital budgeting setup. For multi-period dynamic communication between division manager and headquarters see, for example, Malenko (2012)

The rest of the paper organized as follows: section 2 presents the model, section 3 analyzes properties of optimal contract in general case; section 4 present a particular case, when project qualities are uniformly distributed, and demonstrates implications of optimal mechanism for project selection and managerial compensation scheme; section 5 concludes.

2 Model

Let's consider a risk-neutral headquarters that runs a company with one division, and this division has two possible projects to finance. The quality of these projects is unknown for the headquarters. Therefore the headquarters hires a risk-neutral manager, who knows both project qualities. In addition, we assume that headquarters can finance only one project (for example, due to limited amount of capital available for the company). The headquarters objective is to maximize company's profit. The profit is calculated as revenue from the chosen project minus managerial compensation. There are two instruments that headquarters can use to achieve its goals. First one is project selection rule and the second one is managerial compensation scheme. We will look for optimal contract, that headquarters should offer to the manager. The headquarters should choose project selection rule and managerial compensation as a functions of manager's report about project qualities to form a contract (as usual, applying revelation principle, we restrict our attention only to direct revelation mechanisms).

We assume that i^{th} project's cash flow equal to t_i if it is financed, and zero otherwise. Here t_i stands for i-th project quality. t_1 and t_2 are independently drawn from $[0, \bar{t}]$ according to the same cdf: $\Phi(t) = \Phi_1(t) = \Phi_2(t)$. Our additional assumption is that realized cash flow is not contractible. For example, one of justifications for such assumption can be the fact that projects are long-term and cash-flow will be realized only in distant future, when this particular manager is likely to leave the company. This assumption is quite plausible for some industries like pharmaceutical one.

We denote project selection rule by $p(\hat{t}_1, \hat{t}_2)$, where $\hat{t}_1$,$\hat{t}_2$ are reports of the manager about first and second project qualities. $p(\hat{t}_1, \hat{t}_2) = 1$ means that the first project (with quality t_1) is financed and $p(\hat{t}_1, \hat{t}_2) = 0$ means that capital is allocated to the second project. Note that project selection rule, that we consider, is deterministic[2].

[2]We do not consider probabilistic project selection rules. Using such rule headquarters can specify some probability to choose the project. In contrast, using deterministic project selection rules, headquarters simply selects one of the two projects. Some rationale for this restriction can be provided by the fact, that in a paper by Bernardo, Luo, Wang (2006) the authors look for solution in

Let's denote compensation of the manager by $a(\hat{t}_1, \hat{t}_2)$. In this case manager's utility function is given by

$$U(\hat{t}_1, \hat{t}_2, t_1, t_2) = a(\hat{t}_1, \hat{t}_2) + p(\hat{t}_1, \hat{t}_2)\delta_1 t_1 + (1 - p(\hat{t}_1, \hat{t}_2))\delta_2 t_2 \quad (1)$$

From this function we see that manager gets utility not only from salary $a(\hat{t}_1, \hat{t}_2)$ but also from the fact that particular project is financed: $\delta_i t_i$. We can interpret this term in the utility as the fact that manager can extract private benefits from the projects and amount of this private benefits is proportional to project cash flow t_i. δ_1 and δ_2 will be referred as marginal private benefits of the projects. Note, that in case $\delta_1 \neq \delta_2$ manger can extract more private benefits from one project than from another, if their qualities are the same. However, since the amount of private benefits depends on the quality of the project, manager sometimes prefer project with lower δ, depending on particular realization of (t_1, t_2).

Since our setup is essentially symmetric with respect to the first and second projects, without loss of generality we can assume that $\delta_1 \geq \delta_2$.

The fact that manager can extract private benefits from the projects creates conflict of interest between the headquarters and the manager, since manager wants

class of mechanisms with probabilistic project selection rule, but they find that the optimal contract has project selection rule that is deterministic. Proposition 5 also provides some motivation for our restriction to deterministic project selection rules to overstate quality of the project that she "likes" more, to obtain capital for it. The idea that manager can extract private benefits from the projects is similar to "empire-building" preferences, usually used in the literature. Classical "empire-building" preferences are characterized by the fact that the more capital is allocated to the division of the manager, the more private benefits manager gets. However, in our setup all available capital is allocated to the selected project, and thus amount of capital is fixed. Instead in our model manager receives more private benefits in more profitable divisions. These private benefits is not necessarily monetary ones: for example, it may be reputational effect from running profitable divisions.

In our model manager has some reservation utility $\bar{U} \geq 0$. In addition, we assume that manager learns project qualities, before she is offered the contract. Therefore, in our mechanism manager has to receive utility above reservation one for any realization of (t_1, t_2). To simplify analysis, let's assume that $\bar{U} = 0$

Headquarters objective function is:

$$\Pi = \int_0^{\bar{t}} \int_0^{\bar{t}} [p(t_1, t_2)t_1 + (1 - p(t_1, t_2))t_2 - a(t_1, t_2)] \, d\Phi(t_2) \, d\Phi(t_1) \tag{2}$$

Headquarters tries to maximize its profit, which is given by revenue from the selected project minus payment to the manager.

Now we can state our mechanism design problem:

$$\int_0^{\bar{t}} \int_0^{\bar{t}} [p(t_1,t_2)t_1 + (1-p(t_1,t_2))t_2 - a(t_1,t_2)]\, d\Phi(t_2)\, d\Phi(t_1) \to \max_{p,a}$$

s.t.

(IC) $(t_1,t_2) = argmax_{\hat{t}_1,\hat{t}_2}\{p(\hat{t}_1,\hat{t}_2)\delta_1 t_1 + (1-p(\hat{t}_1,\hat{t}_2))\delta_2 t_2 \ + a(\hat{t}_1,\hat{t}_2)\}$

(IR) $U(t_1,t_2) \geq 0$

(Non-negative wage) $a(t_1,t_2) > 0\ \forall t_1,t_2$

3 Analysis of the optimal mechanism. General Case

3.1 Characterization of the optimal mechanism

Consider a mechanism design problem, formulated in the previous section. Since $p(t_1,t_2)$ can only take values 0 or 1, two-dimensional space of project qualities (t_1,t_2) can be divided into two sets, where $p(t_1,t_2)$ takes the same value (1 or 0). Let's call S_1 a set of (t_1,t_2) for which $p(t_1,t_2)=1$ and S_2 a set for which $p(t_1,t_2)=0$.

Consider a direct revelation mechanism $\{p(\hat{t}_1,\hat{t}_2),\ a(\hat{t}_1,\hat{t}_2)\}$. I claim that this mechanism satisfy incentive compatibility constraint only if $a(t_1,t_2) = u = const$ for any (t_1, t_2) from S_1 and $a(t_1, t_2) = d = const$ for any (t_1, t_2) from S_2, where u and d are some constants. Indeed, utility function of the manager, given her report $(\hat{t}_1, \hat{t}_2)$, is

$$U(\hat{t}_1, \hat{t}_2, t_1, t_2) = p(\hat{t}_1, \hat{t}_2)\delta_1 t_1 + (1 - p(\hat{t}_1, \hat{t}_2))\delta_2 t_2 \ + a(\hat{t}_1, \hat{t}_2) \tag{3}$$

Assume that $a(t_1, t_2)$ is not a constant on S_1. Then there exists (t_1, t_2) from S_1 such that a manager who observe (t_1, t_2) will be able to choose $(\hat{t}_1, \hat{t}_2)$ from S_1 such that $a(\hat{t}_1, \hat{t}_2) > a(t_1, t_2)$. This report will give her strictly larger utility than telling the truth since $p(\hat{t}_1, \hat{t}_2) = p(t_1, t_2)$ and $a(\hat{t}_1, \hat{t}_2) > a(t_1, t_2)$. Hence this mechanism is not incentive compatible. The similar argument is valid for S_2. Therefore wage schedule of incentive compatible mechanism must be of the following form:

$$a(\hat{t}_1, \hat{t}_2) = p(\hat{t}_1, \hat{t}_2)u + (1 - p(\hat{t}_1, \hat{t}_2))d \tag{4}$$

Using this formula we can rewrite utility function of the manager (3) :

$$U(\hat{t}_1, \hat{t}_2, t_1, t_2) = \delta_2 t_2 + d + p(\hat{t}_1, \hat{t}_2)(\delta_1 t_1 - \delta_2 t_2 + u - d) \tag{5}$$

Now we can derive one more necessary condition for considered mechanism to be incentive compatible. In particular, IC constraint will be satisfied only if $p(t_1, t_2) = 1$

when $(\delta_1 t_1 - \delta_2 t_2 + u - d) > 0$ and $p(t_1, t_2) = 0$ when $(\delta_1 t_1 - \delta_2 t_2 + u - d) < 0$:

$$p(\hat{t}_1, \hat{t}_2) = I\{\delta_1 \hat{t}_1 - \delta_2 \hat{t}_2 + u - d \geq 0\}^3 \tag{6}$$

Indeed, assume that for some (t_1, t_2): $(\delta_1 t_1 - \delta_2 t_2 + u - d) > 0$ and $p(t_1, t_2) = 0$ in this case manager will lie and report $(\hat{t}_1, \hat{t}_2)$ such that $p(\hat{t}_1, \hat{t}_2) = 1$, because in this case she will receive greater utility. Similarly, for any (t_1, t_2) such that $(\delta_1 t_1 - \delta_2 t_2 + u - d) < 0$ and $p(t_1, t_2) = 1$, manager will not report truthfully.

Now consider arbitrary contract, that has wage schedule and project selection rule as in (4) and (6) (u and d are arbitrary). In this case utility of the manager is given by (5). It is easy to see that such contract is incentive compatible. Thus, two necessary conditions, that we derived, are also sufficient.

Proposition 1 summarizes our argument

Proposition 1 *Direct revelation mechanism, formulated in the previous section, is incentive compatible if and only if wage schedule and project selection rule have the following form*

$$a(\hat{t}_1, \hat{t}_2) = p(\hat{t}_1, \hat{t}_2)u + (1 - p(\hat{t}_1, \hat{t}_2))d \tag{7}$$

[3] Here, we use weak inequality to define project selection rule. However, the choice between strict and weak inequality is not important, since probability of event: $\delta_1 t_1 - \delta_2 t_2 + u - d = 0$ is zero. The last statement follows from the fact that distributions of t_1 and t_2 are continuous

$$p(\hat{t}_1, \hat{t}_2) = I\{\delta_1 \hat{t}_1 - \delta_2 \hat{t}_2 + u - d \geq 0\} \tag{8}$$

where u and d are some constants and $I\{\ \}$ is indicator function.

Note that the proof of this proposition presented above, requires that sets $S_1 = \{(t_1, t_2) : p(t_1, t_2) = 1\}$ and $S_2 = \{(t_1, t_2) : p(t_1, t_2) = 0\}$ are non-empty. Indeed, let's assume that S_1 is empty, than, although for some (t_1, t_2) $(\delta_1 t_1 - \delta_2 t_2 + u - d) > 0$, it may be incentive compatible to assign $p(t_1, t_2) = 0$, since there is no $(\hat{t}_1, \hat{t}_2)$ such that $p(\hat{t}_1, \hat{t}_2) = 1$. Similar argument is valid if S_2 is empty. Therefore we need the following lemma to makes our proof accurate:

Lemma 1 *Given our assumptions that project qualities are independent draws from the same distribution, optimal project selection rule must imply that S_1 and S_2 are not empty.*

Proof is in the appendix.

From proposition 1 it immediately follows that optimal wage schedule is

$$a(\hat{t}_1, \hat{t}_2) = d + I\{\delta_1 \hat{t}_1 - \delta_2 \hat{t}_2 + u - d \geq 0\}(u - d) \tag{9}$$

Proposition 1 also reduces our initial mechanism design problem to maximization of headquarters' objective function over u and d given individual rationality (IR) and wage non-negativity constraints.

Let's introduce new parameter

$$h = \frac{d-u}{\delta_1} \tag{10}$$

then from (5) (8) and (9)

$$p(\hat{t}_1, \hat{t}_2) = I\{\hat{t}_1 \geq \frac{\delta_2}{\delta_1}\hat{t}_2 + h\} \tag{11}$$

$$a(\hat{t}_1, \hat{t}_2) = d - \delta_1 h I\{\hat{t}_1 \geq \frac{\delta_2}{\delta_1}\hat{t}_2 + h\} \tag{12}$$

$$U(t_1, t_2) = \delta_2 t_2 + d + I\{t_1 \geq \frac{\delta_2}{\delta_1}t_2 + h\}(\delta_1 t_1 - \delta_2 t_2 - \delta_1 h) \tag{13}$$

in the last expression we use the fact that manager tells the truth in our mechanism, thus $(\hat{t}_1, \hat{t}_2) = (t_1, t_2)$

Consider IR constraint. It says that utility of the manager should be greater, than her reservation utility (zero) no matter what project qualities (t_1, t_2) she observes. Since non-negative wage ensures that manager always receives non-negative utility, IR constraint always holds and we can ignore it.

Let's assume that we have $h > 0$ ($u < d$) in optimal mechanism. To satisfy wage non-negativity constraint it should be that $u \geq 0$. Let's show that this constraint on wage will always bind for the lowest level of payment - u. Indeed, assume $u > 0$.

In this case headquarters can shift wage schedule (u, d) down, holding h constant. New mechanism will be incentive compatible and lead to the same project selection rule (11), but payments to the manager will be lower. Since headquarters prefer to pay the manager as least as possible, it would prefer to shift (u, d) down, holding h constant, up to the point where $u = 0$ and non-negativity constraint on wage starts bind.

By similar argument , we can show that in case when in optimal mechanism $h \leq 0$ $(u \geq d)$, d should be zero. Figure 1 illustrates wage schedule of the optimal mechanism.

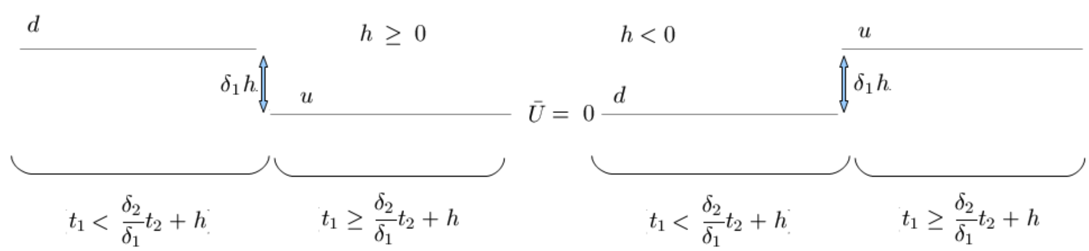

Figure 1: *Optimal wage schedule, when* $\bar{U} = 0$

In summary, given that optimal mechanism implies some particular value of $h = \frac{d-u}{\delta_1}$ it follows that

$$d = \delta_1 h I\{h \geq 0\} \tag{14}$$

$$u = d - \delta_1 h = \delta_1 h(I\{h \geq 0\} - 1) = -\delta_1 h I\{h < 0\} \tag{15}$$

i.e. if $h > 0$, than wage of the manager if the first project is selected is $u = 0$ and if the second one is chosen is $d = \delta_1 h > 0$; if $h < 0$, than $u = -\delta_1 h > 0$ and $d = 0$.

Now turn to objective function of headquarters. Using (11), (12) and (14)

$$\int_0^{\bar{t}} \int_0^{\bar{t}} [p(t_1, t_2)t_1 + (1 - p(t_1, t_2))t_2 - a(t_1, t_2)] \, d\Phi(t_2) \, d\Phi(t_1) =$$

$$= \int_0^{\bar{t}} \int_0^{\bar{t}} [p(t_1, t_2)t_1 + (1 - p(t_1, t_2))t_2 - d + \delta_1 h p(t_1, t_2)] \, d\Phi(t_2) \, d\Phi(t_1) =$$

$$= \int_0^{\bar{t}} \int_0^{\bar{t}} [t_2 - \delta_1 h I\{h \geq 0\} + I\{t_1 \geq \frac{\delta_2}{\delta_1} t_2 + h\}(t_1 - t_2 + \delta_1 h)] \, d\Phi(t_2) \, d\Phi(t_1) \tag{16}$$

So, we have reduced our initial mechanism design problem to unconstraint maximization over parameter h:

$$\int_0^{\bar{t}} \int_0^{\bar{t}} [t_2 - \delta_1 h I\{h \geq 0\} + I\{t_1 \geq \frac{\delta_2}{\delta_1} t_2 + h\}(t_1 - t_2 + \delta_1 h)] \, d\Phi(t_2) \, d\Phi(t_1) \to \max_h \tag{17}$$

Given discussion above we can state

Proposition 2 ***(Characterization of the optimal contract)*** *Optimal contract, consist of project selection rule and managerial compensation, which are given by*

$$p(\hat{t}_1, \hat{t}_2) = I\{\hat{t}_1 \geq \frac{\delta_2}{\delta_1}\hat{t}_2 + h\} \tag{18}$$

$$a(\hat{t}_1, \hat{t}_2) = p(\hat{t}_1, \hat{t}_2)u + (1 - p(\hat{t}_1, \hat{t}_2))d \tag{19}$$

$$d = \delta_1 h I\{h \geq 0\} \tag{20}$$

$$u = -\delta_1 h I\{h < 0\} \tag{21}$$

where h is a solution of maximization problem (17)

This result says that optimal project selection rule prescribes that manager's report about the first project must be compared to the linear function of the report about the second project. The slope of this linear function, is determined completely by the ratio of marginal private benefits from the projects (δ_1 and δ_2), while intercept is optimally chosen by headquarters. Moreover, optimal wage schedule must be represented by two payments - zero and some positive constant. This constant depends on what intercept headquarters has chosen. What project is selected determines which payment manager will receive.

3.2 Full information case

Before we consider properties of the optimal contract, it is useful to consider a full information benchmark. In full information case, headquarters is assumed to know project qualities. Thus, this case is equivalent to the initial mechanism design problem without incentive compatibility (IC) constraint. We can also drop individual rationality constraint (IR), since we have assumed that reservation utility $\bar{U} = 0$ and under this assumption IR never binds due to non-negative wage payment. Clearly, the first-best wage schedule is to pay the manager as least as possible, i.e. zero. First-best project selection rule is to choose best project out of two. Hence

Lemma 2 *First-best project selection rule is*

$$p(t_1, t_2) = I\{t_1 > t_2\} \tag{22}$$

First-best wage is

$$a(t_1, t_2) = 0 \tag{23}$$

On figure 2 first-best project selection rule is depicted. The first project is financed, if (t_1, t_2) lies above the line, and the second project is selected otherwise.

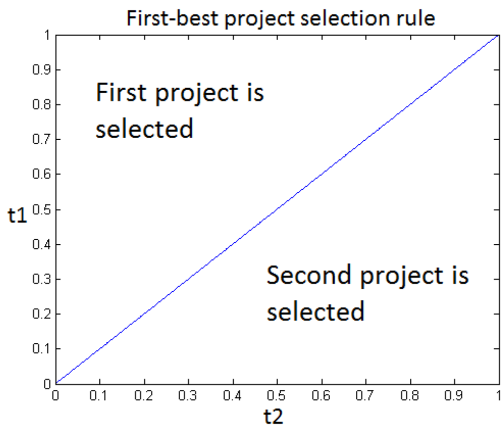

Figure 2: *First-best project selection rule. First project is financed, if (t_1, t_2) lies (weakly) above the line, and the second project is selected otherwise.*

3.3 Properties of the optimal contract

Lemma 3 *When $\delta_1 = \delta_2 = \delta$ optimal mechanism achieves first-best one, which is given by* (22), (23)*. In particular, optimal $h = 0$, $u = d = 0$.*

To see it, note that mechanism designer choose optimal h. Given this choice project selection rule and wage schedule is given by (11), (14), (15):

$$p(t_1, t_2) = I\{t_1 \geq t_2 + h\}$$

$$d = \delta h I\{h \geq 0\}$$

$$u = -\delta h I\{h < 0\}$$

Note that when $h = 0$, project selection rule and wage schedule coincide with first-best ones. Our mechanism with asymmetry of information cannot give greater value of mechanism designer's objective function than in full information case. Thus, $h = 0$ is optimal choice, because it gives the same value of this function as in first-best case.

Lemma 4 *When $\delta_1 \geq \delta_2$ optimal value of h is bounded below and above:* $0 \leq h \leq 1 - \frac{\delta_2}{\delta_1}$.

Proof is in the appendix.

Lemma 4 says that project selection line of optimal mechanism should cross 45 degree line. Example of such project selection rule is depicted on the left panel of figure 3. The slope of this line is determined by the ratio of marginal private benefits (δ_1, δ_2) and intersect – by optimal value of h (see (18)). When difference in marginal private benefits increases, i.e. δ_1 raises, project selection line rotates and becomes flatter. Moreover, optimal choice of h also changes in this case. (see right panel of figure 3).

Using picture on the left panel of figure 3, we can illustrate general tradeoffs that determine our optimal contract. There is two main sources of losses for headquarters, that it tries to minimize using the contract: first one is that we choose wrong projects in areas A and B (we chose second project in area A, while first project is better; and

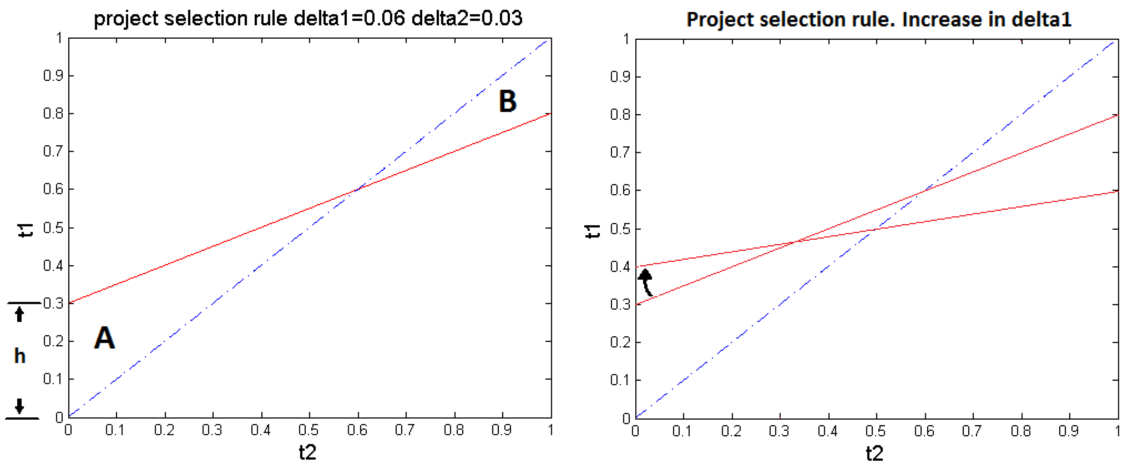

Figure 3: *Examples of possible project selection rules*

we chose first project in area B, where the second project is better); the second one is that we pay salary to the manager. When we pay zero wage to the manager no matter what project is selected, ($h = 0$) area A disappears and we always have bias toward first project. However, it may be optimal for headquarters to pay manager some positive wage if the second project is selected – $d > 0$ (note, that according to lemma 4, h is non-negative when $\delta_1 \geq \delta_2$. Remember that it means that $u = 0$ and $d \geq 0$ – (14), (15) and figure 1). This wage schedule makes manager's bias to the first project less prominent and area A appears. Indeed, since, in case of $d > 0$, we pay manager a constant wage and amount of private benefits that manager gets from the project is proportional to its quality, for low project qualities this constant payment d dominates private benefits of the manager in the first project and, as a consequence, in the region A the second project is selected (although it is worse than the first one). In contrast, in area of high project qualities, private benefits of the manager dominates payment d, that is paid if the second project is selected, and, as a consequence, in the area B the first project is financed (although the second one is better). Clearly, if we will increase payment d to the manager, area A will increase and area B will decrease. Therefore optimal value of d, or equivalently optimal value of h, is determined by tradeoff between losses in areas A and B as well as direct cost of paying wage d to the manager.

The next proposition summarizes this discussion:

Proposition 3 *When $h < 1 - \frac{\delta_2}{\delta_1}$ there is an area of high project qualities, where first project is selected, although the second one is better (area B). When $h > 0$, there is an area of low project qualities, where second project is selected, although first one is better (area A). When $0 < h < 1 - \frac{\delta_2}{\delta_1}$ both areas exist.*

Sensitivity of the optimal contract to the information about t_2

From lemma 4 we can infer that there are some regions, where, given report of $\hat{t}_1$, contract becomes insensitive to the report about the second project - regions A and B on the left panel of figure 4. In this region project selection rule (and thus managerial compensation) is the same for all reports $\hat{t}_2$. Note, that when δ_2

approaches zero project selection line becomes flatter and the total area of this regions tend to increase. In case when manager can not extract any private benefits from the second project ($\delta_2 = 0$), project selection line becomes completely flat and optimal contract is completely insensitive to the report about the second project (right panel of figure 4). Indeed in this case optimal project selection rule, according to (11), is

$$p(\hat{t}_1, \hat{t}_2) = I\{\hat{t}_1 \geq \frac{\delta_2}{\delta_1}\hat{t}_2 + h\} = I\{\hat{t}_1 \geq h\}$$

Also from (12)

$$a(\hat{t}_1, \hat{t}_2) = d - \delta_1 h I\{\hat{t}_1 \geq h\}$$

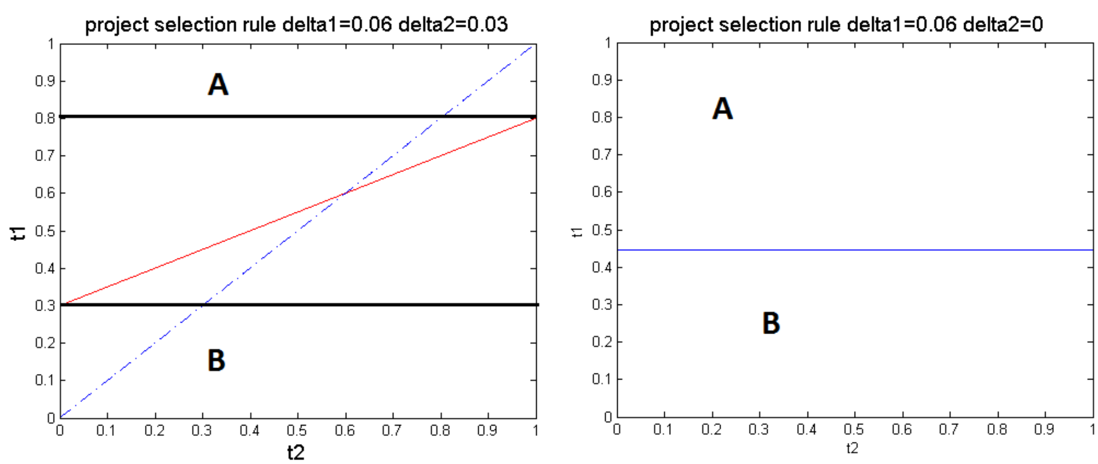

Figure 4: *Optimal project selection rules. In regions A and B optimal contract is insensitive to the information about* t_2

To understand this result we should look at incentive compatibility (IC) constraint. From (5):

$$U(\hat{t}_1, \hat{t}_2, t_1, t_2) = d + p(\hat{t}_1, \hat{t}_2)(\delta_1 t_1 + u - d) \quad (24)$$

As we argued before, the only way to satisfy incentive compatibility is to assign $p(t_1, t_2)$ equal to one, when expression in brackets is positive and zero otherwise. But since this expression in brackets does not depend on t_2, project selection rule can not depend on $\hat{t}_2$. Indeed if $p(\hat{t}_1, \hat{t}_2)$ would depend on $\hat{t}_2$, for a given t_1 manager would report the same $\hat{t}_2$ to maximize her utility, no mater what actual t_2 observes.

Since project selection rule $p(\hat{t}_1, \hat{t}_2) = p(\hat{t}_1)$ does not depend on $\hat{t}_2$, it immediately follows that wage does not depend on it either (see proposition 1).

We can generalize this result

Proposition 4 *In case of multiple projects optimal contract should be insensitive to the managerial reports about projects, from which manager can not extract private benefits, i.e. which δ equal to zero.*

Proof is in the appendix.

Case $\delta_2 = 0$ has one more feature. In this case I can show that if we allow $p(t_1, t_2)$ to be any value between zero and one (such mechanism allow headquarters to define particular probability to chose project, rather than select it or not) and drop non-negativity constraint on wage, the optimal contract will be deterministic, i.e. optimal values of $p(t_1, t_2)$ would be 1 or 0. This result demonstrates that at least for the case $\delta_2 = 0$ deterministic mechanism, that we consider, is optimal in a more general class of mechanisms that allows lotteries as a project selection rule.

Proposition 5 *If $\delta_2 = 0$ and we allow negative payments to the manager, optimal mechanism with probabilistic project selection rules is equivalent to deterministic one, i.e. optimal values of $p(t_1, t_2)$ are either zero or one. Moreover, as proposition 4 claims, in this case optimal contract does not depend on $\hat{t}_2$, i.e. $p(t_1, t_2) = p(t_1)$*

4 Uniform distribution case

Since solving maximization problem (17) is quite complicated task in general case, we assume some particular simple distribution of project qualities t_1, t_2 and investigate implications of optimal mechanism for ex-ante probability of choosing first project and expected payment to the manager. In particular, we assume that project qualities are independent draws from uniform distribution with support $[0, 1]$.

Lemma 5 *When project qualities are distributed uniformly on $[0, 1]$ optimal value of h is given by*

$$h = max\{\frac{1}{1+2\delta_1}(-\frac{\delta_2}{2} + \frac{\delta_1 - \delta_2}{2\delta_1}), 0\} \geq 0 \quad (25)$$

Hence, d is

$$d = \delta_1 h = max\{\frac{\delta_1}{1+2\delta_1}(-\frac{\delta_2}{2} + \frac{\delta_1 - \delta_2}{2\delta_1}), 0\} \geq 0 \qquad (26)$$

and $u = 0$

Proof is in the appendix. It can be seen that there is some region where $h = 0$. Indeed, it is so, when

$$\delta_2 \leq \delta_1 < \frac{2\delta_2}{2 - \delta_2} \qquad (27)$$

Proposition 6 *Ex-ante probability of choosing first project is given by*

$$1 - h(\delta_1, \delta_2) - \frac{\delta_2}{2\delta_1}$$

and it increases with increase in δ_1*, holding* δ_2 *fixed.*

Compensation of the manager, if the second project is chosen (d) is also increases with increase in δ_1*, holding* δ_2 *fixed.*

Ex-ante expected payment to the manager is given by

$$\frac{\delta_2 h(\delta_1, \delta_2)}{2} + \delta_1 h(\delta_1, \delta_2)^2$$

and it increases with increase in δ_1 *up to the point where*

$$\delta_1 = \frac{3\delta_2^2 + 4\delta_2 + 1}{2(1 - \delta_2)^2} \geq \frac{1}{2}$$

Proof is in the appendix. So, the greater the bias of the manager to the first project the greater the ex-ante probability to choose it and the larger the wage of the manager if the second project is financed. This two features has opposite effect on ex-ante expected payment to the manager. Indeed, although, her compensation increases, in case of second project selection, probability to select this project decreases. As the above proposition shows for reasonable[4] values of $\delta_1 < \frac{1}{2}$, the first effect dominates and the greater the difference in managerial preferences over projects, the larger ex-ante compensation headquarters should pay.

[4] $\delta_1 = \frac{1}{2}$ would mean that the division manager can extract private benefits that are equal to half of the cash flow going to headquarters, this assumption is not plausible and reasonable values of δ_1 is significantly smaller

5 Conclusion

In this paper I have studied optimal capital budgeting mechanism within a company, when a division manager has some preferences over projects, that are different from headquarters' ones. I have considered a setup, where the privately informed manager has access to two projects and has some preferences over them. These preferences are represented by the fact that the manager can extract private benefits from the projects. Therefore, a conflict of interests between her and headquarters arises. I have derived optimal contract in this setup, that consist of project selection rule and managerial compensation scheme.

I have shown that optimal project selection rule prescribes for report of the manager about one project to be compared with some linear function of her report about the other project. The slope of this linear function, is determined by the ratio of marginal private benefits of the projects, while its intercept is optimally chosen by headquarters to minimize losses from choosing wrong projects as well as direct costs of paying wage to the manager. I have also shown that, given that marginal private benefits of the first project is grater than marginal private benefits of the second one, there is optimal bias of project selection rule towards second project, when reported projects' qualities are low; and towards first project, when reported projects' qualities are high. In addition, optimal wage schedule prescribes to pay manager some positive constant wage in case, when second project is selected and zero, if the first one is financed.

I have also found that optimal contract cannot depend on information about projects, from which manager cannot extract private benefits. Under additional assumption that project qualities are independent and uniformly distributed, I have derived explicit analytical solution for the optimal contract. In addition, I have demonstrated that ex-ante bias of the manager to one of the projects leads to increase in ex-ante probability to select this project. Moreover, the greater this bias the greater the expected compensation, that headquarters pays to the manager.

Further research on this topic can be devoted to consideration of compensation contracts, that can be written on realized project's cash flow. In this case headquarters receives one more tool to provide incentive for manager to report truthfully. Indeed, from realized cash flow, headquarters would be able to infer probability that manager's report is truthful and link her compensation to this inferred probability.

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6 Appendix

Proof of lemma 1.

The logic of the proof is the following: First, we assume that $S_1 = \emptyset$ and find a contract that gives the maximum headquarter's payoff under this restriction, then we assume $S_2 = \emptyset$ and also find a contract that gives the maximum headquarter's payoff under this restriction. Next, we show that this two contracts leads to the same payoff of the mechanism designer (headquarters). Finally, we construct a contract, such that $S_1 \neq \emptyset$ and $S_2 \neq \emptyset$ that gives greater payoff to the headquarters. The existence of such a contract proves that all contracts corresponding to the cases where $S_1 = \emptyset$ or $S_2 = \emptyset$ are not optimal.

Let's assume that $S_1 = \emptyset$, this means that $p(t_1, t_2) = 0$ for any (t_1, t_2). So, since project selection rule is already assumed, the headquarters can choose only compensation of the manager. Note, that headquarters prefer to pay the manager as least as possible. Thus, wage of the manager should be zero (it cannot be negative due to non-negativity constraint on wages). Hence the contract prescribes to always choose second project and pay manager zero. This contract is incentive compatible. Indeed, no matter what manager reports, the second project is selected and managers receives zero payment. In this case there is no incentives for the manager to lie. Moreover IR and non-negative wage constraints are satisfied, thus such contract is feasible. Clearly, payoff of the headquarters in this case is the expectation of the second project's quality (see (2)).

By analogy if we assume $S_2 = \emptyset$, the headquarters would choose to always pay zero to the manger. Thus, the contract prescribes to always choose first project and pay the manager zero wage. Payoff of the headquarters in this case is the expectation of the first project's quality. Since project's qualities of the first and second projects are independent draws from the same distribution, their expectations are equal. Therefore the two contracts, described above, lead to the same headquarter's payoff.

Let's consider a case when $S_1 \neq \emptyset$ and $S_2 \neq \emptyset$. Under this assumption proposition 1 holds, and (9) and (8) defines incentive compatible contract. Let's assign $u = d = 0$ (we always pay manager zero), in this case IR and non-negative wage constraints are satisfied. Therefore contract is feasible and represented by:

$$p(\hat{t}_1, \hat{t}_2) = I\{\delta_1 \hat{t}_1 - \delta_2 \hat{t}_2 \geq 0\}$$

and

$$u = d = 0$$

.

Let's show that it gives a greater payoff to headquarters than a contract with $S_2 = \emptyset$. Clearly both this contracts are equivalent in terms of managerial compensation (both pay zero). The figure 5 demonstrates project selection line of the contract, that we have constructed. For (t_1, t_2) that lies above the red solid line (area A) first project is selected; and for (t_1, t_2) that lies below this line (area B) second project is selected. Note that since $\delta_1 > \delta_2$ this line is always below the 45 degree line. Also note that if (t_1, t_2) lies below 45 degree line second project is better. The contract with $S_2 = \emptyset$ prescribes to always choose first project. Thus in the area A, both contracts leads to the same project selection rule; while in area B contract, that we have constructed, chooses second project, which is better than the first one. Therefore the project, that we constructed, leads to a greater payoff of the headquarters than contract with $S_2 = \emptyset$. Hence, neither $S_2 = \emptyset$ nor $S_1 = \emptyset$ is an optimal project selection rule.

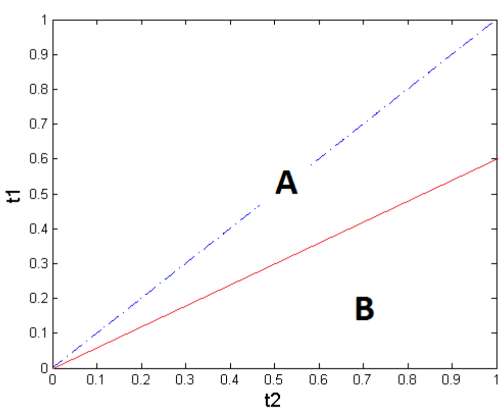

Figure 5: *Project selection rule for a contract, that we have constructed*

Proof of lemma 4.

To understand the logic behind this proposition, look at figure 6, where project selection rules with $\delta_1 \geq \delta_2$ and negative h are depicted. On the left panel h is large and negative. From all points (t_1, t_2), that lies above the solid line, first project is selected. Moreover manager receives $u = \delta_1 |h| > 0$ in this region, while in the region below the solid line she receives $d = 0$ (see (14), (15)). Now it is easy to see that increase in h, while holding h negative is optimal for headquarters(right panel of the figure 6): first, it makes project selection rule close to first best one (45 degree line), thus a region of inefficient project choices reduces; second, it decreases the region where manager receives positive wage . This argument is valid until h reaches zero. Thus when $\delta_1 \geq \delta_2$ h should be non-negative.

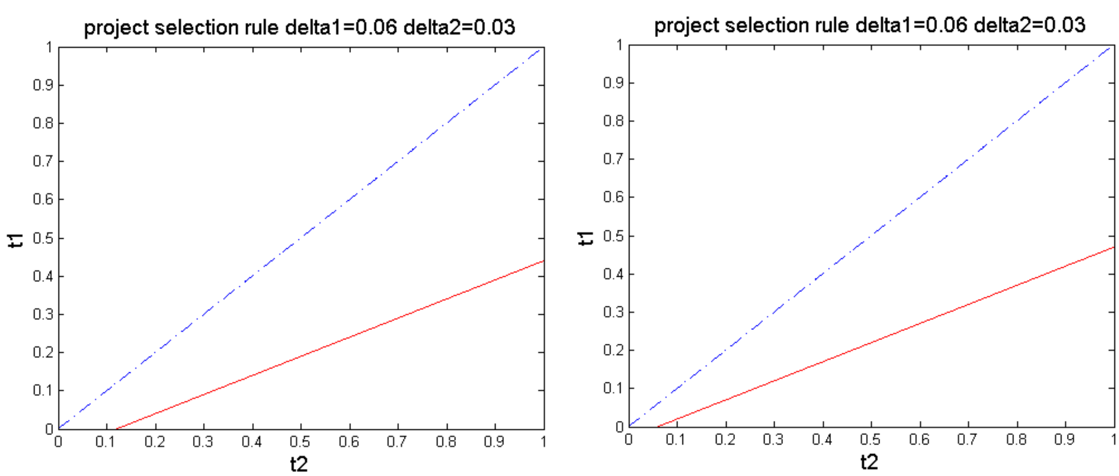

Figure 6: *Project selection rules for $\delta_1 \geq \delta_2$ and negative h*

On the right panel of figure 7 the case when $h = 1 - \frac{\delta_2}{\delta_1}$ is depicted. Picture on the left panel corresponds to $h > 1 - \frac{\delta_2}{\delta_1}$. In this case second project is selected if (t_1, t_2) lies below the project selection line and manager receives $d = \delta_1 h > 0$ (see (14)); otherwise first project is selected and manager receives $u = 0$ (see (15)). Clearly, it is optimal for headquarters to reduce h at least up to $1 - \frac{\delta_2}{\delta_1}$, because in this case project selection rule becomes closer to the most efficient first-beast one and the region where headquarters pays positive wage decreases.

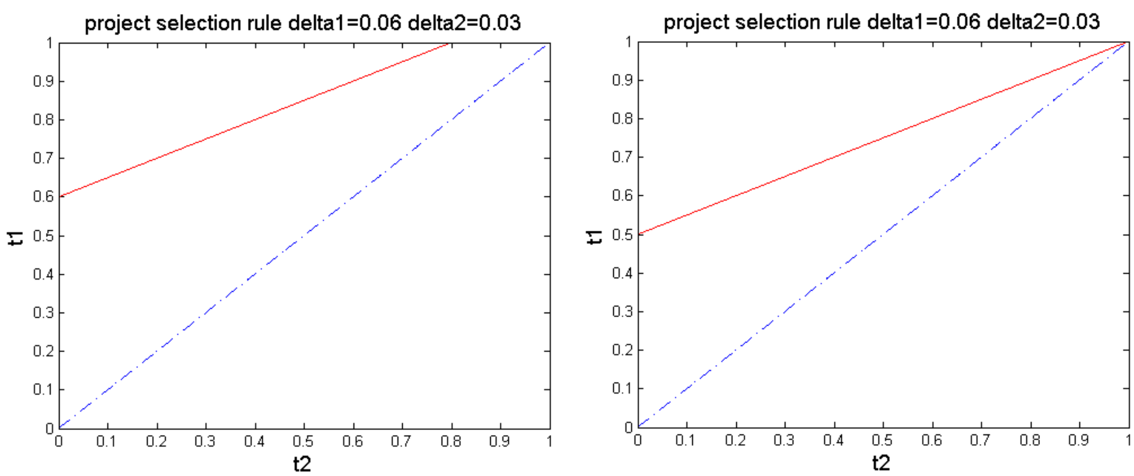

Figure 7: *Project selection rules for* $\delta_1 \geq \delta_2$ *and* $h = 1 - \frac{\delta_2}{\delta_1}$ *(right panel);* $h > 1 - \frac{\delta_2}{\delta_1}$ *(left panel)*

Proof of proposition 4.

In case of multiple projects,optimal contract would define project that should be financed. In this case the whole space of (t_1, t_2) can be divided into regions S_i, where project selection rule select i^{th} project ($i = 1..N$, N - number of projects). By arguments similar to presented at the beginning of section 3, within each of these regions compensation of the manager should be some constant a_i. Therefore utility of the manager must be represented by a function of the following form:

$$U(t_1, t_2) = \sum_i p_i(t_1, t_2)(\delta_i t_i + a_i)$$

where $p_i = 1$, if i^{th} project is selected, and zero otherwise. In this case manager would report truthfully if and only if $p_i = 1$ when $i = \arg\max_j\{\delta_j t_j + a_j\}$. It is easy to see that, if for some project j: $\delta_j = 0$, then its quality cannot influence project selection. Thus optimal contract is insensitive to the report of the manager about t_j.

Proof of proposition 5.

Note, that the division manager and the headquarters are risk neutral, thus their objective functions will include expectations over $p(t_1, t_2)$, when project selection rule is probabilistic. Therefore, using that $\delta_2 = 0$, we can state our mechanism design problem in the following way:

$$\int_0^{\bar{t}} \int_0^{\bar{t}} [p(t_1, t_2)t_1 + (1 - p(t_1, t_2))t_2 - a(t_1, t_2)]\, d\Phi(t_2)\, d\Phi(t_1) \to \max_{p,a}$$

s.t.

(IC) $(t_1, t_2) = argmax_{\hat{t}_1, \hat{t}_2} \{a(\hat{t}_1, \hat{t}_2) + p(\hat{t}_1, \hat{t}_2)\delta_1 t_1\}$

(IR) $U(t_1, t_2) \geq \bar{U} = 0$

where $p(t_1, t_2) \in [0, 1]$ - probability to finance first project.

Let's apply envelope theorem to expression in IC constraint. The objective function of the manager is

$$U(\hat{t}_1, \hat{t}_2, t_1, t_2) = a(\hat{t}_1, \hat{t}_2) + p(\hat{t}_1, \hat{t}_2)\delta_1 t_1$$

We can calculate derivatives with respect to t_1 and t_2.

$$\frac{\partial U(t_1, t_2)}{\partial t_1} = \frac{\partial U(\hat{t_1}, \hat{t_2}, t_1, t_2)}{\partial t_1}|_{(\hat{t_1}, \hat{t_2}) = (t_1, t_2)} = \delta_1 p(t_1, t_2) \tag{28}$$

$$\frac{\partial U(t_1, t_2)}{\partial t_2} = \frac{\partial U(\hat{t_1}, \hat{t_2}, t_1, t_2)}{\partial t_2}|_{(\hat{t_1}, \hat{t_2}) = (t_1, t_2)} = 0 \tag{29}$$

In order for $U(t_1, t_2)$ to exist the following should hold:

$$\frac{\partial U(t_1, t_2)}{\partial t_1 \partial t_2} = \frac{\partial U(t_1, t_2)}{\partial t_2 \partial t_1}$$

From (29) it follows that $U(t_1, t_2) = C + \varphi(t_1)$ where $\varphi(t_1)$ is arbitrary function of t_1. If we plug it in (28) we get that $\varphi'(t_1) = \delta_1 p_1(t_1, t_2)$. Therefore

$$p(t_1, t_2) = p(t_1)$$

-project selection rule is only a function of t_1, not t_2.

Given expression for $p(t_1, t_2)$ and $U(t_1, t_2) = U(t_1) = C + \varphi(t_1)$ we can integrate (28) and get

$$U(t_1, t_2) = U(t_1) = \int\limits_0^{t_1} \delta p_1(s)\, ds + U(0, 0)$$

To satisfy IR constraint we should set $U(0, 0) = 0$. From

$$U(t_1, t_2) = a(t_1, t_2) + \delta_1 t_1 p(t_1, t_2)$$

and

$$U(t_1, t_2) = \int\limits_0^{t_1} \delta_1 p(s)\, ds$$

We can derive

$$a(t_1, t_2) = a(t_1) = \int\limits_0^{t_1} \delta_1 p(s)\, ds - \delta_1 t_1 p(t_1) \tag{30}$$

To plug $a(t_1, t_2)$ into headquarters objective function we need to calculate $\int\limits_0^{\bar{t}} a(t_1, t_2)\, d\Phi(t_1)$. Integration by parts gives

$$\int\limits_0^{\bar{t}} a(t_1, t_2)\, d\Phi(t_1) = \int\limits_0^{\bar{t}} \mu(t_1)\delta_1 p(t_1)\, d\Phi(t_1) - \int\limits_0^{\bar{t}} \delta_1 t_1 p(t_1)\, d\Phi(t_1) \tag{31}$$

Where $\mu(t) = \frac{1-\Phi(t)}{f(t)}$ - hazard rate. $\Phi(t)$ - cdf and $f(t)$ - pdf

Then headquarters objective function is:

$$\int\limits_0^{\bar{t}} m + p(t_1)[t_1 - m - \delta_1 \mu(t_1) + \delta_1 t_1]\, d\Phi(t_1)$$

where $m = \int\limits_0^{\bar{t}} t_2\, d\Phi(t_2)$ - expectation of t_2 Therefore

$$p(t_1) = I\{t_1 - m - \delta_1 \mu(t_1) + \delta_1 t_1 > 0\}$$

or

$$p(t_1) = I\{(1+\delta_1)t_1 - \delta_1\mu(t_1) > m\}$$

From expression above it is clear that optimal project selection rule prescribes to select one of the two projects with probability one, i.e. we have obtained that optimal project selection rule is deterministic. Also note that we have obtained result stated in proposition 4. Indeed, neither project selection rule, nor managerial compensation (see (30)) depend on the report $\hat{t}_2$.

Proof of lemma 5. Our goal is to solve:

$$\int_0^1 \int_0^1 [t_2 - \delta_1 h I\{h \geq 0\} + I\{t_1 \geq \frac{\delta_2}{\delta_1} t_2 + h\}(t_1 - t_2 + \delta_1 h)]\, dt_2\, dt_1 \to \max_h \tag{32}$$

We use lemma 4 to set the limits of integration:

$$\int_0^1 \int_0^1 [t_2 - \delta_1 h I\{h \geq 0\} + I\{t_1 \geq \frac{\delta_2}{\delta_1} t_2 + h\}(t_1 - t_2 + \delta_1 h)]\, dt_2\, dt_1 =$$

$$= \frac{1}{2} - \delta_1 h + \int_0^1 dt_2 \int_{\frac{\delta_2}{\delta_1} t_2 + h}^1 (t_1 - t_2 + \delta_1 h)\, dt_1$$

Let's differentiate the expression above with respect to h:

$$-\delta_1 + \int_0^1 dt_2 [\frac{\delta_1 - \delta_2}{\delta_1} t_2 - h(1+\delta_1) + \delta_1(1 - \frac{\delta_2}{\delta_1} t_2 - h)] =$$

$$= -h(1+2\delta_1) - \frac{\delta_2}{2} + \frac{\delta_1 - \delta_2}{2\delta_1}$$

Equating this expression to zero we obtain candidate for optimal value of h. It is indeed candidate for maximum, since to the left of this value derivative is positive and to the right it is negative. Note, that for some values of parameters δ_1 δ_2, h given by this formula can be negative. Thus according to the lemma 4, optimal value of h is zero in this case. When h, given by the expression above is positive, it is optimal value of h. Thus we obtain (25):

When project qualities are distributed uniformly on $[0,1]$ optimal value of h for $\delta_1 \geq \delta_2$ is given by

$$h = max\{\frac{1}{1+2\delta_1}(-\frac{\delta_2}{2} + \frac{\delta_1 - \delta_2}{2\delta_1}), 0\} \geq 0$$

From (14) and (15) it follows that:

$$d = \delta_1 h = max\{\frac{\delta_1}{1+2\delta_1}(-\frac{\delta_2}{2} + \frac{\delta_1 - \delta_2}{2\delta_1}), 0\} \geq 0$$

and $u = 0$

Proof of proposition 6. Ex-ante probability to choose first project is given by:

$$\int_0^1\int_0^1 p(t_1,t_2)\,dt_2\,dt_1 = \int_0^1\int_0^1 I\{t_1 \geq \frac{\delta_2}{\delta_1}t_2 + h\}\,dt_2\,dt_1 =$$

$$= \int_0^1 dt_2 \int_{\frac{\delta_2}{\delta_1}t_2+h}^1 dt_1 = \int_0^1 dt_2(1 - \frac{\delta_2}{\delta_1}t_2 - h) =$$

$$= 1 - h - \frac{\delta_2}{2\delta_1}$$

To find derivative of this expression, first, we find derivative of h with respect to δ_1: We assume that δ_1 is large enough for h to be positive, otherwise its derivative is zero.

$$\frac{\partial h}{\partial \delta_1} = \frac{-2\delta_1^2(1-\delta_2) + 4\delta_1\delta_2 + \delta_2}{(1+2\delta_1)^2 2\delta_1^2}$$

Thus derivative of ex-ante probability to choose first project is given by:

$$-\frac{\partial h}{\partial \delta_1} + \frac{\delta_2}{2\delta_1^2} =$$

$$= \frac{\delta_2(1+2\delta_1)^2 - 2\delta_1^2(1-\delta_2) + 4\delta_1\delta_2 + \delta_2}{(1+2\delta_1)^2 2\delta_1^2}$$

$$= \frac{\delta_2 + 1}{(1+2\delta_1)^2} \geq 0$$

Therefore ex-ante probability to select first project increases with δ_1.

It is easy to see that d is also increasing function of δ_1. Indeed, if δ_1 is large enough for h to be positive (otherwise derivative of d with respect to δ_1 is zero):

$$d = \delta_1 h = \frac{\delta_1}{1+2\delta_1}(-\frac{\delta_2}{2} + \frac{\delta_1 - \delta_2}{2\delta_1})$$

$\frac{\delta_1}{1+2\delta_1}$ is increasing function of δ_1 as well as $\frac{\delta_1-\delta_2}{2\delta_1}$. Therefore d is increasing function of δ_1.

Since d is a wage, that is paid to the manager if the second project is selected (i.e. when $t_1 < \frac{\delta_2}{\delta_1}t_2 + h$) to derive expected compensation of the manager we need to calculate:

$$\int_0^1 \int_0^1 I\{t_1 < \frac{\delta_2}{\delta_1}t_2 + h\}\delta_1 h \, dt_2 \, dt_1 =$$

$$= \int_0^1 dt_2 \int_0^{\frac{\delta_2}{\delta_1}t_2+h} \delta_1 h \, dt_1 = \int_0^1 dt_2(\frac{\delta_2}{\delta_1}t_2 + h) = \frac{\delta_2 h}{2} + \delta_1 h^2$$

Partial derivative with respect to δ_1 of expression above is given by:

$$\frac{\partial h}{\partial \delta_1}(\frac{\delta_2}{2} + 2h\delta_1) + h^2$$

Using expression for h and $\frac{\partial h}{\partial \delta_1}$ we obtain that this derivative is equal to:

$$\frac{\delta_1^2(-2\delta_1(1-\delta_2^2)+3\delta_2^2+4\delta_2+1)}{4\delta_1^2(1+2\delta_1)^3}$$

This expression is positive if

$$\delta_1 < \frac{3\delta_2^2+4\delta_2+1}{2(1-\delta_2)^2}$$

Since

$$\frac{3\delta_2^2+4\delta_2+1}{2(1-\delta_2)^2} \geq \frac{1}{2}$$

, $\forall\delta_2$. The derivative is positive, when δ_1 is less than $\frac{1}{2}$.