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Design of optimal voluntary payments schemes under conformity

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Abstract

When choosing how to act, a person may care about what actions the others choose and may choose to conform by selecting an action preferred by the others. Such a phenomenon is called normative conformity if the desire to go with the crowd stems from reasons other than informational content of the others' actions, for instance, from morals, social norms, or internal preferences for similarity. I enrich the framework developed in Fischer & Huddart (2008) by allowing agents to have different preferences over actions. I characterize optimal mechanisms in voluntary payments environments, such as donations, contributions for public goods, and tips. I show that when the agents have heterogeneous preferences with conformity the optimal contributions mechanism restricts the range of acceptable payment amounts. In addition to a zero contribution required by voluntary setup the optimal mechanism sets either only one positive payment level or a minimum acceptable payment depending on the distribution of individual components of the preferences.

1 Introduction

Humans are social beings who like to pay attention to what others do. They also know that others may pay attention to what they do. Buying cars or clothes, choosing a school or a restaurant, and voting are examples of decisions that are likely to be influenced by decisions of others. Conformity is a phenomenon of acting similarly to others when being observed or when alone.

Wisdom of the crowd is one source for conformity. Indeed, in looking at decisions of others a person may think that some of them are well informed. Thus, quite rationally, she would be tempted to follow the choices of others in similar decision problems. This phenomenon

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is called informational influence or informational conformity and was first documented by psychologist Sherif (1936). In his experiment the participants had to estimate how much a light dot moved. In fact, the dot didn't move at all and its perceived motion was simply an illusion due to autokinetic effect. Every person was subject to her unique rate of the effect. Participants could revise their estimates observing the estimates of the others and eventually they converged to the same estimate.

In economics, Banerjee (1992) and Bikhchandani, Hirshleifer & Welch (1992) brought attention to informational influence by introducing the information cascades concept. In their models agents receive some private information and act in sequence observing choices made before them. From past choices agents may infer private information of the others which may lead to herding behavior with most of agents ignoring their private information. The convergence of actions in these models crucially depends on the discrete structure of possible choices. Experiments by Anderson & Holt (1997) confirm the existence of information cascades and the literature on informational cascades and herding is quite vast by now.

The second source for conformity is normative influence, or normative conformity. Normative conformity can occur in the absence of any informational exchange and can come from variety of sources. It may arise from internal unwillingness of an individual to deviate from the actions of the others because of moral considerations, religion or social norms. Or it may be driven by a threat of an external punishment for deviation. Asch (1955) explored that phenomenon in attempt of a deep investigation of Sheriff's study. He conducted a similar laboratory experiment with a more clear and easy question, in which subjects were asked to determine which line out of the three presented matched the original one. Asch assumed that conformity would be negligible in that case, but found out that on average people conformed one third of the time. Such behavior could hardly be driven by information concerns because of the simplicity of the question. It supports the existence of normative conformity.

Jones (1984) introduced normative conformity in the economic literature. He concentrated on the analysis of workers' attitudes in a group workplace environment, particularly on the "Hawthorne Puzzle", the desire of workers to conform to each others actions. He built a model with utility functions explicitly reflecting social preferences. This implied conformity by construction and explained emergence of traditions in the overlapping generation model. Importantly, social norms in the model were endogenous and derived from workers' behavior in equilibrium rather than being fixed ad hoc. Many subsequent approaches accepted this idea.

Bernheim (1994) offered another view on conformity. In his model individuals care about their status as well as an "intrinsic" utility. Having heterogeneous statuses they are willing to conform fully because any deviation may seriously impair their perceived status. It is a signaling model but it represents, in fact, normative rather than informational conformity as agents tend to be perceived having a "normal" status. Unlike Jones, Bernheim sets the norm exogenously as an average of possible statuses rather than deriving it from a model. Brock & Durlauf (2001) presented a neat application of the model. They analyzed aggregate behavioral outcomes in presence of social interaction effects but restrict most of their attention to the case of homogeneous agents and binary choice set of possible actions.

Recently, Fischer & Huddart (2008) considered a model with endogenous social norms from the optimal contraction perspective. They treated normative conformity as given and investigated its consequences. In the basic model agents' preferences are two-fold. The first part is conventional and reflects money payoffs. The second part refers to conformity and represents unwillingness of a worker to deviate from both personal norm and endogenous social norm. They analyzed how social norms may affect standard approach but obtained the most results in an absolutely homogeneous case due to complexity of the problem. They demonstrated that it can be optimal to split an organization in order to eliminate the mutual conformity externalities between the agents involved in different tasks.

The crucial difference between modeling informational and normative conformity is timing. Informational conformity is modeled as a dynamic game because its essence is the exchange of information among agents. One advantage of the normative conformity modeling is a possibility to consider a one-shot game instead of a dynamic one. The key feature of normative conformity is the unwillingness to deviate, which can be captured in a static game. This allows for richer models and possibly leads to more plausible results.

In this thesis I concentrate on the normative conformity in a voluntary payments setup. Voluntary payments in my model mean non-obligatory payments which do not imply immediate gains in the form of goods or services for donor. Proper examples are charity donations, tips and voluntary public good contributions. Such transfers involve huge money resources. According to Giving USA Foundation annual charity donations in the United States increased from \$20 bln in 1969 to \$300 bln in 2009, and tipping estimates in the US food industry alone amount to about \$42 bln annually.

Conformity is important in the voluntary payments setting. Alpizar, Carlsson & Johansson-Stenman (2008) conducted a natural field experiment at a national park in Costa Rica. When donors were told that the typical contribution of others is \$2 (a small contribution), the probability of a contribution increased and the conditional contribution decreased, compared with providing no reference information. Providing a high reference level (\$10) increased the conditional contributions. This evidence corresponds to the results of my model.

The basic model is similar to one by Fischer & Huddart (2008). It has donor population of heterogeneous agents, whose preference for donation comes from two sources: individualistic component and social component. The former represents the amount of payment an agent would like to pay apart from social concerns and varies among population. The latter reflects social preferences and constitutes conformity. I look for optimal design in the environment, that is I seek for the mechanism raising the largest possible fund.

While exploiting a similar model my research differs considerably from Fisher&Huddart's one. First, I apply a pure design approach in the voluntary setting while they concentrate on wage/bonus schemes in contracting setting. Second, I analyze heterogeneous agents while hey obtain most results in a homogeneous case.

I show in this thesis that optimal contribution mechanism restricts the possible range of acceptable payments in two important general cases. In the first case, the distribution of individual components of the preferences strictly increases, meaning that the population is generous. Then in addition to a zero contribution required by voluntary setup the optimal mechanism sets only one positive payment level. In the second case, the distribution of the components strictly decreases, meaning that the population is stingy. Then in addition to a zero contribution the optimal mechanism sets a minimum acceptable payment.

In fact, many foundations use such mechanisms though often implicitly to not annoy its donors. Minimal payment varies from \$2 up to \$20 for most charity donations and is often justified by technical reasons. According to my research it may be the best way of fundraising in a stingy population. Also, many organizations sell memorable items indirectly setting the only acceptable payment amount. One example is Colorado State University which sells university license plates for \$100 per item. Probably most graduates purchase no more than one license plate so \$100 turns out to be the only acceptable payment amount. I show that it may be the best way of fundraising in a generous population. Hence, results of my thesis fit the evidence and explain variety of existing contribution schemes.

The paper proceeds as follows. In Section 2 I present the main model for behavior of heterogeneous agents in the presence of conformity in a population. In Section 3 I apply the model to a voluntary payments problem and find optimal mechanisms in the important general cases. In Section 4 I apply the main model to an open voting problem and analyze equilibrium behavior. Section 5 concludes.

2 The main model

The basic model is borrowed from Fischer & Huddart (2008), who studied the notion of conformity as compliance with social norms in a team from the optimal contraction perspective. Fisher&Huddart's model has a continuum of workers in a set I each of whom chooses her level of efforts, action a_i to maximize the utility function:

$$z(a_i) \equiv w_i + b_i h(a_i) - f(a_i - N_i) \tag{1}$$

$$N_i \equiv (1-\alpha_i)A_i + \alpha_i S_a, \qquad S_a \equiv \frac{\int_I a_i \, d_i}{\int_I d_i} \tag{2}$$

Such utility function reflects a two-fold nature of preferences. The first part is conventional and reflects money payoffs. Here w_i is a flat wage and b_i is a monetary bonus for each unit of production $h(a_i)$. The second part refers to conformity and represents unwillingness of a worker to deviate from both personal norm and endogenous social norm. Here $f(a)$ is a convex function with maximum at zero, so N_i is an ideal action for a worker in the absence of bonuses. It depends on A_i, an individual norm, and S_a, social norm in the population. The authors naturally assume this norm to be simply a population average action. Then α is an exogenous rate of conformity varying between 0 and 1 that captures the fundamental trade-off between individualism and conformity. Higher α_i leads to less willingness to deviate from a population average. When $\alpha_i = 0$, there is no conformity and the agent chooses her individual norm. When $\alpha = 1$, there is no individualism and the choice of the agent is driven solely by the crowd. This model captures the essential idea of the conformity but seems to be too complicated. Indeed, while Fisher&Huddart allow for heterogeneous individual norms and rate of conformity, they obtain the most results in the homogeneous case, $A_i = A$, $\alpha_i = \alpha \; \forall \, i \in I$.

I want to capture the conformity idea in the simplest way applicable to voluntary payments problem, which implies heterogeneous donors. So there is a continuum of agents in my main model, i-th agent having its personal norm A_i. These norms are the most preferable actions for agents in the absence of conformity and represents all other concerns. The set of the personal norms is normalized to $[0, 1]$ with distribution function $F(A_i)$ known to everyone. This distribution represents a population attitude towards different possible actions.

Preferences of every each agent constitute her utility function depending on her action a_i:

$$u(a_i) = -(a_i - N_i)^2 \tag{3}$$
$$N_i \equiv (1 - \alpha)A_i + \alpha \mathsf{E}a$$

Here $\mathsf{E}a$ signifies an average population action and is identical to S_a before. Note that I assume a rate of conformity being the same among agents though keep personal norms heterogeneous. Also assume that $\alpha < 1$. These assumptions seem to be quite natural in the setting. I consider the case with heterogeneous α_i but the same A in appendix. The case of dual heterogeneity is intractable and unnecessary for my analysis.

The game proceeds as a typical one-shot game. Each agent chooses independently her action a_i from the set of possible actions $\mathcal{A}$, the same for every agent. Then the actions are observed and payoffs are distributed. I consider a concept of a Nash equilibrium. It is proper to consider just equilibria in pure strategies because in any case almost every agent has a unique best response.

Let us get acquainted with the model and investigate the equilibrium behavior of the agents with unrestricted $\mathcal{A}$.

Theorem 1. *If $\mathcal{A} = (-\infty, +\infty)$ then in the Nash eq'm $\mathsf{E}a = \mathsf{E}A$, i.e. the average action in the population coincides with the average personal norm.*

Proof. F.O.C. to (3) :

$$a_i = N_i = (1 - \alpha)A_i + \alpha \mathsf{E}(a) \tag{4}$$

Applying population mean to the both sides of (5) and using the fact that $\alpha \neq 1$ completes the proof. □

From (5) and Theorem 1 one can derive that in the Nash eq'm

$$a_i^* = (1 - \alpha)A_i + \alpha \mathsf{E}A \tag{5}$$

Note that it differs from simply condition (5) and constitutes an equilibrium behavior. Each individual weighs her personal norm with the average personal norm in the population, which fits common perception of conformity. At the same time conformity here changes just a pattern of the actions but not their average. In the subsequent sections I show that this is an artefact of the unrestricted A and is not true in the general case.

For now, consider two examples:

1. Agents are homogeneous in their personal norm as considered by Fisher&Huddart. Then $A_i = const = A_0 \, \forall \, i$ and according to (6)

$$a_i^* = A_0 \tag{6}$$

In this case conformity doesn't change behavior of the agents in any way. Though the agents don't like to deviate from the crowd there is no reason to do that. Identity of the agents kills any possible effects.

2. Personal norms are distributed uniformly on $[0, 1]$ so

$$a_i^* = (1 - \alpha)A_i + \alpha \, \frac{1}{2} \tag{7}$$

In that case conformity does change behavior of the agents. The chosen actions are uniformly distributed on $[\frac{\alpha}{2}, 1 - \frac{\alpha}{2}]$ concentrating at the middle of the set of possible actions, compared to the distribution of personal norms. Such behavior is demonstrated by the tendency friends exhibit while walking together. Nobody likes to be aside, everyone tends to the middle.

3 Voluntary payments setting

Economic activity involves a lot of monetary transactions. Some of them are obligatory, some are not. Voluntary payments in my model mean non-obligatory payments which do not imply immediate gains in the form of goods or services for a donor. Proper examples are charity donations, tips and voluntary public good contributions. Alpizar et al. (2008) showed that conformity does influence donor's behavior. Thus, one may apply the developed framework to the setting with voluntary payments in attempt to analyze equilibrium behavior and achieve certain goals.

My goal in the analysis is fundraising. First, billions of dollars annually circulate in the industries with voluntary payments. Even a slight relative increase in the revenue leads to considerable profit. Hence, it's interesting to check whether current payment schemes are optimal from a private firm perspective. Second, raising the most revenue may be a social goal. For example, one may use the conformity to override a free-rider behavior in public goods provision. Hence, the goal is appealing from policy implications perspective as well.

3.1 Mechanism design

I consider the following interpretation of the developed model in a voluntary payments setting. Donor population consists of a continuum of agents having personal norms A_i distributed on the interval [0,1] with the density function $f(z)$. Each agent conforms at some extent to the average payment in the population according to (3). A mechanism designer has full information about the population $(f(z), \alpha)$ and offers some mechanism G. I do not allow for monetary transfers among the agents in the mechanism and negative payments. Importantly, as payments are voluntary each agent has an outside option of zero payment.

The goal is to find an optimal mechanism, that is the mechanism raising the most revenue on this population.

3.1.1 Revelation principle

Variety of every possible mechanisms is enormous and maximizing revenue on such a huge class seems to be impracticable. My solution is a successive narrowing of attention. First, I narrow my analysis to the class of direct incentive compatible mechanisms due to revelation principle. Then I exclude classes of mechanisms which can not raise optimal revenue till the maximization is analytically simple.

Lemma 1 (Revelation principle)**.** *For any mechanism G and its equilibrium E_G:*

(a) there exists a direct incentive compatible mechanism D, i.e. a mechanism where each agent has strategies to report any of the types and reporting her true type is an equilibrium E_D.

(b) payoffs in the equilibria E_G and E_D are the same.

Proof. See, for example, Krishna (2002). □

The agent's type in this setting is her personal norm which is private information. Consider any direct incentive compatible mechanism D. It is direct, so it is fully represented by its payment function $d(z_i)$. This function defines payment of every agent according to his called type. Every agent in the mechanism may call any type $z_i \in Z = [0, 1]$ due to (a) (I conform to convenient mechanism design notation here). If agent i with her personal norm A_i announces some type z_i she gets the payoffs, depending on an average payment in the population:

$$U_i(z_i) = -(d(z_i) - (1-\alpha)A_i - \alpha\,\mathsf{E}d)^2$$

The direct mechanism D is also incentive compatible, so its payment function satisfies:

$$\textbf{IC: } A_i \in \underset{z_i \in [0,1]}{\operatorname{argmax}} \{-(d(z_i) - (1-\alpha)A_i - \alpha\,\mathsf{E}d)^2\} \quad \forall A_i \in [0, 1] \tag{8}$$

or equivalently $\forall A_i \in [0, 1]$

$$\begin{cases} \quad\quad\quad\quad \textit{true} & ,\forall z_i: \ d(z_i) = d(A_i) \\ (1-\alpha)A_i + \alpha\,\mathsf{E}d \le \dfrac{d(z_i) + d(A_i)}{2} & ,\forall z_i: \ d(z_i) > d(A_i) \\ (1-\alpha)A_i + \alpha\,\mathsf{E}d \ge \dfrac{d(z_i) + d(A_i)}{2} & ,\forall z_i: \ d(z_i) < d(A_i) \end{cases} \tag{9}$$

Condition (8) restricts class of possible direct mechanisms. Another restriction stemming from voluntariness is a zero payment as an outside option. It follows that an equilibrium payoff of any agent must be no less than her payoff from zero payment:

$$\textbf{IR: } U_i(z_i) = -(d(z_i) - (1-\alpha)A_i - \alpha\,\mathsf{E}d)^2 \ge -((1-\alpha)A_i + \alpha\,\mathsf{E}d)^2 \quad \forall A_i \in [0, 1]$$

or equivalently as $\mathsf{E}d \geq 0 \ \forall A_i \in [0,1]$

$$d(z_i) \leq 2((1-\alpha)A_i + \alpha\,\mathsf{E}d) \tag{10}$$

Denote this as a zero payment condition. Examination of condition (9) gives

Lemma 2. *The payment function $d(z)$ on $[0,1]$ in the direct incentive compatible mechanism D is nondecreasing and piecewise continuous.*

Proof. Assume the contrary. Consider two different agents i and j. Let $A_i > A_j$ and $d(A_i) < d(A_j)$. D is incentive compatible, so (9) holds for A_i at $z_i = A_j$ and (9) holds for A_j at $z_j = A_i$:

$$(1-\alpha)A_i + \alpha\,\mathsf{E}d \leq \frac{d(A_i) + d(A_j)}{2} \leq (1-\alpha)A_j + \alpha\,\mathsf{E}d \tag{11}$$

It follows that $A_i \leq A_j$. Contradiction, $d(z)$ is nondecreasing.
Then $d(z)$ is piecewise continuous as any nondecreasing function defined on compact. □

Further analysis of continuity points leads to

Lemma 3. *If $d(z)$ is the payment function in the direct incentive compatible mechanism D then $d(z) = (1-\alpha)z + \alpha\,\mathsf{E}d$ at every continuity point of increase.*

Proof. Let A_i be the continuity point. Then the statement follows immediately from (11) as A_j approaches continuity point A_i in turn from above and from below. □

Note, that Lemma 3 doesn't deal with constancy periods or break points of the payment function. Analysis of constancy intervals doesn't restrict the payment function because criterion (9) degenerates in that case. Further investigation into break points gives

Lemma 4. *If $d(z)$ is the payment function of the direct incentive compatible mechanism D then $lim_{\epsilon\to 0}\frac{d(z_0+\epsilon)+d(z_0)-\epsilon)}{2} = (1-\alpha)z + \alpha\,\mathsf{E}d$ and $d(z_0)$ is equal to either $lim_{\epsilon\to 0} d(z_0+\epsilon)$ or $lim_{\epsilon\to 0}\, d(z_0-\epsilon)$ at every point of discontinuity z_0.*

Proof. Due to piecewise continuity there exists ϵ_0 such that for every $0 < \epsilon < \epsilon_0$
$d(z_0+\epsilon) > d(z_0-\epsilon)$. Then the first part of the statement follows immediately from (11) as $A_i = z+\epsilon$ and $A_j = z-\epsilon$ and ϵ approaches zero. The second part of the statement follows from the contrary considering in addition limit of $A_i = z_0$ and $A_j = z_0 - \epsilon$, or $A_i = z_0$ and $A_j = z_0 + \epsilon$ □

Note, that Lemma 3 follows from an extension of Lemma 4 to the continuity points of increase.

So far I derived necessary conditions for payment function in a direct incentive compatible mechanism. However, it's easy to see that these conditions are sufficient as well. Really, Lemmas 3 and 4 eliminate incentives of agents to deviate from truthful equilibrium to their neighborhood. But as a payment function is nondecreasing due to Lemma 2, if an agent does not want to deviate to her neighborhood she does not want to deviate any further as well. It follows

Theorem 2. *Payment function $d(z)$ represents some direct incentive compatible mechanism if and only if $d(z)$ is non-decreasing, at any point with non-constant neighborhood*

$$\lim_{\epsilon\to 0} \frac{d(z+\epsilon)+d(z-\epsilon)}{2} = (1-\alpha)z + \alpha\,\mathsf{E}d \tag{12}$$

and at every point of discontinuity $d(z) \in \{lim_{\epsilon\to 0}\, d(z+\epsilon), lim_{\epsilon\to 0}\, d(z-\epsilon)\}$.

Theorem 2 with restriction (10) completely defines class of mechanisms corresponding to any equilibrium in the voluntary payment setting. An example of a typical payment function is presented in Figure 1.

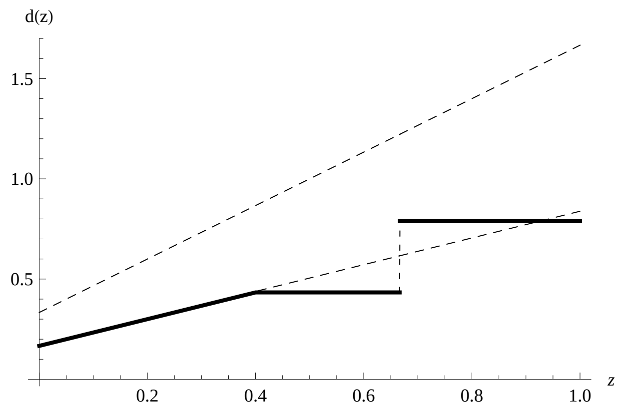


Figure 1: Example of payment function. $\alpha = \frac{1}{3}, \mathsf{E}d = \frac{1}{2}$.

Finally, we have the following maximization problem given personal norms distribution $f(z)$:

$$\mathsf{E}d \equiv \int_0^1 d(z)f(z)dz \to \max_{d(z)} \tag{13}$$

$$\begin{aligned}
\textbf{IC: } & \text{if } d'(z) > 0 \text{ or z is a break point} \\
& \lim_{\epsilon\to 0} \frac{d(z+\epsilon)+d(z-\epsilon)}{2} = (1-\alpha)z + \alpha\,\mathsf{E}d \\
& \text{if z is a break point} \\
& d(z) \in \{lim_{\epsilon\to 0}\, d(z+\epsilon), lim_{\epsilon\to 0}\, d(z-\epsilon)\} \\
\textbf{IR: } & d(z_i) \leq 2((1-\alpha)A_i + \alpha\,\mathsf{E}d) \\
\textbf{Other: } & d(z_i) \geq 0
\end{aligned}$$

This has trivial solution if $\alpha \geq \frac{1}{2}$. In this case conformity becomes overwhelming, payment function of kind $d(z) = const = c_0$ satisfies restrictions and raises infinity revenue as c_0 rockets up. One should not apply the model to that extreme because some of initial assumptions may be violated. Particulary, rate of conformity may fall when an average action of the others becomes extremely high. Hence, I restrict my attention to the case of $\alpha < \frac{1}{2}$ but even that narrower problem is too complicated. Further analysis calls for specific classes of personal norms distribution $f(z)$.

3.1.2 Uniform distribution of personal norms

In this section personal norms are distributed uniformly among population, i.e. $A_i \sim U[0,1]$. This case corresponds to neutral population without any special preferences to low or high contributions. I show that this assumption greatly simplifies analysis and enable us to find the vast class of optimal mechanisms which includes a binary choice scheme and a scheme with a minimum acceptable payment. I approach the main theorem of the section constructing the big classes of mechanisms with the same revenue and then optimizing among these classes.

Lemma 5. *If $d(z)$ is a payment function of the direct incentive compatible mechanism D in the uniform voluntary setting and $d(z)$ has a positive slope somewhere, i.e. $d(z) = (1-\alpha)z + \alpha\,\mathsf{E}d\ \forall z \in [z_1, z_2]$, then there exists revenue equivalent direct incentive compatible mechanism D' with its payment function $d'(z)$:*

$$d'(z) = \begin{cases} d(z), \forall z \notin [z_1, z_2] \\ d(z_1), \forall z \in [z_1, \frac{z_1+z_2}{2}] \\ d(z_2), \forall z \in (\frac{z_1+z_2}{2}, z_2] \end{cases} \tag{14}$$

Proof. It is sufficient to show, that the mechanism D' with its payment function $d'(z)$ is incentive compatible and meets zero payment condition (10). Let's prove that the strategy profile with each individual telling her true type is equilibrium in D'. In fact,

(a) in that profile $\mathsf{E}d' = \mathsf{E}d$ due to uniformity of inner preferences

(b) as D is incentive compatible $d(z)$ satisfies conditions of Theorem 2

(c) as D offers zero outside option $d(z)$ satisfies (10)

Then it follows from (a) and (b) that $d'(z)$ satisfies conditions of Theorem 2 as well, so D' is incentive compatible. Furthermore, it follows from (b) and (c) that $d(z')$ satisfies (10) so D' offers zero outside option. $\square$

An illustration of the transformation in Lemma 5 is represented in the Figure 2.

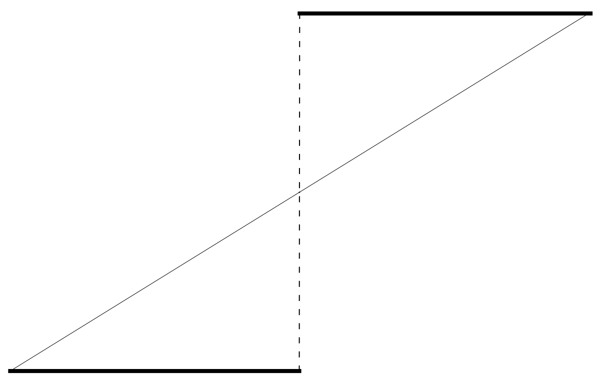


Figure 2: Transformation presented in Lemma 5.

Consecutive iteration of Lemma 5 allows me to narrow the further analysis in the uniform case to the class of direct incentive compatible mechanisms with a piecewise constant payment function because it fully represents variety of possible revenues. I continue the narrowing in

Lemma 6. *If $d(z)$ is a piecewise constant payment function in the direct incentive compatible mechanism D in the uniform voluntary setting and $d(z)$ has two break points nearby, i.e. $\exists\, z_1 < z_2$:*

$$d_0 = \lim_{\epsilon\to 0} d(z_1 - \epsilon) < \lim_{\epsilon\to 0} d(z_1 + \epsilon) = d_1 = \lim_{\epsilon\to 0} d(z_2 - \epsilon) < \lim_{\epsilon\to 0} d(z_2 + \epsilon) = d_2$$

then there exists revenue equivalent direct incentive compatible mechanism D' with zero outside option and a piecewise constant payment function $d'(z)$:

$$d'(z) = \begin{cases} d(z) & , \forall z \notin [z_1, z_2] \\ d_0 & , \forall z \in [z_1, z_1 + \dfrac{d_2 - d_1}{2(1-\alpha)}] \\ d_2 & , \forall z \in (z_1 + \dfrac{d_2 - d_1}{2(1-\alpha)}, z_2] \end{cases} \tag{15}$$

Proof. Logic here is the same as in the previous proof, though calculations are more involved. In fact, as D is incentive compatible then due to Theorem 2
$z_2 = z_1 + \frac{d_2-d_0}{2(1-\alpha)}$. Then in the truthful profile:

(a) $\mathsf{E}d' = \mathsf{E}d + (d_2 - d_1)\frac{d_1-d_0}{2(1-\alpha)} - (d_1 - d_0)\frac{d_2-d_1}{2(1-\alpha)} = \mathsf{E}d$.

(b) $(1-\alpha)(z_1+\frac{d_2-d_1}{2(1-\alpha)})+\alpha\,\mathsf{E}d'$ =due to (a)= $\frac{d_2-d_1}{2}+(1-\alpha)z_1+\alpha\,\mathsf{E}d$ =due to Theorem 2= $\frac{d_2-d_1}{2} + \frac{d_1+d_0}{2} = \frac{d_2+d_0}{2}$.

Analysis of other points is straightforward. Thus $d'(z)$ meets conditions of Theorem 2 so is incentive compatible.

(c) $2\left((1-\alpha)(z_1 + \frac{d_2-d_1}{2(1-\alpha)}) + \alpha\,\mathsf{E}d'\right)$ =due to (b)= $d_2 + d_0 \geq d_2$.

Analysis of other points is straightforward. Thus $d'(z)$ satisfies (10) so D' offers zero outside option. □

An illustration of the Lemma 6 is represented in the Figure 3.

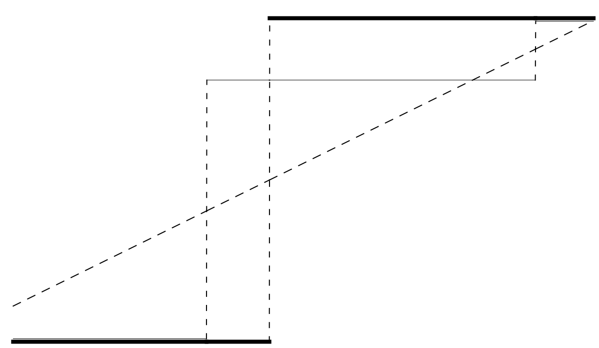


Figure 3: Transformation presented in Lemma 6.

Thus, iteration of Lemmas 5 and 6 restricts my attention to the direct mechanisms with two levels of realized payments in equilibrium and no more than one break point.

Lemma 7. *If $d(z)$ is a payment function of the optimal two-level direct incentive compatible mechanism D with zero outside option in the uniform setting and the break point $z_0 < \frac{1}{2}$ then the first level is equal to zero:*

$$d(z) = \begin{cases} 0 & ,\forall\, 0 \leq z < z_0 \\ d_1 & ,\forall\, z_0 < z \leq 1 \\ 0 \ or \ d_1 & ,if\ z = z_0 \end{cases} \tag{16}$$

Proof. Assume the contrary. Then as payments are non-negative

$$d(z) = \begin{cases} d_0 > 0 & ,\forall\, 0 \leq z < z_0 \\ d_1 & ,\forall\, z_0 < z \leq 1 \\ d_0 \ or \ d_1 & ,if\ z = z_0 \end{cases}$$

Consider the transformation

$$d'(z) = \begin{cases} d'_0 = d_0 - \epsilon & , \forall\, 0 \le z < z_0 \\ d'_1 = d_1 + \dfrac{1 - 2\alpha z_0}{1 - 2\alpha(1 - z_0)}\epsilon & , \forall\, z_0 \le z \le 1 \end{cases}$$

where $\epsilon > 0$ is sufficiently small. See that the mechanism D' with its payment function $d'(z)$ is incentive compatible with zero outside option. In fact, in the truthful profile

$$\begin{aligned} \mathsf{E}d' &= z_0(d_0 - \epsilon) + (1 - z_0)\left(d_1 + \frac{1 - 2\alpha z_0}{1 - 2\alpha(1 - z_0)}\epsilon\right) \\ &= d_0 z_0 + (1 - z_0)d_1 + \frac{1 - 2z_0}{1 - 2\alpha(1 - z_0)}\epsilon \\ &= \mathsf{E}d + \frac{1 - 2z_0}{1 - 2\alpha(1 - z_0)}\epsilon \end{aligned}$$

Then $\frac{d'_1 + d'_0}{2} = \frac{1}{2}(d_0 + d_1 + \frac{2\alpha(1-2z_0)}{1-2\alpha(1-z_0)}\epsilon) =$ using IC of D and Theorem 2 $= (1 - \alpha)z_0 + \mathsf{E}d'$, thus D' is incentive compatible according to Theorem 2.

Moreover, $d'_0 = d_0 + \epsilon > 0$ as ϵ is small enough and $d'_1 = 2\left((1 - \alpha)z_0 + \mathsf{E}d'\right) - d'_0 < 2\left((1 - \alpha)z_0 + \mathsf{E}d'\right)$, so D' satisfies (10) and offers zero outside option.

But $\mathsf{E}d' > \mathsf{E}d$ as $z_0 < \frac{1}{2}$, so D is not optimal. Contradiction. $\square$

An illustration of the Lemma 7 is represented in the Figure 4.

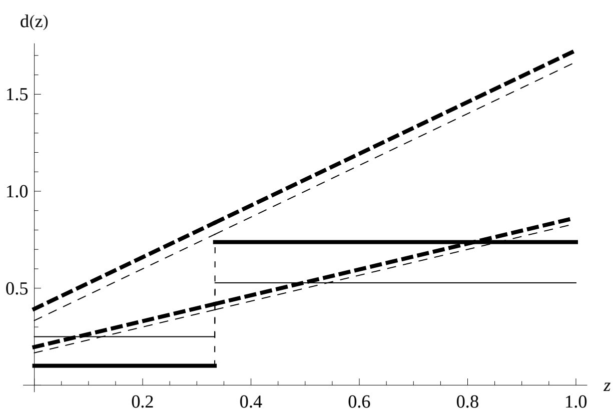


Figure 4: Transformation presented in Lemma 7.

Lemma 8. *If $d(z)$ is a payment function of the optimal two-level direct incentive compatible mechanism D with zero outside option in the uniform setting and the break point $z_0 > \frac{1}{2}$ then the first level is equal to $2\alpha\,\mathsf{E}d$:*

$$d(z) = \begin{cases} 2\alpha\,\mathsf{E}d & , \forall\, 0 \le z < z_0 \\ d_1 & , \forall\, z_0 < z \le 1 \\ 2\alpha\,\mathsf{E}d \text{ or } d_1 & , if\; z = z_0 \end{cases} \tag{17}$$

Proof. Assume the contrary. Then due to (10)

$$d(z) = \begin{cases} d_0 < 2\alpha\,\mathsf{E}d & , \forall\, 0 \le z < z_0 \\ d_1 & , \forall\, z_0 < z \le 1 \\ d_0 \ or \ d_1 & , if \ z = z_0 \end{cases}$$

Consider the transformation

$$d'(z) = \begin{cases} d'_0 = d_0 + \epsilon & , \forall\, 0 \le z < z_0 \\ d'_1 = d_1 - \dfrac{1 - 2\alpha z_0}{1 - 2\alpha(1 - z_0)}\,\epsilon & , \forall\, z_0 \le z \le 1 \end{cases}$$

where $\epsilon > 0$ is sufficiently small. The mechanism D' is incentive compatible for the same reason as in the proof of Lemma 7 and in the truthful profile

$$\mathsf{E}d' = \mathsf{E}d - \frac{1 - 2z_0}{1 - 2\alpha(1 - z_0)}\epsilon$$

Note that $\mathsf{E}d' > \mathsf{E}d$ as $z_0 > \frac{1}{2}$. Thus, $d'_0 = d_0 + \epsilon < 2\alpha\,\mathsf{E}d'$ as ϵ is small enough and $d'_1 < d_1 \le (1 - \alpha)z_0 + \alpha\,\mathsf{E}d < (1 - \alpha)z_0 + \alpha\,\mathsf{E}d'$, so D' satisfies (10) and offers zero outside option.

It follows that $\mathsf{E}d' > \mathsf{E}d$ and D is not optimal. Contradiction. $\square$

An illustration of the Lemma 8 is represented in the Figure 5.

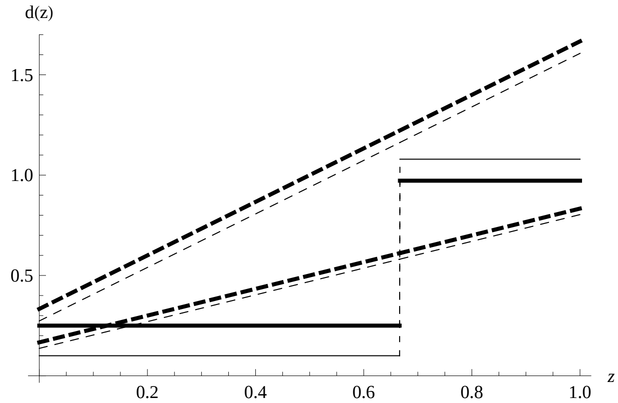


Figure 5: Transformation presented in Lemma 8.

Obviously, if $z_0 = \frac{1}{2}$ one may obtain the same revenue by the mechanisms presented in Lemmas 7 and 8 through the same transformations. So I can seek for the maximum possible revenue and a corresponding optimal mechanism in the binary choice classes presented in these Lemmas.

1.) An optimal mechanism in the class presented in Lemma 7 ($d_0 = 0$, $d_1 > 0$):

$$\max_{z_0 \geq 0} \mathsf{E}d$$
$$s.t. \quad \mathsf{E}d = z_0 * 0 + 2(1 - z_0)((1 - \alpha)z_0 + \alpha \, \mathsf{E}d)$$

The solution to this problem is

$$\mathsf{E}\, d^* = \frac{(1-\alpha)(1-\sqrt{1-2\alpha})^2}{2\alpha^2} \tag{18}$$

$$d_1^* = \frac{1-\alpha}{\alpha}(1-\sqrt{1-2\alpha}) \tag{19}$$

$$1 - z_0^* = \frac{1-\sqrt{1-2\alpha}}{2\alpha} \tag{20}$$

2.) An optimal mechanism in the class presented in Lemma 8 ($d_0 = 2\alpha \mathsf{E}\, d$, $d_1 > d_0$):

$$\begin{aligned} &\max_{z_0 \geq 0} \ \mathsf{E}d \\ s.t. \quad &\mathsf{E}d = z_0 d_0 + (1 - z_0) d_1 \\ &d_0 = 2\alpha \mathsf{E}\, d \\ &\frac{d_1 + d_0}{2} = ((1 - \alpha) z_0 + \alpha \, \mathsf{E}d) \end{aligned}$$

The solution to this problem is

$$\mathsf{E}\, d^* = \frac{(1-\alpha)(1-\sqrt{1-2\alpha})^2}{2\alpha^2} \tag{21}$$

$$d_0^* = \frac{1-\alpha}{\alpha}(1-\sqrt{1-2\alpha})^2 \tag{22}$$

$$d_1^* = \frac{1-\alpha}{\alpha}(1-\sqrt{1-2\alpha}) \tag{23}$$

$$1 - z_0^* = 1 - \frac{1-\sqrt{1-2\alpha}}{2\alpha} \tag{24}$$

Note, that maximum revenue raised is the same in both classes though corresponding mechanisms are different. Thus, both mechanisms are optimal. It's easy to transform the latter mechanism into the mechanism with minimum acceptable payment:

$$d(z) = \begin{cases} \dfrac{(1-\alpha)(1-\sqrt{1-2\alpha})^2}{\alpha}, \forall\, 0 \le z < \dfrac{1-\alpha-\sqrt{1-2\alpha}}{\alpha} \\ (1-\alpha)z + \dfrac{(1-\alpha)(1-\sqrt{1-2\alpha})^2}{2\alpha}, \forall\, \dfrac{1-\alpha-\sqrt{1-2\alpha}}{\alpha} \le z \le 1 \end{cases}$$

Summarizing

Theorem 3. *Maximum revenue in the presented voluntary payment model with uniformly distributed inner preferences is equal to*

$$\pi^* = \frac{(1-\alpha)(1-\sqrt{1-2\alpha})^2}{2\alpha^2} \tag{25}$$

The class of optimal mechanisms is vast. It includes a binary choice mechanism with set of acceptable payments $\{0; d^* = \frac{1-\alpha}{\alpha}(1-\sqrt{1-2\alpha})\}$ *and a minimum payment mechanism which accept any payment not less than* $\frac{1-\alpha-\sqrt{1-2\alpha}}{\alpha}$.

I don't specify the whole class of the optimal mechanisms because it is enormous. In fact, one may obtain any optimal mechanism in the class by backward induction of Lemmas 5 and 6.

Some examples of optimal payment functions are presented in the Figure 6.

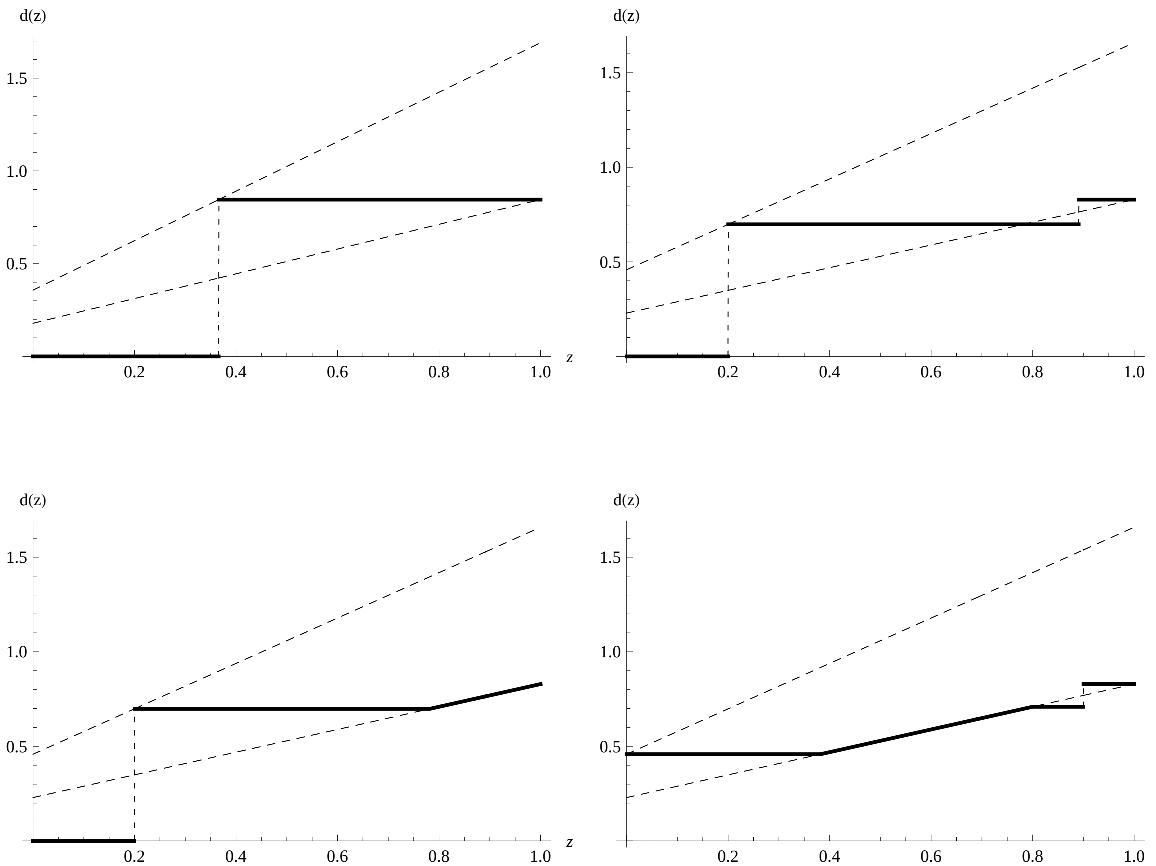


Figure 6: Optimal direct incentive compatible mechanisms in the uniform case with $\alpha = 0.4$ ($\mathsf{E}d \approx 0.57$).

The graphs of some optimal characteristics depending on rate of conformity in population:

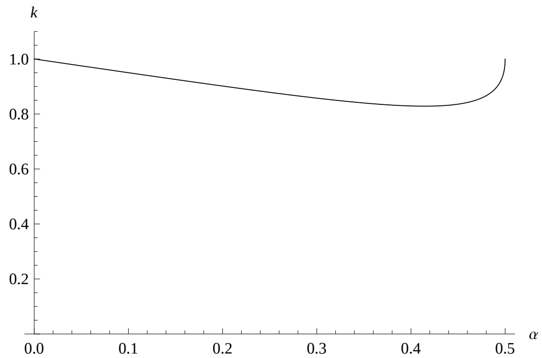


Figure 7: Acceptable payment maximizing revenue depending on the rate of conformity

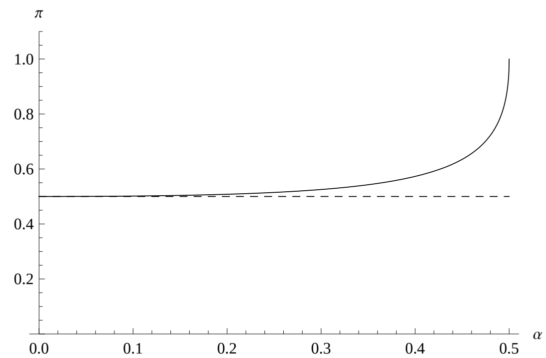


Figure 8: Maximum revenue depending on the rate of conformity

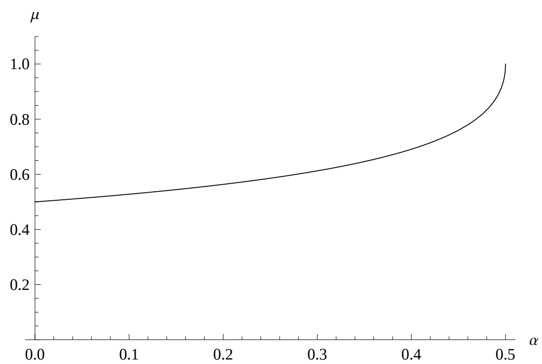


Figure 9: Share of population who contribute depending on the rate of conformity

Note that both π and μ increase along with α. Maximal revenue π^* is everywhere greater than revenue with no restrictions $\pi^{ur} = \frac{1}{2}$ and is twice as large as that amount when α approaches $\frac{1}{2}$.

3.1.3 Arbitrary distribution of personal norms

In this section personal norms are distributed among a population according to an arbitrary density function $f(z)$. This general case is more complicated than the previous one and demands for more sophisticated techniques. I need the generalization of the initial problem. Consider an extension of voluntary payments setting when preferences of agent i correspond to the following utility function

$$U_i(a_i) = -(a_i - (1-\alpha)A_i - \alpha X)^2 \tag{26}$$

where X is a constant parameter and represents conformity reference point for population, exogenous norm. I call it X-generalization of the voluntary payments setting or simply X-setting. I refer to such agent behavior as to X-conformity.

That X-setting refers to the basic voluntary setting in the following sense. Given any direct mechanism D with its payment function $d(z)$ expected revenue $\mathsf{E}d$ in the X-problem is a function of X, $\pi_D(X)$. Then it follows straightforward from definition

Lemma 9. *Expected revenue $\mathsf{E}d$ in the basic voluntary payment problem satisfies the criterion*

$$\mathsf{E}d = \pi_D(\mathsf{E}d) \tag{27}$$

thus it is a fixed point of a revenue function $\pi_D(X)$ and every fixed point represents an expected revenue in some voluntary payment equilibrium in the mechanism D.

Lemma 9 proves to be a powerful tool in the further analysis. Also I need

Lemma 10. *Consider X-problem and any direct mechanism D with its payment function $d(z)$. Then an equilibrium payment of any agent*

$$d_i^*(X) = d(\operatorname*{argmax}_{z_i}(-(d(z_i) - (1-\alpha)A_i - \alpha X)^2)$$

and expected revenue $\pi_D(X)$ in that mechanism are non-decreasing in X and constrained from above.

Proof. Assume the contrary. Then $\exists\, X' > X$ that $z^{*\prime}, z^*$- corresponding equilibrium payments so

$$\begin{cases} U_{iX}(d(z^*)) \geq U_{iX}(d(z^{*\prime})) \\ U_{iX'}(d(z^{*\prime})) \geq U_{iX'}(d(z^{*\prime})) \\ d(z^*) > d(z^{*\prime}) \end{cases}$$

However, it follows from system above that

$$\begin{cases} \dfrac{d(z^{*\prime}) + d(z^*)}{2} \leq (1-\alpha)A_i + \alpha X \\ \dfrac{d(z^{*\prime}) + d(z^*)}{2} \geq (1-\alpha)A_i + \alpha X' \end{cases}$$

which contradicts the fact that $X' > X$. Thus $d_i^*(X)$ and consequently $\pi_D(X)$ are non-decreasing. Observation that revenue function is constrained from above by d(1) as d(z) is non-decreasing completes the proof. □

An important corollary from the last two lemmas is

Lemma 11. *If D is direct incentive compatible mechanism in the X_0-problem and $\pi_D(X_0) > X_0$ then there exists $X_1 > X_0$ such that $\pi_D(X_1) = X_1$. This point corresponds to an equilibrium in the mechanism D with the revenue $X_1 > X_0$ in the main voluntary payment setup.*

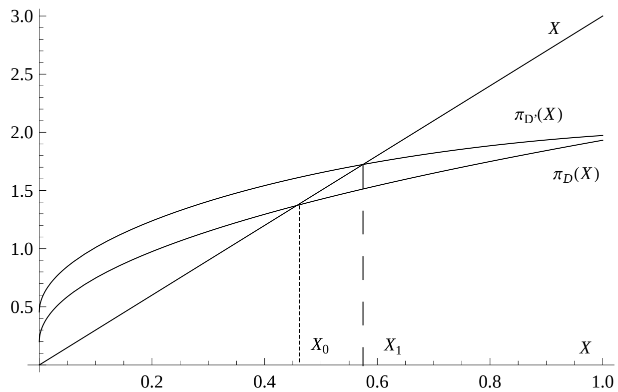


Figure 10: Revenue increase in Lemma 14.

Proof. $\pi_D(X)$ is non-decreasing and constrained from above due to Lemma 10 so there exists $X_1 > X_0$ such that $\pi_{D'}(X_1) = X_1$. This is an equilibrium according to Lemma 9. $\square$

The proof is outlined in the Figure 10. Lemma 11 says that if one can increase the revenue given exogenous norm then one can increase the revenue given endogenous norm by the same direct mechanism. This Lemma constitutes a backbone for analysis in the section. Further, I eliminate non-optimal mechanisms by constructing their transformations with higher revenue.

Some general considerations:

Lemma 12. *If D is an optimal incentive compatible mechanism in the main voluntary payment setup with its payment function $d(z)$ then $d(1) \geq (1-\alpha) + \alpha \mathsf{E} d$.*

Proof. Assume the contrary. Then due to Theorem 2 $\exists\, z_0$ such that $d(z_0) = (1-\alpha)z_0 + \alpha \mathsf{E} d$ and $d(z) = d(z_0)\ \forall z \in [z_0, 1]$. Denote $\mathsf{E} d = X_0$. Due to Lemma 9 $\pi_D(X_0) = X_0$.

Consider the following transformation:

$$d'(z) = \begin{cases} d(z), \forall z \in [0, z_0] \\ (1-\alpha)z + \alpha X_0, \forall z \in (z_0, 1] \end{cases} \tag{28}$$

Due to Theorem 2 direct mechanism D' with payment function $d'(z)$ is incentive compatible in X_0-problem. Obviously $\pi_{D'}(X_0) > \pi_D(X_0) = X_0$. But then according to Lemma 11 there is an equilibrium in D' raising more revenue then X_0 which contradicts optimality of D. Contradiction. $\square$

Lemma 13. *Consider the main voluntary payment setup with strictly monotone preference function. Then if D is an optimal incentive compatible mechanism with its payment function $d(z)$ having exactly one break point z_0 then there exists an optimal D' with the corresponding $d'(z)$ having the same break point and being equal either 0 or $2\alpha \mathsf{E} d'$ for all $0 \leq z < z_0$. That is at least one restriction on the mechanism D' is bounding. Moreover, $D' = D$ if D is not a two-level mechanism with a break point at the median of f(z).*

Proof. Assume the contrary. Denote $\mathsf{E}d = X_0$. Due to Lemma 9 $\pi_D(X_0) = X_0$. Consider possible cases:

(a) $\exists z_1, z_2 : d(z) = d(z_1) = (1-\alpha)z_1 + \alpha \mathsf{E}d \ \forall z \in [z_1, z_0), d(z) = d(z_2) = (1-\alpha)z_2 + \alpha \mathsf{E}d \ \forall z \in (z_0, z_2], d(z) = (1-\alpha)z + \alpha \mathsf{E}d \ \forall z$ in some semi-neighborhood of z_1, z_2

Consider the following transformation depending on sufficiently absolutely small ϵ:

$$d'(z,\epsilon) = \begin{cases} d(z), \forall z \notin [z_1 - \dfrac{\epsilon}{1-\alpha}, z_2 + \dfrac{\epsilon}{1-\alpha}] \\ d(z_1) - \epsilon, \forall z \in [z_1 - \dfrac{\epsilon}{1-\alpha}, z_0] \\ d(z_2) + \epsilon, \forall z \in (z_0, z_2 + \dfrac{\epsilon}{1-\alpha}] \end{cases} \tag{29}$$

Due to Theorem 2 direct mechanism D' with payment function $d'(z)$ is incentive compatible in X_0-problem. See that

$$\begin{aligned} \pi_{D'}(X_0) &= \pi_D(X_0) + \epsilon \left(\int_{z_0}^{z_2} f(z)dz - \int_{z_1}^{z_0} f(z)dz \right) + o(\epsilon) \\ &= X_0 + \epsilon \left(\int_{z_0}^{z_2} f(z)dz - \int_{z_1}^{z_0} f(z)dz \right) + o(\epsilon) \end{aligned}$$

Note that D is incentive compatible and then due to Theorem 2 $z_0 - z_1 = z_2 - z_0$. Then it follows from strict monotonicity that the linear term is non zero, so there exists ϵ either positive and negative such that $\pi_{D'}(X_0) > X_0$. But then according to Lemma 11 there is an equilibrium in D' raising more revenue then X_0 which contradicts optimality of D.

(b) $\exists z_2 : d(z) = d(0) \forall z \in [0, z_0), d(z) = d(z_2) = (1-\alpha)z_2 + \alpha \mathsf{E}d \ \forall z \in (z_0, z_2], d(z) = (1-\alpha)z + \alpha \mathsf{E}d \ \forall z$ in some semi-neighborhood of z_2

Consider the following transformation depending on absolutely small ϵ:

$$d'(z,\epsilon) = \begin{cases} d(z), \forall z \notin [0, z_2 + \dfrac{\epsilon}{1-\alpha}] \\ d(0) - \epsilon, \forall z \in [0, z_0] \\ d(z_2) + \epsilon, \forall z \in (z_0, z_2 + \dfrac{\epsilon}{1-\alpha}] \end{cases} \tag{30}$$

Due to Theorem 2 direct mechanism D' with payment function $d'(z)$ is incentive compatible in X_0-problem. See that

$$\begin{aligned} \pi_{D'}(X_0) &= \pi_D(X_0) + \epsilon \left(\int_{z_0}^{z_2} f(z)dz - \int_{0}^{z_0} f(z)dz \right) + o(\epsilon) \\ &= X_0 + \epsilon \left(\int_{z_0}^{z_2} f(z)dz - \int_{0}^{z_0} f(z)dz \right) + o(\epsilon) \end{aligned}$$

and sign $o(\epsilon) =$ sign ϵ. So there exists ϵ such that $\pi_{D'}(X_0) > X_0$. But then according to Lemma 11 there is an equilibrium in D' raising more revenue then X_0 which contradicts optimality of D.

(c) $\exists z_1 : d(z) = d(z_1) = (1-\alpha)z_1 + \alpha \mathsf{E} d \forall z \in [z_1, z_0), d(z) = d(1) = (1-\alpha)z_2 + \alpha \mathsf{E} d \; \forall z \in (z_0, 1], d(z) = (1-\alpha)z + \alpha \mathsf{E} d \; \forall z$ in some semi-neighborhood of z_1

Applying the similar to point (b) transformation of D one may obtain D' with a higher revenue which contradicts optimality of D.

(d) $d(z) = d(0) \forall z \in [0, z_0), d(z) = d(1) \forall z \in (z_0, 1]$

Consider the following transformation depending on sufficiently absolutely small ϵ:

$$d'(z, \epsilon) = \begin{cases} d(0) - \epsilon, \forall z \in [0, z_0] \\ d(1) + \epsilon, \forall z \in (z_0, 1] \end{cases} \tag{31}$$

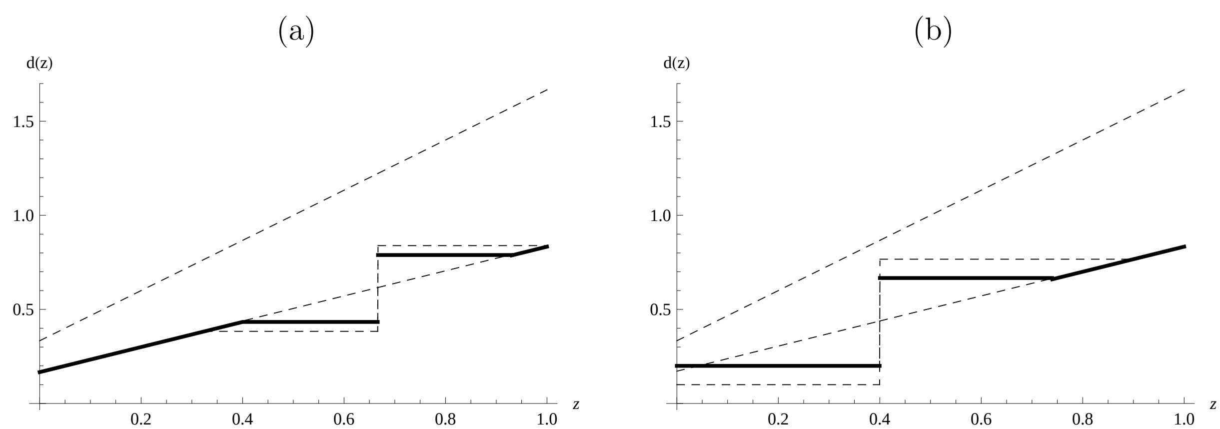
Due to Theorem 2 direct mechanism D' with payment function $d'(z)$ is incentive compatible in X_0-problem. See that

$$\begin{aligned} \pi_{D'}(X_0) &= \pi_D(X_0) + \epsilon \left(\int_{z_0}^{1} f(z) dz - \int_{0}^{z_0} f(z) dz \right) \\ &= X_0 + \epsilon \left(\int_{z_0}^{1} f(z) dz - \int_{0}^{z_0} f(z) dz \right) \end{aligned}$$

In general case the linear term is non zero so there exists ϵ either positive and negative such that $\pi_{D'}(X_0) > X_0$. But then according to Lemma 11 there is an equilibrium in D' raising more revenue then X_0 which contradicts optimality of D. If the linear term is zero that is if z_0 – median, one can set the high enough to obtain $d'(0) = 0$ without any loss in the revenue.

One couldn't apply the transformations above only if the mechanism D is bounded by restrictions. Note that the equalities $d(z) = 2\left((1-\alpha)z + \alpha \mathsf{E} d\right)$ and $d(z) = 0$ are satisfied in the same time if mechanism is incentive compatible. $\square$

An illustration of the transformations used in the proof:



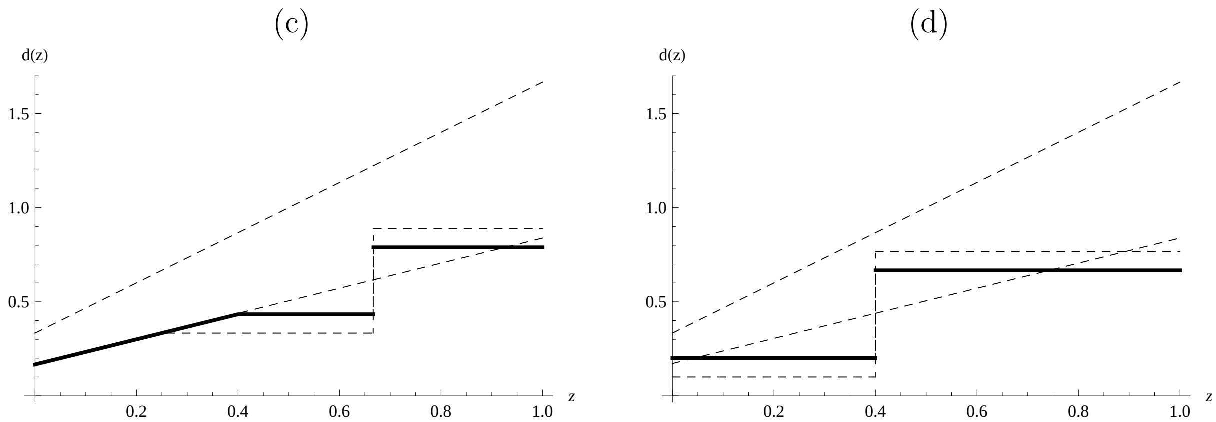


Figure 11: ϵ-transformations used in the proof of Lemma 13

3.1.4 Strictly increasing density function of personal norms

Consider the general setting with strictly increasing density function of personal norms. Such distribution represents generous population as people like to contribute more.

Lemma 14. *If D is an optimal incentive compatible mechanism in the main voluntary payment setup with its payment function $d(z)$ then $d(z)$ is partially constant on every period of increase of population density $f(z)$.*

Proof. Assume the contrary. Then due to Theorem 2 $\exists\, z_0, \epsilon_0 > 0$ such that for $z \in [z_0 - \epsilon_0, z_0 + \epsilon_0]$ holds

$$\begin{cases} d(z) = (1-\alpha)z + \alpha\,\mathsf{E}d \\ f'(z) > 0 \end{cases}$$

Denote $\mathsf{E}d = X_0$. Due to Lemma 9 $\pi_D(X_0) = X_0$.

Consider the same transformation as in Lemma 5:

$$d'(z) = \begin{cases} d(z), \forall z \notin [z_0 - \epsilon_0, z_0 + \epsilon_0] \\ d(z_0 - \epsilon_0), \forall z \in [z_0 - \epsilon_0, z_0] \\ d(z_0 + \epsilon_0), \forall z \in (z_0, z_0 + \epsilon_0] \end{cases} \tag{32}$$

Due to Theorem 2 direct mechanism D' with payment function $d'(z)$ is incentive compatible in X_0-problem. Thus, as $f'(z) > 0$ if $z \in [z_0 - \epsilon_0, z_0 + \epsilon_0]$ it is easy to see that $\pi_{D'}(X_0) > \pi_D(X_0) = X_0$. But $\pi_{D'}(X)$ is non-decreasing and constrained from above due to Lemma 10 so there exists $X_1 > X_0$ such that $\pi_{D'}(X_1) = X_1$. This point corresponds to the equilibrium in the main voluntary payment setup with the greater revenue X_1 due to Lemma 9. Then the mechanism D is not optimal. Contradiction. $\square$

The pattern of the transformation used in the proof is already presented in the Figure 5. Thus, optimal payment function is piecewise constant. Moreover,

Lemma 15. *If D is an optimal direct mechanism in the main voluntary payment setup with strictly increasing population density function then the corresponding payment function has exactly one break point, i.e. consists of two constant levels.*

Proof. Assume the contrary. Let there be at least two break points nearby.

Denote $\mathsf{E}d = X_0$. Consider the same transformation (15) as in Lemma 6:

$$d'(z) = \begin{cases} d(z) & , \forall z \notin [z_1, z_2] \\ d_0 & , \forall z \in [z_1, z_1 + \dfrac{d_2 - d_1}{2(1-\alpha)}] \\ d_2 & , \forall z \in (z_1 + \dfrac{d_2 - d_1}{2(1-\alpha)}, z_2] \end{cases} \tag{33}$$

I proved that mechanism D' is incentive compatible in X_0-problem and has the same revenue as D when personal norms are uniformly distributed. Obviously, $\pi_{D'}(X_0) > X_0$ when population density function is strictly increasing as more people pay higher payment. But then according to Lemma 11 there is an equilibrium in D' raising more revenue then X_0 which contradicts optimality of D.

Let there be no break points at all, $d(z) = d_0 = const$. Then $\mathsf{E}d = X_0 = d_0$. But IR constraint on the agent with type 0 demands $d(z) \leq \alpha \mathsf{E}d \Leftrightarrow d_0 \leq \alpha d_0$ which means $\mathsf{E}d = d_0 = 0$ as $\alpha < \frac{1}{2}$. That contradicts optimality of D.

Hence, the last option left is the optimal function with exactly one break point. □

The main result of this section is

Theorem 4. *Optimal revenue in the voluntary payment model with strictly increasing density function $f(z)$ of agents' personal norms may be raised by the binary choice menu. In this scheme each agent decides whether to pay the only acceptable payment or not to pay at all. Then the acceptable payment, t, share of population paying nothing, s, raised optimal revenue, π^* satisfy:*

$$1 - F(s) - f(s)s - 2\alpha(1 - F(s))^2 = 0 \tag{34}$$

$$\pi^* = \frac{2s(1-\alpha)(1-F(s))}{1 - 2\alpha(1 - F(s))} \tag{35}$$

$$t = 2((1-\alpha)s + \alpha\pi^*) \tag{36}$$

This is the only optimal mechanism except degenerate case $f(z_{med})z_{med} = \frac{1-\alpha}{2}$

Proof. An optimal mechanism has two levels due to Lemma 16. So a two-level mechanism D with either $d(0) = 0$ or $d(0) = 2\alpha\mathsf{E}d$ and optimally chosen break point raises the optimal revenue according to Lemma 13.

Let's prove that $d(0) = 0$. Assume the contrary. Then due to Theorem 2 $\exists z_0$:

$$d(z) = \begin{cases} 2\alpha\mathsf{E}d, \forall z \in [0, z_0) \\ 2(1-\alpha)z_0, \forall z \in (z_0, 1] \\ \alpha\mathsf{E}d \text{ or } 2(1-\alpha)z_0, \text{for } z = z_0 \end{cases} \tag{37}$$

Denote $\mathsf{E}d = X_0$. Due to Lemma 9 $\pi_D(X_0) = X_0$. Denote $z_0' = z_0 - \frac{\alpha}{1-\alpha}X_0$. Consider the following transformation (Fig. 12):

$$d'(z) = \begin{cases} 0, \forall z \in [0, z_0'] \\ 2(1-\alpha)z_0, \forall z \in (z_0', 1] \end{cases} \tag{38}$$

D' is incentive compatible in X_0-problem due to Theorem 2. If $f(z)$ were uniform $\pi_{D'}(X_0) = X_0$ as transformed areas are compensate each other (Fig. 12):

$$S_1 = z_0' * 2\alpha X_0 = z_0' 2(1-\alpha) * (z_0 - z_0') = (\text{upper slope is } 2(1-\alpha)) = S_2$$

Actual density function strictly increases so $\pi_{D'}(X_0) > X_0$ and according to Lemma 11 there is an equilibrium in D' raising more revenue then X_0. It contradicts optimality of D.

Hence, $d(0) = 0$ and the last optimization parameter left is a break point, z_0. Binary structure of $d(z)$ leads to the following problem:

$$\mathsf{E}d = \frac{2(1-F(z_0))z_0(1-\alpha)}{1-2\alpha(1-F(z_0))} \to \max_{z_0 \in [0,1]}$$

$$F.O.C. \quad 1 - F(z_0) - f(z_0)z_0 - 2\alpha(1-F(z_0))^2 = 0$$

$$S.O.C. - \frac{2(1-F(z_0))^2}{f(z_0)(2-4\alpha(1-F(z_0))) - z_0 f'(z_0)} < 0 \; \forall z_0 \in [0,1]$$

Optimal z_0 is exactly the share of population paying nothing s. Then I derive the optimal revenue and the only acceptable payment from the structure of $d(z)$. According to the proof this is the only optimal mechanism except the case when the optimal break point is the median and Lemma 13 gives the whole bunch of optimal mechanisms. First order condition is $f(z_{med})z_{med} = \frac{1-\alpha}{2}$ in this case. □

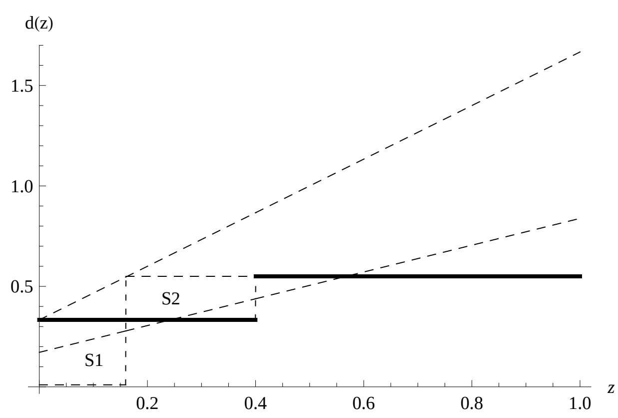


Figure 12: Transformation in Theorem 4.

I show in the section that the optimal mechanism in the general voluntary payments setting with a strictly increasing density function of personal norms is a binary choice menu.

This scheme accepts the only level of donations. Implementation of the scheme may be either explicit or implicit. Many organizations sell memorable items implicitly setting the only acceptable payment. One example is Colorado State University which sells university license plates by \$100 per item. Probably most graduates buy no more than one license plate so \$100 turns out to be the only acceptable payment. One can easily come up with other examples from a day-to-day life.

3.1.5 Strictly decreasing density function of personal norms

Consider the general setting with strictly decreasing density function of personal norms. Such distribution represents stingy population as people like to contribute less.

I start analysis in this section with

Lemma 16. *If D is an optimal direct mechanism in the main voluntary payment setup with strictly decreasing preference density function then the corresponding payment function has no more than one break point.*

Proof. Assume the contrary. Denote $\mathsf{E}d = X_0$. Due to Lemma 9 $\pi_D(X_0) = X_0$. As $d(z)$ has two break points $d(z)$ must intersect a line $(1-\alpha)z + \alpha X_0$ to satisfy Theorem 2 : $\exists z_0, z_2 : d(z) = d(0)\ \forall z \in [0, z_0), d(z) = d(z_2) = (1-\alpha)z_2 + \alpha X_0\ \forall z \in (z_0, z_2], d(0) < d(z_2)$.

It's better to split the function further. Consider the following transformation depending on small positive ϵ (Fig. 13):

$$d'(z,\epsilon) = \begin{cases} d(z), \forall z \notin [z_0 - \dfrac{\epsilon}{2(1-\alpha)}, z_2 - \dfrac{\epsilon}{2(1-\alpha)}] \\ d(z_2), \forall z \in [z_0 - \dfrac{\epsilon}{2(1-\alpha)}, z_2 - \dfrac{\epsilon}{2(1-\alpha)}] \end{cases} \tag{39}$$

D' is incentive compatible in X_0-problem as satisfies Theorem 2. If $f(z)$ were uniform $\pi_{D'}(X_0) = X_0$ as transformed areas are compensate each other (Fig. 13):

$$S_1 = \frac{\epsilon}{2(1-\alpha)}\left(d(z_2) - d(0) - \epsilon\right) = \epsilon\frac{d(z_2) - d(0) - \epsilon}{2(1-\alpha)} = (\text{lower slope is } (1-\alpha)) = S_2$$

As actual density function is strictly decreasing $\pi_{D'}(X_0) > X_0$ and according to Lemma 11 there is an equilibrium in D' raising more revenue than X_0. It contradicts optimality of D. ∎

Lemma 17. *If D is an optimal direct mechanism in the main voluntary payment setup with strictly decreasing population density function then the corresponding payment function has no break points.*

Proof. Assume the contrary that is $d(z)$ has exactly one break point, z_0. Denote $\mathsf{E}d = X_0$. Due to Lemma 9 $\pi_D(X_0) = X_0$. According to Lemma 13 $d(0)$ is equal either 0 or $2\alpha X_0$ (degenerate case can not be optimal because there would be optimal binary choice mechanism having $d'(0) = 0$ which is proven not to be the case).

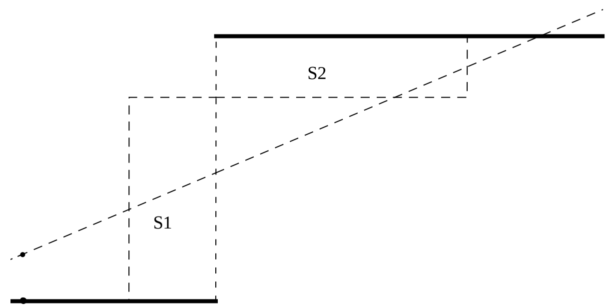


Figure 13: Transformation in Lemma 16.

(a) $d(0) = 0$

$d(1) \geq (1-\alpha) + \alpha X_0 = c(1)$ (against redundancy) according to Lemma 12.

If $d(1) > (1-\alpha) + \alpha X_0 = c(1)$ consider the transformation (Fig.):

$$d'(z) = \begin{cases} 0, \forall z \in [0, z_0 - \dfrac{d(1)-c(1)}{2(1-\alpha)}] \\ c(1), \forall z \in (z_0 - \dfrac{d(1)-c(1)}{2(1-\alpha)}, 1] \end{cases} \tag{40}$$

D' is incentive compatible in X_0-problem as satisfies Theorem 2. If $f(z)$ were uniform $\pi_{D'}(X_0) > X_0$:

$$S_1 = c(1)\frac{d(1)-c(1)}{2(1-\alpha)} > (d(1)-c(1))\frac{2c(1)-d(1)}{2(1-\alpha)} = (d(1)-c(1))(1-z_0) = S_2$$

Actual density function is strictly decreasing which just enhances $\pi_{D'}(X_0) > X_0$ and according to Lemma 11 there is an equilibrium in D' raising more revenue than X_0. It contradicts optimality of D.

If $d(1) = (1-\alpha) + \alpha X_0 = c(1)$ one may apply the same transformation **??** from previous lemma and increase revenue. Contradiction.

(b) $d(0) = 2\alpha X_0$

Consider the following transformation (Fig. 17):

$$d'(z) = \begin{cases} d(z), \forall z \in [0, \dfrac{\alpha}{1-\alpha}X_0] \\ c(z), \forall z \in (\dfrac{\alpha}{1-\alpha}X_0, 1] \end{cases} \tag{41}$$

D' is incentive compatible in X_0-problem as satisfies Theorem 2. Obviously, $\pi_{D'}(X_0) > X_0$ and according to Lemma 11 there is an equilibrium in D' raising more revenue than X_0. It contradicts optimality of D. $\square$

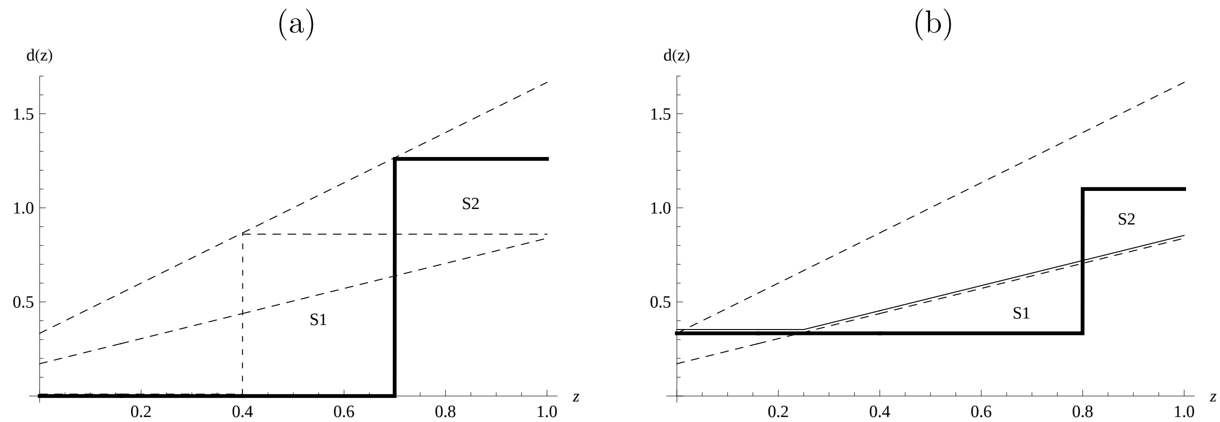


Figure 14: Transformations in Lemma 17

The main result of this section is

Theorem 5. *The optimal mechanism in the voluntary payment model with strictly decreasing density function $f(z)$ of agents' personal norms is implemented by scheme with minimum payment. This scheme accepts any contribution not less than minimum threshold. Everybody pays in the equilibrium. The minimum payment, t, share of population paying exactly t, s, raised optimal revenue, π^*, is derived from:*

$$(1-\alpha(1+F(s)))\,s = \alpha\int_s^1 f(z)z\,dz \tag{42}$$

$$\pi^* = \frac{1-\alpha}{\alpha}s \tag{43}$$

$$t = 2\alpha\pi^* \tag{44}$$

Proof. Denote $\mathsf{E}d = X_0$. It follows from the previous lemma that any optimal payment function $d(z)$ should be like that for some $z_1, z_2 \in [0,1]$:

$$d(z) = \begin{cases} d(0), \forall z \in [0, z_1] \\ c(z, X_0), \forall z \in (z_1, z_2] \\ d(1), \forall z \in [z_2, 1] \end{cases} \tag{45}$$

It is profitable to raise $d(z)$. Consider the following transformation:

$$d(z) = \begin{cases} d(0), \forall z \in [0, \frac{\alpha}{1-\alpha}X_0] \\ c(z, X_0), \forall z \in (\frac{\alpha}{1-\alpha}X_0, 1] \end{cases} \tag{46}$$

D' is incentive compatible in X_0-problem as satisfies Theorem 2. Except the case $z_1 = \frac{\alpha}{1-\alpha}X_0, z_2 = 1$, $\pi_{D'}(X_0) > X_0$ and according to Lemma 11 there is an equilibrium in D' raising more revenue than X_0. That would contradict optimality of D.

Therefore, $z_1 = \frac{\alpha}{1-\alpha} X_0, z_2 = 1$ that is the only optimal mechanism in the setting is a menu with minimum payment with everybody paying in the equilibrium.

X_0 depends on $d(z)$ and must be equal to the integrated profit. It is routine to prove that this leads straightforward to formulas in the statement of the theorem. $\square$

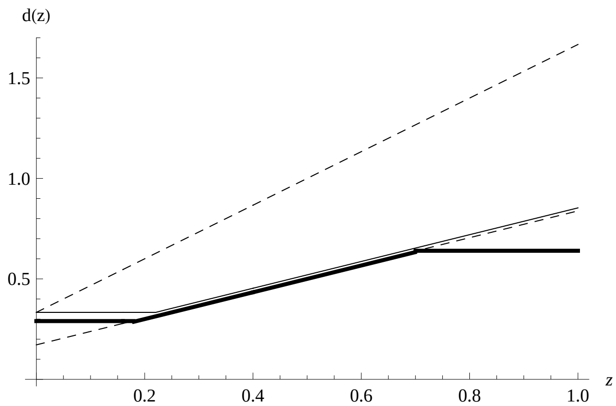


Figure 15: Transformation in Theorem 5.

I show in the section that the optimal mechanism in the general voluntary payments setting with a strictly decreasing density function of personal norms is a minimum payment scheme. This scheme accepts any contributions not less than some threshold. Implementation of the scheme may be either explicit or implicit. Commonly, minimum payment varies from \$2 up to \$20 for any charity organizations though is often justified by technical reasons. One can easily come up with other examples.

3.2 Heterogeneity in rate of conformity (preliminary results)

This section contains preliminary results for another possible heterogeneity in behavior. Assume that rate of conformity is individual but personal norms are the same in the population. This case seems to be rare in practice. However such analysis may be useful at least from the methodology perspective. I find that the optimal mechanism in this setting with both increasing and decreasing population density is a binary choice menu.

So, all agents have the only preferred payment A, but different rate of conformity $\alpha \in [0, 1]$ with c.d.f. $F[0, 1]$ and density function $f[0, 1]$. The main result in the case of decreasing density function of agents' personal norms is

Theorem 6. *Optimal mechanism in the voluntary payment model with strictly decreasing density function $f(z)$ of agents' personal norms is implemented by binary choice menu, i.e. each agent decides whether to pay a possible payment or not to pay at all. The possible payment is equal to $2A$, everybody pays and $\pi^* = 2A$ as well.*

The main result in the case of increasing density function of agents' personal norms:

Theorem 7. *Optimal mechanism in the voluntary payment model with strictly increasing density function $f(z)$ of agents' conformity rate is implemented by binary choice menu, i.e. each agent decides whether to pay a possible payment or not to pay at all. Then the possible payment, t, share of population paying nothing, s, raised optimal revenue, π^*, is derived from:*

$$1 + F(s)(2F(s) - 3) - (1 - s)f(s) = 0 \tag{47}$$

$$\pi^* = \frac{2A(1 - s)(1 - F(s))}{1 - 2s(1 - F(s))} \tag{48}$$

$$t = 2((1 - s)A + s\pi^*) \tag{49}$$

4 Open voting problem (preliminary results)

Another common example of normative conformity is voting, especially an open vote. People often vote for the candidate not because they like and trust him but because they do not want to deviate from the others votes, especially if a winner can take into account their voting choice. I may easily apply the developed model in this setting because voting alternatives are commonly arranged and ordered along one dimension. General politics may be arranged from left-wing to right-wing, tax rates vary between zero and one etc. I analyze just an equilibrium behavior in the simplest case of uniform preferences, though one may try to generalize the model and apply mechanism design as well.

The game proceeds as follows. There are two candidates on public parliament meeting willing to get as many votes as possible. They know that the personal norms, i.e. preferred policies, are distributed uniformly among parliamentarians ($A_i \sim U[0,1]$) who conform at the rate $\alpha < 1$ according to the main model. On the first stage both candidates simultaneously present their policy x and y from the possible set normalized to [0,1]. On the second stage each parliamentarian votes for one of the candidates. I find all Subgame Perfect Nash equilibria in pure strategies in this setting.

The equilibrium without conformity, $\alpha = 0$, consists of candidates choosing $x = y = \frac{1}{2}$ and the shares of votes for the both candidates being equal, which is consistent with the standard voting model. I investigate the possibilities of other eq'a. First, consider the situation with different proposed alternatives. Facing the alternatives x and y with $y > x$ without loss of generality on the second stage of the game each parliamentarian compares

$$u_i(x) = (x - N_i)^2 \qquad vs \qquad u_i(y) = (y - N_i)^2 \tag{50}$$

Let μ be the share of the parliamentarians voting for y. Then $\mathsf{E}(a) = (1-\mu)x + \mu y$ and (24) due to (4) transforms to

$$((1 - 2\alpha(1-\mu))x^2 - 2\alpha(1-2\mu)xy \qquad vs \qquad (1-2\alpha\mu)y^2 - 2(1-\alpha)A_i(y-x) \tag{51}$$

A voter minimizes her losses so if in (50) $vs =$ "<" then the parliamentarian votes for the first candidate, if $vs =$ ">" then the parliamentarian votes for the second candidate, if $vs =$ "=" then a voter is indifferent between the alternatives.

Lemma 18. *If the parliamentarian i votes for the candidate with higher policy in the eq'm then every parliamentarian j with her preferred policy $A_j > A_i$ vote for the same candidate. If the parliamentarian i votes for the candidate with lower policy in the eq'm then every parliamentarian j with her preferred policy $A_j < A_i$ vote for the same candidate.*

Proof. That follows immediately from (50), decision rule and the fact that $A_i \geq 0$ and $\alpha < 1$ □

It follows that there can be two possible types of eq'm.

1. Unanimous voting (don't mix it up with anonymous one).

In an unanimous voting all the parliamentarians vote for the same candidate.

Lemma 19. *The could be no unanimous voting in the equilibrium.*

Proof. Assume the opposite. As the problem is symmetric without loss of generality $y \geq \frac{1}{2}$ and at least for $x < y$ $\mu = 1$ and (50) holds with $\geq \forall A_i$ therefore for $A_i = 0$ then

$$x^2 - 2\alpha xy - (1-2\alpha)y^2 \geq 0 \qquad \forall\, x < y \tag{52}$$

However at $x^* = \alpha y$ (52) holds with an opposite sign strictly for all possible α. So there would be considerable share of voters preferring x^* and increasing with α according to Lemma 18. Contradiction. □

2. Mixing voting.

In this equilibrium each candidate wins some share of votes, $0 < \mu < 1$. According to Lemma 18 there must be an indifferent between the alternatives voter with $A_i = 1 - \mu$. Then from (50) using $x \neq y$

$$2\mu(1 - \alpha + \alpha x - \alpha y) = 2 - 2\alpha - x + 2\alpha x - y \tag{53}$$

That equation corresponds to two possible cases.

First case.

$$1 - \alpha + \alpha x - \alpha y \neq 0 \quad \text{and} \quad \mu = \frac{2 - 2\alpha - x + 2\alpha x - y}{2(1 - \alpha + \alpha x - \alpha y)} \tag{54}$$

μ calls for an extra definition as it can not be beyond $[0, 1]$ however that is not necessary so far. One can see that $\mu \to 1 - x$ as $y \to x + 0$ or $\mu \to 1 - y$ as $x \to y - 0$. Let's call this a mimic strategy.

Second case.

$$1 - \alpha + \alpha x - \alpha y = 0 \quad \text{and} \quad 2 - 2\alpha - x + 2\alpha x - y = 0 \tag{55}$$

One can obtain from the system that $x = 1 - \frac{1}{2\alpha}, y = \frac{1}{2\alpha}$, which is consistent with condition $y > x$. However, then no μ can be supported in the eq'm as each candidate has an incentive to deviate to the mimic strategy in the first case. So there is no ambiguity with $y \neq x$ case and I obtain

Lemma 20. *Given the position x of the candidate his rival may always capture almost all the votes of the parliamentarians with preferred policy A_i from chosen side of the x i.e. $A_i > x$ or $A_i < x$.*

Proof. That follows immediately from the applying the mimic strategy and uniqueness of the μ in that case. □

Then I apply the same reasoning as in case with no conformity and obtain that just one possible eq'm in pure strategies survives with $x = y = \frac{1}{2}$ and $\mu = \frac{1}{2}$. There are no new eq'a in pure strategies in this setting. However it doesn't mean that the standard eq'm survives either. In the eq'm there is no need for voters to deviate. Let's investigate candidates incentives for deviation. Any deviation leads to the first case of mixing voting so it would be reasonable for a candidate if and only if in (54) $\exists\, y \in [\frac{1}{2}, 1] : \mu|_{x=\frac{1}{2}} > \frac{1}{2}$. Analysis of μ depending on y given $x = \frac{1}{2}$ gives the following graphs:

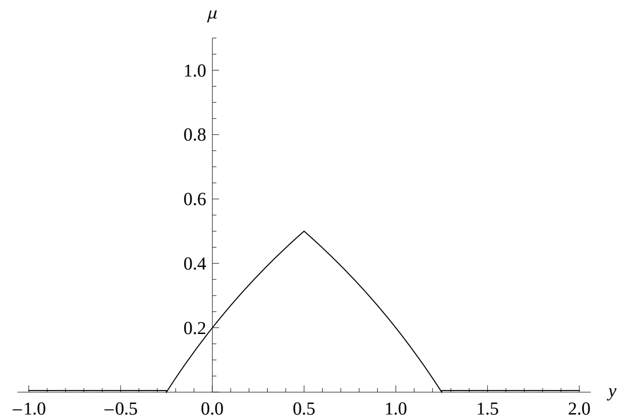


Figure 16: Share for the second candidate depending on her position given the position of her rival being $\frac{1}{2}$ and the rate of conformity being 0.25

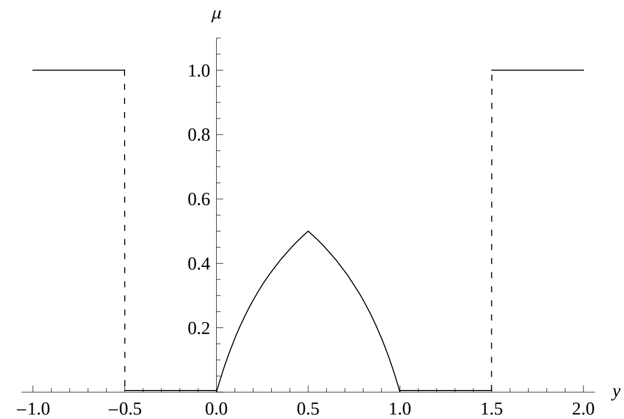


Figure 17: Share for the second candidate depending on her position given the position of her rival being $\frac{1}{2}$ and the rate of conformity being 0.5

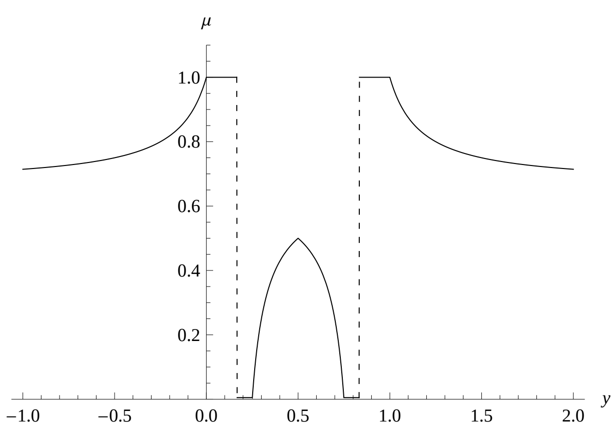


Figure 18: Share for the second candidate depending on her position given the position of her rival being $\frac{1}{2}$ and the rate of conformity being 0.75

The vertical asymptotes are at $y = \frac{1}{2} - \frac{1-\alpha}{\alpha}$ and at $y = \frac{1}{2} + \frac{1-\alpha}{\alpha}$ with μ being equal to 0 at these point analyzing (50) so it is quite clear that there is an incentive to deviate iff $\frac{1-\alpha}{\alpha} < \frac{1}{2} \;\Leftrightarrow\; \alpha > \frac{2}{3}$. As a result

Theorem 8. *An equilibrium in pure strategies in the parliament voting game when $\alpha \leq \frac{2}{3}$ is unique. In the equilibrium both candidates place the same median position and equally divides votes. When conformity in the parliament is too high, $\alpha > \frac{2}{3}$, this equilibrium must be supported by proper beliefs.*

This result seems to be intriguing. When the conformity becomes overwhelming a candidate may deviate from the regular equilibrium to the extreme policy if he thinks that crowd may support him. This may partly explain why unstable political situations often lead to extreme policies. The unstable political situation may raise the rate of conformity in the parliament as well as change candidate beliefs.

5 Conclusion

In the thesis I enrich the framework developed by Fischer & Huddart (2008) by allowing agents to have different preferences over actions. I characterize optimal mechanisms in voluntary payments environments, such as donations, contributions for public goods, and tips. I show that when the agents have heterogeneous preferences with conformity the optimal contributions mechanism restricts the possible range of acceptable payments. In addition to a zero contribution required by voluntary setup the optimal mechanism sets either only one positive payment level or a minimum acceptable payment depending on the distribution of individual components of the preferences. Importantly, maximum revenue raised by the optimal mechanism may be much higher than revenue raised with no restrictions.

Although the main motivation of the research is a voluntary payments problem one may apply the same modeling and results to other voluntary choice settings. The original payment may refer to a rate of participation in social activities, condom usage or blood donations in the society. Then the optimal mechanisms correspond to optimal social restrictions and depend on society's attitude. For example, consider participation in social activities. When society is enthusiastic the policy offering the only acceptable rate of participation is optimal. The government may call for voluntary city cleaning on one particular spring day to implement the policy, one either participate or not. When society is lazy the policy offering the minimum rate of participation is optimal. A minimum plan of social work in a voluntary organization may implement such a policy.

Overall, this study supports the voluntary payment evidence and explains existing variety of contribution schemes. It also constitutes a solid ground for future research on conformity behavior.

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