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The Political Economy of Faculty Selection *

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1 Introduction

Many important decisions are made by committees whose members have limited capacity to assess the true value of their choices. In hiring or senior promotion in academia, those who make the decision do not necessarily have relevant expertize in the exact area of candidate's specialization. Even if they rely on the outside knowledge such as reference letters, they can use the information provided by the outside experts only to some extent; the limit, again, being their own depth and breadth of knowledge in the area.

The preamble to "Principles Governing Research at Harvard" states: "The primary means for controlling the quality of the scholarly activities of this Faculty is through the rigorous academic standards applied in selecting its members." However, the ultimate outcome depends not only on the standards, but also on the organization of the process. In this paper, we build a simple political economy model of faculty selection by a research department. A department is an interval of a one-dimensional line; each department member is characterized by her specialization, which is a point of the interval, and her research capacity at any point of the line, which is maximum at the specialization. A candidate has an ideal specialization, the point of his maximum talent; if a candidate is strategic, he chooses a specialization. The further away from the ideal specialization the candidate's choice is, the lower is his research capacity as a researcher. Yet organizational structure (the size of departments that vote over candidates) and initial allocation of

*A part of this thesis is based on research notes by Irina Khovanskaya and Konstantin Sonin; eventually, both parts will form a joint paper.

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faculty have a major impact on the long-term quality of the department.

Though we do not assume any strategic considerations on the existing faculty part - they are concerned with hiring as capable researchers as possible - our model does have a major political economy flavor. With strategic candidates choosing their research specialization, e.g. topic for their job-market paper, the specialization of the median faculty member might play an important role similar to that of the median voter in the Downsian competition. If the department is narrow, then candidates have incentives to pander to the expected median, departing, at the cost of lower research capacity, from their ideal specializations. In a wider department, these incentives are less pressing, which results in a higher, on average, accepted candidates. If the department is a full circle, as in one of our specifications, and all faculty members are equally likely to be pivotal, there is no incentives to pander at all.

A practical motivation for our paper comes from the choice that creators of research universities, a major concern for governments and private leaders around the world these days, need to make. Should a university start with hiring faculty in many disciplines, or concentrate them around a few narrow areas of research? Given that the existing faculty will make the next round of appointment decisions, the initial choice should play an important role. Equally important is the subdivision of the faculty body into departments that make hiring decisions.[1] Should those who do mathematical statistics, applied mathematics, and pure mathematics form one, two, or three departments? Should tenure promotion be made by the entire tenured faculty of the Graduate School of Arts and Sciences?

In an important early contribution, Carmichael [3] addressed the issue of tenure promotion; he argues for allocating decision-making power to tenured faculty as they, unlike the non-tenured faculty, would not consider strong junior candidates as their potential

[1]The difference might be dramatic. In the U.S., most of the hiring and promotion decisions are made at the departmental level. In Russia, it is often made by subdepartments that are much more narrow.For example, the Department of Mathematics of the Moscow State University (an entity perhaps twice as big as a Department of Mathematics in Princeton or Harvard), for decades made hirings and promotions at the level of Subdepartments, which included, among others, Subdepartments of Higher Algebra, Higher Geometry and Topology, Differential Geometry, Real Analysis, Functional Analysis, Probability, Mathematical Statistics, Differential Equations, etc.

competitors for the limited number of the seats on the department. The model assumes full information with respect to candidates' capacity; neither faculty members, nor candidates differ with respect to their research specialization.

Modeling hiring decisions in both static and dynamic contexts, we try to strike a right balance between the technical complexity of a model as a political economy model and as an organizational economics model. Information aggregation in voting is a major avenue of research for political scientists (Feddersen and Pesendorfer,[4], [5], Feddersen and Sandroni, [6]), but the literature on information aggregation in voting rarely addresses organization issues. Ayres, Rowat, and Zakariya [2] is a rare exception.

Acemoglu, Egorov, and Sonin [1] construct a fully dynamic non-cooperative model of self-selecting governments with potential candidates that differ in their competence. Our current model is a two-dimensional version, with faculty members' and candidates' research specialization being the second dimension. Naturally, this comes at a cost in terms of modeling as a dynamic non-cooperative game.

The rest of the paper is organized as follows. Section 2 introduces the model and contains all basic definitions. A thorough analysis of the model is given in Section 3. The analysis has two components: strategical and dynamical ones which are discussed in Subsections 3.1 and 3.2 respectively. Section 4 contains some concluding remarks.

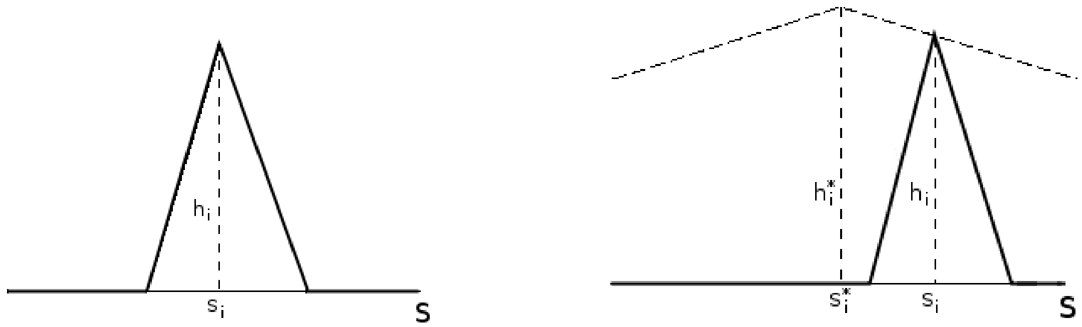
2 Setup

Each professor i has a two-dimensional characteristics (s_i, h_i). The first parameter, $s_i \in S$[2] is her *specialization*; the second parameter, $h_i \in [0, H]$, is her *maximum research capacity.* Given the specialization s_i and maximum capacity h_i, capacity of professor i at specialization $s \in S$ is defined by

$$c_{(s_i,h_i)}(s) = \max\left(h_i - \beta\left|s_i - s\right|, 0\right).$$

Let $I_i = \left(s_i - \frac{h_i}{\beta}, s_i + \frac{h_i}{\beta}\right)$ denote the support of function $c_{(s_i,h_i)}$; we call I_i professor's i *support.*

[2] S can either be $[0, 1]$ or a unit circumference S^1.



(a) $c_{(s_i,h_i)}(s)$ (b) $h_i(s_i)$

Figure 1: Professors and Candidates

Each of candidates $j \in J$ chooses from a range of possible specialties. Candidate j has an *maximum ability specialization* $s_j^* \in S$, in which his maximum research capacity is equal to h_j^*. If j chooses specialty s_j, then his maximum research capacity in s_j is determined by

$$h_j(s_j) = \max\left(h_j^* - \frac{1}{2}\left|s_j^* - s_j\right|, 0\right).$$

Similar to that of professors, research capacity of candidate j at point s is equal to $\max\left(h_j - \beta\left|s_j - s\right|, 0\right)$. β is an exogenous parameter.[3]

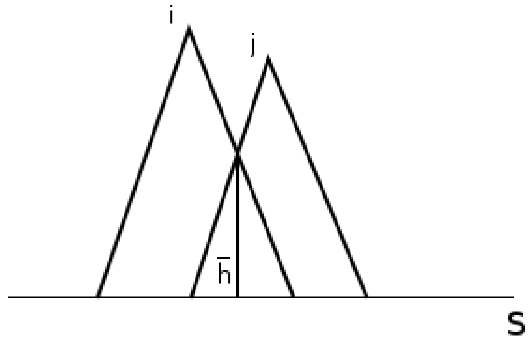
If professor i meets candidate j, i obtains an estimate of j's true research capacity, h_j. After the signal, i knows that h_j exceeds

$$\overline{h}_i(j) = \min\left\{\max_{s\in I_i\cap I_j} \min\left(c_{(s_j,h_j)}(s), c_{(s_i,h_i)}(s)\right), h_i\right\}, \quad \text{where } \max\{\emptyset\} := 0.$$

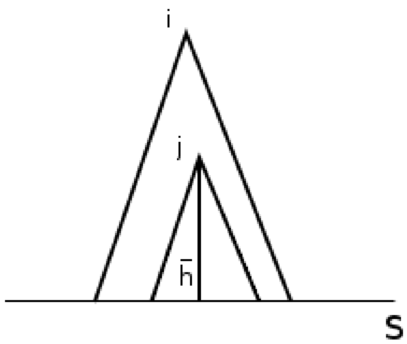
Obviously, if there exists s^* such that $h_i - \beta\left|s_i - s^*\right| = h_j - \beta\left|s_j - s^*\right|$, then $\overline{h}_i(j) = h_i - \beta\left|s_i - s^*\right|$. If there is no such s^* then either $I_i \cap I_j = \varnothing$, and i assumes that $\overline{h}_i(j) = 0$, or $I_i \subset I_j$ (in this case, i estimates that $\overline{h}_i(j) = h_i$), or $I_i \supset I_j$ (then $\overline{h}_i(j) = h_j$).

One assumption calls for an extended discussion. We assume that members of the selection committee may not share their estimates of the candidate's capacity. Indeed,

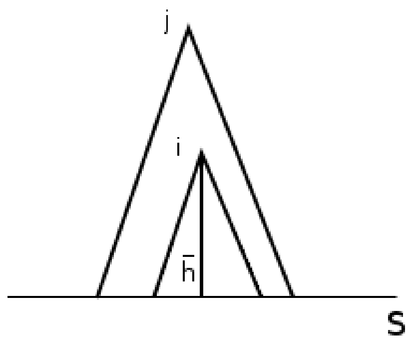
[3]One interpretation is that the higher is β, the more specialized is education.



(a) c^* exists



(b) $I_j \subseteq I_i$



(c) $I_i \subseteq I_j$

Figure 2: Estimation of a Candidate

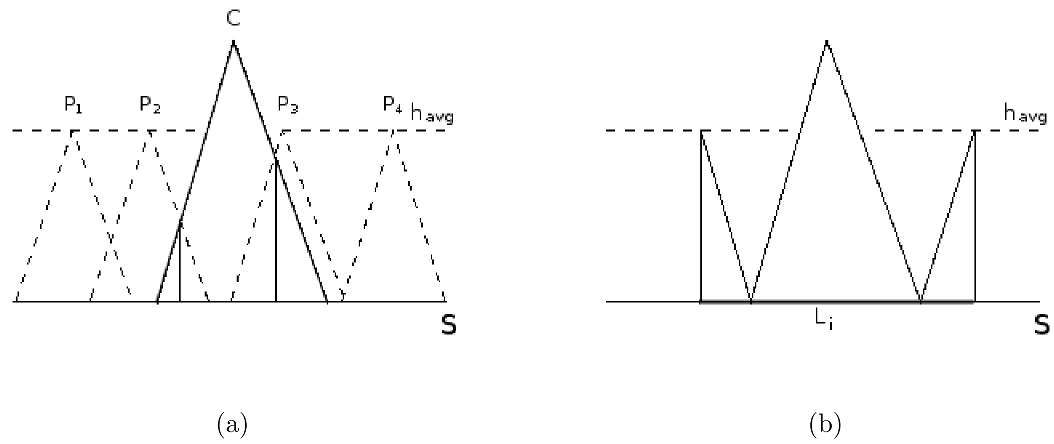


Figure 3: L_i

if information, which we assume available to one committee member can be passed to another, the latter will know the candidate's research capacity for sure. Our assumption is justified by two arguments. First, the premise that the individual's ability to process information is limited by her own specialization and research capacity is basic to our analysis. If an algebraic geometer A tells a biologist that some other algebraic geometer B is good enough, the capacity of the biologist to process this information is the same as his capacity to estimate B's capacity on his own. Second, the critique about individual knowledge could be dealt with by having a multidimensional research space. There, information exchange would not lead to the full candidate's type revelation. However, the cost in terms of increased complexity of the model would be perhaps prohibitive.

For the further discussion we would need a following notation:

Notation 2.1. *Suppose all professors have the same research capacity h_{avg}. Let L_i stand for those professors who estimate Candidate i's research capacity greater then 0:*

$$L_i = \left(-\frac{h_i + h_{\text{avg}}}{\beta} + s_i, s_i + \frac{h_i + h_{\text{avg}}}{\beta}\right) =: (\underline{s_i}, \overline{s_i})$$

3 Analysis

3.1 One Period Game

We are going to analyze the strategic behavior of candidates in the following Bayesian game:

t=0 Two candidate of types (h_i^*, s_i^*) are born. Types are private knowledge and are independently drawn from uniform distribution on $[h_{\min}, H] \times S$, the distribution itself is a common knowledge.

t=1 The candidates simultaneously choose their strategies s_i.

t=2 Each professor estimates the candidates and votes for the one with the higher estimate. If the estimates are equal, then the professor splits his vote and gives each candidate a half.

t=3 The candidate with higher number of votes wins and gets 1, the loser gets 0. They both receive 1/2 in case of a tie.

First, let us explicitly formulate the pay-offs to the both sides with the help of the following lemma.

Lemma 3.1. *Consider two candidates with parameters $(s_i^*, h_i^*)_{i=1,2}$ who choose strategies s_1 and s_2. Suppose that professors are distributed uniformly on S and all have the same research capacity h_{avg}. If $H + h_{\text{avg}} < \beta/2$ and $h_i^* > h_{\text{avg}} + 1/2$ then the candidates' pay-offs are described by the following formulas:*

$$u_1(s_1, s_2) = \frac{1}{2}\left[\operatorname{sgn}\left(|L_1 \cap S| - |L_2 \cap S|\right) + 1\right]$$
$$u_2(s_1, s_2) = \frac{1}{2}\left[\operatorname{sgn}\left(|L_2 \cap S| - |L_1 \cap S|\right) + 1\right]$$

i.e. the first candidate wins with probability one iff $|L_1 \cap S| > |L_2 \cap S|$, ties when $|L_1 \cap S| = |L_2 \cap S|$, and loses with probability one when $|L_1 \cap S| < |L_2 \cap S|$.

Proof. Condition $H + h_{\text{avg}} < \beta/2$ guarantees that $L_1 \cap S \neq S$ and $L_2 \cap S \neq S$ no matter what strategies s_1 and s_2 the players choose. The second condition $h_i^* > h_{\text{avg}} + 1/2$ implies

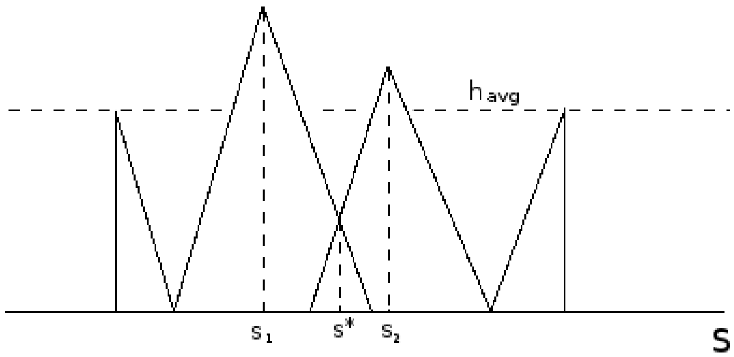


Figure 4: $I_1 \cap I_2 \neq \emptyset$

that $h_i(s_i)$ is always greater then h_{avg} no matter what strategies s_1 and s_2 the players choose.

The statement is obvious when $L_1 \cap L_2 = \emptyset$, when $L_1 \subseteq L_2$ (in this case $I_1 \subseteq I_2$), and when $L_2 \subseteq L_1$ (in this case $I_2 \subseteq I_1$). The only non-trivial case is $L_1 \cap L_2 \neq \emptyset$ and $L_1 \cap L_2 \neq L_1$ and $L_1 \cap L_2 \neq L_2$. Let us examine it in further detail.

First suppose that $S = [0, 1]$ and $I_1 \cap I_2 \neq \emptyset$, then there exists such s^* that $c_{(s_1,h_1)}(s^*) = c_{(s_2,h_2)}(s^*)$ (see Fig. 4).

Without loss of generality let $s_1 < s_2$, then all professors with $\underline{s_1} < s < s^*$ will vote for candidate 1 and all with $s^* < s < \overline{s_2}$ will vote for candidate 2.[4] Thus

$$u_1(s_1, s_2) = \frac{1}{2}\left[\operatorname{sgn}\left(|[\underline{s_1} \vee 0, s^*]| - |[s^*, \overline{s_2} \wedge 1]|\right) + 1\right]$$

But $s^* - \overline{s_1} = \underline{s_2} - s^*$, thus [5]

$$\begin{aligned}\left|[\underline{s_1} \vee 0, s^*]\right| - |[s^*, \overline{s_2} \wedge 1]| &= \left|[\underline{s_1} \vee 0, s^*]\right| + |[s^*, \overline{s_1}]| - |[s^*, \overline{s_2} \wedge 1]| - \left|[\underline{s_2}, s^*]\right| = \\ &= \left|[\underline{s_1} \vee 0, \overline{s_1}]\right| - \left|[\underline{s_2}, \overline{s_2} \wedge 1]\right| = |L_1 \cap S| - |L_2 \cap S|\end{aligned}$$

[4]This is not true when $c_{(s_1,h_1)}(s^*) = c_{(s_2,h_2)}(s^*) > h_{\text{avg}}$. In this case there exists a share of professors around s^*, who are indifferent between the two candidates. These neutral professors are located symmetrically around s^* and therefore do not affect our argument.

[5]When $\overline{s_1} > 1$ or $\underline{s_2} < 0$ this argument is irrelevant, since the candidate situated closer to the middle of the segment will get a higher share of votes. In this particular case the statement of the lemma becomes obvious, therefore we will assume $0 < \underline{s_2} < \overline{s_1} < 1$.

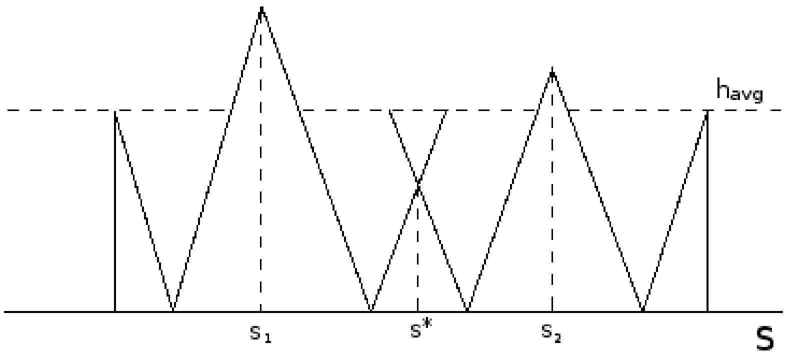


Figure 5: $I_1 \cap I_2 = \emptyset$

where $x \wedge y = \min(x, y)$ and $x \vee y = \max(x, y)$.

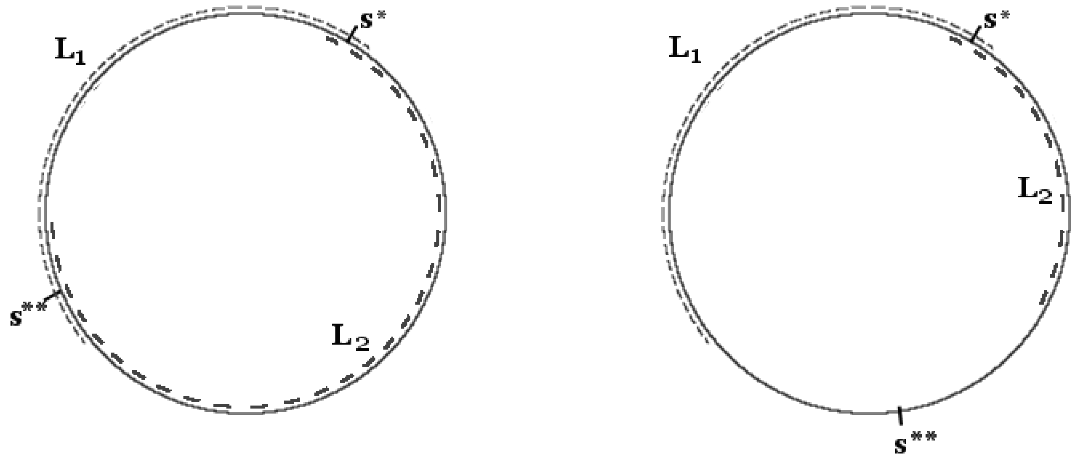
When $S = [0, 1]$ and $I_1 \cap I_2 = \emptyset$ (see Fig. 5), then we simply put $s^* = (\overline{s_1} + \underline{s_2})/2$ and repeat the logic of the previous argument

Now, suppose that $S = S^1$. Then there exist two different points s^* ans s^{**} such that s^* is a "middle" of a connected component of $L_1 \cap L_2$ and s^{**} is either a "middle" of another connected component (Fig. 6(a)), or is equidistant from L_1 and L_2 (Fig. 6(b))

If we are in case (a), then the professors located on the arc $(s^* - L_1 - s^{**})$ will vote for candidate 1 and those on the arc $(s^* - L_2 - s^{**})$ will choose candidate 2. The picture suggests that $|[s^* - L_1 - s^{**}]| - |[s^* - L_2 - s^{**}]| = |L_1| - |L_2|$, the formal proof of this statement can be derived using exactly the same logic as for the case $S = [0, 1]$.

It is easy to see that the same result holds for case (b).

■



(a) (b)

Figure 6: Defining s^* and s^{**}

This lemma has two implications which will be stated below:

Proposition 3.2. *Suppose that* $H + h_{\text{avg}} < \frac{\beta}{2}$ *and* $h_{\min} > h_{\text{avg}} + 1/2$ *. If professors, who evaluate candidates, are distributed uniformly on* S^1 *and all have the same research capacity* h_{avg}*, then*

$$s_i = s_i^* \quad i = 1, 2$$

is a weakly dominant equilibrium.

That is, given that the whole department is participating in hiring process, it is optimal to report the type truthfully for each candidate regardless of the actions and types of the other candidate.

Proposition 3.3. *Suppose that* $H + h_{\text{avg}} < \frac{\beta}{2}$ *and* $h_{\min} > h_{\text{avg}} + 1/2$ *. If professors, who evaluate candidates, are distributed uniformly on* $[0, 1]$ *and all have the same research capacity* h_{avg}*, then*

$$s_i = \begin{cases} s_i^*, & if \quad L(s_i^*, h_i^*) \subseteq [0,1] \\ \frac{h_{\text{avg}} + h_i^* + \frac{s_i^*}{2}}{\beta + \frac{1}{2}}, & if \quad L(s_i^*, h_i^*) \nsubseteq [0,1] \quad and \ s_i^* < \frac{1}{2} \\ \frac{\beta - h_{\text{avg}} - h_i^* + \frac{s_i^*}{2}}{\beta + \frac{1}{2}}, & if \quad L(s_i^*, h_i^*) \nsubseteq [0,1] \quad and \ s_i^* > \frac{1}{2} \end{cases} \quad i = 1, 2$$

is a weakly dominant equilibrium.

We will call this type of equilibrium "pandering" since the transition from the initial point s_i^* to the equilibrium point s_i, is done by moving closer to the median professor ($s = 1/2$). Pandering equilibrium implies deviation from the point of maximum ability specialization for a wide range of types (s, h) which results a social loss in form of decreased research capacity.

Following the spirit of the previous propositions a short-sighted policy maker would be tempted to root out the hierarchy of subdepartments and merge them into one huge department (or even merge all departments into a "general science" department). The next example will reveal an obvious flaw of such policy, the logic behind it is quite similar to that of minority rights protection.

Example 3.4. *Suppose that the department consists of two subdepartments of "math"* $\left[0, \frac{1}{4}\right]$ *and "computer science"* $\left[\frac{3}{4}, 1\right]$. *Professors are distributed uniformly within each of the subdepartments, however, the "CS" subdept. is bigger - it has a total share* $m > 1/2$ *of the whole department professors , the "math" subdept. has a share* $1 - m$ *respectively; and all professors have the same research capacity* h_{avg}. *As usual we will assume that* $H + h_{\text{avg}} < \frac{\beta}{2}$ *and* $h_{\min} > h_{\text{avg}} + 1/2$.

Consider two candidates: a "math" candidate with $(s_1^*, h_1^*) \in [0, 1/4] \times [h_{\min}, H]$ *and a "CS" candidate with* $(s_2^*, h_2^*) \in [3/4, 1] \times [h_{\min}, H]$. *Then, in equilibrium, the "math" candidate will always pander to the "CS" subdepartment, moreover, when* h_1^* *is relatively low such pandering will result in choosing her specialization within "CS" subdept. i.e.* $s_1 \in [3/4, 1]$.

As a result of such pandering the "CS" subdepartment will grow much faster then the "math" one strengthening the inequality between them (m will rise), eventually the "math" subdept. will shrink to zero and the united department will essentially be a "CS" subdept.

Now we will consider different voting rules and investigate their influence on our argument. Remarks 3.5 and 3.6 will show the robustness of our results with respect to a slight altering of the voting procedure.

Remark 3.5. *When $H + h_{\text{avg}} < \beta/2$ and $h_{\min} > h_{\text{avg}} + 1/2$ modification of the voting procedure by adding a lower bound $\underline{h} < h_{\text{avg}}$, does not change the equilibrium in the $S = S^1$ game*[6].

However, the equilibrium of the $S = [0,1]$ game is slightly altered: pandering becomes less strong because the candidates now maximize the length of

$$L_i(s_i, h_i(s_i) - \underline{h}, h_{\text{avg}} - \underline{h}) \cap [0,1]$$

.

Proof. Since $h_{\min} > h_{\text{avg}} + 1/2$, the candidates compete only above the $\underline{\mathrm{h}}$ threshold.

Thus, redefining capacity function

$$c_{(s_i,h_i)}(s) = \max\left(h_i - \beta\left|s_i - s\right| - \underline{\mathrm{h}}, 0\right)$$

and putting a new threshold equal to 0 reduces the game to the one discussed in Lemma 3.1. ∎

Remark 3.6. *Furthermore, if $H + h_{\text{avg}} < \beta/2$ and $h_{\min} > h_{\text{avg}} + 1/2$ introduction of the share α of votes , that a candidate should have in order to be accepted*[7]*, does not change the equilibrium strategies in the game whether $S = [0,1]$ or $S = S^1$ if additionally $\alpha > \frac{1}{2}$. However, this kind of equilibrium is not in weakly dominant strategies, it is ex-post equilibrium.*

Proof. It is easy to show that if the candidate 2 panders then the best response of candidate 1 is also to pander no matter what are their types; hence pandering is an ex-post equilibrium.

In order to see that pandering is not a weakly dominant strategies equilibrium consider a case when $H + h_{\text{avg}} = \beta/2 - \delta$, $s_1^* = \frac{H+h_{\text{avg}}}{\beta} - x$, $h_1^* = H$, $s_2^* = \frac{H+h_{\text{avg}}}{\beta} - \frac{x}{2} - \varepsilon$, $h_2^* = H - \frac{\beta x}{2}$.

Suppose that the candidates decide to choose their ideal specializations, the candidate 1 gets $2(H + h_{\text{avg}})/\beta - x$ share of votes and candidate 2 gets 0 votes.

[6] Professors now have three choices: "candidate 1", "candidate 2", and "no one" if both of his estimates do not exceed $\underline{\mathrm{h}}$

[7] He also needs to beat the other candidate and overcome the barrier $\underline{\mathrm{h}}$

Let us see what happens if the candidate 1 decides to pander, i.e. $s_1 = \frac{H+h_{\text{avg}}}{\beta} - \frac{x}{2\beta+1}$ and the candidate 2 does not change his strategy. Candidate 2 will get $\frac{H+h_{\text{avg}}}{\beta} - \frac{x}{2} + O(\varepsilon)$ share of votes and Candidate 1 gets

$$\text{Share}_1 = \frac{2(H+h_{\text{avg}})}{\beta} - \frac{2x}{2\beta+1} - \left(\frac{H+h_{\text{avg}}}{\beta} - \frac{x}{2}\right) + O(\varepsilon) = \frac{H+h_{\text{avg}}}{\beta} - x\frac{3-2\beta}{2(2\beta+1)} + O(\varepsilon)$$

If $\beta < 3/2$ then for any $\alpha > 1/2$ there exist $x, \delta, \varepsilon > 0$ such that $\frac{2(H+h_{\text{avg}})}{\beta} - x > \alpha$, so choosing her ideal specialization candidate 1 wins, and $\text{Share}_1 < 1/2 < \alpha$, so choosing to pander both of the candidates lose. ■

3.2 Dynamic Model

This section modifies the framework studied so far by introducing dynamics into the model. However, we will have to remove all strategic interaction between the candidates for the sake of simplicity: from now on candidates *will have* to report their type truthfully. As we have seen in Proposition 3.2 this is a reasonable assumption in some particular context, we will assume truth-telling behavior throughout the whole dynamic section even though the context will be different from the one discussed in Proposition 3.2.

A group of researchers called "professors" form the initial staff and initiate the faculty selection process. They face an incoming flow of non-strategic researchers ("candidates") with uniformly distributed parameters that arrive in discrete moments of time one by one and decide on the admission of a new candidate using the $(\alpha, \underline{\text{h}})$ rule, i.e the candidate is given an offer when the share of professors that estimate her research capability as $\underline{\text{h}}$ is greater or equal than α.[8] Once given an offer the candidate becomes a professor enjoying full rights and votes on all next candidates.

Denote by $Y = \{Y_1, Y_2, \ldots, Y_m\}$ the initial faculty staff; $\{Y_i\}_{i=1}^n$ are some known points in $S^1 \times [0, H]$. The sequence of iid random variables $X_1, X_2, \ldots, X_n, \ldots$, where $X_i = (s_{X_i}, h_{X_i})$, defined on a probability space $(\Omega, \mathcal{F}, \mathrm{P})$, with uniform distribution in $S^1 \times [0, H]$ will represent the incoming flow of candidates.

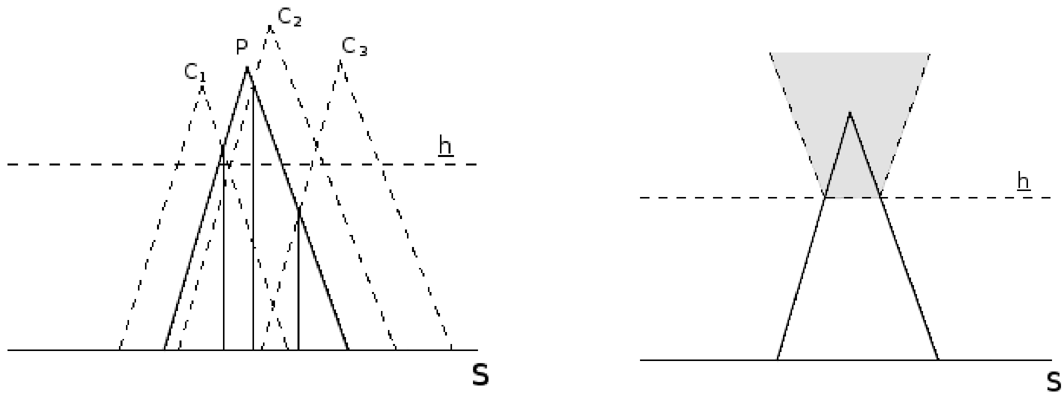
[8] For example in case of $\alpha = \frac{1}{2}$ we get a simple majority rule.

Definition 3.7. ***The acceptance set*** *$C(Y_i)$ (Fig. 7(b)) of professor Y_i is:*

$$C(Y_i) = \begin{cases} \{h \geqslant 2\underline{h} - h_{Y_i} + \beta|s - s_{Y_i}|\} \cap \{h \geqslant \underline{h}\}, & h_{Y_i} \geqslant \underline{h} \\ \emptyset, & h_X < \underline{h}. \end{cases} \quad (1)$$

Essentially $C(Y_i)$ is the set of parameters (research specialization and research capacity) that a candidate should have in order to be approved by the expert Y_i.

It turns out, that $\underline{\text{h}}$, H and β all play the same role in the model, so for further analysis we will put $\underline{\text{h}} = 0$ and $H = 1$, thus, the only parameter that would reflect the relative size of the field S is β.



(a) (b)

Figure 7: Acceptance set

Following the spirit of (1) we can define the acceptance set of the initial staff.

Definition 3.8. ***The acceptance set*** *$C(Y)$ (Fig. 7(b)) of the initial staff Y is:*

$$C(Y) = \{(s, h) \in S^1 \times [0, H] : \#\{i : (s, h) \in C(Y_i)\} \geqslant \alpha m\} \quad (2)$$

Similarly to the acceptance set of the single professor, $C(Y)$ is the set of parameters (research specialization and research capacity) that a candidate should have in order to be hired by the initial staff Y.

Define also the time of arrival of the first accepted candidate:

$$\tau_1 = \inf\{n > 0 : X_n \in C(Y)\}. \quad (3)$$

In a similar manner we can define $C(Y, X_{\tau_1})$ - the acceptance set for the second candidate, τ_2 - the arrival time of the second successful candidate (i.e. $\tau_2 = \inf\{n > \tau_1 : X_n \in C(Y, X_{\tau_1})\}$) and so on.

3.2.1 Finite Sample Properties

The next three lemmas provide a formal proof to the following arguments: first, all the parameters in the acceptance set are equiprobable, second, the larger is the acceptance set the faster the successful candidates arrive, and third which is less intuitive, the larger is the acceptance set the lower is the expected research capacity of the next candidate.

Lemma 3.9. *X_{τ_k} has a uniform distribution in $C(Y, X_{\tau_1}, \dots, X_{\tau_{k-1}})$ conditional on $(Y, X_{\tau_1}, \dots, X_{\tau_{k-1}})$. That is, given the acceptance set the next candidate might have any combination of the parameters (s, h) in this acceptance set equiprobably.*

Proof. For any Borel set $B \subseteq S^1 \times [0, 1]$ calculate $P(X_{\tau_k} \in B | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}})$:

$$\begin{aligned}
&\mathrm{P}(X_{\tau_k} \in B | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}) = \mathrm{E}(\mathbb{I}(X_{\tau_k}) \in B | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}) = \\
&= \mathrm{E}[\mathrm{E}(\mathbb{I}(X_{\tau_k}) \in B | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}, \tau_k) | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}] = \\
&= \mathrm{E}[\mathrm{P}(X_{\tau_k} \in B | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}, \tau_k) | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}] = \\
&= \mathrm{E}\left[\left. \frac{S(B \cap C(Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}))}{S(C(Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}))} \right| Y, X_{\tau_1}, \dots, X_{\tau_{k-1}} \right] = \\
&= \frac{S(B \cap C(Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}))}{S(C(Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}))}
\end{aligned} \tag{4}$$

■

Lemma 3.10. *Successful candidates arrive through time intervals that have a geometric distribution, i.e.*

$$Law(\tau_k - \tau_{k-1} | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}) = Geom(p_k), \quad p_k = \mathrm{Area}(C(Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}))$$

Proof. Let $A = C(Y, X_{\tau_1}, \dots, X_{\tau_{k-1}})$ then

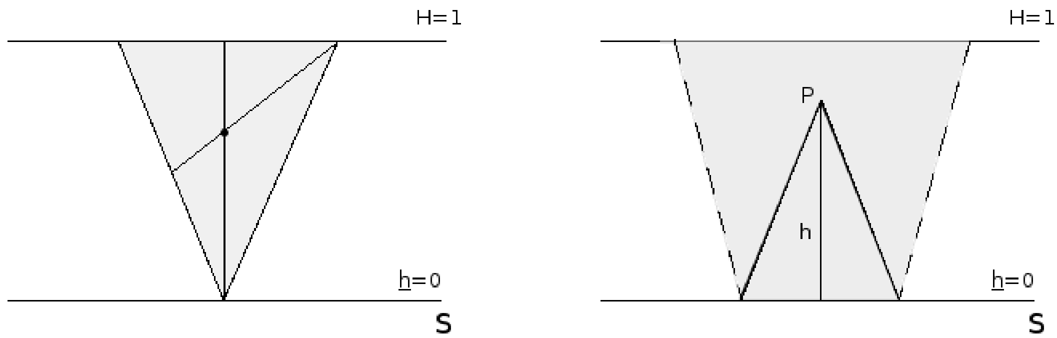
$$\begin{aligned}
&\mathrm{P}(\tau_k - \tau_{k-1} = n | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}) = \\
&= \mathrm{E}[\mathrm{P}(\tau_k - \tau_{k-1} = n | \tau_{k-1}, Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}) | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}] = \\
&= \mathrm{E}[\mathrm{P}(\tau_k = \tau_{k-1} + n | \tau_{k-1}, Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}) | Y, X_{\tau_1}, \dots, X_{\tau_{k-1}}]
\end{aligned}$$

$$\mathrm{P}(X_{\tau_{k-1}+1} \notin A, \ldots, X_{\tau_{k-1}+n-1} \notin A, X_{\tau_{k-1}+n} \in A |\, \tau_{k-1}, , X_{\tau_1}, \ldots, X_{\tau_{k-1}}) =$$
$$= \prod_{i=1}^{n-1} \mathrm{P}(X_{\tau_{k-1}+i} \notin A |\, \tau_{k-1}, X_{\tau_1}, \ldots, X_{\tau_{k-1}}) \cdot \mathrm{P}(X_{\tau_{k-1}+n} \in A |\, \tau_{k-1}, X_{\tau_1}, \ldots, X_{\tau_{k-1}}) =$$
$$= p_k \cdot (1-p_k)^{n-1}$$

where $p_k = \mathrm{P}(X_{\tau_{k-1}+l} \in A |\, \tau_{k-1}, X_{\tau_1}, \ldots, X_{\tau_{k-1}}) = \mathrm{S}(A)$. The result follows from the fact that $X_{\tau_{k-1}+l}$ has a uniform distribution and is independent from $X_{\tau_1}, \ldots, X_{\tau_{k-1}}$. ■

Lemma 3.11. *For high enough β (e.g. $\beta > 6$) smaller acceptance set result in higher average capacity of the next accepted candidate.*

Proof. For such β the acceptance set can only have trapezoid shape or, in extreme, a triangle shape (see Fig. 8(a) and Fig. 8(b)). Hence, for a moment we can replace all the staff by a single fictitious professor with the same acceptance set.



(a) V-shaped (b) W-shaped

Figure 8: Shapes of the acceptance set

Assume now that the "professor" is the one with the lowest possible capacity level, i.e. with triangle shaped acceptance set. Since candidates' parameters are uniformly distributed, the two dimensional average is simply a geometric center of this triangle. Thus $\mathrm{E}\, h_{X_{\tau_1}} = \frac{2}{3}$.

When the "professor" is smarter, then the acceptance set is W-shaped.

The two dimensional average is again a geometric center of the acceptance set. Hence $\mathrm{E}\, h_{X_{\tau_1}} = \frac{2h}{2h+1} \cdot \frac{1}{2} + \frac{1}{2h+1} \cdot \frac{2}{3}$ where h is a capacity of the fictitious "professor". It is clear that $\frac{\mathrm{d}\,\mathrm{E}\, h_{X_{\tau_1}}}{\mathrm{d}h} < 0$. ■

3.2.2 Limit Properties

Definition 3.12. *Given a probability space* $(\Omega, \mathcal{F}, \mathrm{P})$ *a function* $\mu : \Omega \times \mathcal{F} \to \mathbb{R}_+$ *is called a random measure [7, 8, 9] if*

1. *For any* $A \in \mathcal{F}$ *function* $\mu(\cdot, A)$ *is a random variable, i.e. is* $\mathcal{F}|\mathcal{B}(\mathbb{R})$ *measurable*

2. $\mu(\omega, \cdot)$ *is a measure on* $\mathcal{F}$ *(*$\mathrm{P} - a.s.$*)*

Definition 3.13. *For each* $\omega \in \Omega$ *define a sequence of (random) measures* $G_n(\omega)(\cdot)$ *on the cylinder:*

$$G_n(\omega)(A) = \frac{1}{m+n}\left[\#\{i : Y_i \in A\} + \#\{j \leqslant n : X_{\tau_j} \in A\}\right] \quad A \in \mathcal{B}(S^1 \times [0,1])$$

Just like deterministic uniform distribution that places weight 1/(m+n) on each point of its support, distribution G_n puts weight $1/(m{+}n)$ on each of the faculty members. Thus $G_n(\cdot, A)$ shows what share of the current faculty members (initial plus hired candidates) are in the set A.

Definition 3.14. *A corresponding projection of these measures*

$$H_n(\omega)(A) = \frac{1}{m+n}\left[\#\{i : s_{Y_i} \in A\} + \#\{j \leqslant n : s_{X_{\tau_j}} \in A\}\right] \quad A \in \mathcal{B}(S^1)$$

indicates what share of the current faculty members do research within the A field regardless of their research capability.

Alternatively one can also define H_n as $H_n(\omega)(A) = G_n(\omega)(A \times [0,1])$

The question is whether $H_n(\cdot) \xrightarrow{\mathrm{P}-a.s.} H(\cdot)$ for some deterministic measure $H(\cdot)$, and how does $H(\cdot)$ depend on the starting point Y.

Proposition 3.15. *Suppose that* $\alpha = 1$*, that is the voting procedure requires unanimous agreement, then* G_n *converges to a random (non-deterministic) measure* G *that is uniformly distributed over all V-shaped uniform measures in initial W-shaped domain (see Fig. (9)). Moreover:*

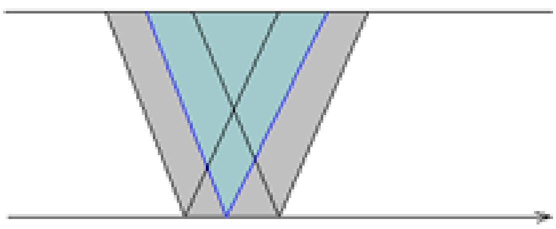
$$\mathrm{P} \lim_{n\to\infty} = \frac{\sum\limits_{k=0}^{\infty} h_{X_{\tau_k}} \cdot \mathbb{I}(\tau_k \leqslant n)}{\sum\limits_{k=0}^{\infty} \cdot\mathbb{I}(\tau_k \leqslant n)} = \frac{2}{3} \tag{5}$$

For $\alpha < 1$ the there does not exist a deterministic limit of G_n (and H_n), i.e. there does not exist a measure $G(\cdot)$ ($H(\cdot)$[9]) that does not depend on ω such that

$$\mathrm{P}\left\{\omega\colon G_n(\omega, A) \to G(A) \quad \forall A \in \mathcal{B}(S^1 \times [0,1])\right\} = 1$$

Proof. At first, notice that with probability one the limit measure's support is V-shaped as shown on Fig. (9) (blue shaded area). The proof is recursive: with probability 1 there will arrive the first candidate outside of the triangle "above" the professor; he will be accepted, since those who arrived before him did not change for acceptance set; moreover, he will shrink the acceptance set with probability one. If the resulting set is V-shaped, than all new candidates from this set will be admitted and the set will stay unchanged. If the resulting set is W-shaped, than we can create a fictitious professor that will have the same acceptance set and start the argument all over.

Figure 9: Limit support



Since the distribution of X_{τ_k} conditional on the history is uniform in the acceptance set, the limit distribution will also be uniform on the V-shaped support almost surely.

Now, let us study the distribution of the limit random measure G over the V-shaped uniform distributions. In order to do this let us compute the probability that $\mathrm{supp}G_n \subset W$ for some W - a W-shaped area inside the initial acceptance set:

$$\mathrm{P}(\mathrm{supp}G_n \subseteq W) = \mathrm{P}(X_{\tau_1} \in W, \ldots, X_{\tau_n} \in W)$$

This probability does not depend on the position of W, since for every sequence $X_{\tau_1}(\omega), X_{\tau_2}(\omega), \ldots, X_{\tau_n}(\omega)$ there exists a one-to-one mapping (a simple horizontal shift)

[9]It means that for any given sub-field A there is no way to predict what share of the faculty will be involved in this particular sub-field in the end of the day.

that depends only on a given W that generates the sequence $X'_{\tau_1}(\omega), X'_{\tau_2}(\omega), \ldots, X'_{\tau_n}(\omega)$ preserving the "most left" W area.

Since $\mathrm{P}(\mathrm{supp} G_n \subseteq W)$ does not depend on position of W, the limit probability will also exhibit this property, moreover, the limit probability will always be greater that zero as soon as W is a true trapezoid, not a triangle. Such a trapezoid can be uniquely characterized by a segment that represents it's smaller base, e.g. $(x, x + \Delta)$. We have proved that the limit distribution does not depend on x, thus, is is uniform.

It means, that in the long-run the faculty will represent the choice of a single professor with the lowest capacity, whose specialization is uniformly distributed in the competency area of the initial professor.

The second statement of the proposition is a straightforward implication of the limit distribution shape.

The non-existence of a deterministic limit when $\alpha < 1$ becomes obvious, when one notices then for any number of admitted candidates N there exists a positive probability that 10^N of the new candidates will arrive in a small neighborhood U located near the boundary of the current acceptance area dramatically changing the distribution the of faculty on S. As a result $\mathrm{P} - a.s.$ convergence is impossible. ■

4 Conclusion

Optimal construction of a modern research university is a multifaceted problem. We suggest a parsimonious model that allows for minimal heterogeneity in committee members' capacities and a simple political framework. The model is rich enough to generate predictions about the impact of departmental subdivision and initial faculty' specialization on long-term dynamics of the departmental quality.

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