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Репутация и мягкие бюджетные ограничения

Препринт #

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We study the role of reputation in dealing with the soft-budget constraint. We examine whether the reputation of a borrower can lead to repayment in an environment where enforcement is weak. We also introduce lenders' reputation and examine how this impacts on the allocation of borrowers. We find that reputation can harden budget constraint and improve welfare, although it can never fully eliminate softness. We also show that lenders who acquire a reputation for being tough can earn higher profits than lenders with reputations for being soft.

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Мы изучаем роль репутации в устранении синдрома мягких бюджетных ограничений. Нас интересует вопрос, может ли репутация заемщика способствовать возвращению долгов в случае ограниченных возможностей для разрешения споров хозяйствующих субъектов. Мы также дополняем модель посредством привнесения репутации заимодавцев и смотрим, каким образом это сказывается на действиях заемщиков. Мы находим, что репутация может сделать бюджетные ограничения более жесткими, но привнесение репутации само по себе не может полностью устранить мягкость бюджетных ограничений. Мы также демонстрируем, что те из заимодавцев, которые приобретают репутацию жестких по отношению к недобросовестным заемщикам, получают более высокую прибыль по сравнению с заимодавцами, которые имеют репутацию мягких по отношению к недобросовестным заемщикам.

1 Introduction

A critical problem in the development of effective financial systems in transition economies is the elimination of the soft-budget constraint syndrome. A soft budget constraint arises when borrowers know (or expect) that they will be bailed out in the face of adverse outcomes. The soft-budget constraint syndrome is a major problem in emerging and transition economies (see [10] and [11] for review of alternative theories). Soft budget constraints introduce weak incentives for restructuring and impede the process of sectoral reallocation (see, e.g. [13]). Hardening budget constraints is often considered a key reform for transition economies. The problem is to figure out how this can be accomplished.

In this paper we examine whether reputation can be a force for mitigating the soft-budget constraint syndrome. In most of the literature on soft budget constraints the financial contracting problem is treated as a one-shot game (see [3]; [5]). In such a setting reputation plays no role. In a dynamic setting borrowers and lenders must consider how their current decisions impact on future opportunities, and this may act to limit opportunistic behavior. This suggests that reputational mechanisms may alter the calculations that borrowers and lenders make in one-shot games (see [7]; [14]). Borrowers may perceive that earning a good reputation may enhance future borrowing opportunities. Lenders that acquire a reputation for being tough may deter opportunistic borrowers and hence lower the rate of future defaults. Acquiring a reputation for being tough may thus enhance profitability for lenders. If lenders with tough reputations earn higher profits then one may expect that they will grow at the expense of lenders with soft reputations. Thus over time, an economy may evolve towards harder budget constraints. Reputational considerations may thus lead to the emergence of hard-budget constraints even without any explicit or implicit government policy to that effect.

Reputation may be especially important for financial contracting in transition economies because of weak legal enforcement. In situations where third-party enforcement is absent self-enforcing contracts are necessary to sustain relationships. Reputation could perhaps play such a role; for example, a borrower may service a debt rather than risk damage to his reputation if this will worsen future borrowing opportunities. Yet the value of reputation in dealing with the soft-budget constraint is not obvious for two reasons. First, Bulow and Rogoff [1] have shown that reputation cannot support sovereign debt when borrowers can retain some of their assets.[1] In such cases the loss of reputation is insufficient to support lending. Early in transition the penalties that lenders can impose on borrowers may be so weak that it resembles the case of sovereign debt, hence reputation may not be a powerful force in this environment. Second, a key feature of transition is a lack of transparency. Financial underdevelopment, weak regulation, and the inheritance of socialist practices combine to make interpretation of financial information very difficult. In such an environment it may be very difficult for a party to develop a reputation because the noise in any communication outweighs the signal. These two features suggest that the effectiveness of reputation as a mitigating device is open to question, especially early in transition. As the transition proceeds, enforcement improves as does transparency. Hence, the role of reputation in financial contracting may as well.

Although the literature on the soft-budget constraint is quite large,[2] there is only one paper we are aware of that studies the role of reputation — Alexeev and Kim [2], which considers the role of a lender's reputation in mitigating the soft-budget constraint. They show that such a reputation may be valuable if it improves the borrower pool a lender faces, and they show that this effect will be more powerful under decentralization than in a centralized system. Their model differs from ours, however, in several ways. First, we consider both borrower's and lender's reputation, and how these interact. Second, in their model tough lenders have no difficulty attracting borrowers. Only experienced entrepreneurs with bad projects avoid tough lenders. In our model even able borrowers can have bad luck. Hence, even able borrowers with sound reputations would prefer to bank with soft lenders, as they will be more lenient in adverse circumstances. Hence, for tough lenders to emerge they will have to attract borrowers – they must offer something in return; in our case a lower interest rate. But this will only make sense for the lenders if their reputations are sufficiently credible to deter the mediocre borrowers who would also prefer lower interest rates. We show that there are equilibria where lenders will value having a tough reputation. Indeed, we show that under plausible conditions lenders with tough reputations will earn higher profits that lenders with soft reputations.

The rest of the paper proceeds as follows. In section 2 we describe the

[1] The reason is that the cost to the borrower of autarky in the future is mitigated by the wealth saved by not servicing the debt.

[2] See [10] and [11] for surveys.

environment. Then (in section 3) we examine the role of borrower's reputation but treat lenders as identical. In section 4 we allow lenders to differ as well and examine the role of lender's reputation. Section 5 offers some concluding thoughts.

2 Environment

We model the interaction between lenders and borrowers as an infinitely repeated discounted game. Thus, time is discrete and the horizon is infinite. At every date there is a $[0,1]$-continuum of risk-neutral lenders and a $[0,1]$-continuum of risk-neutral borrowers. Lenders and borrowers are formed in pairs to undertake investment projects. We assume that in the beginning of every odd-numbered period lenders and borrowers are matched together. A project requires two periods to complete, however, any project can be liquidated after one period. Once a project is either completed or liquidated, the match between a lender and a borrowers dissolves; if a project is liquidated (in the beginning of an even-numbered period), then the pair has to wait until the next period to be matched with other individuals. Thus, every pair stays together for at most two periods. We assume that the matching process is random and uniform; every unmatched borrower has an equal chance to meet every unmatched lender.

To study reputation we assume that agents make choices – effort levels – which effect the results of the project. This introduces a hidden action (moral hazard) element. We also have a hidden information problem. Lenders and borrowers have asymmetric information about their types.[3] Hence, there will be an inference problem: how to distinguish poor borrower types from good borrowers who supply low effort. It is this potential pooling problem that will make reputation potentially important.

Projects are of two types: good and bad. All projects require a unit investment per period; we assume that in every period all lenders posses a unit endowment which they can either consume or invest in a project. Borrowers have no endowment, but they are the only agents that have the technology to undertake projects. A completed good project yields a gross return $X > 2$; gross return to a completed bad project is stochastic. We

[3]In section 3 we consider identical lenders so the hidden information concerns only the borrowers' types. In section 4 we introduce lenders' types so that there is two-sided asymmetric information.

assume that a bad project can be either a success or failure; it yields X with probability p (success) and x, $x < 2$, with probability $1 - p$ (failure). If a project is a success the borrower receives a share, a $(0 < a < 1)$ of the gross return. Also, we assume that $p(1 - a)(X - 2) + (1 - p)(x - 2) > 0$, which implies that all projects are ex ante profitable and, therefore, are financed by lenders.[4]

In the beginning of the first stage (period) of a project the borrower must exert an effort level, either high (E) or low (e), which impacts the quality of the project. Effort is costly. For simplicity we can let the cost of high effort also be indexed by E, and the cost of low effort by e. Furthermore, we assume that $E > e \geq 0$. We assume that the borrowers are of two types — able and mediocre; let λ be the measure of able borrowers. Given effort, able borrowers have (on average) a higher project quality than mediocre borrowers. If an able borrower chooses high effort, then the probability that the project is good is Π; if an able borrower chooses low effort, then the probability that the project is good is π, where $\Pi > \pi$. The probability that a mediocre borrower has a good project is π regardless of her effort.

We understand reputation as a record of past performance. Specifically, we can think of this as a report provided by the lender after a project match is completed. For concreteness, we assume that every lender writes a reference letter for the borrower where the outcome of the project is reported. Lenders always observe outcomes of completed projects; we assume that if a lender liquidates an incomplete project, the lender learns how the project would have fared had it been completed.[5] In other words, a lender can always accurately assess the outcome of a borrowers project, so the reputation of the latter is uncontaminated by the lender's own actions.

All the letters that pertain to the borrower's history are presented to a new lender immediately after the match takes place. Lenders place more weight on recent performance and discount previous letters; for simplicity, we assume that only the most recent letter matters.[6] Reputations are not,

[4]Our main focus here is the soft-budget constraint, that is a decision to refinance a project. Therefore, we do not concentrate on the initial decision to finance; the role of reputation in making such decisions has been studied, most notably by Diamond (1989).

[5]The idea is that when a project is liquidated the lender inspects the assets and can determine whether it would have worked or not. This means that the lender's report is going to be independent of the action that he undertakes (i.e., liquidation or not). This assumption greatly simplifies the analysis.

[6]When lenders are homogeneous nothing significant would change if memories were

however, perfect signals of past performance. When an economy is not fully transparent financial histories are not perfect signals of past behavior. To model the noise introduced by a lack of transparency, we introduce some probability, denoted ϕ, that the letter has been tampered with by the borrower, who can do so at zero resource cost. Because the letters are the only intertemporal link in this economy, the value of ϕ is crucial for the importance of reputation. For example, when is ϕ close to one (low transparency) then almost every borrower with a bad letter will manage to replace it with a good one. Thus, there will be a lot of average borrowers with bad projects, but with good letters. In that case, letters are ineffective as a signalling device (of the type of borrowers) which implies that reputation is not important. When $\phi \to 0$ transparency is high and reputation is more important.

After the first stage of a project is completed, the lender learns some extra information about the project. This extra information concerns the state of the economy or the industry – it is an aggregate, as opposed to, a project-specific signal. Thus, we assume that the lender receives a signal, which is correlated with the outcome of the project. The signal takes two values — with probability p the signal is high ($\overline{\sigma}$) and with probability $1-p$ the signal is low ($\underline{\sigma}$). If the project is bad, then the outcome of the project is perfectly correlated with the the signal — an observed high signal implies that the project will be a success; an observed low signal implies that the project will be a failure. Because a good project is always a success, the return on a good project is independent of the signal. We should emphasize that the lender remains uninformed about the type of the project. If the signal is high, then the lender has nothing to worry about; the project will be a success regardless of type. If the signal is low, then the lender has to really think about what to do with the project — either to refinance or to liquidate — because the project will be a success if it is good and a failure if it is bad. The likelihood of each of the two outcomes (conditional on the observation of the low signal) *depends on the borrower's reputation*; in particular, a good reputation increases the chance that the project is actually good. Thus, even though reputation is irrelevant here for the initial decision to finance a project, it is important for the decision to refinance; some of the bad projects may be denied financing in the second stage and the budget

longer, except that the algebra would be much more complicated. When lenders are heterogeneous then this assumption is less innocuous. With longer memories lenders may engage in strategic liquidation in order to disguise their types. We discuss this below in section 4.

constraint will, then, be hardened.

We think of this signal as an economy-wide or industry-wide piece of information.[7] This information thus contributes to the evaluation of the likelihood that a project will turn out successful to the extent that the project's outcome depends on the general economy. Many projects that have a high potential for success may face problems if there is a negative aggregate shock. The interesting point is that although this shock says nothing about the individual project, the reputation of the borrower may be important for how a lender responds to this signal.

Thus, conditional on reputation and on the observed signal, the lender decides whether to liquidate the project or to invest another unit in order to complete it. The liquidation value of a project is $L < 1$. The value of L is another parameter crucial for the importance of reputation; if L is low (that is if liquidation costs are high), then no project will ever be liquidated even if (after the first stage) it becomes evident to the lender that the project will incur losses. In this case no information the lender has about the borrower (including reputation) is relevant for lender's decision; the soft-budget constraint emerges because refinancing is better than liquidation (see [5] for a similar assumption).

The gains from a project are divided between the lender and the borrower in accordance with a binding financial contract.[8] We assume that in the case of project liquidation or failure lenders can seize all of the residual value of the project.[9] In case of success, borrowers receive a constant share, denoted

[7] One way to think about this is with an agricultural parallel. Consider the projects as crop fields. The lender does not know whether the borrower did a good job planting and fertilizing the crop, but she observes the weather from her office window. If the weather is bad (low signal), the lender concludes that the harvest may be not so great — bad weather increases the likelihood that the harvest will be poor. At this point the lender may want to recall what she has on file about the borrower; if the borrower has a proven record of having good harvests, then the lender might decide that the borrower is good at raising the crop and, even though the weather is bad, the harvest will suffice to pay back another loan. If the record is poor, then the lender might decide that the borrower is indeed a bad farmer and, given bad weather, the chance of good harvest (and, hence, repayment) is too small to provide another loan.

[8] It may seem strange that we assume binding contracts with regard to distribution given that we argued that weak contract enforcement is an important feature of transition. The assumption that repayment to the lender is given is standard in this literature. Still it would be interesting to see how reputation would affect the repayment decision, but that is outside the scope of this paper.

[9] Of course this may be very low if liquidation costs are high. The key point of this

a ($0 < a < 1$) of a project's net return.[10] These assumptions are sufficient to guarantee that no borrower will ever pretend that the project is a failure when it is actually a success; the converse is not possible here because in that case borrowers have insufficient resources to pay back the loan. Notice that even though we eliminate strategic cheating, reputation is still a strategic choice on the part of borrowers (which is consistent with standard models of reputation) because it directly depends on their choice of effort. Given our assumptions, high effort results in a higher probability of project's success and, thus, in a good reputation.

As we have already noted, liquidation costs and transparency are crucial for the value of reputation. We assume that in the early stages of transition liquidation costs are high and transparency is low. In such an environment all debt is similar to sovereign debt — it is prohibitively costly to collect and a failed project with one lender has no consequence for borrowing with other lenders. Bulow and Rogoff [1] show that if direct sanctions are not feasible and the debtor can keep its foreign assets, then a reputation for repayment cannot support sovereign lending. Their argument applies here as well;[11] thus, a borrower's reputation has no importance in early transition. As the transition proceeds, however, liquidation costs decrease and transparency increases; in that situation lenders can recover more of their loans, and a default to any one lender reduces opportunities with other lenders. Thus, reputation acquires value as the transition proceeds.

3 Borrower's reputation

We limit ourselves to the description of stationary equilibria. Given our specified environment, the only equilibria in which reputation is important are those, where able borrowers choose high effort.[12] As we already noted, in any such equilibrium a borrower's reputation signals her type. Therefore,

assumption is that the borrower retains none of the residual value in the case of failure. If liquidation costs are sufficiently high the same is true for the lender.

[10] The assumption a is fixed is standard in the literature on soft-budget constraints. One could relax this assumption by allowing free enty of lenders, and make borrowing costs endogenous. None of our results depend on the assumption of fixed a, however.

[11] The role of the cash-in-advance consumption-insurance contracts in the Bulow-Rogoff model is performed by loans from uninformed lenders.

[12] Because the probability of a project's being good is unaffected by a mediocre borrower's effort, mediocre borrowers will always choose low effort.

a good reputation indicates a higher probability of borrower's being able; a bad reputation raises the chance of borrower being mediocre. For an able borrower the probability that she receives a good letter from the lender is

$$P_{S|A} = \Pi + p(1-\Pi) + \phi(1-p)(1-\Pi). \tag{1}$$

Notice that we take into account the fact that a fraction ϕ of unsuccessful borrowers tamper with their letters. For a mediocre borrower the probability of a good letter is

$$P_{S|M} = \pi + p(1-\pi) + \phi(1-p)(1-\pi). \tag{2}$$

Given (1) and (2), the measure of borrowers of type i, $i \in \{A, M\}$, who have reputation j, $j \in \{S, F\}$, denoted $\mathcal{M}_{ij}$ is:

$$\begin{aligned} \mathcal{M}_{AS} &= \lambda\left(\Pi + p(1-\Pi) + \phi(1-p)(1-\Pi)\right) \\ \mathcal{M}_{AF} &= \lambda(1-\phi)(1-p)(1-\Pi) \\ \mathcal{M}_{MS} &= (1-\lambda)\left(\pi + p(1-\pi) + \phi(1-p)(1-\pi)\right) \\ \mathcal{M}_{MF} &= (1-\lambda)\,(1-\phi)(1-p)(1-\pi). \end{aligned} \tag{3}$$

Straightforward application of Bayes' rule yields the probability that given a good letter, the borrower is able, $\mu_{A|S}$, as the ratio of the measure $\mathcal{M}_{AS}$ of able borrowers with good letters to the measure $\mathcal{M}_S \equiv \mathcal{M}_{AS} + \mathcal{M}_{MS}$ of all borrowers with good letters,

$$\mu_{A|S} = \frac{\mathcal{M}_{AS}}{\mathcal{M}_S}.$$

The measure of borrowers with bad letters, $\mathcal{M}_F \equiv \mathcal{M}_{AF} + \mathcal{M}_{MF}$, is:

$$\mathcal{M}_F = (1-\phi)\,(1-p)\left[\lambda\,(1-\Pi) + (1-\lambda)\,(1-\pi)\right]. \tag{4}$$

Notice that if $\phi = 1$, then $\mathcal{M}_F$ is zero which implies that every borrower comes to a lender with a good letter. In this case $\mu_{A|S} = \lambda$, the objective prior probability that a borrower is able. Thus, if the economy is fully opaque, reputation carries no information about the borrower and, for that reason, has no value.

Given an observed low signal, the lender knows that a project is a success if and only if it is good, and that a project is a failure if and only if it is bad. The probability that a project will be good conditional on a bad letter is,

$$\zeta_{G|F} = \Pi\mu_{A|F} + \pi\mu_{M|F}, \tag{5}$$

and the probability that this same project is bad is,

$$\zeta_{B|F} = (1 - \Pi)\mu_{A|F} + (1 - \pi)\mu_{M|F}. \tag{6}$$

Likewise, given a good letter, the probability that the project is good is,

$$\zeta_{G|S} = \Pi\mu_{A|S} + \pi\mu_{M|S}, \tag{7}$$

and the probability that it is bad is,

$$\zeta_{B|S} = (1 - \Pi)\mu_{A|S} + (1 - \pi)\mu_{M|S}. \tag{8}$$

Initially, as the project is started, the lender's state is defined solely by the borrower's reputation. This follows because all projects are *ex ante* profitable, and lenders provide initial financing to all projects regardless of their reputation. After the first stage, however, lenders acquire additional information about the project; the state becomes a pair — reputation (either F or S) and the observed signal (either $\underline{\sigma}$ or $\overline{\sigma}$). Given the state, the value for a lender is determined from the four-equation system of Bellman equations

$$V' = \beta^2 \left(q_l' + T_l V'\right) \tag{9}$$

where V is the value function, $V \equiv (V_{F\underline{\sigma}}, V_{S\underline{\sigma}}, V_{F\overline{\sigma}}, V_{S\overline{\sigma}})$, T is the transition matrix among states, q is the vector of (expected) project returns, and β, $\beta \in (0, 1)$, is a period discount factor. Because matching is exogenous, the process which governs transition among states is *i.i.d.* Moreover, the signal next-period is independent of today's state and of the matching process. Then, if the observed signal is $\overline{\sigma}$, then the lender knows that the project is a success regardless of its type, and hence, reputation. Because continuation value of a lender is independent of her actions and liquidation of a project always incurs a loss, lenders never liquidate such projects, which implies that $V_{F\overline{\sigma}} = V_{S\overline{\sigma}}$. This allows us to reduce the dimensionality of the state space and write the value function V as $V \equiv (V_{F\underline{\sigma}}, V_{S\underline{\sigma}}, V_{\overline{\sigma}})$, where V satisfies (9) and where

$$T_l = \begin{bmatrix} (1-p)\,\mathcal{M}_F & (1-p)\,\mathcal{M}_S & p \\ (1-p)\,\mathcal{M}_F & (1-p)\,\mathcal{M}_S & p \\ (1-p)\,\mathcal{M}_F & (1-p)\,\mathcal{M}_S & p \end{bmatrix}, \tag{10}$$

and

$$q_l' = \begin{bmatrix} \max\left[\zeta_{G|F}(1-a)(X-2) + \zeta_{B|F}(x-2),\ L-1\right] \\ \max\left[\zeta_{G|S}(1-a)(X-2) + \zeta_{B|S}(x-2),\ L-1\right] \\ (1-a)(X-2) \end{bmatrix}.$$

Because the transition probabilities are independent of the lender's actions, conditional on an observed low signal, the lender liquidates a project of a low reputation borrower and refinances the project of a high reputation borrower if and only if

$$\zeta_{G|F}(1-a)(X-2)+\zeta_{B|F}(x-2) \leq L-1 \leq \zeta_{G|S}(1-a)(X-2)+\zeta_{B|S}(x-2). \quad (11)$$

It is evident from (11) that higher liquidation costs makes refinance of high reputation borrowers (with low signals) more likely.

Before we proceed to the borrower's problem, we impose restrictions that guarantee the unconditional presence of both soft and hard budget constraints. Assume that the lender observes a low signal. Conditional on that observation, we want the lender be willing to liquidate any project regardless of the borrower's type at least when liquidation costs are low enough. Likewise, we want the lender be willing to continue with any project if the liquidation costs are high. This can be guaranteed if the following holds.[13]

Assumption 1. *Sufficient condition for the hard-budget constraint*

$$\Pi(1-a)(X-2)+(1-\Pi)(x-2)<0.$$

Assumption 2. *Sufficient condition for the soft-budget constraint*

$$-1<\pi(1-a)(X-2)+(1-\pi)(x-2).$$

An equilibrium where reputation has value exists only if all able borrowers choose to work hard. The state of an able borrower once the project is chosen is her type, able (A) or mediocre (M) and her reputation, either good (S) or bad (F). Given the state $s \equiv (i,j)$, let d_{Aj}, $j \in \{F,S\}$, be an able borrower's strategy — the probability that the borrower chooses high effort. In terms of this notation, when all able borrowers work hard $d_{AF} = d_{AS} = 1$. Let $f_{j\sigma}$ be the lender's strategy — the probability that given her state, which is the borrower's reputation and the observed signal, the lender

[13]Notice that *Assumption 1* and the sufficient condition for initial financing,

$$p(1-a)(X-2)+(1-p)(x-2)>0,$$

imply that $\Pi < p$, i.e. the probability that the project is good must be less than the probability of success of a bad project.

refinances the project. Because lenders never liquidate apparently successful projects, $f_{F\overline{\sigma}} = f_{S\overline{\sigma}} = 1$. Then, the end-of-even-period value function of an able borrower, $v_A \equiv (v_{AF}, v_{AS})$ solves the following two-equation system:

$$v_A' = \beta^2 (q_b' + T_b v_A')$$

where, given the probability that a high effort borrower's letter is good,

$$P_{S|E} = \Pi + p(1-\Pi) + \phi(1-p)(1-\Pi), \tag{12}$$

and the probability that a low effort borrower's letter is bad,

$$P_{S|e} = \pi + p(1-\pi) + \phi(1-p)(1-\pi), \tag{13}$$

the transition matrix for borrower's state is

$$T_b = \begin{bmatrix} P_{F|E} & P_{S|E} \\ P_{F|E} & P_{S|E} \end{bmatrix},$$

and the vector q is

$$q_b' = \begin{bmatrix} (p + (1-p) f_{F\underline{\sigma}})\, \Pi a(X-2) - E \\ (p + (1-p) f_{S\underline{\sigma}})\, \Pi a(X-2) - E \end{bmatrix}.$$

Thus, the choice of able borrowers to work hard is optimal if

$$p\Pi a(X-2) - E + P_{F|E} v_F + P_{S|E} v_S \geq p\pi a(X-2) - e + P_{F|e} v_F + P_{S|e} v_S. \tag{14}$$

Expression (14) provides a sufficient condition which ensures that borrowers with a bad reputation want to exert high effort. Because the projects of borrowers with a good reputation have fewer chances to be liquidated they are also willing to work hard provided that (14) is satisfied. This condition (14) can be simplified to

$$(\Pi - \pi)\left(p + \beta^2 \Pi (1-\phi)(1-p)^2\right) a(X-2) \geq E - e.$$

It is evident that this condition is more likely to be satisfied when the benefits from high effort increase ($\Pi - \pi$ and X larger) and when the costs of high effort ($E - e$) are lower.

We are now ready to give the main result of this section.

Proposition 1. *Assume that assumptions 1 and 2 hold and that return on high effort exceeds its cost,*

$$(\Pi - \pi)\left(p + \beta^2 \Pi (1-p)^2\right) a(X-2) > E - e. \tag{15}$$

Then there exists a $\overline{\phi} \leq 1$ *such that for all* $\phi \leq \overline{\phi}$ *there exist two values,* $\underline{L}$ *and* $\overline{L}(\phi)$, $0 < \underline{L} < \overline{L}(\phi) < 1$, *such that*

- *i) if* $L < \underline{L}$, *then all able borrowers work hard and no project is ever liquidated,*
- *ii) if* $\underline{L} \leq L \leq \overline{L}(\phi)$, *then all able borrowers work hard and provided that lenders receive a low signal, all projects of borrowers with a bad reputation are liquidated whereas all projects of borrowers with a good reputation are refinanced,*
- *iii) if* $L > \overline{L}(\phi)$, *then provided that lenders receive low signal, all projects are liquidated.*

Proposition 1 yields conditions which ensure that reputation matters. Roughly, these conditions imply that reputation is important if the return to effort is high enough, transparency is high and liquidation costs are in the middle range. This latter condition is hardly surprising. When liquidation costs are very low, lenders will disregard reputation entirely and liquidate any project when they receive a low signal. In that parameter range the economy operates on a *very* hard-budget constraint; a positive fraction of good projects gets liquidated because of the imperfect correlation between the outcome of a project and the signal about that outcome received by lenders. If liquidation costs are too high then the soft-budget constraint emerges; lenders are reluctant to liquidate even if a bad reputation suggests a higher chance that a project is a failure. It is when liquidation costs are in the middle range that reputation matters. In that case a bad reputation induces enough scepticism and the lenders will liquidate, whereas a good reputation creates enough optimism for the lenders to continue financing the project. In this situation the budget constraint can be considered somewhat soft, although a positive measure of good projects is still liquidated.

Notice that when reputation matters here it is because the signal about what is taking place in the economy (or industry) causes the lender to infer a difference in the likely outcomes for borrowers. This aggregate (or industry) signal is the only new piece of information that the lender obtains. Although this signal provides no information about individual projects, it does alter the likelihood of success conditional on reputation. Lenders know that borrowers with good reputations can succeed in bad times, while borrowers with poor reputations will not. Hence, reputation can matter.

Although reputation can be important, there are cases in which it has no value. The following proposition gives sufficient conditions for irrelevance of reputation.

Proposition 2. *Assume that assumptions 1 and 2 hold and that the return on high effort is low,*

$$(\Pi - \pi)\, a(X - 2) < E - e. \tag{16}$$

Then no borrower works hard, and there exists a value $\widehat{L}$ *such that*

- *i) if* $L < \widehat{L}$*, then no project is ever liquidated, and*
- *ii)* $L \geq \widehat{L}$*, then provided that lenders receive low signal, all projects are liquidated.*

The proof of Proposition 2 proceeds by showing that if the return on effort is low, then the prospect of obtaining a good reputation cannot induce borrowers to exert high effort. In this case there is no performance difference between able and mediocre borrowers. Consequently, the reference letter carries no information about borrower's type and, as result, cannot be of any use. Depending on liquidation costs, the economy operates under either hard- or soft-budget constraint — the outcome familiar from the static game (see Dewatripont and Maskin (1995) for similar result).

The last issue we deal with in this section concerns the welfare effects of reputation. In our setting zero transparency is equivalent to borrowers having no reputation. Hence, given the results in Propositions 1 and 2, one might expect that a sufficient increase in transparency would be welfare improving simply because it boosts up the performance of borrowers. This is not, however, the complete story. Sufficient transparency may allow lenders to treat borrowers differently depending on their past. Hence, we examine here the welfare implications of higher transparency conditional on effort exerted by borrowers. What we find is that higher transparency is always good for lenders because it allows them to make more informed decisions. However, higher transparency may be detrimental to borrowers because it reduces the opportunities to pool, and hence be treated in the same way. As a result, social welfare may decrease if reputation is introduced.

Our welfare criterion is the *ex ante* utility of both borrowers and lenders, which is the weighted sum of their value in any given state,

$$W \equiv \sum_{i \in \{A,M\}} \sum_{j \in \{F,S\}} \mathcal{M}_{ij} v_{ij} + (1-p) \sum_{j \in \{F,S\}} \mathcal{M}_j V_{j\underline{\sigma}} + pV_{\overline{\sigma}}.$$

For a given ϕ the following proposition shows how social welfare $W(L;\phi)$ changes with the residual liquidation value L provided that the return on high effort exceeds its cost even if the economy is fully opaque.

Proposition 3. *Assume that assumptions 1 and 2 hold and that return on high effort exceeds its cost,*

$$(\Pi - \pi)\, pa(X-2) > E - e.$$

Then, for all $\phi \in [0,1)$ *there exists* L^*, $\underline{L} < L^* < \overline{L}(\phi)$, *where* $\underline{L}$ *and* $\overline{L}(\phi)$ *are defined in Proposition 1, such that:*

$$\begin{aligned} W(L;\phi) &< W(L;1) \text{ for all } L \in (\underline{L}, L^*) \\ W(L;\phi) &> W(L;1) \text{ for all } L \in (L^*, \overline{L}(\phi)) \\ W(L;\phi) &= W(L;1) \text{ otherwise.} \end{aligned}$$

The interval $L \in (\underline{L}, \overline{L}(\phi))$ is the one where, according to Proposition 1, reputation matters. The point of Proposition 3 is that there is a threshold level of liquidation value, L^*, such that if liquidation values are lower than this threshold, reputation decreases welfare; if liquidation costs are higher than L^*, then reputation is welfare increasing. The threshold L^* is independent of ϕ; what does change with transparency is the magnitude of the effect: the absolute value of the welfare difference, $W_L \equiv |W(L;\phi) - W(L;1)|$, which is a non-increasing function of ϕ.

What is going on here? If liquidation costs are too high (L below the threshold) then the gain to the lenders from greater transparency is small since they will not act on it. So there is no gain to the lenders but clearly some borrowers lose. If L is above the threshold, on the other hand, lenders will benefit from greater transparency. So the fact that (some) the borrowers lose is offset in this case.

An implication of Proposition 1 is that depending on liquidation costs reputation can either harden or soften the budget constraint. In the latter case reputation is always welfare increasing; in the former case welfare can go both ways. Given our normalization of payoffs, softening of the budget constraint increases the likelihood that borrowers will gain if the project turns out to be a success, which accounts for the welfare increase.[14] However, a change in borrowers' payoffs consistent with extra costs in the case of project failure could reverse the outcome, making it consistent with the conventional wisdom that softness is a bad thing.[15]

4 Adding the Lenders' Reputation

Thus far we have focused on the problem of the borrowers reputation alone. In this section we introduce heterogeneity among lenders and allow lenders to develop a reputation. In particular, we consider two types of lenders, tough and soft. Tough lenders have lower liquidation costs than soft lenders; for that reason the former are less likely to re-finance than the latter. Because they re-finance less often than soft lenders, tough lenders can commit to lower interest rates as well, which helps them to attract good borrowers.[16] Borrowers with bad reputations will be less willing to deal with tough lenders, but even borrowers with good reputations would prefer a soft lender if they charged identical interest rates. By deterring such borrowers the tough lenders avoid the cost of refinancing bad borrowers. A better pool of borrowers allows tough lenders to profit from charging lower interest rates.

[14] Recall that all projects are *ex ante* profitable, and our welfare criteria is *ex ante* utility of borrowers and lenders across states.

[15] It should be noted that the typical argument against softness of budget constraints is that it is inefficient and leads to a misallocation of resources. Savings devoted to refinancing projects could be allocated to better opportunties, enhancing growth propects. That effect is not modelled here.

[16] In Alexeev and Kim there are two types of borrowers (entrepreneurs), experienced and inexperienced. The former know the type of project they have drawn, and if the project is good they know they will be not need further finance. Hence, they know that there is no benefit to them of seeking a soft lender. In our model, on the other hand, even good borrowers may benefit from a soft lender, so tough lenders need to differentiate themselves by offering better terms.

Hence, a lender may value the reputation of being a tough to attract a good pool of borrowers. In this section we examine this intuition.

In our notation, a lower interest rate means a higher value of a — the fraction of a project's return retained by the borrower. We assume that there are two values of a, which lenders can offer to borrowers, a_T and a_S, where $a_T > a_S$. In the beginning of every odd-numbered period lenders post a value of a they promise to charge; upon matching with a borrower tough lenders can stand on their promise whereas soft lenders cannot do so and behave opportunistically. Let κ, $0 < \kappa < 1$ be the measure of tough lenders.

We assume that borrowers cannot observe the interest rates charged by lenders (to other borrowers) in the past; these remain private information at all times. All that borrowers can observe prior to matching with lenders is their promise of an interest rate and their reputation. To simplify matters we assume that both borrowers and lenders discount past information, so that only the most recent observation matters.[17] Thus, a lender's reputation takes two values, denoted U and R, where U means that lender has discontinued (liquidated) her most recent project and R means that the most recent project was refinanced. Also, we assume that lenders fully discount the future. If lenders are patient, then their reputation plays two distinct roles. First, the lenders reputation is a signal of their type; it provides information which borrowers can take into account when choosing who to match in a period. Second, this reputation improves the lender's future clientele and, hence, to increase future payoffs. For this reason tough lenders who are stuck with a soft reputation may choose to liquidate apparently successful projects just to gain a tougher reputation. We do not consider strategic liquidation behavior in this paper; here we analyze the role of a lender's reputation as a signal of their type only. As before, let $\beta \in [0, 1)$ be the discount factor of borrowers.

When only borrowers have reputations (as in the model of section 3) all lenders are identical; hence, borrowers are indifferent concerning whom they meet. In that case the assumption of exogenous matching seemed appropriate. Because both borrowers and lenders are now heterogeneous, they may want to target specific groups for matching among themselves. Indeed, reputation matters for the lender precisely because she wishes to deter certain types of borrowers. Hence it is important to endogenize the matching process in this case. To endogenize matching we alter the sequence of actions of the

[17] This simplifies the analysis but we can extend the model to longer memory.

game and allow for a non-uniform matching process.

The timing of events is the following. In the beginning of every odd-numbered period lenders post an interest rate they promise to charge; after that, but before any matching takes place, the borrowers make a decision about the population of lenders they want to be matched with; then matching takes place and matched individuals play the above two-period project game. Because from the borrower's perspective the only difference between lenders is their posted interest rate and their reputation, the borrower is indifferent with regard to lenders with the same interest-reputation pair. We assume that if a borrower targets one particular group of lenders of positive measure, then she gets to meet a lender from that group with probability one; if a borrower is indifferent among several groups, then she has an equal chance to meet a lender from each group.

If search for the borrower is costless it is optimal to all lenders to post a low interest rate in order to attract borrowers.[18] In fact, because reputation is only an imperfect indication of lender's type, given a posted low interest rate, a borrower will be charged a low interest only if a lender turns out to be tough (in which case a lender honors her promise); given a posted high interest rate, a borrower ends up paying the high interest with probability one because, conditional on having located a borrower, all lenders prefer to charge high interest (tough lenders do not violate their promise in that case; other borrowers never observe a break of a promise by soft lenders, which excludes the possibility of future punishment). This means that regardless of reputation, every borrower is willing to target those who promise a low interest rate and, conditional on that strategic choice of borrowers, all lenders make such a promise. In these circumstances the matching decision of a borrower is based entirely on reputation of a lender.

Furthermore, to avoid bottlenecks resulting from matching of populations of unequal sizes, we let lenders form coalitions, where members of each coalition pool outside resources to finance more (less) than one project per member. Given that we have a $[0, 1]$-continuum of lenders, we define coalitions as finite (countable) subsets of the unit interval. We require that coalitions are uniform: all lenders in a coalition have the same type and reputation. As regards the matching process, we assume that at all times every coalition is matched with a homogeneous pool of borrowers, i.e. all borrowers who are

[18] This follows because there is no way for borrowers to force soft lenders to honor their announcements.

matched with a given coalition have the same state. Every borrower has a uniform chance to be matched with every coalition within her targeted group of lenders; given the matching strategy of borrowers, every coalition has a uniform chance to be matched with a pool of borrowers in any given state.

We consider equilibria in which the members of each coalition receives a uniform signal σ and is equally successful in tampering with reputation. That is, each lender in the coalition receives the same signal. Signals might differ across coalitions, however. This formulation is consistent with our interpretation of the signal as an industry specific shock; one can think of the coalitions of lenders as being industrial banks. The latter also adds to the intuition behind uniformity of tampering — a soft bank can either pretend that it is tough or fail to do so.

Given these assumptions one can think of the matching process as follows. In every period nature partitions both borrowers and lenders into a large number (continuum) of groups which, given the borrower's strategy, are uniformly matched among themselves and upon being matched act as single individuals. The partition of lenders is permanent (banks); the partition of borrowers is stochastic. A potentially restrictive side of this construction is that banks always have a homogeneous clientele, however we are confident that we do not miss anything significant here while we avoid the difficulty of dealing with transition of bank's reputation in case a bank liquidates some of the projects and refinances some others.

Homogeneity implies that a state of a matched coalition is a vector $z \equiv (k, n, j, \sigma)$, where k is the type of lenders in the coalition, n is their reputation, j is reputation of borrowers who the coalition is matched with, and σ is the observed signal. Let $f_{knj\sigma}$ be the strategy of the coalition — the probability that given its state z the coalition refinances its clients. Likewise, the state of a matched borrower is a triplet $x \equiv (i, j, n)$, where i is the type of the borrower, j is her reputation, and n is reputation of the coalition that borrower is matched with. Given the state, a borrower chooses her effort; let d_{ijn} be the probability that a borrower chooses high effort. Here we limit ourselves to pure strategy equilibria, the ones in which lenders do not randomize over liquidation decisions and borrowers do not randomize over the choice of effort, i.e. $f_{knj\sigma} \in \{0, 1\}$ for all states z, and $d_{ijn} \in \{0, 1\}$ for all states x. Then, an unmatched borrower's matching strategy can be described by a (multivalued) map τ, which maps an unmatched borrower's state $s = (i, j)$ into the lender's reputation n, $\tau : \{A, M\} \times \{S, F\} \to \{U, R\}$. Given a profile f of lender's refinancing strategies as well as a profile d of borrower's effort choice strategies and provided that all other players follow τ, d, and f, a matching strategy τ is the best response of a borrower. Our normalization of payoffs implies that borrowers are always willing to target some lenders, so that $\tau(s)$ is non-empty for all s.

A reputation for being tough is only meaningful if such lenders have a higher propensity to liquidate. Given τ, tough lenders have a higher propensity to liquidate if the profile of their equilibrium strategies satisfies $f_{Tnj\sigma} \leq f_{Snj\sigma}$ for all n, j, and σ, and the inequality is strict for some pair of states $z_1 \equiv (T, n, j, \sigma)$ and $z_2 \equiv (S, n, j, \sigma)$ such that $n \in \tau(i, j)$ for some i. If tough lenders have a higher propensity to liquidate then a tough reputation means a higher chance that a coalition with such a reputation is indeed tough, and a soft reputation means that a coalition which has soft reputation is indeed soft.

We say that the lenders' reputation is essential if the range of τ is the set of lenders' reputations $\{U, R\}$, and given τ, one can find two states s_1 and s_2, $s_1 \neq s_2$, such that $\tau(s_1) \neq \tau(s_2)$. If the lenders' reputation is essential it means that at least in some of the meetings between borrowers and lenders, the preference of the former was not to randomize among the latter; we show that, generically, this is the case provided that tough lenders have a higher propensity to liquidate. However, if both types have equal propensity to liquidate, then targeting lenders with particular reputation does not increase return of the borrowers. This means that borrowers are indifferent, so that they randomize (which is the case we have already considered in section 3).

What about the borrower's reputation? The borrower's reputation is essential if there is a triplet k, n, σ such that the lender liquidates in state $z_1 = (k, n, j_1, \sigma)$ and refinances in state $z_2 = (k, n, j_2, \sigma)$, where j_1 and j_2 are the two distinct values of borrower's reputation. Here the meaning of essentiality is that borrower's reputation is binding for the lender's decision.

4.1 Search and borrowers reputation

Even though the state space here is the smallest possible, the number of equilibria where reputation is essential is large. Our strategy is to present some of the most interesting equilibria. We start by arguing that many equilibria one might think of are non-generic and for that reason are not interesting. Our next proposition, for example, shows that, provided that the lender's reputation is informative of her type, borrowers will use search strategies which are independent of their types except for a set of measure zero in parameter space.

Proposition 4. *Except for a set of measure zero in the parameter space there are no equilibria in which tough lenders have a higher propensity to liquidate and borrowers use matching strategy which reveals their types.*

The message of this proposition is that when they choose with whom to match, borrowers target lenders according to their own reputation. Because the types of borrowers as well as their matching histories are unobservable by lenders, conditional on reputation, both types of borrowers can secure the same interest rate, which implies that borrowers can separate according to their types only if they are indifferent between the two groups of lenders, i.e. if the expected interest rate paid by tough lenders and by soft lenders is the same. However, as stated in Proposition 4 such an equality of interest rates is non-generic.

Because reputation alone determines the matching decision of borrowers, essentiality of borrowers' reputation is necessary for essentiality of lenders' reputation. We formalize that claim as the following corollary to Proposition 4:

Corollary. *Except for a set of measure zero in the parameter space there are no equilibria in which tough lenders have a higher propensity to liquidate, lenders' reputation is essential, and borrowers' reputation is not essential.*

Notice that this result is rather different from Alexeev and Kim ([2]). They study the reputation of lenders in the absence of a value of this reputation for the borrowers. This is feasible in their model because they assume that there is an exogenous fraction of borrowers who are oblivious of lenders' reputations. These borrowers thus randomize among all lenders. Hence, tough lenders have no problems attracting customers. But this will not work if borrowers care about a lenders's reputations, as well they might if they know that their projects may not succeed. The corollary to Proposition 4 implies that once a lenders' reputation is available to all borrowers, it cannot have any value in isolation from the reputation of the latter.

4.2 An example

We now present an example of an equilibrium where reputation matters for both borrowers and lenders. In such an equilibrium there is a separation based on reputations — borrowers with good reputations meet lenders with tough reputations and borrowers with bad reputations meet lenders with soft ones. We show that such a separation benefits borrowers. This follows because the lender's reputation gives a borrower with a good reputation a better chance to meet a tough lender and benefit from a lower interest rate. It also helps borrowers with bad reputations to meet soft lenders and avoid being liquidated. The effect on lenders, however, is less clear cut because of scale effects. Separation by reputations may leave some lenders with fewer clients. Even though separation implies fewer liquidations by tough lenders, the latter may end up worse off from their tough reputation if transparency is high and bad times are frequent because in that case there is a relatively large number of borrowers with bad letters who choose to bank with soft lenders. If, on the other hand, bad times are sufficiently infrequent (we specify how infrequent below) then tough lenders are better off for having such a reputation.

We summarize these arguments in the following two propositions. Proposition 5 provides sufficient conditions that guarantees the existence of an equilibrium where there is separation by reputations. Proposition 6 then shows that there are conditions such that lenders gain in terms of welfare (profits) from having a tough reputation.

Proposition 5. *Assume that Assumption 1 holds with respect to* $a = a_T$*,*

$$\Pi(1-a_T)(X-2) + (1-\Pi)(x-2) < 0,$$

Assumption 2 holds with respect to $a = a_S$*,*

$$-1 < \pi(1-a_S)(X-2) + (1-\pi)(x-2),$$

$$pa_T < a_S,$$

return on high effort exceeds its cost,

$$(\Pi - \pi)\left[\frac{\frac{\kappa}{2-p}pa_T + (1-\kappa)\,a_S}{\frac{\kappa}{2-p} + 1 - \kappa} + \beta^2(1-\phi)(1-p)\Pi\left(a_T - a_S + \frac{\frac{\kappa}{2-p}}{\frac{\kappa}{2-p} + 1 - \kappa}\left(a_S - pa_T\right)\right)\right](X-2) \geq E - e, \tag{17}$$

and liquidation values L_S and L_T satisfy:

$$L_S < \underline{L}_S,$$

$$\underline{L}_T < L_T < \overline{L}_T$$

where $\underline{L}_S$, $\underline{L}_T$, and $\overline{L}_T$ are constructed in proof. Then:

- *i) borrowers with good reputation meet lenders with tough reputation; borrowers with bad reputation meet lenders with soft reputation,*
- *ii) all able borrowers work hard,*
- *iii) soft lenders refinance all projects and, conditional on observed low signal, tough lenders liquidate all projects of borrowers with bad reputation.*

Proposition 6. *Assume that hypotheses of Proposition 5 hold except that condition (17) on able borrowers effort is replaced by:*

$$(\Pi - \pi)\left[\kappa p a_T + (1-\kappa)\, a_S + \beta^2 (1-\phi)(1-p)^2 \kappa a_T \Pi\right](X-2) \geq E - e. \tag{18}$$

Then, the welfare of borrowers of both types increases when lenders have reputations. Moreover, if the parameters satisfy:

$$\mathcal{M}_F \equiv (1-\phi)(1-p)\left(\lambda(1-\Pi) + (1-\lambda)(1-\pi)\right) < \frac{1}{2}, \tag{19}$$

then the existence of lenders' reputations increases the profits of tough lenders and decreases the profits of soft lenders.

Proposition 6 shows that tough lenders will generate higher profits than soft lenders if the measure of borrowers with a bad reputation is less than one-half. Thus when reputation matters it is valuable to have a tough reputation. But this depends on there being a sufficient fraction of good borrowers in the economy. If most borrowers are bad then having a tough reputation merely deflects borrowers to the soft lenders. When bad borrowers are less common Proposition 6 shows that it is advantageous to be a tough lender. One can observe from (19) that the fraction of borrowers with a bad reputation is small if the aggregate state of the economy is good and there is sufficient transparency. This condition is much more likely to be satisfied in the later stages of transition. Early in transition the lack of transparency and the state of the economy make it unlikely that tough lenders will emerge. Over time, as transparency increases and as the overall state of the economy improves tough lenders will emerge. This suggests that reputation could be a force that encourages lenders to be tough but that it is not an automatic process. It also suggests that over time a virtuous circle may appear. The measure of soft lenders shrinks and the quality of loans improves which further enhances the growth process, and further improves transparency.

Thus, if the economy is already in good shape, then the introduction of a lender's reputation seems to be helpful: it helps tough lenders survive and drives soft lenders out of business.

5 Conclusion

Our goal in this paper was to consider how reputational considerations impact on the development of hard-budget constraints. When only borrowers have reputations (that is when lenders are identical) the effect of reputation on the soft-budget constraint is ambiguous. If liquidation costs are low then introducing reputation may harden budget constraints. If liquidation costs are sufficiently high, however, the opposite occurs.

The analysis becomes more interesting when we introduce a lender's reputation. We show that lenders may want to develop a reputation for being tough in order to attract a better class of borrowers. By doing so they can earn higher profits than lenders with a reputation for being soft. This separation suggests that over time reputational considerations may lead to the development of hard-budget constraints. Two conditions are critical for this outcome to occur. First, the economy must be sufficiently transparent that reputations can be observed. Second, the overall economy must improve so that there are a sufficient level of potentially good borrowers. When both conditions are present then tough lenders earn higher profits and one may expect that their presence in the economy will expand. This suggests that there may be a virtuous circle in economic transition.

Given that (under certain conditions) tough lenders earn higher profits soft lenders may prefer to disguise their own reputations by engaging in strategic liquidations to enhance their future reputations. This is ruled out in our model because of our assumption that memory lasts only one period.

An important topic for future research would be to add longer memories and thus permit strategic liquidation. If soft lenders choose to pool this may enhance the range of parameters in which the hard-budget constraint can emerge. Without further research, however, this can only be considered speculation. Treating reputations in a fully dynamic setting will certainly enhance the analysis. Our model is thus only a first step towards such an analysis.

6 Appendix

Proposition 1. *Assume that assumptions 1 and 2 hold and that return on high effort exceeds its cost,*

$$(\Pi - \pi)\left(p + \beta^2\Pi(1-p)^2\right)a(X-2) > E - e.$$

Then there exists a $\overline{\phi} \leq 1$ *such that for all* $\phi \leq \overline{\phi}$ *there exist two values,* $\underline{L}$ *and* $\overline{L}(\phi)$, $0 < \underline{L} < \overline{L}(\phi) < 1$, *such that*

- *i) if* $L < \underline{L}$, *then all able borrowers work hard and no project is ever liquidated,*
- *ii) if* $\underline{L} \leq L \leq \overline{L}(\phi)$, *then all able borrowers work hard and provided that lenders receive a low signal, all projects of borrowers with a bad reputation are liquidated whereas all projects of borrowers with a good reputation are refinanced,*
- *iii) if* $L > \overline{L}(\phi)$, *then provided that lenders receive low signal, all projects are liquidated.*

Proof: Most of the steps of this proof are carried out in the body of the paper, so we only need to finish some details.

Provided that all borrowers work hard, the probabilities $\zeta_{G|j}$ that given reputation j the project is good are:

$$\zeta_{G|F} = \frac{\lambda\Pi(1-\Pi) + (1-\lambda)\pi\,(1-\pi)}{\lambda\,(1-\Pi) + (1-\lambda)\,(1-\pi)}$$

and

$$\zeta_{G|S} = \frac{\lambda\Pi\left(\Pi+(p+\phi(1-p))(1-\Pi)\right)+(1-\lambda)\,\pi\left(\pi+(p+\phi(1-p))(1-\pi)\right)}{\lambda\left(\Pi+(p+\phi(1-p))(1-\Pi)\right)+(1-\lambda)\left(\pi+(p+\phi(1-p))(1-\pi)\right)}.$$

One can further verify that for all $\phi \in [0,1]$,

$$\pi < \zeta_{G|F} < \zeta_{G|S} < \Pi.$$

Then, it follows immediately that the expected (conditional on observed low signal) payoff of the lender, who is matched with borrower with bad reputation, is smaller than the payoff of the lender who is matched with borrower with good reputation:

$$\zeta_{G|F}(1-a)(X-2)+\zeta_{B|F}(x-2) < \zeta_{G|S}(1-a)(X-2)+\zeta_{B|S}(x-2).$$

Because the continuation value of the lenders is independent of their current actions, conditional on observed low signal lenders refinance all projects if and only if:

$$L-1 < \zeta_{G|F}(1-a)(X-2)+\zeta_{B|F}(x-2). \tag{20}$$

Then, assumption 2 guarantees that there exists $\underline{L} \equiv \zeta_{G|F}(1-a)(X-2)+\zeta_{B|F}(x-2) > 0$ such that (20) holds for all $L \in [0,\underline{L})$. Similarly, lenders liquidate all projects if and only if:

$$\zeta_{G|S}(1-a)(X-2)+\zeta_{B|S}(x-2) < L-1. \tag{21}$$

Assumption 1 guarantees then that there exists $\overline{L}(\phi) \equiv \zeta_{G|S}(1-a)(X-2)+\zeta_{B|S}(x-2) < 1$ such that (21) holds for all $L \in (\overline{L}(\phi),1]$. Because $\underline{L} < \overline{L}(\phi)$ for all $\phi \in [0,1]$, if $L \in [\underline{L},\overline{L}(\phi)]$, then lenders liquidate all projects of borrowers with bad reputation and refinance all projects of borrowers with good reputation. The function $\overline{L}(\phi)$ is decreasing in ϕ, which implies that higher transparency increases the interval $L \in [\underline{L},\overline{L}(\phi)]$, where reputation matters.

To complete the proof we need to find conditions, which ensure that able borrowers work hard. The borrowers work hard if given the continuation value (implied by lenders equilibrium strategy profile) there is no benefit from a unilateral defection. As we say above, it is sufficient that able borrowers with bad reputation do not want to choose low effort even if conditional on low signal lenders liquidate their projects, that is:

$$p\Pi a(X-2)-E+P_{F|E}v_F+P_{S|E}v_S \geq p\pi a(X-2)-e+P_{F|e}v_F+P_{S|e}v_S, \quad (22)$$

where the value function of able borrowers v satisfies the following two-equation system of Bellman equations:

$$\begin{pmatrix} v_F \\ v_S \end{pmatrix} = \beta^2 \left[\begin{pmatrix} p\Pi a(X-2) \\ \Pi a(X-2) \end{pmatrix} + \begin{pmatrix} P_{F|E} & P_{S|E} \\ P_{F|E} & P_{S|E} \end{pmatrix} \begin{pmatrix} v_F \\ v_S \end{pmatrix} \right], \quad (23)$$

and where:

$$\begin{aligned} P_{F|E} &= (1-\phi)(1-p)(1-\Pi), \\ P_{S|E} &= \Pi + p(1-\Pi) + \phi(1-p)(1-\Pi) \end{aligned}$$

and

$$\begin{aligned} P_{F|e} &= (1-\phi)(1-p)(1-\pi), \\ P_{S|e} &= \pi + p(1-\pi) + \phi(1-p)(1-\pi). \end{aligned}$$

One can write (22) as:

$$(\Pi-\pi)pa(X-2) + \left(P_{F|E} - P_{F|e}\right) v_F + \left(P_{S|E} - P_{S|e}\right) v_S \geq E - e,$$

and then observe that:

$$P_{S|E} - P_{S|e} = -\left(P_{F|E} - P_{F|e}\right) = (1-\phi)(1-p)(\Pi-\pi),$$

so that (22) becomes:

$$(\Pi-\pi)\left[pa(X-2) + (1-\phi)(1-p)\left(v_S - v_F\right)\right] \geq E - e.$$

Then, subtracting the first equation from the second equation in (23) yields:

$$v_S - v_F = \beta^2(1-p)\Pi a(X-2),$$

so that (22) simplifies to:

$$(\Pi-\pi)\left[p + \beta^2(1-\phi)(1-p)^2\Pi\right] a(X-2) \geq E - e.$$

One can see that if parameters satisfy:

$$(\Pi - \pi)\left[p + \beta^2(1-p)^2\Pi\right] a(X-2) > E - e,$$

then by continuity there is some $\overline{\phi}$ such that (22) holds for all $\phi \in [0, \overline{\phi}]$. The value of $\overline{\phi}$ can be computed from (22) which then holds at equality. Notice that if parameters satisfy

$$(\Pi - \pi)pa(X-2) \geq E - e,$$

then $\overline{\phi} = 1$, i.e. the benefit of high effort exceeds its cost even if the economy is fully opaque and reputation carries no information about the type of borrowers. In that case able borrowers choose to work hard for all L, regardless of whether lenders liquidate them or not. ■

Proposition 2. *Assume that assumptions 1 and 2 hold and that the return on high effort is low,*

$$(\Pi - \pi)\, a(X-2) < E - e.$$

Then there exists a value $\widehat{L}$ such that no borrower works hard and

- *i) if $L < \widehat{L}$, then no project is ever liquidated, and*
- *ii) $L \geq \widehat{L}$, then provided that lenders receive low signal, all projects are liquidated.*

Proof: Because borrowers get zero if liquidated, the maximum payoff borrowers obtain is when no project is liquidated. The proof proceeds by showing that given that lenders refinance every borrower, condition (16) implies that return on high effort is not sufficient to induce able borrowers work hard.

Indeed, if lenders refinance all borrowers, then able borrowers choose low effort if and only if:

$$\Pi a(X-2) - E + P_{F|E}v_F + P_{S|E}v_S < \pi a(X-2) - e + P_{F|e}v_F + P_{S|e}v_S, \quad (24)$$

where the value function of able borrowers v satisfies the following two-equation system of Bellman equations:

$$\begin{pmatrix} v_F \\ v_S \end{pmatrix} = \beta^2 \left[\begin{pmatrix} \pi a(X-2) \\ \pi a(X-2) \end{pmatrix} + \begin{pmatrix} P_{F|e} & P_{S|e} \\ P_{F|e} & P_{S|e} \end{pmatrix} \begin{pmatrix} v_F \\ v_S \end{pmatrix} \right]. \quad (25)$$

One can rearrange (24) as:

$$(\Pi - \pi)\left[a(X-2) + (1-\phi)(1-p)\left(v_S - v_F\right)\right] < E - e.$$

It follows immediately from (25) that:

$$v_S - v_F = 0,$$

so that (24) becomes:

$$(\Pi - \pi)a(X-2) < E - e.$$

Intuitively, because lenders refinance every borrower, there is no benefit for the borrowers from having good reputation.

Then, provided that all borrowers choose low effort, conditional on observed low signal, the payoff of a lender is:

$$\pi(1-a)(X-2) + (1-\pi)(x-2).$$

Assumption 2 guarantees then that there is a value $\widehat{L}$,

$$\widehat{L} \equiv \pi(1-a)(X-2) + (1-\pi)(x-2),$$

such that if $L \in [0, \widehat{L})$, then lenders refinance all projects and if $L \in (\widehat{L}, 1]$, then lenders liquidate all projects. If $L = \widehat{L}$, then lenders are indifferent, without loss of generality one can assume that lenders liquidate, refinance, or randomize. ■

Proposition 3. *Assume that assumptions 1 and 2 hold and that return on high effort is exceeds its cost,*

$$(\Pi - \pi)\, pa(X-2) > E - e.$$

Then, for all $\phi \in [0,1)$ *there exists* L^*, $\underline{L} < L^* < \overline{L}(\phi)$, *where* $\underline{L}$ *and* $\overline{L}(\phi)$ *are defined in Proposition 1, such that:*

$$\begin{aligned} W(L;\phi) &< W(L;1) \text{ for all } L \in (\underline{L}, L^*) \\ W(L;\phi) &> W(L;1) \text{ for all } L \in (L^*, \overline{L}) \\ W(L;\phi) &= W(L;1) \text{ otherwise.} \end{aligned}$$

Proof: It follows from the proof of Proposition 1 that conditional on low signal lenders liquidate all borrowers if $L > \overline{L}(\phi)$, liquidate all borrowers with bad reputation and refinance borrowers with good reputation if $L \in (\underline{L}, \overline{L}(\phi))$, and refinance all borrowers if $L < \underline{L}$, where $\underline{L}$ and $\overline{L}(\phi)$ are constructed in the proof. (Notice that $\underline{L}$ is independent of ϕ and $\overline{L}(\phi)$ is decreasing in ϕ.) Provided that all able borrowers work hard (which for all $\phi \in [0,1]$ is warranted by the hypotheses of the proposition), the value function of the borrowers is:

$$\begin{aligned} v_{AF} = v_{AS} &= \tfrac{1}{r}\left(p\Pi a(X-2) - E\right) \\ v_{MF} = v_{MS} &= \tfrac{1}{r}\left(p\Pi a(X-2) - E\right) \end{aligned}$$

if $L \geq \overline{L}(\phi)$,

$$\begin{aligned} v_{AF} &= \tfrac{1}{r}\left[\left(1-(1-p)\tfrac{r+(1-\phi)(1-p)(1-\Pi)}{1+r}\right)\Pi a\,(X-2) - E\right] \\ v_{AS} &= \tfrac{1}{r}\left[\left(1-(1-p)\tfrac{(1-\phi)(1-p)(1-\Pi)}{1+r}\right)\Pi a\,(X-2) - E\right] \\ v_{MF} &= \tfrac{1}{r}\left[\left(1-(1-p)\tfrac{r+(1-\phi)(1-p)(1-\pi)}{1+r}\right)\pi a\,(X-2) - e\right] \\ v_{MS} &= \tfrac{1}{r}\left[\left(1-(1-p)\tfrac{(1-\phi)(1-p)(1-\pi)}{1+r}\right)\pi a\,(X-2) - e\right] \end{aligned}$$

if $L \in (\underline{L}, \overline{L}(\phi))$, and

$$\begin{aligned} v_{AF} = v_{AS} &= \tfrac{1}{r}\left(\Pi a(X-2) - E\right) \\ v_{MF} = v_{MS} &= \tfrac{1}{r}\left(\Pi a(X-2) - E\right) \end{aligned}$$

if $L \leq \underline{L}$, where $r \equiv \frac{1}{\beta^2} - 1$. Straightforward manipulations yield the welfare of the borrowers:

$$v(L;\phi) = \begin{cases} v_1 \ \text{ if } L \geq \overline{L}(\phi) \\ v_2(\phi) \ \text{ if } L \in (\underline{L}, \overline{L}(\phi)) \\ v_3 \ \text{ if } L \leq \underline{L} \end{cases}$$

where

$$v_1 = \frac{\lambda}{r}\left(p\Pi a(X-2) - E\right) + \frac{1-\lambda}{r}\left(p\pi a(X-2) - e\right),$$

$$v_2 = \frac{\lambda}{r}\left(1-(1-\phi)(1-\Pi)(1-p)^2\right)\left(\Pi a\,(X-2) - E\right) + \frac{1-\lambda}{r}\left(1-(1-\phi)(1-\pi)(1-p)^2\right)\left(\pi a\,(X-2) - e\right),$$

$$v_3 = \frac{\lambda}{r}\left(\Pi a(X-2) - E\right) + \frac{1-\lambda}{r}\left(\pi a(X-2) - e\right).$$

One can immediately see that borrowers welfare is a discontinuous non-increasing step function. The ex ante welfare of the lenders is:

$$V(L;\phi) = \begin{cases} V_1(L) \text{ if } L \geq \overline{L}(\phi) \\ V_2(L;\phi) \text{ if } L \in (\underline{L}, \overline{L}(\phi)) \\ V_3 \text{ if } L \leq \underline{L} \end{cases}$$

where

$$V_1 = \frac{1}{r}\left(p(1-a)(X-2) + (1-p)\,(L-1)\right),$$

$$V_2 = \frac{1}{r}(p(1-a)(X-2) + (1-p)(\mathcal{M}_F(L-1) + \mathcal{M}_S\left(\zeta_{G|S}(1-a)(X-2) + \zeta_{B|S}(x-2)\right))),$$

$$V_3 = \frac{1}{r}(p(1-a)(X-2) + (1-p)(\mathcal{M}_F(\zeta_{G|F}(1-a)(X-2) + \zeta_{B|F}(x-2)) + \mathcal{M}_S\left(\zeta_{G|S}(1-a)(X-2) + \zeta_{B|S}(x-2)\right)\Big)\Big),$$

which is continuous non-decreasing function of L. It is easy to verify that the expression for V_3 simplifies to

$$V_3 = \frac{1}{r}(p(1-a)(X-2) + (1-p)((\lambda\Pi + (1-\lambda)\,\pi)(1-a)(X-2) + (\lambda(1-\Pi) + (1-\lambda)\,(1-\pi))(x-2)),$$

which does not depend on ϕ. As one would expect, because outside the interval $L \in (\underline{L}, \overline{L}(\phi))$ reputation is irrelevant, transparency has no effect on welfare.

Now assume that $\phi = 1$. Because in that case the measure of borrowers with bad letters is zero, $v_2 = v_3$ and $V_2 = V_3$, which implies that welfare $W(L;1) \equiv v(L;1) + V(L;1)$ is:

$$W(L;1) = \begin{cases} v_1 + V_1(L) \text{ if } L \geq \overline{L}(1) \\ v_3 + V_3 \text{ if } L < \overline{L}(1) \end{cases}.$$

Because $V(L;\phi)$ is continuous and $v(L;\phi)$ is a step function, $W(L;1)$ is discontinuous at $L = \overline{L}(1)$ and $\lim_{L\to\overline{L}(1)-0} W(L;1) > \lim_{L\to\overline{L}(1)+0} W(L;1)$, so that welfare declines as L goes above the threshold $\overline{L}(1)$.

For any $\phi < 1$, $v_1 < v_2(\phi) < v_3$, which implies that welfare $W(L;\phi)$ is discontinuous at both $L = \underline{L}$ and $L = \overline{L}(\phi)$. Because $v(L;\phi)$ is non-increasing, welfare $W(L;\phi)$ drops down at both points of discontinuity. This implies that in some neighborhood $L \in (\underline{L}, L^*)$, $W(L;\phi) < W(L;1)$. On the other hand, for any $L \in (\overline{L}(1), \overline{L}(\phi))$, $V(L;\phi) > V(L;1)$ and $v(L;\phi) > v(L;1)$, which yields $W(L;\phi) > W(L;1)$. Therefore, $L^* < \overline{L}(1)$. The value of L^* can be computed from equating $W(L;\phi)$ and $W(L;1)$, which yields:

$$L^* - 1 = \zeta_{G|F}(1-a)(X-2) + \zeta_{B|F}(x-2) + \\ \mu_{A|F}\left(\Pi a\left(X-2\right) - E\right) + \mu_{M|F}\left(\pi a\left(X-2\right) - e\right).$$

■

Proposition 4. *Except for a set of measure zero in parameter space there are no equilibria in which tough lenders have a higher propensity to liquidate and borrowers use matching strategy which reveals their types.*

Proof: Our first step is, given the profile of liquidation strategies of lenders, to compute the probability that lender's reputation is an accurate indication of her type. The probability that a lender with tough reputation is indeed tough is a ratio of a measure of tough lenders with tough reputation to the measure of all lenders with tough reputation; the probability that a lender with soft reputation is indeed soft is a ratio of a measure of soft lenders with soft reputation to the measure of all lenders with soft reputation. The measure of unmatched lenders in state (k,n), denoted ψ_{kn}, comes from the stationary distribution Ψ implied by the transition matrix for the lenders' reputation,

$$\psi \equiv (\psi_{TU}, \psi_{TR}, \psi_{SU}, \psi_{SR}).$$

Given the matching strategy of borrowers τ, let ν_{nj} be the probability that a coalition with reputation n is matched with a pool of borrowers with reputation j. Because reputation of a borrower is independent of lenders' actions, ν_{nj} depends only on τ and on the measures $\mathcal{M}_{ij}$ of borrowers in all four states (i,j); provided that all able borrowers work hard, $\mathcal{M}_{ij}$ is given by (3). Let us define a function $\mathcal{I}_n(i,j)$, which given τ maps borrower's state into the set of three numbers, $\left\{0, \frac{1}{2}, 1\right\}$:

$$\mathcal{I}_n(i,j) = \begin{cases} 0 \text{ if } (i,j) \notin \tau^{-1}(n) \\ \frac{1}{2} \text{ if } (i,j) \in \tau^{-1}(U) \cap \tau^{-1}(R) \\ 1 \text{ otherwise} \end{cases} .$$

The function $\mathcal{I}_n(i,j)$ summarizes borrower's matching strategy from the viewpoint of a lender. A coalition has no chance to meet those who target other lenders; a coalition meets those who target both groups of lenders with probability one half; a coalition meets all of those who target lenders with a given reputation. Then, our assumption that every coalition has a uniform chance to be matched with a pool of borrowers in any given state implies:

$$\nu_{nj} = \frac{\sum_i \mathcal{I}_n(i,j)\mathcal{M}_{ij}}{\sum_i \sum_j \mathcal{I}_n(i,j)\mathcal{M}_{ij}}.$$

Recall that $f_{knj\sigma}$ is the strategy of a matched coalition — the probability that given an after-the-first-stage-of-a-project state $z \equiv (k,n,j,\sigma)$ a coalition refinances the project. Because lenders cannot change their type, the transition matrix for the state (k,n) of a coalition is block-diagonal:

$$T = \begin{bmatrix} T_T & 0 \\ 0 & T_S \end{bmatrix}, \tag{26}$$

where T_k is a two-by-two transition matrix for reputation:

$$T_k = \begin{bmatrix} \omega_{kUU} & \omega_{kUR} \\ \omega_{kRU} & \omega_{kRR} \end{bmatrix}, \tag{27}$$

$$\omega_{knU} = \sum_j \nu_{nj} \left(p\left(1 - f_{knj\overline{\sigma}}\right) + (1-p)\left(1 - f_{knj\underline{\sigma}}\right)\right),$$

$$\omega_{knR} = \sum_j \nu_{nj} \left(p f_{knj\overline{\sigma}} + (1-p) f_{knj\underline{\sigma}}\right).$$

The matrix T_k shows the transition of reputation of a coalition of lenders of type k; first, it is who this coalition meets, then it is what signal the coalition observes, and then it is what the coalition does in terms of liquidation.

The associated stationary distribution ψ is a solution to $\psi T = \psi$ and to $\psi_{TU} + \psi_{TR} = \kappa$ and $\psi_{SU} + \psi_{SR} = 1 - \kappa$. One can verify that:

$$\psi_{TL} = \kappa \left(1 + \frac{\omega_{TUR}}{\omega_{TRU}}\right)^{-1}, \qquad \psi_{SU} = (1-\kappa)\left(1 + \frac{\omega_{SUR}}{\omega_{SRU}}\right)^{-1},$$

and

$$\psi_{TR} = \kappa \left(1 + \frac{\omega_{TRU}}{\omega_{TUR}}\right)^{-1}, \qquad \psi_{SR} = (1-\kappa)\left(1 + \frac{\omega_{SRU}}{\omega_{SUR}}\right)^{-1}.$$

(Notice that if equilibrium profile of strategies implies $\omega_{kRU} + \omega_{kUR} = 0$, then every distribution $\psi_k \equiv (\psi_{kU} \;\; \psi_{kR})$ is a stationary distribution.) Given that distribution, the measure of lenders with tough reputation is $\psi_U \equiv \psi_{TU} + \psi_{SU}$ and the measure of lenders with a soft reputation is $\psi_R \equiv \psi_{TR} + \psi_{SR}$. Consequently, the probability that a lender with a tough reputation is indeed tough is:

$$\mu_{T|U} = \frac{\psi_{TU}}{\psi_U} \tag{28}$$

and the probability that a lender with soft reputation is indeed soft is:

$$\mu_{S|R} = \frac{\psi_{SR}}{\psi_R}. \tag{29}$$

Because tough lenders have a higher propensity to liquidate:

$$f_{Tnj\sigma} \leq f_{Snj\sigma}, \tag{30}$$

for all n, j, and σ, and there is a pair of states $z_1 \equiv (T, n, j, \sigma)$ and $z_2 \equiv (S, n, j, \sigma)$ such that $\nu_{nj} > 0$ and the inequality in (30) is strict.[19] The latter implies that:

$$\omega_{TRU} \geq \omega_{SRU} \qquad \text{and} \qquad \omega_{SUR} \geq \omega_{TUR}, \tag{31}$$

where at least one inequality is strict. The inequality (31) implies:

$$\mu_{T|U} > \kappa \qquad \text{and} \qquad \mu_{S|R} > 1 - \kappa. \tag{32}$$

As one would expect, because tough lenders have a higher propensity to liquidate, the probability that given tough (soft) reputation, a lender is tough

[19] Notice that $n \in \tau(i,j)$ in the definition of higher propensity to liquidate and $\mathcal{M}_{ij} > 0$ imply that $\nu_{nj} > 0$.

(soft) is greater than the objective prior probability that a randomly drawn lender is tough (soft), so that lenders' reputation is informative of their types.

Given the matching strategy τ and the effort-choice profile of strategies of able borrowers d_A, let v_{ij} be the value of an unmatched borrower in state (i, j). The values v_{ij} satisfy the following four-equation system of Bellman equations

$$v_{Mj} = \beta^2 \left[R_{Mj}\pi(X-2) - e + P_{S|e}v_{MS} + P_{F|e}v_{MF} \right],$$

$$\begin{aligned} v_{Aj} = \\ &\beta^2 \left[R_{Aj} \left(d_{Ajn}\Pi + (1-d_{Ajn})\pi \right) (X-2) - \left(d_{Ajn}E + (1-d_{Ajn})e \right) + \right. \\ &\left. \left(d_{Ajn}P_{S|E} + (1-d_{Ajn})P_{S|e} \right) v_{AS} + \left(d_{Ajn}P_{F|E} + (1-d_{Ajn})P_{F|e} \right) v_{AF} \right], \end{aligned}$$

where

$$R_{ij} = \mu_{T|n}(pf_{Tnj\overline{\sigma}} + (1-p)f_{Tnj\underline{\sigma}})a_T + \mu_{S|n}(pf_{Snj\overline{\sigma}} + (1-p)f_{Snj\underline{\sigma}})a_S \quad (33)$$

is the expected return of a borrower in state (i, j) and $n \in \tau(i, j)$. (Notice that if borrowers use type-dependent matching strategy τ, then borrowers with the same reputation may obtain different returns R_{ij}.)

The final step of the proof is to argue that because types and histories are unobservable, borrowers either use type-independent matching strategy τ (provided that either $R_{Mj} > R_{Aj}$ or $R_{Aj} > R_{Mj}$) or provided that $R_{Aj} = R_{Mj}$, the latter implies a binding restriction on model parameters.

Assume now that borrowers use type-dependent matching strategy, i.e. that there is a j such that $\tau(A, j) \neq \tau(M, j)$. Let us assume that $R_{Mj} > R_{Aj}$, and consider a unilateral defection by an able borrower who has reputation j (here we mean that instead of targeting lenders with reputation n, $n \in \tau(A, j)$, the borrower unilaterally decides to target lenders with reputation n', $n' \in \tau(M, j)$). Because defections by groups of measure zero do not affect stationary distribution ψ and because lenders cannot observe the types of borrowers, such a defection yields one-period return R_{Mj} to the defector. Then, because the probability of a successful letter is independent of lender's actions (i.e. decisions to liquidate/refinance in a given state z) and because letters carry no information about lender's state (the matching history is unobservable), defection has no effect on the continuation value of the defector who can still (by following τ and effort-choice strategies d_A) secure the continuation value (the last line in the second of Bellman equations) in all future periods. Thus, defection yields a higher value to the defector. By a similar argument, if τ is such that for some j, $R_{Aj} > R_{Mj}$, mediocre borrowers will choose to defect. Thus, either borrowers type-independent matching strategy τ or $R_{Mj} = R_{Aj}$.

The latter implies:

$$\mu_{T|U}(pf_{TUj\overline{\sigma}} + (1-p)f_{TUj\underline{\sigma}})a_T + \mu_{S|U}(pf_{SUj\overline{\sigma}} + (1-p)f_{SUj\underline{\sigma}})a_S = \tag{34}$$
$$\mu_{T|R}(pf_{TRj\overline{\sigma}} + (1-p)f_{TRj\underline{\sigma}})a_T + \mu_{S|R}(pf_{SRj\overline{\sigma}} + (1-p)f_{SRj\underline{\sigma}})a_S.$$

As is shown in proof of Proposition 5, because lenders fully discount the future, i.e. $\delta = 0$, their equilibrium liquidation/refinancing decisions are independent of their reputation, i.e. $f_{kRj\sigma} = f_{kLj\sigma}$ for all k, j, and σ. That means that the two expressions in brackets in the first line of (34) are equal to corresponding expressions in the second line of that same equation. Because $\mu_{T|U} > \kappa$ and $\mu_{S|R} > 1-\kappa$, $\mu_{T|U} > \kappa > \mu_{T|R}$ and $\mu_{S|R} > 1-\kappa > \mu_{S|U}$. The latter implies that equation (34) yields a binding restriction on the model parameters.[20] ■

Corollary. *Except for sets of measure zero in parameter space there are no equilibria in which tough lenders have a higher propensity to liquidate, lenders' reputation is essential, and borrowers' reputation is not essential.*

Proof: Inessentiality of borrowers' reputation means that for all types k, reputations n, and signals σ,

$$f_{knj_1\sigma} = f_{knj_2\sigma},$$

which implies that $R_{ij_1} = R_{ij_2}$. By the argument in proof of proposition 3, if either $R_{Mj_1} = R_{Mj_2} > R_{Aj_1} = R_{Aj_2}$ or $R_{Aj_1} = R_{Aj_2} > R_{Mj_1} = R_{Mj_2}$, all borrowers choose to meet lenders with the same reputation and abandon lenders with other reputation, which contradicts the range of τ being in the set $\{U, R\}$. If $R_{MS} = R_{AS} = R_{MF} = R_{AF}$, then (34) implies a binding restriction on the model parameters. ■

Proposition 5. *Assume that Assumption 1 holds with respect to $a = a_T$,*

$$\Pi(1-a_T)(X-2) + (1-\Pi)(x-2) < 0,$$

[20] Notice that the null profile cannot be an equilibrium because this means that lenders liquidate all projects and, hence, incur losses from all projects. This yields lenders a negative value, which is worse than giving no funds to borrowers — an option always feasible for the lenders.

Assumption 2 holds with respect to $a = a_S$,

$$-1 < \pi(1 - a_S)(X - 2) + (1 - \pi)(x - 2),$$

$$pa_T < a_S,$$

return on high effort exceeds its cost, i.e.

$$(\Pi - \pi)\left[\frac{\frac{\kappa}{2-p}pa_T + (1-\kappa)\,a_S}{\frac{\kappa}{2-p} + 1 - \kappa} + \right.$$
$$\left.\beta^2(1-\phi)(1-p)\Pi\left(a_T - a_S + \frac{\frac{\kappa}{2-p}}{\frac{\kappa}{2-p} + 1 - \kappa}\left(a_S - pa_T\right)\right)\right](X-2) \geq E - e,$$

and liquidation values L_S *and* L_T *satisfy:*

$$L_S < \underline{L}_S,$$

$$\underline{L}_T < L_T < \overline{L}_T$$

where $\underline{L}_S$, $\underline{L}_T$, *and* $\overline{L}_T$ *are constructed in proof. Then*:

- *i) borrowers with good reputation meet lenders with tough reputation; borrowers with bad reputation meet lenders with soft reputation,*
- *ii) all able borrowers work hard,*
- *iii) soft lenders refinance all projects and, conditional on observed low signal, tough lenders liquidate all projects of borrowers with bad reputation.*

Proof: We want to construct an equilibrium in which good borrowers meet lenders with tough reputation, bad borrowers meet lenders with soft reputation, tough lenders liquidate bad borrowers with low signal and refinance all other borrowers, and soft lenders refinance all borrowers. The profile f of lenders' refinancing strategies, which corresponds to that situation is:

$$f_{TUF\underline{\sigma}} = f_{TRF\underline{\sigma}} = 0 \text{ and } f_{knj\sigma} = 1 \text{ for all other states } z.$$

Given that profile, the transition matrices for lenders' reputation T_T and T_S are the following:

$$T_T = \begin{bmatrix} 0 & 1 \\ 1-p & p \end{bmatrix} \quad \text{and} \quad T_S = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

Then, a distribution of unmatched lenders in state (k, n) is

$$\psi_{TU} = \kappa \frac{1-p}{1+(1-p)} \quad \text{and} \quad \psi_{SU} = 0, \tag{35}$$

$$\psi_{TR} = \kappa \frac{1}{1+(1-p)} \quad \text{and} \quad \psi_{SR} = 1-\kappa. \tag{36}$$

The probability that given tough reputation a lender is tough, $\mu_{T|U}$, can be computed using (28) and (35),

$$\mu_{T|U} = 1,$$

the probability that given soft reputation a lender is soft, $\mu_{S|R}$, can be computed from (29) and (36),

$$\mu_{S|R} = \frac{1-\kappa}{\frac{\kappa}{2-p} + (1-\kappa)}.$$

Notice that $\mu_{T|U} > \kappa$ and $\mu_{S|R} > 1-\kappa$.

Given the profile f of lenders' strategies, the return of borrowers with good reputation who meet lenders with tough reputation is

$$R_{iS} = a_T; \tag{37}$$

the return of borrowers with good reputation who meet lenders with soft reputation is

$$R_{iS} = (1-\mu_{S|R})a_T + \mu_{S|R}a_S. \tag{38}$$

Because $a_T > a_S$ and $1 > \kappa > 1-\mu_{S|R}$, borrowers with good reputation will choose to meet with lenders who have tough reputation. The return of borrowers with bad reputation who meet lenders with tough reputation is

$$R_{iF} = pa_T; \tag{39}$$

the return of borrowers with bad reputation who meet lenders with soft reputation is

$$R_{iF} = (1-\mu_{S|R})pa_T + \mu_{S|R}a_S = \frac{\kappa p a_T + (1-\kappa)(2-p)a_S}{\kappa + (1-\kappa)(2-p)}. \tag{40}$$

Because $a_S > pa_T$ and $\mu_{S|R} > 1-\kappa > 0$, borrowers with bad reputation will choose to meet with lenders who have soft reputation. Notice here that

$$R_{iS} = a_T > a_S > R_{iF}$$

The Bellman equations of able borrowers (provided that these work hard) are the following:

$$v_{AS} = \beta^2 \left(\Pi R_{AS}(X-2) - E + P_{S|E}v_{AS} + P_{F|E}v_{AF}\right),$$

and

$$v_{AF} = \beta^2 \left(\Pi R_{AF}(X-2) - E + P_{S|E}v_{AS} + P_{F|E}v_{AF}\right),$$

where R_{AS} and R_{AF} are given by (37) and (40) respectively. All able borrowers work hard if a unilateral defection by an able borrower with bad reputation does not lead to a higher return:

$$\Pi R_{AF}(X-2)-E+P_{S|E}v_{AS}+P_{F|E}v_{AF} \geq \pi R_{AF}(X-2)-e+P_{S|e}v_{AS}+P_{F|e}v_{AF},$$

which simplifies to:

$$(\Pi-\pi)\left[\frac{\kappa p a_T + (1-\kappa)(2-p)a_S}{\kappa+(1-\kappa)(2-p)} + \frac{\beta^2(1-\phi)(1-p)\Pi\left((2-p-\kappa)(a_T-a_S)+\kappa(1-p)a_S\right)}{\kappa+(1-\kappa)(2-p)}\right](X-2) \geq E-e.$$

Given borrowers' matching strategy τ, each member of a coalition with a tough reputation provides financing to

$$\Lambda_U \equiv \frac{\mathcal{M}_S}{\psi_U} = \frac{\lambda\left(\Pi + p(1-\Pi) + \phi(1-p)(1-\Pi)\right)}{\kappa\frac{1-p}{1+(1-p)}} + \frac{(1-\lambda)\left(\pi + p(1-\pi) + \phi(1-p)(1-\pi)\right)}{\kappa\frac{1-p}{1+(1-p)}}$$

borrowers and each member of a coalition with soft reputation lends to

$$\Lambda_R \equiv \frac{\mathcal{M}_F}{\psi_R} = \frac{(1-\phi)(1-p)\left[\lambda(1-\Pi)+(1-\lambda)(1-\pi)\right]}{\frac{\kappa}{1+(1-p)}+1-\kappa}$$

borrowers.

Because lenders fully discount the future, they compare one-period gains from refinancing and liquidation of borrowers. Conditional on an observed low signal, the payoff from refinancing is:

$$\Lambda_n\left(\zeta_{G|j}(1-a_k)(X-2)+\zeta_{B|j}(x-2)\right),$$

and the payoff from liquidation is:

$$\Lambda_n\left(L_k-1\right).$$

One can see that profile f of lenders strategies is an equilibrium profile if and only if:

$$\zeta_{G|F}(1-a_T)(X-2)+\zeta_{B|F}(x-2) \leq L_T-1 \leq \zeta_{G|S}(1-a_T)(X-2)+\zeta_{B|S}(x-2) \tag{41}$$

$$L_S-1 \leq \zeta_{G|F}(1-a_S)(X-2)+\zeta_{B|F}(x-2) \tag{42}$$

where the probability that given reputation j of a borrower her project is good, $\zeta_{G|j}$, and the probability that her project is bad, $\zeta_{B|j}$, are given by (3) and (5)-(8). Because $\zeta_{G|j}$ and $\zeta_{B|j}$ are independent of liquidation costs L_k, expressions (41)-(42) pin down the cutoff values $\underline{L}_S$, $\underline{L}_T$ and $\overline{L}_T$ right away. Assumption 1 guarantees that $\overline{L}_T<1$; assumption 2 guarantees that $\underline{L}_S>0$. Notice that as $\phi\to 1$, both $\zeta_{G|S}$ and $\zeta_{G|F}$ approach λ, the objective probability that a randomly drawn borrower is able. In that case $\underline{L}_T\to\overline{L}_T$, which implies that if economy is opaque, then the region in parameter space consistent with the above equilibrium is small. ■

Proposition 6. *Assume that hypotheses of Proposition 5 hold except that condition (17) on able borrowers effort is replaced by:*

$$(\Pi-\pi)\left[\kappa p a_T+(1-\kappa)a_S+\beta^2(1-\phi)(1-p)^2\kappa a_T\Pi\right](X-2)\geq E-e. \tag{43}$$

Then, bringing in lenders reputation increases welfare of both types of borrowers. Moreover, if parameters satisfy:

$$\mathcal{M}_F \equiv (1-\phi)(1-p)\left(\lambda(1-\Pi)+(1-\lambda)(1-\pi)\right)<\frac{1}{2}, \tag{44}$$

then lenders reputation increases profits of tough lenders and decreases profits of soft lenders.

Proof: Because lenders fully discount the future, given hypotheses of the proposition they use identical liquidation strategies, $f_{TUF\underline{\sigma}} = f_{TRF\underline{\sigma}} = 0$ and $f_{knj\sigma} = 1$ for all other states, regardless of whether their reputation is available or not. Assume that lenders reputation is unavailable. Given that in this case borrowers randomize among lenders, Bellman equations of able borrowers (provided that they work hard) are the following:

$$\begin{array}{l} v_{AS} = \beta^2 \left(r_S \Pi(X-2) - E + P_{S|E} v_{AS} + P_{F|E} v_{AF} \right) \\ v_{AF} = \beta^2 \left(r_F \Pi(X-2) - E + P_{S|E} v_{AS} + P_{F|E} v_{AF} \right) \end{array}, \tag{45}$$

where

$$r_S = \kappa a_T + (1-\kappa)\, a_S, \tag{46}$$

$$r_F = \kappa p a_T + (1-\kappa)\, a_S, \tag{47}$$

the probability of success $P_{S|E}$ and the probability of failure $P_{F|E}$ conditional on high effort are given by (12). Notice that one-period returns in (46)-(47) are different from those in Proposition 3 because lenders are now heterogeneous with respect to liquidation costs. The welfare of able borrowers is then:

$$w_A = \mathcal{M}_{AS} v_{AS} + \mathcal{M}_{AF} v_{AF},$$

where the measure of able borrowers with good reputation $\mathcal{M}_{AS}$ and the measure of able borrowers with bad reputation $\mathcal{M}_{AF}$ are given by (3). One can verify that:

$$w_A = \lambda \frac{\beta^2}{1-\beta^2} \left[\mathcal{M}_{AS} \left(r_S \Pi(X-2) - E \right) + \mathcal{M}_{AF} \left(r_F \Pi(X-2) - E \right) \right].$$

Likewise, Bellman equations of mediocre borrowers are:

$$\begin{array}{l} v_{MS} = \beta^2 \left(r_S \pi(X-2) - e + P_{S|e} v_{MS} + P_{F|e} v_{MF} \right) \\ v_{MF} = \beta^2 \left(r_F \pi(X-2) - e + P_{S|e} v_{MS} + P_{F|e} v_{MF} \right) \end{array}, \tag{48}$$

where probability of success $P_{S|e}$ and the probability of failure $P_{F|e}$ conditional on low effort are given by (13). The welfare of mediocre borrowers is:

$$w_M = \mathcal{M}_{MS} v_{MS} + \mathcal{M}_{MF} v_{MF},$$

where the measure of able borrowers with good reputation $\mathcal{M}_{AS}$ and the measure of able borrowers with bad reputation $\mathcal{M}_{AF}$ are given by (3). One can verify that:

$$w_M = (1-\lambda)\frac{\beta^2}{1-\beta^2}\left[\mathcal{M}_{MS}\left(r_S\pi(X-2)-e\right)+\mathcal{M}_{MF}\left(r_F\pi(X-2)-e\right)\right].$$

The total welfare of borrowers is then $w = w_A + w_M$.

The condition that when borrowers randomize all able borrowers work hard is:

$$r_F\Pi(X-2)-E+P_{S|E}v_{AS}+P_{F|E}v_{AF} \geq r_F\pi(X-2)-e+P_{S|e}v_{AS}+P_{F|e}v_{AF},$$

which can be simplified to:

$$(\Pi-\pi)\left[\kappa p a_T+(1-\kappa)\,a_S+\beta^2\,(1-\phi)\,(1-p)^2\,\kappa a_T\Pi\right](X-2)\geq E-e.$$

Assume now that lenders reputation is available, so that borrowers use matching strategy described in Proposition 5. Then, the Bellman equations of both able and mediocre borrowers are identical to those in (45) and (48) except that one-period returns r_S and r_F are replaced by R_S and R_F respectively, where

$$R_S = a_T$$

and

$$R_F = \frac{\frac{\kappa}{2-p}pa_T+(1-\kappa)\,a_S}{\frac{\kappa}{2-p}+1-\kappa}.$$

Consequently, the welfare of able borrowers now is:

$$W_A = \lambda\frac{\beta^2}{1-\beta^2}\left[\mathcal{M}_{AS}\left(R_S\Pi(X-2)-E\right)+\mathcal{M}_{AF}\left(R_F\Pi(X-2)-E\right)\right].$$

and welfare of mediocre borrowers is:

$$W_M = (1-\lambda)\frac{\beta^2}{1-\beta^2}\left[\mathcal{M}_{MS}\left(R_S\pi(X-2)-e\right)+\mathcal{M}_{MF}\left(R_F\pi(X-2)-e\right)\right].$$

Because $R_S > r_S$ and $R_F > r_F$ for all $\kappa \in (0,1)$, $W_A > w_A$ and $W_M > w_M$, so that lenders reputation increases welfare of all borrowers.

The condition that all able borrowers work hard in that case is given by (17) in Proposition 5. One can verify that it is implied by (43), which is not surprising given that lenders reputation increases borrowers payoffs.

Let us now look at lenders profits. Assume first that lenders reputation is not available, so that borrowers randomize. In that case profits of tough lenders are:

$$\kappa[p(1-a_T)(X-2)+(1-p)((L_T-1)\mathcal{M}_F+ \\ \left(\zeta_{G|S}(1-a_T)(X-2)+\zeta_{B|S}(x-2)\right)\mathcal{M}_S)],$$

and profits of soft lenders are:

$$(1-\kappa)\left[p(1-a_S)(X-2)+(1-p)((\zeta_{G|F}(1-a_S)(X-2)+ \\ \zeta_{B|F}(x-2))\mathcal{M}_F+\left(\zeta_{G|S}(1-a_S)(X-2)+\zeta_{B|S}(x-2)\right)\mathcal{M}_S)\right],$$

where the probability that given reputation j of a borrower her project is good, $\zeta_{G|j}$, and the probability that her project is bad, $\zeta_{B|j}$, are given by (3) and (5)-(8).

Assume now that lenders reputation is available. Then, given borrowers matching strategy, profits of tough lenders are:

$$\psi_{TR}\left[p(1-a_T)(X-2)+(1-p)(L_T-1)\right]\Lambda_R+\psi_{TU}[p(1-a_T)(X-2)+ \\ (1-p)\left(\zeta_{G|S}(1-a_T)(X-2)+\zeta_{B|S}(x-2)\right)]\Lambda_U,$$

which simplifies to:

$$\mu_{T|R}\left[p(1-a_T)(X-2)+(1-p)(L_T-1)\right]\mathcal{M}_F+\mu_{T|U}[p(1-a_T)(X-2)+ \\ (1-p)\left(\zeta_{G|S}(1-a_T)(X-2)+\zeta_{B|S}(x-2)\right)]\mathcal{M}_S,$$

where the measure of lenders of type k with reputation n, ψ_{kn}, and the number of borrowers per lender with reputation n, Λ_n, are defined in proof of Proposition 5. Recall that the probability that given soft reputation a lender is tough is:

$$\mu_{T|R}=\frac{\frac{\kappa}{2-p}}{\frac{\kappa}{2-p}+(1-\kappa)}<\kappa,$$

and the probability that given tough reputation a lender is tough is:

$$\mu_{T|U}=1>\kappa.$$

Then, condition (44) implies that:

$$\mathcal{M}_F\equiv(1-\phi)(1-p)\left(\lambda(1-\Pi)+(1-\lambda)(1-\pi)\right)<\mathcal{M}_S.$$

Because given the equilibrium profile of lenders strategies, individual rationality implies that:

$$L_T - 1 < \zeta_{G|S}(1 - a_T)(X - 2) + \zeta_{B|S}(x - 2),$$

one obtains that lenders reputation increases profits of tough lenders.

Likewise, profits of soft lenders are:

$$\mu_{S|R}[p(1 - a_S)(X - 2) + (1 - p)(\zeta_{G|F}(1 - a_S)(X - 2) + \zeta_{B|F}(x - 2))]\mathcal{M}_F + \mu_{S|L}[p(1 - a_S)(X - 2) + (1 - p)(\zeta_{G|S}(1 - a_S)(X - 2) + \zeta_{B|S}(x - 2))]\mathcal{M}_S,$$

where

$$\mu_{S|R} = \frac{1 - \kappa}{\frac{\kappa}{2-p} + (1 - \kappa)} > 1 - \kappa, \tag{49}$$

and

$$\mu_{S|L} = 0 < 1 - \kappa.$$

Then, because given observed low signal the probability that borrower with bad reputation has good project is smaller than the probability that borrower with good reputation has good project:

$$\zeta_{G|F} < \zeta_{G|S},$$

one can see that bringing in lenders reputation decreases profits of soft lenders. ■

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