Favoritism in Repeated Procurement Auction

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Abstract

In this paper, we investigate the interaction between two firms engaged in a repeated procurement relationship modelled as a multiple criteria auction, and a procurement official (agent) whose duty is to decide on a scoring rule.

We find that with favoritism, the procedure selects projects with an extreme design. In the one-shot game, the equilibrium bribe is maximal when the cost of favoritism is zero. As the cost increases, competition for favors weakens.

When the firms meet repeatedly on different markets, the optimal collusive scheme, in the absence of corruption, entails an allocation rule contingent on firms' costs and on the government's preferences. Our main finding is that with corruption a non-contingent allocation rule where firms take turn in winning independently of the true government preferences and the firms' costs is ex-post efficient. The 'environment' adapts to the cartel: in equilibrium, the contract is fine-tailored to the in-turn winner. Favoritism not only increases the gains from collusion, it also solves basic problems of implementation for the cartel.

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1 Introduction

In this paper we investigate the relationship between favoritism and collusion in repeated procurement. A main motivation for the paper is the mounting body of evidence that collusion and corruption often go hand in hand in public procurement.

In France, practitioners and investigators in courts of accounts, competition authorities, and in the judiciary have long been aware of the close links between collusion and corruption in public tenders. The testimony of J. C. Mery provides suggestive evidence of those links (*Le Monde*, September 22 and 23, 2000).[1] A recent judgment in 'Les Yvelinnes' (Cour d'Appel de Versaille, January 2002) provides a vivid illustration as well. According to a judge currently investigating a major corruption case in Paris, there exists in France, almost not a single case of large stake collusion in public procurement without corruption.[2] Beside empirical motivations, there are theoretical motivations for investigating the links between favoritism and collusion. In particular there is a well-known tension facing a cartel between the efficiency goal and incentives related to revelation of private information. A fair amount of attention has been given to the theoretical problem facing a cartel that operates within a (imperfectly or privately observable) environment (e.g. a price cartel on a market whose demand is subjected to shocks cf. Green and Porter 1984, or to shock to costs as in Athey et al. 2002). A central result of our analysis is that with favoritism it is optimal to rely on a very simple rent-sharing rule: a non-contingent in-turn rule so firms take turn to win in a pre-determined manner. As a consequence we find that the cartel is efficient even in the presence of imperfect public information. More strikingly yet, private information about shocks to costs either is revealed at no cost or irrelevant to ex-post efficiency: the cartel

[1] J. C. Mery, a City Hall official, admitted that for ten years (1985-94) he organized and arbitrated collusion in the allocation of most construction and maintenance contracts for the Paris City Hall. In exchange, firms were paying bribes used to finance political parties. The contracts in question were on average very profitable: they generated up to 30 percent profit in an industry that averages 5 percent. Mr. Mery also claimed that he had always managed to allocate the contracts to the lowest price bidder. Both these features suggest that the firms were not competing with each other, but were instead implementing some kind of market sharing agreement.

[2] The case concerns the procurement of a 4.3 billion euros construction market (see *Le Monde*, January 26, 2000).

needs not adapt to the 'environment', the 'environment' adapts to the cartel!

One key characteristics of repeated public procurement is that there exists some uncertainty about the public demand for e.g. construction work. This uncertainty can be expressed as randomness of the current public preferences. In each period the government, the firms and the agent share common priors about the true preferences. But at the beginning of the period the procurement agent observes a private signal of those preferences. His function is to devise and announce the scoring rule that reflects the (current) public preferences. The presence of asymmetric information between the government and its agent implies that the agent has some discretion when deciding the scoring rule. On the other hand, firms have a stake in influencing that decision because a scoring rule that emphasizes an aspect in which a firm has a comparative advantage implies higher expected profit. The conjunction of these two features creates a risk of favoritism i.e. a risk that the agent is bribed to bias the scoring rule in favor of some firm.

We model the procurement procedure as a multiple criteria auction along the lines developed by Che (1993). The true criteria is a stochastic variable with a new realization in each period while the scoring rule is a decision variable of the agent. Before the official auction game, the firms compete in corrupt deals in auction-like procedure. A corrupt deal includes a bribe and a demanded scoring rule.

We first investigate the stage game and show that, when the agent is corrupt, the equilibrium is characterized by favoritism: the scoring rule always fully favors either one of the firms. When the agent's cost of favoritism is zero, the agent collects all the rents. A positive cost of favoritism reduces competition for favors between firms so the auction winner ends up with a strictly positive profit.

In the repeated interaction context, we first show that when the agent is honest, uncertainty about future scoring rule does not prevent efficient collusion. The optimal mechanism entails an allocation rule that is contingent on firms' cost and the announced scoring rule. When the agent is willing to take bribe in exchange for selecting a scoring rule collusion becomes much more profitable. The cartel earns maximal income when the firms take turn in winning in a predetermined manner i.e. this allocation rule depends neither on the firms' costs nor on the true preferences. It also keeps to a minimum the agent's rents. Moreover, the asymmetry induced by this rule between the in-turn firm and the out-of-turn firm also implies that uncertainty about the agent's willingness to take bribe (an instance of imperfect public information) does not induce inefficiencies in the cartel.

The equilibrium allocation pattern is consistent with empirical findings that the winner is the most efficient firm and that its profit often are larger than the average in the branch (30% contra 5%) as in the case with the court case concerned with the series of constructions contracts in Paris.

2 The model

There are two firms characterized by their cost function

$$c\left(q_{At}, q_{Bt}, \theta_i^t\right) = \frac{\theta_i^t q_A^2}{2} + \frac{\left(1-\theta_i^t\right) q_{Bt}}{2},\ i = 1, 2$$

where $\theta_i^t \in \left\{\underline{\theta_i}, \overline{\theta_i}\right\}$; $\underline{\theta_i} = \left(1 - \overline{\theta_i}\right)$ is a cost parameter and q_{At} and q_{Bt} are two quality components.[3] We denote Δ_i the cost differential : $\Delta_i = \overline{\theta_i} - \underline{\theta_i}$. The cost parameters θ_i^t, $i = 1, 2$ is common information among firms but the government and its agent only know their distribution. In each period, there is a new draw of $(\theta_{1t}, \theta_{2t})$ but for simplicity, we assume that the firms always have opposite comparative advantages : $\theta_{iAt} = \underline{\theta_i} \Leftrightarrow \theta_{-iAt} = \overline{\theta_{-i}}$.[4] So when $i = 1, 2$ has a cost advantage in the production of quality A, the other firm has the advantage in the production of quality B :

$$\begin{aligned} c\left(q_{At}, q_{Bt}, \theta_i^t\right) &= \frac{\underline{\theta_i} q_A^2}{2} + \frac{\overline{\theta_i} q_B^2}{2} \\ c\left(q_{At}, q_{Bt}, \theta_{-i}^t\right) &= \frac{\overline{\theta_{-i}} q_A^2}{2} + \frac{\underline{\theta_{-i}}^t q_B^2}{2} \end{aligned}$$

The government derives utility from the project:

[3] An extension to k quality components is developped in sec. 3.1.5

[4] Allowing the firms to have identical cost structure in some periods would not affect the main results of the analysis. The case when in some period, one of the firm is more efficient on all accounts is not covered by the analysis.

$$U_G = \alpha_t q_{At} + (1 - \alpha_t) q_{Bt} - p^t,$$

where p_t is the price paid to the firm and $\alpha_t \in [0, 1]$ is the true preference parameter (social value).[5] It is a random variable, assumed to be uniformly distributed on $[0, 1]$. The government does not know the true α_t. It hires an agent who receives a signal $\sigma(\alpha_t)$ of the true α_t at the beginning of each period. For simplicity we assume that the signal is fully informative. [6]

The Auction rule

The procedure we consider is a first score auction (FSA). At the beginning of each period the agent announces a scoring rule which is a function of both price and quality: $S(q_{it}, p^t)$ where $\mathbf{q}_{it} = (q_{iAt}, q_{iBt})$. The firms simultaneously submit a bid in a sealed envelop $(\mathbf{q}_{it}, p_{it})$. The contract is awarded to the firm whose offer maximizes (among submitted offers) the announced scoring rule:

$$i^{*t} = \arg\max_{i=1,2} S\left(q_{it}, p_i^t\right)$$

subject to a 'reserve score' constraint $S(q_{it}, p_i^t) \geq 0$. The winner is due to deliver the quality (q_{i*At}, q_{i*Bt}) at price p_{i*}^t. In case of tie in scores the project is awarded to the firm whose quality score is highest.

We consider a scoring rule of the type: $S(q_{At}, q_{Bt}, p^t) = s(q_{At}, q_{Bt}) - p^t$ and more precisely we limit attention to

$$S(q_{At}, q_{Bt}, p) = \widehat{\alpha} q_{At} + (1 - \widehat{\alpha}) q_{Bt} - p^t;\ \widehat{\alpha} \in [0, 1]$$

where $\widehat{\alpha}$ is a parameter announced by the agent (see the timing below).

The firm's utility-if-win is

$$U_i = p_i^t - c_i(\mathbf{q}_{it}),\ i = 1, 2.$$

Utility-if-lose is normalized to zero. The game is infinitely repeated with the two same firms but in each period there is a new draw of the parameter α_t. The firms discount future gains

[5] $\alpha = 0$ must be understood as no social value of q_A above a minimal level and similarly for $\alpha = 1$.

[6] We discuss this assumption in Section 5.

with a common factor δ.

Corruption

The agent is opportunistic. He accepts bribes in exchange for announcing a demanded scoring rule. The agent's utility is

$$U_0 = w + b^t - m_t \left(\alpha_t - \widehat{\alpha}_t\right)^2 .$$

where w is a wage that we normalize it to 0, b is the bribe and $m_t \left(\alpha - \widehat{\alpha}\right)^2$, $m^t \in \mathbb{R}^+$ is a term that captures moral and other costs e.g. expected punishment (the government can engage a procedure to find out its true preferences. One motivation for taking the punishment as a function of the distance $\left(\alpha - \widehat{\alpha}\right)^2$, is that the larger the distance to the true preferences the larger presumably the chance that it be detected. The type of the agent i.e. m_t is private information to the agent.[7]

Corruption is modelled as a procedure where the firms compete in corrupt deals. The firms simultaneously and secretly submit a deal offer (b_i, α_i) including a bribe and demanded scoring rule. The bribe is only paid by the official auction's winner if the announced scoring rule corresponds to the one he demanded. We assume that when the agent is honest he disregards the deal offers.

3 Analysis

3.1 The stage game

We start the analysis by investigating the stage game with symmetric firms i.e. $\theta_{1A} = \theta_{2B} = 1 - \theta_{1B} = 1 - \theta_{2A}$ and assuming that the type of the agent is $m \in \{0, \infty\}$ with

$prob\{m = 0\} = p$ known to both firms.

The timing is as follows:

step 1: The agent learns $\sigma(\alpha_t) = \alpha_t$, the firms submits $((b_1, \alpha_1), (b_2, \alpha_2))$

step 2: The agent makes an announcement $a = \widehat{\alpha}$, $\widehat{\alpha} \in [0, 1]$;

[7]When the firms interact repeatedly with the same agent this uncertainty reflects organizational conjuncture e.g. internal anti-corruption campaign.

step 3: The firms submit $(\mathbf{q}_{it}, p_i)$;

step 4: The winner is chosen as the firm whose offer maximizes the announced scoring rule. If the agent is corrupt $(m = 0)$, the winner pays the bribe he offered whenever $\widehat{\alpha} = \alpha_{i^*}$. Otherwise no bribe is paid.

Formally, a strategy of a firm is collection of functions (B, D, Q_A, Q_B, P) that determines a deal offer (b_1, α_1): $b_i = B(p, \theta_i)$, $\alpha_i = D(p, \theta_i)$ and a contract offer (q_A, q_B, p_i): $q_A = Q_A(\widehat{\alpha}, \theta_i); q_B = Q_B(\widehat{\alpha}, \theta_i)$, $p_i = P_i(\widehat{\alpha}, \theta_i, b_i)$. A strategy for the agent is a function that determines his announcement: $\widehat{\alpha} = A(b_1, b_2, \alpha, m)$. We call this game $\Gamma 2$.

There is a unique symmetric Bayesian equilibrium of the game $\Gamma 2$ characterized by

Proposition 1 *i.* $b_1^* = b_2^* = \frac{\Delta}{2\underline{\theta}\overline{\theta}}$;

ii. Competition in price;

iii. $q_J^*(\theta_i) = \arg\max\left[\widehat{\alpha} q_A + 1 - \widehat{\alpha} q_B - c(q_J, \theta_i)\right]$, $i = 1, 2$, $J = A, B$,

vi When $m = \infty$ *we have design efficiency and no bribe paid:* $\widehat{\alpha}^* = \alpha$, *when* $m = 0$, $\widehat{\alpha}^* \in \{0, 1\}$, $q_{i^*} \in \left\{\left(\frac{1}{\underline{\theta}}, 0\right), \left(0, \frac{1}{\underline{\theta}}\right)\right\}$, *the winner pays a bribe* $b^* = \frac{\Delta}{2\underline{\theta}\overline{\theta}}$.

All proofs are gathered in the appendix

We first note that the firms' equilibrium strategy does not on p the probability that the agent is of the corrupt type. By assumption, there is no loss in offering a corrupt deal to the honest agent. He simply turns it down.

When the agent is honest, he announces the true scoring rule and no bribe is paid. We know from Che (1993) that the investigated scoring rule induces a problem separable in qualities and price. For any $a = \widehat{\alpha}$ the equilibrium quality values are given by

$$\begin{aligned} q_J^*(\theta_i) &= \arg\max s(q_J) - c(q_J, \theta_i), \ j = A, B \\ q_A^*(\theta_i) &= \frac{\widehat{\alpha}}{\theta_i^t}, \ q_B^*(\theta_i) = \frac{(1 - \widehat{\alpha})}{(1 - \theta_i)}, i = 1, 2. \end{aligned}$$

When the agent is honest $\widehat{\alpha}^* = \alpha$ the firm that has the cost advantage for the socially most valuable quality component wins. The price paid is the price that equals the winning firm's score with the second score. For $\alpha > 1/2$, the winning firm's payoff is $\pi_{i^*} = p_{i^*} - c\left(q_{i^*}, \theta_i^t\right)$:

$$\pi_1 = \frac{1}{2}(2\widehat{\alpha} - 1)\frac{\Delta}{\underline{\theta}\overline{\theta}}. \tag{1}$$

When the agent is corrupt $\widehat{\alpha}^* \in \{0,1\}$. The selected scoring rule is the one that maximizes the winners' rents. In the official auction the firms compete in price. By symmetry competition in bribes dissipates the rents so the corrupt agent captures them all. In proposition 2 bellow we show that positive profit-if-win appears as soon as $m > 0$.

3.1.1 Costly corruption: $0 < m < \infty$

The next proposition extends the result of proposition 1 to case when the cost of favoritism is known to all players and varies from zero to infinity. Result 1.ii and 1.iii carry over immediately. Our main results are

Proposition 2 *For any level of m there is a symmetric Nash-Bayesian equilibrium. The equilibrium scoring rule always is extreme $\widehat{\alpha}^* \in \{0,1\}$. We have $b_1^* = b_2^* = b^*(m)$:*

i. for $m \in \left]0, \frac{4}{5}b_{\max}\right)$, $b^* = b_{\max} - m$, where $b_{\max} = \frac{\Delta}{2\underline{\theta}\overline{\theta}}$;

ii. for $m \in [\frac{4}{5}b_{\max}, \frac{4}{3}b_{\max})$, $b^* = m/4$;

iii. for $m \in [0, \frac{4}{3}b_{\max})$ the agent announces $\hat{\alpha}(\alpha) = \begin{cases} 0, & \alpha < 1/2 \\ 1, & \alpha > 1/2 \end{cases}$;

vi. $m \in [\frac{4}{3}b_{\max}, +\infty)$: $b^* = \frac{4}{9}\frac{b_{\max}^2}{m}$ and the agent announces $\hat{\alpha}(\alpha) = \begin{cases} 0, & \alpha < \frac{2}{3}\frac{b_{\max}}{m} \\ 1, & \alpha > 1 - \frac{2}{3}\frac{b_{\max}}{m} \\ \alpha, & \alpha \in \left[1 - \frac{2}{3}\frac{b_{\max}}{m}, \frac{2}{3}\frac{b_{\max}}{m}\right] \end{cases}$

.

Unlike in the zero cost case, a positive m creates a situation where there is a continuous trade-off between the size of the bribe and the probability for winning: given b_{-i}^* a marginal decrease in b_i (away from its equilibrium value) only marginally reduces the interval of values of the true α for which the agents chooses to favor firm i. Therefore, in equilibrium the firms earn strictly positive rents. Three regions can be distinguished. In the first region $m \in [0, 4b_{\max}/5]$, a strictly positive m effectively reduces competition for favors between firms. The equilibrium bribe is continuously decreasing in m.

In the second region, $m \in]4b_{\max}/5, 4b_{\max}/3]$, the need to compensate the agent for his cost contributes to determining the equilibrium bribe so the equilibrium bribe is rising in m. When m grows beyond $4b_{\max}/3$, bribing the agent when the true score is in the neighborhood of

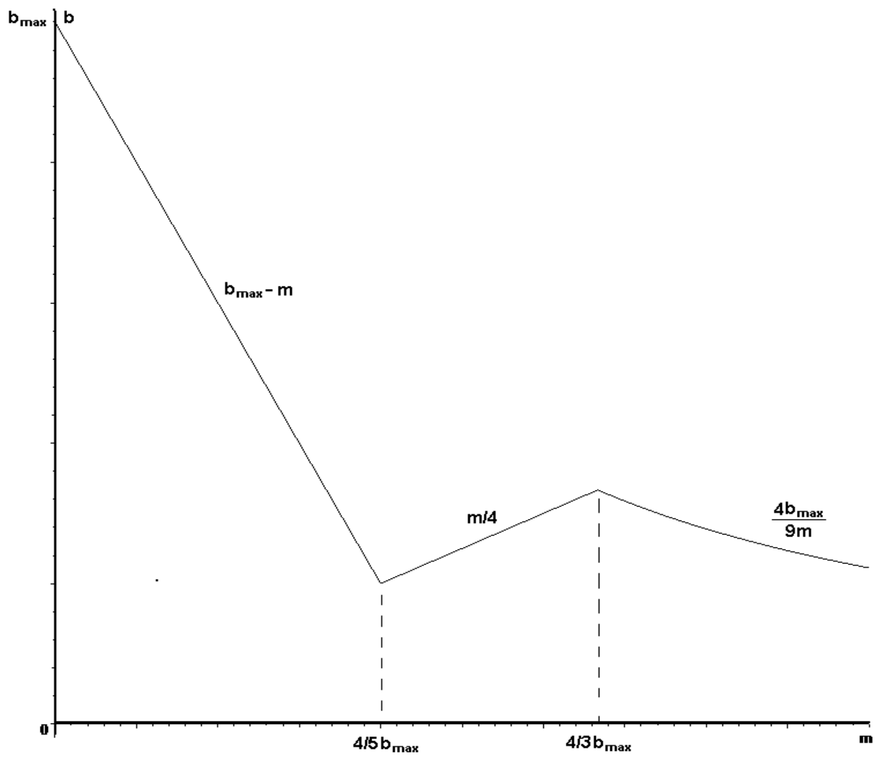

Figure 1:

1/2 becomes very expensive. So a no favoritism region appears in the neighborhood of 1/2. As the cost m increase so does the region where favoritism effectively is eliminated.

3.1.2 More than 2 firms and 2 quality components

In this section we show how the results in proposition 1 extend to a more general setting with $n-$firms, $k-$quality components. There are n firms indexed i and k quality components indexed j. Let $\theta_i = (\theta_{i1}, ...\theta_{ik})$ denote firm i's technology with $\theta_{ij} \in \{\underline{\theta}, \overline{\theta}\}$, $\forall i = 1, .., n$ and $j = 1, ..., k$. The auction scoring rule is given

$$S(\mathbf{q}, p) = \widehat{\alpha}_1 q_1 + \widehat{\alpha}_2 q_2 + ... + \left(1 - \sum_{j=1}^{k-1} \widehat{\alpha_j}\right) q_k - p^t;\ \widehat{\alpha_j} \geq 0 \forall j = 1...k, \sum_{j=1}^{k-1} \widehat{\alpha_j} \leq 1$$

where $\mathbf{q} = (q_1, ...q_k)$. Assume further that for all i we have that $\theta_{ij} = \underline{\theta}$ for *at most* one j i.e. a firm has a comparative advantage in at most one component.[8] We denote I the set of components q_j such that $\theta_{ij} = \underline{\theta}$ for *at most one firm* i. To simplify the presentation of the results we focus on the case when the agent is corrupt: $m = 0$. Denote this game Γn. Then if $|I| \geq 2$

Proposition 3 *Γn has a Bayesian equilibrium characterized by*

i. favoritism: $\widehat{\alpha} = (\widehat{\alpha_1}..., \widehat{\alpha_k})\,; \widehat{\alpha_j} \in \{0, 1\}$ *and* $\sum_{j=1}^{k} \widehat{\alpha_j} = 1$,

ii. $b_j^* = \frac{\Delta}{2\underline{\theta}\overline{\theta}}$, *for* $\forall q_j \in I$, $b_m^* = 0$, $q_m \notin I$

iii. price competition;

vi. Socially inefficient design.

The separability in the determination of p and q_{ij}^* and in the determination of the q_{ij}^* for any firms implies that the equilibrium of Γn is a straightforward extension of the equilibrium of $\Gamma 2$ with $|I|$ firms and $|I|$ components. We first note that $|I| \geq 2$ secures that there will be competition for the scoring rule between at least two firms. Since each firm has at most advantage in one component j, it offers a bribe for $\widehat{\alpha_j} = 1$. Next, since the firms are symmetric the value of obtaining a favorable scoring rule is the same for all and is given by the associated profit-if-win:$\frac{\Delta}{2\underline{\theta}\overline{\theta}}$. A number of $|I|$ firms participates to the bribe auction. With probability$\frac{1}{|I|}$ the agent favors anyone of the bribe bidders. As result we have favoritism in equilibrium with the announced scoring rule giving weight to one single component.

[8] We briefly below address the case when a firm may be efficient at producing several quality components.

4 Repeated interaction: collusion

We now proceed to investigate a situation when the firms meet repeatedly but on public markets administered by different branches of the public administration e.g. different local government. The game $\Gamma 2$ above is infinitely repeated with the same two firms but with different (short-run) agents.

Information assumption

In each period there is a new draw of $(\theta_{1t}, \theta_{2t},\ \alpha^t)$. Most of the results pertain to the symmetric information case with perfectly negatively correlated costs: $\theta_{1A} = \theta_{2B}$. After each period, the submitted contracts offers are publicly observed. The deal offers remain private information to the involved parties. The true value of α is never reveled. The public history of the game up to period t is denoted H_t.

The formal description of the repeated game is the same as that of the stage game when indexing all arguments and functions with time and including the argument H_{t-1} to all the functions except $\widehat{\alpha_t} = A(b_{1t}, b_{2t}, \alpha_t, m_t)$ since the agent is short-run.

The next proposition characterizes the optimal collusive scheme under the assumption that transfers between firms are precluded.

Proposition 4 *i. For $m = \infty$, there exists $\delta \leq 1$, such that collusion is a Nash equilibrium of the repeated game. Any optimal collusive scheme entails an allocation rule contingent on the stochastic government preferences: for $\alpha^t > \widetilde{\alpha}(\alpha^t \leq \widetilde{\alpha})$ the firm with advantage in quality A(B) is designated as the in-turn winner.*

ii. For $m = 0$, there exists $\delta \leq 1$ such that collusion is a Nash equilibrium of the repeated game. Ex-post efficiency is achieved by a collusive scheme that entails a non-contingent (pre-determined) in-turn allocation rule and the scoring rule is fine-tailored to the in-turn winner. The agent earns a bribe $b^* = \varepsilon$.

Proposition 4.i shows that when the agent is honest any optimal incentive scheme selects the winner as a function of the stochastic scoring rule. This follows from the fact that the cartel's income is maximized when the firm that has the comparative advantage implements the contract. In the case when firm are symmetric i.e $\theta_{1A(1B)} = \theta_{2B(2A)}$, the cutting point is $\widetilde{\alpha} = 0.5$. If instead we had $\theta_{1A} = 0.1,\ \theta_{1B} = 0.9$ and $\theta_{2A} = 0.6,\ \theta_{1B} = 0.4$, the cutting value that maximizes the cartel's gain would be $\widetilde{\alpha} \approx 0.3$. Hence, the contingent allocation rule is a function of both on the true alpha and on the firms' cost structure.

When the agent is corrupt, a main concern for the cartel is to contain competition in bribes which is very costly as we know from proposition 1. Proposition 4 ii shows that competition for favors can be eliminated by opting for a simple non-contingent in-turn allocation rule. In each period, the agent is bribed by the pre-determined in-turn-winner to fine-tailor the scoring rule to him which maximizes the cartel's payoff. The out-of-turn firm may deviate and (unobserved) bribes the agent to announce a rule favorable to itself. This is immediately detected however - the scoring rule is not the equilibrium one - and punished by reverting to the equilibrium of $\Gamma 2$ which yields zero payoff to the firms from next period on.This explains why the bribe can be kept to a minimum. Moreover with favoritism the gains from collusion are higher than with an honest agent: the scoring rule is fine-tailored to maximize the winner's profit. And the threat payoffs are lower than in the absence of corruption because competition in bribes dissipates the rents.

Proposition 4 capture the central result of this paper. In the first part of proposition, it is established that efficient collusion requires a contingent allocation rule: the cartel adapts to the environment including the random public preferences and the cost structure. While in the second part, we show that with favoritism ex-post efficiency can be achieved with an allocation rule that is not contingent on either cost nor the true alpha. The remaining of the analysis aims at demonstrating the validity of this central result when modifying the model along various lines.

In the equilibrium depicted in proposition 4 ii. the cartel's rents are maximized by an allocation rule independent of the true alpha. This is not surprising since with $m = 0$ none of the players cares about the true preferences. The next proposition shows that the result in $4ii$ holds true over some interval including strictly positive m.

Proposition 5 *For* $m \in \left[0, \frac{1}{4\theta}\right]$, *the pre-determined in-turn rule is optimal. As* m *grows larger the firms maybe better off in a collusive mechanism with a contingent allocation rule.*

Proposition 5 shows that the non-contingent collusive scheme achieves ex-post efficiency even when corruption is costly to the agent. Ex-post efficiency pertains to the cartel and means that in equilibrium the winner is the most efficient firm relative to the current the scoring rule while the bribe is minimized.

The results in proposition 4 and 5 have been establish in the context of firms' symmetric information. We next show how the result in 4.ii extends to an asymmetric information setting.

4.0.3 Asymmetric information about costs

Consider a model with k-components as in section 3.2.2 but with only two firms. We call this game $\Gamma 2(k)$. This model provides a suitable setting for deriving and interpreting our asymmetric information results. In each period there is a draw of $\boldsymbol{\theta}_i^t = \left(\theta_{i1}^t, ., \theta_{il-1}^t, \theta_{il+1}^t .., \theta_{ik}^t\right), \ i = 1, 2$ where $\theta_{ij}^t \in \left\{\underline{\theta}, \overline{\theta}\right\} \ j = 1, ...k, \ i = 1, 2$. With probability p we have $\theta_{ij} = \underline{\theta}$and with complementary probability, $(1-p), \ \theta_{ij}^t = \overline{\theta}$. We assume that both firms have one component $\theta_{il}^t = \theta_{-im}^t = \underline{\theta}$ with probability 1. That is in each period each firm is efficient in the production of .at least one component. The firms only observes its own vector of cost.[9] In the corruption game the deal offers are richer than in the basic model, we let firm offer a menu of pairs including a bribe and scoring rules. We also assume that the agent receive no bribe when the scoring rule induces fierce competition i.e. when both firms are efficient at (all) the valued component(s).

The stage game is as in section 3.2.2 with $n = 2$ and asymmetric information as above specified. In the appendix, we show that there is no gain in asking for complicated scoring rules instead the single peaked scoring rule is optimal. In equilibrium, the firm offer a menu of bribes and single peaked scoring rules one for each component they are efficient at producing. The equilibrium bribe is the same across the menu and for both agent. It entails zero rents when favoritism occurs and the competitive payoff otherwise i.e. when firms tie on all components.

Proposition 6 *There exists $\delta < 1$, such that collusion is a Nash equilibrium of the repeated*

[9] A richer model with each firm being represented by a vector of probability $(p_{i1}, ..., p_{ik})$ generates the same qualitative results.

game. Optimal collusion can be achieved with the simple scheme where the firms take turn in a pre-determined manner.

Proposition 6 shows that favoritism eliminates the incentive issues related to the revelation of private information about costs.[10] The in-turn firm has an incentive to demand the scoring rule that maximizes its gains which reveals all the needed information. Ex-post efficiency is secured provided only that each firm has an advantage in some component over the other which is a reasonable assumption when the number of components is large enough.

The mechanism in proposition 6 secures ex-post efficiency. In equilibrium the winner is the most efficient firm given the scoring rule. We have ex-ante efficiency when the scoring rule is optimal i.e. favors the least cost component(s) among all components *and* all the firms and the winner is the most efficient firm given the scoring rule. In the case when $\theta_{ij}^t \in \{\underline{\theta}, \overline{\theta}\}$ $j = 1, ...k,$ $i = 1, 2$ ex-post and ex-ante efficiency coincide. For $\underline{\theta}_{ij} \neq \underline{\theta}_{ik}$ this is not true anymore and ex-ante efficiency may revive the old tension between incentives and efficiency.

The simple allocation rule has an additional virtue:

Remark 1 *The efficiency result in proposition 4 ii carries over to the case when the firms face some uncertainty about the agent' type.*

To see this consider the case when the agent is honest (or subject to internal anti-corruption controls) and announces the true alpha with some probability p. This implies that the out-of-turn firm wins with probability $p/2$. So the announced scoring rule is an imperfect signal of the firms' bribing behavior. However, efficiency can here be obtained in a scheme that exploits the asymmetry of the non-contingent in-turn-rule. This contrast, with typical imperfect public information situations (e.g. Green and Porter (1984) or Radner et al. (1986)). Indeed, in each period we only need to secure the incentive of a single player: the out-of-turn firm. For a discount factor sufficiently close to 1, this can be done if when a

[10] Generally, providing incentives to reveal private information is costly. For discount factors strictly smaller than unity, the equilibrium is bounded away from efficiency. Either the mechanism falls short of full sorting implying that we do not have ex-post efficiency. Or we have costly in-equilibrium punishment.

firm wins out-of-turn, its turn is passed for a (number of) period(s). In equilibrium, no firm defects, yet both firms are punished with some probability but at no cost for the cartel.[11]

Proposition 4. reveals a many sided complementarity between favoritism and collusion. It also reveals a conflict of interest. The agent would clearly prefer that there be no collusion so he can earn the rents associated with favoritism under competition (see proposition 1.iv).

5 Discussion and Policy implications

The analysis shows that incentives in favoritism and collusion are intimately linked. A major issue and source of conflicts for real life cartels is the tension between flexibility to adapt to imperfectly or privately observed variations in the environment and the incentive constraints. Our most striking result is that, under some reasonable assumptions favoritism effectively eliminates the tension by adapting the environment to the cartel's needs. Favoritism increases the gains of the cartel as the equilibrium scoring rule maximizes the winner's rent. Hence, favoritism facilitates collusion in many ways.

The allocation pattern emerging from the analysis: in-turn rule that allocates the contract to the most efficient firm while generating large profits is very close to the patterns observed in Paris hall case mentioned in the introduction. When reviewing the cases when agents have been convicted for favoritism, it appears strikingly clear that the cost of favoritism is very low. The only instances of conviction for favoritism in France, pertain to cases where the agent explicitly required a technology that only one single firm could offer.(references to be added). Interestingly people have argued that the fact that the contract were allocated to the most efficient firm, was an indication that there was no collusion. The present analysis shows that it is sufficient that a firm has an advantage in some component to obtain this outcome in a collusive equilibrium with favoritism.

Policy implications The first and most central policy implication is that the risks of collusion and favoritism must be addressed simultaneously. In particular, an institutional

[11] When the discount factor is low, we must bec careful that the scheme does not create incentives for the in-turn firm to trigger a punishment against the out-of-turn firm in order to secure itself the in-turn position for many periods.

structure that leaves the investigation of collusion to Competition Authorities and that of corruption to criminal courts makes efficient prosecution very difficult. The results also provides some insight into the effectiveness of some much advocated anti-corruption policies.

Controls

Our results in proposition 2 show that in a one-shot situation, over a significant interval, an increase in m i.e. stricter controls and/or more severe punishments, has no effect on the cost of favoritism to society. It only affects the allocation of rents between the firms and the agent. It only makes favoritism less expensive for firms. As the cost increases beyond that interval, the occurrence of favoritism does decrease - but only in the region where the quality of different components is near to equally valued. With additional increases of those costs the no-corruption region grows and eventually favoritism is eradicated.

In the repeated context, an increase in m increases the cost of favoritism but over a significant interval, it has no effect on social efficiency as the non-contingent rule remains optimal .

Anti-corruption campaign

A widely spread idea is that random (imperfectly observable e.g. secrete or internal) anti-corruption campaigns can disrupt cooperation and/or induces costly inefficiencies. The argument is that it leads to behavior that looks like a deviation but is not while their possible occurrence can be exploited to cover true defections. A surprise of this analysis is that uncertainty about the agent's type which can capture random anti-corruption campaign does not disrupt the cartel, full efficiency can be preserved. Of course during the campaign we have no favoritism but it is back as soon as the campaign stops. Again this follows from exploiting the asymmetry of the non-contingent in-turn rule.

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A Proof of proposition 1

Proof:

At *step 3* For any $a = \widehat{\alpha}$, the firms make their bid as follows. We know from Che (1993) that the investigated scoring rule induces a problem separable in qualities and price. When $a = \widehat{\alpha}$ the equilibrium quality values are given by

$$\begin{aligned} q_J^*(\theta_i) &= \arg\max s(q_J) - c(q_J, \theta_i)\,,, \;\; j = A, B \\ q_A^*(\theta_i) &= \frac{\widehat{\alpha}}{\theta_i^t},\; q_B^*(\theta_i) = \frac{(1-\widehat{\alpha})}{(1-\theta_i)}, i = 1, 2 \end{aligned}$$

Let $q_i^* = (q_A^*(\theta_i), q_B^*(\theta_i))$ and $\theta_1 = \underline{\theta},\; \theta_2 = \overline{\theta}$.

We first derive the optimal price bids in the absence of corruption (no stage 1) for $\widehat{\alpha} < \frac{1}{2}$. We are in a complete information context. By a standard argument, firm 2 bids the lowest price that just secures non-negative profit

$$p_2^* = c(q_2^*, \theta_2) = \frac{\widehat{\alpha}^2}{2\overline{\theta}} + \frac{(1-\widehat{\alpha})^2}{2\underline{\theta}} \tag{2}$$

Firm 1's best response is given by the second score so $S(q_1^*, p_1^*) = S(q_2^*, p_2^*)$:

$$p_1^* = c(q_2^*, \theta_2) + (s(q_1^*) - s(q_2^*)) = p_2^* + \frac{\Delta}{\underline{\theta}\overline{\theta}}(2\widehat{\alpha} - 1) \tag{3}$$

We call these bids the competitive bids.

The presence of corruption introduces an element of asymmetric information. For any given scoring rule that favors the opponent $-i$, firm i does not in general know the true total cost of $-i$ i.e. it does not know whether i will have to pay a bribe or not. We propose that disadvantaged firm (facing a disadvantageous scoring rule) always bid competitively i.e. as if there was no corruption. Equivalently it expects that firm $-i$'s bribe is not so large that it induces a negative profit if win against a competitive bid. We show below that this is the only Bayesian equilibrium.

We first depict the equilibrium associated with this strategy for the disadvantaged firm. Assume $\widehat{\alpha} > \frac{1}{2}$ so firm 1 has the comparative advantages. Then by assumption firm 2's price bid is given by (2) and firm 1's best response is given by (2). Assume instead that

$p_2 = c(q_2^*, \theta_2) + \Delta > p_2^*$ in which case firm1's best response is $p_1 = p_2 + \frac{\Delta}{\underline{\theta}\overline{\theta}}(2\widehat{\alpha} - 1) > p_1^*$ which secures win with the highest profit. But firm 2 could increase its payoff by lowering its price and win. As long as the lowest bound on the support of firm 1's score associated with positive profit is above the highest score achievable by firm 2, the bids are the competitive bids. When so is not the case e.g. b_1 is so large that $p_1 = c(q_1^*, \theta_1) + b_1 > c(q_2^*, \theta_2)$, firm 2 wins when offering p_2^*. We below show that this cannot be part of an equilibrium of the whole game.

At *step 2* the agent chooses his announcement. When $m = \infty$ he never announces $\widehat{\alpha} \neq \alpha$. So $\widehat{\alpha}^*(b_1, b_2) = \alpha \ \ \forall b_1, b_2$.

When $m = 0$ the agent chooses $\widehat{\alpha}$ to maximize his payoff which is equivalent to the bribe. Denote $\widehat{i} = \arg\max\{b_1, b_2\}$ and $i^* = \arg\max\{S(q_1^*, p_1^*), S(q_2^*, p_2^*)\}$. We propose that

- For $i^*(\alpha_i) = i, \ i = 1, 2,$ the agent announces $\widehat{\alpha}^* = \alpha_{\widehat{i}}$. In case of ties he chooses each with equal probability.

- For $i^*(\alpha_i) \neq i, \ i = 1, 2, \ \widehat{\alpha}^* = \alpha$ and for

- $i^*(\alpha_i) \neq i$, but $i^*(\alpha_j) = j, \ \widehat{\alpha}^* = \alpha_j$.

In the first case both firms make bribe deal offers with a scoring rule that secures win. The agent chooses the scoring rule so the highest bribe bidder wins. In the second case the bribe deal do not secure gain so the agent can't hope to collect a bribe from the favored firm. The agent then chooses to announce the true alpha. Finally there may be a case when one of the firms makes an error. The agent then favor the firm that made a consistent offer since he can't collect a bribe from the other anyway.

At *step 1* the firms make their corrupt deal offers. We first assume the firms know that the agent is corrupt. Assume that the winning firm's expects the disadvantaged firm to bid competitively it payoff is $\pi_{i^*} = p_{i^*} - c\left(q_{i^*}, \theta_i^t\right)$

$$\begin{aligned} \pi_1 &= \frac{\widehat{\alpha}^2}{2\overline{\theta}} + \frac{(1-\widehat{\alpha})^2}{2\underline{\theta}} + \frac{\Delta}{\underline{\theta}\overline{\theta}}(2\widehat{\alpha} - 1) - \frac{\widehat{\alpha}^2}{2\underline{\theta}} - \frac{(1-\widehat{\alpha})^2}{2\overline{\theta}} \\ &= \frac{1}{2}(2\widehat{\alpha} - 1)\frac{\Delta}{\underline{\theta}\overline{\theta}}. \end{aligned} \tag{4}$$

We note that

$$\frac{\partial \pi_1}{\partial \alpha} = \frac{\Delta}{\underline{\theta}\overline{\theta}} > 0, \tag{5}$$

So firm 1(2) maximizes profit-if-win with $\alpha_1(\alpha_2) = 1(0)$ and $\alpha_1^*(\alpha_2^*) = 1(0)$.Next since the firms are symmetric we have $\pi_1(\alpha = 1) = \pi_2(\alpha = 0) = \frac{\Delta}{2\underline{\theta}\overline{\theta}}$. The bribe is the price paid to win the auction which has the same value for both. We shall assume that when it is payoff indifferent a firm prefers strictly to have the contract. Then we must have $b_1^* = b_2^* = \frac{\Delta}{2\underline{\theta}\overline{\theta}}$. Assume instead that $b_1 = b_1^* + \varepsilon$ firm one wins the bribe auction for sure. The agent announces $\widehat{\alpha} = 1$. We first show that $p_2 = p_2^*$ which implies that firm 1 loses the official auction. When it observes $\widehat{\alpha} = 1$ firm 2 knows at step 3 that either $b_1^* = b_2^*$ and the agent selected 1 by chance in which case $p_2 = p_2^*$ is an optimal strategy. Or $b_1 > b_1^*$ in which case $p_2 = p_2^*$ secures win ($b_1^* + \varepsilon$ is sufficient to win in bribes). With $p_2 < p_2^*$ firm 2 can never win the official auction since by assumption the agent breaks ties in score to the advantage of the higher quality proposal. Hence $p_2 = p_2^*$ is a dominating strategy and by symmetry $b_1^* = b_2^* = \frac{\Delta}{2\underline{\theta}\overline{\theta}}$ when $m = 0$.

Finally since offering a bribe to an honest agent is inconsequential, the firms make their bribe offers as-if the agent was corrupt. Hence expectation of competitive bidding are indeed the only rational expectation and the depicted equilibrium is unique. *End proof*

B Proof of proposition 2

Since we are looking for symmetric equilibria with $m > 0$ let us do all analysis for a firm which has comparative advantage for high values of α or which has a lower value of parameter $\theta = \underline{\theta}$. For simplicity let us assume that firm i has type θ and, hence, the firm j has type $1 - \theta$. This firm we call as i and her opponent as j.

Let us suppose that we are checking an equilibrium where firms j and i submit $(b^*, 0)$ and $(b^*, 1)$ at step 1 of stage game. At step 2 the equilibrium behavior of agent is to make the following announcement[12]

$$\hat{\alpha}(\alpha) = \begin{cases} 0, & \alpha < \min\left\{\frac{1}{2}, \sqrt{\frac{b^*}{m}}\right\} \\ 1, & \alpha > \max\left\{\frac{1}{2}, 1 - \sqrt{\frac{b^*}{m}}\right\} \\ \alpha, & \text{o/w} \end{cases}.$$

At steps 3 and 4 when $\hat{\alpha} > 1/2$ firm i wins an auction with bid $(q_{Ait}, q_{Bit}, p_i) = \left(\frac{\hat{\alpha}}{\theta}, \frac{1-\hat{\alpha}}{1-\theta}, \frac{\hat{\alpha}^2+(2\hat{\alpha}-1)(1-3\theta)}{2(1-\theta)\theta}\right)$ and when $\hat{\alpha} < 1/2$ firm j wins an auction with bid $(q_{Ajt}, q_{Bjt}, p_j) = \left(\frac{\hat{\alpha}}{(1-\theta)}, \frac{1-\hat{\alpha}}{\theta}, \frac{\hat{\alpha}^2+(1-2\hat{\alpha})(2-3\theta)}{2(1-\theta)\theta}\right)$. Also at step 4 when $\hat{\alpha} \in \{0, 1\}$ the agent receives bribe of size b^* from a corresponding firm.

Now we check that the bribe level given by (**??**) constitutes the equilibrium by checking all possible deviations. We can separate three different situations. Each of them we will analyze separately.

When $\hat{\alpha}$ is announce to be higher than 1/2 the firm i receives profit

$$\Pi_i(\hat{\alpha}) = (2\hat{\alpha} - 1)b_{\max} \tag{6}$$

Situation 1: $0 < m < b_{\max}/2$. Here the bribe level is equal to $b^*(m) = b_{\max} - m$ and firm i collects positive profit. Now let firm i asks for $\alpha > 1/2$ and $b < (2\alpha - 1)b_{\max}$ (any higher level of bribe gives non positive profit to firm i). Firm i wins a share of cases $s(\alpha, b)$ according to

$$s(\alpha, b) = 1 - \frac{m\alpha^2 + b_{\max} - m - b}{2m\alpha}.$$

Notice that the above expression works for values of b which yields values $s(\alpha, b)$ within interval $[0, 1]$.

All of this gives the following formula for expected profit $\pi_i(\alpha, b)$ of firm i

$$\pi_i(\alpha, b) = [\Pi_i(\alpha) - b]\, s(\alpha, b) = [(2\alpha - 1)b_{\max} - b]\left[1 - \frac{m\alpha^2 + b_{\max} - m - b}{2m\alpha}\right].$$

The function $\pi_i(\alpha, b)$ is quadratic in b and takes unconditional maximum at

$$\tilde{b}(\alpha, m) = (b_{\max} - m)\alpha - \frac{(1 - \alpha^2)m}{2}.$$

When $\tilde{b}(\alpha, m) > (2\alpha - 1)b_{\max}$ the maximum achievable expected profit is zero and when $\tilde{b}(\alpha, m) > (2\alpha - 1)b_{\max}$ it is positive. Now let us study the function of difference between

[12] Notice that we do not need to specify an optimal behavior for $\alpha = 1/2$ because this event happens with zero probability.

$\tilde{b}(\alpha, m)$ and $(2\alpha - 1)b_{\max}$

$$\omega(\alpha, m) = \tilde{b}(\alpha, m) - (2\alpha - 1)b_{\max}.$$

The partial derivative of $\omega(\alpha, m)$ with respect to α is equal to $-b_{\max} - (1-\alpha)m < 0$. This means that this difference is decreasing in α. Also the difference is positive for $\alpha = 1/2$ because $\tilde{b}(\alpha = 1/2, m) = b_{\max}/2 - 7m/8 > 0$ and it is negative for $\alpha = 1$ because $\tilde{b}(\alpha = 1, m) = b_{\max} - m < b_{\max}$. Hence the expected profit can be strictly positive for $\alpha \in (\alpha^*(m), 1]$, where $\alpha^*(m) \in (1/2, 1)$ is a solution of system $\omega(\alpha^*(m), m) = 0$.

Now let us introduce the following function which stands for the highest possible profit for given $\alpha \in [\alpha^*(m), 1]$

$$\psi(\alpha, m) = \pi_i(\alpha, \tilde{b}(\alpha, m)) = \frac{(2b_{\max} - 2\alpha b_{\max} + m\alpha^2 - m - m\alpha)^2}{8m\alpha}$$

We know that $\psi(\alpha^*(m), m) = 0$ and clearly it is nonnegative on segment $[\alpha^*(m), 1]$. Now we need to find such α for which the function $\psi(\alpha, m)$ reaches the highest value. Let us estimate the second derivative of the above function

$$\frac{\partial^2 \psi(\alpha, m)}{\partial \alpha^2} = \frac{4b_{\max}^2 - 4mb_{\max} + m^2 + 3\alpha^4 m^2 - 4\alpha^3 m^2 - 4\alpha^3 m b_{\max}}{4m\alpha^3}$$

One can check that it is strictly positive on $\alpha \in [1/2, 1]$. Given $\psi(1, m) = m/2$ and $\partial\psi(1, m)/\partial\alpha = b_{\max} - m/2$ we get that the maximum of expected profit of firm i is reached under choice $\alpha = 1$ and $b = \tilde{b}(\alpha = 1, m) = b_{\max} - m$. This finishes the proof for situation $0 < m < b_{\max}/2$.

C Proof of proposition 3

Most of the reasoning carries over immediately from the proof of proposition 1. We note that firm i with $\theta_j = \underline{\theta}$, and $\theta_{-j} = \overline{\theta}$, maximizes its profit with a scoring rule $\alpha^j = (\alpha_1, ..., \alpha_n)\,; \alpha_i = 1$ $i = j$ and $\alpha_i = 0$ for $i \neq j$. Firm i's profit is $\pi_i\left(\alpha^J\right) = \frac{\Delta}{2\underline{\theta}\overline{\theta}}$ if $\theta_{kj} = \overline{\theta}$ for all firms $k \neq i$. If $\theta_j = \overline{\theta}$ for more than one firm. Given the optimal choice of quality levels we have that for $\alpha = \alpha^j$, $q_{kj}^* = \frac{1}{\underline{\theta}}$ and $q_{k-j}^* = 0$ for all firms k; $\theta_{kj} = \underline{\theta}$. And we are dealing with a symmetric first price auction between identical firms so the profit if win is zero.

A number $|I|$ of firms are alone to be efficient in a quality component at step 1, they all submit a bribe deal $\left(\alpha^k, \frac{\Delta}{2\underline{\theta}\overline{\theta}}\right)$ where α^k is a vector of zero except for the one dimension of quality in which they are efficient. At step 2 the agent selects randomly among the demanded α^k.

D Proof of proposition 4

4.i In each period $\Gamma 2$ is repeated with $m = \infty$. When the history of the game H_{t-1} is characterized by the winning firm being the one that has the comparative advantage and the winning score being the reserve score, we say $H_{t-1} = H^*$. We propose the following strategies for the players (We skip the corrupt deal submission step 1 since it is known that the agent is honest.):

i. If $H_{t-1} = H^*$ and the announced scoring rule gives advantage to firm i, then firm i submits an offer including the efficient qualities and a price so that its offer scores zero.

ii. If $H_{t-1} = H^*$ and the announced scoring rule gives advantage to firm i, firm $-i$ submits an offer that scores zero.

iii. When the history of the game is $H_{t-1} \neq H^*$, the firms plays the equilibrium depicted in proposition 1ii..

vi. The agent's strategy is at *step 2* to announce the true scoring rule.

Let 'in' and 'out' respectively stand for 'in-turn' and 'out-of-turn'. We first note that by construction competitive bidding is a best response to the other firm's strategy after $H_{t-1} \neq H^*$.

We now consider the subgame following $H_{t-1} = H^*$.When complying with the strategies i. and ii. above the expected per period of in-firm 1 is given by $S(q_1^*, p_1^*) = 0 \Leftrightarrow p_1 = s(q_1^*) = \frac{\widehat{\alpha}^2}{\underline{\theta}} + \frac{(1-\widehat{\alpha})^2}{\overline{\theta}}$

$$\begin{aligned} E\pi^c &= prob\{\alpha \geq .5\} \left(\frac{E(\alpha; \alpha \geq .5)^2}{2\underline{\theta}} + \frac{(1 - E(\alpha; \alpha \geq .5))^2}{2\overline{\theta}} \right) \\ &= \frac{1}{2} \left(\frac{\left(\frac{3}{4}\right)^2}{2\underline{\theta}} + \frac{\left(\frac{1}{4}\right)^2}{2\overline{\theta}} \right) = \frac{1}{4} \left(\frac{9}{16\underline{\theta}} + \frac{1}{16\overline{\theta}} \right) \approx > \frac{1}{8\underline{\theta}} \end{aligned}$$

By symmetry firm 2's expected payoff when it is the in-firm is identical with that of firm 1. Assume firm 2 is the in-turn winner $\widehat{\alpha} \in [0, .5]$ firm 1's maximal (w.r.t. $\widehat{\alpha}$) defection payoff is

$$\pi^d = \frac{(.5)^2}{2\overline{\theta}} + \frac{(.5)^2}{2\underline{\theta}} = \frac{\overline{\theta} + \underline{\theta}}{8\overline{\theta}\underline{\theta}}$$

Defection in any period t is immediately detected since the in-firm loses so $H_t \neq H_t^*$ and it is followed by a breakdown of cooperation. Collusion is sustainable provided that the following condition holds:

$$IC : \frac{\delta}{(1-\delta)} E\pi^c \geq \pi^d(\widetilde{\alpha}) + \frac{\delta}{(1-\delta)} E\pi^{ne}$$

This constraint says in a period where $\alpha = \widetilde{\alpha}$, the out-of-turn firm's payoff is zero in the current period and its discounted expected collusive payoff in future periods. If he defects he wins the defection payoff under the first period and the expected competitive payoff thereafter. We have that the non-cooperative expected payoff is

$$E\pi^{ne} = \frac{1}{2}\frac{\Delta}{2\underline{\theta}\overline{\theta}}\left(2E\left(\alpha \mid \alpha \geq .5\right) - 1\right) = \frac{1}{8}\frac{\Delta}{\underline{\theta}\overline{\theta}}$$

We compute the incentive constraint using the conservative approximation $E\pi^c \approx \frac{1}{8\underline{\theta}}$

$$\begin{aligned} \frac{\underline{\theta} + \underline{\theta}}{8\overline{\theta}\underline{\theta}} &< \frac{\delta}{(1-\delta)}\left[\frac{\overline{\theta}}{8\underline{\theta}\overline{\theta}} - \frac{\Delta}{8\underline{\theta}\overline{\theta}}\right] \\ \frac{\left(\underline{\theta} + \overline{\theta}\right)}{\underline{\theta}} &< \frac{\delta}{(1-\delta)} \end{aligned}$$

for extreme values like $\underline{\theta} = 0.1$ and $\overline{\theta} = 0.9$, the IC is satisfied provided $\delta \geq 0.91$. In equilibrium the cartel's cost are minimized and the price is determined by the reserve score. Hence, the proposed scheme is optimal. *End proof*

4.ii In each period $\Gamma 2$ is repeated with $m = 0$. Let $H_{t-1} = H^*$ denote a history of the game up to period characterized by each firm winning in-turn with its most preferred scoring rule with an offer that scores zero in all periods precedings t. We propose the following strategies for the players:

i. At *step 1* the in-firm submits $b = \varepsilon$ associated with its preferred scoring rule. The out-firm submits $b = 0$ associated with its preferred scoring rule.

ii. At *step 3* if the announced scoring rule is the one preferred by the in-turn-winner, the out-of-turn firm submits an offer that scores at most zero while the in-turn winner submit an offer that scores zero. If the scoring rule is different from the equilibrium one, both firms submit competitive offers as in proposition 1.

iii. The agent's strategy is at *step 2* to announce the scoring rule associated with the largest bribe offer.

We first consider the subgame following $H_{t-1} \neq H^*$.By construction it is a best response for all players to play the Nash equilibrium which is also an equilibrium of the repeated game. It yields zero payoff to both firms but large bribe income to the agent.

We now consider a subgame following $H_{t-1} = H^*$. We proceed by backward induction: at *step* 3 we may be in either one of two subgames. Suppose first $\widehat{\alpha} = \alpha_{in}$ where α_{in} denote the in-turn firm preferred scoring rule. The in-firm expects the out-firm to submit a bid that scores at most zero. For instance, if the out-firm is 2 it submit $q_A^2 = \frac{1}{\theta}$ and $p^* \geq \frac{1}{\theta}$. The in-firm submits $q_A^1 = \frac{1}{\underline{\theta}}$ and $p_1^* = \frac{1}{\underline{\theta}}$ and earns the maximal feasible payoff of $\pi = \frac{1}{2\underline{\theta}}$. The out-firm earns zero. If the out-firm submits $p = \frac{1}{\theta} - \varepsilon$ its gain is $\frac{1}{2\overline{\theta}} - \varepsilon > 0$. However from the next period on the firm revert to the zero payoff competitive equilibrium of proposition 1ii. So the out-firm complies with the collusive strategy whenever

$$\frac{\delta}{(1-\delta)} \frac{1}{4\underline{\theta}} \geq \frac{1}{2\overline{\theta}} - \varepsilon \tag{7}$$

which is satisfied for $\underline{\delta} = \frac{2\underline{\theta}}{\overline{\theta}} / \left(1 + \frac{2\underline{\theta}}{\overline{\theta}}\right)$ which is largest when the cost difference is small, when $\underline{\theta} = \overline{\theta} = .5$, $\underline{\delta} = .666$. When $\widehat{\alpha} \neq \alpha_{in}$, the in-firm expects the out-firm to bid competitively so it is a best response to bid competitively as in proposition 1. The same holds for the out-firm.

At step 2 the agent decides whether which scoring rule to announce. Since the agent is in a single shot situation, the argument developed in the proof of proposition 1 carries over fully.

At step 1 the firms make their bribe offers. The in-firms expects the out-firm to offer

$b = 0$. It is sufficient to offer $b = \varepsilon$ to bribe the agent so he announces the in-firm preferred scoring rule. The out-firm can defect and offer $b = 2\varepsilon$ associated with its most preferred scoring rule α_{out}. It knows that the agent would respond by announcing that α_{out} so the out-firm would be winning the official auction. We know from above that the out-firm would be facing competition at step 3 though so its gain is $\frac{\Delta}{2\bar{\theta}\underline{\theta}}$ in the current period and zero afterwards since from period $t+1$ the firm revert to the equilibrium of proposition 1.ii. Since

$\frac{\Delta}{2\bar{\theta}\underline{\theta}} < \frac{1}{2\bar{\theta}} - \varepsilon$, the satisfaction of 7 secures incentives to comply at the bribing stage. Hence for δ satisfying 7 the proposed strategy do form an equilibrium.

The cartel's gain is maximized. In each period, the scoring rule is the most favorable to the winner, the price is the highest feasible and the bribe is ε. *End proof*

D.1 Proof of Proposition 5

The agent has private information about the true alpha. Within the frame of the game, the agent's only choice is the announcement of the scoring rule. The bribing game is the only means by which the firms can influence the agent and e.g. induce revelation. The agent's optimal behavior is as in proposition 2: If no bribe is offered, the agent announces the true alpha. If the firms offer bribes, he accepts the highest bribe and announces accordingly provided the bribe covers the cost. Otherwise if the highest bribe does not cover the cost, he announces the truth.

The cartel may either stay with the non-contingent allocation rule as in 4.i but the firm will have to pay a bribe to cover costs. Or the cartel can opt for a contingent allocation rule in which case, the mechanism must secure the revelation of information.

We next show that for $m < \frac{1}{4\underline{\theta}}$, the simple non-contingent rule with $b = m$, dominates any contingent allocation rule. First, we note that $b = m$ secures that the agent's cost always is covered: $\max_{\alpha} m(\alpha - 1)^2 = m$. In such a case the agent's cost, the agent only rejects the in-turn firm's offer if the other firm overbid in bribe so we have a perfect public information as in 4.i. This is not the optimal non-contingent rule since the probability that the cost is maximal is zero. Our point is to show that the fixed rule mechanism with $b = m$ does better than any contingent one under some interval which holds true a fortiori for more efficient fixed rules. The collusion IC constraint becomes

$$\frac{\delta}{(1-\delta)}\frac{1}{2}\left(\frac{1}{2\underline{\theta}}-m\right)\geq\frac{1}{2\overline{\theta}}-m-\varepsilon+\frac{\delta}{(1-\delta)}E\pi_{ne} \tag{8}$$

which obviously requires a higher discount factor than the one satisfying the IC constraint in 4.ii. in particular we note that positive cost of favoritism imply that the NE payoffs are strictly positive: $E\pi_{ne}=\frac{\Delta}{2\underline{\theta}\overline{\theta}}-m$. The one period profit-if-win is trivially decreasing in m and $m=\frac{1}{4\underline{\theta}}$

$$\frac{1}{2\underline{\theta}}-m=\frac{1}{2\underline{\theta}}-\frac{1}{4\underline{\theta}}=\frac{1}{4\underline{\theta}}$$

In a contingent scheme the agent announces the true alpha if no bribe are offered. The payoff to the cartel are as in 4.i. Since bribery is secrete, there is an incentive to defect by bribing the agent so he announces a favorable scoring rule. For a discount factor that is not arbitrary close to one such a mechanism is deemed to be plagued by in equilibrium punishment. So the cartel would be less efficient than in 4.i. where the expected per period payoff $E\pi_c\approx\frac{1}{8\underline{\theta}}$ which is less that what the fixed rule mechanism achieves on average for the in-turn firm under the investigated interval : $\left(\frac{1}{4\underline{\theta}}-\frac{1}{2}m\right)$.

Alternatively bribe competition reveals the information about the true alpha necessary to secure efficient favoritism: the contract is fine-tailored to the firm whose cost structure is most efficient given the true preferences of the government. The competitive equilibrium bribe is a function of m see proposition 2.

The mechanism is briefly described. First we have a round of competition in bribes that determines the in-turn winner. For m not too large $(m>\frac{4\pi_c}{3})$ the contract is fine-tailored to the designated winner. Then, we have collusion in price sustained by a threat to revert to the stage game equilibrium. The profit-if-win is $\frac{1}{2\underline{\theta}}$. The expected discounted payoff net of bribe is the same as in the pre-determined in-turn rule. Each firm wins with its preferred contract on average half of the time. Using our result from proposition 2, we know that the equilibrium bribe is $b^*>m$ for $m<\frac{1}{4\underline{\theta}}$ so the proposed fixed in-turn rule again dominates.

end proof

D.2 Proof of Proposition 6

The stage game equilibrium.

We shall assume that firms don't pay the agent when he selects a rule that induces fierce competition i.e. a rule that puts weight on a component such that both firm are good(bad) at it.

We propose the following strategy:

For the firms:

i. Step 1: Submit a deal offer including a menu a single peaked scoring rules, one for each low cost component, associated with the same bribe in each pair $b = \frac{\Delta}{2\underline{\theta}\overline{\theta}}$.

ii. Step 3: submit the competitive offer in quantity and price associated with the announced scoring rule.

iii. Step 4 Pay the proposed bribe if the announced scoring rule is among demanded ones and the firm did not face fierce competition on any of the components.

For the agent:

Step 2 Announce the selected scoring rule among demanded one if any otherwise announce the true alpha. Selection: within the submitted menu of deals, pick up the one that is associated with the highest bribe under the constraint that this scoring rule is only demanded by 1 firm. In case of tie in deals select at random among the scoring rules only demanded by one firm. In case no scoring rule is only demanded by one firm reject the deal offers .

We show that these strategy form a Nash equilibrium of the stage game.

At step 3 the firms submit the competitive offers given the scoring rule. These are calculated as in proposition 1.i. and 3. By construction in the subgame these is a best response to the strategy of the other firm.

At step 2 The agent makes the announcement that maximizes his bribe revenue. If he selects a component demanded by both he knows he will get nothing so he selects a scoring rule demanded by one firm. Since both firms offer the same bribe, if two or more deals are demanded by a single firm, he is indifferent and chooses at random. If the firms tie on all components, he cannot expect any bribe so he announces the true alpha.

At step 1 the firms makes their deal offers.

Consider a case where firm 1 has $\theta_{11} = \theta_{12} = \underline{\theta}$and all the other components are high cost.

What scoring rule maximizes firm 1 profit? With probability p, firm 2 is also good at the one of those components and has demanded a scoring rule that favors that component. The agent rejects the offer and he earns zero (if the other firm makes an acceptable deal offer) or the competitive payoff associated with the true alpha. With probability $(1-p)^2$ the other firm is high cost on both components. The profit-if win is $\left(\left(\alpha^2+(1-\alpha)^2\right)\frac{\Delta}{\underline{\theta}\overline{\theta}}\right)$. And the expected payoff when only submitting this deal offer is

$$\pi_1(\alpha)=(1-p)^2\left(\left(\alpha^2+(1-\alpha)^2\right)\right)+q\pi_{ne}$$

where q is small when the other firm follows the proposed strategy. If the firm instead demands one single peaked scoring rule its the profit if win is $\frac{\Delta}{2\underline{\theta}\overline{\theta}}$, and the expected payoff is

$$\pi_1(\alpha)=(1-p)\frac{\Delta}{2\underline{\theta}\overline{\theta}}.$$

So the single peaked is clearly optimal. But the agent can do much better when offering a menu of single peaked scoring rule. The profit-if-win is the same but the winning probability increases.

When both firms adopt this strategy. The probability that the firm faces no competition in bribe is very small (more precision here). It is the probability that the set of scoring rules demanded by the other firm is strictly included in the firm's own set. When this probability is close to 0, since the profit-if-win is the same for both firms and they firms submit the same bribe equal to $\frac{\Delta}{2\underline{\theta}\overline{\theta}}$ which yields a zero expected payoff from the auction net of the bribe.

The repeated game

The agent's firms' strategy are the same as in proposition 4.ii with the deal offers being the menu of single peaked scoring rule as in the stage game. If the scoring rule does not favor the in-turn firm, they submit competitive offers. If those do not tie they revert to the stage game equilibrium from that period on. The agent's strategy is as in the stage game.

We first show that when the firm faces no competition in price, the optimal scoring rule always is single peaked. Consider the case when the firm is good at three component and faces no competition it payoff is:

$$\frac{\alpha_1^2}{2\underline{\theta}}+\frac{\alpha_2^2}{2\underline{\theta}}+\frac{(1-\alpha_1-\alpha_2)^2}{2\underline{\theta}}$$

Taking the derivative with respect to α we obtain that the derivative is positive for $\alpha_i \geq .3$ and $\alpha_i + \alpha_j > 0.6$ for some j. Using the same reasoning as above, this is always true.

The IC constraint in periods where the scoring rule favors the in-turn firm is the same as in the game with symmetric information:

$$\frac{\delta}{(1-\delta)}\left(\frac{1}{4\underline{\theta}}\right) \geq \frac{1}{2\overline{\theta}} - \varepsilon +$$

There is also with a small probability that the scoring rule does not favor the in-turn firm. This can happen because of defection or because they tied on all components. In such a subgame we assume that the firm play according to the pre-determined rule. The out-of-turn firm submits an offer that scores zero. An the in-turn firm wins with a lower profit. If the out-of-turn firm defects and wins the firms revert to the non-cooperative equilibrium from next period on. The IC constraint for the out of turn firm efficient according to the scoring rule is:

$$\frac{\delta}{(1-\delta)}\left(\frac{1}{4\underline{\theta}}\right) \geq \frac{1}{2\underline{\theta}} - \varepsilon +$$

Collusion can be sustained but we have an instance of ex-post inefficiency. When both firms have an advantage in some component over the other, this subgame never occurs in equilibrium. Hence, there exist $\delta < 1$, such that collusion is an equilibrium. In this equilibrium the cartel rents are lower than with symmetric information because we allow for the case when both firms have an identical cost structure in which case collusion . *End of proof.*