The effects of tax and monetary government's policy and

the society losses

1. Introduction

The evaluation of changing in economic government's policy as well as of the associated society losses is an important part of a macroeconomic investigation. Here we consider two instruments of economic policy: taxation and printing money. How do these instruments influence the rate of capital growth, employment, outcome, etc.? Given a state of economy which of them brings less of losses to the society? There is a lot of literature studying these classical questions. In our long-run approach we pay special attention to structure of losses and suggest its description in the form of sum of surpluses of all agents (including the government). We indicate the conditions under which the losses function monotonously increases in the both instruments, and they exert the contraction influence on the economy (as this follows from the results of comparative statics). A calculation experiment with a model calibrated on Russian data confirms the theoretical assumptions and conclusions. In particular, we found that the society gains 0,22% of GDP if the rate of inflation reduces from present 12% to 10% , and the losses of the budget income are compensated by the equivalent increasing the physical taxes.

The point of departure was the paper by S. Movshovich (2000), where the author succeeded in getting the precise expressions of comparative statics and evaluating of the influence of policy steps on the economy. We use here some his ideas of marginal society losses and the key notations. Our results of comparative statics (Table 1) often coincide with those of Movshovich (that is not surprising since the models essentially differ by the capital market). Besides that paper, some well-known results on costs of inflation and policy effects were important for us. Here is a brief review of them.

A huge literature is devoted to the questions of inflation costs, and policy effects in the long- and the short-run settings. A big review "Costs of inflation" was written by J. Driffill and others (1990) in Handbook of Monetary Economics; a fresher review see R. Lucas (2000) in "Econometrica". Lucas verified the Bailey's (1956) result that the consumer's losses from inflation can be determined as the corresponding area under the inverse money demand function. Working with a steady growth model and basing on American statistics Lucas found in particular, that the reducing of the nominal interest rate from 14% to 3% would yield a benefit in real income less than 0.1%, that the effects of distorting taxation appear only at very low interest rates, and that the optimal interest rate is positive but close to zero. He concludes the "real money balances are a very minor "good" in the U.S. economy". The similar evaluations for Russian economy gave different results for different periods of time during the last decade. Varshavsky (1996) found that inflation had positive correlation with GDP in 1992-1996 years (the reducing inflation with lack of collecting taxes called for shortage of liquidity and decreasing the outcome). For the later period Drobyshevsky [2000] found visible losses from inflation. Our calculations also confirm sensitive losses from inflation, and essentially more than that from taxes.

The topic of policy effects appeared also long ago. One of the first was the paper by Tobin (1965), who argued that the increasing rate of inflation leads to the faster growth of capital. He considered a model with lump-sum taxes in which the consumer going away from inflation invested more into capital. The subsequent authors (Feldstein (1980), Turnovsky and Brock (1981), Turnovsky (1987)), emphasizing the interaction between the (distorting) tax structure and inflation in determining its effects on capital accumulation came to the opposite result. They found that the increasing the rate of inflation calls forth decreasing the long-run value of capital stock. Given our assumptions the same inference is obtained here. The influence of the inflation rate on capital growth on transition paths to the long-run steady state was studied in the delicate Fisher's work (1979). He demonstrated, using the CRRA family of utility functions, that capital accumulation is faster the higher the growth rate of money supply. That was done in the framework of the original Sidrauski's model with lump-sum taxes. The detailed investigation of these questions is contained in Turnovsky (2000). Our paper touches only the uniqueness and local stability results for steady states, and emphasizes the notion of society losses, and comparative efficiency of policy instruments.

A decentralized macro dynamic model with a representative consumer, a producer and the government together called *the society* serves as a research object. The government uses distorting taxes: income and inflation taxes from the consumer, added value tax from the producer, and also tax on profit. After collecting taxes (not before) the government determines the size of public good. In equilibrium the consumer's and the producer's demands of public good coincide with the government's supply. The balance is provided with the help of individual consumer and producer prices of public good. In the **theoretical** model the sum of these prices determines marginal utility value of public good and is an alternative to the distorting taxation. However, a government lives mostly at the account of distortions. And the society generally suffers losses from policy steps using distorting instruments. The idea of society losses is the following. Let a permanent deviation from a given steady state result from a policy step. Every agent gets some real gain or loss. We set the society gain from this step equal to the algebraic sum of the government's and the consumers' gain (profit of the producers is divided between them). The society losses have the opposite sign. Saying more precisely, this value expressed in implicit way from the stationary equilibrium conditions as function of policy is called by definition *the society losses*. The next Section contains a theorem giving a structural characterization of the society losses. The theorem affirms that the surplus of the society is added (conventionally) from the consumer's, the producer's, and the government's surpluses. If for example, the rate of inflation increased the consumer's surplus (that could be gained by reducing the rate of inflation to the initial value) is equal to the corresponding area under the inverse money demand function (Cagan's result (1956)). The producer's and the government's surpluses are equal correspondingly to the analogous areas under the inverse outcome function and the costs of government's expenditure. The society losses from the increasing distorting taxation are measured in the similar way.

A particular (**practical**) case, when public good does not enter the utility and production function, is considered more in detail. We give the conditions on the utility function that provide the losses function really increases in tax and inflation. At the same time the steady state values of capital, labor, outcome, real money balances decrease. Table 1 demonstrates the signs of derivatives of all variables on policy parameters. They show, by the way, that given our assumptions about utility functions the less distorting taxation (and the more - market regulators of Lindahl type) the more the society gains. A more interesting conclusion concerns the comparative efficiency of taxes and printing money from the point of view of filling the budget income and the society losses. The tax on profit brings no losses and most effective. As to other taxes our calculation on the model calibrated on Russian data shows the distorting taxation (such as income and added value one) is more effective than printing money (given 12% present rate of inflation). So we can support the declared government's policy towards the reducing inflation.

The mentioned results have the long-run character. They are obtained at the analysis of stationary equilibrium solutions. The approach is grounded on the results of uniqueness and local asymptotic stability of stationary solutions given in Section 5. The last Section contains a calculation experiment.

2. Model

The theoretical variant of the model describes a closed economy with three agents: a representative consumer, a producer, and a government, related by their budget constraints and called *the society*. The representative consumer chooses consumption, labor supply, the desired level of public good, and extent of investment to capital and real money balances solving the following intertemporal optimization problem:

$$max \int_0^\infty u(c, l, m, g_c)e^{-\gamma t}dt$$

$$\dot{k} + \dot{m} + n(k + m) = (1 - \tau)\,[wl + (1 - \alpha)\Pi + r_k k - q_c g_c] - \pi^e m - c + e,$$

with initial data k_0, m_0 . Here c, l, m, k, e, Π are per capita values, namely, c = real consumption, l = labor supply, m = real money balances, k= capital stock, e = an exogenous donation, Π = real profit, and g_c= consumer's demand for public good, α = profit tax rate, n= rate of growth of the population, π^e= expected rate of inflation, r_k = rate of interest for capital, w= real wage rate, τ = income tax rate, q_c= real price of public good for the consumer. For convenience, we suppose that the payments for public goods are extracted from the sum imposed by tax.

The instant utility function u is supposed to be concave, and $u_c > 0, u_l < 0, u_m > 0, u_g > 0$ and $u_{cc} < 0, u_{ll} < 0, u_{mm} < 0, u_{gg} < 0$. We assume that $n + \gamma > 0$.

The producer maximizes the real after-tax profit, determining the optimal values of capital, labor supply, and the desired level of public good. In per capita terms his problem has the form:

$$Max\ (1 - \alpha)[(1 - \beta)(F(k, l, g) - \delta k) - wl - r_k k - q_p g_p] \text{ on } k, l, g.$$

Production function $F(k, l, g)$ is concave, twice-differentiable, increasing in its arguments, and, generally, non-homogenous; β = added value tax rate, δ = rate of capital depreciation; g_p= producer's demand for public good (considered rather as the productive infrastructure) payable for price q_p. We suppose that this payment is extracted from the profit imposed by tax.

The government assigns the tax rates α, β, τ, the rate of monetary growth η ($\eta = \dot{M}/M$, where M is the nominal money balances), the prices of public good for the consumer q_c and the producer q_p. Having collected the all receipts to the budget the government determines the state expenditure g^s from equation:

$g^s = \tau[wl^s + r_k k^s + (1-\alpha)\Pi - q_1 g_c] + \beta(F(k^d, l^d, g_p) - \delta k^d) + \alpha\Pi + \eta m^s + q_c g_c + q_p g_p$,

the real money supply m^s satisfies the equation:

$$\dot{m} = m(\eta - \pi - n).$$

Thus the public good is financed partly at the expense of taxes and partly on tariffs q_c and q_p.

We shall be interested in the perfect foresight equilibrium. In particular, this means that the expected rate of inflation π^e will coincide with the real one denoted by π ($\pi = \dot{P}/P$, P is prices of good). For accommodation of the equilibrium the following regulators are used: real wage rate (labor supply), interest rate on capital (capital balance), individual prices on public good (equality $g_c = g_p = g$). The resource balance will follow from the consumer's and the government's budget constraints.

Let us denote:

$1-\theta = (1-\tau)(1-\beta)$, $\rho = (1-\tau)\, r_k$, $\omega = (1-\tau)\, w$, $\upsilon_c = (1-\tau)q_c$, $\upsilon_p = (1-\tau)q_p$.

Then the *perfect foresight equilibrium* (p.f.e.) conditions take the form:

$\dot{\psi} = \psi(\gamma + n - \rho)$,

$\dot{m} = m(\eta - \pi - n)$,

$\dot{k} + nk = (1-\alpha)(1-\theta)(F(k, l, g) - \delta k) + \alpha(\omega l + \rho k + \upsilon_p g) - \eta m - c - (\upsilon_c + \upsilon_p)g + e$,

$\dot{k} + nk = F(k, l, g) - \delta k - c - g + e$,

$u_1 = \psi$,

$u_2 = - u_1 \omega$,

$u_3 = u_1(\rho + \pi)$,

$u_4 = u_1 \upsilon_c$,

$(1-\theta)(F_1 - \delta) = \rho$,

$(1-\theta)F_2 = \omega$,

$(1-\theta)F_3 = \upsilon_p$.

Initial data: k_0 is given from the past, m_0 and ψ_0 are determined endogenously. One should add the transversality conditions at infinity for variables k and m:

$$\lim \psi_t k_t e^{-\gamma t} = 0, \ \lim \psi_t m_t e^{-\gamma t} = 0 \text{ при } t \to \infty.$$

Since tax rates τ and β enter the equilibrium conditions together parameter θ represents the both as one tax rate.

Further we assume that all policy parameters are constant in time and shall study the effects of permanent and expected policy deviations and the corresponding society losses on the equilibrium stationary solutions. As a foundation of such approach we produce in Section 5 the uniqueness and local asymptotic stability of stationary solutions results.

The conditions of stationary perfect foresight equilibrium are:

$$u_2 = - u_1\omega, \tag{1a}$$

$$u_3 = u_1(\eta + \gamma), \tag{1b}$$

$$u_4 = u_1\, \upsilon_c, \tag{1c}$$

$$(1-\theta)(F_1 - \delta) = \gamma + n, \tag{1d}$$

$$(1-\theta)F_2 = \omega, \tag{1e}$$

$$(1-\theta)F_3 = \upsilon_p, \tag{1f}$$

$$F(k, l, g) - \delta k - nk + e = c + g\,, \tag{1g}$$

$$c + \eta m + (\upsilon_c + \upsilon_p)g + nk = (1-\theta)\{(F-\delta k) - \alpha[(F-\delta k) - (F_1 - \delta)k - F_2\, l - F_3 g]\} + e. \tag{1h}$$

At a steady state $\eta = \pi + n$, that is the rate of inflation π is determined by the rate of money growth η, and $\rho = \gamma + n$. The variables ρ and π are excluded from system (1).

If system (1) has a solution the equilibrium government's expenditure g satisfies the budget equation:

$$g = \tau\,[wl + r_k\, k + (1-\alpha)\Pi - q_p\, g] + \beta(F(k, l, g) - \delta k) + \alpha\Pi + \eta\, m + (q_c + q_p)g,$$

It can be written in the form (which is the obvious implication of (1g) and (1h)):

$$g = \theta(F - \delta k) + \alpha(1-\theta)[(F - \delta k) - (F_1 - \delta)k - F_2\, l - F_3 g] + \eta\, m + (\upsilon_c + \upsilon_p)g.$$

The paper does not touch the question of existence of equilibrium. One can note, in particular, that $\upsilon_c + \upsilon_p$ should be less than 1 (if $\eta = \pi + n \geq 0$). It follows that
u_g/u_1 and $(1-\theta)F_g$ should be sensitively less than 1 in equilibrium and fall to zero when η grows.

3. The society losses

We call a government's *policy* a triple of constants (θ, η, α), that is the united (distorting) tax rate, the rate of money emission, and the tax rate on profit. Let given a policy $(\theta_0, \eta_0, \alpha_0)$ system (1a)-(1h) have a solution $(\omega_0, c_0, l_0, m_0, k_0, g_0, \upsilon_{c0}, \upsilon_{p0})$ with $e_0 = 0$. When the policy moves to $(\theta_0 + \Delta\theta, \eta_0 + \Delta\eta, \alpha_0 + \Delta\alpha)$ and say, $\Delta\theta \geq 0, \Delta\eta \geq 0, \Delta\alpha \geq 0$, the solution of (1a)-(1h) changes, and generally, $\Delta g = g - g_0 > 0$, and $\Delta u < 0$. Some compensation $\Delta e = e - e_0 > 0$ leaves the consumer indifferent between policies $(\theta_0, \eta_0, \alpha_0)$ and $(\theta_0 + \Delta\theta, \eta_0 + \Delta\eta, \alpha_0 + \Delta\alpha)$. Since Δe is now endogenous one more equation serves to find it:

$$u(c, l, m, g) = u(c_0, l_0, m_0, g_0). \tag{1j}$$

Changing the initial values $m_0 + \Delta m = m$ and $k_0 + \Delta k = k$ also requires the compensation to the consumer equal to $\Delta m + \Delta k$. The equivalent payment x on $[0, \infty]$ is such that $\Delta m + \Delta k = x\int_0^\infty e^{-\gamma t}dt = x\gamma^{-1}$.

Now we define the notion of the society losses. The consumers and the government as independent agents influence to each other by their choices. Every policy step of the government brings to the both sides a gain or a loss of some quantity of resource per unit of time. The algebraic sum of these surpluses is taken as the society losses:

$$E = \gamma\,(\Delta m + \Delta k) + \Delta e - \Delta g.$$

The government's interest is considered here as self-valuable on a level with the consumer's (besides the consumers the producers and the whole economic mechanism use the public good for their functioning). Besides, the definition implies that the public good is considered as equally valuable as the private good. (This presupposes that the government is able and wants to produce such a public good).

We solve system (1a)-(1j) for $(\omega, c, l, m, k, g, e, \upsilon_c, \upsilon_p)$ in terms of (θ, η, α) and substitute them into the formula for E:

$$E(\theta,\eta,\alpha) = \gamma\,[m(\theta,\eta,\alpha) - m_0 + k(\theta,\eta,\alpha) - k_0] + e(\theta,\eta,\alpha) - g(\theta,\eta,\alpha) + g_0.$$

Function $E(\theta,\eta,\alpha)$ is called *the society losses (SL)* of changing policy from $(\theta_0, \eta_0, \alpha_0)$ to $(\theta, \eta, \alpha) =(\theta_0+\Delta\theta, \eta_0+\Delta\eta, \alpha_0+\Delta\alpha)$. Partial derivatives E_η, E_θ, and E_α are *marginal society losses (MSL)*. *Relative marginal losses* (from inflation) per 1 ruble of additional receipts to the budget income are equal (by definition) to:

$$[\gamma(\partial m/\partial\eta + \partial k/\partial\eta) + \partial e/\partial\eta - \partial g/\partial\eta]/(\partial g/\partial\eta),$$

and analogously for θ and α. (see S. Movshovich [1]). Note that the last definition has the original sense till $\partial g/\partial\eta > 0$, but this is difficult to guaranty. (In the calculation experiment such a sign takes place, and we evaluate there the values M_η и M_θ).

Here we deal with the notions of SL and MSL.

We denote: $Y = F - \delta k$, $\upsilon = \upsilon_1 + \upsilon_2$.

Proposition 1. *The MSL have the form:*

$$E_\theta(\theta,\eta,\alpha) = -\,\eta\,\partial m/\partial\theta - \theta\,\partial Y/\partial\theta - \upsilon\,\partial g/\partial\theta,$$

$$E_\eta(\theta,\eta,\alpha) = -\,\eta\,\partial m/\partial\eta - \theta\,\partial Y/\partial\eta - \upsilon\,\partial g/\partial\eta,$$

$$E_\alpha(\theta,\eta,\alpha) = 0.$$

Proof. Equations (1j), (1a) and (1b), and also (1g) give the following relations between differentials:

$$dc = \omega\, dl - (\eta + \gamma)dm - \upsilon_c\, dg\,,$$

$$de - dg = dc + (\delta + n)dk - dF.$$

Let us find the expression for E_η. We differentiate function $E(\theta,\eta,\alpha)$ on η and substitute there the private derivatives from the two last relations taking into account equations (1a)-(1f). We get:

$$E_\eta(\theta,\eta,\alpha) = \gamma\,(\partial m/\partial\eta + \partial k/\partial\eta) + \omega\partial l/\partial\eta - (\eta + \gamma)\partial m/\partial\eta - \upsilon_c\partial g/\partial\eta + (\delta + n)\partial k/\partial\eta - \partial F/\partial\eta = -\,\eta\partial m/\partial\eta$$
$$+ (1-\theta)[(F_1 - \delta)\partial k/\partial\eta + F_2\,\partial l/\partial\eta + F_3\partial g/\partial\eta] - \upsilon_p\partial g/\partial\eta -$$
$$\partial(F - \delta k)/\partial\eta = -\,\eta\,\partial m/\partial\eta - \theta\,\partial(F - \delta k)/\partial\eta - \upsilon\partial g/\partial\eta.$$

So $E_\eta(\theta,\eta,\alpha) = -\,\eta\,\partial m/\partial\eta - \theta\partial(F - \delta k)/\partial\eta - \upsilon\partial g/\partial\eta$.

The formula for E_θ is derived analogously. Equality $E_\alpha = 0$ is obvious, when production function F is homogenous, since in this case system (1) does not include α (see (1h)); in the general case the equality $E_\alpha = 0$ will be established in the next Section. •

Remark. If we were interested only in the consumer's losses the expression for E_η (and the similar for E_θ) would have the form:

$$E_\eta(\theta,\eta,\alpha) = -\,\eta\,\partial m/\partial\eta - \theta\partial(F - \delta k)/\partial\eta + (1 - \upsilon)\partial g/\partial\eta\,. \bullet$$

Now I give a structural characterization of society losses. The losses (= the surplus the society gets when policy returns to its initial value) is equal (conventionally) to the sum of the consumer's, producer's and government's surpluses.

Theorem 1. *Let policy $\xi_0 = (\theta_0, \eta_0, \alpha_0)$ change to $\xi = (\theta, \eta, \alpha)$ and $\overline{\xi} = \xi - \xi_0$. Then society losses (or gains) from policy variation $\overline{\xi}$ are equal*

$$E(\xi) = E^F + E^m + E^g,$$

where E^F, E^m and E^g are correspondingly the surpluses of producer, consumer, and government which they gain (or loose) when policy ξ comes back to ξ_0 and equal:

$$E^F = \int_0^1 (Y(\xi_0 + \varepsilon\overline{\xi})\,\overline{\theta}\, d\varepsilon + \theta_0 Y(\xi_0) - \theta Y(\xi),$$

$$E^m = \int_0^1 m(\xi_0 + \varepsilon\overline{\xi})\,\overline{\eta}\, d\varepsilon + \eta_0 m(\xi_0) - \eta\, m(\xi),$$

$$E^g = \int_0^1 g(\xi_0 + \varepsilon\overline{\xi})\, d\upsilon(\varepsilon) + \upsilon_0 g(\xi_0) - \upsilon\, g(\xi)$$

The following two corollaries give simpler formulae for losses from changing the rates of inflation and taxes.

Corollary 1. *Let $\xi_0 = (\theta_0, \eta_0, \alpha_0)$, $\xi_1 = (\theta_0, \eta_1, \alpha_0)$, and $\eta_1 > \eta_0$. Then society losses from increasing the rate of money growth from η_0 to η_1 are equal:*

$$E(\xi_1) = \theta_0 (Y_0 - Y_1) + E^m + E^g, \tag{2}$$

where

$$E^m = \int_{\eta_0}^{\eta_1} m(\theta_0, \eta, \alpha_0) d\eta + \eta_0 m_0 - \eta_1 m_1, \quad E^g = \int_{\eta_0}^{\eta_1} g(\theta_0, \eta, \alpha_0) d\upsilon(\eta) + \upsilon_0 g_0 - \upsilon_1 g_1.$$

Corollary 2. *Let $\xi_0 = (\theta_0, \eta_0, \alpha_0)$, $\xi_1 = (\theta_1, \eta_0, \alpha_0)$, and $\theta_1 > \theta_0$. Then society losses from increasing the tax rate from θ_0 to θ_1 are equal:*

$$E(\xi_1) = E^F + E^g + \eta_0 (m_0 - m_1), \tag{3}$$

where

$$E^F = \int_{\theta_0}^{\theta_1} Y(\theta, \eta_0, \alpha_0) d\theta + \theta_0 Y_0 - \theta_1 Y_1,$$

$$E^g = \int_{\theta_0}^{\theta_1} g(\theta, \eta_0, \alpha_0) d\upsilon(\theta) + \upsilon_0 g_0 - \upsilon_1 g_1.$$

In Corollary 1 value E^m is the Cagan's (1956) welfare losses from inflation. Some difference consists in that ordinarily the losses according to Cagan are depicted in axes {nominal interest rate; real money balances (or their relation to consumption or output)}. Here the nominal interest rate can be defined as $\eta + \gamma$. So it is not difficult to recount E^m into the Cagan's losses. Besides, in Cagan's model money was superneutral that implied $E^g = 0$ and $E^F = 0$, that is real variables did not depend on the rate of inflation. So the losses from inflation reduced to value E^m. In a more general setting as here the losses from inflation are given in Corollary 1. Fig.1 depicts the losses by the shaded area.

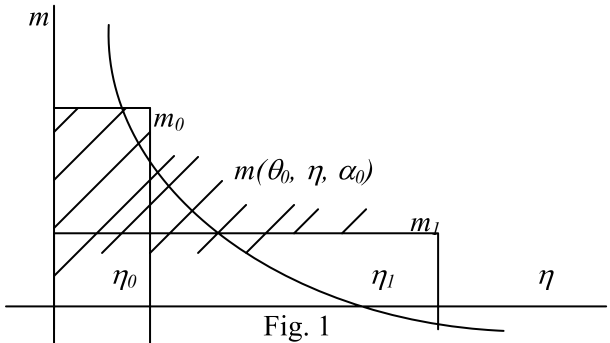

Fig. 1

The items E^F and E^g show the analogous areas generated by changing the production outcome and the costs of government's expenditure.

Note that public good g can enter the model in different ways, for example, as a fixed share of output (from the side of the government), to which the agents' demand at appropriate individual prices compare in equilibrium. In such a case the calculation is somewhat simplified.

4. Characterization of marginal values of the variables

Here we find the expressions and signs of derivatives of all variables of the model on the policy parameters. In fact I consider more in detail a particular case when the government's expenditure does not influence the production and utility functions (the limit case of very low elasticity of those functions on g). In this case $\upsilon \equiv 0$ and so $E^g = 0$. There is no solvent demand for public good and so the changing of g is not perceived as a loss.

Let us express differentials $dk, dl, dm, d\omega$ through policy differentials $d\theta, d\eta, d\alpha$ from system (1). We denote (as in Movshovich [1]):

$$r_1 = (u_{11}\omega^2 + 2u_{12}\omega + u_{22})/u_1, \quad r_2 = [u_{11}(\eta+\gamma)^2 - 2u_{13}(\eta+\gamma) + u_{33}]/u_1,$$

$$s = (-u_{11}\omega(\eta+\gamma) - u_{12}(\eta+\gamma) + u_{13}\omega + u_{23})/u_1, \qquad r = (r_1 r_2 - s^2)^{-1}.$$

The strict concavity property of u implies that $r_1 < 0$, $r_2 < 0$. Signs of r and s are not determined automatically. We come back to them below. However, one can note that for separable in money utility function the inequality $r > 0$ holds; for totally separable utility function $s > 0$.

We get from equations (1a), (1b), and (1j) the relations:

$$dc = \omega\, dl - (\eta+\gamma)\, dm,$$

$$dl = -rr_2\, d\omega - rs\, d\eta,$$

$$dm = rr_1\, d\eta + rs\, d\omega.$$

Let us consider two cases:

a) $n \neq 0$ and F is homogenous production function, and

b) $n = 0$, F is a non - homogenous function.

a) In this case $R = F_{11}F_{22} - F_{12}^2 = 0$ and in order to express the differentials dk, dl we proceed to capital/labor terms. Denote: $f(\tilde{k}) = F(k/l, 1)$, $\tilde{k} = k/l;$ then we have:

$$F(k, l) = f(\tilde{k})l;\ F_1(k, l) = f'(\tilde{k}),\ F_2(k, l) = f(\tilde{k}) - \tilde{k} f'(\tilde{k}).$$

Then equations (1d), (1e), (1g), (1h) take the form:

$$(1-\theta)(f'(\tilde{k}) - \delta) = \gamma + n, \tag{1'd}$$

$$(1-\theta)[f(\tilde{k}) - \tilde{k} f'(\tilde{k})] = \omega, \tag{1'e}$$

$$(f(\tilde{k}) - \delta\tilde{k} - n\tilde{k})l + e = c + g, \tag{1'g}$$

$$c + \eta m + n\tilde{k} l = (1-\theta)(f(\tilde{k}) - \delta\tilde{k})l + e, \tag{1'h}$$

The first two equations give:

$$f''(\tilde{k})d\tilde{k} = (1-\theta)^{-2}(\gamma + n)d\theta,$$

$$-\tilde{k} f''(\tilde{k})d\tilde{k} = (1-\theta)^{-2}\omega\, d\theta + (1-\theta)^{-1}d\omega.$$

We find from them:

$$d\tilde{k} = f''(\tilde{k})^{-1}(1-\theta)^{-2}(\gamma + n)\, d\theta,$$

$$d\omega = -(1-\theta)^{-1}(\tilde{k}\gamma + \omega)d\theta.$$

Note that the capital/labor ratio does not depend on the monetary factor.

Substituting $d\omega$ into expressions for dl and dm we get:

$$dl = rr_2(1-\theta)^{-1}(\tilde{k}\gamma + \omega)d\theta - rs\, d\eta,$$

$$dm = -rs(1-\theta)^{-1}(\tilde{k}\gamma + \omega)d\theta + rr_1\, d\eta.$$

Now the formulae for the derivatives take the form:

$$\partial m/\partial\eta = rr_1,\ \ \partial m/\partial\theta = -rs(1-\theta)^{-1}(\tilde{k}\gamma + \omega),$$

$$\partial m/\partial\alpha = 0;$$

$$\partial F/\partial\eta = lf'(\tilde{k})\partial\tilde{k}/\partial\eta - rs f(\tilde{k}) = -rs f(\tilde{k}),$$

$$\partial F/\partial\theta = lf'(\tilde{k})f''(\tilde{k})^{-1}(1-\theta)^{-2}\gamma + rr_2(1-\theta)^{-1}(\tilde{k}\gamma + \omega) f(\tilde{k}),$$

$$\partial F/\partial\alpha = 0.$$

The "natural" reaction of variables – the decreasing in inflation and taxes prompts the "appropriate" signs of r and s entering the derivatives. For example, the real money balances obviously decreases in the rate of inflation: $\partial m/\partial\pi < 0$. But this is true only if $r > 0$. Besides, this agrees with decreasing supply of labor and outcome in taxes θ. $\partial m/\partial\theta < 0$ and $\partial F/\partial\eta < 0$. Given $r > 0$ the positive sign of s implies inequalities $\partial m/\partial\theta < 0$ and $\partial F/\partial\eta < 0$. Movshovich [1] shows that given $r > 0$ inequality $s > 0$ holds if the consumer decreases supply of labor in response on the increasing non-labored income. A technical condition for $s > 0$ is the following: for separable in money utility function the condition $u_{12} \leq 0$ holds (that is marginal utility of consumption increases with leisure, that is standard and seems to be acceptable condition). Later we give an additional support for assumption $s > 0$ from the side of losses.

Thus one can fix that inequalities $r > 0$ and $s > 0$ imply inequalities:

$$\partial l/\partial\eta < 0,\ \partial l/\partial\theta < 0,\ \partial m/\partial\eta < 0,\ \partial m/\partial\theta < 0,\ \partial F/\partial\eta < 0,\ \partial F/\partial\theta < 0.$$

b) Non-homogenous production function. In this case $R = F_{11}F_{22} - F_{12}^2 > 0$ and one can straightforwardly express the differentials $dk, dl, dm, d\omega$ through policy differentials $d\theta, d\pi, d\alpha.$ Equations (1d), (1e) give:

$$F_{11}dk + F_{12}\,dl = (1-\theta)^{-2}\gamma\, d\theta,$$

$$F_{21}dk + F_{22}\,dl = (1-\theta)^{-1}d\omega + \omega(1-\theta)^{-2}d\theta\,,$$

Solving this system for dk and dl we get the expressions for dk and dl:

$$Rdk = (1-\theta)^{-2}\,(\gamma F_{22} - F_{12}\omega)\,d\theta \;-\; F_{12}(1-\theta)^{-1}d\omega, \tag{4a}$$

$$Rdl = F_{11}(1-\theta)^{-1}d\omega + (1-\theta)^{-2}\,(F_{11}\omega \;-\; F_{21}\gamma)d\theta. \tag{4b}$$

We denote

$$S = -\,Rrr_2 - F_{11}(1-\theta)^{-1}\,.$$

Since $r > 0$ (the assumption holds), $R > 0,$ and $r_2 > 0$ inequality $S > 0$ holds. Comparing the earlier equality $dl = -\,rr_2\,d\omega \;-\; rs\;d\eta$ with the two previous equalities we get the expression for $d\omega$:

$$S\,d\omega = \;Rrs\;d\eta \;+ (1-\theta)^{-2}\,(F_{11}\omega \;-\; F_{21}\gamma)\;d\theta \tag{4c}$$

Remind that:

$$dm = rr_1\;d\eta + rs\;d\omega, \tag{4d}$$

Let us show that $\partial m/\partial\eta < 0$. Using inequality $S > -\,Rrr_2$ and definition of r we obtain from (4c) and (4d):

$$\partial m/\partial\eta = rr_1 + S^{-1}R(rs)^2 < r(r_1 - s^2/r_2) = (r_1r_2 - s^2)^{-1}(r_1r_2 - s^2)/r_2 = 1/\,r_2 < 0\,.$$

Inequality $\partial\omega\,/\partial\theta < 0$ follows directly from (4c) (we remind that $F_{21}>0$). Then (4b) implies:

$$\partial l/\partial\theta = -\,rr_2\,\partial\omega\,/\partial\theta \;<\; 0.$$

Let us check that $\partial k/\partial\theta < 0$:

$$\partial k/\partial\theta = R^{-1}(1-\theta)^{-2}[\,(\gamma F_{22} - F_{12}\omega) - F_{12}(1-\theta)^{-1}\,(F_{11}\omega \;-\; F_{21}\gamma)S^{-1}] = R^{-1}(1-\theta)^{-2}[\gamma\,(F_{22} + (F_{12})^2(1-\theta)^{-1}\,S^{-1}) - F_{12}\omega\,(1 + (1-\theta)^{-1}F_{11}\,S^{-1})] < 0;$$

the last inequality is true because $(1-\theta)^{-1}F_{11}\,S^{-1} > -1.$

Besides that, given $r > 0\ (R \neq 0)$ relations (4a)-(4d) give:

$s > 0 \;\Rightarrow\; \partial\omega\,/\partial\eta > 0,\ \partial k/\partial\eta < 0,\ \partial l/\partial\eta < 0$ and $\partial m/\partial\theta < 0;$

$s < 0 \;\Rightarrow$ the opposite signs of all these derivatives.

Now let us collect all signs of derivatives in a table.

Table 1 that shows the distorting effects of the government's policy.

$(\,r > 0\,,\; s > 0)$

x	x_π	x_θ	x_α
k	< 0	< 0	0
l	< 0	< 0	0
m	< 0	< 0	0
c	-	-	0
ω	≥ 0	< 0	0

$F-\delta k$	< 0	< 0	0
g	–	–	≥ 0
E	> 0	> 0	0

We fix the result in the following

Proposition 2. *Let $r > 0$, $s > 0$. Then $E_\eta > 0$, $E_\theta > 0$; function of society losses $E(\theta, \eta, \alpha)$ strictly increases in θ, η and indifferent in α.*

The positive signs of E_η and E_θ are "natural" and so the positive signs of r and s which they provide are expected at the proper calibration. One could say that the domain of feasible consumer's preferences must be described basing on the "natural" properties of the consumer's behavior. However the budget does not necessarily get a positive receipt from increasing inflation or taxes (as well as private consumption). The signs of derivatives $\partial g/\partial\eta$ and $\partial g/\partial\theta$ except $\partial g/\partial\alpha$ are generally uncertain that can be seen from the relations:

$$\partial g/\partial\alpha = (1-\theta)(F - F_1k - F_2l) \geq 0 \text{ by force of concavity of } F(k, l),$$

$$\partial g/\partial\eta = -\alpha l\partial\omega/\partial\eta + \theta\partial F/\partial\eta + \partial(m\eta)/\partial\eta,$$

$$\partial g/\partial\theta = (1-\alpha)(1-\theta)^2F - \alpha l\partial\omega/\partial\theta + \theta\partial F/\partial\theta + \pi\partial m/\partial\theta.$$

In particular, $dg/d\eta > 0$ only if $\partial(m\eta)/d\eta$ is strictly positive, i.e. the equilibrium point is strictly on the increasing part of the Lafer curve. The expression for $\partial g/\partial\theta$ also contains the items of different signs. So $\Delta g < 0$ is quite possible, in this case function E shows the summary society losses (consumer's and government's).

5. Dynamics

We replaced the question of influence of policy on equilibrium paths by the analysis of its effects on stationary equilibrium paths. Now we produce the uniqueness and local asymptotic stability results for some foundation of such a passage. For simplicity we give these results for simpler model. Namely, suppose that,

g does not enter utility function and production function,

full employment of the increasing population,

tax on profit α is equal to 0 (this simplification is not essential).

Then the model in per capita terms takes the following form.

The consumer's problem:

$$max \int_0^\infty u(c,m)e^{-\gamma t}dt$$

$$\dot{k} + \dot{m} + n(k+m) = (1-\tau)\,[wl + \Pi + r_k k] - \pi^e m - c + e,$$

The producer's problem:

$Max[(1 - \beta)(f(k) - \delta k) - wl - r_k k]$ on k.

The government imposes the tax rates τ and β, the rate of money growth η, and determines the state expenditure g from the expression:

$g = \tau[\,wl^s + r_k k^s + \Pi] + \beta\,(f(k^d) - \delta k^d) + \eta\, m^s.$

If g is defined so the resource balance holds in equilibrium due to the consumer's budget constraint. After the corresponding substitutions from algebraic relations the equilibrium conditions are described by the following system of differential equations (we denote the dual variable by q):

$$\dot{q} = q[n + \gamma - (1-\theta)(f'(k) - \delta)], \tag{5a}$$

$$\dot{m} = m[\eta - u'_m/q + (1-\theta)(f'(k) - \delta) - n], \tag{5b}$$

$$\dot{k} = (1-\theta)(f(k) - \delta k) - c(m, q) - nk - \eta m + e, \tag{5c}$$

where $c(m, q)$ is the implicit function from the relation

$$u'_c(c, m) = q. \tag{5d}$$

Given θ and η stationary solution (q^*, m^*, k^*) can be found from the static system of equations:

$$(1-\theta)(f'(k^*) - \delta) = n + \gamma, \tag{6a}$$

$$\eta + \gamma = u'_m(c(m^*, q^*), m^*)/q^*, \tag{6b}$$

$$(1-\theta)(f(k^*) - \delta k^*) - c(m^*, q^*) - nk - \eta m^* + e = 0. \tag{6c}$$

Proposition 3. *Let $u_{cm} \geq 0$. Then given policy θ and η there exists unique solution (q^*, m^*, k^*) of system (4a)-(4c).*

Proof. Since function f is strictly concave (6a) gives unique value $k^*(\theta)$. Substitute it in (6c). We denote the implicit functions $q(m)$ expressed from (6b) and (6c) correspondingly ${}_b q(m)$ and ${}_c q(m)$.

${}_b q'(m) = - (u_{mc} c_m + u_{mm}) / [u_{mc} c_q - (\eta + \gamma)]$,

${}_c q'(m) = - (c_m + \eta)/c_q$.

Note that $c_m(m, q) = - u_{cm} / u_{cc} \geq 0$ and $c_q(m, q) = 1/u_{cc} < 0$. After substituting them in the previous equalities we get the relations:

${}_b q'(m) = - (- u_{cm} u_{mc} + u_{cc} u_{mm})/[u_{mc} - (\eta + \gamma)u_{cc}] < 0$,

${}_c q'(m) = - (- u_{cm} + \eta u_{cc}) > 0$.

Thus function ${}_b q(m)$ strictly decreases, while ${}_c q(m)$ strictly increases in m. They intersect each other (here we suppose it) only in one point (q^*, m^*). •

Proposition 4. *Let $u_{cm} \geq 0$. Then there exists a unique perfect foresight path asymptotically converging to the stationary state (q^*, m^*, k^*).*

We fix stationary solution (q^*, m^*, k^*) and produce the linearization of the system (5) around it. The linear differential system has the form:

$$\dot{\bar{q}} = 0 \cdot \bar{q} + 0 \cdot \bar{m} + [- q^*(1-\theta)f''(k^*)]\, \bar{k},$$

$$\dot{\bar{m}} = [(m^*(\eta + \gamma + c_m)/q^*]\, \bar{q} + [- m^*(u_{mc}c_m + u_{mm})/q^*]\, \bar{m} + [(1-\theta)f''(k^*)m^*]\, \bar{k},$$

$$\dot{\bar{k}} = [- 1/u_{cc}]\, \bar{q} + [- (c_m + \eta)]\, \bar{m} + \gamma\, \bar{k}.$$

The coefficients of the system can be calculated directly except the first item in the second equation where we used the relations $c_m(m, q) = -u_{cm}/u_{cc}$, $c_q(m, q) = 1/u_{cc}$, and also system (6). Let us calculate the determinant of its matrix, denote it A:

Det $A = -q^*(1-\theta)f''(k^*)\{-m^*(\eta+\gamma+c_m)(c_m+\eta)/q^* - m^*(u_{mc}c_m+u_{mm})/(u_{cc}q^*)\} < 0.$

We used here the inequality $c_m \geq 0$ and the properties of concave function u, in particular, in the last item:

$$m^*(u_{mc}c_m+u_{mm})/(u_{cc}q^*) = m^*(-u_{cm}u_{mc}+u_{cc}u_{mm})/(u_{cc}^2q^*) > 0.$$

Since Det A is equal to the product of the roots of A one can conclude that either all three roots are negative (two of which may be complex with negative real parts) or only one. If we have three negative roots the sum of them, that is the trace of A should be negative. Let us calculate the trace of A.

$$Trace\ A = \gamma - m^*(u_{mc}c_m+u_{mm})/q^* > \gamma > 0.$$

So there is at least one positive root. Together with the previous observation this implies that there is only one negative real root $\lambda_1 < 0$ and the unique equilibrium path converging to the stationary state (q^*,m^*,k^*) of the form [1]:

$$q(t) = q^* + (q(0) - q^*)\, e^{\lambda_1 t},$$

$$m(t) = m^* + (m(0) - m^*)\, e^{\lambda_1 t},$$

$$k(t) = k^* + (k_0 - k^*)\, e^{\lambda_1 t}.$$

Here the capital evolves continuously from its given initial stock k_0, while initial values $q(0)$ and $m(0)$ are determined endogenously.•

With the help of the initial jump in $q(0)$ one can evaluate the wealth effect of changing in policy $\bar{\xi} = \xi - \xi_0$. The initial jump of the real money stock is determined by an appropriate initial jump in the price level (the nominal money stock is predetermined by the given initial value and further by the rule of money emission).

We saw that an increase in the monetary growth rate or/and (distorting) tax rate reduces the long-run capital stock k^*, real monetary stock m^* (outcome, labor supply). This changes initial values $q(o)$, $m(0)$, and ultimately influences the short-run behavior. We do not touch this topic and proceed to calculations.

[1] Strictly speaking one should check that any other solution of the linear system does not satisfy the transversality conditions for m and k.

6. Calculations

Here we produce a numerical experiment with the model described and analyzed in the previous Sections.[2] In fact we shall deal with a model where the government's expenditure does not enter the consumer's utility and the producer's production function. Besides, we suppose that the rate of growth of population $n = 0$.

Since marginal contribution to the budget income may turn out to be negative we define the relative marginal losses as follows:

$$M_\pi = [\gamma(\partial m/\partial\pi + \partial k/\partial\pi) + \partial e/\partial\pi - \partial g/\partial\pi] \,/\, |\partial g/\partial\pi| \,.$$

When $\partial g/\partial\pi < 0$ the value M_π shows the summary (the consumer's and the government's) gains from **decreasing** inflation per one additional ruble to the budget.

We choose the types of utility and production functions. Let us take the consumer's utility function of CRRA type (that is a constant relative risk aversion function), namely:

$$U(c, l, m) = \frac{[c^{z_1}(T-l)^{z_2} m^{z_3}]^{1-\sigma} - 1}{1-\sigma}, \text{ where } z_1 + z_2 + z_3 = 1,\ \sigma > -1,$$

and the production function of the standard form:

$$F(k, l) = x_1 k^{x_2}\, l^{x_3}, \text{ where } x_2 + x_3 \le 1,\ x_1 > 0.$$

The parameters $z_1, z_2, z_3, \sigma, x_1, x_2, x_3$, and the rate of time preference γ are to be evaluated. After substituting functions U and F in system (1) and normalizing it in terms of shares of outcome (the latter becomes equal to 1) we get the following system of equations:

$$z_2 c = z_1 \omega (T - l),$$

$$z_3 c = z_1 m(\pi + \gamma),$$

$$(1 - \theta)(x_2 - \delta k) = \rho k,$$

$$(1 - \theta)\, x_3 = l\omega,$$

$$1 = g + c + \delta k,$$

$$c + \pi m = (1 - \alpha)(1 - \theta) + \alpha\, l\omega + [\alpha\rho - (1 - \alpha)(1 - \theta)\delta]k.$$

With the help of this system we will find the following unknown parameters and variables: ($z_1, z_2, z_3, x_1, x_2, x_3, g, l$). To this end we fix some basic values taken directly or approximately from Russian sources. In particular, the tax rates (with the exception of privileges and nuances) are taken from the Russian Tax Legislation (site www.nalog.ru):

α = 24% = tax on profit,

β = 18% =- added value tax,

τ = 13% = income tax rate,

π = 12% = rate of inflation (for 2003 year).

The evaluation of the interest rate r for Russian economy is the question of a separate investigation. For example, the return of Russian government's bonds (ГКО, ОФЗ) was about 9% per year, hence the real

[2] The calculations were made by A. Skvortsov, a student of NES.

return was negative (9 – 12 = - 3); at the same time the return of many Russian Share Funds (ПИФов) в то же время доходность многих ПИФов surpassed sometimes 30% per year (correspondingly with the real value 30 – 12 = 18). The rates of credits and deposits essentially changed during the 2003 year and from a bank to another one. As the basic value we took the rate of return of the Central Bank of RF. The rate of return changed during 2003 year twice: from 21% till 18%, and then from 18% till 16%. We took as the nominal interest rate the average value equal to 18%. Then real interest rate $r \approx 18 - 12 = 6$ % per year.

Some general macroeconomic parameters were taken from the hand book GosComStat for 2001 year:

the depreciation of capital including the circulating means was about 9%;

GDP (bill. of rubles) = 9040821;

the fixed capital including uncompleted construction (bill. of rubles) =20928833;

the wages of employees (bill. of rubles) = 4069112;

the money balances (M2) in 2001 year changed from 1154 bill. to 1612 bill. of rubles, the average per year = 1318 bill.

After taking these values to GDP we get the following values:

$$k = 2{,}499,\ c=0.497,\ w=0{,}450,\ m=0{,}146.$$

The calibration of the model gave the following values of the utility function parameters:

$$z_1=0.316,\ z_2 = 0{,}668,\ z_3=0{,}016,\ \sigma= 07.$$

One can see that the consumer is most sensitive to the labor supply, then to the consumption, and essentially less sensitive to the transaction services of real money balances he possesses.

For the production function the calibrated parameters are the following:

$$x_1 = 0{,}797,\ x_2=0{,}408,\ x_3 = 0{,}368.$$

Here the share of labor in outcome is equal to 0,408, and the share of capital is 0,368; (so the production function is non-homogenous; we remind that the rate of growth of the population, and consequently, of the labor supply is equal to zero).

The rate of time preference γ is equal to the after tax real interest rate :

$\gamma= 0{,}052$.

The aggregated tax $\theta = 1 - (1-\beta)(1-\tau)= 0{,}287.$

The corresponding equilibrium steady state takes the following values:

$$c=0{,}497,\ l=0{,}67,\ m=0{,}146,\ k = 2{,}499,\ \omega=0{,}391,\ g = 0{,}278.$$

The full time T = 3, 35. Note that only the ratio of the labor time to the absolute time matters: l/T =1/5 ≈ the ratio of labor hours to the full number of hours in a year less holidays and days of rest).

Given the steady state one can calculate the auxiliary parameters r_1, r_2, s, r, R, and S (see Section 4):

$$r_1 = -\ 0{,}454,\ r_2 = -1{,}239,\ s = 0{,}136,\ r = 1{,}836,\ R = 12\times 10^{-3},\ S = 0{,}082.$$

One can see that the signs of all these parameters agree with those marked before in Section 4. In particular, the assumption about the signs: $r > 0$ and $s > 0$, fulfills. So, all the results concerning the comparative statics and the society losses obtained in Section 4 should be confirmed here.

Now we give some calculation results.

Таблица 2 (marginal effects; x= a variable, x_π, x_θ = the derivatives).

x	x_π	X_θ
k	-0.384	-5.169
l	-0.192	-1,087
m	-0,825	-0,119
c	0,077	-0,405
ω	0,037	-0,438
E	0,133	0,294
M	19,037	0,788

The comparing tables 1 and 2 shows that the calculations really confirm the theoretical results. Taxes τ and β (through the aggregated tax θ) exert the contraction influence on all variables, especially visible on capital and labor supply (actually the desire to work and invest decreases when the taxes grow). Unlike θ the influence of the inflation tax rate on the consumption and real wage rate is positive. The increasing inflation implies the decreasing labor supply which in turn implies the growth of the real wage rate. The exogenous donation e the consumer receives for supporting his welfare level is spent on the consumption, and gives a positive effect in the consumer demand. The high relative marginal losses from inflation M_π= 19,037 follows from a very small value of its denominator g_π = 0,007. Thus at 12% inflation rate the increasing of the budget income at the expense of growing inflation costs 20-fold losses for the consumer, and vice versa, a small decreasing of the budget income would lead to 20-fold gains for the consumer! (Of course, this is true only in the relative marginal sense, the marginal losses E_π are quite moderate, and the absolute integral values will be insignificant, see the next item).

Another picture comes out for marginal losses from taxes. According Table 2 given taxes level θ = 28,7% an additional ruble to the budget income costs 1,79 rubles for the consumer. The marginal income to the budget from taxes $g_\theta = 0{,}372$. If we understand values $1/M_\pi$, $1/M_\theta$ as efficiency of tax levies then for taxes τ and β this value 24 times more than for the inflation tax. Point θ = 28,7%, obviously, is located sufficiently far from the point of maximum of the corresponding Laffer curve.

The next two tables present the solutions and marginal evaluations for the neighboring steady states depending on changing the inflation and tax rates. (The range of changing the tax and inflation rates was chosen so that the neighboring equilibrium states do not deviate from the original more than 20% in any of six parameters. This was done in order for the initially calibrated model remained valid).

Таблица 3 (rate of inflation π changes from 10% to 16%).

	10	**11**	**12**	**13**	**14**	**15**	**16**
Y	0,774	0,774	0,774	0,775	0,775	0,775	0,775
m	0,166	0,155	0,146	0,138	0,131	0,124	0,118
g	0,277	0,278	0,278	0,278	0,279	0,279	0,279
E_π	0,144	0,139	0,133	0,128	0,123	0,119	
M_π	10,02	13,48	19,05	29,48	56,1	266,7	
g_π	0,014	0,01	0,007	0,0043	0,0022	0,0004	-0,000
Seniorage	0,017	0,017	0,018	0,018	0,018	0,019	

Таблица 4 (tax rate θ changes from 22 to 36%).

	22	**25**	**28**	**30**	**32**	**36**
Y	0,785	0,781	0,776	0,772	0,769	0,761
m	0,158	0,153	0,147	0,144	0,14	0,133
g	0,229	0,251	0,273	0,288	0,303	0,333
E_θ	0,224	0,254	0,286	0,309	0,334	0,391
M_θ	0,485	0,598	0,749	0,881	1,051	1,596
g_θ	0,463	0,424	0,382	0,351	0,318	0,245

One can see from Table 3 that the influence of inflation on outcome Y, budget income g, and seniorage is insignificant, while the real money balances m visibly decrease. The value E_π is positive and monotonously decreases in inflation, that is the function of society losses $E(\pi)$ looks like an increasing concave function. The value of relative marginal losses M_π reaches its pike at the inflation rate ≈ 15% and then fall. (At this point the denominator of M_π equal to $|\partial g/\partial \pi|$ is close to zero). Obviously, we are near the maximum point of the Laffer curve as function of budget income $g(\pi)$.

Table 4 shows that the tax θ exerts more perceptible influence on the real variables. The outcome, real money balances, and marginal budget income fall, while the budget income, marginal and relative marginal losses grow. Obviously, in this range we are far from the maximum of the corresponding Laffer curve.

Thus, we come to the following conclusions:

choosing the more effective (per one additional ruble to the budget) way of filling up the budget, one should choose first the non-distorting taxation of the profit (α) that brings no losses, then income + added value taxation (θ), and at last the inflationary taxation (π).

the society (and the consumers) would gain from reducing inflation with compensation of losses of the budget income by means of some increasing the physical taxes. (Note that in the majority of developed countries the inflation rate fluctuates ordinarily around 2-3%).

Integral value of society losses

Let us calculate the society losses from finite changing tax and inflation rates. We remind the corresponding formulae:

$$E_1 = \int_{\pi_0}^{\pi_1} m(\theta_0, \pi, \alpha_0)d\pi + \pi_0 m_0 - \pi_1 m_1 + \theta_0(Y_0 - Y_1)$$ = losses from changing inflation from π_0 to π_1;

$$E_2 = \int_{\theta_0}^{\theta_1} Y(\theta, \eta_0, \alpha_0)d\theta + \theta_0 Y_0 - \theta_1 Y_1 + \pi_0(m_0 - m_1)$$ = losses from changing of taxes from θ_0 to θ_1.

We calculate the society losses from the reducing inflation on 2%, that is, from 12% to 10% (planned for 2004 year). The losses of the budget income from inflation will be compensated by some increasing of taxes. The approximate calculations on the formulae give the following losses:

E_1 = - 0,0022, and Δg = 0,277- 0,278 = - 0,001.

This means that the reducing inflation from 12% to 10% the society gains 0,2% GDP, while the budget income looses 0,1% GDP.

Now, we increase θ from the current 28,7% to 28,8%. Then the formula of losses from taxes gives: $E_2 \approx 0$, and Δg = 0.279 - 0,278 = 0,001. The summary result from this operation is: $E_1 + E_2 \approx$ - 0,0022. Thus, the society gains 0,22% GDP, while the budget income remains the same.

Of course, the losses of the budget income from the declared reducing the rate of inflation can be compensated at the account of growth that is not studied here. We tried only to show of principle possibility of advantageous reducing inflation for the society.

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