Contents

Chapter 1

Introductory part

1.1 Introduction

The scientific researches and developing of new technologies (R&D) bring in more advantage to society, than profit benefited by innovator itself. That means positive externality and thus a capability of underinvestment in R&D. Objective of the government is to eliminate such market failures whenever possible.

In spite of the fact that there is a huge number of literature dedicated to the role of the government in stimulation technological progress and scientific developments, the answer to a question: "under what conditions such interference is efficient?" still is not settled.

The different aspects of stimulation of innovatory process by government were already considered in a series of works. Romer, 1990 [18], Segerstrom, Annant, Dinopolus 1990 [13] analyzed a capability of firm stimulation by government to allocate more resources on R&D. Grossman, Helpman 1991 [7] [8], Aghion and Howitt 1992 [3] examined influence of competitiveness of markets on efficiency of stimulation. Segerstrom 2000 [16] investigated dependence on parameters of capital accumulation and mean costs on innovation. So the effectiveness of government interference in economy is determined by composite combination of characteristics of economic medium.

In this work the model of endogenous growth with step-by-step innovations and its modifications are investigated. For the first time within the framework of unified model such characteristics as competitiveness on the markets, ease of imitations or rigidity of the patent legislation and austerity of its adherence, and minimum size of innovations possible in industry are selected.

Three different schemes of subsidizing innovators are analyzed:

(1) transfers to firms, competing at the same technological level;

(2) grants, proportional to investments in R&D;

(3) grants for implementation of new inventions:

(3i) the grants for implementation of already existing technology on the firm, lagging in technological development,

(3ii) the grants for innovation to firms which were on the same technological level with other competitors, and managed to step ahead of the others ("to be pulled out forward"), (3iii) the grants to the leader in industry to engage its innovative efforts.

The work is organized in the following way:

In the introductory part the review of the literature on the subjects of the work and description of Aghion's 2001 model (Aghion et al., 2001 [1]), taken as a base for further research, are placed. The advantages of this model are: it allows to observe features of innovatory process depending on competitiveness on the markets and on potentials of imitations in economy.

The second chapter is dedicated to analysis of government subsidization to competing on the same technological level innovators at the expense of the profit tax for all firms in economy. The problem investigated is: "under what conditions such policy is effective?" In particular, it is shown, that the best effect such scheme gives in case of a strict competition on the markets of finished products, under condition of relative difficulty of imitations, and is ineffective in remaining cases. The size of innovations is also important: for large innovations the scheme is more effective, than in case of gradual innovations of a small-scale size.

In the third chapter the subsidizing of **efforts**, is reviewed: the grants are proportional to investments in innovations/imitations. The application of the second scheme always gives a positive outcome. Thus it is most effective for industries with gradual (small) innovations in case of a mild competition. It is interesting to note, that this scheme is most effective when the initial growth rate had rather small value.

Financing **achievements** — the issue of the fixed grant to firms successfully realizing new technology is reviewed in the fourth chapter. Two cases are investigated separately: case when firm introduces already known, but new concretely for this firm technology (imitation), and case, when firm invents something essentially new (innovations themselves). The analysis of these schemes demonstrates, that encouraging of imitations(at the expense of tax for all producers) can not stimulate economic growth. At the same time encouraging of innovation made by the leading firm in industry has a positive effect for economic growth rate. Thus the maximum effect is reached in case of a rigid competition on the markets characterized by strict patent policy.

Though, on average, subsidizing of efforts or the grants for the introduced innovations, from the idealized point of view, are more effective, however their application is connected with a number of difficulties. First of all it is connected with the fact that it is impossible to determine exactly, how much resources were spent on R&D or how much justified an innovation was. Therefore introduction of these schemes can result in distortions of stimulus of innovators.

Other difficulty is, that any government reallocating is connected with a capability of rent seeking by the officials. That means, that not all collected taxes can be effectively reallocated.

Besides, the very simple frame of economy is supposed in the model: there are only two firms in each industry. Therefore it fails to receive estimations for efficiency of government stimulation of a firm occupying a "mean" position in industry.

Another assumption of the model is that for firms only their relative position in industry is matter.

Thus the absolute level of development of technology is not important. However in real life capabilities for innovations for firms depend on absolute values of technological level.

Due to these difficulties and simplifications obtained results can differ from an actual situation.

1.2 Literature Review

The fact, that for technological growth both innovation, and imitation are important, is widely known (see Dinopolus, 1991 [6], Grossman and Helpman, 1991 [8], Segerstrom, 1991 [14], Aghion and Howitt 1998 [2]). The interaction between innovations and imitations has several aspects. On the one hand, they intensify one another: successful innovator opens additional capabilities for imitations, while imitations propagate and accelerate application of innovations. On the other hand, they suppress one another: the fast process of innovations often makes investments in imitations meaningless, and too intensive imitations reduce attractiveness of innovative activity by augmenting competition on product markets (PMC) and reducing rent from innovation.

In early-Shumpeterian approach to models of endogenous economic growth the main attention was given to monopolistic rent, received by successful innovator. Actually, incentives to realize innovations depend not on the rent, obtained in case of success, but on the *relative rent*, that is a difference between the rent in case of successful innovation and the rent in the case then innovation is not carried out. Firm already being a monopolist, does not invest in innovations because of Arrow effect: as it already receives a monopoly rent it has less incentives to innovate, than the outsider, and if R&D technology has a constant return to scale, innovations are provided only by firms - outsiders (Aghion and Howitt, 1992 [3]). In such models the increase of product markets competition results in reduction of monopolistic rent, and this leads to decrease of incentives to innovate and, consequently to decrease of equilibrium growth rate in an economy.

Similarly decrease of costs of imitations and soft patent policy should reduce incentives to innovate, reducing anticipated duration of obtaining of innovative rent (Zeng, 1993, Zeng, 2001 [19, 20] or Davidson and Segerstrom, 1998 [12]).

However, the increase of markets competition, even if it reduces total profit in industry, can also stimulate R&D, augmenting *relative* profit of the leader, that is strengthen motives to innovate in order *to avoid* competition.

Empirical data which are finding out a direct correlation between product markets competition (PMC) and productivity growth rate in industry is possible to find in several works (Nickell, 1999 [11], Blundell et al. 1995 [5]). For explanation of this tendency Barro and Sala-i-Martin, 1995 [4] ch. 7, and Segerstrom and Zolnierek, 1999 [15] have offered a model which is taking into account the fact, that leader has smaller, in comparison with technologically-backward firm, costs on realization of innovations.

Technological advance and scientific developments influence not only on profit of producers, but also create favorable conditions for further developments and increase of public welfare. Thus, there is a possibility of "market failures" — underinvestment in R&D. Such market failures can justify government interference into economy to correct them.

It was established by Romer, 1990 [18], Segerstrom, Annant, Dinopolus, 1990 [13], Grossman and Helpman, 1991 [7], and Aghion and Howitt, 1992 [3] that subsidizing can stimulate firms to devote more resources on R&D, and, consequently, stimulate long-term economic growth.

In many respects the problem of efficiency of government subsidizing of R&D does not have definite solution till now.

Long-term effect from R&D grants was discussed in the work by Segerstrom, 2000 [16]. The generalized version of Howitt's model (Howitt, 1999 [10]) was reviewed. It was established that R&D grants can both stimulate and suppress economic growth. Thus the suppression of growth occurs in a wide range of parameters.

However in Howitt's model it is impossible to observe relation of this outcome to such characteristics as a level of product market competition and patents protection policy in the country. But it would be interesting to understand how hard (or, to the contrary, soft) patent policy (hardening of intellectual property rights) in a country can influence outcomes of the government programs of stimulation of innovations.

The model offered by Aghion (Aghion et al., 2001 [1]) allows to include these characteristics in consideration, however introduction of the government in this model has not been done yet. The further chapters of this work will be based on the Aghion's model, therefore I would like to discuss it now in more details.

1.3 Aghion's Endogenous Growth Model

(Aghion et al., 2001 [1])

In this model innovations are a stochastic process. It's intensity depends on efforts, chosen by the independent agents. These efforts, in turn, are associated with costs spend on research and development (R&D).

1.3.1 Base model

Consumers

The economics consists of continuum of industries $i \in [0,1]$. In each industries there are two producers: A and B. Consumers live indefinitely long and maximize objective function specifying intertemporal preferences:

$$U = \int_0^\infty u(t)e^{-rt}dt \longrightarrow \max, \tag{1.1}$$

$$u(t) = \int_0^1 \ln Q_i(t)di - L(t),$$

$$s.t.: \quad p_{Ai}q_{Ai} + p_{Bi}q_{Bi} = 1, \tag{1.1*}$$

$Q_i(t)$ — consumption of goods produced by industry i at time t:

$$Q_i(t) = f(q_{Ai}, q_{Bi}) = (q_{Ai}^\alpha + q_{Bi}^\alpha)^{1/\alpha},$$

q_{Ai} and q_{Bi} - outputs of each firm in industry.
p_{Ai} and p_{Bi} - normalized prices of the goods issued by each firm.
Parameter α represents a degree of substitutability between two products in one industry or, using other interpretation, is a measure of competition on the markets: $\alpha = 1$ corresponds to perfect competition, $\alpha = 0$ — absence of competition.
$L(t)$ - labor supply, $r > 0$ - discounting factor.

Actually, budget constraint is less strict then (1.1*). But as logarithmic preferences (1.1) guess that in equilibrium individuals spend equal sums of money on production of each industry Q_i. This sum is normalized on a unit.

Thus demand functions for goods of each firm in industry i

$$q_{Ai} = \frac{p_{Ai}^{1/\alpha-1}}{p_{Ai}^{\alpha/\alpha-1} + p_{Bi}^{\alpha/\alpha-1}}, \qquad q_{Bi} = \frac{p_{Bi}^{1/\alpha-1}}{p_{Ai}^{\alpha/\alpha-1} + p_{Bi}^{\alpha/\alpha-1}}. \tag{1.2}$$

Competition on product markets

For each firm labor is the sole production factor. The production function is characterized by constant return to scale. The salary $w = 1$ is exogenous for producers. The price competition between firms results in Bertran equilibrium. Elasticity of demand on production of firms

$$\eta_j = \frac{1 - \alpha\lambda_j}{1 - \alpha}, \quad j = A, B, \tag{1.3}$$

where λ_j is the firm's income:

$$\lambda_j = p_j q_j = \frac{p_j^{\alpha/\alpha-1}}{p_A^{\alpha/\alpha-1} + p_B^{\alpha/\alpha-1}}. \tag{1.4}$$

Thus, equilibrium price

$$p_j = \frac{\eta_j}{\eta_j - 1} c_j = \frac{1 - \alpha\lambda_j}{\alpha(1 - \lambda_j)} c_j. \tag{1.5}$$

Profit in equilibrium

$$\pi_j = \frac{\lambda_j}{\eta_j} = \frac{\lambda_j(1 - \alpha)}{1 - \alpha\lambda_j}. \tag{1.6}$$

At a given degree of substitutability between goods (competitiveness on the markets α) the profits of firms are determined by their relative costs: $z_j = c_j/c_{-j}$.

Costs are determined by a level of technological development, namely: $c_j = w\Lambda$, Λ — cost of a unit of production issued, which decrease γ times with each "step" of development of technology. Thus, relative costs of firm leading n steps is $z_j = \gamma^{-n}$.

Solving (for example, numerically) system (1.3)–(1.6) we shall get that the profit of firm is a function of its relative costs and parameter α:

$$\pi_j = \phi(z_j, \alpha).$$

The function $\phi(z_j, \alpha)$ has following properties:

- $\frac{\partial \phi(z, \alpha)}{\partial z} < 0$, $\alpha \in (0, 1)$;
- $\phi(z, 0) = 1/2$;
- $\phi(z, \alpha) + \phi(1/z, \alpha) > 2\phi(1, \alpha)$;
- $\phi(0, \alpha) = 1$;
- $\lim_{z \to \infty} \phi(z, \alpha) = 0$;
- $\alpha = 1 : \quad \phi(z, 1) = \begin{cases} 0 & z \geqslant 1, \quad \text{follower}, \\ 1 - z & z < 1, \quad \text{leader}; \end{cases}$
- $\phi(1, \alpha) = \frac{1 - \alpha}{2 - \alpha}$;
- $\left.\frac{\partial \phi(z, \alpha)}{\partial z}\right|_{z=1} = -\frac{\alpha}{4 - \alpha^2}$.

Expenditures on Research and Development

Innovations are a stochastic process and occur with intensity defined by efforts x, which firms spent for them. By definition x is a probability of successful innovation in a unit of time. To ensure this level of effort x innovator should spend $\psi(x)$. Thus, $\psi(x)$ is a cost function of efforts. The process of imitations goes with intensity $(x+h)$ when costs remaining the same: $\psi(x)$. That is parameter $h \geqslant 0$ measures so-called "spillover" effect (spreading effect). It characterizes relative "ease" of imitations. This parameter can be also interpreted as a measure of rigidity of legislative or administrative obstacles (concerning patent right and regulation) which limit direct usage of technological discoverings of a competitor.

Let's x_0, x_n and $\widetilde{x}_n$ be efforts of two equivalent firms, effort of n-steps-ahead leader (on innovation) and effort of n-steps-behind follower (on imitation) accordingly. Let V_0, V_n and $\widetilde{V}_n$ be an anticipated utilities from being in each of these states. In fig. 1.1 the scheme of possible changes of relative position (n) of firm in industry is shown.

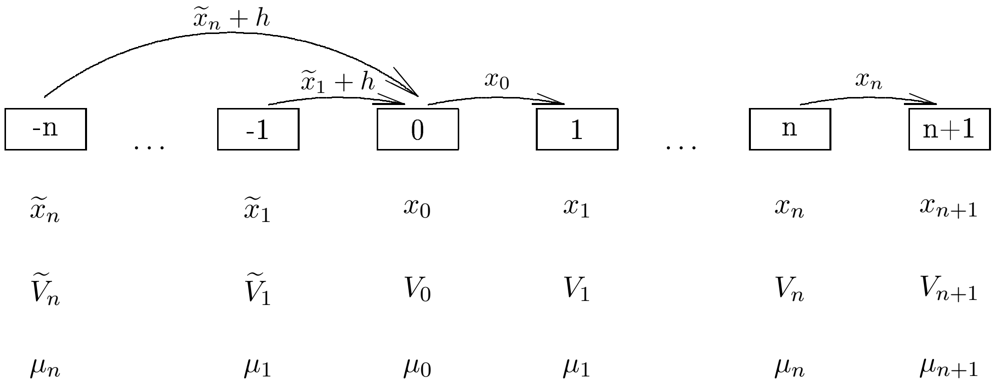

Figure 1.1: Potential opportunities of firms in different states "n". μ_j — occupancy of state j

The values V_j are determined with the help of Bellman's equations. The graphical representation of them gives possible gains of firms and probabilities of their implementation: (see. fig. 1.2).

For example, the Bellman's equation for V_n looks like:

$$V_n = \max_x \{(\pi_n - \psi(x))dt + e^{-rdt}[xdtV_{n+1} + (\widetilde{x} + h)dtV_0 + (1 - xdt - (\widetilde{x_n} + h)dt)V_n]\}. \quad (1.7)$$

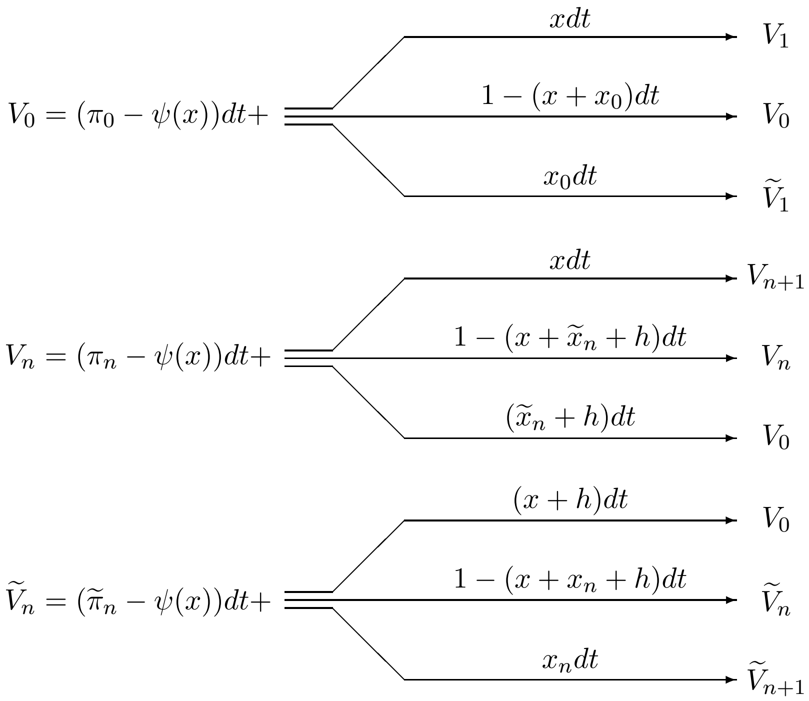

Figure 1.2: A graphical representation of Bellman's equations for value functions in each state.

Let

$$\psi(x) = \frac{1}{2}\beta x^2, \qquad \beta > 0.$$

Decomposing exponent and neglecting the second order on dt from Bellman's equations one can get the system:

$$rV_0 = \pi_0 - \frac{1}{2}\beta x_0^2 + x_0(V_1 - V_0) + \widetilde{x}_0(V_0 - \widetilde{V}_1), \tag{1.8}$$

$$rV_n = \pi_n - \frac{1}{2}\beta x_n^2 + x_n(V_{n+1} - V_n) + (\widetilde{x}_n + h)(V_0 - V_n), \tag{1.9}$$

$$r\widetilde{V}_n = \widetilde{\pi}_n - \frac{1}{2}\beta \widetilde{x}_n^2 + x_n(\widetilde{V}_{n+1} - \widetilde{V}_n) + (\widetilde{x}_n + h)(V_0 - \widetilde{V}_n), \tag{1.10}$$

$$x_0 = \frac{V_1 - V_0}{\beta}, \tag{1.11}$$

$$x_n = \frac{V_{n+1} - V_n}{\beta}, \tag{1.12}$$

$$\widetilde{x}_n = \frac{V_0 - \widetilde{V}_n}{\beta}. \tag{1.13}$$

The values V_j and x_j can be obtained by the numerical series solution of system (1.8)—(1.13).

Industrial structure in stationary state

Let's call occupancy of state n, μ_n, a share of industries in state n (the gap between leader and follower makes up n steps). On fig. 1.3 the channels of inflows and outflows of industries into each state and intensity of these processes are represented.

The outflow of industries from each state should be equal to inflow into this state in stationary situation.

$$2\mu_0 x_0 = \sum_{n\geqslant 1} \mu_n(\widetilde{x}_n + h) \tag{1.14}$$

$$\mu_1(x_1 + \widetilde{x}_1 + h) = 2\mu_0 x_0 \tag{1.15}$$

$$\mu_n(x_n + \widetilde{x}_n + h) = \mu_{n-1} x_{n-1} \tag{1.16}$$

The system (1.14)—(1.16) describes industrial pattern of economics in stationary situation.

Growth rate in stationary state

Whole output in the economy:

$$y = \ln Y = \int_0^1 \ln Q_i(t) di.$$

The growth rate is determined as

$$g \equiv \frac{dy}{dt} = \lim_{\triangle t \to \infty} \frac{\triangle \ln Q_i}{\triangle t},$$

as all industries are identical and for Q_i the ergodic theorem is valid.

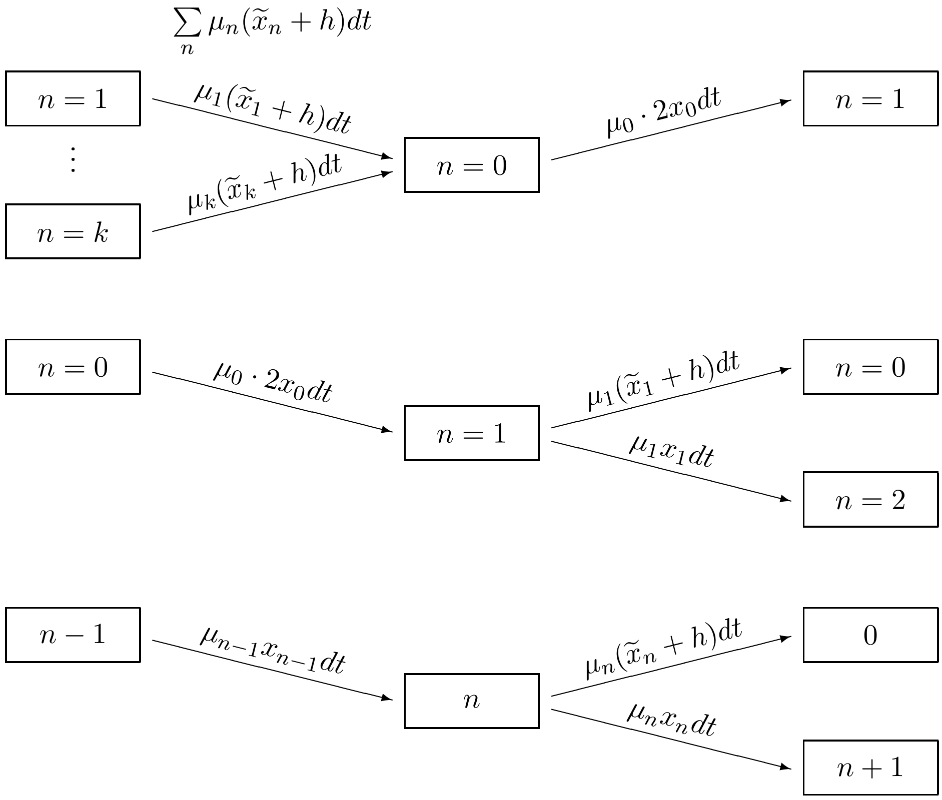

Figure 1.3: Dynamics of population occupancies of system states

It is possible to get, that

$$g = \sum_{p \geqslant 1} \mu_p(\widetilde{x}_p + h)p \ln \gamma = (2\mu_0 x_0 + \sum_{k \geqslant 1} \mu_k x_k) \ln \gamma. \qquad (1.17)$$

The set of equations (1.8)–(1.17) describes economy completely.

The complete solution of this model is possible only numerically, however its main properties can be obtained from the analytical analysis of extreme cases.

1.3.2 Case of large innovations
$(\gamma \to \infty \quad \Leftrightarrow \quad \max n = 1)$

We shall consider a special case, which is characterized by the greatest possible technological leading of one step. We approach to such situation, when the size of innovatory step is large ($\gamma \to \infty$), and leading on one step already raises profit up to the greatest possible value ($\phi(\gamma^{-1}, \alpha) \xrightarrow{\gamma \to \infty} 1, \quad \forall \alpha > 0$) (see fig. 1.4). Therefore, there are no incentives to prolong innovations ($x_1 = 0$). A firm which is one step ahead of the competitor does not conduct new researches.

In this limiting case it is possible to receive(get?) an analytical solution of the model. The set of

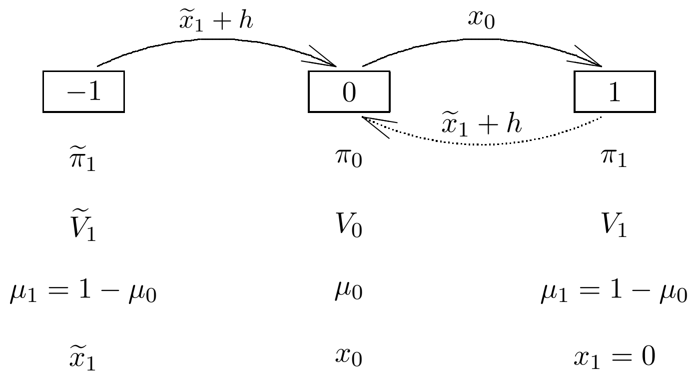

Figure 1.4: Economy with greatest possible leading in one step

equations (1.8)—(1.13) is getting simpler:

$$rV_0 = \pi_0 - \frac{1}{2}\beta x_0^2 + x_0(V_1 - V_0) + \widetilde{x}_0(V_0 - \widetilde{V}_1), \tag{1.18}$$

$$rV_1 = \pi_1 + (\widetilde{x}_1 + h)(V_0 - V_1), \tag{1.19}$$

$$r\widetilde{V}_1 = \widetilde{\pi}_1 - \frac{1}{2}\beta\widetilde{x}_1^2 + (\widetilde{x}_1 + h)(V_0 - \widetilde{V}_1), \tag{1.20}$$

$$x_0 = \frac{V_1 - V_0}{\beta}, \tag{1.21}$$

$$x_1 = 0, \tag{1.22}$$

$$\widetilde{x}_1 = \frac{V_0 - \widetilde{V}_1}{\beta}. \tag{1.23}$$

By some transformations equations for efforts in states 0 and -1 will be:

$$\frac{1}{2}x_0^2 + x_0(r + h) = \frac{1}{\beta}(\pi_1 - \pi_0), \tag{1.24}$$

$$\frac{1}{2}\widetilde{x}_1^2 + \widetilde{x}_1(r + h + x_0) = \frac{1}{\beta}(\pi_0 - \widetilde{\pi}_1) + \frac{1}{2}x_0^2. \tag{1.25}$$

Directly from these equations it can be seen that:

- $\alpha \uparrow \implies (\pi_1 - \pi_0) \uparrow \implies x_0 \uparrow$:
  the increase of competitiveness on product markets results in increase of incentives to innovate in order *to avoid* competition, and so, intensification of innovational efforts.

- $h \uparrow \implies x_0 \downarrow$:
  simplification of imitations reduces incentives to innovate.

- $(\pi_1 - \pi_0) > (\pi_0 - \widetilde{\pi}_1) \implies x_0 > \widetilde{x}_1$:
  firms located on leading edge of technology have more incentives to realize innovations, than firms lagging on their technological development.

Probability of the situation than in industry there are two firms on leading edge of technology (occupancy of a zero state):

$$\mu_0 = \frac{\widetilde{x}_1 + h}{2x_0 + \widetilde{x}_1 + h}. \tag{1.26}$$

The economic growth rate also can be obtained analytically:

$$g = 2\mu_0 x_0 \ln\gamma = \frac{2x_0(\widetilde{x}_1 + h)}{2x_0 + \widetilde{x}_1 + h}\ln\gamma = g(r, h, \beta, \pi_1 - \pi_0, \pi_0 - \widetilde{\pi}_1). \tag{1.27}$$

- $\left.\frac{\partial g}{\partial \alpha}\right|_{\alpha=0} > 0$: Even some competition is good for growth.
- Dependence on h represents the special interest:
  firstly: $\left.\frac{\partial g}{\partial h}\right|_{h=0} > 0$,
  on the other hand: $g(h = \infty) = 0$.

  As g is continuous on h, it means, that there is a best value of ease of imitations h, at which the economic growth rate reaches its maximum value.

1.3.3 Case of small innovations
($\gamma \to 1 \quad \gamma = 1 + \varepsilon$)

We shall now consider another special case, which is characterized by minimum size of innovations (γ aims at its lower boundary, unit: $\gamma \to 1$). For the solution the method of asymptotic decomposition will be used. Thus the cases $\alpha < 1$ and $\alpha = 1$ should be considered separately (as the profit function $\phi(\cdot, \alpha)$ smooth at $\alpha < 1$ and has a kink at $\alpha = 1$).

Case when $\alpha < 1$

Let $\varepsilon = \gamma - 1 > 0$.

Then

$$V_0 = \frac{1}{r}\phi(1, \alpha) + O(\varepsilon^2), \tag{1.28}$$

$$V_n = \frac{1}{r}\phi(1, \alpha) + n\beta\eta\varepsilon + O(\varepsilon^2), \tag{1.29}$$

$$\widetilde{V}_n = \frac{1}{r}\phi(1, \alpha) - n\beta\eta\varepsilon + O(\varepsilon^2), \tag{1.30}$$

where $$\eta = \frac{1}{\beta(r+h)}\left(-\frac{\partial\phi(1, \alpha)}{\partial z}\right) > 0. \tag{1.31}$$

The innovatory efforts will be the following:

$$x_0 = \eta\varepsilon + O(\varepsilon^2), \tag{1.32}$$

$$x_n = \eta\varepsilon + O(\varepsilon^2), \tag{1.33}$$

$$\widetilde{x}_n = n\eta\varepsilon + O(\varepsilon^2). \tag{1.34}$$

The probability of industry to be in zero state, can be determined as:

$$\mu_0 = \breve{\mu}_0 + O(\varepsilon),$$

$$\breve{\mu}_0 = \frac{1}{1 + 2f(h/(\eta\varepsilon))},$$

where $f(\zeta)$ [1] has a property:

$$-1 < \frac{\zeta f'(\zeta)}{f(\zeta)} < 0$$

And growth rate of production volume:

$$g = \left(2\mu_0 x_0 + \sum_{n=0}^{\infty} \mu_n x_n\right) \ln\gamma = (1+\mu_0)\eta\varepsilon^2 + O(\varepsilon^3) \approx \frac{2(1+f(\zeta))}{\zeta(1+2f(\zeta))} h\varepsilon, \tag{1.35}$$

$$\text{where} \quad \zeta = h/\eta\varepsilon.$$

Resume:

- The influence of increase of competitiveness on economic growth rates. From the one hand:
  $\alpha \uparrow \implies \eta \uparrow \implies x_0, x_n, \widetilde{x}_n \uparrow \implies g \uparrow$(effort effect).
  On the other hand:
  $\alpha \uparrow \implies \eta \uparrow \implies \mu_0 \downarrow$ (since $\frac{\partial \mu_0}{\partial \eta} < 0$) $\implies g \downarrow$(composition effect)
  Total effect:
  $\alpha \uparrow \implies \eta \uparrow \implies \zeta = \frac{h}{\eta\varepsilon} \downarrow \implies g \uparrow$ (since $\frac{\partial g}{\partial \zeta}\Big|_{h=const} < 0$). That is the growth rate positively depends from α.

- Let's consider now increase of intensity of imitations.
  $h \uparrow \implies \eta \downarrow \implies x_0, x_n, \widetilde{x}_n \downarrow \implies g \downarrow$(effort effect).

  On the other hand, the increase of intensity of imitations augments occupancy of neck-and-neck state.
  $h \uparrow \implies \zeta = \frac{h}{\eta\varepsilon} \uparrow \implies f(\zeta) \downarrow \implies \widetilde{\mu}_0 \uparrow \implies g \uparrow$(composition effect)
  While intensity of imitations rises
  $h \uparrow \implies \zeta = \frac{h}{\eta\varepsilon} \uparrow \implies g \downarrow$

- Generally, total effect is ambiguous. However, it is possible to conclude, that:
  $\frac{\partial g}{\partial h}\Big|_{h=0} > 0,$
  $g(h \longrightarrow \infty) \longrightarrow 0$
  $g(h \longrightarrow 0) \longrightarrow g_0 > 0$
  As g - is continuous on h, it means, that there is a best value of h, at which the economic growth rate reaches its maximum value.

Case when $\alpha = 1$

Let $\varepsilon = \gamma - 1$, $\varepsilon \to 0$

Then profit of firms

$$\pi_0 = O(\varepsilon^2), \tag{1.36}$$

$$\pi_n = 1 - \gamma^{-n} + O(\varepsilon^2) = n\varepsilon + O(\varepsilon^2), \tag{1.37}$$

$$\widetilde{\pi}_n = O(\varepsilon^2). \tag{1.38}$$

[1] the description of a function $f(\zeta)$ see Aghion 2001

With these suppositions, it is easy to get, that

$$V_0 = O(\varepsilon^2), \tag{1.39}$$

$$V_n = n\beta\eta\varepsilon + O(\varepsilon^2), \tag{1.40}$$

$$\widetilde{V}_n = O(\varepsilon^2), \tag{1.41}$$

where $$\eta = \frac{1}{\beta(r+h)} > 0. \tag{1.42}$$

The efforts on innovation will be as following

$$x_0 = \eta\varepsilon + O(\varepsilon^2), \tag{1.43}$$

$$x_n = \eta\varepsilon + O(\varepsilon^2), \tag{1.44}$$

$$\widetilde{x}_n = O(\varepsilon^2). \tag{1.45}$$

The probability of being in zero state, $\mu_0 = \breve{\mu}_0 + O(\varepsilon)$, is

$$\breve{\mu}_0 = \frac{1}{1 + 2\eta\varepsilon/h}. \tag{1.46}$$

And economic growth rate is

$$g = \left(2\mu_0 x_0 + \sum_{n=1}^{\infty} \mu_n x_n\right) \ln\gamma \approx \frac{2(1+\frac{\eta\varepsilon}{h})}{\frac{h}{\eta\varepsilon}(1+2\frac{\eta\varepsilon}{h})} h\varepsilon. \tag{1.47}$$

From this expression it is easy to get that

- $\left.\dfrac{\partial g}{\partial h}\right|_{h=0} > 0,$
- $g(h \longrightarrow \infty) \longrightarrow 0,$
- $g(h \longrightarrow 0) \longrightarrow g_0 > 0,$

As g is continuous on h, it means, that there is a best value h, at which the economic growth rate reaches its maximum.

All these results have been obtained by Aghion, Howitt et al. and more in-depth interpretation can be found in their article (Aghion and Howitt et al., 2001 [1]). In the context of this work the description of the model itself is important.

All remaining parts of the work represent independent research operating this model as the base.

Chapter 2

When do subsidies to competing at the same technological level innovators stimulate technological progress?

In this chapter we shall address the following problem: under what conditions government interference, in particular, subsidizing of competing at the same technological level innovators, can stimulate technological progress?

Let π be a profit of a firm if there are no taxes. The source of means on the subsidy is the profit taxes imposed on all firms in the economy. The tax rate is τ. The disposable profit of a firm now will be $(1-\tau)\pi$.

In the Aghion's model [1] the greatest contribution to economic growth is given by the industries, in which firms compete at the same technological level (this industry is in "zero state"). We call this situation "neck-and-neck" competition (see [1]). Let's estimate, how do subsidizing of such firms will influence economic growth rate. Let S be the size of the subsidy. Disposable profit of firms in neck-and-neck state (with subsidy) is $(1-\tau)\pi_0 + S$.

The changes in Bellman equations will cause changes in system (1.8)—(1.11). Now it can be written

[1] described in the introductory part of this work and used here as a base one.

as:

$$rV_0 = (1-\tau)\pi_0 + S - \frac{1}{2}\beta x_0^2 + x_0(V_1 - V_0) + \widetilde{x}_0(V_0 - \widetilde{V}_1), \tag{2.1}$$

$$rV_n = (1-\tau)\pi_n - \frac{1}{2}\beta x_n^2 + x_n(V_{n+1} - V_n) + (\widetilde{x}_n + h)(V_0 - V_n), \tag{2.2}$$

$$r\widetilde{V}_n = (1-\tau)\widetilde{\pi}_n - \frac{1}{2}\beta \widetilde{x}_n^2 + x_n(\widetilde{V}_{n+1} - \widetilde{V}_n) + (\widetilde{x}_n + h)(V_0 - \widetilde{V}_n), \tag{2.3}$$

$$x_0 = \frac{(V_1 - V_0)}{\beta}, \tag{2.4}$$

$$x_n = \frac{(V_{n+1} - V_n)}{\beta}, \tag{2.5}$$

$$\widetilde{x}_n = \frac{(V_0 - \widetilde{V}_n)}{\beta}, \tag{2.6}$$

$$2\mu_0 S = \tau(2\mu_0\pi_0 + \sum_{k=1}^{\infty} \mu_k(\pi_k + \widetilde{\pi}_{-k})), \tag{2.7}$$

where the last equation represents budget constraint for the government (money spent for subsidies are equal to taxes collected).

At arbitrary acceptable values of parameters it is possible to solve the system numerically only. Therefore we shall provide solutions for two particular relevant cases: namely, for cases of "large" ($\gamma \to \infty$) and "small" ($\gamma \to 1$) innovations. From this analysis we will identify main characteristics and features of the solution.

2.1 Subsidy in case of large innovations: the scheme is effective on competitive markets when patent policy is strict ($\gamma \to \infty \quad \Leftrightarrow \quad \max n = 1, \quad x_1 \equiv 0$)

Let's consider a situation, for which the greatest possible technological difference makes one step. If size of innovatory step is great ($\gamma \to \infty$), then profit of one-step leader grows up to the greatest possible value ($\phi(\gamma^{-1}, \alpha) \overset{\gamma\to\infty}{\longrightarrow} 1, \quad \forall \alpha > 0$) and there are no other incentives to continue innovations. The firm located on the leading edge of technology does not conduct new researches any more.

In this case it is possible to get analytical results of the model.

The set of equations(2.1)—(2.6), (2.7) will be following:

$$x_1 = 0, \tag{2.8}$$

$$\beta\widetilde{x}_1 = V_0 - \widetilde{V}_1, \tag{2.9}$$

$$\beta x_0 = V_1 - V_0, \tag{2.10}$$

$$rV_0 = (1-\tau)\pi_0 + S - \frac{1}{2}\beta x_0^2 + x_0[\beta x_0] - \beta\widetilde{x}_0\widetilde{x}_1, \tag{2.11}$$

$$rV_1 = (1-\tau)\pi_1 - \beta(\widetilde{x}_1 + h)x_0, \tag{2.12}$$

$$r\widetilde{V}_1 = (1-\tau)\widetilde{\pi}_1 - \frac{1}{2}\beta\widetilde{x}_1^2 + \beta(\widetilde{x}_1 + h)\widetilde{x}_1, \tag{2.13}$$

$$S = \frac{\tau}{2\mu_0}[\mu_0\pi_0 + \mu_1(\pi_1 + \widetilde{\pi}_1)]. \tag{2.14}$$

Equations for efforts in states "0" and "-1" are:

$$\frac{1}{2}x_0^2 + x_0(r+h) = \frac{1}{\beta}(1-\tau)(\pi_1 - \pi_0) - \frac{S}{\beta} =$$
$$= \frac{1}{\beta}(\pi_1 - \pi_0 + \frac{\tau}{2}(\pi_0 - (2 + \frac{\mu_1}{\mu_0})\pi_1 - \frac{\mu_1}{\mu_0}\widetilde{\pi}_1)), \tag{2.15}$$
$$\frac{1}{2}\widetilde{x}_1^2 + \widetilde{x}_1(r+h+x_0) = \frac{1}{\beta}(1-\tau)(\pi_0 - \widetilde{\pi}_1) + \frac{S}{\beta} + \frac{1}{2}x_0^2 =$$
$$= \frac{1}{\beta}(\pi_0 - \widetilde{\pi}_1 + \frac{\tau}{2}(-\pi_0 + \frac{\mu_1}{\mu_0}\pi_1 + (2 + \frac{\mu_1}{\mu_0})\widetilde{\pi}_1). \tag{2.16}$$

To analyze the effect from such scheme of subsidizing, we calculate how do efforts of agents will change with tax rate τ:

$$\frac{\partial x_0}{\partial \tau} = \frac{\pi_0 - (2 + \frac{\mu_1}{\mu_0})\pi_1 - \frac{\mu_1}{\mu_0}\widetilde{\pi}_1}{2\beta(x_0 + r + h)} < 0, \tag{2.17}$$
$$\frac{\partial \widetilde{x}_1}{\partial \tau} = \frac{-\varkappa\pi_0 + \nu\pi_1 + \varsigma\widetilde{\pi}_1}{2\beta(\widetilde{x}_1 + x_0 + r + h)(x_0 + r + h)}, \tag{2.18}$$
$$\varkappa = \widetilde{x}_1 + r + h, \tag{2.19}$$
$$\nu = 2x_0(1 + \frac{\mu_1}{\mu_0}) + x_1(2 + \frac{\mu_1}{\mu_0}) + (r+h)\frac{\mu_1}{\mu_0}, \tag{2.20}$$
$$\varsigma = x_0(1 + \frac{\mu_1}{\mu_0}) - \widetilde{x}_1\frac{\mu_1}{\mu_0} + (r+h)(2 + \frac{\mu_1}{\mu_0}). \tag{2.21}$$

The formulas for an estimation of occupancy of zero state and economic growth rates will stay the same (in terms of x_j):

$$\mu_0 = \frac{\widetilde{x}_1 + h}{2x_0 + \widetilde{x}_1 + h}, \tag{2.22}$$
$$g = 2\mu_0 x_0 \ln\gamma = \frac{2x_0(\widetilde{x}_1 + h)}{2x_0 + \widetilde{x}_1 + h}\ln\gamma. \tag{2.23}$$

As follows from (2.17) instead of enhancing innovational efforts in industries with neck-and-neck competing innovators, subsidy suppress them. As for imitational efforts of technologically lagging firm they can both decrease, or increase (see (2.18)). The profit in the zero state, π_0 in the expression for a derivative of efforts of "catching up" firm on τ is included with sign "$-$"$\dot{\text{T}}$he larger the relative profit is, the larger the profit tax is. And this reduces relative profit and incentives to imitate. Other profits enter with sign "+". Their values augment distributed money, so enhance incentives to imitate.

So, we've got

Proposition 2.1. The introduction of a tax (and corresponding subsidy):

(i) results in reduction of efforts on innovations;

(ii) can augment imitational efforts (if $\pi_0 < \frac{\mu_1}{\mu_0} = \frac{2x_0}{x_1+h}$), as well as reduce them;

(iii) increases costs of imitations (as well as the reduction of costs on innovations) which results in increase of occupancy of zero state.

Let's examine a question how does the introduction of a tax and a subsidy influences economic growth rate. For this purpose we shall calculate a derivative of a growth rate under the tax rate (when tax rate is equal zero):

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} = \overbrace{\frac{\partial g}{\partial x_0}}^{>0}\overbrace{\frac{\partial x_0}{\partial \tau}}^{<0} + \overbrace{\frac{\partial g}{\partial \widetilde{x}_1}}^{>0}\overbrace{\frac{\partial \widetilde{x}_1}{\partial \tau}}^{?} \tag{2.24}$$

Let's consider behavior of this expression in some limiting cases.

- Let parameter of ease of a imitation be large:

$$h \gg \widetilde{x}_1, \quad x_0 \quad \Longrightarrow \quad \mu_0 \longrightarrow 1 \quad \Longrightarrow \quad \frac{\partial \widetilde{x}_1}{\partial \tau} < 0 \quad \Longrightarrow \quad \left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} < 0,$$

  that is

  Proposition 2.2. If patent policy is soft and rights to intellectual property are badly protected then effect from subsidizing innovators competing on the same technological level is negative.

- Let now imitations be hindered, and markets be monopolized: $h \longrightarrow 0; \quad \alpha \longrightarrow 0. \quad \Longrightarrow$
  $\Longrightarrow \quad (\pi_1 - \pi_0) \approx (\pi_0 - \widetilde{\pi}_1), \quad \Longrightarrow \quad x_0 \approx \widetilde{x}_1, \quad \Longrightarrow \quad \frac{\mu_1}{\mu_0} \approx 2.$
  $\Longrightarrow \quad \left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \approx \frac{-\pi_0}{9\beta(x_0 + r)} < 0.$

  Proposition 2.3. If competition on the markets is relatively low (monopolistic power and profit of each of firm, competitive on leading edge of technology are sufficiently large) then an introduction of the subsidies to neck-and-neck innovators is inefficient.

- However, there is also such area of parameters, where a derivative $\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} > 0.$
  Let imitations be hindered, and markets be competitive. $h \longrightarrow 0; \quad \alpha \longrightarrow 1. \quad \Longrightarrow$
  In this case non-zero profit is received only by the actual leader: $\widetilde{\pi}_1 \approx 0, \quad \pi_0 \approx 0; \quad \pi_1 \approx 1.$
  So, the efforts on imitation practically are not applied: $x_1 = 0 \quad \widetilde{x}_1 \approx 0; \quad x_0 > 0:$
  In each industry step-by-step only one leader remains , who receives monopoly profit, and nobody attempts to imitate his achievement: $\Longrightarrow \quad \mu_0 \approx 0; \quad \mu_1 \approx 1$. Further economic growth is intercepted. In this case

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \approx \frac{\pi_1}{2\beta(x_0 + r)} \left(\frac{\mu_1}{\mu_0} r - 2x_0\right) \overset{\frac{\mu_1}{\mu_0} \longrightarrow \infty}{\widehat{\longrightarrow}} \infty. \tag{2.25}$$

  Proposition 2.4. At highly competitive markets (low profit of each neck-and-neck firm) the introduction of the subsidies to neck-and-neck innovators is effective.

  This fact has the intuitive economical content. In a situation with highly competitive market (absolutely substituted goods) and large size of innovations, jump from "lagging" in the "competitive" condition demands costs and does not give a scoring in the profit. Therefore, lagging firm do not undertake efforts, in order to overtake the leader. If "charge-free" imitations in industry are absent ($h = 0$) then one leader (not interested in further innovations, as he already receives monopolistic profit) remains in industry. The development of economy is intercepted. If in this situation the indicated policy (subsidy to neck-and-neck competitors) is applied [2], the investments in imitation become attractive to firms, and this will urge forward development.

[2] we consider, that the firms in this case though compete under the price, lowering the price down to a marginal cost and receive zero profit on realization of production, nevertheless subsidy take to itself.

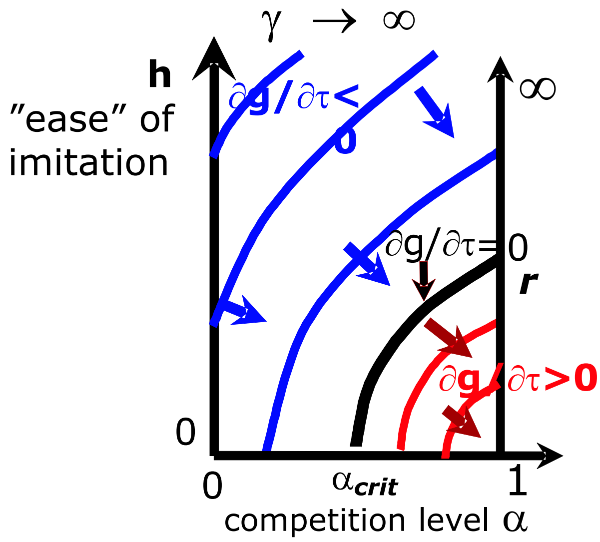

Figure 2.1: Change of economic growth rate while introducing minimal subsidy to neck-and-neck innovators (case of large innovations)

Specially this will be notable at the initial phase, while the fraction of leaders μ_1 is great enough and the concentration of industries with an evenly developed technologies, μ_0, is low. Then growth gradually (step-by-step) slow down, aims at some equilibrium value.

It is possible to show, that at the zero initial tax rate, its increase has a positive effect to equilibrium growth rate, however after some critical value, its further increase though can increase growth rate in short-run, results in decrease of an equilibrium (long-term) growth rate.

Propositions 2.1–2.4 can be represented graphically (see fig. 2.1).

- For an intermediate level of competitiveness on the markets and ease of imitations it can be shown (substituting (2.17) and (2.18) in (2.24)), that the introduction of the subsidy is effective when

$$\pi_0(\alpha) < \pi_1 \left(\frac{\mu_1}{\mu_0} - 2\frac{2x_0^3 + (\widetilde{x}_1 + h)^2(x_0 + \widetilde{x}_1 + r + h)}{2x_0^2(2\widetilde{x}_1 + r + h) - (\widetilde{x}_1 + h)^2(x_0 + \widetilde{x}_1 + r + h)} \right). \tag{2.26}$$

  Substituting expressions for μ_0 and μ_1 in 2.26 it is possible to show, that essential (but not sufficient) conditions to fulfill this inequality are:

$$x_1 + h < x_0, \tag{2.27}$$

$$\frac{(x_1 + h)^2}{2x_0^2} \frac{x_0 + \widetilde{x}_1 + r + h}{2\widetilde{x}_1 + r + h} < 1. \tag{2.28}$$

  The expression (2.27) means, that total intensity of imitations (which depends both on efforts of imitator x_1, and on "spontaneous" imitations h) should be less then the intensity of innovations.

Thus, the basis of the efficiency of the subsidy is the strong easing (in a limit - down to zero point) of the process of imitations which, in turn, could originate at the expense of reduction of the profit in neck-and-neck state, as well as due to parameter h (describing ease of imitations) which aims to zero. In this situation attractiveness of possession of higher technology (due to subsidizing) results in increase in growth rate.

The policy of subsidizing competing on the same technological level innovators has legible alternative: to increase attractiveness of possession of high technology instead of the introducing subsidy it is possible to soften too rigid markets (decrease competition). Other alternative is: "softening" attitude to imitation (increase h). Efforts in this direction also results in increase of growth rate in Case of too strict patent policy (but can result in decrease in other situations). However the question: whether it is necessary to apply the subsidies, or to correct institutions directly? — demands separate consideration out of the framework of this paper.

2.2 Subsidy in case of small innovations: the efficiency is reduced

$(\gamma \to 1, \quad \gamma = 1 + \varepsilon)$

Let's consider a case, which is characterized by minimum size of innovations (γ aims at the lower boundary, $\gamma \to 1$). We use a method of asymptotic decomposition on small parameter ε.

Equations set (2.1)—(2.6) will be transformed into:

$$rV_0 = (1-\tau)\pi_0 + S - \frac{1}{2}(V_1 - V_0)^2 + (V_1 - V_0)(\widetilde{V}_1 - V_0), \tag{2.29}$$

$$rV_n = (1-\tau)\pi_n - \frac{1}{2}(V_{n+1} - V_n)^2 + (V_0 - V_n)(V_0 - \widetilde{V}_n) + \beta h(V_0 - V_n), \tag{2.30}$$

$$r\widetilde{V}_n = (1-\tau)\widetilde{\pi}_n - \frac{1}{2}(V_0 - \widetilde{V}_n)^2 + (V_{n+1} - V_n)(\widetilde{V}_{n+1} - \widetilde{V}_n) + \beta h(V_0 - \widetilde{V}_n). \tag{2.31}$$

It is necessary to considered cases $\alpha < 1$ and $\alpha = 1$ separately, as the profit function $\phi(\cdot, \alpha)$ is smooth at $\alpha < 1$ and has a kink at $\alpha = 1$.

Case $\alpha < 1$

Let $\varepsilon = \gamma - 1$. Then, using asymptotic decomposition of profit functions $\phi(z, \alpha)$ and value functions V_j, neglecting quadratic on ε terms, we receive profits:

$$\pi_0 = \phi(1, \alpha) + O(\varepsilon^2), \tag{2.32}$$

$$\pi_n = \phi(1, \alpha) - n\phi(1, \alpha)'\varepsilon + O(\varepsilon^2), \tag{2.33}$$

$$\widetilde{\pi}_n = \phi(1, \alpha) + n\phi(1, \alpha)'\varepsilon + O(\varepsilon^2). \tag{2.34}$$

Subsidy, allowing (2.32)—(2.34) and (2.7), will be:

$$S = \frac{\tau}{\mu_0}\phi(1, \alpha) + O(\varepsilon^2). \tag{2.35}$$

The solution of system(2.29)—(2.31) looks like:

$$V_0 = \frac{1-\tau}{r}\phi(1,\alpha) + \frac{\triangle}{r} + O(\varepsilon^2), \tag{2.36}$$

$$V_n = \frac{1-\tau}{r}\phi(1,\alpha) + \frac{\delta}{r} + n\beta\eta\varepsilon + O(\varepsilon^2), \tag{2.37}$$

$$\widetilde{V}_n = \frac{1-\tau}{r}\phi(1,\alpha) + \frac{\delta}{r} - n\beta\eta\varepsilon + O(\varepsilon^2). \tag{2.38}$$

Let's remark, that when

$$S > S_0 \approx \frac{1-\tau}{(r+h)}(-\frac{\partial\phi(1,\alpha)}{\partial z})\varepsilon, \tag{2.39}$$

V_1 becomes less then V_0 and the incentives to innovations in zero state certainly vanish, and it means, that the long-run equilibrium growth rate comes to zero. (When everyone, who can, will "catch up" the leaders, than nobody will invest in innovations) [3].

Therefore we shall suspect, that the subsidy S is small enough ($S = s\varepsilon < \frac{\beta\eta(r+h)}{r}\varepsilon$).

Then at $S \ll \frac{2r^2\beta}{3}$

$$\triangle \approx S, \tag{2.40}$$

$$\delta = \frac{h}{r+h}S, \tag{2.41}$$

$$\eta = \frac{1-\tau}{\beta(r+h)}\left(-\frac{\partial\phi(1,\alpha)}{\partial z}\right) = (1-\tau)\eta^{Ag}. \tag{2.42}$$

The efforts on innovations will be

$$\&x_0 = \eta\varepsilon - \frac{\triangle-\delta}{\beta r} \qquad < x_n^{Ag}(1-\tau) \quad < x_n^{Ag}, \tag{2.43}$$

$$x_n = \eta\varepsilon \qquad = x_n^{Ag}(1-\tau) \quad < x_n^{Ag}, \tag{2.44}$$

$$\widetilde{x}_n = n\eta\varepsilon + \tfrac{\triangle-\delta}{\beta r} \qquad > x_n^{Ag}(1-\tau), \tag{2.45}$$

$$\tfrac{\triangle-\delta}{\beta r} = \tfrac{S}{\beta(r+h)}.$$

The probability of industry being in the zero state, $\mu_0 = \hat{\mu}_0 + O(\varepsilon)$, can also be determined analytically:

$$\hat{\mu}_0 = \frac{1}{1+2(1-\zeta)f(\zeta+d\zeta)} > \widetilde{\mu_0}^{\mathrm{Ag}}, \tag{2.46}$$

$$\zeta = \frac{h}{\eta\varepsilon} = \frac{\beta h(r+h)}{-(1-\tau)\phi'\varepsilon} > \zeta^{Ag}, \tag{2.47}$$

$$d\zeta = \frac{\triangle-\delta}{r\beta\eta\varepsilon} = \frac{S}{-(1-\tau)\phi'\varepsilon}. \tag{2.48}$$

The economic growth is asymptotically equal to

$$g = (2\mu_0 x_0 + \sum_{n=0}^{\infty}\mu_n x_n)\ln\gamma = ((1-d\zeta)\mu_0 + 1)\eta\varepsilon^2 + O(\varepsilon^3). \tag{2.49}$$

[3] though it is necessary to note, that at any tax rate < 50%, as the number of firms in states $n > 0$ will become even less, the inequality (2.39) will be broken

Thus the effect from the introducing of the subsidy is two-fold. On the one hand, reduction of incentives results in reduction in efforts on innovations ($x_0 < x_0^{Ag}$, $x_n < x_n^{Ag}$). On the other hand, the process of imitations can be boosted ($\widetilde{x}_n > \widetilde{x}_n^{Ag}$).

However total effect from the introducing of the subsidy is negative.

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau \longrightarrow 0} \longrightarrow \frac{-\eta\varepsilon^2}{1+2(1-d\zeta)f(\cdot)}\left[2-\frac{2\pi_0 f'(\cdot)d\zeta(1-d\zeta)}{-\phi'\varepsilon}+(1-d\zeta)(1+2f(\cdot))\right] < 0. \tag{2.50}$$

That is

Proposition 2.5. In case of very small innovations (while there are imperfections of competition on the markets) subsidy to neck-and-neck innovators slows down economic growth.

Case when $\alpha = 1$

We shall suspect $\varepsilon = \gamma - 1$, $\varepsilon \to 0$.

In this case profit of a firm (the formulas coincide with (1.36)—(1.38)) will be

$$\pi_0 = O(\varepsilon^2), \tag{2.51}$$

$$\pi_n = 1 - \gamma^{-n} + O(\varepsilon^2) = n\varepsilon + O(\varepsilon^2), \tag{2.52}$$

$$\widetilde{\pi}_n = O(\varepsilon^2). \tag{2.53}$$

From system (2.29)—(2.31) it can be received, that

$$V_0 = \frac{\triangle}{r} + O(\varepsilon^2), \tag{2.54}$$

$$V_n = n\beta\eta\varepsilon + \frac{\delta}{r} + O(\varepsilon^2), \tag{2.55}$$

$$\widetilde{V}_n = \frac{\delta}{r} + O(\varepsilon^2), \tag{2.56}$$

where $\triangle = S$, $\delta = \frac{h}{r+h}S$, $\eta = \frac{1-\tau}{\beta(r+h)} = (1-\tau)\eta_0$.

The efforts on innovations will be

$$x_0 = \eta\varepsilon - \frac{S}{\beta(r+h)} + O(\varepsilon^2), \tag{2.57}$$

$$x_n = \eta\varepsilon + O(\varepsilon^2), \tag{2.58}$$

$$\widetilde{x}_n = \frac{S}{\beta(r+h)} + O(\varepsilon^2). \tag{2.59}$$

The probability of being in the zero state, $\mu_0 = \check{\mu}_0 + O(\varepsilon)$ is equal to

$$\hat{\mu}_0 = \frac{\zeta + d\zeta}{2+\zeta+d\zeta} > \widetilde{\mu}_0^{Ag}, \tag{2.60}$$

$$\zeta = \frac{h}{\eta\varepsilon} = \frac{\beta h(r+h)}{(1-\tau)\varepsilon} > \zeta^{Ag}, \tag{2.61}$$

$$d\zeta = \frac{S}{(1-\tau)\varepsilon}. \tag{2.62}$$

Probabilities for industries of being in other states:

$$\mu_k = \left(\frac{1}{1+\zeta+d\zeta}\right)^k \mu_0. \tag{2.63}$$

Value of the subsidy:

$$S = \frac{\tau}{2}\frac{1+\zeta+d\zeta}{\zeta+d\zeta}\varepsilon. \tag{2.64}$$

The economic growth is asymptotically equal to

$$g = (2\mu_0 x_0 + \sum_{n=0}^{\infty}\mu_n x_n)\ln\gamma = ((1-2d\zeta)\mu_0)+1)\eta\varepsilon^2 + O(\varepsilon^3) \tag{2.65}$$

Total effect from the introducing of the subsidy

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau\longrightarrow 0} \longrightarrow \frac{\eta\varepsilon^2}{\zeta^2(2+\zeta)^2}\left[1-6\zeta^2-5\zeta^3-2\zeta^4\right]. \tag{2.66}$$

Let's consider behavior of this expression in some limiting (extreme) cases.

- Let the parameter of ease of imitations be large: $h \gg x_1 = \eta\varepsilon, \quad h \gg x_0 \quad \Longrightarrow \zeta \longrightarrow \infty \quad \Longrightarrow \quad \left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} < 0.$

  Proposition 2.2a. If patent policy is soft and the protection of intellectual property rights is weak then effect from subsidizing neck-and-neck innovators is negative.

- Let now imitations be hindered: $h \longrightarrow 0; \quad \Longrightarrow \quad \zeta = \frac{\beta r}{1-\tau}\frac{h}{\varepsilon},$

  a) If $h \gg \varepsilon$: (namely $h(r+h) > \frac{\varepsilon}{6\beta}$), then $\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} < 0.$

  b) If $h(r+h) \ll \frac{\varepsilon}{6\beta}$, then $\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} > 0.$

  Proposition 2.6. In case of very small innovations on the absolutely competitive markets subsidizing of neck-and-neck innovators can be effective only for the case of very rigid patent policy.

Thus, the introducing of the subsidy in case of small innovations is effective only at very high competition on the markets and low speed of spontaneous imitations. (Parameter h, describing ease of imitations, should be at least six times less then x_n — efforts, directed on innovations).

In other words, if size of innovations decreases, then an area of parameters, in which subsidizing of neck-and-neck innovators is effective, will be more narrow.

The ineffectiveness of subsidizing in case of very small innovations is intuitively clear from following reasons. The decrease of size of innovations has an effect for the relative profit from innovations: when ε approaches zero the relative profit from next innovations approaches zero too. The subsidy, on the opposite has limited value (proportional to profit in zero state). At further reduction of ε the relative profit from innovations becomes less then subsidy received and incentives to prolong innovations vanish at all. That is the firms which have caught up the leader, practically cease to make innovations. The growth is slowed down. Though the introduction of the subsidy to neck-and-neck innovators increases incentives to imitate (and it boosts development in the short-run), nevertheless equilibrium (long-run) growth rate is lowered.

In case of "not so, but small" innovations an area, where the subsidy has a positive effect, exists (it is in the same region, where such area for large innovations is) but if size of innovations decreases this area will decrease too. If profit is equal to zero, and imitations are rather difficult, the subsidy can increase an equilibrium growth rate, that is to promote imitations in the case when they are *really* insufficiently intensive. If the parameter of innovations' smallness, ε, has lower limit, (that an order of magnitude of the relative profit from innovations $\pi_1 - \pi_0 = \beta\eta\varepsilon$ and profit in zero state $\pi_0 = \phi(1, \alpha)$ are comparable) then it results in similar outcomes.

As the conclusion it should be noted that even if the introduction of the minimum subsidy (the introduction of minimum tax at the initial tax rate equal to zero) has a positive effect for growth rate, the further increase of scales of government interference though should increase growth rate in short-run, will result in decrease (after achievement of some critical value) of an equilibrium (long-term) growth rate.

Chapter 3

Subsidy proportional to investments in R&D (partial costs compensation)

In this chapter the results of partial compensation of costs of innovations will be considered.

Let π be a profit of a firm before the introduction of tax, $\psi(x) = \frac{1}{2}\beta x_0^2$ be a costs function of innovation. Source of means on the grants is the profit tax. The tax rate is τ. The size of grants is proportional to R&D (innovation or imitation) costs. Proportionality coefficient is s. Allocated profit minus costs is: $(1-\tau)\pi - (1-s)\psi(x)$.

This scheme can be interpreted as effective reduction of the profit tax for firms, allocating a considerable proportion(part) of their profit in R&D.

The changes in Bellman's equation will cause changes in the system (1.8)—(1.11). Now it will be recorded as:

$$rV_0 = (1-\tau)\pi_0 - (1-s)\frac{1}{2}\beta x_0^2 + x_0(V_1 - V_0) + \widetilde{x}_0(V_0 - \widetilde{V}_1), \tag{3.1}$$

$$rV_n = (1-\tau)\pi_n - (1-s)\frac{1}{2}\beta x_n^2 + x_n(V_{n+1} - V_n) + (\widetilde{x}_n + h)(V_0 - V_n), \tag{3.2}$$

$$r\widetilde{V}_n = (1-\tau)\widetilde{\pi}_n - (1-s)\frac{1}{2}\beta \widetilde{x}_n^2 + x_n(\widetilde{V}_{n+1} - \widetilde{V}_n) + (\widetilde{x}_n + h)(V_0 - \widetilde{V}_n), \tag{3.3}$$

$$x_0 = \frac{(V_1 - V_0)}{(1-s)\beta}, \tag{3.4}$$

$$x_n = \frac{(V_{n+1} - V_n)}{(1-s)\beta}, \tag{3.5}$$

$$\widetilde{x}_n = \frac{(V_0 - \widetilde{V}_n)}{(1-s)\beta}, \tag{3.6}$$

$$\tau\left(2\mu_0\pi_0 + \sum_{k=1}^{\infty}\mu_k(\pi_k + \widetilde{\pi}_{-k})\right) = \frac{s\beta}{2}\left(2\mu_0 x_0^2 + \sum_{k=1}^{\infty}\mu_k(x_k^2 + \widetilde{x}_{-k}^2)\right), \tag{3.7}$$

where last equation represents budget limitation for the government .

The solution for arbitrary permissible parameters is possible only numerically. Therefore I shall consider some limiting cases, from the analysis of which the characteristics and features of the solution will be clear. It again will be the cases of very large ($\gamma \to \infty$) and very small ($\gamma \to 1$) innovations.

As well as in the previous chapter, case of small tax rate τ and small grant will be considered in order to evaluate whether the grants can be useful in principle. Both for the case of large, and for the case of small innovations it will be shown, that the subsidizing of efforts has a positive effect on economic growth rates, however a size of this effect hardly depends on characteristics of an economy.

3.1 Case of large innovations: the grants are most effective when markets are competitive and patent policy is rigid

If the size of innovation is large ($\gamma \to \infty$), leading in one step will raise profit of the leader up to the greatest possible value ($\phi(\gamma^{-1}, \alpha) \overset{\gamma\to\infty}{\longrightarrow} 1, \quad \forall \alpha > 0$), incentives to keep on innovations will vanish. Firm located a step ahead of the competitor will not conduct new researches ($x_1 \equiv 0$). In this case the greatest possible technological leading makes one step ($\max n = 1$).

The solutions for levels of efforts chosen in states "0" and "-1" are coming out from a set of equations:

$$\frac{1}{2}x_0^2 + x_0(r+h) = \frac{1-\tau}{(1-s)\beta}(\pi_1 - \pi_0), \tag{3.8}$$

$$\frac{1}{2}\widetilde{x}_1^2 + \widetilde{x}_1(r+h+x_0) = \frac{1-\tau}{\beta(1-s)}\pi_0 + \frac{1}{2}x_0^2, \tag{3.9}$$

$$\tau = \overbrace{\frac{\beta}{2}\frac{x_0^2(\widetilde{x}_1+h) + x_0\widetilde{x}_1^2}{x_0 + \pi_0(\widetilde{x}_1+h)}}^{\delta<\frac{1}{2}} s = \delta s. \tag{3.10}$$

The formulas for occupancy of the zero state and growth rate will stay the same (in terms of x_j):

$$\mu_0 = \frac{\widetilde{x}_1 + h}{2x_0 + \widetilde{x}_1 + h}, \tag{3.11}$$

$$g = 2\mu_0 x_0 \ln\gamma = \frac{2x_0(\widetilde{x}_1+h)}{2x_0 + \widetilde{x}_1 + h}\ln\gamma. \tag{3.12}$$

The efforts of agents while introducing minimum tax will increase. The value of a derivative of a growth rate under the rate of compensation of costs s numerically is equal to

$$\left.\frac{\partial g}{\partial \tau}\right|_{s=0} = \frac{2(1-2\delta)}{\Delta\beta[2x_0 + \widetilde{x}_1 + h]^2} \times \\ \times \left((1-\pi_0)(\widetilde{x}_1+h)^2(x_0+\widetilde{x}_1+r+h) + 2x_0^2(\pi_0(\widetilde{x}_1+r+h) + x_0 - x_1)\right). \tag{3.13}$$

This expression is greater than zero at any acceptable values of parameters. So, we can formulate

Proposition 3.1. In case of large innovations the subsidizing of efforts (grant, proportional to investments in R&D) has a positive effect on economic growth rate.

To present picture more precisely, we shall analyze behavior of this expression in the main limiting cases.

- Let parameter of ease of imitations be large: $h \gg \widetilde{x}_1, h \gg x_0$
$$\Longrightarrow \quad \left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \longrightarrow \frac{2(1-\delta)(1-\pi_0)}{\beta(r+h)} \longrightarrow 0,$$
that is: if intensity of imitations is considerable then subsidizing will be ineffective.

- Let now imitations be hindered and markets be monopolized: $h \longrightarrow 0, \quad \alpha \longrightarrow 0. \quad \Longrightarrow$
$\Longrightarrow \quad (\pi_1 - \pi_0) \approx (\pi_0 - \widetilde{\pi}_1) \approx 1/2, \quad \Longrightarrow \quad x_0 \approx \widetilde{x}_1 \approx \sqrt{1/\beta}, \quad \Longrightarrow \quad \delta \approx 3/2,$
$\Longrightarrow \quad \left.\frac{\partial g}{\partial s}\right|_{s=0} \approx \frac{1}{18\sqrt{\beta}}.$
If competition at markets is low enough (the large monopoly power and, accordingly, sufficiently large profit of each of neck-and-neck firms), then introduction of the grant gives a positive effect.

- Let now imitations be hindered, and markets be competitive: $h \longrightarrow 0; \quad \alpha \longrightarrow 1. \quad \Longrightarrow$
$\Longrightarrow \quad \widetilde{\pi}_1 \approx 0, \quad \pi_0 \approx 0; \quad \pi_1 \approx 1$: the non-zero profit is received only by the actual leader;
$\Longrightarrow \quad x_1 = 0 \quad \widetilde{x}_1 > 0; \quad x_0 > 0 :$
$\Longrightarrow \quad \left.\frac{\partial g}{\partial s}\right|_{s=0} \approx 6\frac{(\sqrt{2}-1)(1-(2-\sqrt{2})x_0^2)}{(3+2\sqrt{2})x_0\beta} \ (\sim \frac{1}{\sqrt{\beta}})$
subsidizing will give the best outcome.

Extending these results, we shall formulate

Proposition 3.2. In case of large innovations subsidizing of efforts (grant, proportional to investments in R&D) will give maximum outcome if markets are competitive and patent policy is rigid. The efficiency of subsidizing of efforts will approach zero otherwise: on monopolized markets in case of easy imitations.

Generalization of propositions 3.1 and 3.2 and exhaustive picture describing the size of effect from introduction of minimum subsidy, are shown in a fig. 3.1.

We have shown, that at the initial tax rate equal to zero, its increase will have a positive effect for an equilibrium growth rate. But after some critical value further increase of tax rate and subsidy will result in decrease of an equilibrium (long-term) growth rate.

Thus, of grants proportional to investments in R&D are efficient doe to low initial intensity of imitations: both investments of firms and spontaneous imitations are small. Low investments are caused by low profit in zero state because of a rigid competition. Spontaneous imitations are weak

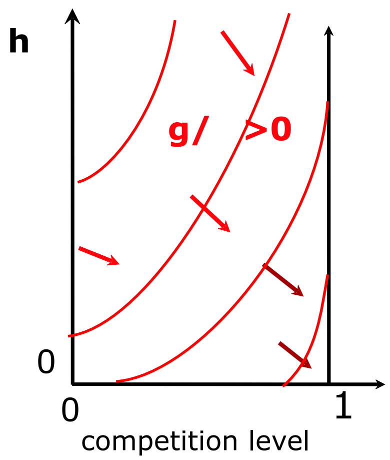

Figure 3.1: Changes of economic growth rate if minimum grant proportional to investments in R&D is introduced (case of large innovations)

because the parameter describing ease of imitations, h, aims to zero point. In this situation the subsidizing of efforts is effective, because actually tax burden fells upon a monopoly (for which in any case there are no incentives to prolong innovations), but collected means are distributed between those who invest substantially in technological advance.

3.2 Case of small innovations: the grants are most effective when competition and patent policy are gentle ($\gamma \to 1, \quad \gamma = 1 + \varepsilon$)

Let's consider a case, which is characterized by minimum size of innovations (γ aims to its lower boundary, $\gamma \to 1$). We use a method of asymptotic decomposition on small parameter ε.

Cases $\alpha < 1$ and $\alpha = 1$ we shall consider separately, as the profit function $\pi = \phi(\cdot, \alpha)$ is smooth at $\alpha < 1$ and has a kink(fracture) when $\alpha = 1$.

Case $\alpha < 1$

Let $\varepsilon = \gamma - 1$. Then, using asymptotic decomposition of a profit function $\phi(z, \alpha)$ and value functions V_j, neglecting quadratic on ε members it can be obtained that

$$\pi_0 = \phi(1, \alpha) + O(\varepsilon^2), \tag{3.14}$$

$$\pi_n = \phi(1, \alpha) - n\phi(1, \alpha)'\varepsilon + O(\varepsilon^2), \tag{3.15}$$

$$\widetilde{\pi}_n = \phi(1, \alpha) + n\phi(1, \alpha)'\varepsilon + O(\varepsilon^2). \tag{3.16}$$

The solution of system (3.1)–(3.6) looks like:

$$V_0 = \frac{1-\tau}{r}\phi(1, \alpha) + O(\varepsilon^2), \tag{3.17}$$

$$V_n = \frac{1-\tau}{r}\phi(1, \alpha) + n\beta(1-\tau)\eta^{Ag}\varepsilon + O(\varepsilon^2), \tag{3.18}$$

$$\widetilde{V}_n = \frac{1-\tau}{r}\phi(1, \alpha) - n\beta(1-\tau)\eta^{Ag}\varepsilon + O(\varepsilon^2). \tag{3.19}$$

The efforts on innovation and imitations will be

$$x_0 = \tfrac{1-\tau}{1-s}\eta^{Ag}\varepsilon + O(\varepsilon^2) \approx \eta\varepsilon, \tag{3.20}$$

$$x_n = \tfrac{1-\tau}{1-s}\eta^{Ag}\varepsilon + O(\varepsilon^2) \approx \eta\varepsilon, \tag{3.21}$$

$$\widetilde{x}_{-n} = n\tfrac{1-\tau}{1-s}\eta^{Ag}\varepsilon + O(\varepsilon^2) \approx n\eta\varepsilon, \tag{3.22}$$

where $\eta^{Ag} = \dfrac{1}{\beta(r+h)}\left(-\dfrac{\partial\phi(1,\alpha)}{\partial z}\right), \quad \eta = \dfrac{1-\tau}{1-s}\eta^{Ag}.$

Probability of being in the zero state, $\mu_0 = \hat{\mu}_0 + O(\varepsilon)$, and economic growth rate:

$$\hat{\mu}_0 = \frac{1}{1 + 2f(\zeta)}, \tag{3.23}$$

$$g = \left(2\mu_0 x_0 + \sum_{n=0}^{\infty}\mu_n x_n\right)\ln\gamma = (\mu_0 + 1)\eta\varepsilon^2 + O(\varepsilon^3) \approx 2\frac{1 + f(\zeta)}{1 + 2f(\zeta)}\eta\varepsilon^2, \tag{3.24}$$

$$\zeta = \frac{h}{\eta\varepsilon} = \frac{1-s}{1-\tau}\zeta^{Ag}.$$

From budget constraint a relationship between the sizes of the grant and tax is

$$\frac{\tau}{(1-\tau)^2} = \frac{s}{(1-s)^2}b, \tag{3.25}$$

$$b = \left(2 + \sum_{k=1}^{\infty}(k^2-1)\mu_k\right)\frac{(-\phi_0')^2}{\phi_0}\frac{\varepsilon^2}{4\beta(r+h)^2}\left(<\frac{1}{2}\right), \quad \frac{(-\phi_0')^2}{\phi_0} = \frac{\alpha^2}{(2-\alpha)(2+\alpha)^2(1-\alpha)}. \tag{3.26}$$

If $\varepsilon \to 0$ the tax rate τ will be less then "the rates of subsidizing" s. In this case efforts of all firms and economic growth rate will increase.

Thus, we can formulate

Proposition 3.3. In case of small innovations subsidizing of efforts (grants proportional to investments in on R&D) has a positive effect on economic growth rate.

Effect from the grant is always positive. But we are interested in the size of this effect. To what extent will growth rate increase if minimum grant is introduced? We can get this number.

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau\longrightarrow 0} = 2\frac{1+\eta+f(\zeta)(3+2\eta+2f(\zeta)+\frac{\zeta f'(\zeta)}{f(\zeta)})}{(1+2f(\zeta))^2}\frac{1-b}{b-\tau}\eta_0\varepsilon^2. \tag{3.27}$$

It is also interesting to compare this increase to an initial value of a growth rate:

$$\left.(\frac{\partial g}{\partial \tau})/g\right|_{\tau\longrightarrow 0} = \frac{1+\eta+f(\zeta)(3+2\eta+2f(\zeta)+\frac{\zeta f'(\zeta)}{f(\zeta)})}{(1+2f(\zeta))(1+f(\zeta))}\frac{1-b}{b(1-\tau)}. \tag{3.28}$$

Considering some limiting cases for these expressions:

- Let $h \gg \widetilde{x}_1, x_0, \quad \alpha \ll 1, \quad \alpha \to 1. \quad \Longrightarrow$
$$\Longrightarrow \quad b \to \text{const} \times \frac{\varepsilon^2}{h^2} \to 0, \quad f(\zeta = \frac{h}{\eta\varepsilon}) \to 0, \quad \Longrightarrow$$
$$\Longrightarrow \quad \left.\frac{\partial g}{\partial \tau}\right|_{\tau\longrightarrow 0} \to 2(1+\eta)\frac{1-b}{b-\tau}\eta_0\varepsilon^2 \to \mathbf{C}\frac{h}{\beta\alpha} \to \infty;$$
$$\left.(\frac{\partial g}{\partial \tau})/g\right|_{\tau\longrightarrow 0} \to (1+\eta)\frac{1-b}{b} \to \mathbf{C}\frac{h^2}{\varepsilon^2\alpha^2} \to \infty.$$
- Let now $h \longrightarrow 0; \quad \alpha \ll 1, \quad \alpha \to 1 \quad \Longrightarrow$
$$\Longrightarrow \quad b \sim \alpha^2 \to 0; \quad \eta_0 \to \frac{\alpha}{r\beta}; \quad \zeta = \frac{h}{\eta\varepsilon} \longrightarrow 0; \quad f(\zeta) \to \widehat{f}(\approx 0.8). \quad \Longrightarrow$$
$$\Longrightarrow \quad g'_{tau}|_{\tau\longrightarrow 0} \longrightarrow 2\frac{1+\eta+\widehat{f}(3+2\eta+2\widehat{f}+\frac{\zeta\widehat{f}'}{\widehat{f}})}{(1+2\widehat{f})^2}\frac{1-b}{b}\eta_0\varepsilon^2 \longrightarrow \frac{1}{\alpha} \longrightarrow \infty$$
$$\left.\frac{g'_\tau}{g}\right|_{\tau\longrightarrow 0} \longrightarrow \frac{1+\eta+\widehat{f}(3+2\eta+2\widehat{f}+\frac{\zeta\widehat{f}'}{\widehat{f}})}{(1+2\widehat{f})(1+\widehat{f})}\frac{1-b}{b} \longrightarrow \frac{1}{\alpha^2} \longrightarrow \infty$$

All this forms

Proposition 3.4. In case of small innovations, if competition on markets is sufficiently low (the large monopoly power and, accordingly, large profit of each of neck-and-neck firms) the efficiency of subsidizing efforts will increase while softening of patent policy and reduction competitiveness on the markets.

Another limiting cases:

- Let $h \gg \widetilde{x}_1, x_0, \quad \alpha \gg 0(\to 1 \quad ?), \quad \Longrightarrow$

$$\Longrightarrow \quad b \to \frac{\varepsilon^2\alpha^2}{18\beta(r+h)^2(1-\alpha)}; \quad \Longrightarrow \quad f(\zeta) \to 0. \quad \Longrightarrow$$

$$\Longrightarrow \quad \frac{\partial g}{\partial \tau}\Big|_{\tau \longrightarrow 0} \to 2(1+\eta)\frac{1-b}{b-\tau}\eta_0\varepsilon^2 \sim (r+h)(1-\alpha);$$

$$(\frac{\partial g}{\partial \tau})/g\Big|_{\tau \longrightarrow 0} \to (1+\eta)\frac{1-b}{b} \sim (r+h)^2(1-\alpha).$$

- Let now $h \longrightarrow 0; \quad \alpha \gg 0(\alpha \to 1), \quad \Longrightarrow$

$$\Longrightarrow \quad b \to \frac{\varepsilon^2}{18\beta r^2(1-\alpha)}; \quad \eta_0 \to \frac{1}{3r\beta}; \quad \zeta = \frac{h}{\eta\varepsilon} \longrightarrow 0; \quad f(\zeta) \to \widehat{f}(\approx 0.8). \quad \Longrightarrow$$

$$\Longrightarrow \quad g'_\tau|_{\tau \longrightarrow 0} \longrightarrow 2\frac{1+\eta+\widehat{f}(3+2\eta+2\widehat{f}+\frac{\zeta\widehat{f}'}{\widehat{f}})}{(1+2\widehat{f})^2}\frac{1-b}{b}\eta_0\varepsilon^2 \sim r(1-\alpha);$$

$$\frac{g'_\tau}{g}\Big|_{\tau \longrightarrow 0} \longrightarrow \frac{1+\eta+\widehat{f}(3+2\eta+2\widehat{f}+\frac{\zeta\widehat{f}'}{\widehat{f}})}{(1+2\widehat{f})(1+\widehat{f})}\frac{1-b}{b} \sim r^2(1-\alpha).$$

Proposition 3.5. In case of small innovations, if competition on markets is high enough (the low profit of each neck-and-neck firms) then efficiency of subsidizing efforts will increase while softening of patent policy and strengthening of competitiveness on the markets.

However, if α is close to unity then the above described decomposition is not correct, therefore outcome of Proposition 3.5 is accurate only partly.

The case $\alpha = 1$ should be considered, as well as in previous parts of work, separately.

Case $\alpha = 1$

Assuming $\varepsilon = \gamma - 1$, $\varepsilon \to 0$.

In this case profit of a firm (the formulas coincide with (1.36)–(1.38)):

$$\widetilde{\pi}_n \simeq \pi_0 \simeq O(\varepsilon^2), \tag{3.29}$$

$$\pi_n = 1 - \gamma^{-n} + O(\varepsilon^2) = n\varepsilon + O(\varepsilon^2). \tag{3.30}$$

The solution of system (3.1)–(3.6) looks like:

$$\widetilde{V}_n \simeq V_0 \simeq O(\varepsilon^2), \tag{3.31}$$

$$V_n = (1-\tau)n\beta\eta^{Ag}\varepsilon + O(\varepsilon^2). \tag{3.32}$$

The efforts on innovations will be

$$x_0 = \frac{1-\tau}{1-s}\eta^{Ag}\varepsilon + O(\varepsilon^2), \tag{3.33}$$

$$x_n = \frac{1-\tau}{1-s}\eta^{Ag}\varepsilon + O(\varepsilon^2), \tag{3.34}$$

$$\widetilde{x}_{-n} = +O(\varepsilon^2), \tag{3.35}$$

where $\eta^{Ag} = \dfrac{1}{\beta(r+h)}$, $\eta = \dfrac{1-\tau}{1-s}\eta^{Ag}$.

Probability of being in the zero state, $\mu_0 = \breve{\mu}_0 + O(\varepsilon)$, or in state "$k$" (relative quantity of industries in the conforming condition) is:

$$\breve{\mu}_0 = \frac{1}{1 + 2\frac{\eta\varepsilon}{h}} = \frac{1}{1 + 2\frac{1}{\zeta}}, \quad \mu_k = \left(\frac{1}{1+\zeta}\right)^k \mu_0, \tag{3.36}$$

where $\zeta = \dfrac{h}{\eta\varepsilon}$.

The economic growth is asymptotically equal to

$$g = (2\mu_0 x_0 + \sum_{n=0}^{\infty} \mu_n x_n) \ln\gamma = 2\frac{1 + \frac{\eta\varepsilon}{h}}{1 + 2\frac{\eta\varepsilon}{h}} \eta\varepsilon + O(\varepsilon^3). \tag{3.37}$$

From budget constraint a relationship between the sizes of the grant and tax rate

$$\frac{\tau}{1-\tau} = \frac{s}{1-s} \overbrace{\frac{h}{2(h+r)}}^{b<0.5}. \tag{3.38}$$

Tax rate will be less then "the rates of subsidizing" s. In this case efforts of all firms and growth rate will increase. Effect from the grant is always positive. The size of the effect is

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau \longrightarrow 0} = 2\frac{1 + 2\frac{\eta\varepsilon}{h} + 2\frac{\eta^2\varepsilon^2}{h^2}}{(1 + 2\frac{\eta\varepsilon}{h})^2}(1 + 2r/h)\eta_0\varepsilon, \tag{3.39}$$

$$\left.(\frac{\partial g}{\partial \tau})/g\right|_{\tau \longrightarrow 0} = \frac{(1 + 2\frac{\eta\varepsilon}{h} + 2\frac{\eta^2\varepsilon^2}{h^2})}{(1 + 2\frac{\eta\varepsilon}{h})(1 + \frac{\eta\varepsilon}{h})}(1 + 2r/h). \tag{3.40}$$

In limiting cases:

- Let parameter of ease of a imitations be large: $h \gg \widetilde{x}_1, x_0, \quad \alpha = 1$

  (т. е. $\zeta \gg 2 \Longleftrightarrow h(h+r)\beta \gg 2\varepsilon$), $\quad \Longrightarrow$

  $g'_\tau|_{\tau\to 0} \to 2\eta_0\varepsilon \to 0,$

  $\left.\frac{g'_\tau}{g}\right|_{\tau\to 0} \to 1.$

  So effect from the grant is zero (as well as equilibrium growth rate: you see their ratio is equal to unity)

- Let now imitations be hindered: $h \longrightarrow 0, \ \alpha = 1$

  (т. е. $\zeta \ll 2 \Longleftrightarrow h(h+r)\beta \ll 2\varepsilon$), $\quad \Longrightarrow$

  $g'_\tau|_{\tau\to 0} \to (1 + 2\frac{r}{h})\eta_0\varepsilon \to \frac{\varepsilon}{\beta h} \to \infty,$

  $\left.\frac{g'_\tau}{g}\right|_{\tau\to 0} \to 1 + 2r/h.$

These results can be formulated in

Proposition 3.6. In case of small innovations, if competition on markets is perfect (profit of each neck-and-neck firms is equal to zero) then the efficiency of subsidizing efforts will increase while patent policy is strengthen.

Let's remark, that Propositions 3.5 and 3.6 somewhat contradict one another. This is possible to explain by the fact that they are correct in different ranges of values of parameters. Proposition 3.5 is

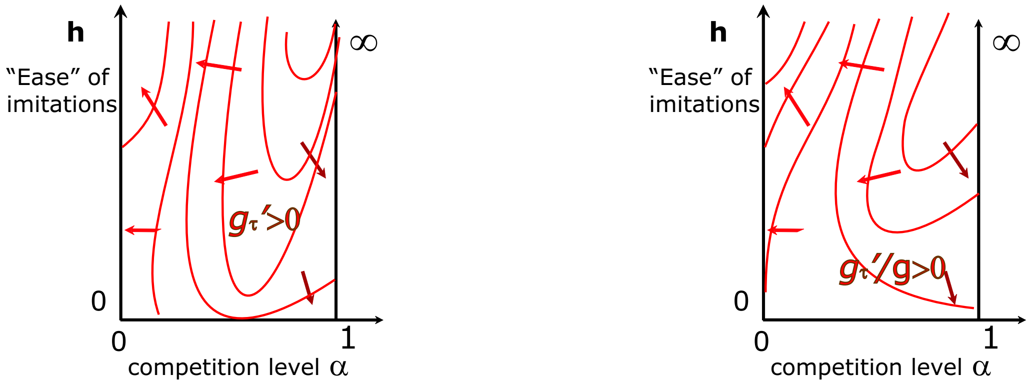

Figure 3.2: Change of a growth rate and ratio between change of a growth rate and growth rate if minimum grant proportional to R&D investments is introduced (case of small innovations)

correct in case $\alpha < 1$, namely, when dependence of profit of firms from its state has a kink and equals to zero in zero state.

The general picture is complex enough (see fig.3.2). In case of perfect competition the efficiency of subsidizing is increased with complication of imitations. If firms have a certain level of the monopolistic power the efficiency will increase while imitations will become easier.

Let's remark, if the level of growth initially is optimal, the subsidizing won't give appreciable outcomes (gives outcomes, but gentle). It is necessary to correct something only if without the correction not everything is all right

Intuitively it is possible to explain these results: if profits of firms are equal to zero because of too rigid competition and if imitations are rather difficult (imitations are insufficiently intensive), the grant will stimulate them. As a consequence, the grant will increase equilibrium rate of growth, influencing on a weak places in economy.

On the markets with gentle competition innovations can appear to be the weak place (if imitations are facilitated, they will occur spontaneously and it won't necessary to put means on them) . In this case grant will acts at the expense of a stimulation of innovations.

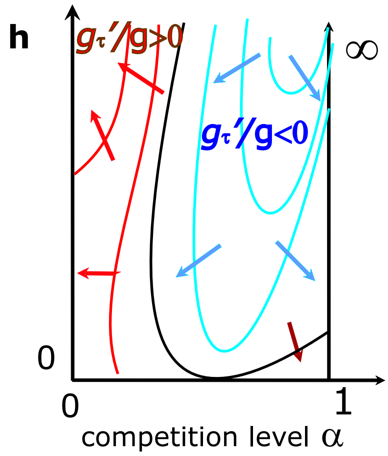

Figure 3.3: Change of growth rate if minimum grant proportional to investments in R&D is introduced if there are losses during reallocating of the public funds (case of small innovations)

Once again it should be remarked, that the minimum grant will be effective if the initial tax rate is equal to zero. However after some critical value the further increase of scales of government interference, though can increase growth rates in the short run, will result in decrease of an equilibrium (long-run) economic growth rate.

This circumstance is especially important in case of losses during reallocation of means through government budget (see fig.3.3). If the collected taxes can be redistributed effectively only partly, the areas with small positive effect will turn into areas with negative effect and there will be no sense to recommend such policy.

Chapter 4

Reward for successful innovation

In this part grants, which are given to firms which has already made successful innovation will be considered.

The scheme looks like this: innovator selects the level of efforts x, and corresponding level of costs $\psi(x)$. Additional incentive for him is the grant S, which he will receive in case of success. The probability of success for innovator is proportional to a level of efforts: $Prob(\textit{success of innovator}) = xdt$. The probability of successful imitation depends on two parameters: applied efforts x and ease of imitations h: $Prob(\textit{success of imitator}) = (x+h)dt$.

Let π be a profit of a firm before taxes. Source of means on the grant are the profit taxes of all firms. The tax rate is τ. The collected sum is distributed between firms, which just now have made the next technological achievement.

The changes in Bellman's equation will cause changes in system (1.8)–(1.11). Now it can be recorded as:

$$rV_0 = (1-\tau)\pi_0 - \frac{1}{2}\beta x_0^2 + x_0(V_1 - V_0 + S_0) + \widetilde{x}_0(V_0 - \widetilde{V}_{-1}), \tag{4.1}$$

$$rV_n = (1-\tau)\pi_n - \frac{1}{2}\beta x_n^2 + x_n(V_{n+1} - V_n + S_n) + (\widetilde{x}_{-n} + h)(V_0 - V_n), \tag{4.2}$$

$$r\widetilde{V}_{-n} = (1-\tau)\widetilde{\pi}_{-n} - \frac{1}{2}\beta \widetilde{x}_{-n}^2 + x_n(\widetilde{V}_{-(n+1)} - \widetilde{V}_{-n}) + (\widetilde{x}_{-n} + h)(V_0 - \widetilde{V}_{-n} + \widetilde{S}_{-n}), \tag{4.3}$$

$$x_0 = \frac{(V_1 - V_0 + S_0)}{\beta}, \tag{4.4}$$

$$x_n = \frac{(V_{n+1} - V_n + S_n)}{\beta}, \tag{4.5}$$

$$\widetilde{x}_{-n} = \frac{(V_0 - \widetilde{V}_{-n} + \widetilde{S}_{-n})}{\beta}, \tag{4.6}$$

$$\tau\left(2\mu_0\pi_0 + \sum_{k=1}^{\infty}\mu_k(\pi_k + \widetilde{\pi}_{-k})\right) = \left(2\mu_0 x_0 S_0 + \sum_{k=1}^{\infty}\mu_k(x_k S_k + \widetilde{x}_{-k}\widetilde{S}_{-k})\right), \tag{4.7}$$

where last equation represents budget constraint for the government.

It is suspected that the grants for innovations and grants for imitations can be different. Each of them can be evaluated separately. Efficiency of each kind of stimulation will be received and they will be compared one to another.

In this case government does not take any risk of paying something in case of a failure of innovator/imitator and rewards only actual achievements.

The solution for arbitrary permissible values of parameters is possible only numerically. Therefore I shall consider some limiting cases, from the analysis of which the characteristics and features of the solution will be clear. It again will be the cases of very large ($\gamma \to \infty$) and very small ($\gamma \to 1$) innovations.

As well as in the previous chapters, I shall considered a case of a small tax rate τ and small grant to evaluate, whether the grants can be useful in principle. Almost apparently, that this scheme (at least from the idealized point of view) should have good efficiency (on the average), at least as contrasted to first scheme discussed in chapter 2, where the grants were given practically for "position" of firms in industry, and were connected with efforts only indirectly.

Other problem, which will be discussed, is following: "what is more effective to stimulate: innovations or imitations(simulation)?"

Both for a case of large, and for a case of small innovations it will be shown, that the subsidizing of innovators usually brings greater effect, than stimulation of imitators. At the same time size of effect depends hardly on the characteristics of an economy.

4.1 Case of large innovations: the encouraging of imitations is inefficient at mild patent policy

If the size of innovatory step is large ($\gamma \to \infty$), leading on one step will raise profit of the leader up to the greatest possible value ($\phi(\gamma^{-1}, \alpha) \stackrel{\gamma\to\infty}{\longrightarrow} 1, \quad \forall \alpha > 0$), incentives to innovate will vanish. The grants will be the only incentive to prolong innovations. Thus all leaders (irrespective of number of steps they lead) are equally interested in innovations. All lagging firms also are identical from the point of view of applied efforts.

The solutions for levels of efforts in "0", "1" and "-1" states can be obtained from a set of equations

$$\frac{1}{2}x_0^2 + x_0(r+h) = \frac{1-\tau}{\beta}(\pi_1 - \pi_0) + \frac{S_0}{\beta}(1 + x_1 + \widetilde{x}_{-1}) - \frac{\widetilde{S}_{-1}}{\beta}x_0 + \overbrace{\frac{S_1^2}{2\beta^2}}^{=x_1^2/2}, \tag{4.8}$$

$$\frac{1}{2}\widetilde{x}_{-1}^2 + \widetilde{x}_{-1}(r+h+x_0) = \frac{1-\tau}{\beta}\pi_0 + \frac{1}{2}x_0^2 + \frac{\widetilde{S}_{-1}}{\beta}(x_0 + r). \tag{4.9}$$

The budget constraint will look like

$$\tau(\pi_0 + \pi_1\frac{x_0}{\widetilde{x}_{-1}+h}) = S_0 x_0 + \frac{x_1 S_1 + \widetilde{x}_{-1}\widetilde{S}_{-1}}{\widetilde{x}_{-1}+h}. \tag{4.10}$$

It seems apparent, that such scheme should give better outcome, than stimulation of efforts or grants for a position of firms. The question is: what is more effective to stimulate, innovation or imitation? To answer this question we shall analyze three different situations:

1) Stimulation of imitations: $\widetilde{S}_{-1} \neq 0, S_1 = S_0 = 0$.

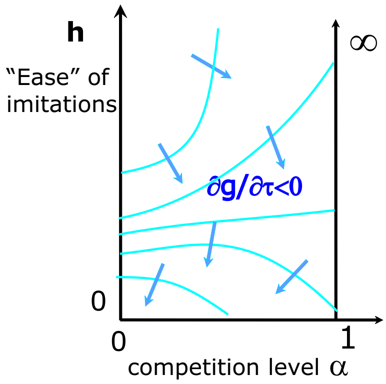

Figure 4.1: Change of economic growth if minimum tax upon all firms and grants to successful imitators is introduced: ($\widetilde{S}_{-1} \neq 0$) (case of large innovations)

2) Stimulation of innovations on the leading edge of technology: $\widetilde{S}_{-1} = S_1 = 0, S_0 \neq 0$.

3) Stimulation of innovations made by industry leaders: $\widetilde{S}_{-1} = S_0 = 0, S_1 \neq 0$.

Let's consider them.

Case 1. Stimulation of imitations: $\widetilde{S}_{-1} \neq 0, \quad S_1 = S_0 = 0$.

In this situation the grant is given to a firm, which has just catched-up with the competitor.

Outcomes of this scheme are negative. (see fig. 4.1).

Both on the markets with hard competition and on the markets where firms enjoy a monopolistic power the efficiency of this scheme depends on ease of imitations. So, if imitations are hard, this scheme will give negative outcome

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \longrightarrow \left(-\frac{2}{9\beta} + \frac{2r}{9\sqrt{\beta}}\right) \ln \gamma \quad \text{при} \quad \alpha \longrightarrow 1, \quad (r+h) \longrightarrow 0, \tag{4.11}$$

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \longrightarrow -\frac{1}{9\beta} - \frac{1}{36\sqrt{\beta}} \ln \gamma \quad \text{при} \quad \alpha \longrightarrow 0, \quad (r+h) \longrightarrow 0. \tag{4.12}$$

If imitations are facilitated, decreasing of a growth rate will be even bigger.

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \longrightarrow -\frac{4}{\beta} \ln \gamma \quad \text{при} \quad \alpha \longrightarrow 1, \quad (r+h) \longrightarrow \infty, \tag{4.13}$$

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \longrightarrow -4h(r+h) \ln \gamma \quad \text{при} \quad \alpha \longrightarrow 0, \quad (r+h) \longrightarrow \infty. \tag{4.14}$$

These results form

Proposition 4.1. In case of large innovations encouraging of imitations has negative effect for economic growth rates. At mild patent policy (facilitated imitations) if competition on the markets is gentle then a slowdown of a growth rate will be most significant (see fig. 4.1).

Intuitively these results can be interpreted as follows. Stimulation of imitations is inefficient in case of the large innovations, as it beats incentives to make innovation (the more the size of innovations, the more strongly negative effect from introduction of such scheme of stimulation is). That is the acceleration of growth originating from strengthening of imitations does not compensate a slowdown

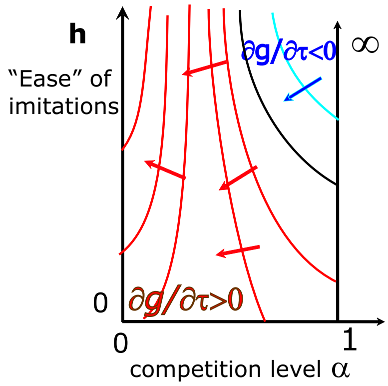

Figure 4.2: Change of a growth rate when minimum tax upon all firms and grants to innovators on leading edge of technology are introduced: $\widetilde{S}_{-1} = S_1 = 0, S_0 \neq 0$ (case of large innovations)

of growth rate coming from a reduction of number of innovations. This occurs because grants depend not only on efforts of imitators. They are also proportional to spontaneous imitations. That means that actually the reallocation of resources in the party of the lagging producers does not affect their incentives, that can not be effective. The taxes, on the opposite, directly negatively affect incentives of innovators. Therefore the damage from this scheme the larger, the easier the imitations are (the larger the share of resources reallocated not affecting incentives is).

Case 2. Stimulation of innovations on the leading edge of technology: $S_0 \neq 0$

In this case the grants are given to firms which have just now became a leader.

Omitting cumbersome calculations, we shall pass to results (see fig.4.2).

On the markets with hard(perfect) competition the efficiency of this scheme depends on ease of imitations. So, if imitation are rather complicated this scheme will give positive outcome

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \longrightarrow \frac{\ln \gamma}{9}, \quad \text{when} \quad \alpha \longrightarrow 1, \quad (r+h) \longrightarrow 0.$$

If imitations are facilitated, change of the growth rate will be negative:

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \longrightarrow \frac{2(1-h)}{\beta h(r+h)} \ln \gamma, \quad \text{when} \quad \alpha \longrightarrow 1, \quad (r+h) \longrightarrow \infty.$$

On hardly monopolized markets the effect is always positive, and increases while imitations become easier:

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \longrightarrow \frac{\ln \gamma}{2} \quad \text{when} \quad \alpha \longrightarrow 0, \quad (r+h) \longrightarrow 0,$$

$$\left.\frac{\partial g}{\partial \tau}\right|_{\tau=0} \longrightarrow 2 \ln \gamma \quad \text{when} \quad \alpha \longrightarrow 0, \quad (r+h) \longrightarrow \infty.$$

Once again summarizing these results, we shall make out

Proposition 4.2. In case of large innovations encouraging of innovations on the leading edge of technology can influence growth rate both positively and negatively. On the markets with a hard competition at mild patent legislation the growth rate is slowed down. On monopolized markets the efficiency of the scheme rises if intensity of spontaneous imitations increases (see fig. 4.2).

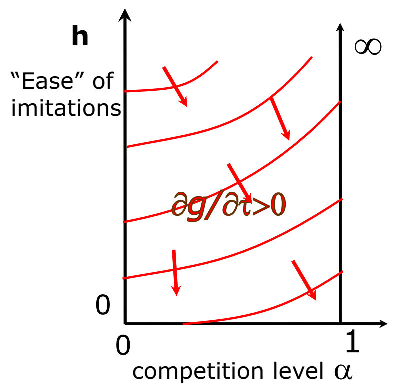

Figure 4.3: Change of a growth rate if minimum tax upon all firms and grants for prolongation of innovations by the leaders ($S_1 \neq 0$) are introduced (case of large innovations)

Intuitively these outcomes can be interpreted in the following way. The given scheme is invoked to promote innovations among firms, competing at the same technological level. It is particularly effective, if firms in this state receive a non-zero profit: a firm making innovation get as an award means collected through taxes from it and from the competitor, and also taxes collected from leaders in other industries.

If, because of a high competition, profits of neck-and-neck firms, are equal to zero, and the relative ease of imitations makes this state most common(stocked), the means for the grants will be collected only from leaders. That reduces incentives to become such leader, that is to stop innovations. And this effect can not be balanced by the promised grant, as its size is limited to tax base (the number of industries in which there is a leader) because the number of industries with non-zero mean profit is smaller, than the number of industries with neck-and-neck firms, in a direction of which the reallocation is going).

Case 3. Stimulation of prolongation of innovations by leaders: $S_1 \neq 0$

In this situation the grants are given to leading firms making next technological step.

The results of application of this scheme we shall formulate in

Proposition 4.3. In case of large innovations encouraging of prolongation of innovative made by leaders in industry always gives a positive outcome. The application of the scheme imost effective in case of rigid patent policy when the competition on the markets is nearly perfect (see fig. 4.3).

As it was already marked, if innovations are large then the profit of a leader will raise up to the greatest possible value during for one step. Thus incentives to prolong innovations are missed. Only grants make incentives to prolong innovations.

Monopolists who are not engaged in innovations without stimulation will prolong their researches and will give the contribution to technological advance if there are grants.

If competition is nearly perfect anticipated profit of leaders won't change, because all taxes collected are reallocated through grants to leaders (but some time later, that is why if the discount rate is large, this effect will be more moderate) As a result, imitators and the innovators-competitors practically do not reduce the efforts.

But if firms on the same technological level have a monopolistic power (monopolistic competition), and have some profit in this state, the taxes will be collected not only from leaders, but also from neck-and-neck firms, that is incentives to imitate will decrease. As a result, the positive effect of this scheme is more gentle. However effect from ascending number of innovations nevertheless predominates.

4.2 Case of small of innovations: the efficiency increases if patent policy becomes tougher

Case of small innovations is even more complex to analyze. Again the method of asymptotic decompositions is used, and cases $\alpha < 1$ and $\alpha = 1$ are considered separately. Only main conclusions from the analysis are represented below.

Case 1. Stimulation of imitations: $\widetilde{S}_{-1} \neq 0, \quad S_1 = S_0 = 0$.

In this case grants are given to firms, which have just now catched up the competitor.

Proposition 4.4. In case of small innovations the encouraging of imitations has a negative effect for economic growth rates, probably, except for an area, where there are no imitations at all (gentle competition, difficult imitations). If patent policy is soft and competition on the markets is gentle then deboosting of growth rate will be most significant (see fig. 4.4).

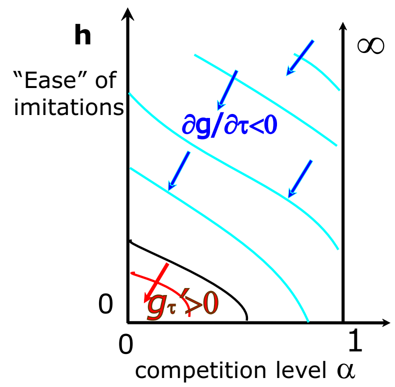

Figure 4.4: Change of a growth rate if minimum tax for all firms and grants to successful imitators are introduced: $\widetilde{S}_{-1} \neq 0, \quad S_1 = S_0 = 0$ (case of small innovations)

Case 2. Stimulation of innovations on the leading edge of technology: $S_0 \neq 0$.

In this situation the grant is given to firms which have just now became a leader.

Proposition 4.5. In case of small innovations the encouraging of innovations on the leading edge of technology has ambiguous, but very small effect. This effect can be both positive or negative, but in any case has the smaller size the smaller the size of innovations is (see fig. 4.5).

This stimulation involve a very small circle of a firms. That's why the outcome of this scheme is almost negligibly small.

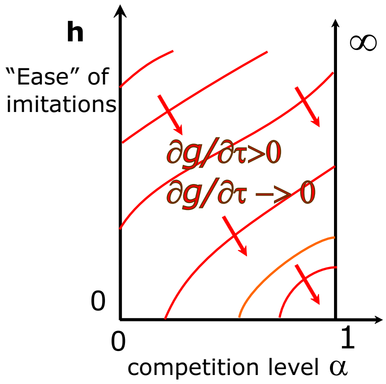

Figure 4.5: Change of economic growth rate if minimum tax and grants to successful innovators on the leading edge of technology are introduced: $\widetilde{S}_{-1} = S_1 = 0, S_0 \neq 0$ (case of small innovations)

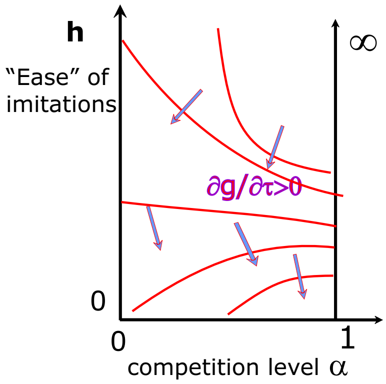

Figure 4.6: Change of economic growth rate if minimum tax and grants for prolongation of innovations by the leaders are introduced: $\widetilde{S}_{-1} = S_0 = 0, S_1 \neq 0$ (case of small innovations)

Case 3. Stimulation of prolongation of innovations by leaders: $S_1 \neq 0$

In this situation the grant is given to the firms-leaders making the next technological step. This is the most successful scheme (see fig. 4.6):

Proposition 4.6. In case of small innovations encouraging of prolongation of innovations by industry leaders always gives a positive outcome. The application of the scheme reaches its peak efficiency in case of strict patent policy if the competition on the markets is close to perfect.

In conclusion of the chapter once again it should be noted that all results are obtained for "minimum grant", if the initial tax rate is equal to zero. It helps to reveal: whether there is a probability of effective interference of government into an economy in principle.

However the further increase of taxes and grants is possible only up to a definite limit. After some critical value the father increase of grant's size though can increase growth rates in short-run, will result in decrease of an equilibrium (long-term) growth rate. The optimal size of government interference is not obtained in this work. It is a subject of separate research.

It is necessary to note, that if from the collected means only a part can be expended effectively, the areas with a small positive effects on all pictures can turn into areas with negative effect. That means that there is no sense to recommend policy with a minor positive effect, as well as policy with negative effect.

Comparison of different schemes. Practical guidelines. Concluding remarks

The purpose of this work is to answer the question: "How do government transfers to competing innovators influence technological growth? Whether they are necessary? Under what conditions?"

The results of this work should be concerned with practical guidelines to state programs of stimulation of technological advance.

The efficiency of different schemes of stimulation of innovations has been obtained in the context of Aghion's endogenous growth model with step-by-step innovations. The results of this research are presented in table 1.

In the initial model innovations in industries with neck-and-neck firms, give the greatest contribution into a development of an economy. The first group of results deals with the first scheme of stimulation: government reallocation of means collected through profit tax in all economy to neck-and-neck innovators (see column 4 of table 1). It has been obtained, that such grants can both stimulate, and slow down technological advance. Namely: (i) the introduction of such grant is good for growth in case of hard competition on the markets and strong patent policy (relatively cost-intensive imitations). (ii) At the same time, such subsidizing is inefficient in case of soft competition and relatively easy imitations. (iii) There is a critical value of competition level, or profits of neck-and-neck firms: if the profit of such firm is less than this critical value, the subsidizing will result in acceleration of economic growth rate; if profits of such firms are greater than critical value, the subsidizing will be inefficient.

The results obtained can be used for the analysis of preferential guidelines of state policy. For example, if some part of economy is characterized by large innovations, sufficiently competitive markets and strongly protected patents for a stimulation of growth rate when it will be possible to recommend: (i) increase of "attractiveness" of possession of high technology (subsidizing), and (ii) facilitation of imitations ("softening" of an attitude to imitations, encouraging imitations, weakening patents protection, patent acquisition by government with their following free (or discounted in price) distribution).

While solving a question: to stimulate neck-and-neck innovators or not their profit can be selected as a decisive parameter. For an estimation of a critical value of this profit it is necessary to know such parameters as (i) the distribution of firms by their levels of technological development; (ii) relative ease of imitations; (iii) expenditures of firms on imitation and innovations; (iv) relative size of innovatory

Table 1. Summary of results. Effective outcomes from different subsidizing schemes.

Ease of imitations	Size of innovations	Competitiveness of markets	Subsidizing of neck-and-neck innovators	Subsidies proportional to costs		Grants for realised improvements in technology. Award for:			
				sign	size of effect, g'	imitations	Innovation made in neck-and-neck position		Innovation made by leader
							sign	size, g'	
1	2	3	4	5	6	7	8	9	10
$h \to 0$	$\gamma \to 1$	$\alpha \to 0$	$(-)$	$(+)$	$1/\alpha \to \infty$	(-0)	$(+0)$		$(+)$
	$(\gamma = 1 + \varepsilon)$	$0 < \alpha < 1$	$(-)$ if $\alpha < \alpha^{\gamma}_{\text{crit}}$ $(+)$ if $\alpha > \alpha^{\gamma}_{\text{crit}}$	$(+)$	$(1-\alpha) \to 0$, if $\alpha \to 1$	$(-)$	$(+0)$		$(+)$
		$\alpha = 1$	$(+)$	$(+0)$	$\varepsilon \to 1$	$(-)$	$(+0)$		$(+)$, max
	$\gamma \to \infty$	$\alpha \to 0$	$(-)$	$(+)$	$(18\sqrt{\beta})^{-1}$	$(-)$	$(+)$	$1/2 \ln \gamma$	$(+)$
		$0 < \alpha < 1$	$(-)$ if $\alpha < \alpha^{\infty}_{\text{crit}}$ $(+)$ if $\alpha > \alpha^{\infty}_{\text{crit}}$	$(+)$		$(-)$	$(+)$		$(+)$
		$\alpha = 1$	$(+)$	$(+)$	$(\sqrt{\beta})^{-1}$	$(-)$	$(+)$	$1/9 \ln \gamma$	$(+)$, max
$h \to \infty$	$\gamma \to 1$	$\alpha \to 0$	$(-)$	$(+)$	$h/\alpha \to \infty$	$(-)(-)$	$(+0)$		$(+)$
		$\alpha = 1$	$(-)$	$(+0)$	$\varepsilon \to 0$	$(-)(-)$	(-0)		$(+)$, min
	$\gamma \to \infty$	$\alpha \to 0$	$(-)$	$(+0)$		$(-)(-)$	$(+)$	$2 \ln \gamma$	$(+)$
		$\alpha = 1$	$(-)$	$(+0)$		$(-)(-)$	$(-)$	$\frac{1-h}{h^2} \ln \gamma$	$(+)$, min

The magnitude of the outcome is defined as a change in economic growth rate while a minimal subsidy is introduced: $g' = \frac{\partial g}{\partial \tau}\Big|_{\tau \to 0}$

$(+)$ — Positive outcome from the scheme: $g' > 0$

$(-)$ — Negative outcome from the scheme: $g' < 0$

$(-)(-)$ —Very negative outcome from the scheme : $g' \ll 0$

$(+0)/(-0)$ — Positive / negative, but negligibly small outcome

max /(min) — The best / the worst outcome (with fixed size of innovations).

h — ease of imitations. $h \to 0$: harden (costly) imitations, strict patent policy. $h \to \infty$: easy (costless) imitations, nearly all imitations are spontaneous.

γ — size of innovations. $\gamma = 1 + \varepsilon$: small, gradual innovations. $\gamma \to \infty$: large innovations.

α — competitiveness of markets. $\alpha \to 0$ — weak competition. $\alpha = 1$ — perfect competition.

neck-and-neck state — two innovators compete at the same technological level.

step. If the profits of firms are rather small (markets are competitive), their subsidizing will have a positive effect on economic growth rate. If the profit is sufficiently high (mild competition at the markets), the subsidizing will lead to opposite outcome.

For an economy, in which industries are mainly characterized by small innovations the conditions on efficiency of the grants are harder. In most cases subsidizing will be inefficient, that means that it is necessary to look for other ways to stimulate of such industries.

It is useful to note that in the above described case the grant is given for **position** of firm on the market at the given moment: it has neck-and-neck competitor. At the same time tax is imposed on profit of all firms.

It is useful to compare the described scheme of subsidizing to other schemes. First, the stimulation of *efforts* is almost always[1] better, than the subsidizing of firms owning to their definite position on the market (compare column 5 of tables 1 with column 4). It is even better only to compensate a part of expenditures on actually successful innovations thus to avoid financing "undeserved achievements", such as: the grants for innovations/imitations, which cost practically nothing to the producer. (compare columns 8 and 10 of tables 1 to column 4).

Let's address the scheme of subsidizing **efforts**. According to this scheme, grants to firms are proportional to their spendings on innovation and/or imitation. As innovations and imitations are stochastic processes, in such scheme the government undertakes a part of risk, by financing efforts, instead of their outcomes. (Partial compensation of investments in innovations, can give different result in contrast to the first scheme.) As well as in the first scheme, the compensation of efforts is effective in case of the competitive markets with the hindered innovations. But also it is important to remark, that this scheme can give considerable positive outcomes on monopolized markets, especially for industries with rather small size of innovations.

As an example we shall consider, which guidelines can be given based on the analysis of the first scheme, say, for Russia.

In Russia the majority of markets are estimated as the markets with a weak internal competition. At the same time copyrights in our country are often violated. So imitations are not too cost-intensive process. It can mean that giving subsidies according to the first scheme can appear wasteful in our country [2]. That means that it will be more convenient to recommend "stimulation of concrete achievements", namely, innovations. [3]

[1] There are situations, in which this scheme of subsidizing (as well as other schemes reviewed) does not give appreciable outcomes

[2] On the other side, there are complexities with dissemination of information about new technologies, that handicaps an effective imitation. In this case guidelines should be "a little bit corrected".

[3] At the same time, in case of incomplete information it is better in general to refuse stimulation and to undertake directly "correcting of parameters of a system": maintenance of competition at the markets, development of laws and creation of institutes which will contribute to protection of intellectual property rights. However, it is another question. You see, putting in order institutions and economy can be done both directly and indirectly. For example, it is widespread enough the judgement, that not only the well tuned economy has a positive effect for growth rates, but there is also

On an example of financing **achievements** (issue of fixed grant to firms making a successful innovation/imitations) another problem is reviewed: what is more effective to stimulate, innovations or imitations? The answer is well-defined: within the framework of this model it is more effective to stimulate innovations (compare column 7 to columns 8 and 9 of table 1). Intuitively these results can be interpreted in the following way: stimulation of imitations is inefficient, as it bates incentives to make innovations. (the more the size of innovations is, the more strong negative effect from the introduction of such scheme of stimulation is). The origins of this situation are: grants, which are given to imitators, depend not only on their efforts. They are also proportional to spontaneous imitations. So the reallocating of means actually is transfered to the party of lagging producers, not affecting their incentives, that can not be effective. But taxes collected for these subsidization directly affect and has a negative effect on incentives of innovators. Therefore the damage from attempts to stimulate imitators is higher the easier the imitations are (the large part of means is reallocated not encompassing incentives).

So, the second set of conclusions is: the stimulation of imitations can occurs more preferential than stimulation of innovations, if imitations are hindered. On the contrary: if imitations are relatively easy and firms have some monopoly power on the markets (monopolist competition), the best outcome will be reached by stimulation of innovations.

Though we have legible enough directions how to operate, in practice the implementation of the majority of good theoretical guidelines is integrated with a number of difficulties. That is why less effective, but more robust schemes are selected more often. For example it is known, that it is possible to evaluate positions of the firm on the market (wether this firm has the close competitors or not, whether it is an indisputable leader in the industry or lags on the level of development). On the other hand, the estimation of R&D expenditures of a firm invokes a set of difficulties. First of all, firm rarely reports precise size of its spendings Firm can record as expenses on innovation and development expenditures, actually not being those. (For example, whether costs presently spent, say, for purchasing of new computers could be considered as the expenses on R&D?)

The same way the estimation of actual successes of firms often is a trade secret and also badly yields to an estimation.

Though on average, from the idealized point of view, subsidizing of efforts or grants for the actual inventions is more effective, however operational use of those schemes is more complex, than application of the first of the reviewed schemes.

The other difficulty is that any government reallocation of resources is connected with a capability of rent seeking by the officials. This means, that not all means collected through taxes will be reallocated effectively.

In the model the very simple economy structure is supposed: in each industry there are only two firms. Therefore it fails to receive estimations for efficiency of government stimulation of a firm

inverse influence: if on any causes the economy starts to grow faster, many gears adjust "by themselves" occupying a "mean" position in industry.

One of the assumptions of the model is that for firms only their *relative* position in industry is important. Thus the absolute level of development of technology has no value. However in real life a capabilities for innovations for firms do depend on absolute values of a technological level.

Because of all these difficulties the obtained results have to be applied gently at real-life situations.

This report is based on the work done during my study at NES, in which I have had the opportunity to interact with a wide variety of persons. In particular the interest shown by those who attend our project meetings and April 2003 presentation of the main results of this work, their questions and fruitful critics were highly stimulating. Special acknowledgements are to Victor Polterovich, my scientific supervisor, for suggestion of an interesting subject for researches, encouraging discussions and especially for saving me from unsolvable problems. I would like to thank Alexander Tonis, our research assistant, for critical comments on the manuscript.

And last, but not least I would like to thank all my family, without their help this work could not appear in the way it looks now: Jakov Zhabitskij for technical support and help on using LaTeX; Tatiana Zhabitskaja, our redactor, who have read the Russian version of the paper and helped me to do it more comprehensive; Michail Zhabitskij, my husband, for continuous assistance in preparing this manuscript, his help with many of the drawings and translation of this work into English, but most of all for his optimism and readiness to support at any time.

Bibliography

[1] Aghion Ph., Harris Ch., Howitt P., Vickers J. (2001). *Competition Imitation and Growth with Step-By-Step Innovation.* Review of Economic Studies (2001), v. 68, pp. 467-492.

[2] Aghion Ph., and Howitt P. (1998). *Endogenous Growth Theory.* The MIT Press, Cambridge, Massachusetts, p.694.

[3] Aghion Ph., and Howitt P. (1992). *A Model of Growth through Creative Destriction.* Econometrica, 1992, **60**, pp. 323-351.

[4] Barro R.J. and Sala-i-Martin X. (1995). *Economic Growth.* New York: McGraw-Hill.

[5] Blundell R.J., Griffith R. and Van Reenen J. *Dynamic Count Data Models of Technological Innovation.* Economic Journal, 1995, **105**, pp. 333-344.

[6] Dinopoulos E. (1991) *Schumpeterian Product Evolution and Vanishing Growth.* Memo, The University of Florida.

[7] Grossman G.M. and Helpman E. (1991). *Innovation and Growth in the Global Economy.* Cambridge. Massachusetts: MIT Press.

[8] Grossman G.M. and Helpman E. (1991). *Quality Ladders in the Theory of Growth.* Review of Economic Studies, 1991, 58, pp. 43-61.

[9] Grossman G.M. and Helpman E. (1991). *Quality Ladders and Product Cycles.* Quarterly Journal of Economic, 1991, 106, pp. 557-586.

[10] Howitt P. (1999). *Steady Ehdogenous Growth with Population and R&D Inputs Growing.* Journal of Political Economy, 1999, **107**, pp. 715-730.

[11] Nickell S. (1996). *Competition and Corporate Perfofmance.* Journal if Political Economy, 1996, **104**, pp. 724-746.

[12] Davidson C., Segerstrom P. (1998). *R&D Subsidies and Economic Growth.* RAND Journal of Economics. 1998, **29**, pp. 548-577.

[13] Segerstrom P., Annant T., Dinopolus E. (1990). *A Shumpeterian Model of the Product Life Cicle.* American Economic Review, 1990, 80, pp. 1077-1092.

[14] Segerstrom P. (1991). *Innovation, imitation and economic growth.* Journal of Political Economy, 1991, 99, pp. 807-827.

[15] Segerstrom P. S. and Zolnierek J.M. (1999). *The R&D incentives of Industry Leaders.* International Economic Review, 1999, **40**, pp. 745-766.

[16] Segerstrom P. (2000). *The long-Run Growth Effects of R&D Subsidies.* Journal of Economic Growth, 2000, v. 5, pp. 277-305.

[17] Cohen W.M. and Levinthal D.A. (1990). *Innovation and Learning: The Two Faces of R&D.* The Economic Journal, 1990, 99(397), pp. 569-596.

[18] Romer P.M. (1990). *Endogenous Technological Change.* Journal of Political Economy, 1990, 98(5), part 2, pp. 71-102.

[19] Zeng J. (1993). *Innovative vs. imitative R&D and Economic Growth.* (Unpublished, University of Western Ontario).

[20] Zeng J. (2001). *Innovative vs. Imitative R&D and Economic Growth.* The Journal of Development Economics, 2001, v. 64, pp. 499-528.