Васин А.А., Гонтмахер К.Е., Сосина Ю.В. Об оптимальном распределении информационных ресурсов в избирательной компании. / Препринт # 2003/. М.: Российская экономическая школа, 2003. — 14 с. (Англ.)

В современной теоретической литературе отмечается важная роль информации в избирательных кампаниях. Политическая реклама может существенно влиять на предпочтения избирателей. Однако для эффективной рекламы требуются большие финансовые либо административные ресурсы. Проблема оптимального распределения таких ресурсов представляет большой интерес для различных политических субъектов. В настоящей работе эта проблема рассматривается с точки зрения игроков: Власти, т.е. политической силы, контролирующей правительство ко времени выборов, и Оппозиции, интересы которой противоположны интересам Власти.

Рассматривается страна с несколькими политическими партиями. Каждая партия характеризуется количеством "сознательных" сторонников, исходным информационным рейтингом и лояльностью по отношению к Власти, то есть долей депутатов, поддерживающий Власть в парламентской фракции партии. Предполагается, что политическая реклама аддитивно меняет информационные рейтинги партий. Итоговые информационные рейтинги определяют долю голосов каждой партии среди "впечатлительных" избирателей. Целью Власти является получение доли голосов, которая бы позволила сохранить контроль над правительством и провести желательные для нее законы через парламент. Оппозиция пытается предотвратить формирование необходимого большинства.

В работе исследованы соответствующие оптимизационные и теоретико-игровые модели. Показано, что отдельной партии обычно невыгодно тратить ресурсы на антирекламу конкурентов. Однако антиреклама партии с низкой лояльностью может входить в оптимальную стратегию Власти, а антиреклама партии с высокой лояльностью – в оптимальную стратегию Оппозиции. В типичном случае каждый игрок предоставляет ресурсы для рекламы только одной партии.

Данная работа была выполнена в рамках проекта GET. Мы благодарим Григория Косенка за полезные рекомендации и замечания.

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The role of information in electoral campaigns is widely recognized in the modern literature. Political advertising may essentially influence voters' preferences, but usually requires large financial or administrative recourses. The problem of the optimal distribution of these resources in the electoral campaign is of great interest for different political subjects. The present paper considers this problem from the point of view of two players: Power, that is the political group that controls the government at the time of election, and Opposition, whose interests are contrary to the interests of Power.

We consider a country with several political parties. Every party is characterized by the number of its conscious supporters, the initial informational rating and its loyalty to Power, i.e. the share of Power's supporters in the parliament fraction of the party. We assume that advertising and discredit additively change informational ratings of the parties. The final informational ratings determine vote shares for each party. Within electoral campaign, Power aims to get certain share of votes in order to maintain its control over the government and push the desirable laws through the parliament. Opposition tries to prevent formation of the necessary majority.

Our main conclusions on the optimal strategies of the players are as follows. Any party typically wouldn't spend resources on discredit of competitors, but use it on advertising itself. However, discredit of the parties with low loyalties may be optimal for Power and, inversely, discredit of the parties with high loyalties may be optimal for Opposition. Moreover, each player typically provides only one party with the resource for advertising.

1. INTRODUCTION

The role of information in electoral campaigns is widely recognized in the modern literature. Political advertising may essentially influence voters' preferences, but usually requires large financial or administrative recourses. The problem of the optimal distribution of these resources in the electoral campaign is of great interest for different political subjects. The present paper considers this problem from the point of view of two players: Power, that is the political group that controls the government at the time of election, and Opposition, whose interests are contrary to the interests of Power.

The theoretical literature on electoral competition considers two main types of agents: political parties and voters. A strategy of the party is its political program. Usually the program is represented by the point in the set of possible political platforms. Each voter is characterized by his bliss point in the same set, and the whole population – by the distribution over the set of political programs. A standard assumption is that each individual votes for the party with the program closest to his bliss point. Two kinds of political parties are considered in the literature. Downs (1957), Cox (1985, 1987) et al. study the models with opportunistic parties that aim to win elections or to get the maximal share of votes and do not care about the final policy. These papers find Nash equilibria in the corresponding games. The known result of such sort is the Median Voter Theorem and its generalizations. Baron (1994) considers the model with one more type of voters – impressionable individuals, who are sensitive to political advertising. This model assumes that distribution of voters of this kind is proportional to distribution of money spent by political parties on advertising. The money comes from another type of agents – lobbies, who are interested in the final policy and finance the parties depending on their programs. Baron shows that existence of lobbies and impressionable voters implies divergence of political programs of opportunistic parties.

Wittman (1990), Ortuno-Ortin and C. Schultz (2000) assume that the parties are ideological, that is, they have preferences over the policy space and represent the interests of competing groups in the society. The objective of each party is to implement the political platform that maximises its utility. The resulting platform corresponds to the convex combination of political programs of the parties with the weights proportional to the shares of votes. The latter paper provides a theoretical model that casts some light on the issue of public funding of political parties. They show that the public funding that proportionally depends on the vote share (the system which prevails in many European countries) makes the parties' policies converge.

It's worth to mention paper by Federsen, Sened, Wright (1990) that considers sophisticated voters, who care about the final outcome of elections and vote strategically. The corresponding models show that sophisticated voters typically choose either extremely left or extremely right parties.

Now consider the mentioned models and concepts with respect to the Russian political system (which is probably similar to other countries in transition). In this discussion we proceed from several empirical researches [9-16]. They show that the conjecture on "rational" behavior of voters who aim to influence the outcome of elections or study the programs of the parties and choose one closest to the bliss point fails for Russian voters. (Let us note that according to the known voting paradox, a rational individual shouldn't take part in elections with many voters if he or she is interested in the outcome and the process itself doesn't touch his (her) utility). One more distinguishing feature of the electoral competition in Russia that does not meet the theoretical models is great expenses of resources on discredit of competitors.

According to the empirical studies, the overwhelming majority of Russian voters belongs to the following two types: "conscious" and "impressionable". Voters of the first type are strongly tied to the existing political parties. Any radical change of the platform may rather repel than attract such voters. As to impressionable voters, their preferences are primarily determined by advertising in mass media. So their behavior corresponds to the mentioned models, except for that some of them a priori reject certain parties. (The label that attracts one part of the population repels another part.) According to the data [10, 13-16], the share of impressionable voters is evaluated from 40 to 70%.

Thus the parties are rather passive in the competition. According to their preferences, they are probably opportunistic, but their roles are given, and one thing they can do is to play them as impressively as possible. The active players are elite groups that have financial and administrative resources. Usually the most powerful player is the group that controls the government. Below we name this group Power. Another group (Opposition) includes financial oligarchs opposed to the government. (Actually, Power and Opposition are often heterogeneous, but here we ignore this circumstance). As Zadorin [8,9] mentions, there are no active players who are toughly tied to the interests of the majority of the population in Russia now. Within electoral campaign, Power usually aims to get a certain share of votes in order to maintain its control over the government and push the desirable laws through the parliament. Opposition tries to prevent formation of the necessary majority. So the interaction may be considered as a two-player zero sum game.

The present paper describes formally and studies the corresponding model. Every party is characterized by the number of its conscious supporters, the initial informational rating and its loyalty to Power, i.e. the share of Power's supporters in the parliament fraction of the party. Proceeding from the discussed relation between a party and its conscious supporters, we assume that the loyalty doesn't depend on the strategies of Power and Opposition. We assume that advertising and discredit additively change informational ratings of the parties. The final informational ratings determine vote shares for each party. We investigate models with convex and concave functions that transform informational ratings into voting ratings. A standard assumption in the theoretical literature is that the shares are proportional to the informational ratings (see Baron, 1994). Besides this case, we consider the variant where the voting rating of the party exponentially depends on its informational rating. Section 3 below provides some arguments in favor of such assumption.

Our main conclusions relate to the optimal distributions of the budgets for advertising and discredit among political parties. We show that each player typically provides only one party (the strongest ally) with the resource for advertising, while the budget for discredit is in general distributed among several most popular hostile parties and equalizes their ratings. Each player should strictly control the employment of its resource: distribution of it among the loyal parties may be inefficient since their incentives for advertising and discredit may sufficiently differ from the player's interests.

The rest of the paper is organized as follows. Section 2 describes the formal model and mathematical problems we aim to study. Section 3.1 considers the problem of the optimal distribution of the resources by Power. Section 3.2 investigates the game "Power – Opposition" under fixed budgets on advertising and discredit and different assumptions on the order of actions. Section 3.3 studies the optimal distribution of the player's budgets between advertising and discredit and considers the model with linear and exponential transforming functions. Section 4 concludes with discussion of political implications of our results.

2. The game "Power – Opposition"

We consider a country with n political parties. Each party j is characterized by its loyalty l_j, the share sc_j of its supporters among conscious voters and initial informational rating r_{j0} among impressionable voters. Let α be the share of impressionable voters, a_{j+} and a_{j-} denote resp. amounts of advertising and discredit of party j. The final informational rating of the party is $r_j = r_{j0} + a_{j+} + a_{j-}$. We assume that the final ratings determine the distribution of impressionable voters over parties as follows:

$$si_j = f(r_j) \Big/ \sum_{k=0}^{n} f(r_k),\ j = 0,\ldots,n,$$

where si_0 is the share of abstinent and si_j is a share of votes for party j among impressionable voters. Function $f(\cdot)$ is nonnegative and monotonously increasing on the set of possible ratings. It characterizes the efficiency of advertising and discredit, so below we call it an efficiency function.

Under any given amounts of advertising and discredit, the total share of supporters of Power in the parliament depends on the informational ratings as follows:

$$l(r) = \sum_{j=1}^{n} \frac{l_j \left(\alpha f(r_j) \Big/ \sum_{k=0}^{n} f(r_k) + (1-\alpha) sc_j \right)}{\sum_{j=1}^{n} \left(\alpha f(r_j) \Big/ \sum_{k=0}^{n} f(r_k) + (1-\alpha) sc_j \right)} \tag{1}$$

Let $a_{j+}^P \geq 0$, $a_{j-}^P \geq 0$ denote the amounts of advertising and discredit of party j by Power, B_+^P, B_-^P be the total amounts of advertising and discredit. Formally a strategy of Power is a vector $a^P = \left(a_{j+}^P, a_{j-}^P,\ j = 1,\ldots,n,\ B_+^P, B_-^P\right)$ that meets the budget constraints:

$$\sum_{j=1}^{n} a_{j+}^P \leq B_+^P, \qquad \sum_{j=1}^{n} a_{j-}^P \leq B_-^P, \tag{2}$$

$$B_+^P + \gamma B_-^P \leq B^P, \tag{2'}$$

where γ shows the related cost of discredit with respect to advertising, B^P is the total resource of Power for the electoral campaign. (Of course, for any reasonable strategy either $a_{j+}^p = 0$ or $a_{j+}^p = 0$ for any j.)

Similarly, a strategy of Opposition is a vector $a^O = \left(a_{j+}^O, a_{j-}^O,\ j = 1,\ldots,n,\ B_+^O, B_-^O\right)$ that meets constraints:

$$\sum_{j=1}^{n} a_{j+}^{O} \le B_{+}^{O}, \qquad \sum_{j=1}^{n} a_{j-}^{O} \le B_{-}^{O}, \tag{3}$$

$$B_{+}^{O} + \gamma B_{-}^{O} \le B^{O}, \tag{3'}$$

Strategies a^P and a^O determine the vector of ratings of the parties $r(a^P, a^O) = r + a_{+}^{P} - a_{-}^{P} + a_{+}^{O} - a_{-}^{O}$ and the value $l(a^P, a^O)$ according to (1).

Below we discuss several optimization problems and games related to the given model:

1) For a given strategy of Opposition and fixed budgets B_{+}^{P}, B_{-}^{P} for advertising and discredit, to find the optimal strategy of Power that maximizes the share of its supporters in the parliament:

$$a^{P*}(a^O) \to \max_{a^P} l(a^P, a^O) \tag{4}$$

under constraint (2). A similar problem for Opposition is to find

$$a^{O*}(a^P) \to \min_{a^O} l(a^P, a^O) \tag{5}$$

under (3).

2) Under given budgets B_{+}^{P}, B_{-}^{P}, B_{+}^{O}, B_{+}^{-}, to find the optimal maximin and minimax strategies of Power and Opposition:

$$\bar{a}^{p} \to \max_{a^P} \min_{a^O} l(a^P, a^O) \overset{def}{=} v^{l}(B^P, B^O) \tag{6}$$

$$\bar{a}^{O} \to \min_{a^O} \max_{a^P} l(a^P, a^O) \overset{def}{=} v^{u}(B^P, B^O) \tag{7}$$

If there is no saddle point then to study the game in mixed strategies.

3) Under a given strategy of Opposition, to find the optimal distribution of the budget of Power between advertising and discredit, i.e. to study the problem (4) under constraints (2), (2'). To examine the game "Power − Opposition" including the distribution of the budgets between advertising and discredit for the both players.

Under a given budget of Opposition, to find the minimal budget of Power that provides a necessary majority l^* in the parliament:

$$B^{Pu} = \min\{B^P \ s.t.\ v^u(B^P, B^O) \ge l^*\}. \tag{8}$$

3. STUDY OF THE MODELS

3.1. Optimization problem

Let $s=(1-\alpha)\sum_{j=1}^{n} sc_j$ denotes the share of conscious voters who take part in the elections, $l^c=\sum_{j=1}^{n} l_j sc_j \Big/ \sum_{j=1}^{n} sc_j$ – an average loyalty of these voters to Power, $m_0 = sl^c/(\alpha+s)$, $s_0 = s/(s+\alpha)$, $m_j=(\alpha\cdot l_j+sl^c)/(\alpha+s)$ – a modified loyalty of party j, $\sigma \overset{def}{=} \sum_{1}^{n} f(r_j)+f(r_0)\cdot s_0$. Then we can rewrite the payoff function (1) as follows :

$$l(r)=\left(\sum_{j=1}^{n} f(r_j)\cdot m_j + f(r_0)\cdot m_0\right)\Big/\sigma .$$

Lemma 1. $\dfrac{\partial l}{\partial r_j}=\dfrac{\partial l}{\partial a_{j+}}=-\dfrac{\partial l}{\partial a_{j-}}=\dfrac{f'(r_j)\cdot(m_j-l)}{\sigma}$.

Thus the marginal efficiency of advertising is proportional to the difference between the modified loyalty of the party and the average loyalty.

Consider problem (4). Any fixed strategy a^0 changes initial ratings of the parties. Otherwise the problem is equivalent to the optimization problem without Opposition.

Proposition 1. Under the optimal strategy a^{p*}, $a_{j+}^{p*}>0$ only for the parties with $m_j>l$ and $j\in Arg\max_i f'(r_i)\cdot(m_i-l)$, and $a_{j-}^{p*}>0$ only for the parties with $m_j<l$ and $j\in Arg\max_i f'(r_i)\cdot(l-m_i)$.

Now consider the case where Power distributes a fixed resource d on advertising between parties 1 and 2, other components of the strategy are fixed. Thus $a_+ \overset{def}{=} a_{1+}=d-a_{2+}$. Then

$$\frac{dl(a_+)}{da_+}=\frac{f'(r_1+a_+)\cdot(m_1-l(a_+))-f'(r_2+d-a_+)\cdot(m_2-l(a_+))}{\sigma}.$$

Lemma 2. If the marginal efficiency of advertising $f'(r)$ doesn't decrease in r, $m_i\ge l(a_+)$ for any $a_+\in[0,d]$ $i=1,2$ then the function $l(a_+)$ is quasi-convex and reaches its maximum either at $a_+=0$ or at $a_+=d$. If $f'(r)$ doesn't increase in r then $l(a_+)$ is quasi-concave.

Proof. Assume that $m_1\ge m_2$. Let us show that if $l'(\bar{a})>0$ for some $\bar{a}\in[0,d]$ then $l'(a)>0$ for any $a>\bar{a}$. Indeed, $f'(r_1+a_+)$ doesn't decrease in a_+, $f'(r_2+d-a_+)$ doesn't increase in a_+, $(m_1-l(a_+))/(m_2-l(a_+))$ doesn't decrease in a_+ since $l(a)$ increases in a_+. The proof of the second proposition is analogous.

Consider a similar problem where Power distributes a fixed resource d on discredit of parties 1 and

2, $a_- \overset{def}{=} a_{1-} = d - a_{2-}$. Then

$$\frac{dl(a_-)}{da_-} = \frac{f'(r_1 - a_-)\cdot(l(a_-) - m_1) - f'(r_1 - d + a_-)\cdot(l(a_-) - m_2)}{\sigma}$$

Lemma 3. If the marginal efficiency of advertising $f'(r)$ doesn't increase in r, $m_i \le l(a_-)$ for any $a_- \in [0, d]$, $i = 1,2$ then the function $l(a_-)$ is quasi-convex and reaches its maximum either at $a_- = 0$ or at $a_- = d$. If $f'(r)$ doesn't decrease in r then $l(a_-)$ is quasi-concave.

Now let us return to the general optimization problem (4).

Proposition 2. If the marginal efficiency of advertising $f'(r)$ doesn't decrease in r and $l_i \ne l_j$ $\forall i \ne j$ or $f''(r) > 0$ then under the optimal strategy Power advertises at most one party. If the marginal efficiency of advertising $f'(r)$ doesn't increase in r and $l_i \ne l_j$ $\forall i \ne j$ or $f''(r) < 0$ then Power discredits at most one party. The marginal efficiencies of advertising and discredit are proportional to the costs, i.e.

$$(a_{i+} > 0, a_{j-} > 0) => f'(r_i)(m_i - l) = \gamma \cdot f'(r_j)(l - m_j).$$

Consider an important case where each party is either loyal or disloyal to Power:

$$l_i = 1, \quad i = 1,..,k, \quad l_i = 0, \quad i = k+1,..,n. \tag{9}$$

Assume that in every group parties are ordered by decrease of their initial ratings: $r_1 \ge ... \ge r_k$, $r_{k+1} \ge ... \ge r_n$.

Proposition 3. If the marginal efficiency of advertising $f'(r)$ increases in r then only party 1 may be advertised by Power, and there exists such $j \ge k+1$ that Power discredits only parties $k+1,..,j$, i.e. $a_{j-} > 0 => a_{i-} > 0$ for $i = k+1,.,j-1$. If the marginal efficiency of advertising $f'(r)$ decreases in r then Power doesn't discredit any party except n, and there exists such j that Power advertises only parties $j,...,k$.

Proof. Note that the difference $m_j - l(a^p)$ doesn't change its sign for any a^p for each party $i = 1,..,n$ in the case.

Now consider the problem (5) for Opposition. Since this problem is symmetric to the problem (4) of Power, we obtain the following

Proposition 4. If the marginal efficiency of advertising $f'(r)$ increases in r then only party $k+1$ may be advertised by Opposition, and there exists such $j \le k$ that Opposition discredits only parties $1,.., j$. If the marginal efficiency of advertising $f'(r)$ decreases in r then Opposition doesn't discredit any party except k, and there exists such $j \ge k+1$ that Opposition advertises only parties $j,...,n$.

One important issue is whether each player may just distribute its resource among parties or should specify the amounts on advertising and discredit for each party. In order to answer this question consider an optimization problem of a party which can spend it's resource on advertising itself or discredit of its competitors. The party aims to maximize its share of deputies in the parliament. Proceeding from (1), this share s_i depends on the strategy as follows:

$$\frac{\partial s_i(r)}{\partial a_{i+}} = \frac{f'(r_i)\left(\frac{\alpha + (1-\alpha)sc_i}{\alpha + s} - s_i\right)}{\sigma}; \quad \frac{\partial s_i(r)}{\partial a_{j-}} = \frac{f'(r_j)(s_i - \frac{(1-\alpha)sc_i}{\alpha + s})}{\sigma^2}.$$

Consider the case (9) with loyal and disloyal parties. Then the average loyalty depends on advertising and discredit as follows:

$$\frac{\partial l}{\partial a_{i+}} = \frac{f'(r_i)}{\sigma}\left(\frac{\alpha + (1-\alpha)\sum_{p\le k} sc_p}{\alpha + s} - \sum_{p\le k} s_p\right), \quad \frac{\partial l}{\partial a_{j-}} = \frac{f'(r_j)}{\sigma}\left(\sum_{p\le k} s_p - \frac{(1-\alpha)\sum_{p\le k} sc_p}{\alpha + s}\right).$$

The differences of the marginal efficiencies for the party and Power are

$$\frac{\partial s_i(r)}{\partial a_{i+}} - \frac{\partial l}{\partial a_{i+}} = \frac{f'(r_i)}{\sigma}\sum_{p\le k, p\ne i}\left(s_p - \frac{(1-\alpha)sc_p}{\alpha + s}\right),$$

$$\frac{\partial l}{\partial a_{j-}} - \frac{\partial s_i}{\partial a_{j-}} = \frac{f'(r_j)}{\sigma}\sum_{p\le k, p\ne i}\left(s_p - \frac{(1-\alpha)sc_p}{\alpha + s}\right).$$

For each p, the difference on the right side corresponds to the share of the impressionable voters supporting party p. Thus Power has stronger incentives for discredit of disloyal parties and less incentives for advertising loyal parties then its allies. Moreover, in contrast to Power one loyal party may be interested in discredit of another loyal party. So Power (as well as Opposition) should strictly control the employment of its informational resource by its allies.

3.2. The game with fixed budgets

Now consider problems (6), (7) under condition (9). First study the case where Power sets its strategy after Opposition. Assume that the budgets for advertising B_+^p, B_+^0 and discredit B_-^p, B_-^0 are fixed, and let us find the optimal distributions among the parties. First consider the variant where marginal efficiency of advertising $f'(r)$ increases in r. According to Proposition 3, under these conditions Power advertises only one party $j \le k$ with the maximal rating after the impact of Opposition.

Proposition 5. The optimal strategy of discredit by Opposition is to reduce and equalize the ratings of loyal parties with j maximal initial ratings, i.e. under the optimal strategy a_-^0,

$r_{1o} - a_{1-}^o = r_{2o} - a_{2-}^o = ... = r_{jo} - a_{j-}^o > r_{j+10}$ for some $j < k$ or

$$r_{1o} - a_{1-}^o = r_{2o} - a_{2-}^o = ... = r_{ko} - a_k^o. \quad (10)$$

The optimal response of Power is to invest the whole resource B_+^p in advertising of any party $i \in \{1,.., j\}$. The optimal strategy of advertising by Opposition is to spend the whole resource B_+^0 on the party $k+1$. The optimal response of Power is to reduce and equalize the ratings of disloyal parties with maximal ratings, i.e. under the optimal a_{-}^{p}, $r_{k+1}+B_{+}^{0}-B_{-}^{p}\geq r_{k+a}$ or $r_{k+1}+B_{+}^{0}-a_{k+1-}^{p}=r_{k+2}-a_{k+2-}^{p}=...=r_{j}-a_{j-}^{p}$ for some $j\geq k+2$.

Note that in general there is no saddle point in pure strategies. In particular, in the game where Opposition distributes only discredit and Power distributes only advertising, so the saddle point in mixed strategies corresponds to the minimax value in pure strategy: the optimal strategy of Opposition source equalizes informational ratings of j parties with the highest initial ratings among loyal.

According to Proposition 5, the game permits decomposition: one may consider separately the game where Power distributes only advertising and Opposition – only discredit among parties $1,...,k$ and another game, where Opposition distributes only advertising and Power – only discredit among parties $k+1,...,n$. The combination of the optimal strategies in these games forms the optimal strategy in the original game.

Now consider the problem (6) where Opposition sets its strategy after Power. The game is symmetric to the previous one: in order to obtain the maxi-min strategies it suffices to change the names of the players and the set of loyal parties in Proposition 5.

Note that in general there is no saddle point in the pure strategies. In particular in the game where Opposition distributes only discredit and Power distributes only advertising, according to the maxi-min strategies, Opposition equalizes ratings after Power invests its budget to party 1. Under the mini-max strategies Opposition first reduces and equalizes ratings of the most popular allies of Power and then Power increases the rating of one of these parties. The final outcomes may essentially differ.

Consider the latter game in mixed strategies. As usually, a mixed strategy of a player is a probabilistic distribution on the set of its pure strategies. The payoff function is an expected value of the payoff in the original game. Proceeding from the general theory (see Mayerson,1990), the saddle point in mixed strategies exists in the game under consideration.

Proposition 6. The optimal strategy of Opposition is pure and the same as in Proposition 5: it is determined by condition (10). The optimal strategy of Power is to choose any party $i\in(1,...j)$ with probability $1/j$ and set $a_{i+}^{p}=B_{+}^{p}$.

Proof. It suffices to check that, under the given strategy of one player any pure strategy of the other player is not better then his strategy specified in the Proposition.

Now let us briefly discuss the case where the marginal efficiency $f''\leq 0$ decreases. Let us start with the problem (7) under conditions $B_{+}^{p}=0,\ B_{-}^{0}=0$. According to Proposition 3, the optimal response of Power to a given strategy of Opposition is to spend the whole budget B_{-}^{p} on discredit of the disloyal party with the minimal rating. The optimal strategy of Opposition in this case is to distribute its budget in order to increase and equalize the ratings of disloyal parties with j minimal initial ratings, that is similar to the strategy (10). Other results in Propositions 5 and 6 may be reformulated for the case with decreasing marginal efficiency in the same way, since the problems are symmetric in some sense.

Our analysis shows that the optimal strategies of the players essentially differ for convex and concave efficiency functions f. Proceeding from the empirical evidence, we propose that the actual marginal efficiency of advertising doesn't decrease in the wide practically important range. Moreover, our conclusions about the optimal strategies for convex functions correspond to the common sense much better, than the conclusions for the concave functions. So below we continue our study for convex and linear efficiency functions.

Consider the problem of the optimal distribution of the budget B^p between advertising B_+^p and discredit B_-^p. We limit our investigation with the case where Power sets its strategy after Opposition. For given budgets B_+^0, B_-^0 of Opposition, the optimal minimax distributions a_+^0 and a_-^0 are determent by Proposition 5 and do not depend on the values of B_+^p, B_-^p. Proceeding from this proposition, the marginal efficiencies of advertising and discredit are

$$\frac{\partial l}{\partial B_+^p} = \frac{\partial l}{\partial a_{1+}} = \frac{f'(r_1)\cdot(m_1 - l)}{\sigma}, \ \frac{\partial l}{\partial B_-^p} = \frac{\partial l}{\partial a_{k+1}} = \frac{f'(r_{k+1})\cdot(l - m_{k+1})}{\sigma} \qquad (11)$$

So the marginal efficiency of the resource a redistributed from discredit to advertising is $\frac{\partial l}{\partial a} = \frac{f'(r_1 + a)\cdot(1 - l(a)) - f'\left(r_{k+1} - \left(B_+^P - a\right)/(\gamma \cdot j(a))\right)\cdot l(a)}{\sigma}$.

In general, we cannot say much on solution of the optimization problem. On one hand, $f'(r_1 + a)$ may grow faster in a then $f'\left(r_{k+1} + \left(B_+^P - a\right)/(\gamma \cdot j(a))\right)$ because of the coefficient $1/j$. On the other hand, $(1 - l(a))/l(a)$ decreases in $l(a)$, so for $l(a)$ close to 1 the derivative is negative. So the problem is probably multi-extremal.

3.3. Models with liner and exponential efficiency functions

Let $f(r) = r$, $l_1 > l_2 \geq l_3 \geq ... \geq l_{n-1} > l_n$. Then by Propositions 1, 2, $a_{1+}^p = B_+^p$, $a_{n-}^p = B_{n-}^p$. In this case these strategies form a saddle point in the game with fixed budgets. Consider the problem of the optimal distribution of the budget B^p between advertising B_+^p and discredit B_-^p. Let $B_+^p \overset{\text{det}}{=} a$, $B_-^p = \left(B^p - a\right)/\gamma$. Under fixed B_+^0, B_-^0,

$$l(a) = \frac{\sum_{i=1}^{n} m_i r_i + m_0 r_0 + m_1 a - m_n \left(B^P - a\right)/\gamma}{\sigma},$$

$$\frac{dl(a)}{da} = \frac{(m_1 - l(a)) - \gamma\cdot(l(a) - m_n)}{\sigma} = \frac{(m_1 + \gamma\cdot m_n) - l(a)\cdot(1+\gamma)}{\sigma}.$$

Lemma 4. The function $l(a)$ is monotonous in a.

Proof. Let $l(0) < (m_1 + \gamma m_n)/(1+\gamma)$. Function $\sigma(a)$ is positive, bounded for $a \in \left[0, B^P\right]$ and reaches its minimum $L_{\min} > 0$ in this segment. Consider the equation

$$\frac{du(a)}{da}=\frac{(m_1+\gamma\cdot m_n)-u(a)\cdot(1+\gamma)}{L_{\min}},\ a\in\left[0,B^P\right],\ u(0)=l(0).$$

Its solution monotonously increases in a and tends to $(m_1+\gamma m_n)/(1+\gamma)$ as a tends to ∞. Besides that, $u(a)>l(a)$ for any a. Thus $l(a)\uparrow a$ for $a\in\left[0,B^P\right]$ in this case.

If $l(0)>(m_1+\gamma m_n)/(1+\gamma)$ then in a similar way one may check that $l(a)\downarrow a$ for $a\in\left[0,B^P\right]$.

Proposition 7. Under a fixed strategy of Opposition, the payoff function $l(a)$ is monotonous and reaches its maximum at $B_+^P=B^P$ if $l(0)<(m_1+\gamma m_n)/(1+\gamma)$. Under the opposite inequality it reaches its maximum at $B_+^P=0$.

For Opposition, under a fixed distribution of the budget of the Power, the payoff function increases in B_+^O and reaches its minimum at $B_+^O=0$ if $l(0)<(m_1+\gamma m_n)/(1+\gamma)$. Under the opposite inequality the payoff function decreases in B_+^O and reaches its minimum at $B_+^O=B^O$.

Thus, there always exists a saddle point in the game. If $m_1-\bar{l}>\bar{l}-m_n$ where $\bar{l}=\sum_{i=2}^{n-1}r_im_i\Big/\sum_{i=2}^{n-1}r_i$ is the average loyalty of parties except 1 and n, then Power and Opposition should invest the whole budget in resp. advertising and discredit of party 1. Under the opposite inequality, they should invest resp. in discredit and advertising of party n.

Now consider the model with the exponential efficiency functions. According to proposition 2, under fixed budgets B_+^p, B_-^p and strategy of Opposition, Power advertises only one party and distributes B_-^p in order to reduce the maximal efficiency of advertising by Opposition. Under condition (9), Power advertises the loyal party with the maximal rating (party 1). In general the optimal party for advertising may be determined as follows. Let $l_a=\sum_{j=1}^{n}l_j\cdot e^{r_{j0}}/\sum_{j=1}^{n}e^{r_{j0}}$ denote the average loyalty before the impact of Power. Then the optimal party is $i=\arg\max_p\,(l_p-l_a)/(e^{B_+^p}+e^{-r_{p0}})$. If is easy to check that this condition is equivalent to maximizing of the loyalty if Power spends B_+^p on advertising of party *p*. Consider the problem of the optimal distribution of the budget B^p between advertising and discredit for the case (9) with loyal and disloyal parties. Assume that Power distributes the budget after opposition, and the distributions among parties correspond to the saddle point in mixed strategies described in Section 3.2. (Alternatively, one nay suppose that each player knows the distribution of the budget for discredit of his competitor when he distributes his budget for advertising.) According to (11), the marginal efficiency $v'(a)$ of redistribution of the budget from discredit to advertising in this case is proportional to

$$e^{r_1+a_1}\cdot\sum_{k+1}^{n}e^{r_i+a_i}-e^{r_{k+1}+a_{k+1}}\cdot\sum_{1}^{k}e^{r_i+a_i}\ , \qquad (12)$$

where $a_1 = a$, $r_{k+1} + a_{k+1} = r_{k+2} + a_{k+2} = ... = r_{k+j(a)} + a_{k+j(a)}$, $a_{k+i} < 0$ for *i*=1,…,*j(a)*, $-\sum_{1}^{j(a)} a_{k+i} = B - a$, $a_i = 0$ for the rest *i*.

Proposition 8. The function $v(a)$ is quasi-convex in *a*, so, under any distribution of B^0, the optimal distribution for Power is either $B_+^p = B^p$ or $B_-^p = B^p$. In the specified variant of the game "Power-Opposition", there are three possible types of saddle points: ($B_+^p = B^p, B_-^0 = B^0$), ($B_-^p = B^p, B_+^0 = B^0$) and ($\overline{B}_+^0, \overline{B}_-^0$) such that any of the responses $B_+^p = B^p$ or $B_-^p = B^p$ gives the same average loyalty.

Finally, let us briefly discuss the problem (8) of determination the minimal budget of Power that provides the necessary majority in the parliament. Propositions 5-8 permit to find the optimal strategies of the players and the corresponding loyalty $v(B^0, B^p)$ for any fixed budgets B^0, B^p. Under a given budget B^0, the value $v(B^0, B^p)$ monotonously increases in B^p. Thus, the minimal budget $B^p(B^0, l^*)$ for a given B^0 and necessary loyalty l^* is determined as a solution of the equation

$$v(B^0, B^p) = l^*$$

Since the value $v(B^0, B^p)$ monotonously decreases in B^p, under uncertainty with respect to B^0, Power should determine its budget proceeding from the worst (maximal) possible budget of Opposition.

4. CONCLUSION

In this paper we studied several optimization problems and games related to the distribution of the informational resources in the electoral campaign. We considered the case where one player (Power) distributes its resource among political parties with different initial ratings and loyalties to Power and aims to maximize the share of its supporters in the parliament. We showed that solution of the problem strongly depends on the properties of efficiency functions that transform informational ratings of the parties into distribution of impressionable voters. Proceeding from the empirical evidence and common sense, we argued that the efficiency functions are convex for practically used values of advertising and discredit. We showed that in this case the player should advertise only one allied party, and distribute its resource for discredit among hostile parties in order to reduce and equalize marginal efficiencies of their advertising. In particular, if any party is either loyal or disloyal to Power, then the player advertises the ally with the maximal initial rating and aims to equalize ratings of the most popular hostile parties. We considered the problem of distribution of the budget between advertising and discredit. For the examples with linear and exponential transforming functions, we showed that the optimal distribution is always to spend the whole budget for one purpose: either for advertising or for discredit.

We also studied the game between two players (Power and Opposition) where the purpose of the second player is opposite to the first one: he aims to reduce the share of supporters of Power in the parliament. We showed that under fixed budget the optimal strategies are symmetric in some sense: Opposition advertises the most popular disloyal party and equalizes ratings of the most popular supporters of Power. However, the outcome depends on the order of actions. In particular, if strategies are set simultaneously, there is no saddle point in pure strategies, and each player randomly chooses the party to advertise among the parties with maximal ratings after their discredit by the competitor.

Our analysis shows that each player should not distribute its resource among his allies but strictly control its employment: an allied party typically is more interested in its own advertising and less interested in discredit of disloyal parties than the player.

Of course, the interaction in the actual electoral campaign is more sophisticated than our model: it is dynamic, and the electorate is more heterogeneous. In particular, some groups of impressionable voters *apriori* reject some political parties irrespectively of their advertising. Nevertheless, our model casts some light on the properties of the optimal distribution of informational resources. For a qualified researcher, it is possible to adjust the model and the results to the more complicated situation.

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