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TAXATION OF FIRMS UNDER INCOMPLETE INFORMATION

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The paper considers the social welfare optimization problem for a one-good economy with sales tax and presumptive tax dependent on the production capacity of the firm. The price is exogenous. The government knows the production capacities of every firm but does not know its costs. The purpose is to characterize the optimal tax structure, tax rates and auditing strategy that maximize the welfare function dependent on net tax revenue and production volumes, under the given penalty for tax evasion.

The first model considers the case where every firm has the same production capacity with a constant marginal cost. Though the government has no information on the type of each agent, the optimal tax system includes only a presumptive tax dependent on the market price. So there is no need to audit firms. In the general model, a firm controls several units with different production costs. We show that, whenever the agents can adjust the property structure to the tax system, a similar result holds, and the optimal presumptive tax is proportional to the production capacity of the firm.

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Данная статья посвящена задаче оптимизации общественного благосостояния для однопродуктовой экономики, в которой используются налог с продаж и вмененный налог на производственную мощность фирмы. Цена на товар задана экзогенно. Государству известны производственные мощности фирм, но не известны функции затрат. Целью работы является определение оптимальных налоговых ставок и стратегии аудита, обеспечивающих максимальное значение функции благосостояния, зависящей от налогового дохода и объема производства.

Первая модель рассматривает случай, когда производственные мощности фирм одинаковы, а предельные затраты постоянны. Хотя государство не располагает информацией о типе каждого агента, оптимальная налоговая система включает лишь вмененный налог, зависящий от рыночной цены. Таким образом, нет необходимости проводить налоговый аудит. В общей модели каждая фирма владеет несколькими производственными единицами с различными издержками. В работе показано, что и в этом случае справедлив аналогичный результат, если фирмы могут приспосабливать структуру собственности к налоговой системе. При этом оптимальный вмененный налог оказывается пропорциональным производственной мощности фирмы.

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TAXATION OF FIRMS UNDER INCOMPLETE INFORMATION

1. Introduction.

The problem of optimal taxation under tax evasion is of general interest and of special importance for economies in transition where tax evasion is typical for economic agents. One known result related to this problem is the Second Welfare Theorem (Debreu, 1954). Consider an economy with risk-neutral agents and a fully informed government on the type of each agent, including the production functions of producers and the utility functions of consumers. Proceeding from this theorem, the maximal welfare for such an economy may be obtained by means of a type-specific lump-sum tax.

Several authors (see Myles, 1996, and references there) consider this result as of only theoretical interest. They argue that the necessary information on the type of an agent is never available without high costs and requires regular renovation. However, in practice some kind of a lump-sum tax – a fixed presumptive tax – is now widely applied in the taxation of small and medium businesses in several countries (see Thieben, 2001, for the data on Ukraine). In this case some characteristics of the production capacity used by a firm or the number of employees determine the type of an agent. Depending on the characteristics used to evaluate the production capacity, this tax may depend on the natural resources employed (for instance, the size and the location of the land), or on the value of some observable input with a small elasticity of substitution (the square of a shop or a café, the number of employees and so on). In particular, Ukraine uses now a fixed tax dependent on the type of production and the number of employees, the trade permit for individuals providing certain cervices and the market fee for every occupied trade place for selling agricultural products. In 2002 similar variants of taxation for small business were widely discussed in Russia. Such taxes seem to be especially attractive for countries with widespread tax evasion and corrupted tax inspectors (see Bardhan and Yakovlev on the latter issue).

The present paper studies the case where the government has incomplete information on the firms' cost functions. Our purpose is to provide a theoretical foundation for the use of a presumptive tax dependent on the production capacity of firms under informational asymmetry between the government and the firms. We show that, under certain conditions, such a tax is more efficient than a sales or a turnover tax and permits to reach the maximal welfare.

We consider an economy with heterogeneous firms under informational asymmetry between the government and the agents. The production capacity of any firm consists of several productive units that may differ in production costs. The government knows the total production capacity of every firm but does not know its cost function. The distribution of productive units over the cost is given for the whole economy.

Each firm chooses the total production volume and the registered amount of production. The rest is sold at the informal market for cash. The government can set a tax dependent on the production capacity and the production volume of a firm. However, for the latter value it has either to proceed from the registered production or to conduct a costly audit. (We assume that auditing the costs of the firms is very expensive, so a profit tax is not implementable in the economy.) Without any auditing by the government, a firm has an incentive to sale its production unregistered. In order to prevent tax evasion, the government organizes tax inspection and penalizes the detected tax evasion. So the government strategy includes also the audit rule that determines the audit effort dependening on the registered production volume. Detected tax evasion monotonously depends on this effort and the amount of unregistered production. The market price is given exogenously. The penalty for evasion is proportional to the detected unpaid tax.

Under a given strategy of the government, each firm aims to maximize its expected after tax and penalty profit. The purpose of the government is to reach the maximal social welfare dependent on net tax revenue, total production volume and profits of the agents.

Our purpose is to study the above tax optimization problem in a principal-agent framework. That is, to find the strategy of the government that maximizes social welfare under optimal behavior of the agents and some participation constraints. Section 2 studies a basic model where every firm has a unit of production capacity with a fixed marginal cost. In contrast to the usual setting of the tax enforcement problem (see Sanches and Sobel, Cowell and Gordon, Chander and Wilde et al.), we permit to exclude from the economy the firms that cannot pay taxes. We determine the optimal auditing strategy under some exogenously given tax rates. This strategy turns out to be a "cut-off" rule with respect to the registered production volume. Actually, this auditing strategy determines the optimal level of exclusion for firms with high production costs.

Section 3 studies the tax optimization problem and shows that the optimal presumptive tax permits to maximize net tax revenue without any need for a sales tax. Section 4 explores a possibility to generalize this result for firms with heterogeneous production capacities. We show that the proposition holds for an economy with a variable property structure, where agents adjust the distribution of productive units across firms to the government tax policy. Section 5 discusses some implementation problems, in particular, related to firm-specific risks.

Our paper shows that the optimal structure of a tax system, which considers tax evasion may essentially differ from the one determined without tax evasion. In particular, our results are in contrast to Myles (see p.231) who shows that input taxes are never included in the optimal tax system for an economy without tax evasion. The previous literature on the optimal taxation under tax evasion considers an individual income tax (Mirrlees, Chander and Wilde, Mookhergee and Png, Vasin and Vasina) and sales taxes on different commodities (Cremer and Gahvari). The participation constraints and (hence) the results in these papers are quite different from the present one.

Our analysis is limited by the assumption that the market price is given exogenously. This is the case if we consider a small economy producing an export good. In general it is necessary to consider a general equilibrium model and compare possible distortionary effects and audit costs in order to determine the optimal tax. This is the subject of the complementary paper in this issue.

2. Basic Model and Optimal Behavior of Firms.

We consider an industry where each firm has the same production capacity $\overline{V} = 1$. Firms differ in their marginal production cost $c \geq 0$ that does not depend on the output V. This cost determines the type of each firm and is its private information. The distribution $G(c)$ of production costs in the economy is known to the government. It sets a sales tax with rate τ and a lump-sum tax T on every capacity unit. Each firm chooses an output $V \leq \overline{V}$ and a volume of registered sales $V_r \leq V$. The amount $V_u = V - V_r$ is sold at an informal market. By selling in the informal market, the firm is able to evade the sales tax. The market is competitive, and the price p in both sectors is the same and exogenous.

The government spends an effort $e(V_r) \leq 1$ on auditing a firm with registered output V_r. The cost of each audit is de. An audit detects some random volume of unregistered production with the mean $V_{du} = H(V, V_u, e)$. Consider two examples.

A) $e(V_r)$ is a checked production volume. This amount is randomly chosen from the total production volume *V*. Then the expected amount of the detected unregistered production is $V_{du} = V_u \min\{1, \frac{e(V_r)}{V}\}$.

B) $e(V_r)$ is a probability of an audit that determines the whole amount V_u with a fixed cost. Then $V_{du} = e(V_r)V_u$.

Proceeding from these examples, we assume below that $H(V, V_u, e)$ is concave in V_u under a fixed V_r, $H(1, V_u, e) = eV_u$. The fine for evasion from the sales tax is proportional to the detected underpayment and is equal to $(1+\delta)\tau p V_{du}$.

The firm's problem

Firms are risk neutral and maximize expected profit under a given government strategy. The following proposition characterizes the optimal output and the optimal volume of registered sales of every firm. For any cost c, these values give a solution to the problem

$$\left(V^c, V_r^c\right) \to \max_{V\in[0,1], V_r\in[0,V]} \{V(p-c) - \tau p V_r - kH(V, V_u, e(V_r))\} \overset{def}{=} I_{at}^c \qquad (1)$$

where $k \overset{def}{=} (1+\delta)\tau p$.

Proposition 1. For any c and $e(.)$, the optimal production volume $V^c(e(.))$ is either 0 or 1. And for each firm such that $V^c(e(.)) = 1$, the optimal registered volume $V_r^c(e(.)) = V_r(e(.))$ does not depend on the type c of the firm.

Proof of Proposition 1. Since the function H is concave in V_u under a fixed V_r, the income (1) is convex in V_u and reaches its maximum either at V_u=0 or at $V_u = 1 - V_r$. Under $V_u = 0$, the maximal income is $\max\limits_{V_r} V_r(p - c - \tau p)$. Under $V_u = 1 - V_r$ the maximal income is $p - c - \min\limits_{V_r} (\tau p V_r + e(V_r)k(1 - V_r))$. Note that the latter value is never less than the former one if $p - c - \tau p > 0$. Thus, the optimal volume V_r in the second case is

$$V_r(e(.)) \to \min_{V_r} (\tau p V_r + e(V_r)k(1 - V_r)) \qquad (2)$$

and does not depend on c. The minimized value on the right side is the effective tax paid to the budget under a given audit rule. Let $T(e(.))$ denote this minimal value. Then, for any firm such that $p - c < T(e(.))$, the optimal strategy is $V^*(e(.)) = V_r^*(e(.)) = 0$, and for any firm such that $p - c > T(e(.))$, $V^*(e(.)) = 1$, $V_r^*(e(.))$ is a solution of (2). *Q.T.D.*

Thus, under a given auditing rule the set of firms splits into two parts: firms in the first part choose the same output $V = 1$ and the same registered volume V_r; whereas, the rest of the firms do not produce. Formally,

$V^c(e(.)) = 0$ iff $p - c < T(e(.))$. Otherwise, $V^c(e(.)) = 1$. (3)

We assume that, under equality, firms prefer to produce.

3. The Optimal Auditing Rule and the Optimal Taxes.

The government's problem

First, consider the tax enforcement problem under an exogenous sales and a lump-sum tax rates, τ and T, respectively, and study the case where $T = 0$, so the tax $T(V_r) = \tau p V_r$. The strategy of the government is the auditing rule $e(V_r)$. The aim of the government is to maximize, under the constraint (1), the welfare function $W(R; (V^c, I_{at}^c, c \geq 0)$ that increases in net tax revenue

$$R(e(.)) = \int [\tau p V_r^c + (1+\delta)\tau p H(V^c, V_u^c, e(V_r^c)) - de(V_r^c)] dG(c),$$

production volumes and after tax profits of producers. Proceeding from (3), we can express the net tax revenue as follows:

$$R(e(.)) = T(e(.))G(p - T(e(.))) - de(V_r(e(.)))G(p - T(e(.)) - de(0)(1 - G(p - T(e(.)).$$

The next proposition describes the optimal auditing rule under a given effective tax $T(e(.)) = \theta \le \tau p.$ This value determines the profits and production volumes for all firms according to (3).

Proposition 2. Consider auditing rules $e(.)$ such that $T(e(.)) = \theta$. Then the optimal rule that minimizes auditing costs is

$$e_\theta^*(V_r) = \max\left\{\frac{\theta - p\tau V_r}{k(1 - V_r)}, 0\right\} \tag{4}$$

This rule means that any firm that voluntarily pays the effective tax, or some greater value, is not audited. Any other firm is audited with an effort that makes tax evasion unprofitable. For any firm with zero registered production this effort should be minimal.

Proof of Proposition 2. Consider a firm with production volume $V = 1$. The optimal registered volume under strategy (4) is $V_r^* = \theta/(p\tau)$. Indeed, for any $V_r < V_r^*$, the effective tax is $p\tau V_r + kV_u e_\theta^*(V_r) = \theta$.

Now consider any audit rule $e(.)$ such that $T(e(.)) = \theta$. We search for the rule with the minimal audit cost. Such rule is a solution to the problem

$$\min_{e(.)}\{de(V_r(e(.))G(p - \theta) + de(0)(1 - G(p - \theta))\}$$

under the condition $T(e(.)) = \theta$.

Consider two cases. If $V_r(e) \ne 0$, then $V_r = 0$ is not better for a taxpayer under the given auditing rule. Hence, $e(0)k \ge \tau p V_r(e) + e(V_r(e))k(1 - V_r(e)) \ge \theta$, so $e(0) \ge \dfrac{\theta}{k}$. If $V_r(e) = 0$, then $e(0) = \dfrac{\theta}{k}$ in order to provide the effective tax. Since, in both cases, $e(0) \ge \dfrac{\theta}{k}$ and $e(V_r(e)) \ge 0$, the rule (4) is optimal. *Q.T.D.*

Under this strategy, the net revenue is the following function of the effective tax

$$R(\theta) = \theta G(p - \theta) - d\frac{\theta}{k}(1 - G(p - \theta)),$$

production volumes and profits of producers are determined according to (3).

Proposition 3. The optimal auditing rule is the cut-off rule determined by (4) for θ^* that is a solution of the problem

$$\theta^* \to \max_{\theta\in[0,p\tau]} W(R(\theta);V^c(\theta),I_{at}^c(\theta),c\geq 0). \quad (5)$$

Now, consider a similar problem with a positive lump-sum tax rate T. We assume that the alternative net profit of any firm is $I_{alt}=0$. The optimal strategy of any participating firm does not depend on T and is given by Proposition 1. However, under a given effective sales tax $\theta = T(e(.))+T$ and zero alternative net profit, it is unprofitable to participate if $c > p-\theta$. Consider the following timing of interaction.

1. The price p is exogenously given.
2. The government sets the tax rates τ and T and the audit rule $e(V_r)$.
3. Each firm decides whether to register.
4. Every registered firm pays the lump-sum tax T.
5. Every registered firm chooses its strategy (V^c, V_r^c) and pays the sales tax $\tau p V_r^c$.
6. The government audits firms according to the rule $e(V_r)$ and collects fines for evasion.

Under these conditions, any firm can estimate its expected net profit at the stage 3. Assume that unregistered firms cannot produce. Then, firms with high costs $c > p-\theta$ do not register. In contrast to the previous case, there are no expenses related to their audit: the audit rule $e_\tau(V_r)$ turns to be costless. The net revenue under a given effective tax θ is $R(\theta)=\theta G(p-\theta)$, and $V^c=1$, $I_{at}^c = p-\theta-c$ for any firm with $c\leq p-\theta$. Thus, we obtain the following proposition.

Proposition 4. Under some given tax rates $T>0$ and $\tau\in[0,1]$, the maximum welfare is

$$\max_{\theta\in[T,p\tau+T]} W(\theta G(p-\theta);V^c(\theta),I_{at}^c(\theta),c\geq 0). \quad (6)$$

The optimal auditing rule is determined by Proposition 2, for the corresponding effective sales tax $\theta^* - T$.

We assumed above that the tax rates τ, T were given exogenously. Now, let us discuss the general welfare optimization problem. Note that net tax revenue R, the profits and production volumes of firms are determined by the total effective tax if the government employs the optimal audit rule and $T>0$. Proceeding from Proposition 4, the maximal welfare value is

$$\max_{\theta\in[0,p]} W(\overline{R}(\theta);V^c(\theta),I_{at}^c(\theta),c\geq 0).$$

Let $\theta(p)$ denote the optimal effective total tax for this problem. Then, any rates $T>0, \tau\in[0,1]$ such that $T+\tau p\geq\theta(p)\geq T$ are formally optimal if the audit rule $e(.)$ is determined by Proposition 2, for the effective sales tax $\theta(p)-T$.

Thus, there are many optimal strategies for the government in this model. Under any of them, the behavior of the agents is as follows: firms with high costs $c > p - \theta(p)$ do not register and take part in production, whereas the rest pay the lump-sum tax T and the effective sales tax $\theta - T$ that saves them from audit.

However, this setting of the problem does not take into account the expenses of sales tax collection, accounting and audit organization. The only variant where such costs are unnecessary is to set $T = \theta(p), \tau = 0$.

On the other hand, Proposition 4 shows that a tax authority that does not control tax rates but sets the audit rule has certain discretion over the effective tax and may use it in order to maximize its welfare function.

Before we continue the discussion about implementation problems, consider more general models.

4. Models with heterogeneous production capacities.

Now assume that a firm may own several productive units, every unit has the same capacity equal to one. First consider the case where the government knows only the total production capacity x^a of each firm a while the cost function is a private information of the firm. A strategy for the government includes the same components: a sales tax rate τ, a presumptive tax $T(x^a)$ dependent on the capacity x^a, and an audit rule $e(V_r^a, x^a)$ which determines the effort of inspection dependent on the capacity and the registered production of a firm. The firm's problem is similar to (1):

$$(V^a, V_r^a) \to \max_{V \le \sum x_i^a, V_r \in [0,V]} \{Vp - C^a(V) - \tau p V_r - (1+\delta)\tau p H(V, V_u, e(V_r, x^a))\} \qquad (7)$$

The only difference is that the cost of production $C^a(V)$ is non-linear: if a firm owns production units with marginal costs $c_1^a \le c_2^a \le ... \le c_{x^a}^a$ then, for any $V \in [n, n+1], n \le x^a - 1$, the cost of production is $C^a(V) = \sum_{i=1}^{n} c_i^a + (V - n)c_{n+1}^a$.

The following statement generalizes Proposition 1 for the present set-up.

Proposition 5. Under a given government strategy, the solution to the problem (7) meets the following conditions:

1) every firm employs the most efficient productive units;
2) if $C^{\prime a}(V^a)$ is the marginal production cost for capacity used by firm a then all units with this cost are completely employed by this firm;

3) under a given total volume, the optimal registered volume does not depend on the production costs of the firm and is a solution of the problem

$$\left(V_r^a\right) \to \min_{V_r \in [0, V^a]} \{V_r + (1+\delta) H(V^a, V^a - V_r, e(V_r, x^a))\}. \tag{8}$$

Let us note that, in general, firms with the same structure x may have different optimal volumes, depending on the structure of their costs.

The government seeks to maximize the social welfare. Below we assume that it depends on the net tax revenue R and the total production volume V of the economy. Consider the following approach. Since the government knows the total distribution of productive units over a cost, it may find the optimal lump-sum tax per unit as in the previous model, as if every unit is managed separately:

$$T^* \to \max_{T \in [0,p]} W(TG(p-T), G(p-T)) \tag{9}$$

Results in section 3 show that the presumptive tax $T(x^a) = T^* \cdot x^a$ that linearly depends on the production capacity of a firm provides the maximal welfare under the whole set of government strategies if every firm owns the same capacity with a constant marginal cost. The question is whether this holds true for the general case.

There are several possible settings of the government problem depending on whether the firms have a possibility of changing their structures according to the government strategy. The first variant is where the property distribution is fixed. Then, timing is the same as in the previous section. At the stage 3, every firm solves the problem (7) under a given price and government strategy. The right side of (7) is the maximal profit of the firm after sales tax. The firm participates if the presumptive tax does not exceed this value.

The following examples show that the conjecture on the optimal tax fails in this case: the sales tax may increase net tax revenue and the optimal lump-sum tax may be non-linear under non-linear cost functions.

Example 1. Assume that there are M_1 productive units with the cost $c_1 = p/2$ and M_2 units with the cost $c_2 = p$ in the economy, $M_1 < 2M_2$. Every firm owns two units. The ratio of the numbers of firms with the structures (c_1, c_1) and (c_1, c_2) is $M_1 : 2M_2$. Then the optimal presumptive tax that maximizes net tax revenue is $T(2) = p/2$ (since $(M_1 + 2M_2)p/2 > M_1 p$), and the revenue is $\hat{R} = (M_1 + 2M_2)M_1/(M_1 + M_2)p/4$. If the firms could exclude inefficient units, then the optimal tax would be $T(n) = np/2$ and the revenue would be $R^* = M_1 p/2 > \hat{R}$. The same result is available with the sales tax rate $\tau = p/2$ and $T = 0$ if audit is costless. Thus, sales tax may increase the revenue if the firms cannot change their structures.

Example 2. Now assume that there are units with the cost $c_1 = p/2$ and units with the cost $c_2 = 0$. M_2 firms own two different units and $(M_1 - M_2)/2$ firms have the costs $(p/2, p/2)$, $M_2 < M_1 < 2M_2$. In addition, there is a large number of firms with one productive unit with the cost $p/2$. Then the optimal presumptive tax is non-linear: $T(1) = p/2$, $T(2) = 3p/2$.

In the long-run prospect, we may expect that agents will adjust the property structure to the tax system in order to maximize their total after tax profit. Such adjustment may include exchange of productive units, splits and mergers and exclusion of the inefficient productive units (since they are accumulated in unregistered firms). Let us find out if the linear lump-sum tax, determined by (7), is optimal under this wider set of firms′ strategies.

In general, this is not true. Moreover, the government can get the maximal net revenue equal to the maximum profit in the economy if it may enforce the merger of all firms in one monopoly (by setting $T(n) = \infty$ for any $n < M$). In this case, the optimal tax is $\max_{V \le M}(pV - C(V))$, where

$$C(V) = M \int_0^{G^{-1}(V/M)} c\,dG(c).$$

Such a global merger eliminates the informational asymmetry between the government and the agents. However, shortcomings of the monopolistic structure of the economy are well known. So let us bound our examination to the case where the government seeks to preserve the competitive market and permits only micro changes in the property distribution. We assume that $T(n) = \infty$, for any $n > \hat{n}$, where $M/\hat{n}$ is very large.

Theorem 1. Under the above assumptions, the optimal government strategy includes only a presumptive tax that is the linear tax determined according to (9). The optimal sales tax rate is 0, so no audit is necessary to get the optimal welfare.

Proof. Consider any government strategy $s_G = (T(x), \tau, e(V_r, x))$. For any firm with capacity x and cost function $C(V)$, let $\tau(V, x)$ be the effective sales tax corresponding to the solution of the problem (8). Then, the optimal production volume is a solution of the problem

$$v(x, C(.)) = Arg\max_V\{(pV - C(V)) - \tau(V, x)\}.$$

Since the firms exchange productive units in order to maximize the total after tax profit, there exists c^* such that all units with marginal costs $c < c^*$ are employed and all units with costs $c > c^*$ are either excluded or do not produce. Let us prove that, under the optimal property distribution,

$$(p - c^*)V(x, C(.)) \ge T(x) + \tau(V(x, C(.)), x) \quad (10)$$

for any registered firm with capacity x and cost function C. Suppose this is not the case, so there exist a registered type (x, C) that does not meet (10). Then the agents can exchange productive units in such way that some firms of this type include only units with the cost c^*. Since the total cost of production does not change, the total profit of agents also does not change after such redistribution if they keep the same strategies. Next, let the agents not register all the firms with capacity x completed by the units with the cost c^*. Then, the total profit will increase, since the inequality (10) does not hold. This contradicts the optimality of the original distribution.

Thus, for every set of registered firms of the same type, the effective tax meets condition (10). Summing over all such types, we obtain that

$$R = \sum_{x,C(.)} N(x, C(.))\{T(x) + \tau(V(x, C(.)), x)\} \le V(p - c^*) \le MG(c^*)(p - c^*), \qquad (11)$$

where $N(x, C(.))$ is the number of firms of this type and V is the total production volume.

Finally, if the government sets the linear presumptive tax $T(x) = x(p - c^*)$ then, under optimal behavior of agents, the revenue is equal to the right side of (11), and the productive volume is $MG(c^*)$. Optimization by c^* gives the maximal tax revenue equal to (9) and completes the proof.

We assumed above that the government can not distinguish productive units at all. Now consider the case of incomplete information. Let there exist several types of production capacities $i \in I$. Every type i is characterized by the total size M_i and the unit cost distribution $G^i(c)$. For this type it determines the share of productive units, such that the cost of one unit of the good does not exceed c. These functions are known to the government. For every firm, it also knows its structure $x^a = (x_i^a, i \in I)$, where x_i^a is the production capacity (that is, the number of productive units) of type *i*. A firm knows precisely the cost of production $C^a(V), V \le \sum_i x_i^a$, for every type, while the government does not know this cost function. A strategy for the government includes the same components: a sales tax rate τ, a presumptive tax $T(x^a)$ and an audit rule $e(V_r^a, x^a)$ which now may depend on the capacity structure of the firm. The firm's problem is:

$$\left(V^a, V_r^a\right) \to \max_{V \le \sum x_i^a, V_r \in [0,V]} \{Vp - C^a(V) - \tau p V_r - (1+\delta)\tau p H(V, V_u, e(V_r, x^a))\} \qquad (12)$$

Under a given government strategy, the solution to the problem (12) meets the following conditions similar to 1)-3):

1') every firm employs the most efficient productive units of every type;

2') if $c_i^a(V_i^a)$ is the marginal production cost for capacity of the type i used by firm a then all units with this cost are completely employed by this firm;

3') under a given total volume, the optimal registered volume does not depend on the production costs of the firm and is a solution of the problem (8).

Consider the welfare optimization problem. In the case where each firm owns one productive unit, the optimal strategy of the government is determined according to the results in Section 3: sales tax and audit are unnecessary, and the optimal presumptive taxes T_i^* for every productive unit of type $i \in I$, are obtained as a solution of the problem

$$(T_i^*, i \in I) \to \max_{T_i \in [0,p], i \in I} W(\sum_i M_i T_i G_i(p - T_i), \sum_i M_i G_i(p - T_i)) . \tag{13}$$

Now consider the general case where each firm may own several productive units of different types. Consider on economy with a variable property structure where agents can exchange productive units of the same type and exclude unprofitable firms in order to maximize the total after tax profit under a given strategy of the government.

The following theorem shows that a presumptive tax that linearly depends on the production capacities of the firms provides the maximal welfare for this economy.

Theorem 2. Under the above assumptions, the optimal government strategy includes only the presumptive tax $T(x^a) = \sum_i T_i^* x_i^a$ where $T_i^*, i \in I$ are determined according to (13). The optimal sales tax rate is 0, so no audit is necessary to get the optimal welfare.

The proof is quite similar to the one for the previous theorem. Under any strategy of the government, agents can costlessly exchange productive units of the same type. Hence, for every type *i,* there exists c_i^* such that all units with marginal costs $c < c_i^*$ are employed and all units with costs $c > c_i^*$ are either excluded or do not produce. Moreover, under the optimal property distribution,

$$pV(x, C(.)) - \sum_i V_i(x, C(.)) c_i^* \geq T(x) + \tau(V(x, C(.)), x) \tag{14}$$

for any registered firm with capacity x and cost functions $C_i, i \in I$. Otherwise the agents can exchange productive units in such way that some firms of this type include only units with the costs c_i^* for every $i \in I$. Then, the total profit will increase if the agents do not register such firms, since the inequality (14) does not hold. This contradicts the optimality of the original distribution.

Thus, for every type of registered firms, the effective tax meets condition (14). Summing over all such types, we obtain that

$$R = \sum_{x,C(.)} N(x,C(.))\{T(x)+\tau(V(x,C(.)),x)\} \le p\sum_i V_i - \sum_i V_i c_i^* \le \sum_i M_i G_i(c_i^*)(p-c_i^*)\,, \quad (15)$$

where $N(x,C(.))$ is the number of firms with these parameters and V_i is the total production volume for the capacity of type i.

Finally, if the government sets the linear presumptive tax $T(x) = \sum_i x_i(p-c_i^*)$ then, under optimal behavior of agents, the revenue is equal to the right side of (15), and the productive volume is $\sum_i M_i G_i(c_i^*)$. Optimization by c_i^* gives the maximal tax revenue equal to (13) and completes the proof.

6. Discussion.

According to the Second Welfare theorem, if the government has complete information on the cost function of every firm, then it may reach the maximal welfare by means of a type-specific lump sum tax. In the present paper, we have studied a welfare optimization problem under informational asymmetry between the government and the agents. Theorems 1 and 2 above show that a similar kind of tax – a presumptive tax dependent on production capacities of a firm - is also optimal, if producers can adjust their property structure to the tax enforcement strategy of the government in order to maximize the total after tax profit. Any efficient exchange market of productive units provides a possibility of such adjustment. Taxation and audit of sales turn out to be unnecessary for the government in this case. Moreover, the optimal presumptive tax in this case is linear with respect to the structure of production capacities.

Another advantage of such taxation is that it facilitates the redistribution of the property to the more efficient owners. Note that in our model, a productive unit may be profitless and excluded from the production process for two different reasons: 1) some local conditions increase the cost of production; 2) poor management. In the latter case, the more efficient owner will buy out and employ this unit. In the former case, the unit stays out of work for a long time. This is a signal for the government to differentiate the tax and reduce the rate for this unit.

This scheme of taxation is distortionary in the long-run prospect since the agents have an incentive to look for substitutes for the taxed inputs. In general it is necessary to compare the losses of welfare related to the distortion and to the audit and tax evasion in order to find the optimal tax enforcement strategy. Note that in practice any taxation of firms is distortionary. For instance, profit tax requires an equivalent wage tax and induces tax evasion activity that may have strong distorting effects (see Jung et al.).

Implementation of the considered presumptive tax meets several difficulties. In practice, a firm determines its structure under different uncertain factors. In particular, the price of the good typically changes rather frequently, and the firm cannot adjust its property structure to these variations. On the other hand, for many goods this price is known at the time of production. In this case, the considered taxation scheme may be implemented if the government can account the employed capacities of every type at any period and sets the tax depending on these values.

The agents also face risk factors that are unknown at the time of production. If the variance of the profits due to these factors is large enough then some share of potentially good firms may be unable to pay the tax and become tax bankrupts. One way to reduce this share is to permit the coverage of the tax debt with the property of the firm. The firm manages this property and can buy it out until the share of the government exceeds some threshold. It is possible to regulate the intensity of tax bankruptcy by the choice of this value.

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