

Francisco Marhuenda<sup>1</sup>, Alexander Vasin<sup>2</sup>, Polina Vasina<sup>3</sup>

## **OPTIMAL CHOICE OF THE TAX SYSTEM UNDER TAX EVASION**

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<sup>1</sup> Departamento de Economía, Universidad Carlos III de Madrid. E-mail: [marhuend@eco.us3m.es](mailto:marhuend@eco.us3m.es)

<sup>2</sup> Faculty of Computational Mathematics and Cybernetics, Moscow State University and New Economic School, Moscow. E-mail: [vasin@cs.msu.su](mailto:vasin@cs.msu.su)

<sup>3</sup> Computational Center of the Russian Academy of Sciences

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We consider taxation of enterprises in a one-good economy and search for the optimal combination of taxes and tax rates. In the general equilibrium framework we investigate the social welfare optimization problem under a given net tax revenue necessary for budget expenses. The welfare is measured by the total production volume. The firms use labour and same heterogeneous natural resource (land) for production. The paper considers sales tax, profit tax and a presumptive tax dependent on the amount of the employed natural resource, and determines the optimal tax rates. For any Leontiev-type production function, we show that the presumptive tax is more efficient than the sales tax. We determine the marginal welfare loss for profit tax under tax evasion and obtain formal conditions that characterize its optimal combination with the presumptive tax.

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Рассматривается проблема выбора налогов и налоговых ставок для налогообложения предприятий в однопродуктовой экономике. В рамках модели общего равновесия исследуется задача максимизации общественного благосостояния, измеряемого общим объемом выпуска, при заданной величине чистого налогового дохода, необходимого для покрытия расходов бюджета. Предприятия наряду с рабочей силой используют неоднородный природный ресурс (земля). В работе определяются оптимальные ставки вмененного налога, зависящего от количества используемого ресурса, налога с продаж и налога на прибыль. Для производственной функции леонтьевского типа доказано, что вмененный налог эффективнее налога с продаж. Найдены предельные избыточные тяготы налога на прибыль при возможности уклонения от этого налога и получены формальные условия, характеризующие его оптимальное сочетание с вмененным налогом.

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## 1. Introduction.

The problem of optimal taxation is of general interest and of special importance for economies in transition. One group of theoretical results for this problem concerns social welfare losses related to distortionary effects for different types of taxes (see Movshovich et al. (1997), Levin and Movshovich (2000), Myles (1996)). A general conclusion in this literature is the superiority of profit tax in comparison with indirect taxes in solution of social welfare optimization problem. In particular, the former paper shows that the marginal excess burden (MEB) of the profit tax is zero (since it does not cause any distortions) while the indirect taxes produce substantial MEBs.

Meanwhile, economic statistics shows that profit tax is less significant for the budget than indirect taxes in the majority of the countries. For instance, VAT, excise and sales taxes provide about 60% of the budget income in Russia and China, more than 30% in Greece and Ireland, more than 20% in Denmark, Finland, Netherlands and Norway. The share of profit tax is between 2% in Germany and 16,6% in Australia and typically does not exceed 10% in the economically developed countries (OESR).

Another important issue is the use of input taxes. In his recent textbook, Myles claims (p.231): “input taxes have been employed in many countries but they would be not form a part of an optimal tax system for a competitive economy”. However, in practice some kind of an input tax – a fixed presumptive tax – is now widely applied in the taxation of small and medium businesses in several countries (see Thieben, 2001, for the data on Ukraine). In this case some characteristics of the production capacity used by a firm or the number of employees determine the type of an agent. Depending on the characteristics used to evaluate the production capacity, this tax may depend on the natural resources employed (for instance, the size and the location of the land), or on the value of some observable input with a small elasticity of substitution (the square of a shop or a café, the number of employees and so on). In particular, Ukraine uses now a fixed tax dependent on the type of production and the number of employees, the trade permit for individuals providing certain services and the market fee for every occupied trade place for selling agricultural products. In 2002 similar variants of taxation for small business were widely discussed in Russia. Such taxes seem to be especially attractive for countries with widespread tax evasion and corrupted tax inspectors (see Bardhan and Yakovlev on the latter issue).

In order to answer these questions, the important questions are: is the latter proposition true? What is the reason of the mentioned contradictions between the theoretical conclusions and the empirical evidence? Why is profit tax less popular in practice than indirect taxes inspite of its

theoretical superiority? And under what conditions presumptive taxes dependent on input may become an effective tool for tax collection?

The present paper studies the social welfare optimization problem under a fixed net tax revenue. We consider taxation of enterprises in a one-good economy in the general equilibrium framework. The firms use labour and some heterogeneous resource (such as land, or trade squares, or oil fields) for production. We consider sales tax, profit tax and a presumptive tax dependent on the amount of the employed resource, and search for the optimal tax rates. For every firm, the government knows the amount of the employed resource but does not know its quality.

In contrast to the previous models (Movshovich et al. (1997), Levin and Movshovich (2000), Myles (1996)), we take into account a possibility of tax evasion and a need for costly audit in order to enforce sales and profit tax payments.

Section 2 describes a basic model without tax evasion and establishes a usual result on the advantage of profit tax. Section 3 considers the model with the Leontiev-type production function. In this case the optimal amount of production is determined by the amount of the employed resource and we show that sales tax is unnecessary in the optimal tax system since it may be efficiently substituted by the presumptive tax. (This conclusion generalizes our previous result obtained for the partial equilibrium model; see the complementary paper Marhuenda et al., 2001). In Section 4 we study the same model with a profit tax as a source of the budget income. We show that under tax evasion the optimal marginal tax rate is actually not constant but is maximal till some value of income and zero above this threshold. The marginal loss of the welfare for this tax is positive and increases in the cost of audit. We characterize the optimal combination of profit tax and presumptive tax that minimizes the social welfare loss.

Then we consider a model with a production function of a general type and study a similar problem. We also distinguish the case where presumptive tax is more efficient than the sales tax, and vice versa. Section 4 discusses the case with complete information on the quality of the input resource and shows that presumptive tax is more efficient than sales tax for any production function. However, this result does not hold if firms use some other heterogeneous resource, and its quality is private information of the firm.

## **2. Basic model.**

We consider a one-good economy where production requires two factors: the labour and some resource (for instance, land). This resource is heterogeneous, and function  $D(q)$  with density  $d(q)$  characterizes the distribution of the total amount of the resource over quality  $q$ . For every unit of the resource with quality  $q$ , production function  $f(q,l)$  characterizes the output

depending on the amount  $l$  of labour. We implicitly assume that the production function of any firm is linear in the amount of the resource. The labour supply  $L(w)$  monotonously increases in wage  $w$ ,  $L(0) = 0$ ,  $L(w) \rightarrow L_{\max}$  as  $w$  tends to  $\infty$ .

The government sets taxes in order to collect net revenue  $R$  necessary to cover fixed budget expenses. Below we consider sales tax with rate  $\tau$ , presumptive tax with rate  $T$  per one unit of the employed resource, and profit tax with rate  $t$ . For any resource unit involved in production, a manager solves the profit maximization problem:

$$l(q, w, \tau) \rightarrow \max_l [(1 - \tau)f(q, l) - wl] \quad (1)$$

The marginal productivity of labour does not increase in  $l$ , so  $l(q, w, \tau)$  does not increase in  $w$  and in  $\tau$ .

The rates  $T$  and  $t$  do not influence its solution. However, only those resource units that bring the positive after tax profit are involved in production. Since the profit increases in  $q$ , the minimum quality  $q^*(\tau, T)$  of the employed resource meets equation

$$(1 - \tau)f(q^*, l(q^*, w(\tau, T), \tau)) = w(\tau, T)l(q^*, w(\tau, T), \tau) = T \quad (2)$$

(In this section we assume that producers do not evade from taxes).

The equilibrium wage  $w(\tau, T)$  equalizes the supply and the demand at the labour market:

$$\int_{q^*(\tau, T)} l(q^*, w(\tau, T), \tau) d(q) dq = L(w(\tau, T)) \quad (3)$$

So tax rates should provide the net tax revenue  $R$ . First consider the case where  $t = 0$ . Then  $R = R(\tau, T) + t \left\{ \int_{q^*(\tau, T)} [f(q, l(q, w(\tau, T), \tau)) - w(\tau, T)l(q, w(\tau, T), \tau)] d(q) dq - R(\tau, T) \right\}$  (4)

where  $R(\tau, T) = \tau \int_{q^*(\tau, T)} f(q, l(q, w(\tau, T), \tau)) d(q) dq + T(1 - D(q^*(\tau, T)))$ .

The social welfare is measured by the total production volume minus the disutility of the labour for workers:

$$W = \int_{q^*(\tau, T)} f(q, l(q, w(\tau, T), \tau)) d(q) dq - \int_0^{L(w(\tau, T))} w(L) dL \quad (5)$$

where  $w(L(w)) \equiv w$ .

The purpose of the government is to set tax rates  $\tau, T, t$  that maximize the welfare (5) under condition (4).

A standard result of the tax optimization theory is that the social welfare reaches its maximum when the budget revenue  $R$  is provided by profit tax. Let us ascertain this proposition for the model (1-3).

**Lemma 1.**  $\partial q^* / \partial \tau > 0$ ,  $\partial q^* / \partial T > 0$ ,  $\partial w / \partial \tau < 0$ ,  $\partial w / \partial T < 0$ ,  $\partial(w(\tau, T)/(1-\tau))/\partial \tau > 0$ ,  
 $\forall q \quad dl(q, w(\tau, T), \tau)/d\tau < 0$ ,  $d[f(q, l(q, w(\tau, T), \tau)) - l(q, w(\tau, T))w(\tau, T)]/d\tau < 0$ .

**Proposition 1.** The optimal tax system for the model (1-5) includes only profit tax, that is, the welfare reaches its maximum under  $T = 0$  and  $\tau = 0$ .

**Proof.** Assume from the contrary that, for the optimal tax rates,  $\tau > 0$  or  $T > 0$ . If  $t < 1$  then we can increase this rate and reduce resp.  $\tau$  or  $T$  according to equation (4'). Then the welfare would increase since  $q^*(\tau, T)$  increases in  $\tau$  and  $T$  and the welfare  $W$  decreases in these tax rates according to (5).

If  $t = 1$  then reduction of  $\tau$  or  $T$  decreases  $q^*(\tau, T)$  and increases simultaneously  $R$  and  $W$ . In the both cases we obtain the contradiction to our assumption on the optimality of the tax rates.

Intuition for this result is rather obvious: according to (1)-(5) profit tax does not produce any distortions in decisions of agents, while the presumptive tax decreases the amount of the employed resource, and sales tax, besides that, reduces the amount of the employed labour force.

However, this theoretical intuition does not correspond to practice since the model ignores tax evasion activity of producers. Below we consider the relevant modification of the model.

### 3. A Model with Leontiev-type Production Function.

First consider the problem for production function  $f(q, l) = \min(1, ql)$ , where  $q$  characterizes labour productivity.

**Proposition 2.** For Leontiev-type production function,  $l(q, w) = 1/q$  and  $f(q, l^*(q, w)) = 1$  for any  $q > q^*(\tau, T)$ , the minimum employed quality and the equilibrium wage meet conditions

$$q^*(\tau, T)(1 - \tau - T) = w(\tau, T), \quad \int_{q^*(\tau, T)} \frac{d(q)}{q} dq = L(w(\tau, T)),$$

the net tax revenue and the welfare are determined as

$$R = (T + \tau)(1 - D(q^*(\tau, T))), \quad W = 1 - D(q^*(\tau, T)) - \int_0^{L(w(\tau, T))} w(L) dL.$$

**Proof** is straightforward from the system (1)-(5).

This proposition shows that the presumptive tax and the sales tax are equivalent in this case (as well as in the previous paper) since the amount of the employed resource determines the production volume. A similar proposition is true if we consider a possibility of evasion from the sales tax and assume that the actual production volume may be verified by the costly audit and the

penalty for evasion is proportional to the unpaid tax. Under any strategy of the government, the effective sales tax per unit of the employed resource (including the tax paid by a firm and the expected penalty for evasion) does not depend on the quality  $q$ . So it is possible to increase the presumptive tax and collect the same gross revenue without and audit costs. (See Marhuenda, et al, 2004, for the formal model and proof.)

Now let us study this model with the presumptive and profit taxes. Then the equilibrium wage, the marginal quality and the social welfare do not depend on the profit tax rate and meet similar conditions:

$$q^*(T)(1-T) = w(T), \quad \int_{q^*(T)} \frac{d(q)}{q} dq = L(w(T)), \quad (6)$$

$$W = 1 - D(q^*(T)) - \int_0^{L(w(T))} w(L) dL$$

In contrast to the previous section, we take into account a possibility of tax evasion and audit costs. A strategy of a firm managing a unit with quality  $q$  includes the employed labour  $l$  and the reported profit  $Pr$ . A strategy of the government includes, besides tax rates, a probability of audit  $\pi(Pr)$  dependent on the reported profit. An audit always reveals the actual profit, and the penalty is proportional to the unpaid tax with coefficient  $1 + \delta$ . Since every firm aims to maximize the expected after tax and penalty profit, its strategy is a solution of the problem

$$(l, Pr)(q, w, t, \pi) \rightarrow \max_{l, Pr} \{f(q, l) - wl - T - tPr - (1 + \delta)\pi(Pr)t \cdot \max[0, f(q, l) - wl - T - Pr]\},$$

if the maximal profit is non-negative, otherwise the resource of such quality is not used.

**Proposition 3.** For any strategy of the government for any employed  $q$ , the optimal value  $l(q, w, t) = l(q, w)$  is determined according to (1) under  $\tau = 0$  and maximizes the before tax profit. The minimum quality of the employed resource  $q^*(T)$  and the wage  $w^*(T)$  do not depend on  $t$  and  $\pi(Pr)$  and meet system (2), (3) with  $\tau = 0$ :

$$f(q^*, l(q^*, w^*)) - w^* l(q^*, w^*) = T \quad (8)$$

$$\int_{q^*} l(q, w^*) d(q) dq = L(w^*) \quad (9)$$

The government aims to maximize the total social welfare and provide fixed net tax revenue  $R$ . The total welfare is

$$W = \int_{q^*(T)} f(q, l(q, w^*(T))) d(q) dq - \int_0^{L(w^*(T))} w(L) dL - c \int_{q^*(T)} \pi(Pr(q, w^*(T), t, \pi(\cdot))) d(q) dq,$$

where  $c$  is a cost of one audit.

The net revenue is

$$R = (1-t)T(1-D(q^*(T))) + t \int_{q^*(T)} [Pr(q, w(T), t, \pi(\cdot)) + (1+\delta)\pi(Pr(\cdot))(f(q, l(q, w^*(T))) - w^*(T)l(q, w^*(T))) - T - Pr(\cdot)] d(q) dq - c \int_{q^*(T)} \pi(Pr(\cdot)) d(q) dq$$

Note that, under given rate  $T$ , the profit before tax may be considered as an exogenously given function of  $q$  and the government's problem is a partial case of the welfare maximization problem with a given distribution of the income before tax studies by Sanchez and Sobel, 1993. Their result implies the following proposition.

**Proposition 4.** Under fixed tax rates  $T$  and  $t$ , the optimal auditing rule  $\pi(Pr)$  belongs to the class of "cut-off rules": there exists such threshold  $\overline{Pr}$  that every report  $Pr < \overline{Pr}$  is audited with probability  $1/(1+\delta)$  that makes tax evasion unprofitable, and every report  $Pr \geq \overline{Pr}$  is not audited.

Let  $Y(q, T) = f(q, l(q, w^*(T))) - w^*(T)l(q, w^*(T))$  denote the profit before tax for a unit with quality  $q \geq q^*(T)$ ,  $\bar{q}$  be such quality that  $Y(\bar{q}, T) = \overline{Pr}$ . Under the cut-off rule, for every unit with quality  $q \leq \bar{q}$ , the manager reports the actual profit, and for every unit with quality  $q > \bar{q}$ ,  $Pr(q, w(T), t, \pi(\cdot)) = \overline{Pr}$ .

Proceeding from this result, we can characterize a government strategy by triplet  $(T, t, \bar{q})$ . Under a given  $T$ , the welfare optimization problem may be reformulated as follows:

$$(t, \bar{q}) \rightarrow \min(D(\bar{q}) - D(q^*(T))) \quad (10)$$

under condition



$$R = t \left[ \int_{q^*(T)}^{\bar{q}} Y(q, T) d(q) dq + (1 - D(\bar{q})) Y(\bar{q}, T) \right] + (1 - t) T (1 - D(q^*(T))) - \bar{c} (D(\bar{q}) - D(q^*(T))), \quad (11)$$

where  $q^*(T)$ ,  $w^*(T)$  are determined from system (8,9); is the  $\bar{c} = c/(1 + \delta)$ . Expected audit cost per one unit for  $q \in [q^*, \bar{q}]$ . Components  $t$  and  $\bar{q}$  should minimize the total audit cost under condition (10). Assume that the profit tax rate is limited by  $t^* \leq 1$ .

Proposition 5. A solution of the problem (10,11) is  $(t^*, \bar{q}(T))$  where  $\bar{q}(T)$  is a minimal root of the equation (11) under  $t = t^*$ .

Proof. Let  $R(t, \bar{q})$  denote the right-hand side of (11). This equation determines  $\bar{q}(t)$  as an implicit function. According to the known theorem,

$$\frac{d\bar{q}}{dt} = - \frac{(\partial R / \partial t)}{(\partial R / \partial \bar{q})}.$$

Since  $\bar{q}$  is a minimum root of the equation (11),  $\partial R / \partial \bar{q} > 0$ . Let us show that  $\partial R / \partial t > 0$ . Then  $dW(t, \bar{q}(t)) / dt = -\bar{c} d(\bar{q}) d\bar{q} / dt < 0$ , so  $W$  reaches its maximum at  $t = t^*$ . Indeed,  $\partial R / \partial t > \int_{q^*(T)} (Y(q, T) - T) d(q) dq > 0$  since  $Y(q, T) > T$  for any  $q > q^*(T)$ .

Propositions 4,5 show that the optimal way to collect profit tax (if we like to maximize the total welfare) is to implement the extremely regressive tax schedule: the profit below the threshold level  $\bar{Pr}$  should be taxed with the maximal marginal rate  $t^*$ , and above the threshold the marginal rate is zero and the tax is actually flat.

Thus, the welfare optimization problem takes the form:

$$T \rightarrow \max_{q^*(T)} \int f(q, l(q, w^*(T))) d(q) dq - \int_0^{L(w^*(T))} w(L) dL - \bar{c} (D(\bar{q}(T)) - D(q^*(T)))$$

where  $q^*(T)$ ,  $w^*(T)$  and  $\bar{q}(T)$  are determined by conditions (8), (9) and (11) under  $t = t^*$ .

In order to find the optimal strategy of taxation, let us determine and compare the marginal welfare losses for the presumptive tax and profit tax denoted by  $dW / dR^T$  and  $dW / dR^I$  resp. The marginal loss shows the welfare reduction if the additional amount  $dR$  of the net tax revenue is provided by the corresponding tax. Formally,  $dW / dR^I = \frac{dW / d\bar{q}}{dR / d\bar{q}}$ , where  $\bar{q}$  and  $T$  are considered as independent variables when we compute the derivatives. We obtain:

$$\begin{aligned}
\frac{dW}{d\bar{q}} &= -\bar{c}d(\bar{q}), \frac{dR}{d\bar{q}} = t^* (1 - D(\bar{q}))Y_q'(\bar{q}, T) - \bar{c}d(\bar{q}) = \\
&= t^* (1 - D(\bar{q}))(f_q'(\bar{q}, l(q, w^*)) + f_l'(\bar{q}, l(q, w^*))l_q'(\bar{q}, w^*)) - w^*(T)l'(\bar{q}, w^*) - \bar{c}d(\bar{q}); \\
\frac{dW}{dT} &= \left( \frac{\partial W}{\partial q^*} + \frac{\partial W}{\partial w^*} \cdot \frac{dw^*}{dq^*} \right) \frac{dq^*}{dT}; \quad \frac{\partial W}{\partial q^*} = -f(q^*, l(q^*, w^*))d(q^*) + \bar{c}d(q^*) \\
\frac{\partial W}{\partial w^*} &= \int_{q^*} f_l' l_w'(q, w^*)d(q)dq - w^* L'(w^*); \quad \frac{dq^*}{dT} = (f_q'(q^*, w^*)l(q^*, w^*) \frac{dw^*}{dq^*})^{-1}; \\
\frac{dw^*}{dq^*} &= \frac{l(q^*, w^*)d(q^*)}{\int_{q^*} l_w'(q, w^*)d(q)dq - L'(w^*)}. \tag{12}
\end{aligned}$$

First consider the case with the Leontiev-type production function  $f(q, l) = \min(1, ql)$ . Then  $q^*(T)$  meets equation  $1 - w^*(q^*)/q^* = T$  (8') where  $w^*(q^*)$  is determined by  $\int_{q^*} d(q)/q dq = L(w^*)$  (9').

The welfare and net tax revenue are resp.

$$\begin{aligned}
W &= (1 - D(q^*(T))) - \int_0^{L(w^*(q^*(T)))} w(L)dL - \bar{c}(D(\bar{q}) - D(q^*(T))), \\
R &= t \left[ \int_{q^*(T)}^{\bar{q}} (1 - w^*(q^*(T))/q)d(q)dq + (1 - D(\bar{q}))(1 - w^*(q^*(T))/\bar{q}) \right] + \\
&\quad + (1 - t)T(1 - D(q^*(T))) - \bar{c}(D(\bar{q}) - D(q^*(T)))
\end{aligned}$$

The marginal loss of the welfare for profit tax is

$$-\frac{dW/d\bar{q}}{dR^t/d\bar{q}} = \bar{c}d(\bar{q}) \left/ \left[ t^* (1 - D(\bar{q})) \frac{w^*(T)}{\bar{q}^2} - \bar{c}d(\bar{q}) \right] \right.$$

The similar value for the presumptive tax is

$$\begin{aligned}
-\frac{dW}{dR^T} &= -\frac{dW/dT}{dR/dT} \text{ where } -\frac{dW}{dT} = \frac{dq^*}{dT} [d(q^*)(1 - \bar{c}) + w^*(q^*) \frac{dw^*}{dq^*}], \\
-\frac{dR}{dT} &= (1 - t)(1 - D(q^*(T))) - \frac{dq^*}{dT} \left\{ d(q^*)(T - \bar{c}) - \left[ \frac{1 - D(\bar{q})}{\bar{q}} + t \int_{q^*}^{\bar{q}} \frac{d(q)}{q} dq \right] \frac{dw^*}{dq^*} \right\}, \\
\frac{dw^*}{dq^*} &= -\frac{d(q^*)}{q^* L'(w^*)}, \quad \frac{dq^*}{dT} = \frac{q^{*2}}{w^*(1 + d(q^*)/L'(w^*))}.
\end{aligned}$$

Finally, we obtain

$$-\frac{dW}{dR^T} = \frac{(1 - \bar{c} - (1 - T)/L'(w^*))q^*}{(1 - t)(1 - D(q^*))(1 - T)(1 + d(q^*)/L'(w^*))/d(q^*) - ((T - \bar{c})q^* + t[\frac{1 - D(\bar{q})}{\bar{q}} + \int_{q^*}^{\bar{q}} \frac{d(q)}{q} dq]) / L'(w^*)}$$

The standard optimization theory implies the following proposition.

**Proposition 6.** The optimal taxation strategy meets the following condition: either  $dW/dR^t = dW/dR^T$ , or  $dW/dR^t > dW/dR^T$  and  $T = 0$ , or  $dW/dR^t < dW/dR^T$  and  $\bar{q} = q^*$ .

Let us compare the marginal welfare losses under  $q' = q^*$  and very large  $L'(w^*)$ . In this case

$$-\frac{dW}{dR^t} \approx \left( \frac{K(1 - T)t}{\bar{c}} - 1 \right)^{-1},$$

$$-\frac{dW}{dR^T} \approx \left[ \frac{(1 - T)(K(1 - t) + 1)}{1 - \bar{c}} - 1 \right]^{-1},$$

where  $K = (1 - D(q^*)) / (q^* d(q^*))$ . For typical statistic distributions,  $K$  reduces in  $q^*$ . Then the ratio of the marginal losses  $|W_{R^t}| / |W_{R^T}|$  exceeds 1 if and only if

$$\frac{\bar{c}(1 - t + 1/K)}{(1 - \bar{c})t} > 1. \quad (13)$$

If the ratio exceeds 1 under  $T = 0$  then the optimal way to finance the budget under small  $R$  is to use the presumptive tax. Moreover, as  $T$  increases,  $q^*$  also increases and the inequality holds true. So the presumptive tax stays the only source of the optimal budget financing. (In the more general case, in particular, under a fixed  $L'(w^*)$ , the effect of the rate  $T$  increasing is ambiguous.)

If the inequality inverse to (13) holds under  $T = 0$  then the profit tax is the most efficient source of the budget financing under small  $R$ . However, as  $\bar{q}$  increases, the marginal loss  $|W_{R^t}|$  also increases while the marginal loss  $|W_{R^T}|$  stays constant (under  $L'(w^*) \approx \infty$ ) or reduces. So the combination of the taxes with equal marginal welfare losses is, probably, the most efficient way of the budget financing.

#### 4. Discussion of the model with other production functions.

Proposition 2 and our conclusion on the equivalence of the presumptive and sales taxes strongly relies on the following two properties of the Leontiev-type production function: under

any tax rates, 1) the optimal production volume per employed resource unit does not depend on  $q$ ; 2) the optimal amount of labour per employed resource unit does not depend on  $\tau$ .

The first property implies that the sales tax does not depend on  $q$  and may be efficiently substituted by the presumptive tax per unit of the employed resource. According to the second property, sales tax does not cause any distortions in employment decisions. Consider two classes of production functions.

- a) Let  $q \geq q_{\min}$ ,  $f(q_{\min}, l^*(q_{\min}, w(0), 0)) \geq R$ . Then the necessary budget revenue may be obtained by means of the presumptive tax  $T = R$  without any distortions since any resource unit is profitable under this tax. On the other hand, sales tax reduces the demand for labour and the welfare if  $f'' < 0$  and the increase of the profit tax revenue requires additional audit costs. Thus, the presumptive tax is the optimal source of the budget financing in this case. More generally, the marginal loss of the social welfare is minimal for this tax if density  $d(q^*)$  is close to zero. In this case the reduction of the employed resource under increasing  $T$  is also close to zero according to the system (12).
- b)  $f(q, l) = a(q) \min(1, ql)$ , where  $a(q)$  increases in  $q$ . In this case the second property holds while the first fails. Then under sufficiently small audit cost, sales tax is the more efficient tool of financing than the presumptive tax because the minimum employed quality  $q^*(\tau)$  under this tax is less than  $q^*(T)$  under the presumptive tax, and there are no other distortionary effects besides unemployment of some part of the resource and the corresponding reduction of  $w$ . Sales tax may turn the optimal source of budget financing in this case because the audit costs related to this tax are usually lower than the audit costs related to profit tax: auditing of production expenses requires large additional efforts.

We assumed above that the government has no information on the quality of a particular resource unit. Proceeding from the given model, this assumption may look strange: it suffices to organize the profit audit for all units in one period of taxation in order to determine the maximal profits and thus ascertain the quality for every resource unit. After that the presumptive tax rates may be specified according to this information, and the government can get the same net tax revenue without audit costs.

However, the given model does not take into account other factors that influence production function. In particular, there exist random factors that make the production function in the future period a stochastic rather than deterministic function. So if the distribution of  $q$  is independent for

different periods of the time (and the actual quality is known before involving a resource unit in the production process) then the given model is appropriate for such economy.

In practice the quality of a resource unit includes both stochastic and deterministic components. The existing method of the analysis of the financial and production activity of enterprises and organizations (Методика проведения анализа финансово-хозяйственной деятельности предприятий и организаций, 1997) worked out for tax inspectorates aims to distinguish the deterministic component. According to this method, enterprises are ranked according to the difference between declared profit (and some other parameters) and typical values calculated from the past activities of similar enterprises. It is proposed to audit enterprises where this difference exceeds some threshold. Thus, the threshold level of the profit is established for every homogeneous group of enterprises. So this method of taxation combines the presumptive tax and profit tax approaches.

Our previous paper (Marhuenda et al, 2004) discusses other issues of the presumptive taxation related to stochasticity of the production function and redistribution of the productive resource to the more efficient owners.

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