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TAX OPTIMIZATION UNDER TAX EVASION

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Цель статьи - сформулировать и исследовать проблему налоговой оптимизации в постановке, подходящей для переходных экономик. Для таких экономик как российская характерны широкое распространение уклонения от налогов и коррупция налоговой администрации. Поэтому задача налоговой оптимизации должна решаться одновременно с проблемой налогового принуждения, включая проблему эффективной организации налоговой инспекции. Исследование посвящено налогообложению фирм. Предлагается разработать методы расчета оптимальной налоговой ставки, стратегии аудита и механизма стимулирования эффективной работы налоговой инспекции с целью максимизации чистого налогового дохода.

Мы изучаем оптимальное соотношение между вмененным налогом и налогом с продаж при различных ограничениях на штраф за уклонение. Мы обнаружили, что уклонение от налогов может увеличить чистый налоговый доход, если наказание за уклонение относительно мало. Другой интересный вывод состоит в том, что при общих предположениях, не стоит затрачивать ресурсы на поиск честных инспекторов.

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The paper aims to formulate and examine the tax optimization problem in the setting that is appropriate for economies in transition. The typical features of such economies as Russian are a wide spread tax incompliance and corruptibility of the tax administration. Thus, tax optimization should be studied simultaneously with tax enforcement problem including organization of tax inspection. The research is devoted to taxation of firms. The purpose is to characterize the optimal tax structure, tax rates, auditing strategy and incentive mechanism that stimulates an efficient work of tax administration in order to maximize the net tax revenue.

We study optimal relations between lump sum and sales taxes under different constraints on the penalty for tax evasion. We find out that tax evasion may increase net tax revenue if the penalty for tax evasion is sufficiently soft. Another interesting conclusion is that, under general assumptions, it is not worth to spend resources on selection of honest auditors.

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## 1. Introduction

### a) Statement of the Problem

The model includes the government, tax inspectorates and firms. Production capacities of firms are random variables with a known distribution. Each firm chooses the total production volume and the registered amount of production. The rest is sold at the informal market for cash.

The government sets a lump-sum tax and sales tax rate, so without audit a firm has an incentive to sale unregistered production. In order to prevent tax evasion the government organizes tax inspection and sets a penalty for detected tax evasion.

We consider several possibilities of tax inspection organization.

- 1) The government hires honest inspectors who do not collude with taxpayers.
- 2) The government hires dishonest inspectors who collude with taxpayers whenever there is a possibility of mutually beneficial collusion. In this case, the cost of audit is less then in the previous case.
- 3) The government can use the both types of inspectors with corresponding costs respectively for auditing and reviewing.

In all these cases audit is costless for inspectors and the government controls the resource on audit of every taxpayer.

For each variant our purpose is to study tax optimization problem, that is to find the government strategy that maximizes the net tax revenue under Nash equilibrium (NE) behavior of agents and participation constraints. . We consider 2 types of participation constraints for taxpayers and inspectors.

- (1) Expected incomes of agents under their NE behavior should exceed given alternative incomes. Otherwise the agents would not take part in this interaction.
- (2) An income of every firm is a random variable. We require that under NE behavior and the worst random outcome the income should exceed the

minimum possible value. The sense of this restriction is that the decrease of income below this value leads to destruction of a firm.

We assume that, under a fixed strategy of the government, behavior of the inspectorate and the firm corresponds to the Nash equilibrium in the related game. Each agent aims to maximize its expected income.

We consider three variants of penalty functions: a) penalty is proportional to unpaid tax, b) pure penalty is proportional to hidden income, c) penalty is bounded because of the given minimal income of an agent.

In the present model the strategy of the government includes:

- 1) tax rates, and also penalties for evasion, if they are not given exogenously,
- 2) a strategy of tax inspection organization, including the system of inspectors selection, existence or absence of reviewing, payment rule, a system of premiums and penalties to auditors;
- 3) a strategy of audit: a rule of allocation resources on auditing and reviewing, which depends on information received from taxpayers and auditors.

The government aims to maximize net tax revenue under the specified constraints. The interests of firms-producers are reflected in the model by the alternative income value.

## **b) Objectives of the study**

The main objectives are as follows:

- to solve the specified maximization problem of the net tax income within the model with two possible levels of production volumes and homogeneous group of enterprises;
- to examine the impact of parameters on decision, in particular, to evaluate the role of lump sum tax and to find out if this tax should be maximal possible and what limits its value;

- to determine the conditions under which the solution of this model corresponds to the honest behavior of the agents and to point out the possible reasons of the preference for tax evasion.

### **c) Importance of the study**

In the short-term prospect, Russian economy meets the following dilemma. On the one hand, there are important reasons (huge foreign debt and hard social problems) to increase budget expenses. On the other hand, by general opinion, the tax rates should be reduced because it is impossible to work honestly under the present rates.

A possibility to solve this dilemma relates to the fact that the essential part of the economy does not pay taxes now. The level of tax collection in Russia is very low (<50% in 1999, see Summers (2000)).

However, the IMF specialists reasonably argue that simple reduction of tax rates would not increase tax revenue. Those who do not pay would not pay, and those who pay would pay less. The real way to solve the problem is simultaneous optimization of the tax system and corresponding reorganization of the tax inspectorate. Such reorganization is necessary to increase the efficiency of tax collection.

The problem is complicated by corruption in fiscal bodies. This phenomenon is widespread in transition economies. In such countries as India or Taiwan surveys show that more than half of interviewees usually pay bribes to tax officials (Keen, Hindriks, Muthoo (1998), Mookherjee, Png (1995)). There exists a widespread opinion that the level of corruption within fiscal bodies in Russia is relatively substantial. The most important reason for that seems to be a very low salary of tax inspectors. This leaves strong incentives for accepting bribes from firms evading much greater amount of the tax liabilities. In Russia the salary of tax inspectors was about \$100-\$200 in 1996 and decreased since that time. The problem is not only that

the inspectors should not take bribes but that they have to make essential efforts in order to reveal tax evasion, especially in firms incorporated in shadow economy. Statistics shows that this work may be dangerous for their lives: several hundreds attempts by criminals to impact on inspectors were registered in 1998. Thus, it is necessary to provide fiscal bodies with sufficient resources for their activity and create appropriate incentives for their efficient work.

We propose to study related theoretical problems with focus on taxation of enterprises. The reason for that is the structure of the Russian budget income where the individual taxes play the minor role, and it is unlikely to change it in the nearest future. Moreover, statistic data on district tax inspectorates shows that typically 10-20 largest enterprises give 80-90% of the total tax revenue.

## **2. Survey of Literature**

The recent paper by Hindriks et al. (1999) seems to be rather close to our setting of the problem. The interaction between the government, the taxpayer and the auditor is described as follows.

1. The government sets a tax and penalty schedules for the taxpayer and the payment rule for the auditor depending on the reported income and revealed misreporting.

2. The taxpayers' income is determined according to a given probability distribution.

3. The auditor finds out the actual income and bargains with the taxpayer on the reported income. If collusion is profitable then they share the surplus at a given ratio. Otherwise, the auditor sends his report on the income without collusion.

4. The taxpayer can complain if she does not agree with the report.

5. In this case an external honest auditor reviews the first audit. If the taxpayer does not complain then reviewing happens with a given probability  $\pi$  (which is called «the level of residual honesty»).

6. Transfers between the government and the agents take place.

Under these assumptions the optimal government strategy that maximizes the tax revenue is collusion proof. The most important conclusion is that the maximal tax revenue is  $\pi E\theta$  where  $E\theta$  is the expected income of taxpayer. This result sounds rather pessimistic with respect to Russia where after 7 years of reforms, the level of residual honesty seems to be extremely low.

On the other hand, Vasin and Panova (1999) obtained the more positive results. Proceeding from their paper, we propose that it is possible to collect some taxes irrespective of the level of honesty. The reason of such difference in the results is that taxpayers do not independently declare their incomes in the model by Hindriks et al. This is in contrast to the model by Vasin and Panova. There the government can establish a threshold tax value such that any firm is free of audit if it pays this tax. Note that this approach is similar to the cut-off rule studied by Scotchmer (1987) and Sanchez and Sobel (1993).

Papers on tax enforcement typically consider direct taxes. Reinganum and Wilde (1985), Border and Sobel (1987), Sanches and Sobel (1993), Chander and Wilde (1992), Vasin and Panova (1998) study the problem under fixed taxes and fines for evasion.

Chander and Wilde (1998) consider a more general problem of income tax enforcement. They introduce the notion of efficient scheme including tax, penalty and auditing probability functions  $(t, f, p)$  such that any other scheme does not allow to increase the expected payment of any taxpayer without increasing probabilities of auditing for some reported income. For different objectives of the tax authority the optimal scheme must be efficient. The problem of the tax authority is the problem of

optimal mechanism design where a mechanism is a scheme  $(t, f, p)$ . The revelation principle holds so that it is possible to restrict attention to incentive compatible direct revelation schemes. They find that for any efficient scheme the payment function must be non-decreasing and concave, the tax function is non-decreasing with non-increasing average tax rate. These properties imply that there is no redistribution among the taxpayers. The audit probabilities are determined wholly by the marginal payments rates and are non-increasing. Regressivity implies that the inability of the government to costlessly observe true incomes severely restricts its ability to redistribute through direct taxation. The regressivity result is known from another consideration which takes into account the supply side effect (Mirrlees (1971) and others).

Mookherjee and Png (1989,1990) study the tax enforcement problem in the setting close to the contract theory. They consider the moral hazard problem and permit arbitrary tax and penalty schedules that meet participation constraints. Their results show that such approach has some disadvantages: the optimal penalty schedule they find is to fine a tax evader with the whole income irrespective of the amount of the concealed income. Obviously, such rule cannot be realized in practice.

We are aware of one model of tax enforcement optimization for indirect taxes by Cowell and Gordon (1995). They compare different audit strategies available to a tax authority attempting to collect indirect taxes. The authors model tax evasion as follows: taxpayers choose between taxable activities on the regular market and unreported activities on an informal market. If an individual is audited and found to be undertaking irregular activities she/he is fined and made to repay the evaded tax. One possible strategy is to audit randomly, with some fixed probability that any taxpayer is investigated. An alternative policy is to take into account what the authority knows about each taxpayer. Cowell and Gordon study a simple form of



this approach where the authority conditions the audit probability on reported turnover via a cut-off rule: those reporting less (no less) than a certain amount are always (never) audited. Cowell and Gordon establish conditions under which the optimal random audit is better than the optimal cut-off rule, and vice versa. However, as D. Siniscalco notes in his discussion of the Cowell and Gordon model, the optimal audit strategy in general does not belong to any of the specified classes.

However, these studies do not take into account an important aspect of the work of any tax inspection, which is the possibility of corruption. In fighting the phenomenon of collusion between inspectors and evaders it is possible to improve situation by introduction of a flexible payment rule. Under such rule inspectors get some part of collected fines as a premium. This idea was developed in the work by Vasin and Panova (1998).

The role of giving incentives to the state officials in fighting corruption was recognized within the economic literature (see e.g. Bardhan ,1997). Some countries have accepted systems of tax enforcement which include a bonus to the tax officer based on the amount of taxes he or she collects, which significantly improved tax compliance. Besides Vasin and Panova (1998), a theoretical base for that was given in Hindriks, Keen, Muthoo (1999). They show that the honest implementation of progressive tax schedule may require paying commission on high income reports.

However, the direct application of the obtained results to the corporate taxation seems to be unreasonable. For instance, Chander and Wilde (1998), Hindriks et al. (1999), Vasin and Panova (1998) consider the problem under a fixed tax base. Their models do not catch an important issue of production activity stimulation. They also do not reflect dependence of audit results on inspector's effort. On the other hand, the papers by Mirrlees (1971, 1986) and Mookherjee and Png (1989) devoted to tax optimization do not take corruption into account. Thus,

the problem of tax optimization under corruptible tax inspection stays open at the present time.

### 3. A formal model and research results

This section describes and studies several models of tax optimization taking tax evasion into account.

#### 3.1. A Basic Model without Corruption with Two Possible Production Capacity Levels

Let us formally describe a basic model without corruption. We consider an industry with two possible production capacity values. Each firm has a high production capacity  $V_H$  with probability  $q$  and low capacity  $V_L$  with probability  $1-q$ ,  $V_H > V_L$ . The capacity  $\bar{V} \in \{V_L, V_H\}$  of each firm is its private information. The marginal production cost  $c$  is constant and common for all firms. Each firm chooses output  $V$  and volume of registered sales  $V_r$ . Both  $V$  and  $V_r$  are from the set  $\{V_H, V_L\}$ . Thus, there exist three possible strategies  $s^1 = (V_L, V_L)$ ,  $s^2 = (V_H, V_L)$ ,  $s^3 = (V_H, V_H)$  for a firm with  $\bar{V} = V_H$  and the unique strategy  $s^1$  for a firm with  $\bar{V} = V_L$ . The volume  $V_r$  is sold at the regular market, the tax is  $T(V_r) = T + tpV_r$  ( $T$  is a lump sum tax paid by any firm in the industry,  $t$  is a tax rate per unit of the production sold at a regular market). The amount  $V_u = V - V_r$  (illicit sales) is sold at the informal market (for cash). By selling in the informal market the firm is able to evade sales tax.

The government spends effort  $e(V_r)$  on auditing a firm with registered output  $V_r$ . The cost of each audit is  $de(V_r)$ . An audit detects the volume  $V_{du} = \min\{V_u, \frac{e(V_r)V_u}{V}\}$  of unregistered production. Below we consider several variants of penalties for evasion. The first is the variant (a) (see the Introduction) where the

fine for evasion is a surcharge  $s > 0$  on the unpaid tax and is equal to  $f(V_r, V_{du}) = (1 + s)tpV_{du}$ .

The market is competitive, and the price  $p$  in both markets is constant and exogenous.

### *A firm's problem*

Firms are risk neutral. Each firm with capacity  $V_H$  sets its strategy  $(V, V_r) \in \{s^1, s^2, s^3\}$  to maximize its profit:

$$(V, V_r) \rightarrow \max \{pV - T - tpV_r - cV - (1 + \delta)tpV_{du}(V, V_r, e(V_r))\} \quad (1.1)$$

under a given government strategy  $s_G = (T, t, e(V_r))$ .

### *A government's problem*

The government aims to maximize net tax revenue

$$R(T, t, e) = (1 - q)(T_L - de) + q(T(V_r) + F(V_r, V_{du}(V, V_r, e)) - de(V_r)) \quad (1.2)$$

under condition (1.1) and the following participation constraints:

$$I_{Lat} \geq I_{\min} \quad (1.3)$$

$$(1 - q)I_{Lat} + qI_{Hat}(s_G) \geq I_{alt} \quad (1.4)$$

where  $I_{Lat}$  (respectively  $I_{Hat}$ ) is the income after tax and audit for a firm with low (respectively, high) capacity,  $I_{Hat}(s_G)$  denotes the optimal income under government strategy  $s_G$ . The latter value is a solution of the problem (1.1).

In these inequalities  $I_{alt} > I_{\min}$ . Their meaning was discussed in the Introduction. Note that it is unnecessary to introduce condition  $I_{Hat} \geq I_{\min}$  since a firm with high capacity can use strategy  $(V_L, V_L)$ . Now we shall describe optimal behavior of firms under a given strategy of the government. Obviously the optimal effort  $e(V_H) = 0$ . Let  $e = e(V_L)$ .

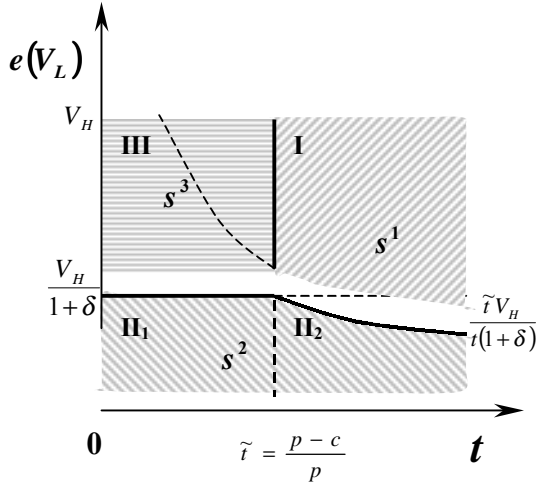
### *Proposition 1.1.*

For any strategy  $s_G = (T, t, e)$ , the optimal strategy of a firm with the high capacity is

$$\begin{cases} s^3, & \text{if } t < (p-c)/p, \quad e > V_H/(1+\delta); \\ s^2, & \text{if } t < (p-c)/p, \quad e < V_H/(1+\delta) \text{ or } t > (p-c)/p, \quad e(V_L) < (p-c)V_H/(tp(1+\delta)); \\ s^1, & \text{if } t > (p-c)/p, \quad e > (p-c)V_H/(tp(1+\delta)). \end{cases}$$

Figure 1 illustrates this result.

Figure 1.



Now we'll determine the optimal government strategy for the problem (1.2) depending on exogenous parameters of the model. Let us introduce the following notation. Let  $T_L$  (respectively  $T_H$ ) denote a total tax on a firm with the low (respectively high) production volume. Then  $T_L = tpV_L + T$ ,  $T_H = T_L + \Delta T$ , where  $\Delta T = tp(V_H - V_L) = tp\Delta V$ . Let  $I_L = (p-c)V_L$ ,  $\Delta I = (p-c)\Delta V$ .

Proposition 1.1. shows that in the area I, where

$$t > \tilde{t} \stackrel{\text{def}}{=} \frac{p-c}{p}, \quad V_H \geq e > \tilde{t} V_H / (t(1+\delta)),$$

the optimal strategy is  $s^1 = (V_L, V_L)$ . An agent's optimal income is  $I^1 = I_L - T_L$ , and the net revenue is  $R^1 = T_L - de$ . The participation constraints take forms

$$I_L - T_L \geq I_{\min} \tag{1.3.I}$$

(it is the same in all areas) and

$$I_L - T_L \geq I_{alt}, \tag{1.4.I}$$

so (1.4.I)  $\Rightarrow$  (1.3.I).

In the area II the optimal strategy is  $s^2 = (V_H, V_L)$  that is tax evasion. An agent's optimal income is  $I^2 = I_L + q\Delta I - T_L - (1+\delta)q\Delta Te/V_H$ , the net revenue is  $R^2 = T_L + q(1+\delta)\Delta Te/V_H - de$ . The participation constraint is

$$I^2 \geq I_{alt}. \quad (1.4.II)$$

In the area III  $t < \tilde{t}$ ,  $V_H \geq e > \tilde{t}V_H/(t(1+\delta))$ , the optimal strategy is  $s^3 = (V_H, V_H)$ , an agent's optimal income is  $I^3 = I_L + q\Delta I - T_L - q\Delta T$ , the net revenue is  $R^3 = T_L + q\Delta T - (1-q)de$ . The participation constraint takes the form

$$I_L + q\Delta I - T_L - q\Delta T \geq I_{alt}.$$

Now, let us study the tax optimization problem. Note that if we exclude incentive constraints, that is, optimize by agents' strategy, then we obtain the first best solution. It corresponds to the strategy  $s^3 = (V_H, V_H)$ . The problem is to find

$$\max (T_L + q\Delta T) \quad (1.2^*)$$

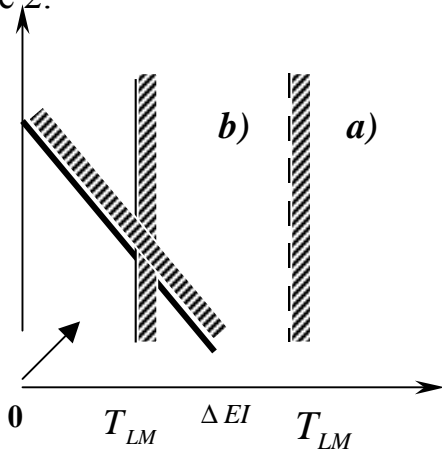
under constraints

$$T_L \leq I_L - I_{\min} \stackrel{def}{=} T_{LM}, \quad \Delta T \geq 0, \quad (1.3^*)$$

$$T_L + q\Delta T \leq I_L + q\Delta I - I_{alt} \stackrel{def}{=} \Delta EI \quad (1.4^*)$$

where  $\Delta EI$  is an expected surplus profit of an agent before tax (see Figure 2).

Figure 2.



*Proposition 1.2.*

If  $T_{LM} > \Delta EI$  (1.5)

then the optimal net revenue  $R^* = \Delta EI$  is the same for the problems (1.1)-(1.4) and (1.2\*)-(1.4\*), and the optimal government strategy is  $\Delta T = 0$  (that is,  $t = 0$ ),  $T = T_L = \Delta EI$ ,  $e \equiv 0$ .

Proceeding from the previous discussion of conditions (1.3), (1.4), the inequality (1.5) shows that the expected surplus profit is less than that maximal lump-sum payoff that does not undermine activity of a firm under unfavorable conditions. Proposition 1.2 means that in this case it is possible to get the first best outcome by means of the lump sum tax. It is unnecessary to collect any other taxes and organize audits.

Now consider the case where  $T_{LM} < \Delta EI$ . Then the first best revenue value is the same but requires a combination of the both kinds of taxes. Let us find a solution of the tax optimization problem (1.1)-(1.4) with additional constraint  $F \leq F_{\max}$ .

*Proposition 1.3.*

If  $T_{LM} < \Delta EI$  and  $t_{\max}$  is sufficiently large then the optimal government strategy is to set  $F = F_{\max}$ , audit effort  $e^* = V_H(\Delta EI - T_{LM})/(qF_{\max})$  and lump sum tax  $T_L = T_{LM}$ . Under this strategy, the optimal strategy of agents is  $s^2 = (V_H, V_L)$ , and the revenue  $R(t_{\max})$  approximates the first best result  $\Delta EI$  as  $t_{\max}$  tends to infinity. The maximal revenue under honest behavior of taxpayers is

$$\max(\Delta EI - (1-q)d \frac{V_H}{1+\delta}, T_{LM}) < \Delta EI.$$

A similar proposition holds under the penalty constraint b). Note that in all three cases the tax optimization problem in the area II can be set in the same way if we introduce  $y = eF/V_H$  - an actual expected payment to the budget from the additional income. Then the problem is

$$T_L + qy - de \rightarrow \max \quad (*)$$

under  $T_L \leq T_{LM}$ ,  $T_L + qy \leq \Delta EI$ ,  $y \leq \Delta I$ ,  $y \leq \Delta T$  and the penalty constraint. Consider the problem without the latter.

Let  $F = F_{\max}$ ,  $T_L = T_{LM}$ ,  $y = (\Delta EI - T_{LM}) / q = \Delta I + (I_{\max} - I_{alt}) / q < \Delta I$ , then  $R(S_G) = \Delta EI - dyV_H / F_{\max} \geq \Delta EI - d\Delta I / F_{\max} \rightarrow \Delta EI$ , as  $F_{\max} \rightarrow \infty$ . Let us show that it is possible to meet the penalty constraints. In the case a):  $\Delta T = F_{\max} / (1 + \delta) > y$ , in the case b):  $\Delta T = F_{\max} - \delta\Delta V > y$ , if  $F_{\max}$  is sufficiently large. The case c) is discussed below.

Implementation of the specified optimal strategy assumes very large tax rate and penalty for evasion. The previous model assumes that auditor's effort strictly determines the result of audit. However, in practice this result is usually random. It is reasonable to consider the value  $V_{du}(V, V_r, e)$  as an expected value of detected unregistered production and assume that by chance any volume up to  $V - V_r$  may be detected. Then the participation constraint (1.3.II) takes another form:

$$I_L - T_L \geq I_{\min}, \quad I_H - F - T_L \geq I_{\min} \quad (1.3.II')$$

Here we consider  $e/V_H$  as a probability to detect tax evasion under  $s^2 = (V_H, V_L)$ .

Now the tax optimization problem for area II is (\*) plus  $T_L + F \leq T_{LM} + \Delta I$  and the penalty constraint.

We bound our study with the case where the cost of audit is sufficiently small:

$$q\Delta I \geq dV_H. \quad (**)$$

#### *Proposition 1.4.*

Under this condition and irrespective of the penalty constraint, the optimal government strategy in the area II is

$T_L^2 = I_L - I_{alt} = T_{LM}$ ,  $F^2 = \Delta I$ ,  $e^2 = V_H(\Delta EI - T_{LM})/(\Delta I q)$ . Under this strategy,  $R^2 = \Delta EI - de^2$ , and this strategy is optimal in the whole if  $R^2 = \max\{R^3, T_{LM}\}$ , that is,  $R^2$  is greater than the maximal revenue under honest behavior of firms.

Under  $F = (1 + \delta)\Delta T$ ,  $R^2 > R^3$  if  $I_{alt} - I_{\min} > \frac{\delta + q}{1 - q} q \Delta I$ .

Under  $F = \Delta T + \delta \Delta V$ ,  $R^2 > R^3$  if  $I_{alt} - I_{\min} > q^2 \Delta I + \delta \Delta V$ .

Finally,  $R^2 > T_{LM}$  if  $d$  is sufficiently small.

Note that since the tax rate is bounded, we cannot reach the first best outcome.

Thus, in this model in general the honest behavior of agents is not optimal.

The reason is the inflexible dependence of the penalty for evasion on the unpaid tax:

a)  $F(V, V_r) = (1 + \delta) \cdot (T(V) - T(V_r))$  or b)  $F = \Delta T + \delta \Delta V$ .

Note that this rule corresponds to the legislation of several European countries. Moreover, important theoretical results are based on this relation. In particular, Sanchez and Sobel (1992) prove optimality of the auditing “cut-off” rule for this kind of penalty.

Now let us consider c)  $I_H - (T_L + F) \geq \bar{I}_{\min}$ . Condition c) is similar to (1.3.II') but in this case it relates to any (not necessary NE) behavior of a firm, and characterizes the social norm, not the condition of a firm's survival. Naturally,  $\bar{I}_{\min} < I_{\min} < I_{alt}$ .

*Proposition 1.5.*

Under penalty constraint c)  $F(V_H, V_L) = I_H - \bar{I}_{\min} - T_L$ , the optimal strategy  $s_G$  is  $\Delta T^* = (\Delta EI - T_{LM})/q$ ,  $e^* = V_H \Delta T^* / (I_H - \bar{I}_{\min} - T_{LM})$  if  $q/(1 - q) > V_H d / (I_H - \bar{I}_{\min} - T_{LM})$ , otherwise  $\Delta T^* = 0$ ,  $e^* = 0$ . In all the cases  $T_L^* = T_{LM}$ .

### 3.2. A Basic Model with Dishonest Auditors

This model is similar to the basic model with two production levels in Section 1. The difference is that inspectors are independent players (as well as firms) and



aim to maximize their expected incomes. Thus, an audited firm can bribe an inspector whenever there exists a possibility for mutually beneficial collusion. The government sets a regular salary  $s$  that is paid if an auditor confirms the low production volume, and a premium  $\Delta s$  is paid for revealing of unregistered production  $\Delta V = V_H - V_L$ . Then, under a given fine  $F$  for evasion, the minimal bribe acceptable for an auditor who detected tax evasion is  $b_{\min} = \Delta s$ , and the maximal bribe profitable for tax evader is  $b_{\max} = F$ . Collusion is possible if  $b_{\min} < b_{\max}$ , in this case  $b = \gamma b_{\max} + (1 - \gamma)b_{\min}$ , where  $\gamma \in (0, 1)$  characterizes the bargaining power of an auditor.

The government strategy  $s_G$  includes in this case 5 components:  $T, t, e, s$  and  $\Delta s$ . The penalty  $F$  is given exogenously (expect for the case c)), first we'll discuss the case a)  $F = (1 + \delta)\Delta T$ . (Recall that  $\Delta T = tp\Delta V$ ,  $\Delta I = (p - c)\Delta V$ ,  $T_L = tpV_L$ , etc.)

We assume that  $\Delta s \leq F$ . Otherwise the following kind of collusion is profitable for agents: irrespective of the audit result, an inspector reports tax evasion, and then he shares the surplus with the audited firm.

Examining the tax enforcement problem, it is reasonable to consider two types of the government strategies.

If  $\Delta s = F$  then there is no collusion and the problems of a firm and a government are similar to the first basic model (see 1.1-1.4). The only difference is that the cost of audit to the government depends on its outcome:  $d = \bar{d} + s$  if there is no detected evasion, and  $d = \bar{d} + s + \Delta s$  if an auditor detects tax evasion. Thus, net tax revenue is  $R(s_G) = T_L - (\bar{d} + s)e$  under firm's strategy  $(V_L, V_L)$  or  $(V_H, V_L)$ ,  $R(s_G) = T_L + q\Delta T - (1 - q)(\bar{d} + s)e$  under strategy  $(V_H, V_H)$ . Participation constraints for a firm stay the same (see 1.3, 1.4). As to auditors, let us require that their expected income under the optimal strategy of a firm is not less than  $s_{alt} > 0$ , where  $s_{alt}$  is their alternative occupation salary. Thus,  $s + qF \geq s_{alt}$  under  $(V_H, V_L)$ .

Under any strategy  $s_G$  such that  $\Delta s < F$ , net revenue of the government is the same under a fixed firm's strategy, and a firm with high capacity gets

$$\begin{aligned} V_L(p - c - tp) &= I_L - T_L \text{ under } (V_L, V_L), \\ V_H(p - c - tp) &= I_H - T_L - \Delta T \text{ under } (V_H, V_H), \\ I_H - T_L - \frac{e}{V_H}(\gamma F + (1 - \gamma)\Delta s) &\text{ under } (V_H, V_L). \end{aligned}$$

Participation constraints for an auditor are

$$\begin{aligned} s &\geq s_{alt} \text{ under } (V_L, V_L) \text{ or } (V_H, V_H), \\ s + qb &\geq s_{alt} \text{ under } (V_H, V_L). \end{aligned} \tag{2.1}$$

Let us note that the first best outcome  $R^* = \Delta EI = I_L + q\Delta I - I_{alt}$  is the same as in the first model since auditors produce nothing and  $s_{alt} > 0$ . Hence, Proposition 1.2 also holds for the present model.

Now consider the case where  $\Delta EI > T_{LM}$ . In order to find the optimal  $s_G$ , it suffices to solve the tax optimization problem for each kind of government strategies and to take the best solution. The next proposition simplifies this task: it shows that it suffices to consider such strategies that make bribing unprofitable.

For every tax optimization problem discussed in Section 1, the corresponding model with dishonest auditors is constructed as above in this section, including an extended government strategy and participation constraint (2.1) for auditors. The next result is valid for all discussed types of penalty constraints.

*Proposition 2.1.*

For any strategy  $s_G = (T_L, \Delta T, e, s, \Delta s)$  there exists a strategy  $\bar{s}_G$  such that  $\Delta \bar{s} = \bar{F}$  and  $R(\bar{s}_G) = R(s_G)$  where  $R(\bar{s}_G)$  is the maximal net revenue under the optimal firm's strategy and participation constraints.

Now let us show that, under equal alternative salaries, the optimal net revenue in the model with dishonest auditors is the same as with honest auditors.

*Proposition 2.2.*

Let the cost of audit in the model with honest auditors be  $d = \bar{d} + s_{alt}$ . Then for every penalty constraint a)-c) the optimal revenues  $R^{a*}$ ,  $R^{b*}$  and  $R^{c*}$  are the same in the corresponding models with honest and dishonest auditors.

*Corollary 1.*

The optimal tax rates, penalties, efforts and firms' strategies in the models with honest auditors are also optimal in the corresponding models with dishonest auditors. In particular, whenever the honest behavior of a firm is optimal in the first model (for instance under the penalty constraints b, c), it is also optimal in the second model.

*Corollary 2.*

In general, selection of honest auditors requires additional costs, that is,  $\bar{d} + s_{alt}$  is greater in the first model. Then it is always optimal for the government revenue to employ dishonest auditors. Kofman, Lawarree (1996) obtained a similar result for a different model with a possibility of extortion and more strict participation constraints.

Note that in the case b) inspector's income per audit may essentially vary depending on the capacity of an audited firm. In particular, under conditions of Proposition 1.3,  $F$  tends to infinity together with  $t_{max}$ , and a regular salary tends to  $-\infty$ . (However,  $e$  tends to zero, so an auditor can audit many firms per unit of the time.) The negative salary means that an auditor should buy a license for audit from the government. But even if we require  $s \geq 0$ , Proposition 2.2. and the corollaries hold whenever the optimal behavior of taxpayers is honest, in particular, under the penalty constraints b) and c).

### 3.3. A Model Where Audit Is Costly for Inspectors

#### 3.3.1 A Model without Corruption

This model is similar to the basic model in Section 1. The difference is that the government sets only the maximal effort  $e_M \leq V_H$  on auditing of a firm with the reported low output, and each auditor chooses himself his effort  $e \in [0, e_M]$  on the audit of such firm. The cost of audit to an inspector is equal to his effort. A probability to detect tax evasion and, hence, the expected detected unregistered production  $V_{du}$  depend on  $e$ :  $V_{du} = V_u e/V_H$ . The cost of the audit to the government is  $\bar{d}e_M$ , besides that it pays a regular salary  $se_M$  and a premium proportional to the detected unregistered production. If  $V_{du} = \Delta V$  then the premium is  $\Delta s$ . In the present variant auditors do not take bribes, so the total cost of audit of one firm to the government is  $(d + s)e_M + \Delta s \frac{eV_{du}}{V_H \Delta V}$ . Let  $\xi_1$  denote the share of tax evaders (who play  $(V_H, V_L)$ ),  $\xi_2$  - the share of  $(V_L, V_L)$  players among all firms,  $\xi_3$  - the share of  $(V_H, V_H)$  players. Then an average income per audit of an auditor with a strategy  $e$  is

$$I_i(\xi, e) = se_M + \frac{q\xi_2}{1 - q + q(\xi_1 + \xi_2)} \frac{e}{V_H} \Delta s - e, \quad e \leq e_M.$$

An expected after tax and penalty profit of a firm is

$$I^1(e, s_G) = I_L - T_L \text{ for strategy } (V_L, V_L),$$

$$I^2(e, s_G) = I_L - T_L + q(\Delta I - \frac{e}{V_H} F) \text{ for } (V_H, V_L),$$

$$I^3(e, s_G) = I_L - T_L + q(\Delta I - \Delta T) \text{ for } (V_H, V_H),$$

as well as in the basic model.

A strategy of the government is  $s_G = (T_L, \Delta T, e_M, s, \Delta s)$ , and net revenue under any given strategies of all agents is

$$R = T_L + q\Delta T_{\xi_3} + \frac{e}{V_H} q\xi_2 (F - \Delta s) - (d + s)e_M (1 - q + q(\xi_1 + \xi_2)). \quad (3.2)$$

We assume that, under a given strategy of government  $s_G$  agents use Nash equilibrium strategies, that is,

$$e^* \rightarrow \max_{e \in [0, e_M]} I_i(\xi^*, e, s_G) \quad (3.3)$$

$$\xi_i > 0 \Rightarrow I^l = \max_{j=1,2,3} I^j(e^*, s_G).$$

Participation constraints are:

$$\begin{aligned} I_i(\xi^*, e^*, s_G) &\geq s_{alt}, \\ \xi_i^* > 0 \Rightarrow I^l(e^*, s_G) &\geq I_{alt}, \\ I_L - T_L &\geq I_{\min}. \end{aligned} \quad (3.4)$$

We shall consider the same variants a, b, c of the penalty constraints as in Sections 1, 2. Under all these constraints, we aim to find the optimal strategy  $s_G^*$  that maximizes the revenue  $R$ .

Note that if  $\Delta T > \Delta I$  then  $I^1 > I^3$ . However, if  $\xi_1 > 0$  then the strategy  $s_G$  is not optimal. If  $\xi_1 = 1$  then  $R \leq T_{LM}$ , and this value may be obtained as a lump sum tax. If  $I^1 = I^2$  then it is possible to increase tax revenue by the slight reduction of the effort  $e_M$ : then all firms will play  $(V_H, V_L)$ .

If  $\xi_2 = 0$  then  $e^* = 0$ , but in this case  $I^2 > I^1$ . Thus,  $\xi_2 > 0$  at any NE, and we are to consider two variants of possible optimal behavior of firms: 1)  $\xi_2 = 1$ ; 2)  $\xi_1 > 0, \xi_2 > 0$ .

If  $\frac{e}{V_H} F < \Delta T$  then  $I^2 > I^3$ ,  $\xi_2 = 1$ . In this case if  $q\Delta s/V_H < 1$  then  $e^* = 0$ , and the revenue is less than  $T_{LM}$ .

If  $\xi_1 > 0$  then in order to provide  $e > 0$  it is necessary to set  $\Delta s \geq V_H(1 - q + q\xi_2)q\xi_2$ , otherwise careful audit is unprofitable for inspectors.

Consider any strategy  $s_G$ . If, under NE behavior of agents,  $e < e_M$ , then it is possible to reduce  $e_M$  keeping all constraints and increasing net revenue. Thus, we may assume  $e_M^* = e^*$  at the optimum.

Finally, note that, under the optimal strategy, the participation constraint (3.4) for inspectors holds as equality. Otherwise it is possible to increase the net revenue by reduction of the salary  $s$ .

Let us summarize these results.

*Proposition 3.1.*

In order to find the optimal strategy  $s_G$ , it suffices to compare  $T_{LM}$  with solutions of the following optimization problems:

$$1) T_L + e_M q F / V_H - d e_M - (e_M + s_{alt}) \rightarrow \max ,$$

under constraints

$$0 \leq F e_M / V_H \leq \Delta T ;$$

$$T_L \leq T_{LM} , \quad T_L + q F e_M / V_H \leq \Delta E I$$

and the penalty constraint. In this case all firms play strategy  $(V_H, V_L)$ .

$$2) T_L + q \Delta T (1 - \xi_2) + \xi_2 q F e_M / V_H - (d e_M + e_M + s_{alt}) (1 - q + q \xi_2) \rightarrow \max ,$$

under

$$0 \leq F e_M / V_H = \Delta T ;$$

$$T_L \leq T_{LM} , \quad T_L + q F e_M / V_H \leq \Delta E I$$

and the penalty constraint. In this case  $\xi_2 \in [0,1]$ .

#### 4. Policy Conclusions

Proceeding from the obtained results, we can make several conclusions on the optimal tax enforcement strategy of the government.

According to the known Welfare theorem, if the government has a complete information on fixed production capacity and cost of a firm, it is optimal to impose a type-specific lump sum tax, and not to organize audit. However, if the capacity

and/or the cost is random, it may be optimal to combine a lump sum tax with other types of taxes, even if the government knows the type of a firm, that is, its capacity/cost distribution. In this case the maximal lump sum tax is limited, on the one hand, by the value  $T_{LM}$  that can undermine a firm's activity under unfortunate production conditions, on the other hand, by the value  $EI - I_{alt}$ , that is the surplus expected profit before tax with respect to the profit from alternative activity. Proposition 1.2 shows that if this surplus value is relatively small then lump sum tax is still optimal. But if the profit distribution is widely dispersed then it is necessary to consider other variants of taxation. Our results show that it is always optimal to set the maximal lump sum tax under the mentioned constraints. If the penalty for evasion is a surcharge on unpaid tax or the fine is proportional to unregistered production then tax evasion may be optimal for the government if the penalty for evasion is sufficiently soft (in particular,  $\delta\Delta T$ , where  $1 + \delta < (1 - q)\Delta Iq / (\Delta Iq + I_{min} - I_{alt})$ , or  $\delta\Delta V$ , where  $\delta\Delta V < (1 - q)\Delta Iq - (\Delta EI - T_{LM}) = I_{alt} - I_{min} - q^2\Delta I$ ).

If the penalty is sufficiently strict ( $\delta$  meets the opposite inequality) in these cases or the only penalty constraint is that the total payoff from a taxpayer is uniformly limited (the case c) then honest behavior of taxpayers is always optimal. The optimal sales tax rate is either 0 or such that with account of lump sum tax it leaves firms with their alternative profits. The optimal audit rule in this case is to apply such minimal effort that makes tax evasion unprofitable.

With respect to tax inspection organization, an interesting conclusion is that it is never optimal to spend resources on selection of honest auditors if dishonest are cheaper (Proposition 2.2). Another conclusion related to dishonest inspectors is that permission of corruption never increases the optimal revenue in the discussed models (Proposition 2.1).

When we take into account that the result of audit depends on the inspector's effort, and this effort is costly to auditor we find out that some level of tax evasion is necessary to create incentives for careful audit, and the greater is the necessary effort the less is the optimal net revenue. Thus, in this case the optimal behavior of taxpayers is heterogeneous (Section 3).

## **5. Bibliography**

1. P.Bardhan, 1997, "Corruption and Development", *Journal of Economic Literature*, XXXV, 1320-1346
2. P. Chander, L. Wilde, 1998, "A General Characterization of Optimal Income Tax Enforcement", *Review of Economic Studies*, 65, 165-189.
3. F. Cowell, G. F. Gordon, 1995 "Auditing with "ghosts"". In: "The Economics of Organized Crime", 184-198.
4. J.Hindriks, M.Keen, A.Muthoo, 1999, "Corruption, Extortion and Evasion", forthcoming in *Journal of Public Economics*.
5. Klitgaard, 1988, *Controlling Corruption*, Berkeley, CA: University of California Press.
6. F. Kofman and J. Lawarree, 1993, "Collusion in Hierarchical Agency" *Econometrica*, p.629-656.
7. F. Kofman and J. Lawarree, 1996, "On the Optimality of Allowing Collusion" *Journal of Public Economics*, p.383-407.
8. J.-J. Laffont and J.Tirole, 1991, "The Politics of Government Decision Making: A Theory of Regulatory Capture" *Quarterly Journal of Economics*, Vol.106, 1089-1127.
9. J. Mirrlees, 1971, "An Exploration in the Theory of Optimal Income Taxation", *Review of Economic Studies*, 328, 175-208



10. J. F. Reinganum, L. L. Wilde, 1985, "Income tax compliance in a principal-agent framework", *Journal of Public Economics* 26, 1-18.
11. Sammers, 2000
12. I. Sanchez, J. Sobel, 1993 "Hierarchical design and enforcement of income tax policies", *Journal of Public Economics*, 50, 345-69
13. J. Tirole, 1992, "Collusion and the Theory of Organization", in: J.-J. Laffont, ed., *Advances in Economic Theory, Sixth World Congress*, vol.2 (Cambridge University Press, Cambridge)
14. A. A. Vasin, E.I. Panova, 1993, "Tax Collection and Corruption in Fiscal Bodies", Final Report on EERC Project

## 6. Appendices.

*Proof of Propositions 1.4 and 1.5 .* Let  $eF/V_H = y$ , then we can reformulate the optimization problem as follows

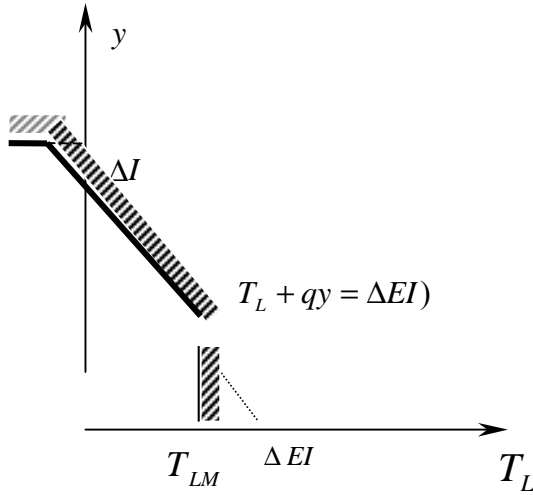
$$T_L + qy - de \rightarrow \max$$

under  $T_L \leq T_{LM}$ ,  $y \leq \Delta I$ ,  $T_L + qy \leq \Delta EI$ ,  $T_L + F \leq T_{LM} + \Delta I$ . Under any permissible  $T_L, y$ , the optimal  $e$  turns the latter relation into equity. Thus, we should maximize

$$T_L + qy - dyV_H / (T_{LM} - T_L + \Delta I)$$

at the area represented at Figure 3.

Figure 3.



The condition (\*\*\*) implies that  $R(T_L, y)$  increases in  $y$  under any permissible  $T_L$ . On the line  $T_L + qy = \Delta EI$ , the revenue is  $\Delta EI - de(y)$ , where the effort  $e(y) = V_H y / (\Delta I + T_{LM} - \Delta EI + qy)$  increases in  $y$  and reaches its maximum at  $T_L = T_{LM}$ ,  $y = \frac{(\Delta EI - T_{LM})}{q}$ . On the line  $y = \Delta I$ ,  $R_{T_L} \geq 0$  since  $(\Delta I + T_{LM} - T_L)^2 \geq q(\Delta I)^2 \geq d\Delta I V_H$ . Thus,  $R^2 = \Delta EI - dV_H (\Delta EI - T_{LM}) / (\Delta I q)$ .

Note that, irrespectively of penalty constraints, strategy  $s^3$  strictly dominates  $s^1$  iff  $t < \tilde{t}$ , that is,  $\Delta T < (p - c)\Delta V$ . Strategy  $s^1$  is never optimal for the government since the corresponding revenue  $R^1 < T_{LM}$ , and the latter value may be

obtained under  $s_G = (T_{LM}, t = 0, e = 0)$  and  $s^2$  without any expenses on audit. In order to find the optimal government strategy it suffices to compare  $T_{LM}$  with the maximal revenues in the areas II and III.

In the case a)  $F = (1 + \delta)\Delta T$ ,  $R^3 = \max\left(0, \Delta EI - (1 - q)\frac{dV_H}{1 + \delta}\right)$ ,  $R^2 > R^3$  iff

$$\frac{\Delta EI - T_{LM}}{\Delta I q} < \frac{1 - q}{1 + \delta} \text{ that is equivalent to } I_{alt} - I_{min} > q\Delta I(\delta + q)/(1 + \delta).$$

$$\text{In the case b) } F = \Delta T + \delta\Delta V, \quad R^3 = \max(T_L + q\Delta T - (1 - q)de) \quad (1.6)$$

under constraints

$$\frac{e}{V_H}(\Delta T + \delta\Delta V) \geq \Delta T \text{ where } F = \delta\Delta I,$$

$$T_L \leq T_{LM}, \quad T_L + q\Delta T \leq \Delta EI, \quad \Delta T \leq \Delta I. \quad (1.7)$$

Then  $e^* = V_H \frac{\Delta T}{\Delta T + \delta\Delta V}$ , and net revenue reaches its maximum either when

$T_L + q\Delta T \leq \Delta EI$ ,  $T_L = T_{LM}$ ,  $\Delta T = \Delta I - (I_{alt} - I_{min})/q < \Delta I$ , or at  $T_L = T_{LM}$ ,  $\Delta T = 0$ . In the former case  $R^2 > R^3$  iff

$$\frac{1}{\Delta I q} < \frac{(1 - q)}{\Delta EI - T_{LM} + \delta V} \Leftrightarrow q^2\Delta I - (I_{alt} - I_{min}) + \delta V < 0.$$

In the case c)  $F + T_L \leq I_H - \bar{I}_{min}$ , we aim to maximize the same value (1.6)

under constraints  $F + T_L \leq I_H - \bar{I}_{min}$  and  $\frac{e}{V_H}F \geq \Delta T$ . Thus,  $e^* = V_H \frac{\Delta T}{F}$  is the same,

$F^*(T_L) = I_H - \bar{I}_{min} - T_L$ . If  $\Delta T^* > 0$  then  $T_L^* + q\Delta T^* = \Delta EI$  and the optimal  $T_L^*$

minimizes the audit cost proportional to  $\frac{\Delta EI - T_L}{I_H - \bar{I}_{min} - T_L}$ . Since  $I_{alt} > I_{min} > \bar{I}_{min}$ ,

$\Delta EI = I_L + q\Delta I - I_{alt} < I_H - \bar{I}_{min}$ , and  $T_L^* = T_{LM}$ . The optimal additional tax either

$\Delta T^* = (\Delta EI - T_{LM})/q$  or  $\Delta T^* = 0$ . In the former case  $R^2 > R^3$  iff

$$\frac{1}{I_H - \bar{I}_{min} - T_{LM}} > \frac{1}{\Delta I} \Leftrightarrow I_{min} < \bar{I}_{min} \text{ that never happens.}$$

*Proof of Proposition 2.1.*

Consider the optimal strategy of a firm that realizes  $R(s_G)$ . If it is  $(V_L, V_L)$  or  $(V_H, V_H)$  then it suffices to set  $\Delta\bar{s} = F$ , and do not change other components of the strategy  $s_G$ . Then tax evasion is also unprofitable, because the effective payment of a firm under evasion increases. Thus, every agent acts as under  $s_G$ , and the revenue does not change.

If  $(V_H, V_L)$  is optimal then  $\bar{s}_G$  slightly differs for different penalty constraints. For  $F = (1 + \delta)\Delta T$  let  $\Delta\bar{s} = \gamma F + (1 - \gamma)\Delta s$ ,  $\Delta\bar{T} = \Delta s / (1 + \delta)$  (hence  $\Delta\bar{s} = \Delta\bar{F}$ ), other components staying the same. Since the effective payment  $\Delta\bar{s}$  of a firm with high capacity related to the tax and auditing effort  $e$  do not change,  $(V_H, V_L)$  stays the optimal strategy. Agents get the same incomes under any state of nature, and the net revenue does not change.

For the other types of penalty constraints the idea is similar: to keep the effective payment from a tax evader to an auditor at the same level. If  $F = \Delta T + k\Delta V$  then let  $\Delta\bar{s} = \gamma F + (1 - \gamma)\Delta s = \bar{F}$  and  $\Delta\bar{T} = \Delta T - (F - \bar{F})$ . If  $T_L + F \leq I_H - I_{\min}$  then the same change of the penalty does not require any adjustment of other components.

*Proof of Proposition 2.2.*

Proceeding from Proposition 2.1, it suffices to consider the case where there is no bribing, and  $\Delta s = F$ . Then every optimization problem for dishonest auditors differs from the corresponding problem for honest in additional constraints (2.1). Hence, the optimal result in the second case is never less than in the first case. Now let us prove the inverse inequality. Proceeding from Propositions 1.3.-1.5, we should consider two possibilities.

The optimal revenue corresponds to the honest behavior of taxpayers. Then  $R = T_L + q\Delta T - (1 - q)de$ , and it suffices to keep the same first three components of the

strategy and to set  $s = s_{alt}$ ,  $\Delta s = F$  in order to obtain the same result in the model with corruption.

2) The optimal revenue corresponds to the strategy  $(V_H, V_L)$ . Then  $R = T_L + \left(\frac{qF}{V_H} - s_{alt} - \bar{d}\right)e$ . Let us keep the same  $T_L$ ,  $F$  and  $e$ , and set  $s = s_{alt} - qF$ ,  $\Delta s = F$ . Then an average income of an inspector is  $qF + s = s_{alt}$ , so the strategy meets all constraints and provides the same revenue.