## Tax Enforcement for Heterogeneous Firms ${ }^{1}$


#### Abstract

The paper considers a model of taxation of firms with lump-sum and sales taxes under informational asymmetry between the government and agents. The government knows the production capacities of every firm but does not know its costs. The cost distribution is given for every type of production capacities. The purpose is to characterize the optimal tax structure, tax rates and auditing strategy that maximize net tax revenue under the given participation constraints and penalty for tax evasion.

The first model considers the case where every firm has the same production capacity and constant marginal cost. We determine the optimal auditing strategy under exogenously given tax rates. This strategy turns out to be a "cut-off" rule with respect to registered production volume. We also show that a flexible lump-sum tax dependent on the market price is optimal under this setting of the problem.

In the general model a firm controls several units with different production costs. We show that, whenever the agents can adjust the property structure to the tax system, sales tax is unnecessary, and the optimal lump-sum tax is proportional to the production capacity of the firm.


## 1. Introduction.

In the short-term prospect, Russian economy meets the following dilemma. On the one hand, there are important reasons (huge foreign debt and hard social problems) to increase budget expenses. On the other hand, the general opinion is that the tax rates should be reduced because it is impossible to work honestly under the present ones. A possibility to solve this dilemma relates to the fact that a large part of the economy does not pay taxes now. Thus, a simultaneous optimization of the tax system and of the mechanisms to enforce it may essentially increase the tax revenue.

Because of the widespread tax evasion and inefficient work of the tax inspection (see the interview of I. Hakamada in "Delovie lyudi", N2, 1998, and Yakovlev, 2000, on this issue) lump-sum taxation seems to be an attractive variant for Russia. In pracice a lump-sum tax is a tax on some observable and relatively constant input. This might be the total square of

[^0]the shop, the number of oil towers or the square of the oil field used by the company etc. Let us note that criminal groups often use this form of taxation for enterprises under their control, so many Russian firms are used to it.

This paper considers several models in order to find the optimal lump-sum tax depending on some observable inputs (in particular, production capacities) used by the producers. We also examine whether any combination of lump-sum and sales taxes may increase net tax revenue.

We consider an economy with heterogeneous firms under informational asymmetry between the government and the agents. The government knows the production capacities of every firm but does not know its costs. The cost distribution is given for every type of production capacities. Each firm chooses the total production volume and the registered amount of production. The rest is sold at the informal market for cash.

The government sets a lump-sum tax and sales tax rates. Without any audit by the government, a firm has an incentive to sale unregistered production. In order to prevent tax evasion the government organizes tax inspection and penalizes detected the tax evasion.

Our purpose is to study the above tax optimization problem in a principal-agent framework. That is, to find the strategy of the government that maximizes net tax revenue under optimal behavior of the agents and some participation constraints. Section 3 studies a basic model where every firm has a unit of production capacity and a fixed cost. The latter value is unknown to the government. The difference in costs can be connected with local conditions and/or qualification of the managers or owners of the firm. In contrast to the usual setting of the tax enforcement problem (see Cowell and Gordon, Sanches and Sobel, Chander and Wilde et al.), we do not consider unsystematic or firm specific risks and permit to exclude from the economy the firms that cannot pay taxes. In Section 3, we determine the optimal auditing strategy under some exogenously given tax rates. This strategy turns out to be a "cutoff" rule with respect to the registered production volume. Actually, this auditing strategy determines the optimal level of exclusion for firms with high production costs.

Section 4 studies the tax optimization problem and shows that the optimal lump-sum tax permits to maximize the net tax revenue without any need for a sales tax. Section 5 provides a generalization of this result for firms with heterogeneous production capacities. Section 6 discusses some implementation problems, in particular, related to firm-specific risks.

## 2. Survey of the Literature.

Papers on tax enforcement typically consider direct taxes. Reinganum and Wilde (1985), Border and Sobel (1987), Sanches and Sobel (1993), Chander and Wilde (1992), Vasin and Panova (1998) study the problem under fixed taxes and fines for evasion.

Chander and Wilde (1998) consider a more general problem of income tax enforcement. They introduce the notion of an efficient scheme, which consists of a tax, penalty and auditing probability functions ( $t, f, p$ ) such that any other scheme does not allow to increase the expected payment of any taxpayer without increasing probabilities of auditing for some reported income. For different objectives of the tax authority the optimal scheme must be efficient. The problem of the tax authority is to design the optimal mechanism, where a mechanism is a scheme $(t, f, p)$. The revelation principle holds so that it is possible to restrict the attention to incentive compatible direct revelation schemes. They find that, for any efficient scheme, the payment function must be non-decreasing and concave. In addition, the tax function is non-decreasing with non-increasing average tax rate. These properties imply that there is no redistribution among the taxpayers. The audit probabilities are determined completely by the marginal payments rates and are non-increasing. Regressivity implies that the inability of the government to costlessly observe true incomes severely restricts its policies to redistribute through direct taxation. The regressivity result is known from another considerations which take into account the supply side effect (Mirrlees (1971) and others).

Mookherjee and Png $(1989,1990)$ study the tax enforcement problem in a setting close to Contract Theory. They consider the moral hazard problem and permit arbitrary tax and penalty schedules that meet the participation constraints. Their results show that such an approach has some disadvantages: the optimal penalty schedule they find is to fine a tax evader with the whole income irrespective of the amount of the concealed income. Obviously, such a rule cannot be realized in practice.

We are aware of one model of tax enforcement optimization for indirect taxes by Cowell and Gordon (1995). They compare different audit strategies available to a tax authority attempting to collect indirect taxes. The authors model tax evasion as follows: taxpayers choose between taxable activities on the regular market and unreported activities on an informal market. If an individual is audited and found to be undertaking irregular activities she/he is fined and made to repay the evaded tax. One possible strategy is to audit randomly. The probability that any taxpayer is investigated is fixed. An alternative policy is to take into account what the authority knows about each taxpayer. Cowell and Gordon study a simple form of this approach where the authority conditions the probability of auditing on the reported turnover via a cut-off rule: those reporting less (no less) than a certain amount are always (never) audited. Cowell and Gordon establish conditions under which the optimal random auditing is better than the optimal cut-off rule, and vice versa. However, as $D$.

Siniscalco notes in his discussion of the Cowell and Gordon model, the optimal audit strategy in general does not belong to any of the specified classes.

However, the direct application of the obtained results to the corporate taxation seems to be unreasonable. First, the sales tax is linear by its nature. Second and most important, none of the mentioned papers permits the government to set taxes so that some taxpayers cannot pay them. More precisely, the literature discussed above assumes that every taxpayer should have a non-negative after tax income under any state of Nature and any behavior (or at least under optimal behavior, as in the second model in Chander, Wilde, 1996). Under this setting it does not matter if all taxpayers have the same random income distribution or differ in this distribution. Below, we assume that firms differ in their production costs. In contrast to the mentioned papers, taxpayers leave the economy if their optimal behavior, under a given government strategy, leads to a negative after tax profit. Thus, a government strategy determines the level of production activity and changes the distribution of costs for operating firms. We show how to find the optimal level and the corresponding tax policy proceeding from the technological characteristics of the economy.

## 3. Basic Model and the Optimal Auditing Rule.

We consider an industry where each firm has the same production capacity $\bar{V}=1$. Firms differ in their marginal production cost $c$ that does not depend on the output $V$. This cost determines the type of each firm and is its private information. The distribution $G(c)$ of production costs of the economy is known to the government. Each firm chooses an output $V \leq \bar{V}$ and a volume of registered sales $V_{r} \leq V$. The amount $V_{u}=V-V_{r}$ is sold at an informal market. By selling in the informal market, the firm is able to evade tax. The market is competitive, and the price $p$ in both sectors is the same and exogenous.

The government spends an effort $e\left(V_{r}\right) \leq 1$ on auditing a firm with registered output $V_{r}$. The cost of each audit is $d e$. An audit detects some random volume of unregistered production with the mean $V_{d u}=H\left(V, V_{u}, e\right)$. This function meets the following conditions: 1) $H\left(V, V_{u}, e\right)$ is concave in $V_{u}$ under a fixed $V_{r}, H\left(1, V_{u}, e\right)=e V_{u}$. As a particular case, we consider $V_{d u}=V_{u} \min \left\{1, \frac{e\left(V_{r}\right)}{V}\right\}$. Here, $e\left(V_{r}\right)$ means the checked production volume. This section studies the tax enforcement problem under an exogenous sales and a lump-sum tax rates, $t$ and $T$, respectively, so the total tax is $T+t p V_{r}$. First, consider the case where $T=0$, so the tax $T\left(V_{r}\right)=t p V_{r}$. The fine for evasion from the sales tax is proportional to the
detected underpayment: $f\left(V_{d u}\right)=(1+\delta) t p V_{d u}$. The strategy of the government is the auditing rule $e\left(V_{r}\right)$.

## The firm's problem

Firms are risk neutral and maximize expected profit:

$$
\begin{equation*}
\left(V^{c}, V_{r}^{c}\right) \rightarrow \max _{V \in[0,1], V_{r} \in[0, V]}\left\{V(p-c)-t p V_{r}-(1+\delta) t p H\left(V, V_{u}, e\left(V_{r}\right)\right)\right\} \tag{1}
\end{equation*}
$$

under a given government strategy. The alternative net profit of a firm is $I_{\text {alt }}=0$. Thus, it is unnecessary to consider participation constraints in this case, because every firm can choose $V=0$ if it is unprofitable to produce under the current government strategy.

## The government's problem

The aim of the government is to maximize net tax revenue:

$$
R(e(.))=\int\left[t p V_{r}^{c}+(1+\delta) t p H\left(V^{c}, V_{u}^{c}, e\left(V_{r}^{c}\right)\right)-d e\left(V_{r}^{c}\right)\right] d G(c)
$$

under the constraint (1).
Proposition 1. For any $c$ and $e($.$) , the optimal production volume V^{c}(e()$.$) is either 0$ or 1 . And for each firm such that $V^{c}(e())=$.1 , the optimal registered volume $V_{r}^{c}(e())=.V_{r}(e()$. does not depend on the type $c$ of the firm.

Proof of Proposition 1. Since, the function $H$ is concave in $V_{u}$, under a fixed $V_{r}$, the income (1) is convex in $V_{u}$ and reaches its maximum either at $V_{u}=0$ or at $V_{u}=1-V_{r}$. Under $V_{u}=0$, the maximal income is $\max _{V_{r}} V_{r}(p-c-t p)$. Under $V_{u}=1$, the maximal income is $p-c-\min _{V_{r}}\left(t p V_{r}+e\left(V_{r}\right) k\left(1-V_{r}\right)\right)$. Note that the latter value is never less than the former one if $p-c-t p>0$. Thus, the optimal volume $V_{r}$ in the second case is

$$
\begin{equation*}
V_{r}(e(.)) \rightarrow \min _{V_{r}}\left(t p V_{r}+e\left(V_{r}\right) k\left(1-V_{r}\right)\right) \tag{2}
\end{equation*}
$$

and does not depend on $c$. The minimized value on the right side is the effective tax paid to the budget under a given audit rule. Let $T(e()$.$) denote this minimal value. Then, for any firm$ such that $p-c<T(e()$.$) , the optimal strategy is V^{*}\left(e()=.V_{r}^{*}(e())=\right.$.0 , and for any firm such that $p-c>T(e()),. V^{*}\left(e()=1,. V_{r}^{*}(e(\right.$.$) is a solution of (2).$

Thus, under a given auditing rule the set of firms splits into two parts: firms in the first part choose the same output $V=1$ and the same registered volume $V_{r}$; whereas, the rest of the firms do not produce, that is, set $V=V_{r}=0$.

Moreover, the proof shows that every firm with output $V=1$ minimizes the effective tax paid to the budget:

$$
\begin{equation*}
V_{r}(e(.)) \rightarrow \min _{V_{r}}\left(V_{r}+(1+\delta) H\left(1,1-V_{r}, e\left(V_{r}\right)\right)\right. \tag{3}
\end{equation*}
$$

Let $T(e()$.$) denote this minimum value. Then V^{c}(e())=$.0 iff $p-c<T(e()$.$) .$ Otherwise, $V^{c}(e())=$.1 . (We assume that, under equality, firms prefer to produce).

Proceeding from these results, we can express the net tax revenue under optimization as follows:
$R(e())=.T(e()) G.(p-T(e()))-.d V_{r}(e()) G.(p-T(e())-.d e(0)(1-G(p-T(e()).$.
Now, let us characterize the optimal auditing rule. The next proposition describes the optimal rule under a given effective tax $T(e())=.\tau \leq t p$.

Proposition 2. Consider auditing rules $e($.$) such that T(e())=.\tau$. Then the optimal rule that minimizes auditing costs is

$$
\begin{equation*}
e_{T}^{*}\left(V_{r}\right)=\max \left\{\frac{\tau-p t V_{r}}{k\left(1-V_{r}\right)}, 0\right\} \tag{4}
\end{equation*}
$$

where $k=(1+\delta) t p$.
This rule means that any firm that voluntarily pays the effective tax, or some greater value, is not audited. Any other firm is audited with an effort that makes tax evasion unprofitable. For any firm with zero registered production this effort should be minimal.

Under this strategy, the net revenue is the following function of the effective tax:

$$
R(\tau)=\tau G(p-\tau)-d \frac{\tau}{k}(1-G(p-\tau))
$$

Proof of Proposition 2. Consider and audit rule $e($.$) such that T(e())=.\tau$. We search for the rule with the minimal audit cost, that is the auditing that is a solution to the problem

$$
\min _{e(.)} d e\left(V_{r}(e(.)) G(p-\tau)+d e(0)(1-G(p-\tau))\right.
$$

under the condition $T(e())=.\tau$.
Consider two cases. If $V_{r}(e) \neq 0$, then $V_{r}=0$ is not better for a taxpayer under the given auditing rule. Hence, $e(0) k \geq t p V_{r}(e)+e\left(V_{r}(e)\right) k\left(1-V_{r}(e)\right) \geq \tau$, so $e(0) \geq \frac{\tau}{k}$. If $V_{r}(e)=0$, then $e(0)=\frac{\tau}{k}$. Since, in the both cases, $e(0) \geq \frac{\tau}{k}$ and $e\left(V_{r}(e)\right) \geq 0$, the rule specified in Proposition 2 is optimal.

Proposition 3. The optimal auditing rule is the cut-off rule determined by (3) for $\tau^{*}$ that is a solution of the problem

$$
\begin{equation*}
\tau^{*} \rightarrow \max _{\tau \geq 0} R(\tau) \tag{5}
\end{equation*}
$$

Now, consider a similar problem with a positive lump-sum tax rate $T$. The optimal strategy of any participating firm does not depend on this value and is given by Proposition 1. However, under a given effective sales tax $\tau=T(e()$.$) and zero alternative net profit, it is$ unprofitable to participate if $c>p-\tau-T$. Consider the following timing of interaction.

1. The price $p$ is exogenously given.
2. The government sets the tax rates $t$ and $T$ and the audit rule $e\left(V_{r}\right)$.
3. Each firm decides whether to register.
4. Every registered firm pays the lump-sum tax $T$.
5. Every registered firm chooses its strategy $\left(V^{c}, V_{r}^{c}\right)$ and pays the sales tax $t p V_{r}^{c}$.
6. The government audits firms according to the rule $e\left(V_{r}\right)$ and collects fines for evasion.
Under these conditions any firm can estimate its expected net profit at the stage 3. Assume that unregistered firms cannot produce. Then, firms with high costs $c>p-\tau-T$ do not register. In contrast to the previous case, there are no expenses related to their audit. The net revenue under a given effective sales tax $\tau$ is $R=\sum_{i} M_{i} T_{i}^{*} G_{i}\left(p-T_{i}^{*}\right)$.

Proposition 4. Under some given tax rates $T>0$ and $t \in[0,1]$, the maximum net revenue is

$$
\begin{equation*}
\max _{\tau \leq t p}(T+\tau) G(p-T-\tau) \tag{6}
\end{equation*}
$$

the optimal auditing rule is determined by Proposition 2, for the corresponding effective sales $\operatorname{tax} \tau^{*}$.

The one-dimensional optimization problems (4), (5) can be easily solved for typical probabilistic distributions such as lognormal.

## 4. Tax optimization problem.

We assumed above that the tax rates $t, T$ were given exogenously. Now, let us find the optimal rates that maximize net tax revenue.
Proposition 5. The maximal net tax revenue is

$$
\max _{T \in[0, p]} T G(p-T)
$$

Let $T^{*}(p)$ denote the optimal effective total tax. Then, any rates $T>0, t \in[0,1]$ such that $T+t p \geq T^{*}(p)$ are optimal if the audit rule $e($.$) is determined by Proposition 2,$ for the effective sales tax $\tau=T^{*}(p)-T$.

Thus, there are many optimal strategies for the government. Under any of them, the behavior of the agents is as follows: firms with high costs $c>p-T^{*}(p)$ do not register and
take part in production, whereas the rest pay the lump-sum tax $T$ and the effective sales tax $\tau$ that saves them from audit.

Note that this setting of the problem does not take into account the expenses of tax collection, accounting and audit organization. The only variant where such costs are unnecessary is to set $T=T^{*}, t=0$.

Before we continue the discussion about implementation problems, consider a more general model.

## 5. General model.

Let there exist several types of production capacities $i \in I$. Every type $i$ is characterized by the unit cost distribution $G^{i}(c)$. It determines the probability that the cost of one unit of the good does not exceed $c$ for this type. The functions are known to the government. Every firm is characterized by its structure $x^{a}=\left(x_{i}^{a}, i \in I\right)$, where $x_{i}^{a}$ is the production capacity of the type $i$. A firm knows precisely the cost of production $c_{i}^{a}(V), V \leq x_{i}^{a}$, for every type.

We assume that the government knows the structure of production capacities of every firm, but does not know its cost functions. A strategy for the government includes a sales tax rate $t$, a lump-sum tax $T\left(x^{a}\right)$ dependent on the structure $x^{a}$, and an audit rule $e\left(V_{r}^{a}, x^{a}\right)$ which determines the effort of inspection dependent on the structure and the registered production of a firm. The firm's problem is similar to (1):

$$
\begin{equation*}
\left(V^{a}, V_{r}^{a}\right) \rightarrow \max _{V \leq \sum x_{i}^{a}, V_{r} \in[0, V]}\left\{V p-C^{a}(V)-t p V_{r}-(1+\delta) t p H\left(V, V_{u}, e\left(V_{r}, x^{a}\right)\right)\right\} \tag{1’}
\end{equation*}
$$

The only difference is that the cost of production $C^{a}(V)$ is non-linear.
The following statement generalizes Proposition 1 for the present set-up.
Proposition 6. Under a given government strategy, the solution to the problem ( $1^{\prime}$ ) meets the following conditions:

1) any firm uses the most efficient productive units of every type;
2) if $C_{i}^{a}\left(V_{i}^{a}\right)$ is the marginal production cost for capacity of the type $i$ used by firm $a$ then all units with this cost are completely employed by this firm;
3 ) under a given total volume, the optimal registered volume does not depend on the production costs of the firm and is a solution of the problem

$$
\begin{equation*}
\left(V_{r}^{a}\right) \rightarrow \min _{V_{r} \in[0, V]}\left\{V_{r}+(1+\delta) H\left(V^{a}, V^{a}-V_{r}, e\left(V_{r}, x^{a}\right)\right)\right\} \tag{2'}
\end{equation*}
$$

Let us note that, in general, firms with the same structure $x$ may have different optimal volumes.

The government seeks to maximize net tax revenue. There are several possible settings of the government problem depending on whether the firms have the possibility of changing their structures according to the government strategy. The first variant is where the property distribution is fixed. Then, timing is the same as in the previous section. At the stage 3 , every firm solves the problem ( $1^{\prime}$ ) under a given price and government strategy. The right side of ( $1^{\prime}$ ) is the maximal profit of the firm after sales tax. The firm participates if the lumpsum tax does not exceed this value.

However, it is often reasonable to assume that the agents adjust their property structure to the tax system. The second variant is where every firm can exclude some (less efficient) productive units from its structure at the stage 3 . Consider the case where the government uses only a lump-sum tax that is linear with respect to the capacity structure:

$$
T(x)=\sum_{i} T_{i} x_{i} .
$$

Let us describe the optimal government strategy in this context. For every type $i$ of capacities, let $T_{i}^{*}(p)$ denote the optimal effective tax per unit for capacities of this type. This value is a solution of the problem

$$
T_{i}^{*} \rightarrow \max _{T \in[0, p]} T G^{i}(p-T)
$$

Proposition 7. The optimal linear lump-sum tax in this variant is $T\left(x^{a}\right)=\sum_{i} T_{i}^{*} x_{i}^{a}$. Under this strategy, the net tax revenue is

$$
\begin{equation*}
R=\sum_{i} M_{i} T_{i}^{*} G_{i}\left(p-T_{i}^{*}\right), \tag{7}
\end{equation*}
$$

where $M_{i}$ is the total production capacity of type $i$.
Proposition 5 shows that $R$ is the maximal net revenue under the whole set of government strategies if every firm owns the same capacity with a constant marginal cost. The question is if whether this holds true for the general case.

Consider first an economy with one type of capacities. Then, the structure of any firm is given by the number of productive units. The following examples show that the sales tax may increase net tax revenue and the optimal lump-sum tax may be non-linear under nonlinear cost functions.

Example 1. Assume that there are $M_{1}$ productive units with the cost $c_{1}=p / 2$ and $M_{2}$ units with the $\operatorname{cost} c_{2}=p$ in the economy. Every firm owns two units. Under the random distribution in pairs, the ratio of the numbers of firms with the structures $\left(c_{1}, c_{1}\right)$ and $\left(c_{1}, c_{2}\right)$ is $M_{1}: 2 M_{2}$. If $M_{1}<2 M_{2}$. If the firms cannot exclude inefficient units, then the optimal
lump-sum tax is $T(2)=p / 2\left(\right.$ since $\left.\left(M_{1}+2 M_{2}\right) p / 2>M_{1} p\right)$. The revenue in this case is $\widehat{R}=\left(M_{1}+2 M_{2}\right) M_{1} /\left(M_{1}+M_{2}\right) p / 4$. If the firms could exclude inefficient units, then the optimal tax would be $T(n)=n p / 2$ and the revenue $R^{*}=M_{1} p / 2>\hat{R}$. The same result is available with the sales tax rate $t=p / 2$ and $T=0$ if audit is costless. Thus, sales tax may increase the revenue if the firms cannot change their structures.

Example 2. Now assume that there are $M_{1}$ units with the $\operatorname{cost} c_{1}=p / 2$ and $M_{2}$ units with the cost $c_{2}=0, M_{2}<M_{1}<2 M_{2}$. Matching in pairs is not random: $M_{2}$ firms own two different units and the rest $\left(M_{1}-M_{2}\right) / 2$ firms have the costs $(p / 2, p / 2)$. Then, the optimal linear lump-sum tax rate is $T(2)=p / 2$ if the firms can exclude inefficient units (since $\left.R^{*}=\left(M_{1}+M_{2}\right) p / 2>p M_{2}\right)$. But if they cannot exclude those inefficient units, then the optimal lump-sum tax is $T(2)=3 p / 2$, since $\widehat{R}=M_{2} 3 p / 2>R^{*}$. If, in addition, there is a large number of firms with one productive unit with the cost $p / 2$, then the optimal lumpsum tax is non-linear: $T(1)=p / 2, T(2)=3 p / 2$.

In the long-run prospect, we may expect that agents will adjust the property structure to the tax system in order to maximize their total after tax profit. Such adjustment may include splits and mergers and exclusion of the inefficient productive units. Let us find out if the linear lump-sum tax, determined by Proposition 7, is optimal under this wider set of firms' strategies.

In general, this is not true. Moreover, the government can get a maximal net revenue equal to the maximum profit in the economy if it may enforce the merger of all firms in one monopoly (by setting $T(n)=\infty$ for any $n<M$ ). In this case, the optimal tax is $\sum_{i} \max _{V_{I} \leq M_{i}}\left(p V_{i}-C_{i}\left(V_{i}\right)\right)$, where $\quad C_{i}(V)=M_{i} \int_{0}^{G_{i}^{-1}\left(V / M_{i}\right)} c d G_{i}(c)$ and $M_{i}$ is the total capacity of this type.

Such a global merger eliminates the informational asymmetry between the government and the agents. However, shortcomings of the monopolistic structure of the economy are well known. So let us bound our examination to the case where the government seeks to preserve the competitive market and permits only micro changes in the property distribution. We assume formally that $T(n)=\infty$, for any $n>\hat{n}$ where $M / \hat{n}$, is very large. Moreover, if there exists a firm with parameters $\left(x, C_{i}(),. i \in I\right)$, then there are many firms with the same parameters.

Theorem 1. Under the above assumptions, the optimal lump-sum tax is the linear tax determined according to Proposition 7. The optimal sales tax rate is 0 , so no audit is necessary to get the optimal revenue.

Proof. Consider any government strategy $s_{G}=\left(T(x), t, e\left(V_{r}, x\right)\right)$. For any firm with structure $x$ and cost functions $C_{i}\left(V_{i}\right)$, let $\widehat{V}_{r}^{*}$ denote a solution of the problem (2') and let $t(V, x)$ be the corresponding effective sales tax. Then, the optimal production volumes $\left.\left(V_{i}, i \in I\right)\left(x, C_{i}().\right), i \in I\right)$ are a solution of the problem

$$
\left(V_{i}, i \in I\right)=\operatorname{Arg} \max \left\{\sum_{i}\left(p V_{i}-C_{i}\left(V_{i}\right)\right)-t\left(\sum_{i} V_{i}, x\right)\right\}
$$

Now, assume that the firms can change their property distribution. Then, for any $i$ there exists $c_{i}^{*}$ such that all units of the type $i$ with marginal costs $c<c_{i}^{*}$ are employed and all units with costs $c>c_{i}^{*}$ are either excluded or do not produce. Let us prove that, under the optimal property distribution,

$$
\begin{equation*}
p V(x, C(.))-\sum_{i} V_{i}(x, C(.)) c_{i}^{*} \geq T(x)+t(V(x, C(.)), x) \tag{8}
\end{equation*}
$$

for any existing property structure $x$. From the contrary, let there exist a structure $x$ that does not meet (8). Consider the following redistribution of property: let the agents collect all units with the costs $c_{i}=c_{i}^{*}$ in the firms with the structure $x$ until all such structures are completed or all such units of some type are in these firms. Since the total cost of production does not change, the total profit of agents also does not change after such redistribution. Next, let the agents exclude all the firms completed by the units with the costs $c_{i}^{*}$. Then, the total profit will increase, since the inequality (8) does not hold. This contradicts the optimality of the original distribution.

Thus, for every existing structure, the effective tax meets condition (8). Summing all existing structures, we obtain that

$$
\begin{equation*}
R=\sum_{x, C(.)} N(x, C(.))\{T(x)+t(V(x, C(.)), x)\} \leq p \sum_{i} V_{i}-\sum_{i} V_{i} \widehat{c}_{i}\left(V_{i}\right) \tag{9}
\end{equation*}
$$

where $N(x, C()$.$) is the number of firms with these parameters and V_{i}$ is the total production volume for capacities of the type $i, \widehat{c}_{i}\left(V_{i}\right)=G_{i}^{-1}\left(V_{i} / M_{i}\right)$.

Finally, if the government sets the linear lump-sum tax

$$
T(x)=\sum_{i} x_{i} \bar{c}_{i}\left(V_{i}\right)
$$

then, under optimal behavior of agents and irrespective of the property structure, the revenue
is equal to the right side of (9). Optimization by $c_{i}^{*}$ gives the maximal tax revenue equal to (7) and completes the proof.

Note. The proof shows that a possibility to introduce a non-linear sales tax does not violate our conclusion that such a tax is unnecessary for tax revenue maximization.

## 6.Discussion.

According to the known Welfare theorem with tax evasion (see Atkinson, Stiglitz), if the government has a complete information on the cost function of every firm, it is optimal to impose a type-specific lump sum tax, and not to organize any audit. Theorem 1 above shows that a lump-sum tax is also optimal under informational asymmetry between the government and the agents, if the latter can adjust their property structure to the tax enforcement strategy of the government. Moreover, the optimal lump-sum tax in this case is linear with respect to the structure of production capacities.

Another advantage of the lump-sum taxation is that it facilitates the redistribution of the property to the more efficient owners. Note that in our model, a productive unit may be profitless and excluded from the production process for two different reasons: 1) some local conditions increase the cost of production; 2) poor management. In the latter case, the more efficient owner will by out and employ this unit. In the former case, the unit stays out of work for a long time. This is a signal for the government to differentiate the tax and reduce the rate for this unit.

The optimal tax in our model may be interpreted as follows. The government plays the role of a monopolist who owns several inputs. At least, one of them is necessary for the production of the good. The government sets the monopoly prices on these inputs and, thus, maximizes the tax revenue. In general, optimization of total welfare does not correspond to the maximal tax revenue. In this case, the optimal tax rates are equivalent to the lesser prices on the inputs.

This scheme of taxation is distortionary in the long-run prospect since the agents have an incentive to look for substitutes for the taxed inputs. However, in practice any taxation of firms is distortionary. For instance, profit tax requires an equivalent wage tax and induces tax evasion activity that may have strong distorting effects (see Jung et al.) .

Implementation of the considered lump-sum tax meets several difficulties. In practice, a firm determines its structure under different uncertain factors. In particular, the price of the good typically changes rather frequently, and the firm cannot adjust its property structure to these variations. On the other hand, for many goods this price is known at the time of production. In this case, the considered taxation scheme may be
implemented if the government can account the employed capacities of every type at any period and sets the tax depending on these values.

The agents also face risk factors that are unknown at the time of production. If the variance of the profits due to these factors is large enough then some share of potentially good firms may be unable to pay the tax and become tax bankrupts. One way to reduce this share is to permit the coverage of the tax debt with the property of the firm. The firm manages this property and can by it out until the share of the government exceeds some threshold. It is possible to regulate the intensity of tax bankruptcy by the choice of this value.

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