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ANALYSIS OF SECURITY MARKET CRISIS IN TERMS OF STRATEGIES OF THE MAJOR PLAYERS

Working paper # BSP/99/023

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В данной работе представлены три теоретико-игровые модели, которые описывают отдельные аспекты рынка ценных бумаг. Две модели рассматривают статические игры между двумя репрезентативными инвесторами: резидентами и нерезидентами. Первая из этих моделей преобразует ожидания инвесторов в равновесное рыночное значение доходности ценных бумаг. В другой обсуждаются мотивы посылки сигналов в игре, в которой инвестор-нерезидент ведет себя как лидер. В третьей модели рассматривается бесконечно повторяющаяся игра между инвесторами с одной стороны и правительством и Центральным Банком с другой. С использованием результатов первой модели, находится равновесная траектория заимствования посредством выпуска государственных ценных бумаг во времени. Показано, что изменения ожиданий инвесторов приводят к избыточным изменениям в траектории заимствования, что может повлечь кризис на этом рынке. С другой стороны адекватная информационная политика государства может смягчить последствия изменения ожиданий инвесторов на объемы заимствований и помочь избежать кризиса.

Andryakov A.D. Analysis of Security Market Crisis in Terms of Strategies of the Major Players. / Working Paper # BSP/99/023 E. - Moscow, New Economic School, 1999. - 52 p. (Engl.)

This paper considers three game models, which describe different aspects of government security market behavior. Two models are static games between representative resident and non-resident investors. The first of them translates the players' expectations into an equilibrium yield of the security. The other discusses signaling motives in the game between these players on the secondary market with non-resident investor acting as a leader. The third model is a dynamic model of an infinitely repeated game between investors on one side and the monetary authorities on the other side. This model takes the results of the first one and gives an equilibrium time trajectory of the security supply by the Government. It is shown that shocks to investors' expectations produce overshooting effect on the trajectory of borrowing and can cause a crisis. It is also shown that appropriate information policy of the government can soften the overshooting effect and can help in avoiding the crisis.

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CONTENTS

1. INTRODUCTION

One of the main features of Russian financial crisis of 1998 is its comprehensiveness. Present study was stimulated by the crisis in one segment of the financial market, namely in the market of the Russian government security papers — GKO/OFZ. There are different approaches to the analysis of the financial markets performance. System analysis and simulating behaviour of the market major participants are among them. In present study we will try to look into the crises of security markets from another point of view, namely from the point of view of strategies of the major players: the Government, the Central Bank, resident and non-resident investors. In this approach, evolution of the security market can be studied in terms of a number of games among the market participants.

The history of the Russian government security market includes such events as waves of increasing yield of the securities well before the crises, which can be connected to speculative attacks of investors, financial crises in another markets, fall of world oil prices etc. In year before the crisis and especially during its last half the investors' expectations of default were steadily rising which caused increasing withdrawal of investors from the market and finally the government defaulted (de facto) on its securities, GKO/OFZ. In the present study different aspects of these issues will be addressed in three different game models. Though the primary interest of this study was the Russian security market crisis, modelling a security market behaviour in terms of games between the major players has nothing particular to whatever country and therefore the models developed in present study are applicable to any security market.

The paper consists of 6 major parts. The first is the introduction. The second contains review of literature on modelling a financial market behaviour. The third part is devoted to a static game with incomplete information between two players: resident and non-resident representative investors. In the forth one signalling motives are studied in the game between these two players, with the non-resident investor acting as a leader. The fifth part contains a dynamic model of infinitely repeated game between three players: resident and non-resident investors and the monetary authority. Finally, the main results are summarised in the conclusion.

2. REVIEW OF LITERATURE

A vast literature on financial crises contains a bunch of papers devoted to understanding financial crises from a game-theoretic point of view. Some papers representing different approaches in this activity will be discussed here.

One subset of this bunch of the literature deals with models of financial fragility of the economy. A good example of this kind of literature is a paper of R. Lagunoff and S. Schreft (1997). The economy in the model is represented as chains (closed or opened) of investors and entrepreneurs (projects). Until external shocks have hit the economy causing some projects to fail and their entrepreneurs to default on corresponding loans, the economy is stable and all the chains are closed. Shocks lead to break of some linkage and to losses of some investors. The losses can force the investors to withdraw their money from other projects, thus triggering breaks of other chains not directly hit by the shock. This propagation process continues until all the chains hit by the shocks cease to exists and the economy reaches new steady state with less financial links and fewer project in operation. A financial crisis in this model is defined as breakage of financial links in the economy. The more links are broken the more severe financial crises is. This approach does not seem directly applicable for the purpose of describing security market crises. Following this approach security market can be thought as consisting of only one entrepreneur – government and many investors. The key feature of the R. Lagunoff and S. Schreft (1997) model, spread of default wave (contagion), is not relevant in the case of security market crisis and therefore actual-default crisis can not develop.

Another subset of literature discusses contagion in more details in context of evolutionary games. The examples of this bunch of the literature are the papers of S. Morris (1997a,b). The model discussed in Morris (1997a) deals with infinite population of players each of which strategically interacts with a finite subset of other players. Important feature of such models is a form of connections between players. All theses models discuss local interaction of the players, in the sense that each player can interact with other players only within some neighbourhood. Usually the structure of this local interactions is taken to be exogenousely given.. The sets of possible actions of the players are finite and identical for each player. So each player chooses his best strategic response to actions of the other players. The main question studied in local interaction models is under what condition the same strategic behaviour can spread over the whole system (i.e. contagion occurs). The models of this kind constitute completely different approach to understanding financial crises and apart from some technique of calculating best response in dynamic games are not very much interesting in the present study.

Before turning to the models of financial crises driven by non-fundamentals let's give a look at one more model of the crises caused by fundamentals. F. Allen and D. Gale (1996) built a model where financial crisis in form of banking panic occurs from a natural outgrowth of the business cycle. The economy in this model consists of a number of identical investors and perfectly competitive banking industry, which in fact can be easily described by a single representative bank with appropriate features. There are two types of investments: into risky assets and into safe one. Only bank can invest into risky assets, so investors become in fact bank depositors As soon as investors learn about future problems with risky assets they run to bank in attempt to get back their deposits. Since risky assets are illiquid, this massive withdrawal leads to a collapse of the bank and to financial crises. In principal, bank can defend itself against this bank run by holding large reserves. But what is shown in this paper is that there exist equilibrium with possibility of bank run, or, in other words, in some cases bank finds it optimal to face a threat of bank run. In this equilibrium optimal risk sharing among investors and bank and efficient resources allocation take place.

Now we turn to two bunches of literature which generate financial crises from non-fundamentals. One of them discusses the model where crises are generated by herd behaviour of agents. The example of such models can be found in the paper of Chari and Kehoe (1996) which is based on the herd behaviour model of Banerjee (1992). The other bunch of literature deals with the crises driven by self-fulfilling prophesies. We will discuss them briefly on an example of the model given in the paper of Cole and Kehoe (1996).

The model built in Chari and Kehoe (1996) is worthy of a special attention from the point of view of the present study since it has many features which can be useful. This model contains a number of external lenders and a government. The government can be of two types: efficient and inefficient, and the type is its private information. The government is in need of funs for its internal project and can rise these funds only by borrowing them from the external lenders. There is a threshold in amount of funds below which the government project is inoperable. In this case the lending commitments are nullified and the lenders are free to use their funds in other ways. The economy can either be in normal state or in a crisis state. So the lenders make their investment decisions based on their private information about the type of the government and a private signal about current state of the economy. A very important feature of the model is that all lenders are ordered in a sequence and make their decisions sequentially. Since they can observe decisions made be the lenders in the beginning of the sequence they incorporate this information in their decision making. The authors focus on equilibria in which if the economy is in the normal state both types of government repay, while if it is in the crisis state the incompetent government defaults. In this framework there is obvious possibility of herding behaviour within each period. For example, if the prior of lenders that the government is competent is in an intermediate range then the fact that first in sequence agents do not invest can lead to that the other investors will not invest regardless their private signals on economy state. This effect works in the other direction also. The fact that first in the sequence lenders have invested pushes the rest of the sequence to invest. This model has also a dynamic part, which works in the following way. Suppose the economy starts with lenders having intermediate priors about the government and probability of crises being small. Then, until crisis arrives economy follows the path with capital randomly flow into and out of the country based on small changes of the signal realisations by the lenders. When crisis occurs inefficient government defaults on its loans and losses possibility to borrow again until something occurs that changes priors of lenders. The efficient government do not default and this strengthen favourable priors of lenders and creates more steady capital inflow. This part of the model can be very interesting in view of the present study for it gives possibility to analyse such problems as reputation and dynamic inconsistency after being updated to include government signalling and equilibrium with both government not defaulting in crises. It should be stressed once more that the key feature of the model is that the lenders moves sequentially with knowledge about actions of those moved before them.

In the model of Cole and Kehoe (1996), that of self-fulfilling prophecies crises, the lenders move simultaneously. The authors built a dynamic, stochastic general equilibrium model. This model is quite technical and its main goal is to study optimal policy of the government to avoid the crises. But as it is always mentioned in the literature on such kind of crises, there exists a crisis zone where the fears of government default on its debt on the part of lenders can be self-fulfilling and crisis can be triggered by realisation of external stochastic sunspot variables. The probability of such crisis to happen depend on the parameters of the crisis zone and the main attempt of the authors is to defined a proper actions of the government in order to reduce this crisis zone. This kind of models is a bit far from the goals of the present study and we will go in to more details of the model.

Another approach was developed by M.Obstfeld (1996,1998) in his discussion of currency crises with self-fulfilling features. In the analysis of strategic foundations of these models he considered a game between two holders of domestic currency and the government. The holders can either continue to hold or sell the currency to the government for foreign one. It is also clear that when holders sell domestic currency they attack the exchange rate. There are three following states of the world with respect to the government foreign currency reserves: low, intermediate and high reserves states. The states have the following properties: in the low reserves state even one holder can successfully attack the exchange rate, in the intermediate reserves state only common attack of both holders can be successful and in the high reserves state the government can reject any attacks on the exchange rate.

It is obvious that the government possesses this information and the holders do not. The holders' payoffs are such that they get profit from successful attack and bear losses in case of unsuccessful one. The strategic behaviour of the players is analysed in term of Bayesian equilibrium. Although this approach is rather simple it captures many important features of real cases. Notwithstanding that this approach was developed for the currency crises it is very attractive for description of the security market behaviour, because the idea of 2x2 Bayesian games can be directly applicable in the latter case.

The extension of this approach to the case of dynamic repeated game between a strategic money holder and the government can be found in paper of Z.Chen (1995). This paper discusses the currency market with two major players: strategic money holder and the government; and the competitive fence of many small non-strategic money holders. The strategic money holder attacks the exchange rate by short selling domestic currency in order to get a profit in case of successful attack. The government defends the exchange rate by buying out its currency. This fight defines a motion of the exchange rate. The role of the competitive fence is to add a stochastic element into exchange rate motion. The dependence of the major players' equilibrium strategies on the parameters of the model allows to study the effectiveness of various political measures in avoiding the currency crises. Although this model is more specific to the currency market description it contains many elements which can be used in the present analysis of the security market.

Another very interesting piece of literature constitutes two notes of G. Calvo (1998a,b). Despite the fact that this two notes discuss problems caused by the Russian crisis they contain some ideas useful for the present study. The author divide all the investors into two categories with respect to information they have: those informed and those uninformed or less informed. In cases of emerging market economies the classification of investors into informed and less informed groups corresponds very much to the classification into resident and non-resident investors with those non-resident being better informed. The rationale behind this correspondence is that the non-resident investors are more experienced in operating in the financial market and therefore are able to produce more accurate estimations of the market behaviour and thus are better informed.

The idea of dividing investors into informed and non-informed classes is similar to the idea of dividing stock traders into classes of those sophisticated and of so called "noise traders", that can be found in papers by A. Shleifer and co-authors (Shleifer 1990, De Long 1990). These papers are good examples of the literature on noise trader approach to description of financial markets. The basic idea of this approach is existence in the market of sophisticated trader with rational expectation on stock prices and noise traders with peculiar and unpredictable expectations. It is demonstrated in the papers that in a dynamic approach the very existence of the noise traders creates chances for arbitrage for the sophisticated traders in betting against the noise ones. On the other hand the possibility that extreme expectations of the noise traders become even more extreme in future creates an additional risk in betting against them which in turn limits the arbitrage possibilities. This approach was very successful in understanding many puzzles of financial markets which the efficient market approach failed to describe, such as excess volatility of stock prices and others.

It is obvious that the division of investors into sophisticated and noise traders (Shleifer et all) is an extreme case of their division into those informed and less informed (Calvo 1998a,b). In the latter case both categories of investors have rational expectations, but the accuracy of their estimations of the market conditions is different. In other words both informed and less informed investors have some noise admixture in their expectations with informed one having this admixture less than the others. In present study we will follow the line of Calvo (1998a,b) in dealing with investors' expectations.

To summarise this review of game approaches in description of the financial crises we stress the attractiveness of the approach of M.Obstfeld for the purpose of the present study. In the following sections two static models of the government security market will be based on the ideas presented in his papers, reviewed above (Obstfeld 1996,1998). The dynamic model of the security market presented in the last section will be developed as extension of the static game following some lines of the paper by Z.Chen(1995).

3. STATIC GAME BETWEEN RESIDENT AND NON-RESIDENT INVESTOR IN SECONDARY GOVERNMENT SECURITY MARKET

3.1. Basic assumptions

In this section a static model which describes the market behaviour when the monetary authority does not intervene is being built. A particular concern of this model is a dependence of the players behaviour on their expectations. In the case of by-standing authority there are only two categories of players in the market: resident and non-resident investors. Each of these two categories will be described in the game by one representative investor. In this game each player has to decide on his strategic move: maintain his position on the market (generally will be referred as "buy" security) or withdraw from the market (generally will be referred as "sell"). This strategic choice should be done in view of possibility that the government moves result in losses to investors. The examples of such moves are: government default on its obligation on security papers, currency devaluation, etc. Therefore one can define probability of these outcomes with looses to investors as the risk that they face in the market. A bit more formally the basic assumptions are:

- ***Two players:*** resident (R) investor and non-resident one (N)
- ***Two states of the world:*** there will be "default" in future (D) and there will be no (ND). The "default" here is a generalizes term to describe all the cases when the government moves result in losses to investors. As usual the exact state of the world is unknown. Each player has his private estimation of the probability of "default", or, in other words, risk estimations (expectations), which is unknown to the other player.
- ***Expectations.*** Private estimation of the risk (θ) is not unique. It is instead a random value distributed over some interval with some cumulative distribution function F:

$\boldsymbol{\theta_N} \in [\underline{\theta}_N, \overline{\theta}_N] \sim F_N(\theta_N)$, $\Delta\theta_N = \overline{\theta}_N - \underline{\theta}_N$ — for non-resident investor

$\boldsymbol{\theta_R} \in [\underline{\theta}_R, \overline{\theta}_R] \sim F_R(\theta_R)$, $\Delta\theta_R = \overline{\theta}_R - \underline{\theta}_R$ — for resident one.

(It is of course assumed that $0 \leq \underline{\theta}_N, \overline{\theta}_N, \underline{\theta}_R, \overline{\theta}_R \leq 1$)

While particular value of θ_i is private information its distribution is a common knowledge in the game.

- ***Better informed non-resident investors:*** It is also assumed that non-resident investors are more experienced and therefore are able to produce more accurate risk estimations. This results in $\Delta\theta_R > \Delta\theta_N$.

- ***Strategies:*** as it has already been stated above each player has two strategies: "buy" (b) and "sell" (s).

Although payoffs are among basic assumptions their structure requires special discussion.

3.2. Payoff structure

Suppose that the player has initially \$1 of security and \$1 of risk-free asset (with net return r). Suppose also that current yield on security is y, in other words buying a security for \$1 investor gets \$(1+y) at maturity date. Thus his initial wealth W is

$$W=1+y\ +\ 1+r.$$

Consider now investor's wealth resulting from different actions and in different states of the world ("default", "no default")

"<u>No default</u>" state:

own action (partner's action)	Wealth
b(b)	$1+y + 1+r + \varepsilon$
b(s)	$1+y + 1+y$
s(b)	$1+r + 1+r$
s(s)	$1+r + \delta$

Where it is supposed that:

- When both investors buy they reduce yield (or increase price of the security) up to the minimal acceptable value: $r+\varepsilon$, where ε is a minimal risk premium.

- Investors never hold money in their pockets, they always invest them into risk-free asset. So when they sell the security they immediately buy risk-free asset for this money. (This assumption is not crucial, it is just matter of normalisation.)
- When both investors sell they drastically reduce the price of the paper (the price falls to δ: $1>\delta>0$).
- When one investor sells and the other buys they do not affect current value of the security yield (or price).

In the **"<u>default</u>"** state and under the same assumptions wealth is.

own action (partner's action)	Wealth
b(b)	0
b(s)	0
s(b)	1+r + 1+r
s(s)	1+r + δ

So we have the following payoffs:

"<u>no default</u>"

		NR: B	NR: S
R	b	1+y+1+r+ε, 1+y+1+r+ε	2(1+y), 2(1+r)
R	s	2(1+r), 2(1+y)	1+r+δ, 1+r+δ

"<u>default</u>"

		NR: b	NR: s
R	b	0, 0	0, 2(1+r)
R	s	2(1+r), 0	1+r+δ, 1+r+δ

Adding a constant to all the payoffs in all the states does not affect the players behavior. Thus adding $-(1+r+\delta)$ to all the payoffs we get the following:

"<u>no default</u>"

		NR b	**NR** S
R	b	$1+y+\varepsilon-\delta,$ $1+y+\varepsilon-\delta$	$1+2y-r-\delta,$ $1+r-\delta$
R	s	$1+r-\delta,$ $1+2y-r-\delta$	$0,$ 0

"<u>default</u>"

		NR b	**NR** s
R	b	$-(1+r+\delta),$ $-(1+r+\delta)$	$-(1+r+\delta),$ $1+r-\delta$
R	s	$1+r-\delta,$ $-(1+r+\delta)$	$0,$ 0

Introducing for convenience the following definitions:

$a=1+y+\varepsilon-\delta,$

$b=1+2y-r-\delta,$

$c=1+r-\delta,$

$e=1+r+\delta,$

one gets the following payoff matrices:

"<u>no default</u>"

		NR b	**NR** s
R	b	a, a	b, c
R	s	c, b	0, 0

"<u>default</u>"

		NR b	**NR** s
R	b	-e, -e	-e, c
R	s	c, -e	0, 0

where $b>a>c>0$, $e>0$, $e>c$.

3.3. Equilibrium

An investor chooses a particular strategy on the basis of expected payoff that results from this strategy given investor's private estimation of the risk and the fact that the other investor picks up his best strategy. This in fact is a definition of the player's best response function that maps his private risk estimation into a strategy space. The symmetry of the payoffs with respect to the player index results in the same symmetry of the best response functions. If one restrict himself with equilibria in pure strategies only then the best response function for player *i* has the following form:

$$best_i(\theta_i) = \begin{cases} b_i, & \theta_i < \hat{\theta}_i \\ s_i, & \theta_i > \hat{\theta}_i \end{cases} \qquad (1)$$

where $\hat{\theta}_i$ is an equilibrium cut-off value of player's private risk estimation at which the player is indifferent between "buying" and "selling". This form of the best response function leads to a simple definition of the equilibrium in the game. The equilibrium can be characterised by a pair of cut-off values $\hat{\theta}_N$ and $\hat{\theta}_R$ of resident and non-resident investors correspondingly. These equilibrium cut-off values can be easily found in the following way.

Given the form of the best response function the expected payoff of player *i* has the following form:

$$U_i(\bullet, best_{-i}) = \theta_i E_{\theta_{-i}}\, u_i^d(\bullet, best_{-i}) + (1-\theta_i)\, E_{\theta_{-i}}\, u_i^{nd}(\bullet, best_{-i}), \qquad (2)$$

where $\bullet=\{b_i,s_i\}$; $E_{\theta_{-i}}$ - expectation operator over distribution of the other player's θ; u_i^d, u_i^{nd} are payoffs in cases of "default" and of "no default" respectively; $best_{-i}$ is the best response function of the other player. The cut-off value of one player (say *i*) as a function of the other player's (say *-i*) cut-off value is determined from the following equation:

$$U_i(b_i,best_{-i}) = U_i(s_i,best_{-i}) \tag{3}$$

Plugging payoffs and the best response functions in to equation (3) yields the following cut-off functions:

$$\hat{\theta}_R(\hat{\theta}_N) = \frac{b-(b-a+c)F_N(\hat{\theta}_N)}{b+e-(b-a)F_N(\hat{\theta}_N)}, \quad \hat{\theta}_N(\hat{\theta}_R) = \frac{b-(b-a+c)F_R(\hat{\theta}_R)}{b+e-(b-a)F_R(\hat{\theta}_R)} \tag{4}$$

In the equilibrium these two equations hold simultaneously, so one can get equilibrium cut-off values of player's private estimations of the risk. For example the equilibrium cut-off value of the non-resident investor can be derived from the following equation:

$$F_R\left(\frac{b-(b-a+c)F_N(\hat{\theta}_N)}{b+e-(b-a)F_N(\hat{\theta}_N)}\right) = \frac{b-(b+e)\hat{\theta}_N}{b-a+c-(b+e)\hat{\theta}_N} \tag{5}$$

From the symmetry considerations, noticed above, the equilibrium cut-off value of the resident investor is defined similarly.

Formula (5) defines non-resident investor's equilibrium cut-off value, $\hat{\theta}_N$, as an implicit function of the payoffs and the distributions of the risk expectations of both players. It is not so easy to analyse behaviour of $\hat{\theta}_N$, defined by this implicit function. However, there is one immediate and obvious conclusion on the players behaviour, that can be drawn from the formula (5) and the initial assumptions on possible ranges of θ for both players. There are regions of dominance in the game. Namely, there are such values of player's private estimation of the "default" probability, θ, that given this estimation player chooses his action independently of the other player's estimations and actions. For example, if the non-resident investor treats "default" as a very rare event ($\theta_N < \frac{a-c}{a+e}$) he will "buy" the security no matter what the resident investor expects and how he behaves. Figure 1 presents all the regions of dominance, which are either crossed or darkened.

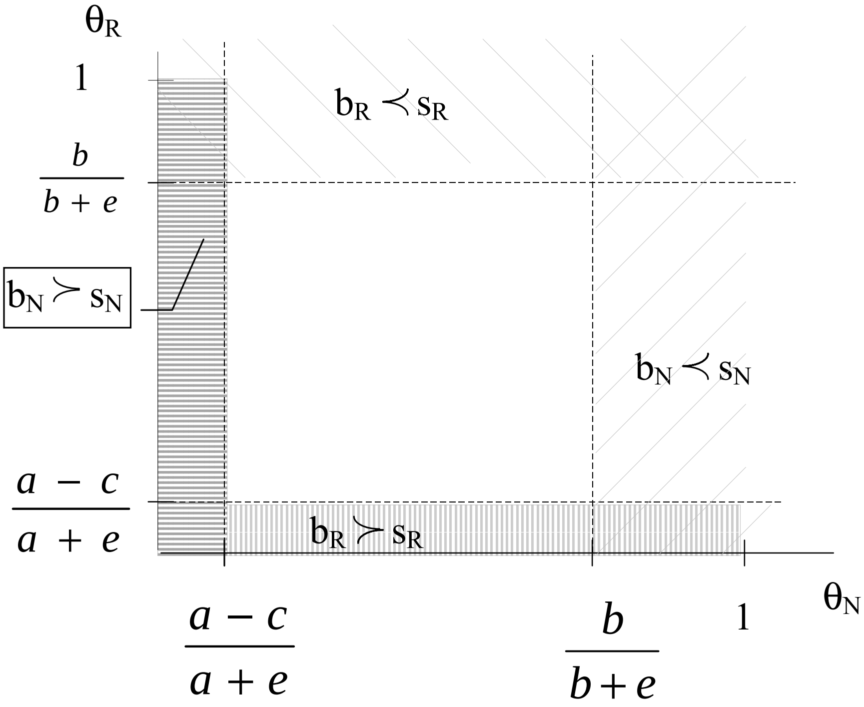

Figure 1. Regions of dominance

3.4. Numerical study

As it has been already mentioned, it is hard to study properties of the equilibrium cut-off values given only its definition as implicit function, formula (5). One needs to choose at least a particular form of the expectation distributions to make the analysis a bit easier. However even the simplest form of the distribution law, uniform distribution, does not make the general analysis tractable. Therefore we resorted to numerical study of the equilibrium cut-off values.

This numerical study was aimed at defining the dependence of the equilibrium cut-off values on the security yield and on differences of the player's expectations. The form of expectation distributions has been chosen uniform for both players, though parameters of these distributions were different in accordance with the assumptions of the model. All the payoffs have been made dependent only on the security yield by making constant all other their determinants:

$\varepsilon = 0.1$ (minimal risk premium)

$r = 0.15$ (risk-free interest rate)

$\delta = 0.1$ (price of the security when both investors sell it)

All the numerical study has been done by means of computer package MAPLE V. Whenever properties of the equilibrium cut-off value are reported this is done for the non-resident investor only, since the resident investor's cut-off behaves similarly.

Figure 2 contains 2-dimesional plot of $\hat{\theta}_N$ dependence on the average risk estimations of both players. The spreads of these estimations were chosen to be: $\Delta\theta_N=0.1$ for the non-resident investor and $\Delta\theta_R=0.2$ for the resident one. Yield on the security was fixed at 0.35. As it can be seen from this plot, the non-resident investor's cut-off value, $\hat{\theta}_N$, is an increasing function of the average risk estimation of the non-resident investor, $\langle\theta_N\rangle$, and a decreasing function of the same value of the resident investor, $\langle\theta_R\rangle$.

Present analysis also shows that the surface of $\hat{\theta}_N(\langle\theta_N\rangle,\langle\theta_R\rangle)$ corresponding to higher value of the yield is placed above the same surface but corresponding to the lower yield. Therefore obvious and well expected conclusion on the positive dependence of the equilibrium cut-off value on the security yield arises.

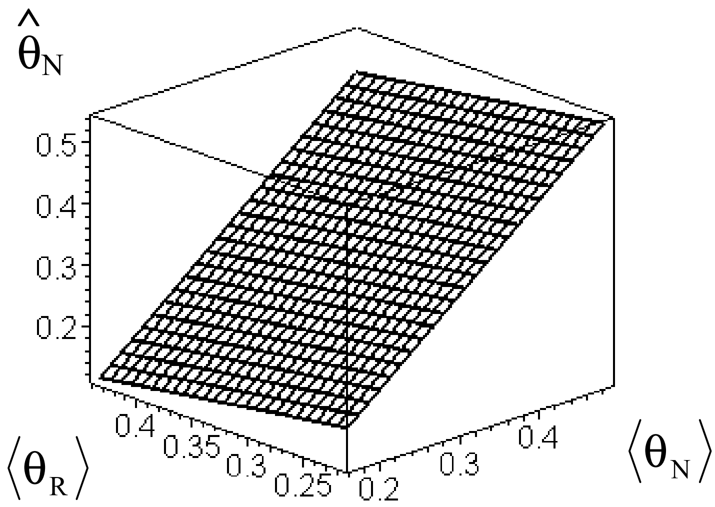

Figure 2. Dependence of the non-resident investor's equilibrium cut-off value on the average "default" probability estimations of both players

Another interesting property of the equilibrium cut-off value is their dependence on the spread of the investor's risk expectations. Fig.3 presents the dependence of the non-resident investor's equilibrium cut-off value, $\hat{\theta}_N$, on the average, $\langle\theta_N\rangle$, and the spread, $\Delta\theta_N$, of his risk expectation distribution. The resident investor's average estimation of the risk was taken equal to that of the non-resident investor, $\langle\theta_R\rangle=\langle\theta_N\rangle$. The spread of this distribution, $\Delta\theta_R$, was taken to be approximately equal to 0.14. The yield on the security was fix at 0.35, as on the previous plot.

As it is well seen from Fig. 3, there are two clear regions with respect to $\langle\theta_N\rangle$ where the non-resident investor's equilibrium cut-off value, $\hat{\theta}_N$, demonstrates different dependencies on the spread, of his risk expectations distribution. The plot

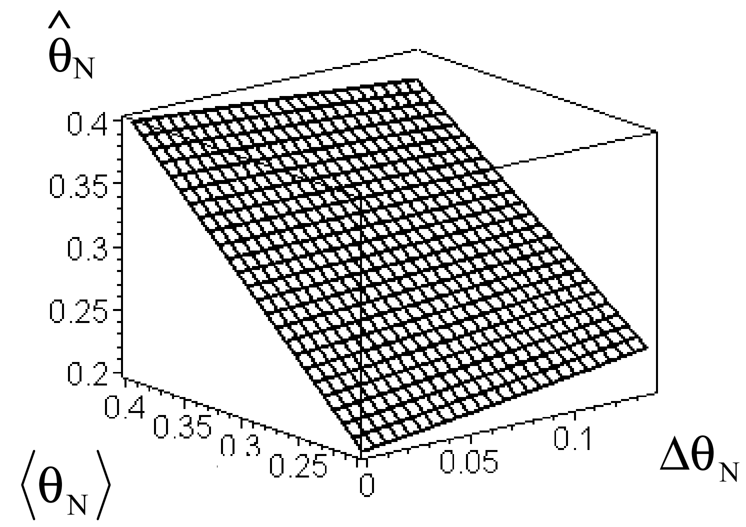

Figure 3. Dependence of the non-resident investor's equilibrium cut-off value, $\hat{\theta}_N$, on the average, $\langle\theta_N\rangle$, and the spread, $\Delta\theta_N$, of his "default" expectations distribution

on Fig. 3 implies that there is some crucial value of the non-resident investor's average estimation of the risk, $\tilde{\theta}_N$, such that for average estimations below this value his equilibrium cut-off, $\hat{\theta}_N$, is an increasing function of the spread, $\Delta\theta_N$, and vice versa in the other region of $\hat{\theta}_N$:

$$\partial\hat{\theta}_N \big/ \partial\Delta\theta_N = \begin{cases} > 0, & \langle\theta_N\rangle < \tilde{\theta}_N \\ < 0, & \langle\theta_N\rangle > \tilde{\theta}_N \end{cases} \tag{6}$$

Knowledge of the investors equilibrium cut-off values and of the distribution laws of their risk expectations allows to define what happens in the market in the equilibrium. In particular, it allows to estimate a fraction of both investors "buying" and a fraction of those "selling" and therefore to find the value of excess demand on the security. In fact all the properties of the equilibrium cut-off values can be translated into the properties of the security excess demand. Figure 4 contains

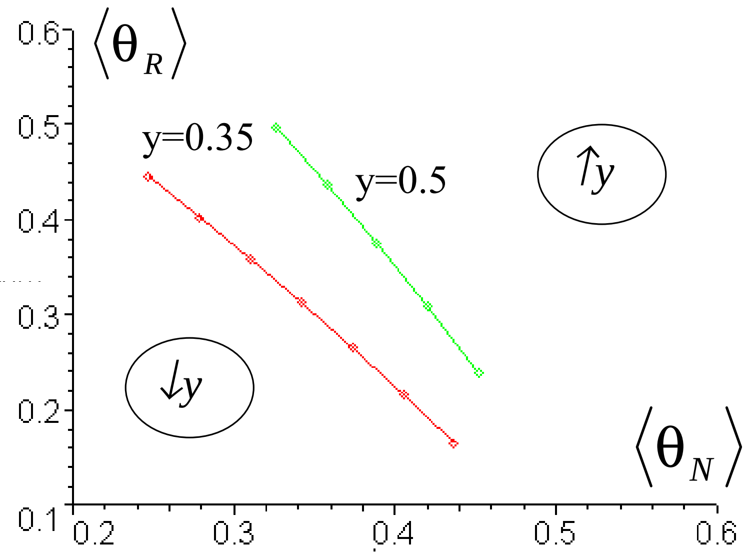

Figure 4. Regions of the excess demand for the security in coordinates of the averages of the investor's expectations distribution

regions of the excess demand for two values of the yield, 0.35 and 0.5. The spreads of the risk expectation distributions are chosen to be $\Delta\theta_N$=0.1 and $\Delta\theta_R$=0.2, as on Fig. 2. The lines on Fig. 4 represent the loci of 0 excess demand for corresponding values of the yield. As one can expect the locus of no excess demand for the higher yield (y=0.5) is systematically shifted into the region of higher average values of the risk expectations, in comparison with the locus of y=0.35. If the players' expectation structures and the value of the yield are such that this point appears below the corresponding line on the plot, then the relative fraction of investors of both types which are "buying" in the equilibrium will be greater than the fraction on those investors "selling" and this will pressure down the yield of the security (or in other terms, will create a tendency of its price rising). So the region below 0 excess demand line is the region of positive excess demand for the security, or true excess demand. The tendency is reversed when the expectations are such that the point on the graph is above the corresponding line. The region above the line corresponds to negative excess demand and is, in fact, region of excess supply of the security.

Fig.4 presents loci of 0 excess demand in coordinates of average risk

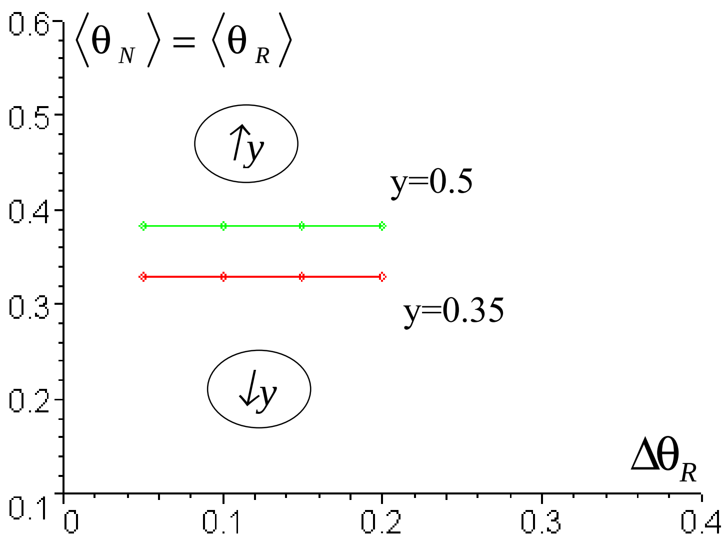

Figure 5. Regions of the excess demand for the security in coordinates of the average and the spread of the resident investor's expectation distribution

expectations. But how these loci depend on the spreads of the expectations? Fig.5 demonstrates the dependence of the loci of 0 excess demand on the average and the spread of the risk expectations of the resident investor. As on Fig.3 the non-resident investor's average estimation of the risk was taken equal to that of the resident investor ($\langle \theta_R \rangle = \langle \theta_N \rangle$). The spread of the non-resident investor's distribution was taken to be $\Delta\theta_N$=0.05. As on the previous figure, two lines are presented on Fig.5, which correspond to two values of the security yield, 0.35 and 0.5. All the qualitative behaviour of the excess demand regions observed on the previous plot preservers also on this plot. The main conclusion which can be drawn from the plot on Fig.5 is that the loci of 0 excess demand almost do not depend on the spread of expectation distribution of one investor given that the spread of the other investor's distribution is fixed and the averages of distributions of both investors are equal.

As one can conclude from Fig.4 and Fig.5 equilibrium in this game demonstrates knife-edge stability with respect to the security yield (or its price). If it happens that excess demand for the paper in equilibrium is 0, then this equilibrium can last forever. If on the other hand there is either excess demand for or excess supply of the security then this equilibrium cannot sustain without intervention of a force other than investors. There is a room here for the authority actions. If it stands by then the yield or the price of the security will change according with the market pressure and the market will move into direction of stable equilibrium. This situation can be welcomed by the authority if there is an excess demand for its security and the yield tends to fall (or the price tends to rise). In case of excess supply for the paper when the yield tends to rise the authority may well find it unacceptable and can try to fix the value of the yield by buying out the excess of the security. Another way to keep the yield constant is to change the expectation structure of the investors such that the equilibrium becomes stable at this yield. The present study shows that the average value and not the spread of the investor's expectations is crucial from this point of view. It should be mentioned that the investors themselves can sometimes find it profitable to change down the security price by influencing the other player's expectations. Unfortunately this model can only point out the direction of this influencing, but does not capture a mechanism of such interactions.

4. SIGNALLING GAME BETWEEN RESIDENT AND NON-RESIDENT INVESTOR ON SECONDARY SECURITY MARKET

4.1. Basic assumptions and the game structure

As it has been already mentioned secondary security market exposes such phenomenon as announcements of market condition evaluations by players. These announcements can be considered as signals which the players send to each other. The most frequent phenomenon of this type in the Russian security market is publication by non-resident investors of their assessment of correspondence of the security yield to the risk estimations. This signal can be considered in a simplified manner as a message on leaving the market if the yield does not cover the risk. There may be different situations with sending such a signal. For example, one is to announce true information about the market and another is to involve the resident investors into the pressure on the security yield by affecting the resident investor's expectations. In the last case the non-resident investors can report wrong information in attempt to move up the yield above the market equilibrium level. In order to discuss the described above phenomena from the point of view of strategic behaviour of the players we develop in this section the model of signalling game between the non-resident and resident investors.

One of the basic assumptions the whole present study is that the non-resident investor has more accurate estimations of the market conditions. In the signalling game model this assumption will be used in its extreme form: the non-resident investor knows exactly what state of the world has realised ("default" or "no default"). Therefore it is natural to assume that he acts as a leader and a signal sender in the game and the resident investor is a follower. All the assumptions of the previous section on the state of the world, strategies of the players, payoff structure etc. will be used in this section.

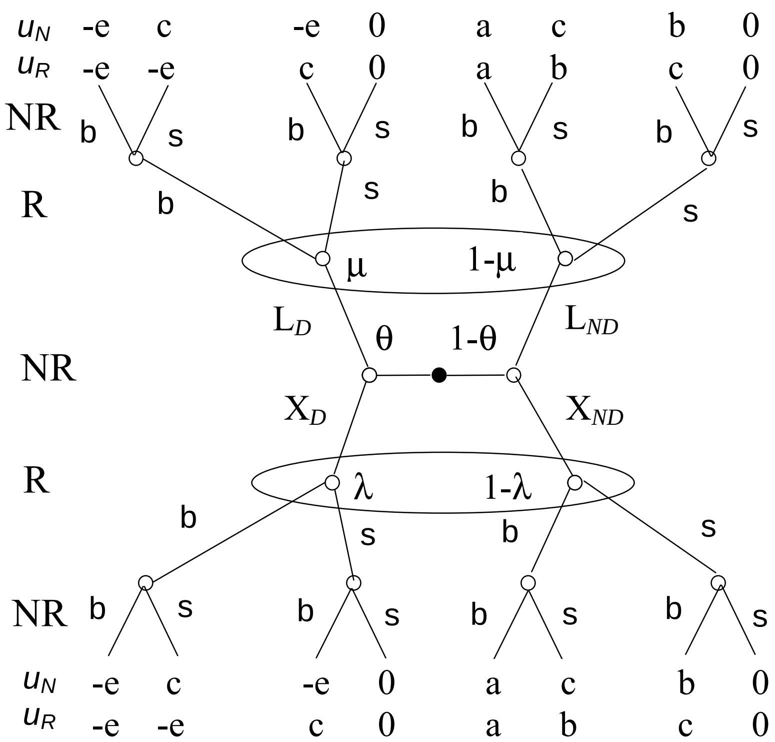

Figure 6. Full game tree

The signalling game has the following 3 stages:

I. The non-resident investor learns the state of the world and decides on whether to send or not to send the signal that he is leaving the market. (The absence of the signal on leaving can be treated as signal on staying.) His strategies at this stage are: to send a signal ("L") and not to send it ("X").

II. Upon receiving the signal the resident investor decides on whether to stay in the market ("buy") or to leave it ("sell")

III. Observing the resident investor's reaction to the signal or its absence the non-resident one decides in his turn on whether to stay or to leave. The important assumption is that this decision may not coincide with the signal sent before.

The full tree of the game is presented on Fig.6.

Given the fact that only perfect Bayesian equilibria in the game will be discussed in the present analysis, this full game can be reduced by solving 3rd stage

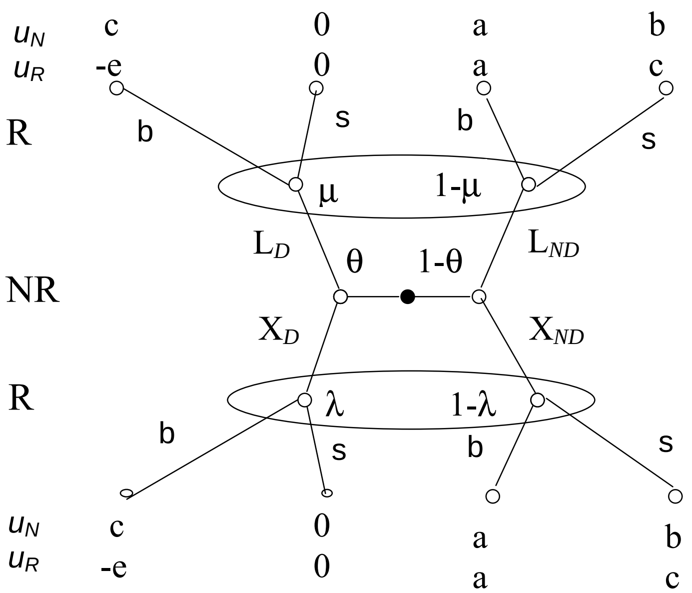

Figure 7. Reduced game tree

non-resident investor's subgames. The reduced game is presented on Fig.7.

On both Fig.6 and Fig.7, θ represents objective probability of the "default" state realisation and μ and λ are beliefs of the resident investor that he receives and does not receive the signal in the "default" state, correspondingly.

4.2. Equilibria

Following the idea of perfect Bayesian equilibrium concept one starts from the last stage of the game and goes backward to its beginning. Following this procedure in analysis of the reduced game the critical values of the resident investor's beliefs $\hat{\lambda}, \hat{\mu}$ has to be defined first. By definition, critical belief is such a belief that having it a player is indifferent to choose any of his strategies. From this definition one gets the following value of the critical belief in case of non-resident investor sending a signal in the "default" state:

$$\hat{\mu} = \frac{a-c}{a-c+e} \tag{7}$$

It is rational for the resident investor to "buy" whenever $\mu < \hat{\mu}$ and to "sell" if $\hat{\mu} < \mu$. Recalling that $a > c, e > 0$ it is obvious that $0 < \hat{\mu} < 1$. Given symmetry of the player's payoffs in cases of sending and not sending the signal by the non-resident investor one can immediately conclude that the values of the critical beliefs are the same: $\hat{\lambda} = \hat{\mu}$.

The critical beliefs of the resident investor defines his rational strategy profile under any belief system (λ, μ). Now one can proceed further and check for any rational strategy profile, which consists of corresponding profiles of both players, whether it is possible to derive through Bayes' rule corresponding belief system (λ, μ). Whenever it is possible the strategy profile and the belief system constitutes perfect Bayesian equilibrium. This checking procedure includes consideration of many cases of various strategy profiles and belief systems. Some cases are very similar to each other for this particular game. Therefore only few really critical cases will be discussed in details below.

- $\lambda > \hat{\mu},\ \mu < \hat{\mu}$, which leads to resident investor playing "sell" in absence of the signal and "buy" upon receiving it. Given the payoffs in the reduced game (see Fig.7) it is obvious that non-resident investor will find it optimal to send the signal from the "default" state and keep silence in the "no default" one. The belief system corresponding to this strategy profile and derived through Bayes' rule is: $\lambda = 0, \mu = 1$, which obviously contradicts to the belief system corresponding to the resident investor's strategy profile assumed above. Therefore the conclusion is that there is no equilibrium in the game where the resident investor "sells" in absence of the signal and "buys" upon receiving it. The symmetric case when the resident investor "buys" in absence of the signal and "sells" in response to it can be considered in the same manner by interchanging the players actions in cases of sending and not sending the signal. The result in this case is also the same: Bayesian belief system contradicts to that implied by the strategy profile. The final conclusion from this discussion is that there is no equilibrium in the game where the resident investor behaves differently in case of receiving and not receiving the signal from the non-resident investor.
- $\lambda, \mu < \hat{\mu}$ which leads to resident investor playing "buy" strategy whether or not the signal is sent by the non-resident one. In this case the non-resident investor is indifferent to sending the signal. This indifference opens a possibility of randomising by the non-resident investor between sending and not sending the signal. Therefore the following mixed strategies (σ,τ) can be defined for him:

$$\sigma X_D + (1-\sigma)L_D, \quad \tau X_{ND} + (1-\tau)L_{ND}, \qquad (8)$$

Choosing appropriate mixed strategies (σ,τ) allows to build such Bayesian belief system (λ, μ) that supports assumed strategy profile. Moreover it turns out that there exists 3 continuums of equilibria where the resident investor "buys" independently of presence of the signal. The non-resident investor's part of the equilibria in the reduced game looks as follows:

$$\begin{cases} \tau > \sigma > 1 - \dfrac{(1-\theta)\hat{\mu}}{(1-\hat{\mu})\theta}(1-\tau) \\ 0 < \tau, \sigma < 1 \end{cases},$$

$$\begin{cases} \tau < \sigma < \dfrac{(1-\theta)\hat{\mu}}{(1-\hat{\mu})\theta}\tau \\ 0 < \tau, \sigma < 1 \end{cases}, \qquad (9)$$

$$\begin{cases} \tau = \sigma \\ 0 \le \tau, \sigma \le 1 \end{cases}.$$

It is obvious that two first classes of equilibria exist only if $\theta < \hat{\mu}$. The symmetric case with the resident investor selling independently of the signal and $\lambda, \mu > \hat{\mu}$ can be considered in a similar way and the results will be the same with the inequality signs changed to opposite in the first two classes of equilibria. Not surprisingly, these equilibria are possible only if $\theta > \hat{\mu}$. There are cases which are very similar to those already discussed, such as $\lambda > \mu = \hat{\mu}$, $\lambda < \mu = \hat{\mu}$ and so on. The equilibria have the same form with the only change that some inequalities become relaxed.

- $\lambda = \mu = \hat{\mu}$, which makes the resident investor indifferent to playing either of his two strategies and gives him a possibility to randomise between his strategies (mixed strategies: α and β). There is one restriction on equilibrium mixed strategies, namely the equilibrium mixtures of the resident investor should be the same in both cases of receiving and not receiving the signal from the non-resident investor. If it will not so the non-resident investor will prefer either sending or not sending signal, which will imply the Bayesian beliefs to be either 1 or 0, which in turn will contradict to $\lambda = \mu = \hat{\mu}$. So in the equilibrium the non-resident investor should be indifferent to sending and not sending the signal and will randomise these strategies. As it can be easily seen the only way to get Bayesian beliefs equal $\lambda = \mu$ is to use identical mixed strategies of the non-resident investor in both "default" state and "no default" one. The requirement that Bayesian beliefs to be equal $\hat{\mu}$ implies that $\hat{\mu} = \theta$. Therefore there is again a continuum of equilibria of the following form:

$$\begin{cases} \alpha = \beta, & 0 \le \alpha, \beta \le 1 \\ \tau = \sigma, & 0 \le \tau, \sigma \le 1 \end{cases} \tag{10}$$

All these results can be summarised in a slightly different way, namely with respect to the probability of the "default" state. If the "default" is probable ($\theta > \hat{\mu}$) then the equilibrium strategy of the resident investor is to "sell" without paying any attention to signalling of the non-resident investor. If, on the other hand, the "default" is not very much probable ($\theta < \hat{\mu}$) then the best strategy for the resident investor is to "buy" no matter what signalling is. The non-resident investor has a wide range of mixed strategies supporting all these equilibria. If the "default" probability reaches a critical value ($\theta = \hat{\mu}$) then the equilibrium response of the resident investor to the signal of the non-resident one is random, given that both players use the same mixed strategies in their respective information sets.

The main conclusion from the analysis of this signalling game is that there is no much sense in sending a signal by the non-resident investor or, in other words the signal is almost not informative. What are the reasons for this outcome? One of the reasons can be found in the structure of the game. The non-resident investor sends the signal and after having observed the reaction to it from the resident one makes a real choice what to do. His real choice may contradict to the signal he sent to the resident investor. Some strategies of such type can cause losses to the resident investor if he will take into account all the signals sent by the non-resident investor. This possibility results in the resident investor defending himself against these losses by choosing the strategy based on the risk estimation and not on the signal of the other player. This outcome coincides with the assumption about rational behaviour of the players. Such an assumption implies that one player can not systematically fool the other one which would have been exactly the case if the resident investor reacted differently to the signal of the non-resident one. In case of critical value of the risk ($\theta = \hat{\mu}$) the resident investor defends itself against fooling signal of the non-resident one by randomising his response to the signal.

5. REPEATED GAME BETWEEN RESIDENT AND NON-RESIDENT INVESTORS AND AUTHORITY IN THE SECURITY MARKET

5.1. Basic assumptions and the game structure

In the two previous sections the game models were developed to describe some aspects of the security market behaviour in cases when the government does not intervene in the market and acts as a bystander. Another common feature of these models is that they are one-shot games. In this section a model of infinitely repeated game between investors and the government will be developed. This model will incorporate some results of the analysis done so far. The government in this model will be treated more generally than the government itself for it will include the Central Bank which is not a part of the government. Thus it will be better called the authority.

One cycle of the game can be described as follows. Suppose at time *t* the security price p_t prevails in the market. The authority supplies amount b_t of the security. For the sake of simplicity we will consider only one type of securities, namely one period bonds, and all players are supposed to be risk-neutral. Investors form some expectations of the probability of “default” , θ_t. The “default” is treated in exactly the same general way as in the previous sections, so it is just a state where investors loose a part of their money. Therefore θ_t can be considered as investors’ estimation of the risk associated with the bond. It is assumed that both resident and non-resident investors have the same risk estimation. Investors react to the amount b_t by buying it and then reselling the bonds among each other. This action according to the model of section 4 produces the pressure on the bond price which in the present model will be allowed to realise in the new value of the price at which the market clears. This new value of the price will prevail in the market in the next period, so it is called p_{t+1}. One comment is required here. Of course in the real life the authority has different means of affecting this equilibrium price or the market response. For example, the government can reduce the amount of borrowing, and the Central Bank (CB) can intervene in the market and buy out some excess of the bonds in order to keep the price under control. Apart from some effects, which are not to be discussed here, this open market operation on the part of CB is equivalent from the point of view of the market response to supplying less bonds at the beginning of the period. This fact is one of the main reasons to combine the government and CB in one entity.

An important point of the model is how the investors form their expectations. It is assumed in the model that they do it by comparing the amount of bonds offered in the current period b_t with some equilibrium amount of borrowing, $\widetilde{b}$. This last value is an important part of the expectation structure and it is assumed to be exogenous to the security market. This equilibrium amount of borrowing summarises effects of the rest of the economy on the security market. Therefore the evolution of the investor’s estimations of risk, which can also be called as their expectations, can be represented as following:

$$\theta_t = \theta_{t-1} + f(b_t, \widetilde{b}) \tag{11}$$

Given this structure of the investor's expectations the amount of the bonds offered by the authority in the current period can be considered in some circumstances as a signal sent by the authority to the investors.

The payoff of the authority in the one-shot game can be defined in the following way. Since it offers the amount b_t of the one period bonds at the current price p_t (all the bonds are supposed to be bought) and it has to repay its loan of the previous period the payoff has the form:

$$\mathrm{W_t} = p_t b_t - b_{t\text{-}1} \tag{12}$$

It is supposed that the government always repays in full its debt on bonds of the previous period. In cases when it cannot do so the "default" is declared and the game stops. It should be noticed that formula (12) describes only part of the authority's welfare. However it can still be used in some particular cases. If for example one assumes that the authority issues the bonds to finance some exogenousely given budget deficit, that is constant, then it will enter in the authority's welfare as constant and can be dropped from the maximisation problem, thus formula (12) can be used. In this case budget deficit will affect the authority's decision only through the bounding the borrowing amount from below. This particular case will be briefly discussed in section 5.3.

The payoffs of the investors have the same structure as in section 4. They always act optimally according to their expectations and produce current equilibrium market price of the bonds. According to findings of section 4 and the assumptions of present model main properties of this price are:

$$p_t \equiv p_t(\theta_t, b_t), \quad \partial p \big/ \partial\theta < 0, \quad \partial p \big/ \partial b < 0 \tag{13}$$

When this one-shot game is repeated infinitely one additional assumption should be made. This assumption implies that the players value future revenue and expenditures less than those of today. This assumption is taken into account by introducing a time discount factor. As usually only subgame perfect equilibria will be of interest in this study. According to this equilibrium concept resident and non-resident investors subgame can be reduced to equilibrium price at each moment and the evolution of their risk estimations. The total payoff of the authority in this case is a sum of the discounted payoffs over all time periods (or game cycles). The goal of the authority is to maximise its total payoff by choosing optimal borrowing scheme at each time period given the equilibrium behaviour of the investors. Therefore the equilibrium in this infinitely repeated game can be found as a solution to an optimal control problem which in the limit of small time interval has the following form:

$$\begin{cases} \mathrm{W} = \int_0^\infty e^{-\delta t} b_t(p_t - 1 + \delta)dt \to \max\limits_{\{b_t\}} \\ \dot{\theta} = f(b_t, \tilde{b}) \\ 0 \le \theta_t \le 1 \\ b_t \ge 0 \end{cases} \tag{14}$$

5.2. Equilibrium

To simplify a bit an analysis of this optimal control problem an additional assumption about linear dependence of the investors' expectations on the volume of borrowing has been made:

$$\dot{\theta} = \alpha(b_t - \tilde{b}), \tag{15}$$

where α is a sensitivity of the investors' expectations to deviation of the volume of borrowing from its equilibrium level. If only internal solution is of interest, which implies that all the restrictions in the optimal control problem are not binding, then the corresponding Hamiltonian looks as follows:

$$H = e^{-\delta t} b_t (p_t - 1 + \delta) + \pi_t \alpha (b_t - \tilde{b}) \tag{16}$$

Applying the maximum principle yields the following first-order condition and two differential equations:

$$\begin{cases} \dfrac{\partial H}{\partial b} = 0 \\ \dfrac{\partial H}{\partial \theta} = -\dot{\pi} \\ \dfrac{\partial H}{\partial \pi} = \dot{\theta} \end{cases} \tag{17}$$

Using the first order condition the system of differential equations can be transformed to another form, which omitting simple intermediate calculations can be expressed as follows:

$$\begin{cases} \dot{\theta} = \alpha(b - \tilde{b}) \\ \dot{b} = \dfrac{-b^2 \alpha p''_{\theta b} + b(\delta p'_{\theta} + \alpha p''_{\theta b} \tilde{b}) + \alpha \tilde{b} p'_{\theta} + \delta(p - 1 + \delta)}{2 p'_b + b p''_{bb}} \end{cases} \tag{18}$$

This system of differential equations describes a dynamics of the game. The system in this general form is too complex to yield a tractable analysis of the dynamics therefore one more simplifying assumption has been made. The equilibrium price of the securities is assumed to be linear dependent on the risk estimations and the volume of borrowing:

$$p(\theta, b) = \bar{r}\bar{\theta}(1 - \gamma b), \;\; \bar{r} = 1 - r, \;\; \bar{\theta} = 1 - \theta, \;\; b < {}^{1}\!/_{\gamma} \tag{19}$$

where r is a risk-free interest rate, γ is a sensitivity of the price to the volume of borrowing and $\bar{\theta}$ is the risk estimation. A natural way to make calculations more tractable is to switch from (θ,b) variables set to $(\overline{\theta},b)$ one. The system of differential equations in this case has the following form:

$$\begin{cases} \dot{\overline{\theta}} = \alpha(\widetilde{b} - b) \\ \dot{b} = \dfrac{b^2\alpha\gamma\overline{r} + 2b\gamma\overline{r}(\delta\overline{\theta} - \alpha\widetilde{b}) + \alpha\widetilde{b}\,\overline{r} - \delta(\overline{r}\overline{\theta} - 1 + \delta)}{2\gamma\overline{r}\overline{\theta}} \end{cases} \tag{20}$$

This system has the following equations of loci $\dot{\overline{\theta}} = 0$ and $\dot{b} = 0$:

$$\begin{cases} \dot{\overline{\theta}} = 0: \ \ b = \widetilde{b} \\ \dot{b} = 0: \ \overline{\theta} = \dfrac{b^2\alpha\gamma\overline{r} - 2b\alpha\gamma\overline{r}\widetilde{b} + \alpha\widetilde{b}\,\overline{r} + \delta(1-\delta)}{\delta\overline{r}(1-2\gamma b)} \end{cases} \tag{21}$$

The discriminant of the numerator of the locus $\dot{b} = 0$ has the following form:

$$D = (\alpha\gamma\overline{r})^2\widetilde{b}(b - 1/\gamma) - \alpha\gamma\overline{r}\delta(1-\delta). \tag{22}$$

It is obviously less than 0 under assumption $b < 1/\gamma$. Therefore the numerator is always positive and the sign of the whole fraction depends only on the sign of the denominator which is positive if $b < 1/2\gamma$. In other cases $\overline{\theta}$ becomes negative that has no economic sense. Another important restriction on the parameters of the model comes from the equation for the $\dot{b} = 0$ locus and the requirement that $\overline{\theta} \leq 1$. It turns out that a necessary condition for the existence of the internal equilibrium solution is $r < \delta$.

Having defined loci of $\dot{\overline{\theta}} = 0$ and $\dot{b} = 0$ one can draw a phase diagram of the system, which is presented on Fig. 8. The direction of the field on this phase diagram clearly demonstrates the possibility of a saddle path behaviour. This observation can be checked by looking at the sign of the determinant of the system linearised around the equilibrium point (E):

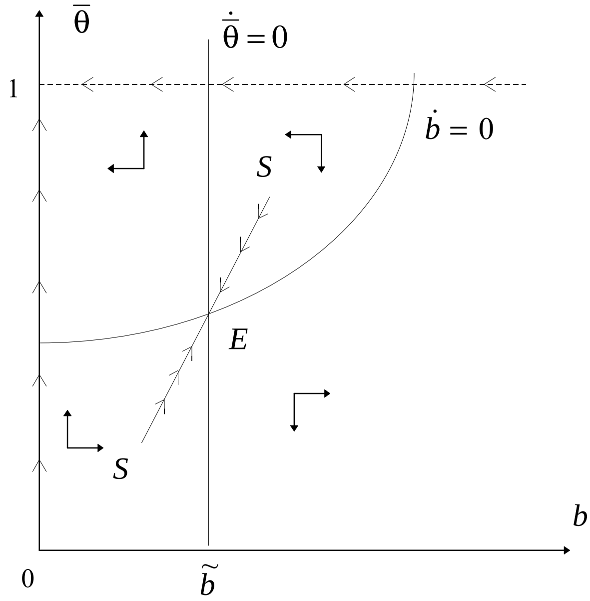

Figure 8. Phase diagram of the dynamic system.

$$\begin{vmatrix} 0 & -\alpha \\ \dfrac{\delta\bar{r}(2\tilde{b}\gamma-1)}{2\gamma\bar{r}\overline{\theta}} & 0 \end{vmatrix} = \begin{cases} <0, & \tilde{b} < \frac{1}{2\gamma} \\ >0, & \tilde{b} > \frac{1}{2\gamma} \end{cases} \tag{23}$$

It is obvious that there is a saddle path behaviour whenever $\tilde{b} < \frac{1}{2\gamma}$. Moreover as it has been already mentioned we are not interested in the area where $b > \frac{1}{2\gamma}$. Therefore the system exposes the saddle path behaviour in the area of interest. This saddle path is schematically drawn as SS line on Fig.8.

Looking at this phase diagram one can conclude that the saddle path divides the whole available phase space into two regions: the one above the saddle path and the other below it. Any trajectory that starts above the saddle path ends up at the point with coordinates: $b = 0$ and $\overline{\theta} = 1$. This point is a boundary equilibrium, which corresponds to the case when the authority does not borrow at all through the security market and the bonds are considered by the investors as equivalent to risk-free assets. Finding itself, for example, at a point above the saddle path and below the $\dot{b}=0$ locus the authority discovers that it can increase its borrowing. Doing so and acting optimally, i.e. following the dynamics of the system, Fig.8, it starts to increase borrowing and still faces the investors' risk estimations moving in favourable direction. This may results in price of bonds increasing and in a possibility for the authority to raise enough money for budget financing with less borrowing, especially in the region above the $\dot{b}=0$ locus. Following this policy the authority ends at the boundary equilibrium. In contrast, the region below the saddle path is that of collapse of the market. Any trajectory that starts there ends up in a finite time at points with $b = 1/\gamma$ and price of bonds equal to 0. If for example the authority faces the market in such a conditions that correspond to a point below the saddle path and to the left from the $\dot{\bar{\theta}}=0$ locus it finds it optimal to increase borrowing. Doing so the authority initially moves the investors' risk estimations into favourable direction, but the rate of change of their expectation does not correspond to the rate of increase in borrowing to make the trajectory stable. This can result in prices steadily falling with time, that can force the authority to increase from period to period its borrowing to raise enough money to finance the budget. As soon as the amount of borrowing increases its equilibrium level $\tilde{b}$ the investors start to raise their risk estimations each period, which causes further fall of bond prices. This causes the authority to further increase their borrowing. Such a process is very much similar to the authority building a financial pyramid each period borrowing more than in the previous one. In the linear prices model such policy of the authority is unsustainable and leads to complete loss of investors' confidence and finally to their withdrawal from the market. In the last period the authority has a huge debt to be repaid to the investors and has no possibility to borrow these money through the bond market. Whether it will or will not default on this debt depends on factors, which are external with respect to the government security market, but it should be stressed that the security market itself collapses and default is very much probable.

The only sustainable borrowing policy of the authority is to preserve the rate of changes in borrowing in correspondence with the rate of changes in the investors' expectations so as to keep always on the saddle path. In this case the market will steadily evolve towards the equilibrium, where it can stay forever.

An important issue is how the equilibrium and the saddle path depend on the parameters of the model, such as sensitivity of the investors expectations to the volume of borrowing α and equilibrium volume of borrowing $\tilde{b}$. The key point in this analysis is how loci of $\dot{\bar{\theta}} = 0$ and $\dot{b} = 0$ depend on these parameters. The easiest answer is for the $\dot{\bar{\theta}} = 0$ locus. It depends only on $\tilde{b}$ because it is exactly the line $b = \tilde{b}$. The $\dot{b} = 0$ locus moves upward and to the left whenever α, $\tilde{b}$ or both, α and $\tilde{b}$, increase which causes internal equilibrium and the saddle path, whenever they exist, to move in direction of increasing b and $\bar{\theta}$. Very important observation is that there exist such values of the parameters α and $\tilde{b}$ that the internal equilibrium does not appear in economically sensible region

After defining these properties of the solution one can analyse the reaction of the system to external shocks which can lead to a new equilibrium or to losses of the equilibrium. Consider a case when unanticipated constant negative shock to the equilibrium volume of borrowing happens: $\tilde{b} \to \tilde{b}'$, when the economy stays in the internal equilibrium. This may mean that the estimation of the economy conditions by the investors changed in the unfavourable direction to the authority. (Among the real reasons for such a change one can identify, for example, deterioration of the country current account due to changes of world prices. This is particularly true for the case of Russian financial crisis when decrease of world oil prices caused additional pressure on the GKO yield.) Clearly this will result in a new equilibrium of the system with lower volume of borrowing $\tilde{b}'$ and lower equilibrium risk estimations, which means higher $\overline{\theta}'$. What is the dynamic of the transition to the new

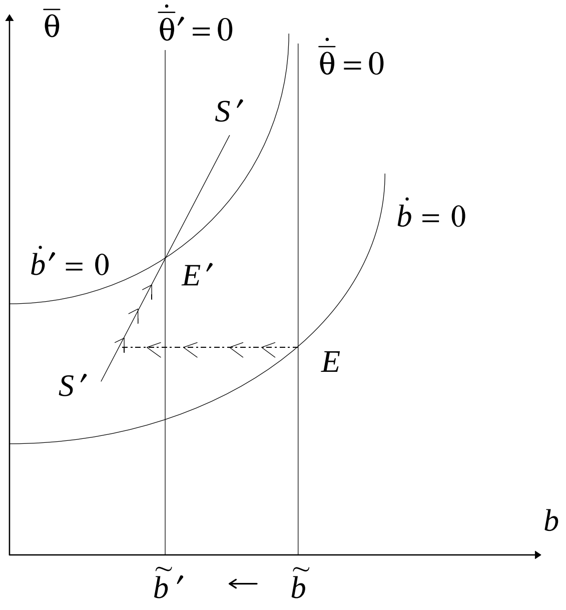

Figure 9. Example of reaction to unanticipated negative $\tilde{b}$ shock

equilibrium? Assuming that the economy always stays on the saddle path, one concludes that at the moment of the shock it jumps to the new saddle path. Given the assumption of the model that investors' expectations can not change immediately the only way to jump to the new saddle path is to change the amount of borrowing b by corresponding value. In cases when the necessary changes in b are not made the system appears in the region of inevitable market collapse, below the new saddle path. Fig. 9 schematically shows the process of the transition. There is obvious effect of overshooting during the transition which results in higher drop in the volume of borrowing during the transition than the drop in respective equilibrium volumes. Similar effect of overshooting but in the opposite direction can be found in case of positive shock to the value of $\tilde{b}$.

The examples of kind considered above have one very important feature that the shocks are unanticipated. In case of anticipated shocks the dynamics is a bit different. Consider now an example when the constant negative shock to the equilibrium volume of borrowing results from the actions of the authority and it announces that these actions will be taken in some near future. Fig.10 shows the dynamics of the transition to the new equilibrium in this case. The key feature of this process is that the system reacts immediately to the announcement of the future actions but not in the scale it would do if the actions were unanticipated. Still this

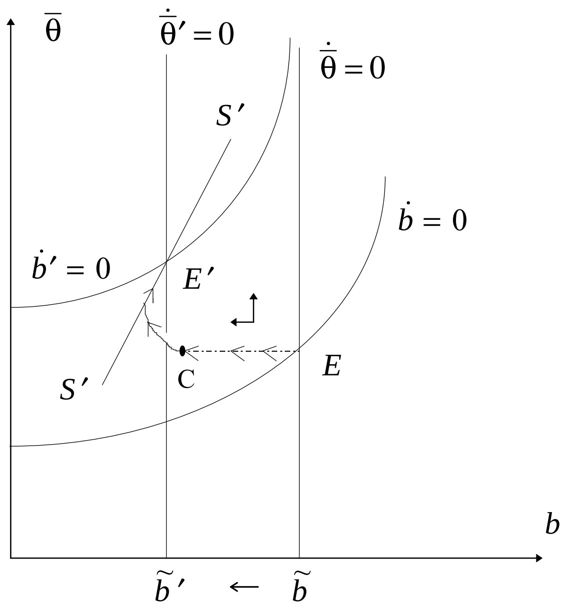

Figure 10. Reaction to anticipated negative shock to $\tilde{b}$

reaction will kick out the system from the equilibrium to the point C on Fig.10 and then the system will evolve according to the current dynamics. This process will continue until the moment when the authority will really make the announced actions. At this moment the dynamics will change and the system will reach the new saddle path and then will evolve along it to the new equilibrium. The main conclusion from this example is that the overshooting will be less in case of anticipated shock to $\tilde{b}$ than in the case of unanticipated one. It might well happen that overshooting will disappear at all. On the other hand if shock is going to be positive then the authority can find it useful to get a possibility to borrow bigger amounts during transition which happens in case of overshooting. These observation leads to a conclusion that within this model the authority has a possibility to control, to some extent, overshooting effect in cases when shocks are due to its actions.

It should be noticed that the overshooting effect in cases of negative shock to the investors' expectations ($\tilde{b} \to \tilde{b}'$) may cause crises, which in this model will look like reaching the boundary (b=0) before getting to the new saddle path and consequently reaching the market collapse area below it. This can happen within quite wide range of the parameter values but there is a range of the parameters where proper information policy can soften this overshooting effect so that crises will not happen.

As it has been already mentioned a positive shock to the sensitivity of the investors expectations to the volume of borrowing α moves $\dot{b} = 0$ locus upward causing the equilibrium to move up as well. The economic interpretation of such a shock may be that the investors get more nervous and sensitive to the borrowing policy of the authority. Why they do so? One of the reasons may be losses experienced in similar markets of another countries. In fact there are a lot of examples when crisis in one market causes waves of increasing security yield in the markets of another countries. Effect of Asian financial crises on the Russian GKO market is one of these examples. If the shock to the sensitivity of the investors' expectations is not accompanied by change in the equilibrium volume of borrowing $\tilde{b}$ then the new equilibrium will differ from the old one only by higher value of $\bar{\theta}$. However the overshooting effect will cause the authority to reduce its borrowing through the security market during the transition to the new equilibrium. This can be easily seen on the Fig.9 by collapsing the loci $\dot{\bar{\theta}} = 0$ and $\dot{\bar{\theta}}' = 0$ into one line. So the shocks of such type do not change the amount of borrowing by the government in the equilibrium, but cost the authority a reduction of borrowing during the transition period. The overshooting effect in cases of shocks to investors' sensitivity can obviously cause a crises of the nature described above. It is clear that such shocks but of the opposite sign give the authority a possibility to enjoy additional amount of borrowing in the transition period. All the considerations about anticipated and unanticipated shocks made above are well applicable here. If the shock results from the authority actions and is of undesired nature it is in the interest of the authority to announce the action well in advance. This announcement will reduce the size of the overshooting effect on the amount of borrowing. The important feature of the system is that, in contrast to the shocks to equilibrium amount of borrowing, it is almost impossible to completely compensate the overshooting effect in response to shocks to the sensitivity α by announcing the corresponding action in advance.

The above example of external shock to the sensitivity of investors' risk estimations to the amount of borrowing can describe such effects observed in the bond market as waves of increasing yield. As it has been shown, the authority should reduce its borrowing in response to the shock if it wants to stay on the equilibrium path. In real life the following is observed: in response to crises in other country market some investors start to sell their security on the secondary market creating the excess supply of the securities that produces the pressure on yield to rise and on price to fall. The authority (CB) supports the price of the paper by buying out the excess of it. This move can be recognised on the Fig.9 as reduction of borrowing in compensation of the shock. After the price has been defended by this reduction of bonds through the open market operations, new bonds are issued and the old equilibrium volume of borrowing is steadily being reached again. This can be recognised on Fig.9 as moving along the new saddle path to the equilibrium. If the authority made all appropriate actions in response to this shock and has successfully defended the market, then it gains higher confidence of the investors in its market. This fact is reflected on Fig.9 as lower risk estimations by the investors (higher $\overline{\theta}$) in the new equilibrium.

As it has been demonstrated in examples of different shocks above, the nature of the system implies that the first immediate reaction to any shocks, in order to get to a new saddle path, is a corresponding change in the amount of the government borrowing. Therefore it should be stated clearly that within the framework of the present model the appropriate borrowing policy is vitally important for staying always on the equilibrium path and the corresponding information policy can only accompany the borrowing one to ease the overshooting effect.

5.3. Possible extensions of the model

One of the features of real economy is that it costs a lot to the authority to borrow less than some fixed amount. In the model described above this feature can be taken into account by changing the restriction $b \geq 0$ to $b \geq b_{\min}$ in the problem formulation (14). This modification does not change the analysis of the problem and the internal equilibrium position. One of consequences of it is that all the trajectories which start above the saddle path end up now at point $b = b_{\min}$ and $\overline{\theta} = 1$, so boundary equilibrium is moved. Of course if it happens that $b_{\min} > \widetilde{b}$ then the equilibrium will not exist any more and the market will always collapse. Another consequence of bounding the amount of borrowing from below is that the authority looses some flexibility of its response to various shocks. Recalling Fig.9 with additional restriction $b \geq b_{\min}$ one can conclude that the bond market becomes more vulnerable to external shocks in a sense that its collapse becomes more probable on one hand. On the other hand the authority has to be much more accurate in its information policy when its actions can negatively affect the investors' expectations because the overshooting effect becomes more serious when the authority can not reduce its borrowing below some level.

Another possible modification to the model is to consider the authority which directly cares about the investors' expectations. The simplest way to make such a modification is to include these expectations in the authority welfare function in the following way:

$$\mathrm{W}_{\mathrm{t}} = p_t b_t - b_{t\text{-}1} + \beta\overline{\theta}, \tag{24}$$

where β describes how much the authority values good investors expectations in comparison with the money obtained through borrowing. This will change in obvious way the objective function of the optimal control problem (14):

$$\mathrm{W} = \int_0^\infty e^{-\delta t}[b_t(p_t - 1 + \delta) + \beta\overline{\theta}]dt \to \max_{\{b_t\}} \tag{25}$$

Subsequently the system of differential equations (20) will change in the following way:

$$\begin{cases} \dot{\overline{\theta}} = \alpha(\widetilde{b} - b) \\ \dot{b} = \dfrac{b^2\alpha\gamma\bar{r} + 2b\gamma\bar{r}(\delta\overline{\theta} - \alpha\widetilde{b}) + \alpha\widetilde{b}\,\bar{r} - \delta(\bar{r}\overline{\theta} - 1 + \delta) + \alpha\beta}{2\gamma\bar{r}\overline{\theta}}, \end{cases} \tag{26}$$

and the expressions for the loci $\dot{\overline{\theta}} = 0$ and $\dot{b} = 0$ transform into:

$$\begin{cases} \dot{\overline{\theta}} = 0: \ b = \widetilde{b} \\ \dot{b} = 0: \ \overline{\theta} = \dfrac{b^2\alpha\gamma\bar{r} - 2b\alpha\gamma\bar{r}\widetilde{b} + \alpha\widetilde{b}\,\bar{r} + \delta(1-\delta) + \alpha\beta}{\delta\bar{r}(1-2\gamma b)} \end{cases} \tag{27}$$

As it can be seen from comparison of the systems (26) and (27) with (20) and (21) the modification (24) does not change very much all the properties of the equilibrium discussed in the previous section. Direct inclusion of the investors' expectations into the authority payoffs in the form (24) only moves $\dot{b} = 0$ locus, saddle path and the internal equilibrium in upward direction. The effect appears somewhat similar to that from positive shock to sensitivity of the investors' expectations α, discussed in the previous section. On one hand it is very natural for such an inclusion that the internal equilibrium occurs at higher $\overline{\theta}$ value and that points of the same $\overline{\theta}$ value on the new saddle path appears to be at lower b values than those on the old saddle path. On the other hand the system becomes more sensitive to shocks of all kinds because now the equilibrium appears to be closer to the boundary than in the original model. All this forces the authority to more cautious behaviour. Needless to say that there exist such values of parameter β which destroy the internal equilibrium and leave the authority with the choice either to borrow 0 (or the minimal amount if it is possible) or to build a pyramid and to collapse the market in nearest future.

There are of course many other extensions of the model which allows to study more tiny effects of real bond market behaviour. Some examples of such extensions can be risk averse players, non-linear prices and introduction of costs of borrowing. All these modifications can be considered in the future research.

6. CONCLUSIONS

The present study is aimed at analysing the government security market crisis in terms of strategies of the major players. It was stimulated by the crisis in the market of the Russian government security papers — GKO/OFZ but the models developed in this study has more general nature and are applicable to any security market. The main results are summarised in this section.

Three game models of the security market have been built in present study. The first one is a static game with incomplete information between two representative players: resident and non-resident investors which have private estimations of risk that they face in the market. The equilibrium in the model is constituted by the risk cut-off value for each player. The property of these cut-off values is that players with risk estimation below it stay in the market and those with higher risk estimations leave the market. It is shown that the risk cut-off values positively depend on the yield of the security and on the average risk estimations of both investors. This model allows to transform the investors' risk estimations into the equilibrium yield of the security.

The second model is a static signalling game with the non-resident investor acting as a leader and a signal sender and the resident one as a follower. There are multiple equilibria in the model but in all the equilibria the resident investor pays more attention to his own risk estimations than to the signal sent by the non-resident one. This outcome coincides with the assumption about rational behaviour of the resident investor and the possibility of the non-resident one to fool his partner with false signal.

The third model is a dynamic repeated game between the resident and non-resident investors and the monetary authority. The authority borrow in the market by supplying one period bonds. The investors act optimally in accordance with the findings of the first model in each cycle of the game and define equilibrium prices of the bonds each period. In the assumption of the linear price it is shown that there may be two equilibria. One is the boundary solution with the authority borrowing the minimal possible amount and the other one is internal equilibrium with saddle path properties. It is demonstrated that the non-equilibrium behaviour of the authority leads either to the boundary equilibrium or to a collapse of the market as the result of financial pyramid built by the authority. In the latter case authority finds itself in conditions with huge debt to be paid to investors and no possibility of future borrowing in the security market. The analysis of reaction of the model to various shocks to the parameters provides clear evidence of the overshooting effect of the following nature: the change in the amount of borrowing during the transition to the new equilibrium exceeds the corresponding change in the equilibrium amounts. The real life examples of such shocks, relevant to the Russian security market, can be Asian financial crises, fall of the world oil prices, etc. It is demonstrated that in some cases it is impossible for the authority to appropriately change the amount of borrowing in order to get to the new saddle path. In such cases shocks can cause the collapse of the market and make a default of the authority on its bond obligations very probable. It is also shown that anticipation of the shocks by the investors leads to the overshooting being less pronounced and therefore allows the authority to control to some extent this dangerous effect. The natural conclusion arises that the corresponding information policy of the authority together with appropriate borrowing policy can help in some circumstances to avoid the security market crisis.

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