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THE CREATION OF FINANCIAL INDUSTRIAL GROUPS
A SIGNALING MODEL

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В работе рассматривается модель, в которой две фирмы, внешне неразличимые но имеющие разный потенциал, желают инвестировать в снижение издержек. Имеется банк, который может дать кредит только одной фирме и предполагается что банк не только дает кредит но и покупает акции предприятия. Чтобы выявить свой тип, фирмы вынуждены подавать сигналы. Рассматривается два типа сигналов: выпуском и экспортом. Ищутся равновесия Нэша и исследуются их свойства.

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There are two firms which are identical for outsiders, but have different potential. Both firms want to invest in cost reduction. The third agent is bank which is able to credit only one firm and wants to invest in the better one. The condition of partial ownership is assumed. To reveal its type each firm has to signal. Two types of signals are considered: by output and by export. The paper considers Nash equilibria and their properties.

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CONTENTS

1. Introduction

Attempts by the Russian Government to promote economic growth have led to the emergence of financial-industrial groups (FIGs). A federal law concerning FIGs was adopted in the middle of 1995 and currently it is claimed that six such conglomerates control over 50% of the Russian economy. While this share is probably overestimated, it points to the growing influence of FIGs in the economic landscape of Russia and provides incentives to look closer at the causes and consequences of FIGs formation.

The standard approach to the analysis of the advantages and disadvantages of such formations appeals to technological complementarities that are realized by the integration of different enterprises, and the cheaper access to funds which can be obtained from banks participating in FIGs. If a bank does not participate in a FIG it is primarily interested in the repayment of loans but it does not care directly about the borrowing enterprises performance in the economy.

Currently there are two major types of FIGs in Russia: industrial and bank-led FIGs. Many of the official FIGs are organized along industrial lines and are comprised of enterprises that were connected in the past (that were part of some ministry, for example). Typically the legal basis for these FIGs is provided by presidential decree. The other type are those FIGs formed by some major bank, such as Menatep, MOST-bank, etc.

Russia is specific in the sense that FIGs which are based on banks (and which are not formed by the President’s decree) almost always are characterized by mutual cross shareholdings. There may be some special benefits that the financial core may provide. One argument, for example, is that banks are reluctant to lend to non-affiliated firms because of the difficulty of ensuring contract fulfillment, and choose instead to purchase shares so that they can enhance their monitoring. From this perspective, incomplete information exposes the bank both to moral hazard and adverse selection.

I believe that FIGs are mainly formed to overcome problems of institutional underdevelopment. First, the Russian legal system does not provide firms with adequate mechanisms to undertake complex transactions, or transactions with new partners. Second, inadequate financial information and collateral system creates an environment of financial underdevelopment. Cross-ownership allows member enterprises to, at least partly, to cope with these problems. Third, cross ownership is very useful in supporting informal profit seeking activities; it opens the possibility that income and expenses can be spread across the different firms in FIG in order to minimize the tax burden on the group as a whole.

Not only firms have incentives to be involved in FIGs, but for banks FIGs are not less interesting to be involved in. Equity participation affords commercial banks greater security via increased ability to monitor enterprises (see [2]). In Russia, it is sometimes the only way to get any information about enterprise's activity. Thus, I assume that banks are very interested in FIGs participation, the only problem which is left is how bank can choose partners.

With absence of the reliable information about candidate firm's activity it is quite difficult to choose between two initially identical firms. I develop below the model when initial statement of the bank about its intentions to invest (participate in FIG) provokes firms to signal about their financial positions. I consider that bank has to choose between two firms which are ex-ante identical and outsider cannot observe whether they are 'good' or 'bad' producers. However, in order to attract investment, these firms start to signal their type. In this paper I consider the standard signaling model as in [1], [3]. The model described in the paper allows to get socially optimal distribution of investment between firms. This model works only when bank intends to buy the share in the firm in which it wants to invest (only in this case the bank has an incentive to invest in the best firm) and does not work in the case of simple lending.

The paper is organized as follows: the first chapter describes the general model and provides all auxiliary results for future development of signaling model disregarding the way of signaling of the firm. The second chapter considers two different types of signals: by export and by output. Third chapter contains conclusions.

2. Chapter 1.

The model

Let us consider two firms compete in quantities and two countries: “home” and “foreign” (the rest of the world).

Let the demand function at home be $q = 1 - p$ and let c represent the constant marginal cost of production ($MC=c$).

Let A be the max. production capacity of the firms and assume that the firms are sufficiently large, that their production choices on domestic market are not constrained by their capacity.

Let the price on the foreign market be $E_1 < 0$. However, the firms have the opportunity to decrease their marginal costs to c_i through appropriate investment in the amount of α rubles, where $c_1 < c_2$ and $c_2 > \tilde{p}$.

Suppose also that neither of these firms have liquid assets on hand for the investment. Based on their differing investment prospects, we call firm 1 the “good” firm and firm 2 the “bad” firm.

There is a bank in this economy which has α available rubles for investment in firms. We assume that the profit from investments is divided equally between the bank and the firm.

We consider a two period model:

In the first period the firms produce their output and sell it at the home and/or at the foreign market and receive their first period profits. Then, the bank decides in which firm it wants to invest, and after the investment takes place we come to the second period.

In the second period the firms again produce and sell output and the realization and distribution of the second period profits takes place.

The problem for each of the firms is to maximize a weighted average of their profits in the first and second periods: $\pi_j = \lambda \pi_j^1 + (1-\lambda)\pi_j^2$ where π_j^i is the profit of firm j in period i; the problem for the bank is to maximize its profit in the second period.

Without investment, in each period the firm *i* chooses its output q_i to maximize its profit and, solving the corresponding Cournot game, we find:

$$q_1 = q_2 = \frac{1-c}{3}$$

$$p = \frac{1+2c}{3},$$

and the profit of each firm in each period is $\pi_i^j = (p-c)q = \frac{(1-c)^2}{9}$.

(See the Appendix for a proof.)

Let $\pi_i^2(k)$ be the gross profit (without deducting for investment and together with the share that will go to the bank) of firm *i* in the second period, under the assumption that the investment took place in firm *k*.

First, consider the situation in which the bank invests in the second firm.

In this case, the results are (see the Appendix for a proof):

$$\begin{cases} q_1 = \dfrac{1-2c+c_2}{3} \\ q_2 = \dfrac{1-2c_2+c}{3} \end{cases}$$

$$p = \frac{1+c+c_2}{3}$$

$$\pi_1^2(2) = \frac{(1+c_2-2c)^2}{9} \text{ and } \pi_2^2(2) = \frac{(1+c-2c_2)^2}{9}.$$

If now we consider the situation in which the bank invests in the first firm and if $c_1 > \tilde{p}$, then the results become (see the Appendix for a proof):

$$\begin{cases} q_1 = \dfrac{1-2c_1+c}{3} \\ q_2 = \dfrac{1-2c+c_1}{3} \end{cases}$$

$$p = \frac{1+c_1+c}{3}.$$

(Note: we assume that $1+c_1 > 2c$ because we want the price p to be higher than the marginal cost of the firm in which the investment was not made.)

$$\pi_1^2(1) = \frac{(1+c-2c_1)^2}{9} \text{ and } \pi_2^2(1) = \frac{(1+c_1-2c)^2}{9}$$

And, if $c_1 < \tilde{p}$, then in equilibrium (see the Appendix for a proof):

$$\begin{cases} q_1 = \dfrac{1-2\tilde{p}+c}{3} \\ q_2 = \dfrac{1-2c+\tilde{p}}{3} \end{cases}$$

$$p = \frac{1+\tilde{p}+c}{3},$$

$$\hat{\pi}_2^2(1) = \frac{(1+\tilde{p}-2c)^2}{9}$$

and $\hat{\pi}_1^2(1) = A(\tilde{p}-c_1) + \dfrac{(1+c-2\tilde{p})^2}{9}$

(Here and throughout the paper, the notation '^' indicates the situation where $c_1 < \tilde{p}$.)

To find the net profit of the firm (in which the investment takes place) in the second period we should subtract the investment α and the half of the excess profit (which goes to the bank) from the profit of this firm.

So, the net profit of the firm *i* in the second period in the case when it receives the investments funds, is:

$$net\pi_i = \frac{\pi_i^2(i) - \alpha - \pi_i^2}{2} + \pi_i^2 = \frac{\pi_i^2(i) + \pi_i^2}{2} - \frac{\alpha}{2} \text{ if } i\text{=2 or both } i\text{=1 and } c_1 > \tilde{p},$$

$$net\hat{\pi}_1 = \frac{\hat{\pi}_1^2(1) + \pi_1^2}{2} - \frac{\alpha}{2} \text{ if } c_1 < \tilde{p}.$$

Let us introduce the following notation:

$B_1 = net\pi_1 - \pi_1^2(2)$,

$\hat{B}_1 = net\hat{\pi}_1 - \pi_1^2(2)$,

$B_2 = net\pi_2 - \pi_2^2(1)$,

$\hat{B}_2 = net\pi_2 - \hat{\pi}_2^2(1)$.

Here B_i can be interpreted as the lost benefit for firm *i*, in the sense that B_i is the difference between the profit of the firm *i* in the second period under the assumption that the investment was made in this firm and its profit in the second period under the assumption that the investment was made in the alternative firm.

We assume that $B_1 > 0$, $\hat{B}_1 > 0$, $B_2 > 0$ and $\hat{B}_2 > 0$, because we are interested in the case when the firm that receives the investment funds is more profitable than it would be if the other firm receives this money.

Remark 1.

We denote $D_i = \pi_i^2(j) - \pi_i^2$, $i \neq j$. This means D_i is the additional payoff which the firm *i* receives in the case when the investment takes place in the other firm in comparison with the case without investments at all. (Analogously if $c_1 < \tilde{p}$: $\hat{D}_2 = \hat{\pi}_2^2(1) - \pi_2^2$.)

Thus, we have: $D_i = \dfrac{(1+c_j-2c)^2-(1-c)^2}{9} = \dfrac{(c_j-c)(2+c_j-3c)}{9} = \dfrac{(c-c_j)(3c-2-c_j)}{9}$;

$\hat{D}_2 = \dfrac{(1+\tilde{p}-2c)^2-(1-c)^2}{9} = \dfrac{(c-\tilde{p})(3c-2-\tilde{p})}{9}$.

Because we have supposed that $1+c_1 > 2c$ and we have 1 > *c*, we have $2+c_1 > 3c$, hence $3c-2-c_1 < 0$. But $c_2 > c_1$, hence $3c-2-c_2 < 0$ and if $\tilde{p} > c_1$, we also have $3c-2-\tilde{p} < 0$. So, we obtain $D_i < 0$ and $\hat{D}_2 < 0$, which means that a firm is worse off if the other firm invests.

Remark 2:

Because $\pi_1^2(1) > \pi_2^2(2)$, $\hat{\pi}_1^2(1) > \pi_2^2(2)$ and $\pi_1^2 = \pi_2^2$ we conclude that the bank will always prefer to invest in firm one (the good firm).

Let us suppose that the quality of firms can not be distinguished by the bank, so, the good firm might be interested in signaling its type. We consider two types of signals: signaling by exports and signaling by output. We consider these two cases separately.

3. Chapter 2.

Case 1. Signaling by exports.

Before trying to describe this situation in general, we would like to prove the following proposition.

Proposition 1.1. There is no pooling equilibrium.
Proof. Let both firms export $\tilde{q}_1 = \tilde{q}_2 = \tilde{q}$. Hence, in the second period the bank chooses the firm for investment randomly, with equal probability. So, the expected profit of firm one in the second period is $\frac{net\pi_1 + \pi_1^2(2)}{2}$ (or $\frac{net\hat{\pi}_1 + \pi_1^2(2)}{2}$ if $c_1 < \tilde{p}$).

If now the first firm increases its export by ε then in the first period it bears additional losses accounted for $\varepsilon(c - \tilde{p})$ but in the second period its profit will be $net\pi_1$ ($net\hat{\pi}_1$ in the case $c_1 < \tilde{p}$). So, it increases its profit by $\frac{B_1}{2}$, ($\frac{\hat{B}_1}{2}$) but $B_1 > 0$ ($\hat{B}_1 > 0$) and therefore even with small ε the first firm can improve its average profit. The proof is completed.

The fact that in this case there is no pooling equilibrium means that investment is always made in the better firm (there is socially optimal investment).

Thus, we see that we can have only separating equilibria, but we should also say several words about what such a separating equilibrium would look like. Firms have losses when they signal by exporting, so , they signal only if their future profit will be high enough to cover these losses. Because the gain from the investment of the first firm is higher than the gain of the second firm, firm 1 is ready to bear higher losses than firm 2. The bank knows this fact too, and so to reveal the good firm it should play the following strategy: Give the money for investment to the firm with larger exports.
Because we are looking for the separating equilibrium in which firms play different strategies the bank can distinguish between them.

Consider a potential equilibrium in which the second firm does not export while the first one exports the amount $\tilde{q}_1$ (such that it is not profitable for the second firm to export more than $\tilde{q}_1$) More formally, this $\tilde{q}_1$ should be such that:

$$\lambda(c-\tilde{p})\tilde{q}_1 = (1-\lambda)B_2 \;\Rightarrow\; \tilde{q}_1 = \frac{(1-\lambda)}{\lambda}\frac{B_2}{c-\tilde{p}}$$

(in the case $c_1 < \tilde{p}$ we have $\hat{\tilde{q}}_1 = \frac{(1-\lambda)}{\lambda}\frac{\hat{B}_2}{c-\tilde{p}}$).

(The assumption that second firm, if it were export $\tilde{q}_1$ will merge with bank with probability 1 (not 0.5) is based on the assumption that second firm will export $\tilde{q}_1 + a$ where a is small.)

Let us find the additional profits which both firms receive in this equilibrium in comparison with the situation without investments at all (we denote these additional profits as E_1 and E_2 (in the case when $c_1 < \tilde{p}$, we call them $\hat{}$

So, in this case, the worse firm has losses comparing with the situation with no bank at all. Thus, the result has obvious explanation: in the process of signaling by export the profit of the second firm at the domestic market does not change in the first period (there is no bad firm at the external market since price there is lower than costs and we have separating equilibrium). In the second period the profit of the second firm decreases because of decreasing costs of the rival firm.

Let $G = E_1 - E_2$ be the difference between the gains of the first firm and the second one (in the case when $c_1 < \tilde{p}$ let $\hat{G} = \hat{E}_1 - \hat{E}_2$).

So, we have:

$$G = (1-\lambda)\left(net\pi_1 - net\pi_2\right) = (1-\lambda)\left(\frac{\pi_1^2(1) - \pi_2^2(2)}{2}\right) =$$

$$= \frac{(1-\lambda)}{2}\left[\frac{\left(1+c-2c_1\right)^2}{9} - \frac{\left(1+c-2c_2\right)^2}{9}\right] = (1-\lambda)\frac{\left(c_2-c_1\right)\left(1+c-c_1-c_2\right)}{9}.$$

We have that if c_2 is only a little bit larger than c_1, then G is small and E_1 is only a little bit larger than E_2 but we have $E_2 < 0$.

So, in this case if $c_1 = c_2 - \varepsilon$ where ε is small we have that $E_1 < 0$ and therefore both firms suffer losses when the bank appears at the market.

$$\hat{G} = (1-\lambda)\left(net\hat{\pi}_1 - net\pi_2\right) = (1-\lambda)\left(\frac{\hat{\pi}_1^2(1) - \pi_2^2(2)}{2}\right) =$$

$$= \frac{(1-\lambda)}{2}\left[\frac{\left(1+c-2\tilde{p}\right)^2}{9} - \frac{\left(1+c-2c_2\right)^2}{9} + A\left(\tilde{p}-c_1\right)\right] =$$

$$= (1-\lambda)\left[\frac{\left(c_2-\tilde{p}\right)\left(1+c-\tilde{p}-c_2\right)}{9} + A\left(\tilde{p}-c_1\right)\right]$$

From this formula we can conclude that even if c_1 is slightly different from c_2 and A is large, then $\hat{G}$ can be also large (because of the possibility for the firm to export) and $E_1 > 0$.

But even in this particular case, with c_1 and c_2 sufficiently close it can be true that $\hat{E}_1 < 0$ and both firms receive lower profits if a bank is available to make an investment loan.

This phenomenon (in which the existence of a bank leads to losses for both firms) can take place because an investment in one of the firms has an external effect on the profit of the other firm; therefore, firms will compete for investment not only to directly receive higher profits, but also to avoid the losses that they will have if the other firm receives the loan. The cost of their competition in the first period can be larger than their profits in the second period, causing both firms to be worse of, while the bank still makes a profit on its loan.

Case 2. Output as a signal

Now, let the total output of firms be observable by all parties, including the bank.
Let $Q_i = q_i + \tilde{q}_i$ be the output of firm *i*; q_i is the sales of firm *i* on the domestic market and $\tilde{q}_i$ is the sales of firm *i* on the foreign market.
Assume that the bank invests in the firm with the larger output. Below, for notational simplicity, we do not write "^" above the variables in the case where $c_1 < \tilde{p}$. We will consider only the case where $c_1 > \tilde{p}$, although all the results obtained below can be easily adopted to the case $c_1 < \tilde{p}$.

Proposition 2.1. There is no pooling equilibrium.
The proof is similar to the proof of proposition 1.1:
Assume that a pooling equilibrium $Q_1 = Q_2$ exists. Hence, the expected profit of firm one in the second period is $(net\pi_1 + \pi_1^2(2))/2$. If it increases its production Q_1 by ε, for example increasing $\tilde{q}_1$ by ε, the firm bears additional losses amounting to $\varepsilon(\tilde{p} - c)$, but it wins the bank loan. Therefore, its expected profit in the second period is $net\pi_1$, i.e. the firm increases its profit by $B_1 / 2$. Since $B_1 > 0$ by choosing ε which is small enough, firm 1 can increase its weighted average profit. Hence, the initial state is not an equilibrium. The proof is completed.

Let us find a separating equilibrium. Since in a separating equilibrium the bank can distinguish between the two firms and, hence, chooses to loan to firm 1, then the second firm maximizes its profit in the first period given q_1 and $\tilde{q}_1$. The first firm chooses q_1 and $\tilde{q}_1$ in such a way:
(i) to maximize its profit in the first period; and
(ii) to make the intervention of the second firm unprofitable.

In this connection some interesting questions arise.

1. Is it possible that $\tilde{q}_1 > 0$ and under which conditions will this take place? In other words, when does firm 1 signaling through total output, choose to export? It happens (and it will be shown later in the text) that it is possible that $\tilde{q}_1 > 0$, i.e. the firm exports in the first period, even when $c_1 > \tilde{p}$, i.e., after investing the firm will not export.
2. Does q_1 increase or decrease compared with the situation without investment? In other words, is it possible that the firm will decrease sales on the domestic market when it has the opportunity to signal its type through total output?
3. Is it possible that $1 - q_1 < c$, i.e., that the price on the domestic market becomes less than costs, leading firm 2 to go exit the market in the first period?
4. Is it possible that $1 - q_1 < \tilde{p}$, i.e. the price on the domestic market becomes less than the price on the foreign market?
5. What are advantages and disadvantages of signaling by output compared with signaling by exports?

Obviously, the bank will be indifferent between these two cases because the cost of signaling is paid by firm 1. Of course, firm 1 is not indifferent between these situations. Moreover, in the case of signaling by exports, the signaling of firm 1 does not have an external effect on the profit of firm 2, while in the case of signaling by output the change in q_1 has an external effect on the profit of firm 2. Thus, firm 2 is not indifferent between these situations, either.

Let us write the problem more formally.
Let q_1 and $\tilde{q}_1$ be the first firm's outputs. Consider two cases: first, when $1 - q_1 > c$ and then, when $1 - q_1 < c$.
1. If $1 - q_1 > c$ we have that firm 2 will also produce and its problem will be

$$(1 - c - q_1 - q_2)q_2 \to \max_{q_2} \Rightarrow \text{F.O.C: } q_2 = \frac{1 - c - q_1}{2}.$$

So, we can find $p = 1 - q_1 - q_2 = \frac{2 - 2q_1 - 1 + c + q_1}{2} = \frac{1 - q_1 + c}{2}$ and the profit of the second firm on the domestic market will be $F_2 = (p - c)q_2 = \frac{(1 - c - q_1)^2}{4}$.

The profit of firm 1 on the domestic market will be $F_1 = (p - c)q_1 = \frac{(1 - c - q_1)}{2} q_1$.

If firm 2 wants to compete with firm 1 for the bank investment, when it chooses q_2 and $\tilde{q}_2 = q_1 + \tilde{q}_1 - q_2$ (in fact, it should be $\tilde{q}_2 = q_1 + \tilde{q}_1 - q_2 + \varepsilon$, but ε can be small), than q_2 in the first period will be chosen from the profit maximization problem:

$(1 - c - q_1 - q_2)q_2 + (q_1 + \tilde{q}_1 - q_2)(\tilde{p} - c) \to \max\limits_{q_2}$, and we have

$$q_2 = \frac{1 - c - q_1 - \tilde{p} + c}{2} = \frac{1 - q_1 - \tilde{p}}{2},$$

$p = 1 - q_1 - q_2 = \frac{1 - q_1 + \tilde{p}}{2}$, and $\tilde{q}_2 = \frac{3q_1 - 1 + \tilde{p}}{2} + \tilde{q}_1$. (We should also note the constraints $q_2 \geq 0$ and $\tilde{q}_2 \geq 0$, but, as will be seen later, these conditions are automatically satisfied.)

Thus, the gain of firm 2 from the intervention into the investment seeking is

$$\tilde{F}_2 = (p - c)q_2 + \tilde{q}_2(\tilde{p} - c) = \frac{(1 - q_1 + \tilde{p} - 2c)(1 - q_1 - \tilde{p})}{4} + \frac{3q_1 - 1 + \tilde{p}}{2}(\tilde{p} - c) + \tilde{q}_1(\tilde{p} - c)$$

and the additional profit from this intervention is

$$\Delta F_2 = \tilde{F}_2 - F_2 = \frac{3}{2} q_1(\tilde{p} - c) + \tilde{q}_1(\tilde{p} - c) + \frac{\tilde{p}^2 - 2\tilde{p} + 2c}{4}.$$

So, the problem for firm 1 is:

$$\begin{cases} \lambda\,[\,F_1 + \tilde{q}_1(\tilde{p} - c)] + (1 - \lambda)B_1 \to \max\limits_{q_1, \tilde{q}_1}, \\ \quad s.t.: \quad (1) \quad \lambda \Delta F_2 + (1 - \lambda)B_2 \leq 0, \\ \qquad\qquad (2) \qquad \tilde{q}_1 \geq 0. \end{cases}$$

It is easy to show that (1) is satisfied with equality. Let us suppose that (2) is not binding and we can omit it. By doing this, we can, firstly, find the conditions under which (2) is not binding and, secondly, find the solution $q_1, \tilde{q}_1$.

From (1) we obtain $\Delta F_2 = -\frac{1 - \lambda}{\lambda} B_2$. Hence,

$\tilde{q}_1(\tilde{p}-c)=-\frac{1-\lambda}{\lambda}B_2-\frac{\tilde{p}^2-2\tilde{p}+2c}{4}-\frac{3}{2}q_1(\tilde{p}-c)$. Substituting this expression into the problem for firm 1, taking the FOC with respect to q_1, and using $F_1=\frac{1-c}{2}q_1-\frac{q_1^2}{2}$, we obtain $q_1=\frac{1-c}{2}-\frac{3}{2}(\tilde{p}-c)=\frac{1+2c-3\tilde{p}}{2}>0$.

Recalling the condition $1-q_1>c$ leads to the following condition on $(\tilde{p},c)$: $\tilde{p}>\frac{4c-1}{3}$, and q_1 can be found from (1).

$$\tilde{q}_1=-\frac{1-\lambda}{\lambda}\frac{B_2}{\tilde{p}-c}-\frac{\Delta F_2}{\tilde{p}-c}=$$
$$=\frac{1-\lambda}{\lambda}\frac{B_2}{c-\tilde{p}}-\frac{1}{\tilde{p}-c}[\frac{3}{2}\frac{(1+2c-3\tilde{p})}{2}(\tilde{p}-c)+\tilde{q}_1(\tilde{p}-c)+\frac{\tilde{p}^2-2\tilde{p}+2c}{4}]=$$
$$=\frac{1-\lambda}{\lambda}\frac{B_2}{c-\tilde{p}}-\frac{3(1+2c-3\tilde{p})}{4}+\frac{\tilde{p}^2-2\tilde{p}+2c}{4(c-\tilde{p})}=$$
$$=\frac{1-\lambda}{\lambda}\frac{B_2}{c-\tilde{p}}-\frac{3[c+2c^2-3\tilde{p}c-\tilde{p}-2c\tilde{p}+3\tilde{p}^2]+\tilde{p}^2-2\tilde{p}+2c}{4(c-\tilde{p})}=$$
$$=\frac{1-\lambda}{\lambda}\frac{B_2}{c-\tilde{p}}-\frac{5c+6c^2-15\tilde{p}c-5\tilde{p}+10\tilde{p}^2}{4(c-\tilde{p})}.$$

So, we have derived the two conditions

$$\frac{1-\lambda}{\lambda}\frac{B_2}{c-\tilde{p}}-\frac{5c+6c^2-15\tilde{p}c-5\tilde{p}+10\tilde{p}^2}{4(c-\tilde{p})}>0 \qquad (*)$$

and

$$\tilde{p}>\frac{4c-1}{3}. \qquad (**)$$

These conditions are sufficient to make $\tilde{q}_1>0$, which means that firm 1, in the process of signaling by total output enters the foreign market. Note that condition (*) always holds if λ is small enough (that is, if future profit has a large weight in the firms objective function), but a situation (if $\tilde{p},c,\lambda$ are high) where this condition does not hold is also possible. In that case $\tilde{q}_1=0$ in equilibrium and there is no participation at the foreign market. In such case, taking $\tilde{q}_1=0$ we can find q_1 simply from (1):

$$\frac{3}{2}q_1 = \frac{(1-\lambda)}{\lambda}\frac{B_2}{(c-\widetilde{p})} + \frac{\widetilde{p}^2 - 2\widetilde{p} + 2c}{4(c-\widetilde{p})} > 0.$$

Moreover, because (*) does not hold we have $\frac{(1-\lambda)}{\lambda}\frac{B_2}{(c-\widetilde{p})} + \frac{\widetilde{p}^2 - 2\widetilde{p} + 2c}{4(c-\widetilde{p})} - \frac{3(1+2c-3\widetilde{p})}{4} < 0$ and we can conclude that $\frac{3}{2}q_1 < \frac{3(1+2c-3\widetilde{p})}{4}$. So, a sufficient condition for $q_1 < 1-c$ is $1-c < \frac{3(1+2c-3\widetilde{p})}{4}$, which can be rewritten as $\widetilde{p} > \frac{4c-1}{3}$.

So, we have that $q_1 < 1-c$ if and only if condition (**) holds. So (**) is a necessary and sufficient condition to have a situation where the second firm will produce in the first period. Moreover, if both (**) and (*) both holds, then $\widetilde{q}_1 > 0$ and firm 1 will export in the first period.

Proposition: Firm 1 increases its sales on the domestic market in comparison with the non-investment case, when there is signaling by total output and when $1-q_1 < c$.

Proof:

If $\widetilde{q}_1 > 0$ and $\widetilde{p} > \frac{4c-1}{3}$, then $q_1 = \frac{1+2c-3\widetilde{p}}{2} = \frac{1-c+3c-3\widetilde{p}}{2} > \frac{1-c}{2} > \frac{1-c}{3}$. Hence, firm 1 increases its sales on the domestic market.

If $\widetilde{q}_1 > 0$ and $\widetilde{p} < \frac{4c-1}{3}$, then $q_1 > 1-c > \frac{1-c}{3}$,

if $\widetilde{q}_1 = 0$ and $\widetilde{p} > \frac{4c-1}{3}$, then q_1 is located in the interval from $\frac{\widetilde{p}^2 - 2\widetilde{p} + 2c}{4(c-\widetilde{p})} \cdot \frac{2}{3}$ to $\frac{3(1+2c-3\widetilde{p})}{4} \cdot \frac{2}{3}$.

Is it possible that $q_1 < \frac{1-c}{3}$? It is possible only if $\frac{\widetilde{p}^2 - 2\widetilde{p} + 2c}{4(c-\widetilde{p})} \cdot \frac{2}{3} < \frac{1-c}{3}$, which can be rewritten as $\widetilde{p}^2 + 2c^2 - 2\widetilde{p}c < 0$. But $c > \widetilde{p}$ and $4c^2 - 4\widetilde{p}c > 0$ leads to a contradiction. Hence, $q_1 > \frac{1-c}{3}$.

If $\tilde{q}_1 = 0$ and $\tilde{p} < \frac{4c-1}{3}$,then $q_1 > 1-c > \frac{1-c}{3}$. So, the proposition is proved.

Now, consider the second case; when $q_1 > 1-c$ which implies that firm 2 goes exit the market in the first period.

In this case, $F_2 = 0$, $F_1 = (1-q_1-c)q_1$, and $\tilde{F}_2$ does not change. $\Delta F_2 = \tilde{F}_2$.

The problem for firm 1 will be

$$\begin{cases} \lambda\,[F_1 + \tilde{q}_1(\tilde{p}-c)] + (1-\lambda)B_1 \to \max\limits_{q_1,\tilde{q}_1}, \\ s.t.: \qquad (1) \quad \lambda \Delta F_2 + (1-\lambda)B_2 \le 0, \\ \qquad\qquad (2) \qquad \tilde{q}_1 \ge 0. \end{cases}$$

As before, we can take (1) as an equality and assume that (2) is not binding. So, from (1) we find

$$\tilde{q}_1(\tilde{p}-c) = -\frac{1-\lambda}{\lambda}B_2 - \frac{(1-q_1+\tilde{p}-2c)(1-q_1-\tilde{p})}{4} - \frac{3q_1-1+\tilde{p}}{2}(\tilde{p}-c) =$$

$$= -\frac{q_1^2}{4} + q_1[\frac{1+\tilde{p}-2c+1-\tilde{p}}{4} - \frac{3}{2}(\tilde{p}-c)] + \text{constant} =$$

$$= -\frac{q_1^2}{4} + q_1\frac{1-3\tilde{p}+2c}{2} + \text{constant}.$$

Substituting this expression into the problem of firm 1 and taking FOC with respect to q_1, we obtain $-2q_1 + (1-c) - \frac{q_1}{2} + \frac{1+2c-3\tilde{p}}{2} = 0$ which can be rewritten as $q_1 = \frac{3}{5}(1-\tilde{p})$.

Conditions for $\tilde{q}_1 = 0$ can be written from (1) analogously with the previous case. The result is an unwieldy formula that does not contain important information for our purpose, so, we do not write it here explicitly.

Remark. $q_1 = \frac{3}{5}(1-\tilde{p})$, hence $\tilde{p} < 1-q$. This means that the price on the domestic market will always be larger than the price on the foreign market.

4. Chapter 3.

Results

1.A signaling game as analyzed here is possible only under a situation where the bank has partial ownership in the firm that it lends to. In the case without partial ownership (with pure lending) the bank is indifferent to which firm to invest.

2. With partial ownership, investment always takes place by the more efficient firm.

3. It is possible that during the “investment-seeking” period, the price set by a ‘good’ firm is lower and its exports are higher than after the investment. Moreover, it is possible, even when firms signal by output, that the ‘good’ firm will export before the investment and will not export after it invest.

4. The ‘bad’ firm usually is worse off in comparison to the case where there is no possibility to invest. The bank, alternatively, always makes a profit. The ‘good’ firm may be either better off or worse off. So, the possibility of investment may benefit the industry or may lead to losses because of the firm’s losses during the investment-seeking period. But because these losses are due to price decreasing (and this price rarely falls lower than marginal costs), the investment usually leads to a benefit for society, where society includes not only the bank and the firms, but also consumers.

5. In the situation with the signaling by export the first firm should export such amount that it is unprofitable for the second firm to compete for investment. At the same time in the situation with the signaling by output the first firm can use two instruments: domestic sales and export.

To increase the possible losses of the second firm from interventions the first firm can either raise export or decrease domestic sales. For the second case (decreasing of domestic sales) the first firm allows the second one to have more profit in the first period. In the exchange the second firm will not compete for investment.

However, this case is impossible and ‘good’ firm increases sales at the domestic market.

So, the ‘bad’ firm has losses in both periods.

Moreover, it is possible the situation when the first firm sales so much at the domestic market in the first period that the price at this market becomes lower than costs and the second firm has to leave the market in the first period. At the same time, the price at the domestic market is always higher than the price at the external market.

Intuitively, price decrease at the domestic market has twofold effect on profit of the first firm.

(i) Lower domestic price affects profit of the first firm directly.

(ii) The profit of the second firm goes down that lowers losses when second firm competes for investment. Second firm will not compete for investment if its possible losses from this competition are higher than some fixed amount (possible losses equal the profit of the second firm if it does not compete for investment minus its profit if it competes).

Thus, to prevent second firm from this competition for investment the first firm should increase its output in the first period and this leads to additional losses for the first firm. This is indirect effect.

Concluding remarks

Described model does not work for the case of debt contract. The main reason is that in the case of debt contract the return of the bank consists of the principal and the interest payments, so, the bank is indifferent to which firm ('good' or 'bad' one) to invest.

Moreover, with underdeveloped legal base in Russia, the probabilities of loan repayment are low for both 'good' firm and 'bad' one. (We assume here that if the bank gives money to the firm but does not control the way of spending them, there is a high probability that the firm will spend money not to the agreed project.) Thus, we consider the situation where the probabilities of non-repayment under pure debt contract are equal (and large) for both firms, but under the partial ownership scheme the debt is always repaid. Because of such situation banks in Russia almost never give credits to firms where they do not have partial ownership.

Since it does not matter for bank which firm to invest in (for the case of debt contract) for social welfare increasing it would be desired to give incentives for bank to invest in 'good' firm. The only possibility to make bank interested is to give it the part of additional firm's profit from the investment. The most simple way of doing that is to sell shares to bank. It is also possible to include the additional profit into the debt contract.

Unfortunately it is difficult to do that since

(i) it is hard to define profit from particular investment

(ii) the firm will try to hide this profit from the bank.

It is possible also to consider the problem where the 'good' firm repays debt in the case of pure debt contract and the 'bad' firm does not do that. In this case firms also have to revel there types and we will get signaling games similar to those we have considered in this paper.

This model can be tested on empirical data. We need to look at output data for firms which are members of Financial Industrial Groups and look for differences in policy of firms before they enter the FIG and after that. Such firms had to lower prices before the 'entry time' (damping policy) and after getting investment through FIG they had to have prices lower than competing firms had but higher than in the first period**.**

5. Appendix

1) In the situation without investment, in each period firm *i* chooses its output q_i to maximize its profit, taking as given the output of its competitor:

$$\left(1-c-q_i-q_j\right)q_i \to \max_{q_i}.$$

So, the F.O.C. is $1-c-2q_i-q_j=0$, and we have the unique symmetric equilibrium: $q_1=q_2=\frac{1-c}{3} \Rightarrow p=1-q_1-q_2=\frac{1+2c}{3}$, and the profit of each firm in each period is $\pi_i^j=\left(p-c\right)q=\frac{(1-c)^2}{9}$.

2) The bank invests in the second firm.

In this case, in the second period firm 1 will solve the following problem:

$$\left(1-c-q_i-q_j\right)q_i \to \max_{q_i} \Rightarrow 2q_i+q_j=1-c.$$

Firm 2, in which the investment took place, will solve:

$$\left(1-c_2-q_1-q_2\right)q_2 \to \max_{q_2} \Rightarrow 2q_2+q_1=1-c_2.$$

Together, these problems lead to:

$$\begin{cases} q_1=\dfrac{1-2c+c_2}{3} \\ q_2=\dfrac{1-2c_2+c}{3} \end{cases}$$

and $p=1-q_1-q_2=\frac{1+c+c_2}{3}$.

Therefore, $\pi_1^2(2)=\left(p-c\right)q_1=\frac{\left(1+c_2-2c\right)^2}{9}$ and $\pi_2^2(2)=\left(p-c_2\right)q_2=\frac{\left(1+c-2c_2\right)^2}{9}$.

3) The bank invests in the first firm and $c_1>\widetilde{p}$.

The first firm solves:

$$\left(1-c_1-q_1-q_2\right)q_1 \to \max_{q_1} \Rightarrow 2q_1+q_2=1-c_1.$$

While firm 2 solves:

$$\left(1-c-q_1-q_2\right)q_2 \to \max_{q_2} \;\Rightarrow\; 2q_2+q_1=1-c.$$

Solving simultaneously yields

$$\begin{cases} q_1=\dfrac{1-2c_1+c}{3} \\ q_2=\dfrac{1-2c+c_1}{3} \end{cases}$$

and $p=\dfrac{1+c_1+c}{3}$.

Therefore, $\pi_1^2(1)=\left(p-c_1\right)q_1=\dfrac{\left(1+c-2c_1\right)^2}{9}$ and $\pi_2^2(1)=\left(p-c\right)q_2=\dfrac{\left(1+c_1-2c\right)^2}{9}$.

4) The bank invests in the first firm and $c_1<\tilde{p}$.

The problem for the first firm now becomes:

$$\left(1-c_1-q_1-q_2\right)q_1+(A-q_1)\tilde{p} \to \max_{q_2} \;\Rightarrow\; 2q_2+q_1=1-\tilde{p}.$$

The problem for the second firm is the same as in case (3).

So, we have $\begin{cases} q_1=\dfrac{1-2\tilde{p}+c}{3} \\ q_2=\dfrac{1-2c+\tilde{p}}{3} \end{cases}$ and $p=\dfrac{1+\tilde{p}+c}{3}$;

$\hat{\pi}_2^2(1)=\left(p-c\right)q_2=\dfrac{\left(1+\tilde{p}-2c\right)^2}{9}$ and

$\hat{\pi}_1^2(1)=\left(p-c_1\right)q_1+(A-q_1)(\tilde{p}-c_1)=\dfrac{\left(1+c-2\tilde{p}\right)^2}{9}+A(\tilde{p}-c_1)$.

6. References

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