Mortgage Contracts and Underwater Default

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Introduction: objectives

- Objectives:
  - Analysis of different mortgage contracts and their comparison
  - These contracts aim to remove the possibility of selective default
  - Option-based approach: options to default and prepay
- Context:
  - When house prices go down, the mortgage may be 'underwater' and borrower can default selectively
  - 2007-2009 crisis highlighted this problem
  - Defaults create feedback loop
  - Foreclosure costs for bank are high: direct and indirect
  - Standard mortgage contracts exacerbate wealth inequality problem (Mian and Sufi)
  - Several contracts were proposed to address this issue

Introduction: standard contract

- Fixed rate mortgage (FRM) contract
- Input: initial balance B_0, mortgage rate m and maturity T
- Balance dynamics

$$dB_t^F = (mB_t^F - c^F)dt$$

and $B_T = 0$ so that

$$B_t^F = B_0 \frac{1 - e^{-m(T-t)}}{1 - e^{-mT}}$$

- Coupon payment is

$$c^F = \frac{mB_0}{1 - e^{-mT}} = \frac{mB_t}{1 - e^{-m(T-t)}}$$

- The house price H_t is stochastic process, we assume that $H_0 = 1$
- When $H_t < B_t^F$ and is sufficiently low, borrower may default strategically

Proposed contracts

- We group the proposed contracts into two broad categories:
- Adjustable Balance Mortgage (ABM), Ambrose and Buttimer (2012)
- Adjustable Payment Rate Mortgage (APRM): two examples:
  - Continuous Workout Mortgage (CWM), Shiller, Wojakowski, Ebrahim, Shackleton (2013, 2019)
  - Shared Responsibility Mortgage (SRM), Mian and Sufi (2016)
- Main idea: balance and mortgage payments are reduced when house prices decline

Proposed contracts

- Economists analyzed this type of contracts mainly from principal-agent and/or equilibrium considerations: Piskorski and Tchistyi (2010, 2011, 2017); Campbell, Clara and Cocco (2018); Greenwald, Landvoigt and Van Niewerburgh (2021); Guren, Krishnamurthy and McQuade (2021).
- In this paper, we consider valuation of these contracts using option-based framework (see, e.g., Kau and Keenan (1995)).
- We formulate and analyze associated optimal timing problems.
- We use American options pricing methodology, while also allowing for mortgage turnover. More precisely, excluding turnover related prepayments, we assume that the bank takes a worst-case approach.

Proposed contracts

- However, it was recognized that borrowers do not always act in a financially optimal manner.
- This led to the popularity of reduced form models for mortgage valuation (see, e.g., Schwartz and Torous (1989)).
- Despite its pitfalls, in order to compare the proposed contracts, we believe the options pricing approach is appropriate.
- Simply put, as the contracts' stated objective is to reduce selective default, we must assume the borrower is sophisticated enough to selectively default.

Adjustable Balance Mortgage

- Input: initial balance B_0, mortgage rate m^A, maturity T, local house index H
- Then define nominal remaining balance $\widehat{B}^A$ and payment rate $\widehat{c}^A$ using

$$\widehat{B}_t^A = \frac{B_0\left(1 - e^{-m^A(T-t)}\right)}{1 - e^{-m^A T}}; \quad \widehat{c}^A = \frac{m^A B_0}{1 - e^{-m^A T}}.$$

- The actual remaining balance B^A is set to

$$B_t^A = \min(\widehat{B}_t^A, H_t), \quad t \leq T$$

- The actual payment rate c^A

$$c_t^A = \frac{m^A B_t^A}{1 - e^{-m^A(T-t)}} = \widehat{c}^A \times \min\left(1, \frac{H_t}{\widehat{B}_t^A}\right), \quad t \leq T$$

- The prepayment amount $\mathcal{B}_t^A = B_t^A$.

Adjustable Payment Rate Mortgage

- Input: initial balance B_0, mortgage rate m^P and maturity T
- Then define nominal remaining balance $\widehat{B}^P$ and payment rate $\widehat{c}^P$ using

$$\widehat{B}_t^P = \frac{B_0\left(1 - e^{-m^P(T-t)}\right)}{1 - e^{-m^P T}}; \quad \widehat{c}^P = \frac{m^P B_0}{1 - e^{-m^P T}}.$$

- We define payment rate

$$c_t^P = \widehat{c}^P \times \min(1, H_t)$$

- The balance of SRM is given by

$$B_t^P = c_t^P \times \frac{1 - e^{-m^P(T-t)}}{m^P} = \widehat{B}_t^P \times \min(1, H_t)$$

- Additional feature: upon prepayment the borrower shares a fraction (e.g. $\alpha = 5\%$) of capital gain

$$\mathcal{B}_t^P := B_t^P + \alpha\,(H_t - 1)^+$$

Model

- We apply risk-neutral pricing under measure Q
- The house price index H follows

$$dH_t/H_t = (r-\delta)dt + \sigma dW_t$$

- No basis risk
- r is constant interest rate (could be also stochastic process)
- δ is 'dividend' yield or utility that house provides to the borrower
- $\sigma > 0$ is constant volatility
- W is SBM under Q
- $T \leq \infty$ is the mortgage maturity date
- We can also allow for non-strategic behavior corresponding to turnover (i.e prepayment/default due to income loss, job relocation, death, divorce, etc.)

Assumptions

In the current paper we assume that the bank is conservative and is prepared for the worst case scenario

- Borrower chooses stopping rule that is worst for bank
- At stopping time τ borrower makes choose between default and prepayment in optimal way, i.e., bank receives

$$\min(H_\tau, \mathcal{B}^i_\tau)$$

Contract	Payment Rate at t	Prepayment Amount at t
FRM	$m^F B_0$	B_0
ABM	$m^A \min[B_0, H_t]$	$\min[B_0, H_t]$
APRM	$m^P B_0 \min[1, H_t]$	$B_0 \min[1, H_t] + \alpha(H_t - 1)^+$

Optimal stopping problem

- We now define the contract/option values as the value functions of corresponding optimal stopping problems.
- As H is a Markov process, the bank assigns the contract a value of

$$V^i(t,h) = \inf_{\tau \geq t} \mathbb{E}_{t,h} \left[\int_t^\tau e^{-r(u-t)} c_u^i du + e^{-r(\tau - t)} \min(H_\tau, \mathcal{B}_\tau^i) \right]$$

  for $h > 0$, where the infimum is taken over all stopping times τ with values greater than t.
- We now further assume that we perpetual contracts, i.e., $T = \infty$.

FRM

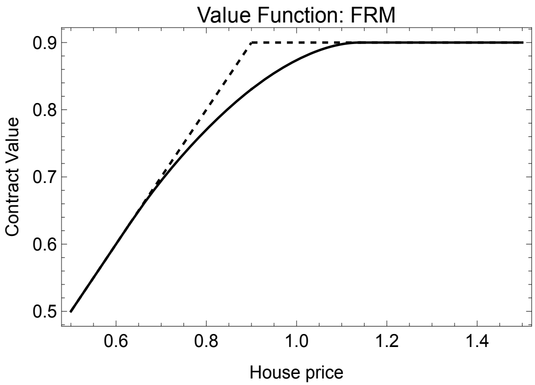

Figure: The value function $V^F(h)$ of FRM (solid line) versus the payoff function $f(h) = \min(B_0, h)$ (dashed line).

ABM

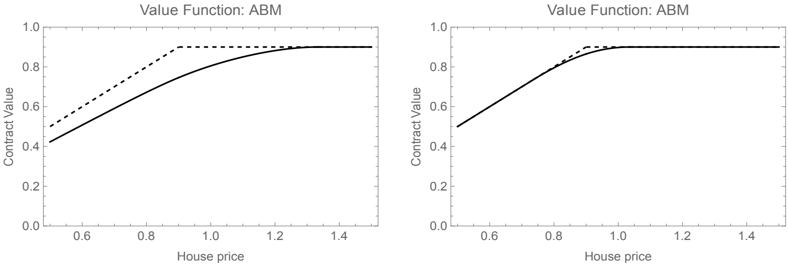

Figure: The value function $V^A(h)$ of ABM (solid line) versus the payoff function $f(h) = \min(B_0, h)$ (dashed line). Left panel: $m^A < \delta$. Right panel: $m^A > \delta$.

APRM

Assume $m^P \leq \delta$ and define critical threshold α^*.
(i) When $\alpha < \alpha^*$ the action regions and value function are

h	< 1	$\in [1, h_2)$	$\in [h_2, h_3]$	$> h_2$
Action	Continue	Continue	Prepay	Continue
$V^P(h)$	$C_1 h^{p_1} + \frac{m^P B_0}{\delta} h$	$\widetilde{C}_1 h^{p_1} + \widetilde{C}_2 h^{-p_2} + \frac{m^P B_0}{r}$	$B_0 + \alpha(h-1)$	$\check{C}_2 h^{-p_2} + \frac{m^P B_0}{r}$

where the constants $C_1, \widetilde{C}_1, \widetilde{C}_2, \check{C}_2$ are all negative and h_2, h_3 are optimal prepayment boundaries.

(ii) When $\alpha \geq \alpha^*$ the action regions and value function are

h	< 1	> 1
Action	Continue	Continue
$V^P(h)$	$K_1 h^{p_1} + \frac{m^P B_0}{\delta} h$	$\widetilde{K}_2 h^{-p_2} + \frac{m^P B_0}{r}$

where the constants $K_1, \widetilde{K}_2$ are all negative.

APRM

We define critical thresholds m^* and α^*, and assume $\delta < m^P < m^*$.
For (i) $\alpha < \alpha^*$ the action regions and value functions are

h	$\leq h_1$	$\in (h_1, 1]$	$\in [1, h_2)$	$\in [h_2, h_3]$
Action	Prepay	Continue	Continue	Prepay
$V^P(h)$	$B_0 h$	$C_1 h^{p_1} + C_2 h^{-p_2} + \frac{m^P B_0}{\delta} h$	$\widetilde{C}_1 h^{p_1} + \widetilde{C}_2 h^{-p_2} + \frac{m^P B_0}{r}$	$B_0 + \alpha(h-1)$

where the constants $C_1, C_2, \widetilde{C}_1, \widetilde{C}_2, \check{C}_2$ are all negative, and h_1, h_2, h_3 are optimal prepayment boundaries.
For (ii) $\alpha \geq \alpha^*$ the action regions and value function are

h	$\leq h_1$	$\in (h_1, 1]$	> 1
Action	Prepay	Continue	Continue
$V^P(h)$	$B_0 h$	$K_1 h^{p_1} + K_2 h^{-p_2} + \frac{m^P B_0}{\delta} h$	$\widetilde{K}_2 h^{-p_2} + \frac{m^P B_0}{r}$

where the constants $K_1, K_2, \widetilde{K}_2$ are all negative, and h_1 is the optimal prepayment boundary.

APRM

Assume $m^P \geq m^*$ and $\alpha < B_0$. Then, the action regions and value function are

h	$\leq h_1$	$\in (h_1, 1]$	$\in [1, h_2)$	$\in [h_2, h_3]$
Action	Prepay	Continue	Continue	Prepay
$V^P(h)$	$B_0 h$	$C_1 h^{p_1} + C_2 h^{-p_2} + \frac{m^P B_0 h}{\delta}$	$\widetilde{C}_1 h^{p_1} + \widetilde{C}_2 h^{-p_2} + \frac{m^P B_0}{r}$	$B_0 + \alpha(h-1)$

where the constants $C_1, C_2, \widetilde{C}_1, \widetilde{C}_2, \check{C}_2$ are all negative, and h_1, h_2, h_3 are optimal prepayment boundaries.

Foreclosure costs

- We assume that upon default of the FRM at time τ, there is a fractional loss ϕ incurred by the bank, so that the bank receives $(1-\phi)H_\tau$.
- Therefore, the FRM has foreclosure-adjusted value

$$V_\phi^F(h) = V^F(h) - \phi h_1 \mathbb{E}^h \left[e^{-r\tau_1(h)} 1_{\tau_1(h)<\tau_2(h)} \right]$$

  for $h > 0$, where $\tau_1(h)$ and $\tau_2(h)$ are the first hitting times to h_1 and h_2, respectively, given $H_0 = h$.
- Now for a given foreclosure percentage cost ϕ and FRM rate m^F, one seeks rates m^A and m^P for which all three contracts have the same value.

$$V_\phi^F(h, m^F) = V^A(h, m^A(\phi)) = V^P(h, m^P(\phi))$$

  and identify the endogenous spread (in bps) as

$$\mathfrak{s}^A(\phi) := 10,000 \times (m^A(\phi) - m^F); \qquad \mathfrak{s}^P(\phi) := 10,000 \times (m^P(\phi) - m^F).$$

Foreclosure costs and mortgage spreads

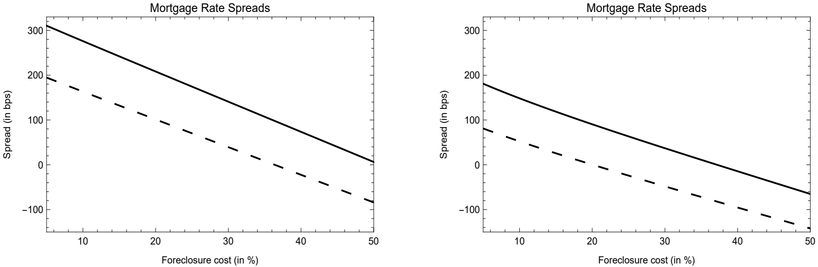

Figure: Endogenous mortgage rate spreads (in basis points), as a function of the foreclosure cost for ABM (dashed) and APRM ($\alpha = 5\%$, solid) for $\delta = 12\%$ (left) and $\delta = 9\%$ (right).

Results

Our main findings are

1) The APRM contract value is insensitive to the capital gain sharing proportion α because, even for small α, high state prepayment is virtually eliminated. Therefore, it is difficult to allow for endogenous α as one cannot invert the contract value in α.

2) For a given common contract rate, the APRM has a lower value than the ABM, even ignoring the capital gain sharing feature, because the APRM lowers payments once H falls below 1, rather than once H falls below B_0.

Results

3) Depending on the benefit rate δ, for relatively low foreclosure costs, the ABM may be more valuable than the FRM in low house price states even at a common contract rate. Furthermore, for all δ the ABM has a lower equivalent foreclosure cost than the APRM.

4) For observed foreclosure costs (e.g. $30\% - 35\%$) the endogenous spread of the ABM is lower than that for the APRM, but both increase substantially with the utility rate δ. However, for low utility rates, at observed foreclosure rates, the ABM actually has a negative endogenous spread.

Reference and future work

References:

- The paper is available on Arxiv. https://arxiv.org/abs/2005.03554

Future work:

- we aim to extend theoretical results to a finite horizon and stochastic interest rates, allow for a jumps in house index,
- to incorporate basis risk between the observed local house price index value and the observed house value.

Thank you!