# Mathematics for Economists-1

## Module 1, 2018-9

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## Course information

**Course Website:**    https://my.nes.ru

**Instructor's Office Hours:**    By appointment

**Class Time:**   TBA

**Room Number:**   TBA

**TAs:** TBA

## Course description

This is the first half of the Math for Economists sequence which aims to provide students with a good command of the basic mathematical tools used in economics. This first part of the sequence is dedicated to static optimization problems, parametric optimization/comparative statics, and fixed point theorems.

## Course requirements, grading, and attendance policies

There will be a midterm (%40) and a final exam (%60). The final will be comprehensive. Following the general policy of NES, students are entitled to a make-up exam if they have missed the final with a valid reason or if they have failed to get a passing grade at the first try. The difficulty of tasks and the grading scheme in the make-up are likely to be different than those in the earlier exams.  In addition, there will be weekly homework assignments, which won't be graded.

## Course contents

1. Equality-constrained optimization (Ch. 5)

2. Inequality-constrained optimization (Ch. 6)

3. Convex optimization (Ch. 7)

4. Quasi-convex optimization (Ch. 8)

5. Parametric continuity, maximum theorem, Brouwer/Kakutani fixed point theorem (Ch. 9)

6. Supermodularity and parametric monotonicity (Ch. 10)

7. Contraction mappings and their fixed points (Ch. 12)

**Description of course methodology**

The instructor will use the traditional methods (i.e., a whiteboard, a marker and verbal discussions) to teach. Students are encouraged to participate in lectures with questions and comments.

## Course materials

**Required Textbook:**

R. Sundaram, "A First Course in Optimization Theory."

(The chapter numbers in course contents refer to this book.)

## Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.

## Sample tasks for course evaluation

**1.** Using the second order conditions, determine if $(x^*, y^*) := (1, 0)$ is a *local* solution to:

$$\min x^3 + (x - x^2)y - y^3 \quad \text{s.t.} \quad x^2 - y^2 = 1, \ (x, y) \in \mathbb{R}^2.$$

**2.** Consider the following (parametric) optimization problem:

$$\max 4\sqrt{x} + y \quad \text{s.t.} \quad 10 - x - y \geq 0, \ y^2 \geq ax, \ \text{and} \ (x, y) \in \mathbb{R}_+^2.$$

Find the set of all solutions for the cases $a = 1/9$ and $a = 81$. Verify your answer. (*Hints:* All critical points are integers. For each value of $a$, there are "basically" two cases to consider. In one of those cases, first order conditions do not really matter.)

**3. (i)** Let $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^m \to \mathbb{R}$ be concave functions. Show that $h(x, y) := f(x) + g(y)$ is a concave function of $(x, y) \in \mathbb{R}^{n+m}$.

**(ii)** Let $u : \mathbb{R} \to \mathbb{R}$ be a strictly concave function. Is $f(x) := u(x_1) + x_2$ a strictly concave function of $x \in \mathbb{R}^2$? How about $g(x) := u(x_1)$ for $x \in \mathbb{R}^2$?

**3.1 (Signalling).** Let $S \subseteq \mathbb{R}$ be a set of actions available to an individual. The action $x$ selected by the individual is observed in the labor market, and rewarded at the wage rate $w(x)$. For example, each $x \in S$ may represent a diploma that brings a wage rate of $w(x)$. However, the market cannot directly observe the individual's type, $\theta$. The types belong to a set $\Theta \subseteq \mathbb{R}$, and higher types correspond to more productive individuals. Let $c(x, \theta)$ denote a real valued function on $\mathbb{R}^2$ that represents the (psychological or monetary) cost of action $x$ to type $\theta$ individual. Finally, let $\pi(x, \theta) := w(x) - c(x, \theta)$ denote the total payoff of action $x$ to type $\theta$ individual.

**(i)** Show that $\pi(x, \theta)$ is a (strictly) supermodular function on $S \times \Theta$ iff so is $-c(x, \theta)$.

**(ii)** Suppose $c$ is a twice continuously differentiable function on $\mathbb{R}^2$ such that $\frac{\partial^2 c(x,\theta)}{\partial x \partial \theta} < 0$. Thus, the marginal cost of an action is decreasing with types in line with our productivity assumption. Show that $-c(x, \theta)$ is strictly supermodular on $\mathbb{R}^2$ (and hence, on $S \times \Theta$). (*Hint.* No need to go into details. Just follow the logic of a theorem stated in class.)

**(iii)** Let $x^*(\theta)$ denote an optimal action for type $\theta \in \Theta$. So, $x^*(\theta)$ maximizes $\pi(\cdot, \theta)$ on $S$, for any given $\theta$. Conclude that $\theta > \theta'$ implies $x^*(\theta) \geq x^*(\theta')$. (*Hint.* You can simply invoke a theorem covered in class, but would be even better to check if you understand the proof of that theorem.)

**(iv)** Suppose $X = \{1, 2\}$, $\Theta = \{1, 2\}$ and $c(x, \theta) = \frac{x}{\theta}$. Say that the market *separates the high type individual from the low type* if (a) $x^*(2) \neq x^*(1)$, and (b) these optimal actions are uniquely defined for both types. What conditions on $w(2)$ and $w(1)$ would guarantee such separation? (*Note:* Congratulations! You just had a brief introduction to signalling models.)