CONTINUOUS-TIME STOCHASTIC PROCESSES

Module 3, 2020–2021

Professor: Andrei Savochkin

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Course information

Course Website: my.nes.ru

Instructor's Office Hours: TBD

Class Time: TBD

Room: TBD

TA: TBD

Course description and prerequisites

This course teaches the basic ideas of the theory of continuous-time stochastic processes and stochastic calculus. The goal is to enable students to understand models in Economics and Finance that rely that use continuous-times stochastic processes as modeling devices. The target audience of the course is students who did not have any formal training in stochastic processes as undergraduates but wish to expand the range of tools and methods that they are familiar with. The prerequisites for the course are basic knowledge of probability theory and good command of mathematical analysis; knowledge of the theory of measure and integration is a big plus but is not strictly necessary.

Due to prerequisites and limited class time, the course is designed to be not 100% formal and rigorous. We will be doing some (easier) proofs to have a better understanding of what claims are true and what are false and master the studied concepts—more than we do in Math-II course. However, many more demanding proofs (e.g., existence theorems) will be omitted. Three lectures are planned to be spent on applications.

Course requirements, grading, and attendance policies

Student's achievements will be evaluated on the basis of problem sets, pop quizzes, and the final exam. The exam and the problem sets are graded on the 0–100 scale; the final score is computed on the same scale and then converted to transcript grades (from 2 to 5+). There will be five small problem sets with the total weight of 15% in the final score. Then, there will be two pop quizzes given without warning during class time that have the weight of 5% each. The remaining weight goes to the final exam. The exam score of at least 20 points is required for getting a passing grade.

The format of the exam is "A4." Each student is allowed to bring to the exam one sheet of paper of A4 size (double-sided) with notes, handwritten or typed.

Course contents

The course covers the following topics.

1. Basic probability concepts
    - σ-fields, measures, convergence, integrals, stochastic processes, filtrations, conditional expectations
2. Continuous-time martingales
    - the concept, stopping time, optional sampling theorems
3. Brownian motion
    - construction, properties, the Ito integral, the martingale representation theorem
4. Stochastic differential equations
    - the concept, definitions of a solution, existence and uniqueness, Levy's characterization of the Brownian motion, Ito diffusions
5. Applications
    - stochastic optimal control, Principal-Agent problem in continuous time, option pricing and the Black-Scholes formula

Description of course methodology

Most of course material will be presented in lectures and sections meetings. Taking notes in class is strongly recommended. Due to the complexity of the studied concepts, it is also recommended to review lecture notes (and read the textbook if something remains unclear) before each class.

Sample task for course evaluation

(Oksendal, Ex. 4.5) Let B_t be a one-dimensional Brownian motion with $B_0 = 0$. Define $\beta_k(t) = E[B_t^k]$ for $k \in \mathbb{Z}$.

Prove that $\beta_k(t) = \frac{1}{2}k(k-1)\int_0^t \beta_{k-2}(s)\,ds$ for all $k \geq 2$.

Course materials

Required textbooks and materials

The required textbook is

- Oksendal B., "Stochastic Differential Equations: An introduction with Applications," 6th edition.

Besides that, we will study one or two papers that will be announced later.

Additional materials

A more rigorous textbook that also covers most of the topics of the course is

- Karatzas, I., and S. E. Shreve, "Brownian Motion and Stochastic Calculus," 2nd edition.

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.