Macroeconomics 4

Module 4, 2019-2020

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Course description

This is the forth part of the sequence of macroeconomics courses at NES. The goal of this module is to introduce the students to the microeconomic foundations of aggregate demand. The course will cover the basic theories of consumption, investment and capital markets. We will start with a baseline model (life cycle/permanent income hypothesis for consumption, Tobin's q-theory for investment and CAPM/CCAPM theory for asset pricing) and study when and why these basic theories fail empirical tests. After that we will consider extensions and modifications of the baseline model which are more successful in replicating the data.

Course requirements, grading, and attendance policies

There will be a midterm quiz (30% of the final grade) and a final exam (70% of the final grade). Passing the course requires all students to take two exams (at their regularly scheduled times), unless an exam is missed for a documented reason approved by NES administration. There will be also several homework assignments during the course, which will not be graded.

Course contents

1. **Consumption**
   (a) The Life Cycle/Permanent Income Hypothesis
   (b) Consumption under uncertainty: Random Walk Hypothesis
   (c) Empirical applications: excess smoothness and excess sensitivity of consumption
   (d) Extensions: precautionary saving, liquidity constraints, time inconsistent preferences, habit formation, durable goods
2. **Asset pricing**
   (a) CAPM and Consumption CAPM
   (b) Lucas model of asset pricing

(c) Stock prices and equity premium puzzle

(d) Term structure of interest rates

3. **Investment**

   (a) A model of investment with adjustment costs, Tobin's q, its empirical testing, Hayashi conditions

   (b) Extensions: non-convex adjustment costs, fixed costs

   (c) Investment under uncertainty, irreversible investments and real options

   (d) The Modigliani-Miller theorem

   (e) Financial market imperfections and investment, the role of asymmetric information

Sample tasks for course evaluation

Problem 1. Durable goods PIH

(problem from Ivan Werning)

Suppose that consumers have the following preferences over durable goods (there are no non-durables in this exercise):

$$E_0 \sum_{t=0}^{\infty} \beta^t u(S_t) \tag{1}$$

where $S_t = (1-\delta)S_{t-1} + c_t$ and S_t is the stock of durables and c_t is the purchase of new durables. Consumers have access to a financial market with no borrowing constraints. Labor income y_t is the only source of uncertainty, the interest rate is constant and equal to r, so $A_{t+1} = (1+r)(A_t + y_t - c_t)$.

1. Show that the budget constraint and the accumulation equation imply that:

$$\tilde{A}_{t+1} = (1+r)\left(\tilde{A}_t + y_t - S_t\left[1 - \frac{1-\delta}{1+r}\right]\right) \tag{2}$$

   where $\tilde{A}_t = A_t + S_{t-1}(1-\delta)$. You can interpret $1 - \frac{1-\delta}{1+r}$ as the shadow cost of renting a unit of a durable good and $\tilde{A}_t$ as a total net wealth.

   Write out the maximization problem the agent faces in terms of $\tilde{A}_t$. Show that the first order condition for optimality is:

$$u'(S_t) = \beta R E_t u'(S_{t+1}) \tag{3}$$

2. Show that if u is quadratic and $\beta(1+r) = 1$ then (3) implies that, $\Delta c_t = u_t - (1-\delta)u_{t-1}$, i.e. the innovations in consumption have a MA(1). Compare with the Random Walk hypothesis for the model with non-durable goods. Interpret.

Problem 2. Lucas tree model.

Consider an economy with a representative agent, in which a random amount of perishable output y_t falls from a fruit tree each period t. Output follows the stochastic process: $\log y_t = \log y_{t-1} + \epsilon_t$, where the i.i.d. shock ϵ_t is drawn from a normal distribution $N(0, \sigma^2)$. The agent maximizes her expected lifetime utility function: $E_t \left\{ \sum_{s=t}^{\infty} e^{-\theta(s-t)} u(c_s) \right\}$, where $\theta > 0$ is the rate of time preference. Assume that there is a competitive stock market in which people can trade shares in the fruit tree, whose price on date t is p_t. This is the ex-dividend price: if you buy a share on date t, you get your first dividend on date $t+1$.

1. Show that the agent will choose optimal contingent consumption plans such that on each date: $p_t u'(c_t) = e^{-\theta} E_t \left\{ (y_{t+1} + p_{t+1}) u'(c_{t+1}) \right\}$. Interpret this equation.

2. Show that in equilibrium, the fundamental price of the tree is: $p_t = E_t \left\{ \sum_{s=t+1}^{\infty} e^{-\theta(s-t)} \frac{u'(y_s)}{u'(y_t)} y_s \right\}$. Interpret this equation.

3. Let $u(c) = c^{1-\gamma}/(1-\gamma)$ for $\gamma > 0$. Show that the normality and i.i.d. assumptions for shocks ϵ_t imply for all $s > t$: $E_t\{y_s^{1-\gamma}\} = y_t^{1-\gamma} e^{\frac{\sigma^2(1-\gamma)^2}{2}(s-t)}$. (Hint: use the properties of log-normal distribution; if $\epsilon \sim N(\mu, \sigma^2)$, then e^ϵ has a log-normal distribution with $E(e^\epsilon) = e^{\mu + \frac{1}{2}\sigma^2}$.)

4. Deduce from part (3) that if $\theta > \sigma^2(1-\gamma)^2/2$, then $p_t = \kappa y_t$. Compute κ and interpret this equation.

Problem 3. PIH

Consider an infinitely lived consumer with quadratic preferences and subjective time discount factor $\beta \in (0, 1)$, who can lend and borrow freely at the market interest rate $r > 0$. Assume that $\beta(1+r) = 1$. The consumer's current income y_t is governed by the following non-stationary stochastic process:

$$y_t = y_{t-1} + \epsilon_t - \theta\epsilon_{t-1}$$

where ϵ_t is i.i.d. process with zero mean and variance σ^2 and $0 < \theta < 1$ is a constant. Denote net assets of the consumer as of the beginning of date t by A_t.

1. Find the optimal consumption on date t for given values of θ, A_t, y_t and ϵ_t.

2. Compute $\Delta c_t = c_t - c_{t-1}$ as a function of ϵ_t. Does this change in consumption react to expected changes in income? Why? Compute marginal propensity to consume (MPC). Explain.

3. Assume that on date t consumer learned that starting from the period $t+2$ his income will be perfectly known and equal to y_{t+2}, so $y_{t+j} = y_{t+2}$ for any $j \geq 2$. How does consumption change on date t? And on date $t+2$? Compute these changes. Explain.

Problem 4. Deep habits

Consider an individual who lives for two periods, $t = 1$, when he is young, and $t = 2$, when old. On each date, this person receives 2 dollars of income, $y_1 = y_2 = 2$, and consumes two goods: Mars and Snickers chocolate bars. Assume that there are deep habits in consumption, in particular the utility from consumption in the second period depends on consumption of Mars in the first period. Thus, the preferences are given by:

$$\log\left(m_1^{\frac{1}{2}} s_1^{\frac{1}{2}}\right) + \log\left((m_2 - \alpha m_1)^{\frac{1}{2}} s_2^{\frac{1}{2}}\right)$$

where m_t and s_t denote respectively consumption of Mars and Snickers in the period $t = 1, 2$.

For simplicity assume that prices of goods are equal to 1 in both periods and that this person can save or borrow at zero interest rate and has zero assets at the beginning of his life.

1. Assume first that there are no habits in utility, i.e. $\alpha = 0$. Find the life-cycle consumption profile, m_1, s_1, m_2 and s_2, for this individual.

2. Assume now that $0 < \alpha < 1$ and deep habits are *internal*, i.e. the utility of this person in the second period depends on his own consumption of Mars in the first period, m_1. Find the life-cycle consumption profile, m_1, s_1, m_2 and s_2, in this model. How does consumption of Snickers and Mars change over time? Compare with the model without habits and explain intuitively.

3. Assume now that $0 < \alpha < 1$ and deep habits are *external*, i.e. the utility of this person in the second period depends on aggregate consumption of Mars in the first period, M_1 (however $m_1 = M_1$ in equilibrium). Find the life-cycle consumption profile, m_1, s_1, m_2 and s_2, in this model. How does consumption of Snickers and Mars change over time? Compare with the model without habits and internal deep habits. Explain intuitively.

Problem 5. True or False

Please explain whether the following statements are True, False, or Partially True. You will be graded based on the quality of your explanation.

1. Certainty equivalence behavior cannot be observed in the model with risk-averse consumers.
2. Excess sensitivity of consumption is an observation that consumption reacts to transitory unexpected changes in income.
3. In the model with prudence motive, higher volatility of consumption implies larger precautionary savings.
4. According to the permanent income hypothesis, consumption of durable goods follows a random walk process.
5. Increasing interest rate results in lower consumption today and higher consumption tomorrow.
6. Let x be a diffusion process given by $dx = axdt + \sigma xdz$, where dz is the increment of a Wiener process, and $y = log(x)$. Then $dy = adt + \sigma dz$.
7. Average q can never be larger than marginal q.
8. Consumption CAPM model predicts that risky assets which have zero correlation of returns with consumption should have the same expected returns as risk-free assets.
9. The model with kinked capital adjustment costs implies intermittent but gradual changes in capital.
10. In the model with irreversible investments, increasing probability of positive future capital returns which does not affect neither expected capital returns nor uncertainty over negative shocks has no any effect on investment.

Problem 6. ***Lucas tree model with options.***

Consider an economy with a representative agent, in which a random amount of perishable output y_t falls from a fruit tree each period t. Output follows a two-state independent identically distributed (i.i.d.) stochastic process: $y_t \in \{y_H, y_L\}$, $y_H > y_L > 0$, with a probability distribution:

$$P(y_t = y_H) = \pi,\ P(y_t = y_L) = 1 - \pi,\ 0 < \pi < 1$$

The agent maximizes her expected lifetime utility function: $E_t \sum_{s=t}^{\infty} \beta^{s-t} \log(c_s)$, where $0 < \beta < 1$. Assume that there is a competitive stock market in which people can trade shares in the fruit tree, whose price in state $i \in \{H, L\}$ is p_i. This is the ex-dividend price: if you buy a share on date t, you get your first dividend on date $t+1$. In addition to trees, agents can trade in risk-free real bonds and call and put options with strike price $\bar{p} = \frac{p_H + p_L}{2}$ (aggregate supply of these assets is zero). Call option gives a right to buy the tree at strike price $\bar{p}$ in the next period (so, it pays $\max\{0, p_i - \bar{p}\}$ in the next period). Put option gives a right to sell the tree at strike price $\bar{p}$ in the next period (so, it pays $\max\{0, \bar{p} - p_i\}$ in the next period).

1. Write down a representative agent's optimization problem.
2. Derive a system of equations that characterize equilibrium allocations and prices of trees, p_t, bonds, R_t^{-1}, call options, co_t, and put options, po_t, in this economy. Interpret these equations.
3. Compute equilibrium prices of trees, risk-free bonds, call and put options as functions of π, β, y_H and y_L.
4. Show that put-call parity is valid in this model:

$$p_i + po_i - \beta y_i = \bar{p} R_i^{-1} + co_i,\ i \in \{H, L\}$$

   where βy_i is a net present expected value of the next period's dividends. Interpret.

Problem 7. ***q-theory of investment.***

Recall the continuous-time version of the q-theory of investment with convex internal adjustment costs and no uncertainty. Assume for simplicity that gross profits of the representative firm per unit of its capital $\pi(K)$ are linear in industry capital K and internal adjustment costs $C(I)$ are quadratic. Assume also that the price of investment goods is constant and equal to p_K. The representative firm maximizes its net discounted profits:

$$\int_{\tau=t}^{\infty} e^{-r(\tau-t)} \left[\pi(K(\tau))k(\tau) - p_K I(\tau) - C(I(\tau))\right] d\tau$$

subject to $\dot{k} = I$.

1. Write down optimality conditions characterizing an equilibrium in this model. Interpret these conditions.

2. Suppose that initially the economy rests in a steady state. How will an unanticipated permanent decrease in the price of investment goods p_K affect the time paths of capital, investment, and q? Use relevant graphs to illustrate your answer. Explain.

3. Suppose that initially the economy rests in a steady state. On date t it becomes known that, starting from date $T > t$, the government will reduce import tariffs on investment goods, so their price p_K will permanently decrease on date T. What will be the effect of this policy on the time paths of capital, investment, and q? Use relevant graphs to illustrate your answer. Explain.

Problem 8. *The term structure and consumption.*

Consider an economy populated by a large number of identical households. A representative household maximizes expected life-time utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $0 < \beta < 1$ and $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.

Each household owns one tree. Thus, the number of households and trees coincide. The amount of consumption that growth in a tree satisfies $c_{t+1} = c^* c_t^{\phi} \epsilon_{t+1}$ where $0 < \phi < 1$, and ϵ_t is a sequence of i.i.d. log-normal random variables: $\log(\epsilon_t) \sim \mathcal{N}(0, \sigma^2)$. Assume that, in addition to shares in trees, one-period and two-period risk-free bonds are traded in this economy.

1. Define a competitive equilibrium. Derive a system of equations that characterize equilibrium.

2. Compute the term structure of interest rates, $\tilde{R}_{jt}$, for $j = 1, 2$.

3. Economist A argues that economic theory predicts that the variance of the log of short-term interest rate is always lower than the variance of the log of long-term interest rate, because short rates are riskier. Do you agree? Justify your answer.

4. Economist B claims that short-term interest rates are more responsive to the state of the economy, i.e. c_t, than are long-term interest rates. Do you agree? Justify your answer.

5. Economist C claims than in economies in which consumption is very persistent ($\phi \approx 1$), changes in consumption do not affect interest rates. Do you agree? Justify you answer and provide economic intuition for you argument.

Problem 9. *Merton's portfolio problem with two risky assets.*

Consider the Merton's model of optimal portfolio and consumption choice in continuous time. An infinitely lived consumer has CRRA preferences $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and subjective discount rate $\rho > 0$. At instant t she decides what amount of her initial wealth x to consume: c, and what fraction, θ, of the remaining wealth to invest in risky asset 1 with expected return r_1 and volatility σ_1. The remaining fraction, $1-\theta$, of wealth is invested in risky asset 2 with expected return r_2 and volatility σ_2. Returns of two assets are correlated with covariance equal to $\nu\sigma_1\sigma_2$. Short sales of assets are allowed, so θ is unrestricted.

An agent maximizes expected life-time utility: $E_t\left\{\int_t^\infty e^{-\rho(s-t)}u(c(s))ds\right\}$ subject to wealth x evolving according to the following diffusion process:

$$dx = \left[(\theta r_1 + (1-\theta)r_2)x - c\right]dt + \theta\sigma_1 x dz_1 + (1-\theta)\sigma_2 x dz_2$$

where $\begin{pmatrix} dz_1 \\ dz_2 \end{pmatrix} \sim N\left(0, \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix} dt\right)$ is the increment of two-dimensional Wiener process. (Note, that dz_1 and dz_2 are correlated and $Edz_1dz_2 = \nu dt$.)

1. Write down this problem as a dynamic programming problem in continuous time.

2. Using Ito's lemma, rewrite this problem as a second-order differential equation. (Note, that dz_1^2, dz_2^2 and dz_1dz_2 are of order dt.)

3. Using educated guess for the value function: $V(x) = \psi\frac{x^{1-\gamma}}{1-\gamma}$, compute optimal θ and c. Verify that a guessed functional form of the value function is correct.

4. Assume that two assets have equal volatility: $\sigma_1 = \sigma_2$. Show that asset with higher expected return will have higher weight in optimal portfolio. What will happen when these two assets become more and more correlated: $\nu \to 1$?

5. Assume now that two assets have equal expected returns: $r_1 = r_2$, and their returns are perfectly correlated (positively or negatively): $\nu = 1$ or $\nu = -1$. Show that optimal portfolio in these two cases is riskless, i.e. its volatility is zero. Explain this result.

Problem 10. ***A model of the housing market.***

Let H denote the stock of housing, I the rate of investment, p_H the real price of housing, and R the rent. Assume that I is increasing in p_H, so that $I = i + I(p_H)$, with a constant autonomous (government-sponsored) part of residential investment i, $I'(\cdot) > 0$, and that $\dot{H} = I - \delta H$. Assume also that the rent is a decreasing function of H: $R = R(H)$, $R'(\cdot) < 0$. Finally, assume that rental income plus capital gains must equal the exogenous required rate of return, r: $(R + \dot{p}_H)/p_H = r$.

1. Are adjustment costs internal or external in this model? Explain.
2. Sketch the set of points in (H, p_H) space such that $\dot{H} = 0$. Sketch the set of points such that $\dot{p}_H = 0$. Why is the $\dot{H} = 0$ locus not horizontal in this model?
3. What are the dynamics of H and p_H in each region of the resulting diagram? Sketch the saddle path.
4. Suppose the market is initially in long-run equilibrium, and that it becomes known that there will be a permanent increase in government-sponsored residential investment i at time T in the future. What happens to H and p_H at the time of the news? How do H, p_H, I, and R behave between the time of the news and the time of the increase?

Course materials

Required textbooks and materials

1. Romer, David, *Advanced Macroeconomics*, McGraw-Hill/Irwin, 4th ed., 2011
2. Blanchard, Oliver J. & Fischer, Stanley, *Lectures on Macroeconomics*, The MIT Press, 1989

Additional materials

1. Ljungqvist, Lars & Sargent, Thomas J., *Recursive Macroeconomic Theory*, The MIT Press, 2nd ed., 2004
2. Attanasio, Orazio P., *Consumption*, in J.B. Taylor & M. Woodford (eds.), *Handbook of Macroeconomics*, Elsevier Science, 1999
3. Caballero, Ricardo J., *Aggregate Investment*, in J.B. Taylor & M. Woodford (eds.), *Handbook of Macroeconomics*, Elsevier Science, 1999
4. Campbell, John Y., *Asset Prices, Consumption and the Business Cycle*, in J.B. Taylor & M. Woodford (eds.), *Handbook of Macroeconomics*, Elsevier Science, 1999

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.