Mathematics for Economists-1

Module 1, 2022-3

Ozgur Evren
New Economic School
oevren@nes.ru

Course information

Course Website: https://my.nes.ru

Office Hours: By appointment

Class Time: TBA

Room: TBA

TAs: TBA

Course description

This is the first half of the Math for Economists sequence which aims to provide students with a good command of the basic mathematical tools used in economics. This first part of the sequence is dedicated to static optimization problems, parametric optimization/comparative statics, and fixed point theorems.

Course requirements, grading, and attendance policies

There will be a midterm (%40) and a final exam (%60). The final will be comprehensive. Following the general policy of NES, students are entitled to a make-up exam if they have missed the final with a valid reason or if they have failed to get a passing grade at the first try. The difficulty of tasks and the grading scheme in the make-up are likely to be different than those in the earlier exams. In addition, there will be weekly homework assignments, which won't be graded.

Course contents

1. Equality-constrained optimization (Ch. 5)

2. Inequality-constrained optimization (Ch. 6)

3. Convex optimization (Ch. 7)

4. Quasi-convex optimization (Ch. 8)

5. Parametric continuity, maximum theorem, Brouwer/Kakutani fixed point theorem (Ch. 9)

6. Supermodularity and parametric monotonicity (Ch. 10)

7. Contraction mappings and their fixed points (Ch. 12)

Description of the course methodology

If external conditions permit, the instructor will use the traditional methods in a classroom (i.e., a whiteboard, a marker and verbal discussions). Otherwise, we will have online classes. In either case, students are encouraged to participate in lectures with questions and comments.

Course materials

Required Textbook:

R. Sundaram, "A First Course in Optimization Theory."

(The chapter numbers in the course contents refer to this book.)

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.

Sample tasks for course evaluation

1. Using the second order conditions, determine if $(x^*, y^*) := (1, 0)$ is a *local* solution to:

$$\min x^3 + (x - x^2)y - y^3 \quad \text{s.t.} \quad x^2 - y^2 = 1,\ (x, y) \in \mathbb{R}^2.$$

2. Consider the following (parametric) optimization problem:

$$\max 4\sqrt{x} + y \quad \text{s.t.} \quad 10 - x - y \geq 0,\ y^2 \geq ax,\ \text{and } (x, y) \in \mathbb{R}^2_+.$$

Find the set of all solutions for the cases $a = 1/9$ and $a = 81$. Verify your answer. (*Hints:* All critical points are integers. For each value of a, there are "basically" two cases to consider. In one of those cases, first order conditions do not really matter.)

3. (i) Let $f : \mathbb{R}^n \to \mathbb{R}$ and $g : \mathbb{R}^m \to \mathbb{R}$ be concave functions. Show that $h(x, y) := f(x)+g(y)$ is a concave function of $(x, y) \in \mathbb{R}^{n+m}$.

(ii) Let $u : \mathbb{R} \to \mathbb{R}$ be a strictly concave function. Is $f(x) := u(x_1) + x_2$ a strictly concave function of $x \in \mathbb{R}^2$? How about $g(x) := u(x_1)$ for $x \in \mathbb{R}^2$?

3.1 (Signalling). Let $S \subseteq \mathbb{R}$ be a set of actions available to an individual. The action x selected by the individual is observed in the labor market, and rewarded at the wage rate $w(x)$. For example, each $x \in S$ may represent a diploma that brings a wage rate of $w(x)$. However, the market cannot directly observe the individual's type, θ. The types belong to a set $\Theta \subseteq \mathbb{R}$, and higher types correspond to more productive individuals. Let $c(x, \theta)$ denote a real valued function on $\mathbb{R}^2$ that represents the (psychological or monetary) cost of action x to type θ individual. Finally, let $\pi(x, \theta) := w(x) - c(x, \theta)$ denote the total payoff of action x to type θ individual.

(i) Show that $\pi(x, \theta)$ is a (strictly) supermodular function on $S \times \Theta$ iff so is $-c(x, \theta)$.

(ii) Suppose c is a twice continuously differentiable function on $\mathbb{R}^2$ such that $\frac{\partial^2 c(x,\theta)}{\partial x \partial \theta} < 0$. Thus, the marginal cost of an action is decreasing with types in line with our productivity assumption. Show that $-c(x, \theta)$ is strictly supermodular on $\mathbb{R}^2$ (and hence, on $S \times \Theta$). (*Hint.* No need to go into details. Just follow the logic of a theorem stated in class.)

(iii) Let $x^*(\theta)$ denote an optimal action for type $\theta \in \Theta$. So, $x^*(\theta)$ maximizes $\pi(\cdot, \theta)$ on S, for any given θ. Conclude that $\theta > \theta'$ implies $x^*(\theta) \geq x^*(\theta')$. (*Hint.* You can simply invoke a theorem covered in class, but would be even better to check if you understand the proof of that theorem.)

(iv) Suppose $X = \{1, 2\}$, $\Theta = \{1, 2\}$ and $c(x, \theta) = \frac{x}{\theta}$. Say that the market *separates the high type individual from the low type* if (a) $x^*(2) \neq x^*(1)$, and (b) these optimal actions are uniquely defined for both types. What conditions on $w(2)$ and $w(1)$ would guarantee such separation? (*Note:* Congratulations! You just had a brief introduction to signalling models.)