Computational Macroeconomics

Module 3, 2024-2025

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Course description

Many, if not most, dynamic models used in modern macroeconomics do not have analytical (closed-form) solutions. For this reason, numerical methods and computer programming have become indispensable tools of the macroeconomic research. In this course we will discuss the main computational algorithms of the dynamic optimization problem. We will start with overview of the basic results from dynamic programming. Then, we will study the main numerical algorithms for its solution with application to a simple neoclassical growth model. In particular, we will consider linear-quadratic approximation, value and policy functions iterations algorithms and their modifications, perturbation and projection methods. Finally, we will discuss more complicated algorithms for solving heterogeneous agents' models. This course requires a basic knowledge of Python, Julia or MATLAB.

Course requirements, grading, and attendance policies

There will be a few (maximum 3) home assignments (50% of the grade) asking for writing a code in Python (or in MATLAB, Julia, Fortran 90 etc.) to solve a simple dynamic programming problem. The exam (50% of the grade) will contain questions on a published macroeconomic article handed out in advance. All these components (including all home assignments), as well as at least 70% attendance, are mandatory for getting a passing grade.

Course contents

1. **Review of dynamic programming:** mathematical preliminaries, contraction mapping theorem, Blackwell's sufficient conditions, theorem of the maximum, dynamic programming under uncertainty

2. **Discrete-state dynamic programming:** value function iteration algorithm and its improvements, policy function iteration, interpolations and splines

3. **Linear approximation methods:** linear-quadratic (LQ) approximation algorithm, first-order perturbation methods

4. **Higher-order perturbation methods**

5. **Projection methods:** finite elements method, spectral methods (Chebyshev polynomials)

6. **Parameterized expectations algorithm**

7. **Heterogeneous agents models and incomplete market economies:** computation of stationary equilibrium, transitional dynamics, aggregate uncertainty in heterogeneous agents models, Krussel-Smith algorithm

Sample tasks for course evaluation

Problem 1.

The Problems 1-4 are based on the following paper: Guerrieri & Iacoviello (2015) *'OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily'*, Journal of Monetary Economics, 70, 22-38
To find a full nonlinear solution of the RBC model with a constraint on investment this paper uses a value function iteration algorithm on a very fine grid of capital stocks.

1. Write down the Bellman equation for the RBC model with a constraint on investment, as described in Section 4.1 of this paper. Be precise in specifying state space, payoff function and feasibility correspondence.

2. Provide a sketch of the value function iteration algorithm to compute value and policy functions of this model.

3. Discuss the main advantages and disadvantages of this approach comparing to the piecewise linear solution proposed in the paper.

Problem 2.

This paper proposes an algorithm to obtain a piecewise linear solution of the models with occasionally binding constraints.

1. Why is the first-order perturbation approach not applicable for the models with occasionally binding constraints?

2. Write down the linearized systems (M1) and (M2) and associated functions f and g for a simple linear model described in Section 2.4. Be precise in specifying the matrices $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}$ and $\mathcal{A}^*, \mathcal{B}^*, \mathcal{C}^*, \mathcal{D}^*, \mathcal{E}^*$.

3. The algorithm in this paper has six main steps. Describe them very briefly. What exactly have we to guess and verify in this algorithm?

4. What are the main sources of approximation error in this algorithm? Which of them are the most problematic?

Problem 3.

This paper uses two approaches to assess numerical accuracy of the linear, piecewise linear and full nonlinear algorithms.

1. What are these two approaches? Describe them very briefly.
2. What is the best algorithm in terms of accuracy? The worst? Explain.
3. Explain why a superior approximation to the solution of the model should yield a higher level of utility.

Problem 4.

Consider a simple model of small open economy. The representative household maximizes an expected life-time utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

subject to budget and borrowing constraints:

$$c_t + b_{t+1} + \psi(b_{t+1}) = y_t + (1+r)b_t$$
$$b_{t+1} \geq -\phi y^{ss}$$

where c_t denotes consumption, b_{t+1} are holdings of risk-free international asset (negative values mean foreign debt), $\psi(b) = \frac{1}{2}\psi b^2$ denotes debt adjustment costs, r is a risk-free international real interest rate (with $r = \frac{1}{\beta} - 1$) and output y_t follows AR(1) stochastic process:

$$\log(y_t) = (1-\rho)\log(y^{ss}) + \rho \log(y_{t-1}) + u_t, u_t \sim N(0, \sigma^2)$$

Assume that $\beta = 0.96$, $\psi = 0.001$, $\phi = 0.1$, $\rho = 0.9$, $\sigma = 0.02$ and $y^{ss} = 5$.

1. Write down the first-order conditions characterizing an equilibrium in this model.

Download here the OccBin toolkit realizing the piecewise linear algorithm. Note that you will need Dynare to use it. Read carefully `readme.pdf` file and look at the examples of models solved in OccBin.

2. Using this toolkit write the code and solve the model both with and without borrowing constraint. Illustrate the impulse responses (both with and without borrowing constraint) of output y_t, consumption c_t, asset holdings b_{t+1} and Lagrange multiplier λ_t to 3 standard deviations negative shock in u_t equal to -0.06. What is about the effects of positive shock of the same size?

3. Explain the differences in the results of these two models. Why does consumption fall more initially but recover faster afterwards after the negative shock in the model with borrowing constraints?

Course materials

Required textbooks and materials

1. Heer, Burkhard & Maussner, Alfred, *Dynamic General Equilibrium Modeling: Computational Methods and Applications*, Springer, 2nd ed., 2009

2. Sargent, Thomas & Stachurski, John, *Quantitative Economics with Python*

3. Ljungqvist, Lars & Sargent, Thomas J., *Recursive Macroeconomic Theory*, The MIT Press, 2nd ed., 2004

Additional materials

1. Marimon, Ramon & Scott, Andrew, *Computational Methods for the Study of Dynamic Economies*, Oxford University Press, 1999

2. Adda, Jerome & Cooper, Russell W., *Dynamic Economics: Quantitative Methods and Applications*, The MIT Press, 2003

3. Judd, Kenneth L., *Numerical Methods in Economics*, The MIT Press, 1998

4. Stokey, Nancy L., Lucas, Robert E. & Prescott, Edward C., *Recursive Methods in Economic Dynamics*, Harvard University Press, 1989

5. DeJong, David N. & Dave, Chetan, *Structural Macroeconometrics*, Princeton University Press, 2nd ed., 2011

I will also provide a reading list of papers applying the quantitative methods discussed in the class, with the rate of about 2-3 per week.

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.