# MATHEMATICAL AND STATISTICAL FOUNDATIONS OF ECONOMICS

**Module 1, 2021–2022**

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## Course information

**Course Website:** my.nes.ru

**Instructor's Office Hours:** TBD

**Class Time:** TBD

**Room:** TBD

**TA:** TBD

## Course description

This is a "practical" course that covers a few selected concepts and tools that are most needed for working with models in Economics and Finance. Although students may be familiar with the introductory concepts from the course, we will soon jump into topics that do not get enough attention in the curriculum of any undergraduate math programs, including the best ones.

In this course, we will very rarely (if ever) discuss proofs. Methods and results will be described in a loose manner — with the goal of starting to think like economists and getting intuitive and practical understanding of how to solve problems. Then we will see how solving mathematical problems gives us important insights about the world around us.

## Course requirements, grading, and attendance policies

Student's achievements will be evaluated on the basis of problem sets, the final exam, and attendance. The exam and the problem sets are graded on the scale of 0 to 100 points. The final score is also computed on the 100-point scale and then converted to transcript grades (from 2 to 5+). There will be four problem sets (possibly, of unequal size and weight) with the 24% weight in the final score, and the adjusted exam score has the remaining weight.

The format of the exam is closed book.

The raw score for the exam is computed as the sum of points earned for each problem. Then, it is adjusted for class participation. In the online teaching format, class participation is determined by participation in the polls. During each lecture, there is one or more polls that require students quickly express their opinion on some question (e.g., "Is this a correct solution of the equation, yes or no?"). If a student misses a substantial number of polls by not answering the question, then

he or she is regarded as missing the class meeting. (Exceptions may be granted by the academic office due to illness, or the instructor in extraordinary circumstances by advance agreement.) For students who miss four or fewer meetings, the adjusted score equals to their raw score. Starting from the fifth, each one missed meeting reduces the adjusted score by 8 points. The adjusted score of at least 20 points is required for getting a passing grade.

## Course contents

In the course, we will discuss the following topics.

1. Constrained optimization in $\mathbb{R}^n$ using the Kuhn-Tucker theorem. Illustration: classical demand theory. Convexity. Hemicontinuity of set-valued mappings and Berge's maximum theorem. The envelope theorem.
2. Risk and expected utility maximization. Finite-dimensional optimization under risk. Illustrations: insurance and portfolio choice.
3. Infinite-horizon optimization with Lagrange multipliers. First order conditions and optimality. Illustration: consumption-savings problem under certainty. Uncertainty in infinite-horizon optimization. Illustration: introduction to consumption-based asset pricing.
4. Dynamic programming.
    - Infinite-horizon optimization problems in recursive formulation and the Bellman equation. The optimality principle.
    - Bellman equation as a fixed point problem. Existence, uniqueness, and properties of the solution. Blackwell's theorem.
    - Bellman equation with uncertainty. Markov chains. Dynamic programming in the Markovian setting.
    - Illustration: optimal stopping problem in the context of job search.
5. Key statistical concepts.
    - Parameter estimation. Confidence intervals. Asymptotic vs. exact methods.
    - Properties of point estimators.
    - Methods of estimation. Method of Moments. Maximum Likelihood method.
    - Statistical tests. Type-I and Type-II errors. Significance level and power of a test. $p$-value.
    - Monte Carlo simulations.
6. Introduction to Bayesian statistics. Conjugate families of distributions.

### Description of course methodology

All course material will be presented in lectures and sections meetings. Taking notes in class is strongly recommended. Reading textbooks in addition to class attendance may be helpful but is not absolutely necessary.

## Sample tasks for evaluation

**Problem 1.** In a modern monarchy, a princess wishes to get married. The candidates arrive strictly one per week, and the type $x_t$ of the candidate in week $t$ is stochastic. The candidates can be of three types: tycoon heirs, middle managers, and fitness instructors. Besides their type, they are indistinguishable, and marrying one of them gives the princess utility $H$, $M$, and $L$, respectively. Each week, she learns the type of the candidate who arrives and has three options:

- marry the current candidate right away,
- get engaged to the current candidate (breaking up the previous relationship if she was engaged),
- reject the current candidate and marry the person to whom she is engaged,
- reject the current candidate and pass on to the next period.

Naturally, the princess cannot be engaged to more than one candidate at any point in time. Once she decides to marry a candidate, she gets the corresponding utility and the game ends. The type of the arriving candidate follows a Markov chain with the transition matrix $P$. At each point in time, the princess evaluates her options according to the discounted expected utility with the week-to-week discount factor $\beta$ (and her utility is zero if she never gets married).

Assume that $H = 60$, $M = 35$, $L = 20$, $\beta = \frac{5}{6}$, and

$$P = \begin{pmatrix} \frac{1}{8} & \frac{1}{2} & \frac{3}{8} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{8} & \frac{7}{8} \end{pmatrix}.$$

Find the expected utility values for the princess conditional on the current state and one of her optimal strategies.

**Problem 2.** Your boss asked you to test some hypothesis at the 4% level. You designed a test, collected a random sample, and obtained the $p$-value of 0.0439. Then, you and your colleague learn that the boss was absent-minded and unintentionally gave both of you the same assignment. As you learn, your colleague also collected a random sample independently from you and obtained the $p$-value of 0.0478. Now it is almost the deadline to report. You do not have time to inspect each other's test design, data, and calculation, but must report your conclusion of whether to accept or reject.

You are considering the following potential plans.

1. The 4% level does not make sense. At the standard 5%, the hypothesis is rejected by your sample and hence you should answer "reject."
2. Although each of the samples does not warrant immediate rejection, the two $p$-values are near the borderline of 4%, so it is more appropriate to coordinate between you and your colleague and reject the hypothesis.

3. Although the $p$-values are above the 4% threshold, it seems that the data does not really support the hypothesis. To convey your doubts, the best action is to coordinate between you and your colleague and for one of you to say "reject" and for the other one to say "accept." With this, the boss will make the decision that suits him the best (or flip a coin to resolve the conflicting evidence).

4. You and your colleague should be honest and both say "accept."

Comment on the validity of each of the four reasonings and pick one that you think is the best.

## Course materials

### Required textbooks and materials

The main source of information in the course is lectures and sections meetings. For some of the topics the following books can be used as supplementary reading

- Ljungqvist, L., and T. J. Sargent, *Recursive Macroeconomic Theory*
- Hogg, R. V., J. W. McKean, and A. T. Craig, *Introduction to Mathematical Statistics*

### Additional materials

As a more technical reference, one can use

- Lucas, R. E., and N. L. Stokey with E. C. Prescott, *Recursive Methods in Economic Dynamics*
- Casella, G., and R. L. Berger, *Statistical Inference*

## Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.