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Money Creation in a Random Matching Model

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Abstract

I study money creation in versions of the Trejos-Wright (1995) and Shi (1995) models with indivisible money and individual holdings bounded at two units. I work with the same class of policies as in Deviatov and Wallace (2001), who study money creation in that model. However, I consider an alternative notion of implementability—the ex ante pairwise core. I compute a set of numerical examples to determine whether money creation is beneficial. I find beneficial effects of money creation if individuals are sufficiently risk averse (obtain sufficiently high utility gains from trade) and impatient.

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1   Introduction

The welfare effects of lump-sum money creation differ depending on the model used. In particular, there seems to be a sharp contrast in results de-

pending on whether or not the model contains heterogeneous agents. Representative-agent models tend to yield results in line with the Friedman rule: the optimal monetary policy is not creation, but destruction financed by taxes. Models with heterogeneous agents do not give a general answer: in some the optimal monetary policy is contractionary, in others it is expansionary.[1] This paper confirms that result in a somewhat new model—actually, the familiar matching model setting of Trejos-Wright (1995) and Shi (1995), but with a notion of implementability that has not been used before to study the effects of money creation. Here a lottery allocation is implementable if it is in the pairwise core (in every meeting).

My work is most closely related to Molico (2006) and to Deviatov and Wallace (2001) who study money creation in versions of the same model. Molico (2006) approximates divisible money and proceeds numerically using a particular bargaining solution—take-it-or leave-it offers by consumers. Hence, his work leaves open whether the results are special to that bargaining solution. Deviatov and Wallace (2001) allow for any outcome which satisfies ex post individual rationality in meetings and work with money holdings in the set $\{0, 1, 2\}$, the smallest set that gives money creation a role in determining the distribution of money holdings. They get an analytical result—money creation is beneficial whenever agents are sufficiently patient—but only because they do not permit those in a meeting to commit to lotteries. Here, I adopt the same set of individual holdings but allow people in a meeting to commit to lotteries, while at the same time requiring that the lottery trades be in the pairwise core for every meeting.

I cannot get analytical results, and, therefore, proceed numerically. For each example studied, I find both the best rate of money creation and the best pairwise-core lottery allocation. In other words, I allow the division of the gains from trade in a meeting to depend both on the money creation rate and on the money holdings of the consumer and producer in the meeting. I find that in general optima do not have take-it-or leave-it offers or any other fixed bargaining rule except in settings where individuals are sufficiently impatient. In that case the optima have binding producer participation constraints in all meetings, which implies take-it-or leave-it offers by consumers.

For many settings I cannot find beneficial money creation. However, when

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[1]Examples of models where it is expansionary include Imrohoroglu (1992), Levine (1991) and a generalization by Kehoe, Levine and Woodford (1992), Deviatov and Wallace (2001), Berentsen, Camera, and Waller (2005), Bhattacharya, Haslag, and Martin (2005), and Molico (2006).

people are both sufficiently impatient and risk averse (obtain sufficiently high utility gains from trade), I find that money creation is beneficial. That is quite different from Deviatov and Wallace (2001), whose results apply only if individuals are sufficiently patient. In all examples where money creation is beneficial there is no randomization in meetings.

The rest of the paper is organized as follows. The next section provides the description of the environment; in section 3 I define implementable allocations; section 4 contains a discussion of some general properties of implementable allocations; in section 5 I describe the algorithm; section 6 presents numerical examples; and section 7 concludes.

2   Environment

The background environment is a simple random matching model of money due to Shi (1995) and Trejos and Wright (1995). Time is discrete and the horizon is infinite. There are $N \geq 3$ perishable consumption goods at each date and a $[0, 1]$ continuum of each of N types of agents. A type n person consumes only good n and produces good $n + 1$ (modulo N). Each person maximizes expected discounted utility with discount parameter $\beta \in (0, 1)$. Utility in a period is given by $u(y) - c(x)$, where y denotes consumption and x denotes production of an individual $(x, y \in \mathbb{R}_+)$. The function u is strictly concave, strictly increasing and satisfies $u(0) = 0$, while the function c is convex with $c(0) = 0$ and is strictly increasing. Also, there exists $\hat{y} > 0$ such that $u(\hat{y}) = c(\hat{y})$. In addition, u and c are twice continuously differentiable. At each date, each agent meets one other person at random.

There is only one asset in this economy which can be stored across periods: fiat money. Money is indivisible and no individual can have more than two units of money at any given time. Agents cannot commit to future actions (except commitment to outcomes of randomized trades). Finally, each agent's specialization type and individual money holdings are observable within each meeting, but the agent's history, except as revealed by money holdings, is private.

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3 Implementable allocations and the optimum problem

The timing in a period is the following. First, there are meetings and trades. Then, the monetary policy is applied. The policy is a probabilistic version of the proverbial helicopter drops of money. Then, the next period begins and the above sequence of actions is repeated.

The pairwise meetings, the inability to commit, the privacy of individual histories, and the perishable nature of the goods imply that any production must be accompanied by a positive probability of receiving money. A trade meeting is a meeting between a potential producer with $i \in \{0, 1\}$ units of money and a potential consumer with $j \in \{1, 2\}$ units of money.

For each trade meeting, a general lottery trade is represented by a probability measure on $\mathbb{R}_+ \times \{0, 1, 2\}$ with the interpretation that if (y, k) is randomly drawn from that measure, then y is produced and consumed and k units of money are transferred from the consumer to the producer. As is obvious and spelled out below, only measures that are degenerate on output can be in the pairwise core. Consequently, any trade in the pairwise core can be described by the quantity of goods, y_{ij}, traded in meetings between producers with i units of money and consumers with j units and by a probability distribution $\left(\lambda_{ij}^0, \lambda_{ij}^1, \lambda_{ij}^2 \right)$, where λ_{ij}^k is the probability that k units of money are transferred and where $\lambda_{ij}^k = 0$ if $k > \min\{j, 2 - i\}$. Finally, let p_i be the fraction of agents in each specialization type who start a date with i units of money and let $p = (p_0, p_1, p_2)$. Then, in terms of p_i and λ_{ij}^k, the transition matrix T for money holdings (implied by the trades) is given by:

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T = \begin{bmatrix}
t_{00} & \frac{1}{N}\left(p_1\lambda_{01}^1 + p_2\lambda_{02}^1\right) & \frac{1}{N}p_2\lambda_{02}^2 \\
\frac{1}{N}\left(p_0\lambda_{01}^1 + p_1\lambda_{11}^1\right) & t_{11} & \frac{1}{N}\left(p_1\lambda_{11}^1 + p_2\lambda_{12}^1\right) \\
\frac{1}{N}p_0\lambda_{02}^2 & \frac{1}{N}\left(p_1\lambda_{12}^1 + p_0\lambda_{02}^1\right) & t_{22}
\end{bmatrix}, \quad (1)
$$

where t_{mm} denotes a diagonal element of T (the probability that an individual leaves the meeting with the same quantity m of money she brought into that meeting). Because T is a transition matrix, t_{mm} can be recovered from the condition that each row of T sums to unity.

I use the formulation of policy introduced by Deviatov and Wallace (2001). As I said, the policy constitutes a probabilistic version of the proverbial helicopter drops of money at a rate. Under policy (α, δ), at each date each person

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not at the upper bound receives a unit of money with probability α and then each unit of money disintegrates with probability δ. As now explained, the α part of the policy resembles lump-sum money creation, while the δ part is a stand-in for the normalization that is equivalent to inflation.

With divisible money and no bound on individual holdings, the standard policy is creation at a rate where money is handed out lump-sum to people. In a broad class of settings, this policy is equivalent to such creation followed by a proportional reduction in individual money holdings (see e.g. Lucas and Woodford, 1994). The proportional reduction is nothing but a normalization of individual holdings. Here, because individual holdings are bounded, such a normalization is necessary. Moreover, because money holdings are indivisible, both parts of the policy must be probabilistic. Also, because of the bound, the α part only approximates lump-sum creation because a person at the upper bound cannot receive such a transfer.

The standard policy (regardless of whether it is followed by a proportional reduction in holdings or not) has two effects. It shifts the distribution of real money balances towards the mean and makes money less desirable to acquire or retain. The above (α, δ)-policy also has these effects. In particular, producers are less willing to produce for money (because they may get a transfer without production and may lose any money acquired) and consumers are more willing to part with money (because they may get a transfer and may lose money they retain).

I also follow Deviatov and Wallace (2001) in their interpretation of sequential individual rationality (which here is a part of the pairwise core notion) as precluding direct taxes. That, among other things, implies that it is not feasible to simply take money from people or to force producers to produce. For that reason I consider only non-negative (α, δ)-policies.

Similar to trades, the creation and destruction parts of the policy yield a pair of transition matrices for money holdings, denoted A and D respectively. According to my description of the policy, they are

$$A = \begin{bmatrix} 1-\alpha & \alpha & 0 \\ 0 & 1-\alpha & \alpha \\ 0 & 0 & 1 \end{bmatrix},$$

and

$$D = \begin{bmatrix} 1 & 0 & 0 \\ \delta & 1-\delta & 0 \\ \delta^2 & 2\delta(1-\delta) & (1-\delta)^2 \end{bmatrix}.$$

Then, our sequence of actions implies that the stationarity requirement is $pTAD = p$.

It is convenient to express individual rationality and pairwise core constraints in terms of discounted expected utilities. Given a meeting of a producer with i and a consumer with j units of money, let $\mu_{ij} \equiv \left\{ \left(\lambda_{ij}^0, \lambda_{ij}^1, \lambda_{ij}^2 \right), y_{ij} \right\}$. Also, let μ denote the collection of all μ_{ij}. For an allocation (p, μ, α, δ) that is stationary, discounted expected utility of an agent who ends up with i units of money at the end of the period, denoted v_i, is constant. Then, the vector $v \equiv (v_0, v_1, v_2)$ satisfies the following three-equation system of Bellman equations:

$$v' = \beta(q' + TAD\, v'),\tag{2}$$

where q, the vector of (expected) one period returns from trade, is given by:

$$q' = \begin{bmatrix} -\frac{p_1}{N}\lambda_{01}^1 c(y_{01}) - \frac{p_2}{N}\left(\lambda_{02}^1 + \lambda_{02}^2\right)c(y_{02}) \\[2mm] -\frac{p_2}{N}\lambda_{12}^1 c(y_{12}) + \frac{p_1}{N}\lambda_{11}^1\left[u(y_{11}) - c(y_{11})\right] + \frac{p_0}{N}\lambda_{01}^1 u(y_{01}) \\[2mm] \frac{p_0}{N}\left(\lambda_{02}^1 + \lambda_{02}^2\right)u(y_{02}) + \frac{p_1}{N}\lambda_{12}^1 u(y_{12}) \end{bmatrix}.\tag{3}$$

Note that an individual with no money can only expect to be a producer, an individual with two units can only be a consumer, and a person with one unit of money can be either a consumer or a producer.

Because T, A, and D are transition matrices and $\beta \in (0,1)$, the mapping $G(x) \equiv \beta(q' + TAD\,x')$ is a contraction. Therefore, (2) has a unique solution which can be expressed as

$$v' = \left(\frac{1}{\beta}I - TAD\right)^{-1}q',\tag{4}$$

where I is the 3×3 identity matrix.

Let

$$\Pi_{ij}^p \equiv \sum_k \lambda_{ij}^k\left(e_{i+k} - e_i\right)ADv' - c(y_{ij})\tag{5}$$

be the expected gain from trade for the producer with i units of money in a meeting with a consumer with j units and let

$$\Pi_{ij}^c \equiv \sum_k \lambda_{ij}^k(e_{j-k} - e_j)ADv' + u(y_{ij})\tag{6}$$

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be the gain from the same trade for the consumer (where e_l is the three-component coordinate vector with indices running from 0 to 2).

The ex ante pairwise core notion of implementability gives rise to the following definition:

Definition 1. *An allocation (p, μ, α, δ) is called ex ante pairwise core implementable if (i) $pTAD = p$, (ii) v (given by 4) is non-decreasing, (iii) the participation constraints*

$$\Pi_{ij}^p \geq 0 \qquad and \qquad \Pi_{ij}^c \geq 0 \tag{7}$$

hold for all i and j, and (iv) for every pair (i, j) that corresponds to a trade meeting, μ_{ij} solves

$$\max_{\mu_{ij}} \Pi_{ij}^c \tag{8}$$

$$\text{subject to } \Pi_{ij}^p \geq \gamma$$

for some γ consistent with the participation constraints (7), where the policy (α, δ) and the value function v are taken as given.

In Definition 1, γ can depend on the meeting (i, j) and on the policy (α, δ). Definition 1 says that an allocation is implementable if (i) it is stationary, (ii) satisfies free disposal of money, (iii) satisfies individual rationality, and (iv) there is no incentive for defections by pairs in meetings.

Finally, my optimum problem is to maximize ex ante utility. That is, the optimum problem is to choose (p, μ, α, δ) from among those that are implementable to maximize $pv' \equiv W$.

It is useful to express the objective W in terms of returns. If I multiply (2) by p and use the fact that $pTAD = p$, then I obtain

$$W = pv' = \frac{\beta}{1 - \beta} pq'.$$

Then, by writing out the product pq', I get

$$W = \frac{\beta}{1 - \beta} \frac{1}{N} \sum_{i=0}^{1} \sum_{j=1}^{2} p_i p_j \left[u(y_{ij}) - c(y_{ij}) \right]. \tag{9}$$

As one would expect, because for every consumer there is a producer, welfare is equal to the net expected discounted utility in all trade meetings.

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4   General results

Because Π_{ij}^c and Π_{ij}^p are strictly concave functions of y, randomization over output cannot be a solution to (8).[2] The proof proceeds by replacing any non-degenerate distribution over output by its mean, which increases the objective Π_{ij}^c and relaxes the constraint $\Pi_{ij}^p \geq \gamma$. Such degeneracy implies that my optimum problem is finite dimensional. This allows me to characterize the ex ante pairwise core in terms of the necessary first order conditions. Because of concavity of Π_{ij}^p and Π_{ij}^c these necessary conditions are also sufficient. If an allocation (p, μ, α, δ) has $y_{ij} > 0$ in all trade meetings,[3] then the first order conditions can be conveniently written as

$$\left[(e_{j-k} - e_j) + \frac{u'(y_{ij})}{c'(y_{ij})} (e_{i+k} - e_i) \right] ADv' \quad \begin{array}{l} \geq 0 \text{ if } \lambda_{ij}^k = \overline{\lambda}_{ij}^k \\ = 0 \text{ if } 0 < \lambda_{ij}^k < \overline{\lambda}_{ij}^k \\ \leq 0 \text{ if } \lambda_{ij}^k = 0 \end{array} \qquad (10)$$

for all pairs (i, j) corresponding to trade meetings and transfers of positive amounts of money k, where $\overline{\lambda}_{ij}^k \equiv 1 - \sum_{s \neq k} \lambda_{ij}^s$.

The first order conditions (10) yield a set of constraints which an ex ante pairwise core implementable allocation must satisfy in addition to the participation constraints in Definition 1. If the value function ADv' implied by an implementable allocation (p, μ, α, δ) is strictly concave, then (10) has implications for the level of output in some meetings. In particular, if $\lambda_{ij}^k > 0$ and $k \geq j - i$ for some positive k, then $y_{ij} \leq y^*$, the unconstrained maximizer of $u(y) - c(y)$. Because the bound on individual holdings is two units, the only meetings in which output can exceed y^* are those between a producer with zero and a consumer with two units of money.

5   The algorithm

Because the beneficial external margin and harmful internal margin effects of money creation are at balance in any optimum, the optima always have

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[2]Berentsen, Molico, and Wright (2002) introduce lotteries in a random matching model of money and give a complete characterization of the ex ante pairwise core for the case of one-unit bound on holdings.

[3]A sufficient condition for this is that ADv', where v is the value function implied by an implementable allocation (p, μ, α, δ), is strictly increasing and that $u'(0) = \infty$ and $c'(0) = 0$.

some binding participation constraints. If individuals are patient enough, in addition to binding participation constraints the optima have randomization over how much money is transferred in meetings. This implies that some of the constraints in (10) are also binding. Because these constraints are complicated functions of an allocation, closed-form solutions for the optima are out of reach even for the case of a two-unit bound on holdings. For this reason I compute a set of examples.

My optimization problem falls within the class of problems generally referred to as "nonlinear programming problems", for which many standard routines are available. However, as one can see, the constraints in (10) are discontinuous.[4] Another difficulty is that the mapping $F(p) \equiv pTAD - p$ is ill-behaved at $\alpha = \delta = 0$.[5] This precludes application of routines which require continuous differentiability of the objective and constraints, such as sequential quadratic programming. I overcome this difficulty by designing a hybrid algorithm which combines genetic and conventional smooth optimization techniques.

There are three main steps in this algorithm. First, create an initial population of allocations. Second, amend the population by replacing the worst allocations by better ones. Third, check if the termination criterion is satisfied for the best allocation in the population. If yes, then terminate; if no, then return to the second step.

In the first step, I create a matrix where each row is an allocation. Allocations in the initial population are picked randomly among those which satisfy ex ante individual rationality. The size of the population is a parameter of the algorithm.

To amend the population (the second step), I use several genetic operators. These operators are called selection, crossover, and mutation. I use standard selection and crossover operators, a subset of those described in Houck, Joines and Kay (1996). However, I modify the standard mutation

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[4]Each constraint in (10) is equivalent to

$$\left[(e_{j-k} - e_j) + \frac{u'(y_{ij})}{c'(y_{ij})} (e_{i+k} - e_i) \right] ADv' + (\text{sign}(\lambda_{ij}^k) - \text{sign}(\overline{\lambda}_{ij}^k - \lambda_{ij}^k))\vartheta_{ij}^k = 0$$

and

$$\vartheta_{ij}^k \leq 0,$$

where $\text{sign}(x)$ is the sign function, and ϑ_{ij}^k is a slack variable.

[5]See Deviatov and Wallace (2001), who study the properties of that mapping.

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operator. The standard operator alters a single allocation (called "the parent") to produce another allocation (called "the child"). The operator I use is a composition of two independent operators.

The first one is applied only if the parent has at least one of the transfer probabilities λ_{ij}^k at its upper or lower bound or if it has $\alpha = \delta = 0$. The operator pushes a random subset of these variables into the interior. If a better allocation is produced, it replaces the parent in the population. This simple mutation deals with discontinuity of the constraints in (10) and with ill behavior of the mapping $F(p)$ at zero.

The second operator alters only those of the transfer probabilities and policy pairs which are already in the interior. There, because all constraints are twice continuously differentiable, application of smooth methods is possible. This leaves a range of possibilities for what this second operator can be. In particular, one can run a few iterations of a sequential quadratic routine or of the BFGS algorithm[6] (as long as these iterations remain in the interior). The operator I adopt makes use of the gradients in the following way.

First, I compute (reduced) gradients of the objective and of all active constraints. Then I compute an orthogonal projection of the gradient of the objective onto the subspace orthogonal to the one spanned by the gradients of the active constraints. After that I randomly pick a search direction in the neighborhood (small cone) of that projection. Going in that search direction is likely to improve the objective and does not violate (at least by much) the active constraints. The child is obtained from the parent by moving along the search direction. However, this procedure often leads to a violation of some constraints even if the parent satisfies all the constraints. In this case the objective implied by the child is reduced by some value which is proportional to the amount by which the constraints are violated. If the penalty parameter is large, even a small violation is costly, and the child dies out of the population quickly. If the parent itself violates the constraints by large amounts, then the search direction is chosen to move the child closer to the feasible region regardless of what happens to the objective. Because the initial population is chosen randomly, this is important in the beginning of search. In other words, the second operator first pushes allocations towards satisfaction of the pairwise core conditions; then it drives the population to the optimum.

The termination criterion in the third step is based on the first order

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[6]See Judd (1998) for further details.

conditions for the Kuhn-Tucker theorem. If the length of the projection of the gradient of the objective onto the subspace orthogonal to that spanned by the gradients of the active constraints is less than the tolerance value, the necessary conditions for the theorem are (approximately) satisfied. Because the probability of selection of parents in the population is an increasing function of the objective, this is sufficient to guarantee that every terminal point is a (local) maximum.

6   The examples

I use the above algorithm to compute examples of optimal allocations. In all the examples, $u(y) = y^\kappa$, $c(y) = y$, and $N = 3$. The examples are computed for various κ and various degrees of patience, r, where $r \equiv \frac{1}{\beta} - 1$.

I find examples where money creation is beneficial provided that individuals are sufficiently risk averse and impatient. However, an interesting finding is that there are no examples where money creation is beneficial and individuals randomize in meetings. It seems that when randomization is a part of optimal trades, randomization itself produces beneficial extensive margin effects that in some sense dominate those of a policy. To see why this conjecture seems plausible, consider allocations under no policy. If $\alpha = \delta = 0$, then stationarity requirements $pTAD = p$ collapse to a single equation:

$$\lambda^1_{11} p_1^2 = \lambda^1_{02} p_0 p_2, \tag{11}$$

which along with:

$$p_0 + p_1 + p_2 = 1, \quad \text{and} \quad p_i \geq 0, \tag{12}$$

yields the set of all stationary distributions. Then, if randomization is not feasible, equation (11) defines a one-dimensional family of stationary distributions on the simplex (12). If the policy is applied, then it shifts the locus of stationary distributions on that simplex. Deviatov and Wallace (2001) show that under some parameters it is feasible to reach out distributions consistent with an increased frequency of trades. That external margin (distribution) effect gives rise to higher ex ante utility in their model.

However, if randomization is feasible, there exist many randomization schemes consistent with any distribution p being a stationary distribution. Thus, an (α, δ)-policy no longer enlarges the set of feasible distributions.

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However, an expansionary policy tends to tighten producer participation constraints. Because the optima tend to have binding producer participation constraints—and, hence, $y_{ij} \leq y^*$—in many meetings (in all meetings if individuals are sufficiently impatient), the policy tends to reduce welfare and, therefore, cannot be optimal.

There are two other features that are common to every example. First, there are no binding consumer participation constraints. (This, however, is not surprising because, as demonstrated in Berentsen, Molico and Wright (2002), money has no value if the gain from trade for consumers is zero.) Second, in a meeting of a producer with no money and a consumer with two units, one unit of money changes hands with probability one. I take advantage of these common features to simplify the description of results in the tables below. I omit the probabilities of transfer of money in meetings of producers with nothing and consumers with two units (λ_{02}^1 and λ_{02}^2). I also suppress superscripts in the notation for the other transfer probabilities (λ_{01}^1, λ_{12}^1 and λ_{11}^1). I attach stars (*) to outputs which correspond to binding producer participation constraints and daggers (†) to the transfer probabilities which correspond to binding first order constraints in (10).

I compute two sets of examples. In the first set I compute examples for moderate risk aversion and patience, i.e. for all combinations of κ and r from $\kappa \in \{0.2, 0.3, 0.4, 0.5, 0.6\}$ and $r \in \{0.01, 0.02, 0.03, ..., 0.25, 0.30, 0.35, 0.40, 0.50, 1.00\}$. Because I keep the cost function constant in all examples, more risk averse individuals derive higher utility gains from trade (i.e. $u(y) - c(y) = y^\kappa - y$ is a strictly decreasing function of κ for all y, $0 < y < \widehat{y} = 1$).

I report a subset of these examples in Tables 1-3 (one table for each value of risk aversion κ). All the results are consistent with the existence of four different regions with respect to the degree of patience r. If r is small enough, then the optima have randomization over the transfers of money in all three trade meetings where transfers of only one unit are feasible. If r belongs to the second region, then the optima have randomization over the transfers of money only in meetings where the consumers have one unit. In meetings of producers with one and consumers with two units, money changes hands with probability one. In the next region the optima have randomization over the transfers of money in meetings where both producers and consumers have one unit. Finally, if r is big enough, one unit of money changes hands with probability one in all trade meetings.

The examples are consistent with the transfer probabilities λ_{12}, λ_{01}, and

λ_{11} being decreasing functions of patience. On the other hand, the optimum quantity of money, $p_1 + 2p_2$, is an increasing function of patience, which is not surprising because less money loosens producer participation constraints (less money implies a higher probability of meeting other producers without money in the future). In all examples where money creation is not beneficial, the fraction of individuals with one unit of money, p_1, is an increasing function of patience and increasing function of risk aversion. That leads me to surmise that higher (utility) gains from trade and greater patience seem to increase the extent to which external margin (distribution) effect is beneficial for trade. However, when money creation is beneficial, the optimal policy is not a monotone function of patience, so that p_1 is not monotone too. Also, there is an increase in p_1 when one moves from the region in parameter space where money creation is not beneficial to the region where it is beneficial.

Examples in the first set are consistent with the optima having at most one nonbinding producer participation constraint, the one in meetings of producers with nothing and consumers with two units of money. In a meeting of a producer with one unit and a consumer with two, lowering the probability of handing over money raises v_2. That is helpful because it loosens producer constraint in the $(1, 1)$ meeting, which, in turn, allows a decrease in λ_{11} and, thus, an increase in p_1 (and, thereby, in the frequency of trade). Because λ_{11} is low, the participation constraint in the $(1, 1)$ meeting is binding and the output is low. Likewise, a smaller probability of giving up money in the $(0, 1)$ meeting lowers v_0 which helps to relax the producer constraint in the $(0, 2)$ meeting. This allows a higher y_{02} which, again, pushes up v_2. This accounts for why y_{02} is so high in some examples. The same kind of effect on v_2 could be achieved with a positive λ_{02}^0, but that would reduce λ_{02}^1 and, hence, the inflow into p_1.

The second set of examples (reported in Table 4) shows optima for low patience and high risk aversion (high utility gains from trade). Here I compute examples for $\kappa = 0.2$ and $r \in \{0.80, 0.85, 0.90, 0.95, 1.00, 1.10, 1.20, 1.50, 2.00, 3.00\}$. If r is low, then the optima have randomization in meetings and no policy. If r is high, then the optima have no randomization because provided that individuals are highly impatient randomization becomes too costly. In that case the optima have large (α, δ)-policies needed to sustain distributions which imply high frequency of trade in meetings. Large (α, δ) is incentive feasible when risk aversion κ is high because given higher utility gains from trade, risk averse individuals are willing to trade money even if acquisition of money is risky due to a high risk δ of subsequent loss of

money. Also, in all examples where money creation is beneficial, the optima have take-it-or-leave-it offers in all meetings — the bargaining rule assumed by Molico (2006).

7   Concluding remarks

This paper adds to the list of models where money creation is beneficial. Because I work with fully decentralized environment, analytical solutions are not feasible, so I do a series of numerical examples. I compute examples for a case of two-unit bound on individual money holdings. That bound is the lowest for which money creation can have a beneficial role. A natural question is what happens for all higher bounds. It is intuitive that as the bound gets larger, randomization plays a smaller and smaller role and, in the limit, no role. The absence of examples where money creation is beneficial and individuals randomize in meetings leads me to surmise that beneficial effects of money creation persist for all higher bounds. Moreover, as the bound becomes large, it seems plausible that money creation will be beneficial for lower risk aversion and greater patience than in case of two-unit bound. That conjecture seems somewhat difficult to verify because the dimensionality of the optimization problem is proportional to the cube of the bound and even for a case of a three-unit bound numerical analysis is demanding. One way to get around the curse of dimensionality is to work with environments with partially centralized markets, where the distribution of money is "manageable".[7] However, that should be done with caution because the external margin beneficial effect of money creation depends on having heterogeneity in the model.

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[7]See e.g. Lagos and Wright (2003) and Berentsen, Camera, and Waller (2005).

References

[1] Aiyagari, R., N. Wallace, R. Wright, 1996, Coexistence of Money and Interest-Bearing Securities, Journal of Monetary Economics 37, 397-420.

[2] Berentsen, A., G. Camera, C. Waller, 2005, The distribution of money balances and the nonneutrality of money, International Economic Review 46, 465-494.

[3] Berentsen, A., M. Molico, R. Wright, 2002, Indivisibilities, lotteries and monetary exchange, Journal of Economic Theory 107, 70-94.

[4] Bhattacharya, J., J. Haslag, A. Martin, 2005, Heterogeneity, redistribution, and the Friedman rule, International Economic Review 46, 437-464.

[5] Camera, G., 2005, Distributional aspects of the divisibility of money: an example, Economic Theory 25, 487-496.

[6] Dawid H, 1999, Adaptive learning by genetic algorithms (Springer Verlag, Berlin).

[7] Deviatov, A., N. Wallace, 2001, Another example in which money creation is beneficial, Advances in Macroeconomics 1, article 1.

[8] Houck, C., J. Joines, M. Kay, 1996, A genetic algorithm for function optimization: a MATLAB implementation, mimeo, North Carolina State University.

[9] Imrohoroglu, A., 1992, The welfare cost of inflation under imperfect insurance, Journal of Economic Dynamics and Control 16, 79-91.

[10] Judd, K., 1998, Numerical methods in Economics (MIT Press, Cambridge).

[11] Katzman, B., J. Kennan, N. Wallace, 2003, Output and price level effects of monetary uncertainty in a matching model, Journal of Economic Theory 108, 217-255.

[12] Kehoe, T., D. Levine, M. Woodford, 1992, The optimum quantity of money revisited, in: Dasgupta P. et al, eds., Economic analysis of markets and games: essays in honor of Frank Hahn, (MIT Press, Cambridge) 501-526.

[13] Lagos, R., R. Wright, 2003, Dynamics, cycles, and sunspot equilibria in 'genuinely dynamic, fundamentally disaggregative' models of money, Journal of Economic Theory 109, 156-171.

[14] Levine, D., 1991, Asset trading mechanisms and expansionary policy, Journal of Economic Theory 54, 148-164.

[15] Lucas, R., M. Woodford, 1994, Real effects of monetary shocks in an economy with sequential purchases, Mimeo, University of Chicago.

[16] Molico, M., 2006, The distribution of money and prices in search equilibrium, forthcoming in International Economic Review.

[17] Shi, S., 1995, Money and prices: a model of search and bargaining, Journal of Economic Theory 67, 467-496.

[18] Trejos, A., R. Wright, 1995, Search, bargaining, money and prices, Journal of Political Economy 103, 118-141.

Table 1: Optima when $u(y) = x^{0.6}$. $\arg\max\left[u(y) - y\right] = 0.2789$.

r	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50	1.00
α	0	0	0	0	0	0	0	0	0	0
δ	0	0	0	0	0	0	0	0	0	0
p_0	.2307	.2689	.3241	.3822	.4285	.4629	.4895	.5222	.5454	.6043
p_1	.5593	.4936	.4263	.3894	.3628	.3417	.3246	.3025	.2950	.2727
p_2	.2100	.2374	.2496	.2284	.2087	.1954	.1859	.1753	.1596	.1230
λ_{01}	$.2693^\dagger$	$.4492^\dagger$	$.8033^\dagger$	1	1	1	1	1	1	1
λ_{12}	$.3896^\dagger$	$.6435^\dagger$	1	1	1	1	1	1	1	1
λ_{11}	$.1548^\dagger$	$.2620^\dagger$	$.4452^\dagger$	$.5758^\dagger$	$.6796^\dagger$	$.7749^\dagger$	$.8640^\dagger$	1	1	1
y_{01}	$.2789^*$	$.2789^*$	$.2789^*$	$.2345^*$	$.1730^*$	$.1331^*$	$.1055^*$	$.0699^*$	$.0498^*$	$.0147^*$
y_{12}	$.2789^*$	$.2789^*$	$.2342^*$	$.1445^*$	$.0973^*$	$.0686^*$	$.0501^*$	$.0285^*$	$.0179^*$	$.0033^*$
y_{11}	$.1108^*$	$.1136^*$	$.1043^*$	$.0832^*$	$.0661^*$	$.0531^*$	$.0432^*$	$.0285^*$	$.0179^*$	$.0033^*$
y_{02}	.6945	.5769	$.3472^*$	$.2345^*$	$.1730^*$	$.1331^*$	$.1055^*$	$.0699^*$	$.0498^*$	$.0147^*$

Table 2: Optima when $u(y) = x^{0.4}$. $\arg\max\left[u(y) - y\right] = 0.2172$.

r	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50	1.00
α	0	0	0	0	0	0	0	0	0	0
δ	0	0	0	0	0	0	0	0	0	0
p_0	.2022	.2437	.2715	.2892	.3197	.3448	.3660	.4046	.4327	.4834
p_1	.6240	.5542	.5012	.4647	.4316	.4079	.3898	.3623	.3411	.3134
p_2	.1738	.2021	.2273	.2461	.2487	.2473	.2442	.2331	.2262	.2032
λ_{01}	$.1419^\dagger$	$.2495^\dagger$	$.3793^\dagger$	$.5081^\dagger$	$.6756^\dagger$	$.8439^\dagger$	1	1	1	1
λ_{12}	$.2795^\dagger$	$.4837^\dagger$	$.7280^\dagger$	$.9735^\dagger$	1	1	1	1	1	1
λ_{11}	$.0903^\dagger$	$.1604^\dagger$	$.2456^\dagger$	$.3294^\dagger$	$.4267^\dagger$	$.5125^\dagger$	$.5884^\dagger$	$.7188^\dagger$	$.8412^\dagger$	1
y_{01}	$.2172^*$	$.2172^*$	$.2172^*$	$.2172^*$	$.2172^*$	$.2172^*$	$.2142^*$	$.1600^*$	$.1255^*$	$.0529^*$
y_{12}	$.2172^*$	$.2172^*$	$.2172^*$	$.2172^*$	$.1614^*$	$.1218^*$	$.0946^*$	$.0616^*$	$.0426^*$	$.0111^*$
y_{11}	$.0702^*$	$.0720^*$	$.0733^*$	$.0735^*$	$.0689^*$	$.0624^*$	$.0556^*$	$.0442^*$	$.0358^*$	$.0111^*$
y_{02}	.6720	.5491	.4738	$.4247^*$	$.3215^*$	$.2573^*$	$.2142^*$	$.1600^*$	$.1255^*$	$.0529^*$

Note that for $r = 0.01$ the set of binding first order constraints (10) includes binding inequality constraint for λ_{02}^1 which is not shown in the table.

Table 3: Optima when $u(y) = x^{0.2}$. $\arg\max\left[u(y) - y\right] = 0.1337$.

r	0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50	1.00
α	0	0	0	0	0	0	0	0	0	.2498
δ	0	0	0	0	0	0	0	0	0	.1763
p_0	.1494	.1925	.2244	.2448	.2600	.2721	.2824	.3022	.3232	.3686
p_1	.7309	.6563	.5992	.5608	.5314	.5077	.4888	.4572	.4302	.3970
p_2	.1197	.1512	.1764	.1944	.2086	.2202	.2288	.2406	.2466	.2344
λ_{01}	$.0453^{\dagger}$	$.0908^{\dagger}$	$.1475^{\dagger}$	$.2021^{\dagger}$	$.2565^{\dagger}$	$.3106^{\dagger}$	$.3611^{\dagger}$	$.4668^{\dagger}$	$.5872^{\dagger}$	1
λ_{12}	$.1525^{\dagger}$	$.2943^{\dagger}$	$.4722^{\dagger}$	$.6435^{\dagger}$	$.8154^{\dagger}$	$.9886^{\dagger}$	1	1	1	1
λ_{11}	$.0335^{\dagger}$	$.0676^{\dagger}$	$.1103^{\dagger}$	$.1513^{\dagger}$	$.1921^{\dagger}$	$.2326^{\dagger}$	$.2704^{\dagger}$	$.3477^{\dagger}$	$.4306^{\dagger}$	1
y_{01}	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.0582^{*}$
y_{12}	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.1337^{*}$	$.1165^{*}$	$.0882^{*}$	$.0659^{*}$	$.0162^{*}$
y_{11}	$.0294^{*}$	$.0307^{*}$	$.0312^{*}$	$.0315^{*}$	$.0315^{*}$	$.0315^{*}$	$.0315^{*}$	$.0307^{*}$	$.0284^{*}$	$.0162^{*}$
y_{02}	.6087	.5243	.4097	.3647	.3338	.3127	.3113	$.2865^{*}$	$.2277^{*}$	$.0582^{*}$

Note that for $r = 0.01$ the set of binding first order constraints (10) includes binding inequality constraint for λ_{02}^{1} which is not shown in the table.

Table 4: Optima when $u(y) = x^{0.2}$ (continued). $\arg\max\left[u(y) - y\right] = 0.1337$.

r	0.80	0.85	0.90	0.95	1.00	1.10	1.20	1.50	2.00	3.00
α	0	0	.2483	.2489	.2498	.2522	.2551	.2647	.2795	.2321
δ	0	0	.1741	.1751	.1763	.1789	.1818	.1901	.2018	.1762
p_0	.3689	.3739	.3660	.3674	.3686	.3708	.3727	.3768	.3810	.3920
p_1	.3770	.3705	.3970	.3970	.3970	.3974	.3977	.3993	.4016	.3908
p_2	.2551	.2556	.2370	.2356	.2344	.2318	.2296	.2239	.2174	.2172
λ_{01}	$.9538^{\dagger}$	1	1	1	1	1	1	1	1	1
λ_{12}	1	1	1	1	1	1	1	1	1	1
λ_{11}	$.6608^{\dagger}$	$.6963^{\dagger}$	1	1	1	1	1	1	1	1
y_{01}	$.1337^{*}$	$.1317^{*}$	$.0640^{*}$	$.0610^{*}$	$.0582^{*}$	$.0531^{*}$	$.0488^{*}$	$.0387^{*}$	$.0282^{*}$	$.0201^{*}$
y_{12}	$.0323^{*}$	$.0291^{*}$	$.0181^{*}$	$.0171^{*}$	$.0162^{*}$	$.0145^{*}$	$.0131^{*}$	$.0100^{*}$	$.0071^{*}$	$.0044^{*}$
y_{11}	$.0213^{*}$	$.0203^{*}$	$.0181^{*}$	$.0171^{*}$	$.0162^{*}$	$.0145^{*}$	$.0131^{*}$	$.0100^{*}$	$.0071^{*}$	$.0044^{*}$
y_{02}	$.1402^{*}$	$.1317^{*}$	$.0640^{*}$	$.0610^{*}$	$.0582^{*}$	$.0531^{*}$	$.0488^{*}$	$.0387^{*}$	$.0282^{*}$	$.0201^{*}$