Centre for
Economic
and Financial
Research
at
New Economic
School

May 2006

Public Markets Tailored for the Cartel: Favoritism in Procurement Auctions

Ariane Lambert Mogiliansky,
Grigory Kosenok

Working Paper No 74

CEFIR / NES Working Paper series

Public Markets Tailored for the Cartel
- Favoritism in Procurement Auctions -*

Ariane Lambert Mogiliansky†, Grigory Kosenok‡

May 2, 2006

Abstract

In this paper, we investigate interaction between two firms engaged in a repeated procurement relationship modelled as a multiple criteria auction, and an auctioneer (a government employee) who has discretion in devising the selection criteria.

A first result is that, in a one-shot context, favoritism turns the asymmetric information (private cost) procurement auction into a symmetric information auction (in bribes) for a common value prize. In a repeated setting we show that favoritism increases the gains from collusion and contributes to solving basic implementation problems for a cartel of bidders that operates in a stochastically changing environment. A most simple allocation rule where firms take turn in winning independently of stochastic government preferences and firms' costs is optimal. In each period the selection criteria is fine-tailored to the in-turn winner: the "environment" adapts to the cartel. This result holds true when the expected punishment is a fixed cost. When the cost varies with the magnitude of the distortion of the selection criteria (compared with the true government's preferences), favoritism only partially shades the cartel from the environment. Nevertheless, even in this case favoritism greatly simplifies matters for the cartel. We thus find that favoritism generally facilitates collusion at a high cost for society. Some policy implications of the analysis are suggested.

Keywords: auction, collusion, favoritism, procurement

JEL: D44, D73, H57

*We are thankful to Olivier Compte, Philippe Jehiel and Paul Milgrom for insightful comments and suggestions as well as to seminar participants at INSEAD and NES for fruitful discussions. Financial support from the New Economic School is gratefully acknowldeged.

†PSE, Paris-Jourdan Sciences Economiques (CNRS, EHESS, ENS, ENPC) Paris, alambert@pse.ens.fr.

‡New Economic School, Moscow, gkosenok@nes.ru

1 Introduction

Many cartels operate in a stochastically changing environment. In particular, this is the case of firms involved in public procurement. The public demand for e.g., construction works typically depends on a number of factors that are difficult to predict. They include social needs, elected representatives' political agenda, internal budget concerns etc... In addition, firms' technology changes with time. Altogether this implies a significant uncertainty about the profitability of future contracts. In face of such an uncertain environment, a cartel of firms must devise a mechanism that while being responsive to changes does not open up for gaming opportunities. In this paper we propose that favoritism can contribute to solving key problems for a cartel of bidders that operate in a stochastically changing environment. A main motivation for the paper is the mounting body of evidence that collusion and corruption often go hand in hand in public procurement.

In France, practitioners and investigators in courts of accounts, competition authorities, and in the judiciary have long been aware of the close links between collusion and corruption in public procurement. The testimony of J. C. Mery provides suggestive evidence of those links (*Le Monde*, September 22 and 23, 2000).[1] A recent judgment in 'Les Yvelines' (Cour d'Appel de Versaille, January 2002) provides a vivid illustration as well. According to a judge investigating a major corruption case in Paris, there exists in France, almost not a single case of large stake collusion in public procurement without corruption.[2] Beside empirical motivations, there are theoretical motivations for investigating the links between favoritism and collusion. In particular, a cartel typically faces a tension between the efficiency goal and the need to provide firms with incentives to reveal private information. A fair amount of attention has been given to the theoretical problems facing a cartel that operates within an imperfectly or privately observable environment. Recently, Athey and al. (2004) show that it can simply be too costly for a price cartel to provide the right incentives for firms to reveal private information about shocks to costs so the optimal mechanism entails price rigidity (see also Green and Porter (1984) for the analysis of a price cartel on a market

[1] J. C. Mery, a City Hall official, admitted that for ten years (1985-94) he organized and arbitrated collusion in the allocation of most construction and maintenance contracts for the Paris City Hall. In exchange, firms were paying bribes used to finance political parties.

[2] The case concerns the procurement of a 4.3 billion euros market for the recontruction of Paris's lycees (see *Le Monde* April 23 2005).

with a demand subject to shocks). Our analysis is concerned with a cartel of bidders that face both incomplete information about demand i.e., government preferences and asymmetric information about shocks to firms' costs. A central result is that favoritism can shade the cartel from hazards in the environment. The cartel can achieve full (cartel) efficiency with a non-contingent allocation rule so that firms take turn to win in a pre-determined manner. The cartel needs not adapt to the environment. Instead the environment adapts to the cartel: the contract is fine-tailored to the pre-determined in-turn winner. This result is established for the case the expected punishment cost is independent of the magnitude of the distortion of government preferences. When the expected punishment varies with that magnitude, favoritism only partially shades the cartel from hazards in the environment. Favoritism still greatly simplifies matters for the cartel. But the equilibrium allocation is bounded away from full cartel efficiency. Favoritism generally exacerbates the social costs of collusion: the selected specification is socially inefficiency and the price paid by the government is higher than in the absence of favoritism.

We model the procurement procedure as a "first score auction". Two firms characterized by a vector of cost parameters compete in scores with offers that include a specification of the project and a price. Public preferences are stochastic. The procedure is administered by an auctioneer who is a government employee. At the beginning of the period the auctioneer, privately observes a signal of public preferences. His duty is to devise and announce a scoring rule that reflects the (current) public preferences. In the absence of favoritism, the procedure selects the socially efficient specification of the project.

The presence of asymmetric information between the government and its auctioneer implies that the auctioneer has some discretion when deciding over the scoring rule. We call favoritism the act of biasing the scoring rule in favor of one of the firms. Corruption is modelled as an auction-like procedure that takes place before the official auction. Firms compete in (menus of) corrupt "deals" including a bribe and a demanded scoring rule. We find that with favoritism the procedure selects a non-standard specification of the project. The intuition is that the associated scoring rule induces minimal competition and thus maximal profit-if-win in the official auction. In the one-shot setting favoritism turns the asymmetric information private cost procurement auction, into a symmetric information auction (in bribes) for a common value prize corresponding to influence over the design of the contract.

We then consider a situation where firms meet repeatedly, each period on a new market (the auctioneers are short-run players). We show that favoritism fully solves the cartel's problems related to stochastic government preferences and privately observable costs. Provided each firm is efficient at producing some specification of the project, the cartel can earn the maximal income in a scheme that selects the winner independently of the true preferences and of firms' costs. The intuition is that at the corruption stage firms submit corrupt deals that truthfully reveal private information about their costs to the auctioneer. This is because a corrupt auctioneer has own incentives to use that information to fine-tailor the scoring rule to the in-turn winner. Firms' main concern is to contain competition in bribes. That is achieved by opting for a fixed in-turn allocation rule which makes any defection immediately observable.

In an extension we investigate a case where the expected punishment for favoritism is a function of the magnitude of the distortion between the announced scoring rule and the true preferences. We find that the central insights from the fixed punishment case carry over. In the stage game competition in bribes does not dissipates all the firms' rents however. And in the repeated setting the cartel may face a problem of imperfect public information. For high cost of punishment, the optimal scheme is contingent on the true government preferences which are never observed. The official auction outcome is bounded away from full cartel efficiency. For low cost of punishment the pre-determined in-turn allocation rule is optimal and full cartel efficiency obtains.

The equilibrium allocation patterns emerging from the analysis is consistent with empirical findings. There exists ample evidence e.g., in developing countries of problems of maintenance of construction objects due to the non-standard design that was selected in the international procurement procedure (see Rose-Ackerman 1999). Evidence from corruption scandals in France also show that the tender winner is the most efficient firm and that its profits often are larger than the average in the branch (30% contra 5%) as in the case with the court case concerned with the series of constructions contracts in Paris.

This paper contributes to a growing literature on corruption in auction.[3] The auctioneer's abuse of discretion to devise the selection rule has been studied in Che and Burget (2004) in the context of a single auction. The present article is most closely related to Compte, Lambert-Mogiliansky and Verdier (2005) and Lambert-Mogiliansky and Sonin (forthcoming

[3] See for instance Laffont J-J. and J. Tirole (1991), Celentani, M. and J. Ganunza, (2002), Che and Burget (2004), Compte, Lambert-Mogiliansky and Verdier (2005).

2006). Both articles are concerned with links between corruption and collusion. They address a cartel's enforcement problem in a one-shot setting and focus on the impact of the auctioneer's abuse of discretion to let firms readjust their bid. In Compte et al. the auctioneer sells an illegal opportunity to resubmit, which is shown to permit sustaining collusion in a single object auction. In Lambert-Mogiliansky and Sonin, the auctioneer abuses a legal right to let all firms simultaneously readjust their offer in the context of a multiple-object auction. As a consequence collusive market-sharing becomes sustainable. The contribution of the present paper is to demonstrate corruption's role with respect to another central problem of a cartel: how to achieve (cartel) efficiency in a stochastically changing environment.

The paper is organized as follows. The model is described in section 2. Section 3 offers an analysis of the one-stage game. In Section 4 we derive our central results. Section 5 proposes an extension to the case with varying punishment cost. Central assumptions are discussed in section 6 where we also suggest policy implications for procurement and control agencies.

2 The model

In each time period a project is allocated. A project allows for a multiplicity of specifications. A specification is a vector $\mathbf{q} = (q_1, ...q_k)$ where q_j represents the level of the j (quality) component. There are two firms indexed i, $i = 1, 2$, which are characterized by their cost function

$$c_i\left(\mathbf{q}; \boldsymbol{\theta}_i^t\right) = \sum_{j=1}^{k} \frac{\theta_{ij}^t q_j^2}{2}$$

where $\theta_{ij}^t \in \left\{\underline{\theta}, \overline{\theta}\right\}$, $j = 1, ..., k$ is firm i's cost parameter associated with quality component q_j in period t. The vector of cost parameters $\boldsymbol{\theta}_i^t = \left(\theta_{i1}^t, ...\theta_{ik}^t\right)$ is firm i's private information. In each period there is a new draw of $(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$. For the sake of convenience we remove the realizations $\boldsymbol{\theta}_i^t = (\overline{\theta}, \overline{\theta}, ..., \overline{\theta})$, $i = 1, 2$ from the support. We also remove the joint realizations where one of the firms is strictly more efficient than the other and when the firms are fully identical.[4] The probability of the realizations left are proportional to the probabilities which we would have if parameters θ_{ij}^t are i.i.d. with $prob(\theta_{ij}^t = \underline{\theta}) = \rho$ across all i, j and t.[5]

[4] Behind practical convenience, the idea is that both firms must be sufficiently efficient (they have low cost on at least one component) and different from each other i.e., they have a comparative advantage in at least one component.

[5] The precise characterization of the probability is rather complex but its details play no role for our results.

The government derives utility from the realization of a project in period t:

$$W\left(\mathbf{q}^t, p^t; \boldsymbol{\alpha}^t\right) = \alpha_1^t q_1^t + ... + \alpha_k^t q_k^t - p^t; \quad (1)$$

$$\text{with } \alpha_j^t \geq 0, \ \forall j = 1...k, \ \sum_{j=1}^{k} \alpha_j^t = 1, \quad (2)$$

where p^t is the price paid to the firm that delivers the project and $\boldsymbol{\alpha}^t = \left(\alpha_1^t, ..., \alpha_k^t\right)$ is a vector of parameters representing the true social preference in period t. A zero value for a component j, $\alpha_j^t = 0$ is understood as no social value of q_j above a *minimal level* that defines a "basic good". The vector $\boldsymbol{\alpha}^t$ is random with a uniform measure on Δ^{k-1}. The government does not know the true $\boldsymbol{\alpha}^t$. It hires an auctioneer who privately observes a signal of the true $\boldsymbol{\alpha}_t$ at the beginning of each period. For simplicity we assume that the signal is fully informative.[6]

The auction rule

At the beginning of each period the auctioneer announces a selection criteria which is a function of both price p and quality $\mathbf{q} = (q_1, ..., q_k)$. We consider a class of selection criteria similar to the government's utility function:

$$S(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) = s(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) - p = \sum_{j=1}^{k} \widehat{\alpha}_j q_j - p, \ \sum_{j=1}^{k} \widehat{\alpha}_j = 1,$$

where $\widehat{\boldsymbol{\alpha}}$ is the vector of parameters *announced* by the auctioneer (see *Timing* below). Throughout the paper we refer to $\widehat{\boldsymbol{\alpha}}$ as the "scoring rule" which is a slight abuse of language since the score of an offer is determined by its price also according to the selection criterion. The firms simultaneously submit in a sealed envelop an offer including a project specification $\mathbf{q}_i^t$ and a price p_i^t. The contract is awarded to firm i^{*t} whose offer maximizes (among submitted offers) the announced selection criteria subject to a "reserve score" normalized to zero:

$$i^{*t} \in \arg\max_{i=1,2} S\left(\mathbf{q}_i^t, p_i^t, \widehat{\boldsymbol{\alpha}}\right) \quad (3)$$

$$s.t \ : \ S\left(\mathbf{q}_i^t, p_i^t, \widehat{\boldsymbol{\alpha}}\right) \geq 0.$$

The winner is due to deliver the specification $\mathbf{q}_{i*}^t$ at price p_{i*}^t. In case of tie in scores the project is awarded to the firm whose "quality score"(i.e., $s(\mathbf{q}, \widehat{\boldsymbol{\alpha}})$) is highest. In case of tie in both price and quality the auctioneer randomizes. We refer to this procedure as a First Score Auction (FSA).

[6] This is a not an assumption crucial to our results.

The firm i's per-period profit-if-win is

$$\pi_i^t = p_i^t - c_i \left(\mathbf{q}_i^t ; \boldsymbol{\theta}_i^t \right). \tag{4}$$

Profit-if-lose is zero. The game is infinitely repeated with the same two firms but with a different auctioneer in each period. The firms discount future gains with a common factor δ. Their payoff for the whole game is the discounted sum of the per period payoffs.

Corruption

The auctioneer is opportunistic. He accepts bribes in exchange for announcing a scoring rule i.e., some $\widehat{\boldsymbol{\alpha}}$. The auctioneer's utility is

$$U = w + b - m,$$

where w is a wage that we normalize to 0, b is a bribe and $m \geq 0$ is a term that captures moral and other costs e.g. expected punishment for distorting the government preferences and for taking bribes.[7] In the basic model expected punishment is a fixed cost. This is consistent with e.g., French legislation (Code Penal 432-14, 432-11). In an extension we consider the special case where $k = 2$ and the expected punishment depends on the magnitude of the distortion of social preferences so $U = b - m \left(\widehat{\alpha}_1 - \alpha_1 \right)^2$. Such a model can be relevant when the magnitude of the distortion significantly affects the probability of detection. We discuss these assumptions in section 6.

Corruption is modelled as a procedure whereby the firms compete in corrupt "deals" where a deal is an offer to pay a bribe in exchange for a specific scoring rule. The two firms simultaneously and secretly submit a menu of deals $M_i = \{ (\boldsymbol{\alpha}_{il}, b_{il}),\ l = 1, ..., n_i \}$, where n_i is freely chosen by firm i. The bribe is only paid by the official auction's winner if the announced scoring rule corresponds to one he demanded *and* the offers submitted in the official auction are not identical i.e., include both the same quality specification and the same price. This rule is not crucial to the main results but it greatly simplifies the analysis as it contributes to aligning firms' and the auctioneer's incentives. It is also consistent with the presumption that fully identical offers reflect fierce competition so favoritism brings no gain to the winning firm.

[7] The government can engage a procedure to find out its true preferences and punish the auctioneer if he distorted them in his announcement.

3 The stage game

The stage game is defined by the following *Timing*:

Step 0: Firms learn privately their cost parameters $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$.

step 1: The auctioneer learns $\boldsymbol{\alpha}$, the firms submit $M_1 = \{(b_{1l}, \boldsymbol{\alpha}_{1l})\}$ and $M_2 = \{(b_{2l}, \boldsymbol{\alpha}_{2l})\})$ respectively;

step 2: The auctioneer makes an announcement $\widehat{\boldsymbol{\alpha}}$, $\widehat{\boldsymbol{\alpha}} \in \Delta^{k-1}$;

step 3: The firms submit $(\mathbf{q}_i, p_i)$, $i = 1, 2$;

step 4: The auctioneer publicly opens the envelops and he selects the firm whose offer maximizes the selection criteria corresponding to the announced scoring rule. The winner i^* pays a bribe b if and only if $(\widehat{\boldsymbol{\alpha}}, b) \in M_{i*}$. Otherwise no bribe is paid.

We first establish a result applying to the First Score Auction described by the *Timing* above when deleting step 1 from consideration and as we show later applying to any subgame starting from step 2. Throughout the paper we consider subgame perfect equilibria.

Lemma 1 *The subgame perfect equilibrium offers of the FSA are characterized by specification efficiency:* $\mathbf{q}_i^* = \arg\max s(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) - c(\mathbf{q}; \boldsymbol{\theta}_i)$.

All proofs are gathered in the appendix.

The result exploits separability between quality and price in the selection criteria. It can be shown that for any offer not including the efficient values for the components, we can find another offer with the same score but that yields a higher expected profit.

The result in Lemma 1 greatly simplifies the forthcoming analysis. Lemma 1 allows us to, *at step 3*, separate between firms' offer of project specification and their price bid.[8] The equilibrium values of the components are the efficient ones corresponding to the announcement $\widehat{\boldsymbol{\alpha}}$

$$\begin{aligned} \mathbf{q}_i^* &= \arg\max s(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) - c(\mathbf{q}; \boldsymbol{\theta}_i) \\ q_{ij}^*\left(\theta_{ij}, \widehat{\alpha_j}\right) &= \frac{\widehat{\alpha_j}}{\theta_{ij}}, \; i = 1, 2, \; j = 1..., k. \end{aligned}$$

When the announcement corresponds to the true government preferences, Lemma 1 implies social efficiency in project specification.

[8] A similar result can be found in Che (1993).

3.1 Favoritism

We now proceed to investigate the one-shot game described in *Timing* above.

Proposition 1 *In any Bayes-Nash equilibrium we have that*

i. For $m \leq \frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}$ *and* $\boldsymbol{\theta}_1 \neq \boldsymbol{\theta}_2$ *the equilibrium scoring rule is* $\widehat{\boldsymbol{\alpha}}^* = (0, ..., 1_j, ...0)$ *for some* $\theta_j; \theta_{1j} \neq \theta_{2j}$ *and* $b^*_{1j} = b^*_{2j} = \frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}$ *for all* j*. When* $m > \frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}$ *or* $\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2$ $(\theta_{1j} = \theta_{2j}$ *for all* $j)$ *there is no favoritism .*

ii. The equilibrium offers are the competitive offers relative to the announced scoring rule.

A first result is that whatever the true government preferences, favoritism always entails an extreme (single-peaked) scoring rule $\widehat{\boldsymbol{\alpha}}^* = \widehat{\boldsymbol{\alpha}}^j = (0, ..., 1_j, ...0)$ for some j.[9] The intuition for the single-peakness result is that the winner's profit is maximal when the scoring rule emphasizes a single component for which he has a comparative advantage. Alternatively, a selection rule including a single-peaked scoring rule induces the "weakest possible competitive pressure" among all FSA generated by any selection rule from the relevant class.

The interpretation of this result is that with favoritism the scoring rule tends to drive to a minimum the weight given to most components while emphasizing quite exclusively a component characterized by weak competition in production.[10] This means that the winning project has a specification that tends to be "non-standard". We note that the true government's preferences have minimal impact on the announced scoring rule. In case of ties in the corruption game, the auctioneer may choose the deal that is most congruent with the true preferences.

Quite remarkably we find that firms' asymmetric information is a minor concern in our context. The intuition is that while an honest auctioneer uses information about costs to minimize firms' rents[11], a corrupt auctioneer has incentives to use that information to devise a scoring rule that *maximizes* the winning firm's rents. In equilibrium the firms' cost structure

[9] We show in section 5 below (Extensions) that this result is robust to other specification of the punishment costs. The general argument is similar to that in Lemma 1. It rests on the separability between bribe and punishment in the auctioneer's utility function (see Lemma 2 in the proof of Proposition 4).

[10] Strictly speaking the interpretation of zero weight as a minimal level is equivalent to assuming that firms are (more or less identical) in the production of a "basic good" while they differ in the production of specifications of the project in excess of the requirement defining the "basic good".

[11] If the honest auctioneer knew the firms' costs, he would simply give the contract to the most efficient firm and pay it its actual cost i.e., he would leave no rents to the winner. Therefore firms are unwilling to reveal their costs.

is revealed in the submitted menus of deals and firms infer all relevant information about each other's cost structure from the announced scoring rule. The equilibrium offers are the (unique) competitive Nash equilibrium offers of the *symmetric* information FSA defined by the announced scoring rule (for details see the appendix). The offered specifications are efficient relative to $\widehat{\boldsymbol{\alpha}}^*$ and the equilibrium price is determined by the second score.

Competition for favors drives up the bribe to $\frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}(\geq m)$ which is the profit that yields with a scoring rule that is most favorable to the winner. Since this profit is the same for the two firms, the auctioneer captures the totality of the winning firm's rents. In the remaining of the paper we assume that $m < \frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}$ so the stage game is characterized by favoritism.[12]

In effect, favoritism turns the asymmetric information (private cost) auction, into a symmetric information common value auction (in bribes) for a prize. The prize is influence over the design of the selection rule which has a common value corresponding to the gain when winning the official auction with a maximally favorable selection criteria. This gain is common knowledge and identical for both firms.[13] The social cost of favoritism is twofold. First, a socially inefficient project specification is selected. Second, the price paid by the government is higher than in the absence of favoritism. The bias in project specification due to favoritism minimizes competition between firms. The equilibrium depicted in Proposition 1 will serve a threat point in the collusive schemes we study next.

4 Repeated interaction

We now proceed to investigate a situation when the two firms interact repeatedly. In each period they meet on a public market administered by a new auctioneer, e.g., different local governments. In each period there is a new draw of $\left(\boldsymbol{\theta}_1^t, \boldsymbol{\theta}_2^t, \boldsymbol{\alpha}^t\right)$. We are interested in collusion between the two firms under the assumption that transfers between them are precluded.

Information assumptions

[12]When reviewing court cases, it appears quite clear that the cost of favoritism is very low. The only instances of conviction for favoritism in France pertain to cases where the auctioneer explicitly required a firm specific technology. (Cour des Graces 2002).

[13]We consider a symmetric case but the logic would be the same if we allowed for some asymmetry in the cost structure. All that a firm needs to know is the other firm's value of winning the contract under the most favorable circumstances i.e., with a selection rule that gives full weight to a component such that the firm has the largest comparative advantage in its production.

At the end of each period the submitted contract offers are publicly observed by the two firms and the active auctioneer. The corrupt deal offers remain private information to the involved parties. The true value of $\boldsymbol{\alpha}$ is never revealed. The public history of the game up to period t is denoted H_t. Each auctioneer is appointed for one period and there is no communication between auctioneers from different periods.

4.1 Collusion

As a benchmark we characterize the optimal collusive scheme under symmetric information and in the absence of favoritism. The timing of the stage game is as follows.

Step 0: Firms learn the cost parameters $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$.

step 1: The auctioneer learns $\boldsymbol{\alpha}_t$ and makes an announcement $\widehat{\boldsymbol{\alpha}}$, $\widehat{\boldsymbol{\alpha}} \in \Delta^{k-1}$;

step 2: The firms submit $(\mathbf{q}_i, p_i)$;

step 3: The auctioneer selects the firm whose offer maximizes the announced selection criteria.

The two firms play this game repeatedly an infinite number of periods each time with a new auctioneer. They discount the future with a common factor δ.

Proposition 2 *There exists $\underline{\delta_0} < 1$, such that for $\delta \geq \underline{\delta_0}$ collusion is a Bayes-Nash equilibrium of the repeated game. An optimal collusive scheme entails an allocation rule contingent on the true government preferences and on the firms' costs. The winner is (one of) the most efficient firm(s) relative to the government's true preferences.*

Proposition 2 establishes that in any optimal collusive scheme the winner's identity depends on the government true preferences and on the firm's cost. This is not surprising since for any given selection criteria, the cartel's income is maximized when the most efficient firm relative to that criteria implements the contract. With symmetric information about costs and in the absence of favoritism the optimal collusive cartel can be implemented. We shall not investigate the asymmetric information optimal cartel, which is a serious enterprise aside the main focus of this paper. We content ourselves with taking notice of the dependence of the optimal symmetric information scheme on firms' costs implies that under asymmetric information any collusive scheme is likely to be plagued by inefficiency. This is due to firms' incentive to distort information and mimic other cost structures to increase the probability to win. Similar issues have been thoroughly investigated in Athey et al. (2004).

4.2 Collusion and Favoritism: A Strategic Complementarity

We now consider a repetition of the game described in *Timing* (Section 3) i.e., we are back in the asymmetric information context. Proposition 3 constitutes the central result of this paper.

Proposition 3 *i. There exists* $\underline{\delta_1} < 1$ *such that for* $\delta \geq \underline{\delta_1}$ *full cartel efficiency is achievable in a Bayes-Nash equilibrium of the repeated game.*

ii. In the official auction firms take turn in winning independently of government preferences and firms' costs.

iii. The equilibrium scoring rule is single-peaked (i.e., $\widehat{\alpha} = (0, ..., 1, ...0)$*) and the winning firm* i^* *pays a bribe* $b_{i^*} = m$.

Full cartel efficiency is defined for the official auction as follows. i. In each period the winner is (one of) the most efficient firms relative to the announced selection criteria; ii. The selection criteria that applies yields the highest gains to the winning firm from among all possible selection criteria. Proposition 3 establishes that with favoritism full cartel efficiency is achievable in spite of asymmetric information.[14] The cartel needs not adapt to the "environment" i.e., to the current cost structure or to the current government preferences. Instead the environment adapts to the cartel: in each period the auctioneer is bribed to fine-tailor the scoring rule to the in-turn winner. The optimal allocation rule is extremely simple: firm take turn for winning in a non-contingent manner. A main concern for the cartel is to contain competition in bribes which can be very costly as we learned from proposition 1. Proposition 3 shows that competition for favors can be eliminated by opting for the simplest non-contingent in-turn allocation rule. At the corruption stage both firms offer a menu of deals each with a single-peaked scoring rule as in proposition 1. The out-of-turn firm offers a zero bribe while the in-turn winner offer a bribe that just covers the expected punishment cost: m. The out-of-turn firm may deviate and (unobserved) bribe the auctioneer to announce a scoring rule favorable to itself. This is immediately detected however - the pre-determined in-turn rule is violated - and punished by reverting to the equilibrium of proposition 1 which yields zero payoff to the firms from next period on. This explains why the bribe can be kept to a minimum of $m \geq 0$. In the official auction the out-of-turn firm submits an offer that

[14] Notice that the firm that wins first has a higher expected discounted payoff. We can equate the two firms' discounted payoffs by randomly designing the first winner.

scores at most zero. Since contract offers become public information any defection at that stage is detected after the official opening and punished similarly.

We thus see that favoritism facilitates collusion in several ways. The gains from collusion are higher than with an honest auctioneer: the scoring rule is fine-tailored to maximize the winner's profit. While the threat payoffs are lower than in the absence of corruption because competition in bribes dissipates the rents. Most importantly we find that favoritism solves key problems for a repeated cartel in a stochastic environment. The auctioneer's self-interested determination of the scoring rule effectively shades the cartel from fluctuations in the profitability of projects due to stochastic government preferences and changing costs. The environment "adapts" to the cartel and ex-post efficiency i.e., efficiency relative to the *announced* scoring rule, is secured. But this comes at a cost, the bribe. The fixed in-turn rule eliminates competition in bribes. For k sufficiently large firms are better off with favoritism than without.[15]

In the equilibrium of the one-shot game the agent is limited to choosing a scoring rule that minimizes competition i.e., a scoring rule emphasizing a (single) component such that firms have differentiated technologies (costs). In the repeated setting with collusion, the agent needs not bother about cost differentials. So in a sense collusion makes favoritism "easier": the agent's equilibrium choice set is larger.

Remark 1 *The social cost of collusion is generically higher with favoritism than without.*

This follows from the fact that favoritism induces a socially inefficient project specification while simple collusion does not (see Proposition 2 above). Since the selected project is always non-standard, the government also pays a higher price than in the case of simple collusion. On the other hand with favoritism efficiency (relative to the announced scoring rule) is secured. The firms that actually implements the contract is cost efficient. This is also true with simple collusion but under symmetric information only. We have not characterized the asymmetric information optimal collusive scheme but as earlier mentioned it is likely to be plagued by some ex-post inefficiency.

[15] For $k = 3$ it can be shown that the per period expected payoff is larger with favoritism than without provided $\overline{\theta} < 3\underline{\theta}$ for all $m \leq \frac{D}{2\underline{\theta}\overline{\theta}}$.

5 Extensions

In this section we extend the analysis by considering the case when the expected punishment for favoritism depends on the magnitude of the distortion of social preferences. We do that in a simpler setting with $k = 2$ so $\alpha_1 = \alpha$ and $\alpha_2 = (1 - \alpha)$. The auctioneer's utility is $U = b - m(\widehat{\alpha} - \alpha)^2$. It is common knowledge that firms are anti-symmetric in costs: $\boldsymbol{\theta}_1 = (\underline{\theta}, \overline{\theta})$ and $\boldsymbol{\theta}_2 = (\overline{\theta}, \underline{\theta})$. The time line of events in stage game is as follows:

step 1: The auctioneer learns α, the firms submit (b_1, α_1) and (b_2, α_2);

step 2: The auctioneer makes an announcement $\widehat{\alpha}$, $\widehat{\alpha} \in [0, 1]$;

step 3: The firms submit $(\mathbf{q}_i, p_i)$;

step 4: The auctioneer selects the firm whose offer maximizes the selection criteria corresponding to the announced scoring rule. The winner i^* pays the bribe he offered whenever the announced scoring rule is $\widehat{\alpha} = \alpha_{i^*}$. Otherwise no bribe is paid.

Proposition 4 characterizes symmetric Bayes Nash equilibria of the stage game described above. We show that for $m \leq \frac{4}{5}\left(\frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}\right)$

Proposition 4 *Any symmetric Bayes-Nash equilibrium is characterized by*

i. The equilibrium scoring rule is $\widehat{\alpha}^*(m, \alpha) = \begin{cases} 1 \text{ for } \alpha \geq 1/2 \\ 0 \text{ for } \alpha < 1/2 \end{cases}$;

ii. The equilibrium bribe is $b_1^* = b_2^* = b^*(m) = \frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}} - m$;

iii. The contract offers are the competitive equilibrium offers relative to the announced scoring rule.

A first important result is that the equilibrium scoring rule is single-peaked as in the fixed punishment case. In the appendix we prove this as Lemma 2. The intuition is that in the corruption game firms compete in the auctioneer's utility levels. This utility is separable in bribe and expected punishment cost. We show that any deal with $\alpha \notin \{0, 1\}$ that achieves a given utility to the auctioneer there exists a deal with $\alpha \in \{0, 1\}$ that achieves the same utility level but yields a higher expected profit for the firm. We also note that, as in Proposition 1, the official auction offers are the (efficient) competitive equilibrium offers.

In contrast with earlier results competition for favors does not dissipate all firms' rents. The intuition is that contingent punishment costs introduces an asymmetry between firms: the firm whose demanded scoring rule is closer to the true one is more attractive to the auctioneer. Firms' incomplete information about the true preferences therefore induces a continuity of the probability to win in the submitted bribe. As a result competition for favor is mitigated.

The results in proposition 4 apply for $m \leq \frac{4}{5}\left(\frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}\right)$. Since $\frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}$ is the competitive profit-if-win associated with the most favorable scoring rule, this range covers most interesting cases.[16] Over that range of value for m the profit-if-win is simply $\pi_{i^*} = m$.

Summing up, with an expected punishment that is a function of distortion, the scoring rule always induces a "non-standard" project but a downright reversal of the true preferences is precluded. The same outcome is obtained with a fixed punishment when assuming auctioneer's weak preference for avoiding downright reversal. What government expenditure concerns there is no advantage in the more sophisticated punishment rule. Finally, because it mitigates competition in favors, some of the rents stay with the firms. We conclude that in the stage game, the sophisticated punishment scheme offers no advantage from the point of view of social efficiency.

We now consider a repeated version of the game described above. As in the case with k-components, the two firms meet in each period with a new (short-run) auctioneer. At the end of each period, the submitted contracts offers are publicly observed by the two firms and the active auctioneer. The corrupt deal offers remain private information to the involved parties. The true value of α is never revealed. There is no communication between auctioneers from different periods.

Proposition 5 *i. For $\delta \geq \underline{\delta_2} \in (0,1)$, there exists a Public Perfect Equilibrium equilibrium of the repeated game with collusion in contract offers and in corruption deals.*

ii. For m small a simple pre-determined in-turn allocation rule is optimal while for m large any optimal collusive scheme entails a contingent allocation rule.

A first important remark is that collusion in contract offers and in bribes is achievable in a simple pre-determined in-turn scheme at $b^* = m$. The reasoning is similar to that in proposition 3. However, for m relatively large, the simple scheme implies a significant loss in revenue for the cartel. This is because in such a scheme the bribe always covers the punishment cost associated with the maximal distortion of the scoring rule relative to the

[16] In an earlier version we investigated the whole solution. For $m > \frac{4}{5}\left(\frac{D}{2\underline{\theta}\overline{\theta}}\right)$ the equilibrium bribe is equal to the expected punishment cost and favoritism occurs less often until as the cost grows it gets fully prohibitive.

true one. The bribe cost can be reduced in a contingent scheme but that may not always be worthwhile because of imperfect public information which induces new inefficiencies.

We first note that once the winner has been designated, collusion in the official auction is sustainable relying on a standard folk theorem argument. This is because offers are ex-post public information. In the scheme of Proposition 5, the announcement resulting from competition in corruption deals determines the winner. The main issue for the cartel is therefore to sustain collusion in corrupt deals in order to contain competition in bribes and make an efficient use of stochastic government preferences as an allocation rule. The problem is that firms don't know the true scoring rule and do not observe the submitted bribe deals. They only observe the announced scoring rule which is an imperfect public signal of firms' action in the corruption game. Therefore firms must sometime be "punished" even when complying (this is similar to Green and Porter 1986, Radner, Myerson and Maskin 1986[17]). In the appendix we provide an example showing that collusion is sustainable in a Public Perfect Equilibrium (PPE) with $b^* = \frac{1}{4}m$. A PPE is a profile of public strategies that, beginning any date t and given any public history till time t, form a Nash equilibrium.[18] Deterrence from defection at the bribing stage is achieved by the threat of competition in the official auction. In case a firm wins twice in a row, it is "punished" by the other firm which then submits an offer that scores more than zero. This reduces the cartel's revenue. In this example there is an equilibrium with a contingent scheme that is bounded away from full cartel efficiency but that dominates the fixed in-turn rule scheme for m not too small.

The main insight from Proposition 5 is that when the simplest in-turn rule is too costly, favoritism allows for a reasonably simple contingent collusive scheme to sustain collusion. In the scheme we investigate, firms win a lower current period payoff when they win for the second period in a row. This is because we need to prevent defection in bribes. Only the preceding period's announced scoring rule matters. So unlike in the case with a fixed punishment cost, favoritism here does not fully shade firms from future hazards in preferences and costs. Yet, matters are greatly simplified for firms. With favoritism the profit-if-win is fully known by force of single-peakness and depends minimally on the environment.

The results in proposition 5 apply to a symmetric information context. We conclude this

[17] A distinction with the mentioned articles is that in our context, it is in each case possible to identify the possible defector and so to limit the in-equilibrium punishment to that player.

[18] A strategy for player i is public if, at each time t, the strategy depends only on the public history and not on i's private history.

section with a few remarks suggesting that the main insights generalize to an asymmetric information context where there is a new draw of $\boldsymbol{\theta}_i$ in each period and it is privately observed by firms. As before we delete the realizations when any one of the firms is inefficient at the production of both components.

First we note that just as in the symmetric information case collusion in contract offers and in bribes is achievable in a simple pre-determined in-turn scheme at $b^* = m$. In a working paper we show that a more sophisticated scheme that blends features of the pre-determined in-turn rule with features of the contingent scheme can achieve collusion in bribes and contract offers while keeping the bribe to $b^* = \frac{1}{4}m$. Although the scheme is not truly complex, the proof is rather laborious and lengthy that is why we chose to leave it out. In order to be able to sustain this scheme the winning firm's profit from the FSA can never be the maximal gain which suggests that for small m the simple predetermined in-turn rule can be optimal.

Our conclusion is that the central insight from proposition 3 i.e., that favoritism facilitates collusion and contributes to resolving implementation problems in face of demand uncertainty (incomplete information about the true government preferences) and privately observed shocks to costs, is robust to accounting for a punishment cost that varies with the magnitude of the distortion.

6 Discussion and Policy implications

The main insights of the analysis can be summarized as follows:

- Favoritism facilitates collusion because
  - It induces the revelation of firms' private information as that information is used by the corrupt auctioneer to *maximize* the winner's rent;
  - It shades firms from fluctuations in government preferences. The selected contract specifications reflect the cartel's interests instead of social preferences;
- Favoritism exacerbates the cost of collusion for society. The contract specification is socially inefficient and the price is higher than with collusion alone.

The analysis thus reveals that favoritism fundamentally perverts the auction mechanism both what concerns the use of firms' private information (about their costs) and that of the agent's private information about government preferences.

A central intermediary result is that the equilibrium scoring rule is extreme i.e., "single-peaked". In the one-shot setting this allows to minimize competition between firms. As a result the selected project tends to be non-standard in the sense that the winning firm is alone to be efficient at its production. In the repeated setting competition is less of an issue because of collusion. As a result the winning firm's rents can be maximize for a larger range of project specifications each of which responding to a single-peaked scoring rule. Most procurement codes include provisions that preclude the use of non-standard (a fortiori firm specific) specifications and that encourage generic technical specification. Interestingly, even for the simplest objects such as print paper one may not be able to define a unique standard (see Compte and Lambert-Mogiliansky 2000). When dealing with complex procurement projects, it is simply not realistic to expect being able to define a unique generic specification. Choices have to be made either by settling for a technical solution or in a scoring rule. Often it is mistakenly believed that a first price auction of a technically specified object precludes favoritism. In Compte and Lambert-Mogiliansky (2000), it is demonstrated that such procedure can be even more vulnerable to favoritism. A technical specification can bias competition at a larger cost for the government than a scoring rule. Generally, the use of a scoring rule (that weights technical components or performance measures) increases competition and thereby reduces the stake of favoritism. Our analysis applies within the spectrum of discretion consistent with typical anti-favoritism provisions. It says that within this spectrum, favoritism results in the selection of a project specification that maximizes the winner's rent. It also says that collusion relaxes a constraint on equilibrium scoring rule (i.e., it needs not minimize competition), which presumably makes favoritism more difficult to detect.

Single-peakness as the solution to rent maximization, obtains from the conjunction of a series of assumptions most of them are standard or reasonable. Two assumptions deserve some comments: separability in costs between components and separability in bribes and punishment cost. There is a natural way to reinterpret the single-peakness result when relaxing the assumption of separability in costs. If we have complementarities in costs, one should group components that are complementary in production into a composite component that is given full weight in a proper manner. Clearly, a more involved cost structure would entail more complex computation of the demanded scoring rule(s) and a more involved operation to compute the scoring rule that maximizes the winner's rent (used in the stage game). A conjecture is that the menu of deal offers is sufficiently rich a message language to allow for quite sophisticated information to be revealed so the auctioneer can minimize competition as in the basic model. With (ex-ante) symmetric firms the prize i.e., winning the contract with minimal competition is the same for both firms in which case most of the results carry over. Some additional analysis may be required if we want to relax the assumption about separability in bribes and punishment. In particular to investigate the case when the auctioneer is not willing to take a bribe so high that it covers the expected cost of all distortions. However, evidence suggests (see footnote 8 and policy implications below) that the expected cost is rather low in which case the problems related to bribe cap would not arise.

Our conjecture is thus that the main insights of the analysis do not depends on the fine details of the model but capture central features of the reality of favoritism in procurement as revealed by empirical evidence. First there exists numerous anecdotal evidence e.g., from developing countries. In one case an Africa country set its telephone specification to require equipment that could survive in frigid climate. Only one telephone company from Scandinavia could satisfy this obviously worthless specification (Rose-Ackerman 1999, p.64. Similarly problems of maintenance of construction objects are often due to the non-standard project specification that was selected by the international procurement procedure. Second, the allocation pattern emerging from the analysis: a pre-determined in-turn rule allocates the contract to the most efficient firm while generating large profits is very close to the patterns observed in Paris Hall case mentioned in the introduction. Interestingly, people have argued that the fact that the contract were allocated to the most efficient firm, was an indication that there was no collusion. The present analysis shows that it is sufficient that each firm has a comparative advantage in some component for this outcome to obtain in a collusive equilibrium with favoritism.

Policy implications

A central message of the analysis is that the risks of collusion and favoritism are linked and must be addressed simultaneously. Yet, the investigation of collusion is often the jurisdiction of Competition Authorities while that of corruption is the jurisdiction of criminal courts. A first recommendation is to develop cooperation to overcome this institutional separation so as to improve efficiency in the prosecution of cases that involve both favoritism (corruption) and collusion.

The analysis confirms earlier results (see e.g., Laffont and Tirole 1993, Compte and Lambert-Mogiliansky 2000, and Che and Burget 2004) that discretion to devise the scoring rule or similarly to define the technical specification is very sensitive to capture because that decision strongly affects firms' payoff. We already noted that most procurement code includes provisions that encourage a standardization wherever it is possible. When that is too costly (or not feasible), the auctioneer's decision should be subjected to close scrutiny. This recommendation is in line with Steven Kelman (1994)[19] who argues in favor of preserved flexibility associated with increased accountability of procurement officials. Concretely this means for instance an obligation to motivate their decisions in writing. Another type of measures recognizes that firms often have a superior information about each other than the government has. They can be in a position to recognize when a scoring rule is fine-tailored to some other firm. A recommendation would be to consider devising a mechanism to reveal this information e.g., by performing an anonymous consultation prior the official submission.[20]

The results suggest that over a significant interval, an increase in m i.e., stricter controls and/or more severe punishments, has no effect on the cost of favoritism to society. Hence, we find that to be any effective the expected punishment has to be very severe. This contrast with the current legislation in the European Union that makes it very difficult to convict for favoritism. A central reason for this is that favoritism is difficult to prove. Indeed, generally any selection criteria would favor some firm(s) at the expense of others. "Deciding to build a swimming pool rather than a stadium is good for firms that have a comparative advantage in building swimming pools." The problem is thus to compare between selection criteria that favor different firms. The honest auctioneer picks up the one that is congruent with public preferences while the corrupt selects another one. But public preferences are seldom so well-defined that congruence can be measured in a way that is non-controversial (which also suggests that a fixed punishment cost model maybe the relevant one). Generally, detecting and proving the occurrence of favoritism is difficult. An implication of the analysis is that attention should be paid to a careful study of allocation patterns over time. Unfortunately courts tend to focus on bribery and few cases of favoritism are brought to court. We thus

[19] Harvard professor Steven Kelman was the director of the Office of Federal Procuremnt Policy under 1993-1997.

[20] This would be most efficient in a one-shot situation or when some firms are excluded from the cartel. At least it would create new problems for a cartel.

suggest that sophisticated economic expertise be given more power in cases where there is a suspicion of favoritism. Indeed, while this is the rule in cases of standard collusion, economist expertise appears to be seldom requested in cases involving corruption and favoritism.

References

[1] Athey S., K. Bagwell and C. Sanchirico (2004) "Collusion and Price Rigidity" *Review of Economic Studies* 71 (2) April.

[2] Burguet R. and Y-K Che (2004), "Competitive Procurement with Corruption", *Rand Journal of Economics* 35, 50-68.

[3] Caillaud Bernard and Philippe Jehiel (1998), "Collusion in Auctions with Externalities" *Rand Journal of Economics,* 680 - 702.

[4] Cartier Bresson Jean (1998), "Corruption et Pratiques anti-concurrentielles: une première reflexion à partir d'une étude de cas", mimeo Paris XIII.

[5] Celentani, M. and J. Ganunza, (2002) "Competition and Corruption in Procurement" *European Economic Review, 43* 1273-1303,

[6] Compte O. and A. Lambert-Mogiliansky (2000) " Efficacite et Transparence dans les Procédure de Spécification sur les marchés Publics" Direction de la Prévision, Ministere de L'Economie des Finances et de l'Industrie. www.enpc.fr/ceras/lambert.

[7] Compte O., A. Lambert-Mogiliansky and T. Verdier (2005), "Corruption and Competition in Procurement", *RAND Journal of Economics* vol 36/1, 1-15

[8] Y-K Che (1993), "Design Comptetition Through Multidimensional Auctions" *Rand Journal of Economics,* Vol. 24/4 668-680.

[9] Fudenberg D. D. Levine and E. Maskin (1994) " The Folk Theorem with Imperfect Public Information" *Econometrica* Vol 62/5, 997-1039.

[10] Graham Daniel A. and Robert C. Marshall (1987), "Collusive Bidder Behavior at Single-Object Second-Price and English Auctions", *Journal of Political Economy*, vol.95, n°6, 1217-1239.

[11] Kelman Steven (1994) "Deregulating Federal Procurement Nothing To Fear but Discretion Itself" in John D; Dulilio Jr Ed., Deregulating The Public Service Can Government be Efficient?, Washington DC: The Brookling Institution 102-128.

[12] Laffont J-J. and J. Tirole (1991) "Auction Design and Favoritism", *International Journal of Industrial Organization* Vol 9, 9-42.

[13] Lambert-Mogiliansky A. and K. Sonin " Collusive Market-Sharing and Corruption in Procurement" *Forthcoming 2006 Journal of Economics and Management Strategy.*

[14] McAfee R. Preston and John McMillan, (1992), "Bidding Rings" *American Economic Review* Vol. 83 No.3, 579-599.

[15] Radner R, R. Myerson and E. Maskin (1986) " An example of a Repeated Partnership Game with Discounting and with Uniformaly Inefficient Equilibriua" *Review of Economic Studies* 53, 59-70;

[16] Rose-Ackerman (1999), Corruption in Government. Cause, Consequences and Reform. Cambridge University press, Cambridge.

A Proof of Lemma 1

For any announcement $\widehat{\boldsymbol{\alpha}}$, the efficient specification for firm i is defined: $\mathbf{q}_i^* = \arg\max s(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) - c_i(\mathbf{q}, \boldsymbol{\theta}_i)$. We claim that in the equilibrium of the FSA both firms offer the efficient specification corresponding to their cost structure. Assume that this was not the case i.e., that, in equilibrium, firm i offers $(\widehat{\mathbf{q}},\widehat{p})$ with $\widehat{\mathbf{q}} \neq \mathbf{q}_i^*$. We first note that under asymmetric information $prob\{\text{win}|\,((\widehat{\mathbf{q}},\widehat{p}))\} > 0$. To show this we order the firms' type according to the number of components for which they have high cost (so each such type corresponds to a group of "elementary types") $\overline{\boldsymbol{\theta}}^0 <, ..., < \overline{\boldsymbol{\theta}}^{k-1}$, where $\overline{\theta}^{k-1}$ is the highest cost type (recall the fully inefficient types have been deleted from the distribution). Let $S^*(\boldsymbol{\theta}) = \max_q s(\mathbf{q}, \widehat{\boldsymbol{\alpha}}) - c_i(\mathbf{q}, \boldsymbol{\theta})$, in our model we have that $prob\left\{\text{win}|\, S^*\left(\overline{\boldsymbol{\theta}}^{k-1}\right)\right\} > 0$. If the offer $(\mathbf{q},p)$ for some $\overline{\boldsymbol{\theta}}^j < \overline{\boldsymbol{\theta}}^{k-1}$ was such that $prob\{\text{win}|\,(\mathbf{q},p)\} < prob\left\{\text{win}|\, S^*\left(\overline{\boldsymbol{\theta}}^{k-1}\right)\right\}$, it could not be an equilibrium offer since the score in decreasing in cost i.e., there would exist another offer that would yield higher expected profit.

We now show that offer $(\mathbf{q}_i^*,p')$ with $p' = \widehat{p} + s(\mathbf{q}_i^*, \widehat{\boldsymbol{\alpha}}) - s(\widehat{\mathbf{q}}, \widehat{\boldsymbol{\alpha}})$ dominates $(\widehat{\mathbf{q}},\widehat{p})$. Note that $S(\mathbf{q}_i^*,p') = S(\widehat{\mathbf{q}},\widehat{p})$ so in particular $prob\{\text{win}|\,(\widehat{\mathbf{q}},\widehat{p})\} = prob\{\text{win}|\,(\mathbf{q}_i^*,p')\}$. Now the expected profit from submitting $(\mathbf{q}_i^*,p')$ is

$$\begin{aligned}
\pi_i\left(\mathbf{q}_i^*,p';\boldsymbol{\theta}_i\right) &= \left[\, p' - c_i(\mathbf{q}_i^*, \boldsymbol{\theta}_i)\right] prob\left\{\text{win}|\,\left(\mathbf{q}_i^*,p'\right)\right\} \\
&= \left[\widehat{p} - c_i(\widehat{\mathbf{q}}, \boldsymbol{\theta}_i) + s(\mathbf{q}^*, \widehat{\boldsymbol{\alpha}}) - c_i(\mathbf{q}_i^*, \boldsymbol{\theta}_i) - (s(\widehat{\mathbf{q}}, \widehat{\boldsymbol{\alpha}}) - c_i(\widehat{\mathbf{q}}, \boldsymbol{\theta}_i))\right] prob\{\text{win}|\,(\widehat{\mathbf{q}},\widehat{p})\} \\
&> \left[\widehat{p} - c_i(\widehat{\mathbf{q}}, \boldsymbol{\theta}_i)\right] prob\{\text{win}|\,(\widehat{\mathbf{q}},\widehat{p})\} = \pi_i(\widehat{\mathbf{q}},\widehat{p};\boldsymbol{\theta}_i)\}.
\end{aligned}$$

The last inequality holds because $s(\mathbf{q}^*, \widehat{\boldsymbol{\alpha}}) - c_i(\mathbf{q}_i^*, \boldsymbol{\theta}_i) > s(\widehat{\mathbf{q}}, \widehat{\boldsymbol{\alpha}}) - c_i(\widehat{\mathbf{q}}, \boldsymbol{\theta}_i)$.

The argument applying to the symmetric information case which we also use below (in Proposition 1, 2 and 4) is even simpler. Consider the case when firm 1 has a cost structure that is more congruent with the announced scoring rule than firm 2. Firm 1 is sure to win when submitting the second highest score (corresponding to firm 2's efficient specification associated with a price bid equal to its cost) because the tie breaking rule favors quality. Suppose firm 2 submits an offer that does not include the efficient specification and firm 1 matches that score. Then firm 2 could switch to an offer that includes the efficient specification to achieve a higher score and win. Suppose now that firm 1 matches firm 2's score with an offer that does not include the efficient specification. Appealing to the argument above (setting the winning probability equal to 1), we see that it cannot be optimal. Firm 1 could earn a higher profit with an offer that scores the same but includes the efficient specification. Similar reasoning applies when firms are identically efficient. Hence, in equilibrium firms submit offers that include the efficient specification. $QE\left(\overline{\theta}-\underline{\theta}\right)$

B Proof of proposition 1

We consider the following strategies for the players:

Firms:

At *step 1* make a menu of deal offer $\left\{\left(\boldsymbol{\alpha}^j, b^*\right)\right\}$ with a deal for *each* component j; $\theta_{ji} = \underline{\theta}$ with $\boldsymbol{\alpha}^j = (0, ..., 1_j, ..., 0)$. The same bribe is offered in each one of the deals belonging to the offered menu. Both firms offer $b^* = \frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}}$.

A *step 3* When the announced scoring rule is single-peaked, firms submit the competitive equilibrium offers while assuming that they are anti-symmetric in cost. When the scoring rule is not single-peaked, they submit the competitive offers under the assumption that their cost structures are identical.

The auctioneer:

At *step 2*

The auctioneer selects from among the submitted corrupt deals that include scoring rules *only demanded by one firm*, a deal associated with the highest bribe provided the bribe covers the cost m. He announces the associated scoring rule.

We below show that the strategies described above form a Bayes-Nash equilibrium of the game. For that purpose we first derive the competitive offers that form the symmetric information Nash equilibrium of the First Score Auction described by step 3 and 4 with no bribes. Consider the cost structures $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$. Define the efficient firm: $i^* = \arg\max s\left(\mathbf{q}_i^*, \widehat{\boldsymbol{\alpha}}\right) - c\left(\mathbf{q}_i^*, \boldsymbol{\theta}_i\right)$ where $\mathbf{q}_i^* = \arg\max_{\mathbf{q}} s\left(\mathbf{q}, \widehat{\boldsymbol{\alpha}}\right) - c\left(\mathbf{q}, \boldsymbol{\theta}_i\right)$ i.e., $q_{ij}^*\left(\theta_i\right) = \frac{\widehat{\alpha}_j}{\theta_{ij}^t}, i = 1, 2,\ j = 1, ...k$. We refer to i^* as firm 1 and appealing to Lemma 1, we focus on the price bids. By a standard argument, firm 2 bids the lowest price that just secures non-negative profit

$$p_2^* = c\left(\mathbf{q}_2^*, \boldsymbol{\theta}_2\right) = \sum_j^k \frac{\widehat{\alpha_j}^2}{2\theta_{2j}}. \tag{5}$$

Denote θ_j^r; $\theta_j; \theta_{j1}^r \neq \theta_{j2}^r$. Let $\beta_{11} = \sum \alpha_j^2\left(\theta_1^r = \underline{\theta}\right)$ and $\beta_{21} = \sum \alpha_j^2\left(\theta_1^r = \underline{\theta}\right)$ and $\beta_0 = \sum \alpha_j^2\left(\theta_1 = \theta_2\right)$. We can rewrite expression (5) $p_2^* = \frac{\beta_{11}}{2\overline{\theta}} + \frac{\beta_{21}}{2\underline{\theta}} + \frac{\beta_0}{2\theta}$ where we do not specify the cost parameter for the cases when firms have identical cost. Firm 2's score can be computed $s(\mathbf{q}_2^*, \widehat{\boldsymbol{\alpha}}) - c_2(\mathbf{q}_2^*, \boldsymbol{\theta}_2) = \frac{\beta_{11}}{2\overline{\theta}} + \frac{\beta_{21}}{2\underline{\theta}} + \frac{\beta_0}{2\theta}$. Firm 1's best response is to bid the lowest price that secures win:

$$p_1^* = c(q_2^*, \theta_2) + s(q_1^*) - s(q_2^*) = p_2^* + \frac{(\overline{\theta} - \underline{\theta})}{\underline{\theta}\overline{\theta}}(\beta_{11} - \beta_{21}). \tag{6}$$

The winner's payoff is

$$\pi_1 = \frac{(\overline{\theta} - \underline{\theta})}{2\underline{\theta}\overline{\theta}}(\beta_{11} - \beta_{21}). \tag{7}$$

We call the bids $(\mathbf{q}_1^*, p_1^*)$, $(\mathbf{q}_2^*, p_2^*)$ defined in Lemma 1 and (5) and (6) the equilibrium competitive bids.

We now proceed to investigate the complete game by backward induction. Step 4 is neglected throughout the appendix since players' move are fully determined by the rules of the game. At *Step 3* firms make their offers. Consider first the subgames when $\widehat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}^j$ for some $j = 1, ..., k$ and say it favors firm 1. Because the auctioneer's strategy calls for never announcing a scoring rule demanded by both, firms infer from $\widehat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}^j$ that they are anti-symmetric in cost with respect to θ_j so in particular when a firm is not favored (firm 2), it infers that its opponent is favored. Assume now that firm 2 assumes that $b_1 < p_1^* - c(\mathbf{q}_1^*, \boldsymbol{\theta}_1)$ where p_1^* is defined in (6). We claim that firm 2 bids p_2^* defined in (5). Assume by contradiction that in equilibrium $p_2 = c(\mathbf{q}_2^*, \theta_2) + x > p_2^*$. Firm 1's best response is $p_1 = p_2 + \frac{(\overline{\theta} - \underline{\theta})}{\underline{\theta}\overline{\theta}}(\beta_{11} - \beta_{21}) > p_1^*$. But firm 2 could lower its price and win a positive payoff - a contradiction. Where b_1 is so large that $p_2^* + s(q_1^*) - s(q_2^*) < c(\mathbf{q}_1^*, \boldsymbol{\theta}_1) + b_1$, firm 1 cannot win with positive profit. We below show that this cannot happen in equilibrium. Note that in a subgame where $\widehat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}^j$ but $\theta_{1j} = \theta_{2j}$ then either both firms are favored or both are non-favored. Since firms make their offer under the assumption that costs are asymmetric, they submit identical offers. In a subgame where $\hat{\boldsymbol{\alpha}} \neq \boldsymbol{\alpha}^j$, firms infer that $(\mathbf{q}_1^*, p_1^*, \hat{\boldsymbol{\alpha}}) = (\mathbf{q}_2^*, p_2^*, \hat{\boldsymbol{\alpha}})$ so profit-if-win is zero and no bribe will be paid. By definition the proposed offers are best response to each other. Hence, the Nash equilibrium offers of FSA without corruption described in Lemma 1 and (5) and (6) are part of an equilibrium.

At *step 2* the auctioneer chooses a deal among the submitted menus (M_1, M_2) with $M_i = \{(\boldsymbol{\alpha}_{im_i}, b_{im_i})\}_{m_i=1}^{m_i \leq k}$. The auctioneer expects firms to ask for scoring rules that emphasize components in which they have low cost. If both ask for the same scoring rule, which can happen since they don't know each other's cost, the auctioneer knows firms will be submitting identical contract offers if that scoring rule is announced. But then he receives no bribe. So he never selects a scoring rule demanded by both. Since $U = b - m$, he selects $\hat{\boldsymbol{\alpha}}_k \in \{\hat{\boldsymbol{\alpha}}_k : \hat{\boldsymbol{\alpha}}_k \in M_i \text{ and } \hat{\boldsymbol{\alpha}}_k \notin M_{-i}\}$ such that $b_k \in max\left\{b_{1m_1}, b_{2m_2}\right\}_{m_i=1}^{m_i \leq k}$ and $b_k \geq m$. If no such deal has been submitted the auctioneer announces the true $\boldsymbol{\alpha}$.[21]

At *step 1* We know from *step 2* that the auctioneer selects a deal associated with the highest bribe among those with scoring rules demanded by one firm only. Firms simply demand scoring rules that maximizes the profit-if-win $\pi_1 = \frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}(\beta_{11} - \beta_{21})$ which is decreasing in β_{21} so firm 1 sets $\beta_{21} = 0$ i.e., all the weight is be put on components $\theta_{1j}^r = \underline{\theta}$. Next, the single term $\beta_{11} = \sum \alpha_j^2 (\theta_1^r = \underline{\theta})$ must be such that $\sum \alpha_j (\theta_1^r = \underline{\theta}) = 1$, implying $\frac{\partial \beta_{11}}{\partial \alpha_j} > 0$ so π_1 is maximized with any $\boldsymbol{\alpha}_1^j = (0, ..., 1_j, ..., 0)$; $\theta_{1j}^r = \underline{\theta}$. The firms don't know each other's cost, so for any give $M_{2(1)}$ the probability that the auctioneer finds a deal with a scoring rule demanded by only one firm increases with the number of submitted deals by firm 1(2). So it is optimal to submit a deal on each low cost component. A firm's profit-if-win with any of the $\boldsymbol{\alpha}^j$ it demands is equal to $\frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}$. The corruption game boils down into a symmetric information common value auction. By a standard argument, firms submit the common value $b_{1j}^* = b_{2i}^* = \frac{(\bar{\theta}-\underline{\theta})}{2\underline{\theta}\bar{\theta}}$ for all j and i. $QE\left(\bar{\theta} - \underline{\theta}\right)$

C Proof of proposition 2

We first introduce a function $G(\boldsymbol{\alpha}, \boldsymbol{\theta}_i)$ representing the social gain corresponding to the efficient specification. By Lemma 1, firms choose the social efficient specification profile when $\widehat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}$. When firm i implements the contract in period t the gain is

$$G(\boldsymbol{\alpha}^t, \boldsymbol{\theta}_i^t) = \sum_{j=1}^{k} \frac{\left(\alpha_j^t\right)^2}{2\theta_{ij}^t}. \tag{8}$$

The object of proposition 2 is a repeated FSA game of complete information, we propose a simple Grim Trigger (GT) strategy for sustaining collusion. As usual it is composed of a *punishment phase* and a *cooperative phase.* In the punishment phase each firm gets the payoff of the stage game Nash equilibrium (defined in the proof of proposition 1). These non-cooperative payoffs can be expressed as the difference between the values of the G function

[21] We do not analyze the case of coincidence of true $\boldsymbol{\alpha}$ with the one in some corrupt deal. Because it has a zero probability and hence it has no effect on a choice of corrupt deals by firms at Step 1.

defined above, for firm 1

$$E\pi^{ne} = E_{\boldsymbol{\alpha}^t,\boldsymbol{\theta}_1^t,\boldsymbol{\theta}_2^t} \left\{ \, G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_1^t) - G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_2^t) | \; G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_1^t) > G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_2^t) \, \right\}, \tag{9}$$

where the notation $E_{\boldsymbol{\alpha},\boldsymbol{\theta}_1,\boldsymbol{\theta}_2}\{.|.\}$ stands to taking the conditional expectation over random variables $\boldsymbol{\alpha}$, $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$. Notice that due to symmetry, firm 2 gets the same payoff $E\pi^{ne}$.

In the *cooperative phase,* firms collude to collect the highest feasible expected profit. This profit is achieved when the firm, say firm i, with the highest value of G given by (8) (the firm with the cost structure $\boldsymbol{\theta}$ the most congruent to $\boldsymbol{\alpha}$) wins the auction and retain the entire social gains from the contract i.e., it earns $G(\boldsymbol{\alpha},\boldsymbol{\theta}_i)$. The expected per period payoff of firm 1 (as well as of firm 2) in the cooperative phase is

$$E\pi^{c} = E_{\boldsymbol{\alpha}^t,\boldsymbol{\theta}_1^t,\boldsymbol{\theta}_2^t} \left\{ \, G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_1^t) | \; G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_1^t) > G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_2^t) \, \right\}. \tag{10}$$

We note that $E\pi^c > E\pi^{ne}$ which follows from $E_{\boldsymbol{\alpha}^t,\boldsymbol{\theta}_1^t,\boldsymbol{\theta}_2^t} \left\{ \, G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_2^t) | \; G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_1^t) > G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_2^t) \, \right\} > 0$.

We now describe a GT strategy. The game starts in the cooperative phase. In the cooperative phase in each period t the firm with the highest value of $G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}^t)$ is designated as the current winner. The winner makes an offer that scores zero while the loser makes an offer that scores negative. In case of tie in $G(\boldsymbol{\alpha}^t,\boldsymbol{\theta}_i^t)$, they toss a coin, so with probability .5 each firm is designated as the in-turn winner. If the actual winner is different from the designated winner in some period t, from the next period on the firms revert to the play of the Nash equilibrium of the stage game (punishment phase).

We now define the range of the discount factor δ for which the GT strategy is an equilibrium strategy. According to the one-stage-deviation principle we check for the highest gain from deviation within a period. This gain is maximal in a period when the designated loser has the highest value of G. This, for example, takes place when at some time period $\boldsymbol{\alpha}$ has only one nonzero component with weight equal to 1, and for instance all components of $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are equal to $\underline{\theta}$. Here $G(\boldsymbol{\alpha},\boldsymbol{\theta}_1) = G(\boldsymbol{\alpha},\boldsymbol{\theta}_2) = 1/(2\underline{\theta})$, and say the tie breaking rule ruled against firm 2. Firm 2 can then achieve a one period profit close to $1/(2\underline{\theta})$ by slightly overbidding firm 1's equilibrium offer. However from the next period on the play yields the non-cooperative Nash payoffs. The IC constraint securing that such a deviation is not profitable is

$$\frac{E\pi^c}{1-\delta} \geq \frac{1}{2\underline{\theta}} + \frac{\delta(E\pi^c - E\pi^{ne})}{1-\delta}.$$

so the GT strategy form an equilibrium with collusion of the repeated FSA for $\delta \geq \underline{\delta_0}$ where $\underline{\delta_0}$ is given by

$$\underline{\delta_0} = \frac{1/(2\underline{\theta}) - E\pi^c}{1/(2\underline{\theta}) - E\pi^c + E\pi^{ne}} < 1.$$

QED

D Proof of Proposition 3

We show that the collusive equilibrium of proposition 3 can be supported by a Trigger strategy with a punishment phase corresponding to the play of the equilibrium of proposition 1. The cooperative phase is characterized by the following:

Firms' strategy

At *step 1* the in-turn-firm submits a menu $M_{in} = \left\{ \left(b^*, \boldsymbol{\alpha}^j \right) \right\}$ with $\theta_{in,j} = \underline{\theta},\ b^* = m$. The out-of-turn firm submits $M_{out} = \left\{ \left(0, \boldsymbol{\alpha}^j \right) \right\}$ for some $\theta_{out,j},\quad b^* = 0$.

At *step 3* For any $\widehat{\boldsymbol{\alpha}}^t$the in-turn firm submits an offer that scores zero. The out-of-turn firm bids to score strictly less than zero.

The auctioneer's strategy

At *step 2*

The auctioneer selects from among the submitted corrupt deals a one associated with the highest bribe. If that bribe covers the costs, he announces the associated scoring rule.

Let $H_{t-1} = H^*$ denote a public history of the game when it is in a cooperative phase i.e., in all $t' = 1, ..., t-1$ the outcome is characterized by the firm wining in alternation i.e., every second period.

The trigger strategy entails that in any subgame following $H_{t-1} \neq H^*$, the firms move to (stay in) the punishment phase. Since it is a Nash equilibrium, conforming is by construction a best response for all players.

We now consider a subgame following $H_{t-1} = H^*$ to show that cooperating according to the strategies defined above is optimal. We proceed by backward induction.

At *step 3* whatever $\widehat{\boldsymbol{\alpha}}^t$, the in-turn firm expects the out-of-turn firm to bid less that zero. The maximal payoff $\pi^c = \frac{1}{2\underline{\theta}}$ yields when the in-turn firm offers the efficient specification and a price so its offer scores just zero. So the proposed strategy is optimal. The out-of-turn firm may deviate. The most profitable deviation occurs when the announced scoring rule is single-peaked and the out-of-turn firm also has low cost on the emphasized component and submits $p = \frac{1}{\underline{\theta}} - \varepsilon$. Its gain is $\pi^d = \frac{1}{2\underline{\theta}} - \varepsilon$. However the in-turn rule is violated and from the next period on the firms revert to the zero payoff competitive equilibrium of proposition 1. So the out-of-turn firm complies with the collusive strategy whenever

$$IC : \frac{\delta}{(1-\delta)} \frac{1}{2} \left(\frac{1}{2\underline{\theta}} - m \right) \geq \frac{1}{2\underline{\theta}} - \varepsilon \tag{11}$$

which is satisfied for $\delta \geq \underline{\delta_1} \leq 1$ with $\frac{\partial \underline{\delta_1}}{\partial m} > 0$.

At *step 2* Since the auctioneer is a short-run player, the argument developed in the proof of proposition 1 carries over. A distinction is that the auctioneer needs not care about avoiding scoring rules submitted by both because firms never submit identical offers in response to single peaked scoring rules..

At *step 1* the firms submit their menu of deals. Since the auctioneer only cares about the bribe the argument of proposition 1 carry over and firms always propose deals with single-peaked scoring rules. The in-turn firm expects the out-of-turn firm to offer $b = 0$. It is sufficient to offer $b = m$ to cover the auctioneer's cost so he announces one of the in-turn firm's preferred scoring rule. The out-of-turn firm can defect and offer $b = m + \varepsilon$ associated with a menu including a most preferred scoring rule $\boldsymbol{\alpha}_{out}$. It knows that the auctioneer would respond by announcing that $\widehat{\boldsymbol{\alpha}} = \boldsymbol{\alpha}_{out}$. But such a deviation only brings profit if the out-of-turn wins the official auction. Since we know that such a win triggers a punishment phase, under (11) defection is not profitable. Hence for δ satisfying 11 the proposed strategies do form a Bayes-Nash equilibrium of the repeated game. The cartel's gain is maximized. In each period, the scoring rule is the most favorable to the winner, the price is given by the reserve score and the bribe is the lowest possible. *QED*

E Proof of Proposition 4

Firms' strategy:

At *step 1* The firms offer a deal for its low cost component with a single-peaked scoring rule emphasizing that component and the bribe $b^* (m)$ defined below.

A *step 3*, the firms submit the competitive offers relative to the announced scoring rule.

The auctioneer's strategy

At *step 2*

The auctioneer selects from among the submitted deals, the one that maximizes his payoff. He announces the associated scoring rule provided the bribe covers the cost.

We below show that the strategies described above form a symmetric Bayes-Nash equilibrium with favoritism. We develop the proof in terms of firm 1 which has its advantage in the production of q_1. Firm 2 is symmetric with advantage in component 2. We proceed by backward induction.

At *Step 3,* In the present context firms know each other costs yet we can apply the same reasoning as in Proposition 1 for $k = 2$, $\alpha_1 = \alpha$ and $\alpha_2 = (1 - \alpha)$ and with $\boldsymbol{\theta}_1 = \left(\underline{\theta}, \overline{\theta}\right)$ and $\boldsymbol{\theta}_2 = \left(\overline{\theta}, \underline{\theta}\right)$. In Proposition 1 firms correctly believe they are asymmetric in cost. here they simply know that.

A step 2 the auctioneer's utility function is $b_i - m\left(\alpha_i - \alpha\right)^2$. So it is optimal to choose a deal among the submitted ones as follows $\widehat{i} = \arg\max_{\{(b_1,\alpha_1)(b_2,\alpha_2)\}} b_i - m\left(\alpha_i - \alpha\right)^2$ s.t. $b_i \geq m\left(\alpha_i - \alpha\right)^2$. In case of ties, he selects each firm with equal probability.The auctioneer announces $\alpha_{\widehat{i}}$ if $i^*\left(\alpha_{\widehat{i}}\right) = \widehat{i}$. If no bribe deal can secure win in the official auction or if $b_{\widehat{i}} < m\left(\alpha_{\widehat{i}} - \alpha\right)^2$, the auctioneer announces the true alpha.

A step 1 We start with a Lemma

Lemma 2 *In a symmetric equilibrium firms always demand the "cartel efficient" scoring rule contingent on their cost structure i.e.,* $\alpha_1^* = 1$ *and correspondingly* $\alpha_2^* = 0$.

For firm 1, the "cartel efficient" scoring rule is defined $\alpha_1^* = \arg\max_{\widehat{\alpha}_1}\left\{\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}}\left(2\widehat{\alpha}_1 - 1\right) - m\left(\widehat{\alpha}_1 - \alpha\right)^2\right\}$. It is the scoring rule that maximizes the cartel's payoff given that there is a cost associated with deviations from the true scoring rule. We know that the auctioneer selects the firm whose deal maximizes $U(b_i, \alpha_i) = b_i - m\left(\alpha_i - \alpha\right)^2$. Suppose by contradiction that an equilibrium offer is (b_1, α_1) with $\alpha_1 \neq \alpha_1^*$. We now construct offer (b_1', α_1^*) with $b_1' = b_1 - m(1/2 - \alpha_1)^2 + m(1/2 - \alpha_1^*)^2$. By construction

$$prob\left(U\left(b_1', \alpha_1^*\right) > U\left(b_2, \alpha_2\right)\right) = prob\left(U\left(b_1, \alpha_1\right) > U\left(b_2, \alpha_2\right)\right) = 1/2.$$

Now

$$
\begin{aligned}
E\pi\left(b_1', \alpha_1^*\right) &= \left[\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - b_1'\right] prob\left(U\left(b_1', \alpha_1^*\right) > U\left(b_2, \alpha_2\right)\right) \\
&= \left[\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - b_1 + m(1/2-\alpha_1)^2 - m(1/2-\alpha_1^*)^2\right]\frac{1}{2} \\
&= \left[\left(\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}}\left(2\alpha_1-1\right) - b_1\right) + \left(2\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - m\alpha_1\right)\left(1-\alpha_1\right)\right]\frac{1}{2} \\
&> \left[\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}}\left(2\alpha_1-1\right) - b_1\right] prob\left(U\left(b_1, \alpha_1\right) > U\left(b_2, \alpha_2\right)\right)
\end{aligned}
$$

where the inequality holds because $m < \frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}}$.

Hence $\alpha_1^* = 1$ and similarly for firm 2: $\alpha_2^* = 0$.

We now consider the determination of b_1 and b_2

$$
\begin{aligned}
E\pi_1\left(b_1\right) &= \left(\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - b_1\right) prob.\left\{U\left(b_1, 1\right) > U\left(b_2, 0\right)\right\} \\
&= \left(\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - b_1\right) prob.\left\{b_1 - m\left(1-\alpha\right)^2 > b_2 - m\left(\alpha\right)^2\right\} \\
&= \left(\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - b_1\right) prob.\left\{\alpha > \frac{1}{2} + \frac{\left(b_2-b_1\right)}{2m}\right\} \\
&= \left(\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - b_1\right)\left(\frac{1}{2} - \frac{\left(b_2-b_1\right)}{2m}\right)
\end{aligned}
$$

Taking the derivative with respect to b_1 an interior solution satisfies

$$
b_1^* = \frac{1}{2}b_2 + \frac{1}{2}\left(\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - m\right)
$$

In a symmetric equilibrium we obtain

$$
b^* = b_1^* = b_2^* = \frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - m,
$$

In a symmetric equilibrium the auctioneer never distorts more than .5 so the highest cost for distortion is $\frac{1}{4}m$. Hence, for $\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}} - m > \frac{1}{4}m \Leftrightarrow m < \frac{4}{5}\frac{\left(\overline{\theta}-\underline{\theta}\right)}{2\underline{\theta}\overline{\theta}}$, the investigated strategies described above including the deal offers $(b^*, 1)$ and $(b^*, 0)$ form a Bayes-Nash equilibrium of the FSA with favoritism. *QED*.

F Proof of proposition 5

In this proof we consider two types of collusion, characterize the condition for their sustainability and compare them in terms of cartel efficiency.

"In turn rule" collusion:

This type of collusion is similar to the one constructed in Proposition 3. The strategies are the same as the ones described in the proof of proposition 3 when putting $k = 2$. Any deviation from those strategies triggers the play of the Nash equilibrium of proposition 4 from the next period on.

As usual we investigate the game by backward induction and focus on incentives to comply in a period following a history of compliance play.

At *Step 3* the designated winner, say firm 1, has no incentive to deviate while the other firm might undercut the offer of firm 1 and get at most $\frac{1}{2\overline{\theta}}$ in the current period and a continuation payoff of $\frac{\delta}{2(1-\delta)}\left\{\frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}} - b^*(m)\right\}$ afterwards. Given that $b^*(m) = \frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}} - m$, the incentive constraint writes

$$\frac{\delta}{2(1-\delta)}\left\{\frac{1}{2\underline{\theta}} - m\right\} \geq \frac{1}{2\overline{\theta}} + \frac{\delta}{2(1-\delta)}m. \tag{12}$$

where the rhs is the compliance payoff and the lhs is the deviation payoff. We show below that deterring deviation at step 1 is more constraining so we postpone the derivation of the limit on the discount factor.

At *step 2* there is no incentive to deviate for the agent for reason similar to those in proposition 4. At *step 1* firm 2 might make a secret bribe bid of $b \in (0, 2m]$ and demand $\hat{\alpha} = 0$. The agent will grant firm 2 the favor of choosing its demanded scoring rule when $\alpha \in [0, b/(2m)]$. Firm 2 then gets a payoff of $\frac{1}{2\underline{\theta}} - b$ and a continuation payoff $\frac{\delta}{2(1-\delta)}\left\{\frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}} - b^*(m)\right\}$. The expected payoff from this deviation is

$$E\pi = \frac{b}{2m}\left\{\frac{1}{2\underline{\theta}} - b\right\} + \frac{b}{2m}\frac{\delta}{2(1-\delta)}m + \left\{1 - \frac{b}{2m}\right\}\frac{\delta}{2(1-\delta)}\left\{\frac{1}{2\underline{\theta}} - m\right\}.$$

In order for $b = 0$ to be an equilibrium strategy, the above expression needs to be maximized at $b = 0$. Since the expression is strictly concave in b we get the following restriction for the discount factor

$$\frac{\partial E}{\partial b}(b = 0) \leq 0 \text{ or } \frac{1}{2\underline{\theta}} + \frac{\delta}{2(1-\delta)}\left\{m - \frac{1}{2\underline{\theta}}\right\} \leq 0 \tag{13}$$

The expression in the bracket is negative and increasing in m. By assumption we have $m < \frac{4}{5}\frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}$ but to simplify the calculation, we check for $m = \frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}$ the constraint is most restrictive then. Thus a conservative formulation of the constraint on the discount factor yields $\delta \geq \underline{\delta}$ where

$$\underline{\delta} = \frac{\overline{\theta}}{(\underline{\theta} + \overline{\theta})} < 1.$$

Contingent rule

We use the notation: $w_{1(2)}(1)$ and $w_{1(2)}(0)$ to denote firm 1(2) continuation after a period when the announcement is $\widehat{\alpha} = 1$ and $\widehat{\alpha} = 0$ respectively. We focus on a smaller set of strategies including $b \in \left\{\frac{1}{4}m, \frac{5}{4}m\right\}$ so in particular we only consider a defection that secures win. To outcompete firm 1 when the true scoring rule is most favorable to it e.g. $\alpha = 1$, firm 2 must offer $\frac{5}{4}m$. This is not crucial to the result but it simplifies the presentation.

Let $H_{t-1} = H^*$ denote a history of the game where in all periods t', $t' = 0, ..t-1$,we have $\widehat{\boldsymbol{\alpha}}^{t'} \in \{0, 1\}$, and $S\left(p^{*t'}, q^{*t'}\right) = 0$ for t' such that $i^{*t'} \neq i^{*t'-1}$ and $S\left(p^{*t'}, q^{*t'}\right) > 0$ otherwise.

We propose the following strategies for the players:

i. If $H_{t-1} \neq H^*$, the firms and the auctioneer play the equilibrium strategies depicted in proposition 4.

ii. If $H_{t-1} = H^*$,

The firms' strategy

At *step 1* Firm 1(2) submits a deal (α_1, b_1) $((\alpha_2, b_2))$ with $\alpha_1 = 1$, $\alpha_2 = 0$ and $b_1 = b_2 = \frac{1}{4}m$.

At *step 3* If $\widehat{\boldsymbol{\alpha}}^t = 1$ and $\alpha^{t-1} = 0$ firm 1submits an offer including the corresponding efficient specification such that the offer scores zero. Firm 2 submits an offer that scores at most zero. If $\widehat{\boldsymbol{\alpha}}^t = \alpha_{t-1} = 1$, firm 2 submits an offer that scores *more* than zero such that it yields a payoff $(1-\delta) w_1(1)$ to firm 1 (the average of continuation payoff when firm 1 wins for the second time in a row) when firm 1 exactly matches firm 2's score. Firm 1 strategy is to submit an offer that score at most firm 2's score. When $\widehat{\boldsymbol{\alpha}}^t = 0$ the strategies are similarly defined with $\widehat{\boldsymbol{\alpha}}^t = 0$, $\alpha_{t-1} = 1$ leading to zero score bids, and $\widehat{\boldsymbol{\alpha}}^t = \alpha_{t-1} = 0$ leading to bids that yield $(1-\delta) w_2(0)$.

The auctioneer's strategy

At *step 2*, the auctioneer selects the corruption deal that maximizes his utility provided the bribe covers expected costs and he announces the corresponding scoring rule. In case of ties he randomizes.

We below show that these strategies form a Perfect Public Equilibrium of the repeated game with the stage game as described in section 5. Collusive bidding at *step 3* is sustainable relying on an argument similar to the one developed in Proposition 3 when setting $k = 2$

and firms are anti-symmetric in costs. The non-favored (say 2) firm's incentives to comply with the collusive strategy is satisfied for $\delta > \underline{\delta}_2$ where $\underline{\delta}_2$ is defined by the following equality $\frac{\delta}{(1-\delta)}\frac{1}{2}w_2(0) = \frac{1}{2\overline{\theta}} - \varepsilon + \frac{\delta}{(1-\delta)}\frac{1}{2}\pi^{ne}$ the lowest compliance payoff is set equal to the defection payoff.

At *step 2* the proposed strategy is optimal for the auctioneer appealing to the same argument as in proposition 4. At *step 1* the firms may consider defection and offer a deal with a bribe equal to $\frac{5}{4}m$. The defection payoff is at most $\pi^d = \pi^c - \frac{5}{4}m$ while the expected compliance payoff is $\frac{1}{2}\left[\pi^c - \frac{1}{4}m\right]$. We first note that for $m \geq \frac{4}{9}\pi^c$ there is no incentive to defect. But for $m < \frac{4}{9}\pi^c$ (recall that we only consider the interval $m < \frac{4}{5}\frac{(\overline{\theta}-\underline{\theta})}{\overline{\theta}\underline{\theta}}$ implying $m < \frac{4}{9}\pi^c$). The following incentive constraint, applies in any period t preceded by $\alpha_{t-1} = 0$:

$$\frac{1}{2}\left[\pi^c - \frac{1}{4}m\right] + \delta\left[\frac{1}{2}w_1(1) + \frac{1}{2}w_1(0)\right] > \left(\pi^c - \frac{5}{4}m\right) + \delta w_1(1)$$

$$\delta\left[\frac{1}{2}w_1(0) - \frac{1}{2}w_1(1)\right] > \left(\frac{1}{2}\pi^c - \frac{9}{8}m\right)$$

$$\delta\left[w_1(0) - w_1(1)\right] \geq \left(\pi^c - \frac{9}{4}m\right) \tag{14}$$

So as $\delta \longrightarrow 1,\ w_1(0) - w_1(1) \rightarrow \left(\pi^c - \frac{9}{4}m\right)$ the continuation payoff following an announcement of $\widehat{\alpha} = 1$ must be lower that the one following $\widehat{\alpha} = 0$. This payoff is achieved by letting firm 2 submit an offer in the official auction that induces a lower profit to firm 1. For $m = \frac{4}{9}\pi^c$ (the highest m for which there is an incentive to deviate), the rhs is equal to 0. Consider the following strategy that satisfies the constraint. The full punishment is taken in the next-following period and "the clock is reset" i.e., the next following payoffs in $t+2$ are determined as if $\widehat{\alpha}_t = 0$. Note that when (14) holds incentives to comply in a period following an announcement of $\alpha = 1$ also are satisfied. This is because the gain from defection are lower then. Hence, for $\delta \geq \underline{\delta}_2$ and $w_1(1) = w_2(0)$ satisfying (14) the proposed strategies form a Perfect Public Equilibrium of the repeated game.

As $m \rightarrow \frac{4}{9}\pi^c$ the equilibrium average expected payoff of the contingent scheme tends toward $\frac{1}{2}(\pi^c - m/4) = \frac{4}{9}\pi^c$ while the average payoff of the fixed in-turn rule is $\frac{\frac{5}{9}\pi^c}{(1+\delta)}$. So for any $\delta > \frac{5}{4} - 1$, the contingent scheme yields a higher expected payoff. We note that the condition $m \geq \frac{4}{9}\pi^c$ is consistent with the condition in proposition 4 $m \leq \frac{4}{5}\pi_{ne}$ when $\frac{4}{9}\frac{1}{2\underline{\theta}} \leq \frac{4}{5}\frac{(\overline{\theta}-\underline{\theta})}{2\underline{\theta}\overline{\theta}}$ which requires $\underline{\theta} \leq \frac{4}{9}\overline{\theta}$.

On the other side when $m \rightarrow 0$ the rhs of (14) tends to π^c implying $\pi_1^{t+1}(1) \rightarrow 0$ corresponding to an average equilibrium expected payoff in the contingent scheme of $\frac{1}{4}\pi^c$

which is strictly smaller than the average payoff in the in-turn scheme $\frac{\pi^c}{(1+\delta)}$. *QED*