Contracting on Time

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Abstract

The paper shows how the time considerations, especially those concerning contract duration, affect incomplete contract theory. Time is not only a dimension along which the relationship unfolds but also a continuous verifiable variable that can be included in contracts. We consider a bilateral trade setting where contracting, investment, trade, and renegotiation take place in continuous time. We show that efficient investment can be induced either through a sequence of constantly renegotiated fixed-term contracts; or through a renegotiation-proof 'evergreen' contract—a perpetual contract that allows unilateral termination with advance notice. We provide a detailed analysis of properties of optimal contracts. (JEL D23, K12, L14)

The central issue of the incomplete contract literature is the provision of incentives for specific investments. Such investments often take an intangible form (e.g., human capital) and may be not verifiable by a third party. As a result, the distribution of gains from the relationship is achieved not through explicit *ex ante* contracting but through *ex post* negotiations, in which the investing party generally does not appropriate the full marginal returns to the investment. The diluted incentives lead to underinvestment—the hold-up problem arises.

Following the influential work by Oliver Hart and John Moore (1988), various solutions to the hold-up problem have been proposed in the literature. Among others, these include (i) the allocation of property rights (Hart, 1995), (ii) the allocation of bargaining power (Philippe Aghion et al., 1994), (iii) option contracts (Georg Noldeke and Klaus M. Schmidt, 1995), and (iv) breach remedies, such as specific performance (Aaron S. Edlin and Stefan Reichelstein, 1996). However, most of this *theoretical* literature is confined to static two-period framework, excluding one of the most important factors in virtually any economic relationship, *time*.

The *empirical* literature, on the other hand, has always stressed the importance of time variables in contracts, especially in the presence of specific investments. The agreement between General Motors and Fisher Body, perhaps the most famous contract in the incomplete contract literature, explicitly mentions time in 5 out of 10 major clauses (Ronald Coase, 2000). The contract provided Fisher brothers with a share of profits as well as a half of the voting rights for 5 years and included an exclusive dealing arrangement for 10 years. The four younger Fisher brothers were guaranteed employment for 5 years, while the two elder brothers had an employment contract for 7 years with an option to quit after 5 years.

Paul L. Joskow (1987) argues that the degree of investment specificity is the crucial factor determining the duration of contracts between coal suppliers and electric utilities. His study of a sample of 277 contracts shows that as specific investments become more important the parties prefer to rely on longer-term contracts. Studies of different industries by Scott E. Masten and Keith J. Crocker (1985), Victor P. Goldberg and John R. Erickson (1987), and Crocker and Masten (1988) all demonstrate that the investment specificity is one of the key determinants of contract duration. They also note that many contracts allow termination prior to the expiration of the contract, usually by means of unilateral options rather than contingent clauses.

Some insight can also be gained by looking at contract duration across industries (Table 1). As in Joskow's sample, contract duration is likely to be longer in industries with a high degree of investment specificity. Stephen Craig Pirrong (1993) provides extensive evidence that the large variation in duration of bulk shipping contracts across industries can be explained by the degree of specificity in the customer-shipper relationship.

It is important to emphasize the dual nature of time in the real world. On the one hand, time is an integral part of environment—a dimension along which the relationship unfolds. On the other hand, time is one of the most important variables in a contract either as the *duration* of contractual obligations or as the *advance notice* time for certain unilateral actions. Time variables are easily verifiable and can be used to reduce contractual incompleteness.

This paper develops a dynamic model that encompasses both functions of time. Parties not only invest, trade, and contract *in* continuous time, they also can contract *on* time. We consider a bilateral trade environment in which at any moment a buyer can purchase a service (or a unit of a good) from a seller. At each moment the seller undertakes a relationship-specific investment, the investment reduces the cost of providing the service and increases the value of the service to the buyer. The seller has no bargaining power, hence, the threat of hold-up undermines incentives to invest. The buyer's outside option follows a stochastic process. When the value of the outside option exceeds the value provided by the seller, the buyer wants to terminate the original relationship.

Consider a fixed-term contract—a contract that stipulates trade for a definite period of time at fixed prices. The seller's incentives to invest at the beginning of a contract are determined only by its duration. If the seller's investment is sufficiently selfish and durable the contract duration can serve as an instrument for the provision of appropriate incentives. A short contract term leads to underinvestment at the first moment of the contract because the seller's total cost savings over the contract term are small. A long contract leads to overinvestment at the first moment of the contract because the seller's total cost savings over the contract term are too large. There exists some intermediate contract duration that provides just enough costs savings and induces optimal investment at the first moment. Allowing renegotiation does not change the seller's investment incentives at the contract's first moment. Anticipating renegotiation, the seller still maximizes her continuation surplus to cut the best deal in renegotiation process.

But the contract that provides correct incentives the moment it is signed becomes suboptimal later on. Suppose, for instance, that the optimal duration of a contract signed in January is nine months. The seller, protected by the contract, invests efficiently in January. In February, however, the contract provides only eight months of protection, which results in underinvestment relative to the optimum. Thus, the first best requires perpetual renegotiation of the fixed-term contract; to restore efficiency the parties constantly need to reinitialize the contract. We show (Proposition 1) that a sequence of fixed-term contracts that constantly replace each other provides incentives for efficient investment, solving the hold-up problem. The duration of the optimal fixed-term contract exceeds the expected duration of the relationship. By extending the contract term beyond the actual duration of the relationship, the fixed-term contract provides protection for the cooperative part of the investment.[1]Such contracts cannot be studied within conventional two-period setting, where the relationship lasts for two periods and the contracts are either one- or two-period long.

This result hinges on the absence of renegotiation costs. Neglecting the costs of bargaining may be an innocuous assumption in the two-period environment where renegotiation takes place only once. But if the renegotiation occurs all the time this assumption is no longer innocent. Is there a renegotiation-proof arrangement that replicates the first-best sequence of the fixed-term contracts? We study 'evergreen' contracts often observed in practice. An evergreen contract has an indefinite duration but includes an option of unilateral termination with advance notice time τ; if the buyer wants to terminate the contract at time t she must notify the seller at time $t - \tau$. The second major result of the paper (Proposition 2) is the characterization of renegotiation-proof evergreen contracts that attain the first best.

How does the optimal termination notice time τ compare to the optimal duration of the fixed-term contract T? Is it longer, shorter, or the same? Our model provides an unambiguous answer: Although both contracts induce the same investment dynamics and the same actual duration of the relationship, the advance notice time in the optimal evergreen contract is always shorter than the nominal duration of the optimal fixed-term contract. This result is based on renegotiation-proofness of the evergreen contract. Under a fixed-term contract the treat of hold-up comes from two sources: continuous renegotiation before the arrival of the outside option and renegotiation at the moment when the alternative provider arrives. The former problem, however, is not relevant under an evergreen contract and the efficient incentives are provided by a short advance notice time. The optimal advance notice time is shorter than the optimal fixed-term contract duration by exactly the expected duration of the relationship.

The rest of the paper proceeds as follows. Section I discusses related literature. Section II lays out the model. Section III analyzes incentives under fixed-term and evergreen contracts. Section IV discusses extensions: (i) nonstationary arrival process of the outside option, (ii) general allocation of bargaining power and investment by both parties. Section V concludes.

I Literature

W. Bentley MacLeod and James M. Malcolmson (1993, 1995) study a dynamic version of the conventional two-period incomplete contract model. They show that the efficient outcome can be implemented if the set of states is rich enough (so that option contracts can be used), or if only one party invests and the other party has no bargaining power, or if specificity of investment is only due to switching costs (so that a fixed-price contract is sufficient). Their analysis focuses on wage dynamics in contracts, especially the nominal rigidities and the prevalence of fixed-price contracts. MacLeod and Malcolmson do not consider contracts on time; in particular, they do not allow contracts to extend beyond the breakup of the relationship, which is crucial in our paper. Another important difference is that there is no uncertainty in the duration of the relationship. Stochastic duration of the relationship in our model allows us to analyze an important trade-off between protection of the seller's investment and the buyer's flexibility.

Noncontractual solutions to the hold-up problem were proposed by Rohan Pitchford and Christopher M. Snyder (2003) and Yeon-Koo Che and Jozsef Sákovics (2004).[2]Pitchford and Snyder allow for the gradual investment by the seller and reimbursement by the buyer after each installment. The seller's threat of withholding future investment induces the buyer to make the necessary repayments. Che and Sákovics consider the following model. In each period, both parties choose how much (more) to invest, and then a (randomly chosen) party offers terms of trade. If the offer is accepted, then trade occurs and the game ends. If the offer is rejected the game moves to the next period without trade and the same process is repeated until there is an agreement. There exists a (Markov Perfect) equilibrium approaching the first best as the parties' discount factors tend to 1.

Both results crucially depend on the assumption that the number of rounds in the investment (and bargaining) game is infinite. We study *contractual* solutions that perform equally well both in finite and infinite horizon settings. Moreover, although the two papers above allow for some investment dynamics they still do not fully depart from the two-period setting—trade occurs only once! While solutions by Pitchford and Snyder and by Che and Sákovics might be relevant in some settings, they do not seem to be applicable to a wealth of real-life economic phenomena that take place within a framework of complex, long-term contracting when parties invest, trade, and contract in continuous time.

Contracts on time are also studied in the literature on sequential innovations and patents. Gerard Llobet et al. (2001) show that if the value of invention were contractible then efficient incentives for researchers could be provided by research prizes. However, if the contracts are incomplete *ex ante* then the innovators can be given patents of certain duration that serve as threat points in negotiations with subsequent innovators. Llobet et al. focus mostly on the issues specific to the R&D literature such as patent breadth and compulsory licensing. Yet, their finding of the optimality of infinitely durable patents is consistent with our results. In the limiting case of purely cooperative investment (i.e., original invention can only be used for further innovations), optimal contract duration in our model is also infinite. Unlike our model, Llobet et al. allows contracting and renegotiation to happen only at two moments: *ex ante*, before research effort and *ex post*, after the research effort.

There also exists a large literature on the dynamics of complete contracts. Vincent P. Crawford (1988) studies the problem with and without risk-neutrality for the case when investments are contractible. Oliver Hart and Jean Tirole (1988) and Patrick Rey and Bernard Salanie (1990) consider the settings with asymmetric information and moral hazard, with and without renegotiation. Milton Harris and Bengt Holmstrom (1987) consider a model with costly recontracting where the duration of contracts becomes endogenous, parties do not renegotiate inefficient contracts immediately but wait until the existing contract's inefficiency outweighs the costs of recontracting.

II Model

We study a continuous time version of the bilateral trade model.[3]There are two risk-neutral agents: a buyer and a seller. At each moment $t \in [0, \infty)$ the buyer can purchase a service (or a unit of a good) from the seller. The value of the service to the buyer is $v(t)$ per unit of time, the cost to the seller is $c(t)$ per unit of time. Both agents have the same time preference rate ρ.

Investment. At each moment t the seller undertakes specific investment $\sigma \geq 0$ at the cost σdt. The investment increases buyer's value by $b(\sigma)dt$ and reduces seller's cost by $s(\sigma)dt$. The total instantaneous returns to the investment are, therefore, $w(\sigma) = b(\sigma) + s(\sigma)$. The effects of investment depreciate over time at the rate δ. Hence, the investment σ undertaken at the moment x increases the value at time t by $b(\sigma)e^{-\delta(t-x)}dt$ and decreases the cost by $s(\sigma)e^{-\delta(t-x)}dt$.

We assume that the effects of investment on value and cost are additively separable over time; the current investment does not affect the quality that has been built through the past investment. Let $\sigma : [0, \infty) \rightarrow R_+$ be a stream of investments. Then, the value and the cost evolve over time according to

$$\begin{aligned} v(t) &= v(0) + \textstyle\int_0^t b(\sigma(x))e^{-\delta(t-x)}dx, \\ c(t) &= c(0) - \textstyle\int_0^t s(\sigma(x))e^{-\delta(t-x)}dx, \end{aligned} \tag{1}$$

where $v(0)$ and $c(0)$ are exogenous initial values.

Equations (1) can also be written in the differential form. The buyer's accumulated utility (due to past investments) $v(t) - v(0)$ evolves according to

$$\frac{d}{dt}[v(t) - v(0)] = -\delta[v(t) - v(0)] + b(\sigma(t)).$$

Outside options. The seller's outside option is normalized to zero. The buyer's outside option follows a Poisson process. Initially (at $t = 0$) the buyer's outside option is trivial. At the moment ξ the buyer's outside option increases to $V > 0$ and remains at this level forever. This change in the value of the buyer's outside option is referred to as 'the arrival of the outside option'. The switching time ξ is a random variable with c.d.f. $F(\xi) = 1 - e^{-\lambda \xi}$, where λ is the Poisson arrival rate. That is, if the outside option has not arrived by time t then the probability of arrival during $[t, t + dt]$ is λdt. We analyze a more general, non-Poisson, arrival process in Section IV.

The stochastic nature of the outside option reflects the fact that the set of alternatives available to the buyer may expand over time. Initially the buyer can acquire the service only from the seller. But as the time goes by there may arrive an alternative supplier able to provide the buyer with a service of (net) value V.

Payoffs. Suppose that the buyer exercises the outside option at the time $\xi \in [0, \infty)$, then the parties' payoffs are given by

$$\begin{aligned} U^B &= \int_0^\xi (v(t) - p(t))\, q(t)\, e^{-\rho t} dt + \int_\xi^\infty V e^{-\rho t} dt, \\ U^S &= \int_0^\xi \left[(p(t) - c(t))q(t) - \sigma(t)\right] e^{-\rho t} dt, \end{aligned}$$

where $q(t) \in \{0, 1\}$ is the quantity traded and $p(t)$ is the price paid at time t.

Assumptions. We impose the following restrictions on the primitives of the model:

*A*1. Inada conditions:

$$\begin{aligned} b'(\sigma) > 0, b''(\sigma) < 0, b'(0) = \infty, b'(\infty) = 0, \\ s'(\sigma) > 0, s''(\sigma) < 0, s'(0) = \infty, s'(\infty) = 0. \end{aligned}$$

*A*2. Boundedness: $b(\infty) < \infty$, $s(\infty) < \delta c(0) < \infty$.

*A*3. Superiority of the outside option: $V > v(0) - c(0) + [b(\infty) + s(\infty)]/\delta$.

*A*4. Gains of trade: $v(0) \geq c(0)$.

Assumptions *A*1 and *A*2 guarantee the existence of finite interior solutions for all maximization problems. *A*3 assures that the outside option dominates trade regardless of the investment level, and $A4$ implies that before the arrival of the outside option trade is efficient at every moment t, $v(t) - c(t) \geq v(0) - c(0) \geq 0$.

Contracts. The payments, the delivery of the service, and the time of payments and delivery are verifiable by a third party. The parties can also contract on messages such as advance termination notices. The investment, the value and the cost of the service, and the value of the outside option are observable but not verifiable.

The parties can costlessly renegotiate contractual obligations at any moment. To focus on the most severe case of the hold-up problem, we assume that all bargaining power in the renegotiation process is allocated to the buyer. That is, under the null contract (no contract) the buyer would expropriate all the surplus and, as a result, the seller would not invest, $\sigma = 0$. In Section IV we generalize the model allowing for an arbitrary allocation of bargaining power.

Timing. The timing of events is illustrated in the Figure 1. If there is a valid contract at time t then the seller provides the service at the specified price $p(t)$; otherwise, the buyer makes a take-it-or-leave-it offer to the seller. Next, the seller makes an investment decision $\sigma(t)$ and incurs investment costs. After that both parties observe whether the buyer's outside option has arrived. When the buyer exercises the outside option the game ends; otherwise it proceeds to $t + dt$.

Each infinitesimal period $[t, t+dt]$ also includes a possibility of renegotiation. Renegotiation can occur for two reasons. First, during this period an additional amount of investment $\sigma(t)dt$ is sunk, so both the buyer's value and the seller's cost have changed. Second, the parties receive new information on whether the buyer's outside option has arrived.

Note that if the outside option has not arrived, the environment at time $t+dt$ is identical to the one at time t (modulo the change in the levels of cost c and value v). The stationarity follows from three features of the setting: (i) constant rate of the arrival of the outside option, (ii) additive separability of the effects of investment on value and cost over time, and (iii) constant depreciation and discount rates.

III Analysis

A First-best outcome

Suppose that the buyer's outside option arrives at $\xi > 0$. Then, in the first-best outcome, the seller provides the buyer with a service at all $t \leq \xi$ (Assumption $A4$). Afterwards, for all $t > \xi$, the buyer exercises his outside option (Assumption $A3$) and the seller does not invest.

At every moment $t \leq \xi$ the seller's investment $\sigma(t)$ is chosen to maximize the expected total welfare. The duration of the relationship $\xi - t$ is a random variable with p.d.f. $\lambda e^{-\lambda(\xi - t)}$ and its expected value is independent of t. For every $t \leq \xi$ the optimal level of investment $\sigma^*(t)$ maximizes

$$-\sigma(t) + \int_t^\infty \lambda e^{-\lambda(\xi - t)} \left(\int_t^\xi w(\sigma(t)) e^{-\delta(x-t)} e^{-\rho(x-t)} dx \right) d\xi,$$

where $w(\sigma(t))e^{-\delta(x-t)}dx$ is the increase in the total welfare at time x owing to investment $\sigma(t)$ at time t. Hence, $\sigma^*(t)$ maximizes

$$-\sigma(t) + \frac{1}{\lambda + \rho + \delta} w(\sigma(t)).$$

From the first-order condition, the optimal level of investment $\sigma^*(t) = \sigma^*$ is

$$\frac{1}{\lambda + \rho + \delta} w'(\sigma^*) = 1. \tag{2}$$

Note that the optimal level of investment does not depend on the current values $v(t)$ and $c(t)$, nor does it depend on the time t.

Thus, the social optimum is: (i) until the arrival of the outside option ($t \leq \xi$) the seller provides a service, $q(t) = 1$, and invests $\sigma(t) = \sigma^*$; (ii) after the arrival ($t > \xi$) the buyer exercises the outside option, $q(t) = 0$, and the seller does not invest, $\sigma(t) = 0$.

Let R be the ratio of the investment's effect on the buyer's utility and the effect on the seller's cost at the socially optimal level of investment σ^*:

$$R = b'(\sigma^*)/s'(\sigma^*).$$

The parameter R captures the degree of marginal 'cooperativeness' of the seller's investment (Ilya Segal and Michael D. Whinston, 2002). If $R = 0$ then the investment is purely 'selfish', if $R = \infty$ the investment is purely 'cooperative' (Yeon-Koo Che and Donald B. Hausch, 1999). Using the definition of R the first order conditions can be rewritten as

$$s'(\sigma^*) = \frac{\lambda + \rho + \delta}{1 + R}. \tag{3}$$

B Fixed-term contracts

A fixed-term contract of duration T signed at moment t_0 is a triple $\langle T, q(t), p(t) \rangle$ where $q(t) = 1$ if $t \in [t_0, t_0 + T]$, otherwise $q(t) = 0$ and $p(t) = 0$. In other words, the contract reads 'the seller provides the service to the buyer from t_0 till $t_0 + T$ at the price $p(t)$'.

The payoff to the seller under such a contract is given by

$$U_{t_0}^S(T) = \int_{t_0}^{t_0+T} [p(t) - c(t) - \sigma(t)]\, e^{-\rho(t-t_0)} dt, \tag{4}$$

where $c(t)$ evolves according to (1). Hence, at the time t_0 the seller chooses $\sigma(t_0)$ to maximize

$$-\sigma(t_0) + \int_{t_0}^{t_0+T} s(\sigma(t_0))\, e^{-(\rho+\delta)(t-t_0)} dt = -\sigma(t_0) + \frac{1 - e^{-(\rho+\delta)T}}{\rho + \delta} s(\sigma(t_0))$$

The investment σ_T at the start of a fixed-term contract of duration T is determined by the first order conditions

$$\frac{1 - e^{-(\rho+\delta)T}}{\rho + \delta} s'(\sigma_T) = 1. \tag{5}$$

The stationarity of the problem implies that the investment σ_T does not depend on the current levels of $v(t)$ and $c(t)$ and is the same for every fixed-term contract of duration T independent of when it is signed. It is important to emphasize that the level of investment is determined only by the duration of the current contract, T, and not by the expected duration of the relationship, λ^{-1}. Though the investment undertaken at time t_0 does affect the cost $c(t)$ and the value $v(t)$ after the expiration of the contract (at $t > t_0 + T$) these future gains are fully appropriated by the buyer who has all bargaining power.

If the contract duration is very short then the seller invests too little compared to the social optimum. If the contract term is too long then the seller overinvests—she ignores the fact that social returns to her investment, as well as the expected duration of the relationship, are short-lived. The fixed-term contract induces the optimal level of investment at the moment t_0 if and only if $T = T^* \geq 0$, where

$$T^* = \frac{1}{\rho+\delta} \ln \frac{1}{1 - \frac{\rho+\delta}{s'(\sigma^*)}} = \frac{1}{\rho+\delta} \ln \left(\frac{\lambda+\rho+\delta}{\lambda - R(\rho+\delta)} \right). \tag{6}$$

The optimal duration T^* is well-defined if and only if the investment is sufficiently selfish and durable over the expected duration of the relationship:

$$R(\rho+\delta)\lambda^{-1} < 1. \tag{7}$$

Otherwise there is no fixed-term contract that implements the first best. The seller's incentives are driven by the self-effects of investment she is going to receive over the duration of the contract. The fixed-term contract works if the self-effects are substantial (relative to cross-effects) and do not depreciate (or get discounted) too quickly. The expected duration of the relationship is also an important factor. The longer the parties are expecting to stay together the higher is the optimal level of investment, and the parties need a longer contract.

The optimal fixed-term contract is not, however, renegotiation-proof. The contract of duration T^* provides correct incentives to invest only at the moment it is signed. At any other time $t' \in (t_0, t_0 + T^*]$ the seller would underinvest because the remaining duration of the contract is shorter than T^*. Hence, the parties want to replace the current contract by a new one that maximizes their welfare at time t'. The stationarity of the problem implies that the duration of new contract will also be T^*. Thus, the contract 'trade at price $p(t)$ until $t_0 + T^*$' is renegotiated at the moment t' to the contract 'trade at price $p'(t)$ until $t' + T^*$'. The price schedule in the new contract may be different from the previous one.

When the outside option arrives the contract is renegotiated for the last time. Exercising the outside option is efficient, so the buyer is better off paying the seller as much as the seller would get under the contract in place. This payment is essentially contingent on the seller's past investment and is equivalent to the net present value of getting a flow of $p(t) - c(t)$ for the duration of contract T^*. Even though investments are not contractible the fixed-term contract provides the seller with marginal returns on investment $s'(\sigma)$ amplified by the length of the contract T^*. In other words, the possibility of renegotiation does not distort the seller's investment incentives. The seller still wants to maximize her continuation surplus $U^S_{t_0}(T^*)$ to get the best position in renegotiation.

To complete the description of the optimal fixed-term contract we need to specify the price schedule $p(t)$. As the seller has zero reservation value and no bargaining power, the seller's participation constraint is binding, $U^S_{t_0}(T) = 0$. Since both agents are risk-neutral and have the same time preference rate, only the total transfer $\int_{t_0}^{t_0+T} p(t)e^{-\rho(t-t_0)}dt$ matters. Therefore, the optimal contract can be supported by a continuum of price schedules. The most straightforward one is

$$p(t) = c(t) + \sigma_{t_0+T^*-t},$$

where $c(t)$ is given by (1) with $\sigma(x) = \sigma_{t_0+T^*-x}$, and $\sigma_{t_0+T^*-x}$ solves (5) for $T = t_0 + T^* - x$.

To see why this schedule results in $U^S_{t_0}(T) = 0$ assume, for the sake of argument, that there is no renegotiation. Then, at each $t \in (t_0, t_0 + T^*]$ the remaining contract duration is shorter than T^* and the seller invests $\sigma(t) = \sigma_{t_0+T^*-t} < \sigma_{T^*} = \sigma^*$. As a result, at every moment $t \in (t_0, t_0 + T^*]$ the price exactly covers the seller's production and investment costs and the seller receives zero payoff. Moreover, the expected payoff under such contract is also zero at every $t \in (t_0, t_0 + T^*]$. Thus, when the contract is renegotiated the buyer does not make any transfers to the seller.

Note that the price schedule is relevant only at $t = t_0$, already at the very next moment $t' = t_0 + dt$ a new fixed-term contract with a new price schedule $p'(t) = c(t) + \sigma_{t'+T^*-t}$, $t \in (t', t' + T^*]$ is signed.

The discussion above is summarized as

Proposition 1 *When $R(\rho + \delta)\lambda^{-1} < 1$ there exists a sequence of continuously renegotiated fixed-term contracts that implements the first best. At any time $t_0 \in (0, \xi)$ the contract reads 'trade at price $p(t)$ until the time $t_0 + T^*$' where*

$$T^* = \frac{1}{\rho + \delta} \ln \left(\frac{\lambda + \rho + \delta}{\lambda - R(\rho + \delta)} \right).$$

When the outside option arrives the contract is renegotiated to the null contract.

The following Corollary explores an important case when $\rho+\delta$ is negligible compared to λ, that is depreciation and discounting are small over the expected duration of the relationship.

Corollary 1 *When $\lambda^{-1}(\rho+\delta)$ tends to zero the optimal contract duration is*

$$T^* = \lambda^{-1}(1+R).$$

This limiting case highlights the intuition behind the Proposition 1. The optimal contract duration T^* exceeds the expected duration of the relationship λ^{-1} by R percent, thus rewarding the *cooperative* part of the seller's investment.

C 'Evergreen' contracts

The analysis above leads to a question: Can the sequence of perpetually renegotiated fixed-term contracts be replaced with a renegotiation-proof contract? Consider an 'evergreen' contract that reads *'at time t the parties trade at price $p(t)$; the contract lasts indefinitely; the buyer may terminate the contract at any moment ξ provided she has notified the seller at time $\eta = \xi - \tau$; once the termination notice is sent parties trade at $\tilde{p}(t;\eta)$'*. The notice is verifiable.

An evergreen contract is renegotiation-proof if it provides efficient incentives. Suppose that at time t the parties sign an evergreen contract that induces the optimal level of investment $\sigma(t) = \sigma^*$. Will the contract be renegotiated if the outside option has not arrived in $(t, t+dt)$? The buyer could send a termination notice (to terminate at $t + dt + \tau$) and then offer a new evergreen contract. The situation at $t+dt$ is, however, identical to the one at t; a contract that provides efficient incentives at $t+dt$ also maximizes the total welfare at $t+dt$. The buyer would initiate renegotiation only if she could redistribute the wealth away from the seller (total welfare is the same and is equal to the first-best level). Thus, as long as the seller's payoff at each moment is equal to zero an evergreen contract is renegotiation-proof until the arrival of the outside option.

Once the outside option arrives the buyer wants to terminate the relationship. The termination increases total welfare and the contract will be renegotiated. The seller's threat point in the renegotiation is exactly her payoff under a fixed-term contract of duration τ. Thus, the evergreen contract with the advance notice time τ signed at the time t_0 is equivalent to the fixed-term contract of duration from t_0 until $t_0 + \xi + \tau$. The only difference is that the duration, $\xi + \tau$, is a random variable with p.d.f. $\lambda e^{-\lambda\xi}$.

At the time t_0 the seller chooses $\sigma(t_0)$ to maximize

$$
\begin{aligned}
&-\sigma(t_0)+\int_{t_0}^{\infty}\lambda e^{-\lambda(\xi-t_0)}\left(\int_{t_0}^{\xi+\tau}s(\sigma(t_0))e^{-(\delta+\rho)(t-t_0)}dt\right)d\xi\\
&=-\sigma(t_0)+\frac{1}{\rho+\delta}\left(1-\frac{\lambda}{\lambda+\rho+\delta}e^{-(\rho+\delta)\tau}\right)s\left(\sigma\left(t_0\right)\right).
\end{aligned}
$$

The investment σ_τ is determined by the first order conditions

$$
\frac{1}{\rho+\delta}\left(1-\frac{\lambda}{\lambda+\rho+\delta}e^{-(\rho+\delta)\tau}\right)s'\left(\sigma_\tau\right)=1. \tag{8}
$$

Again, additive separability implies that optimal investment depends only on the notice time τ and is independent of the current values $v\left(t\right)$ and $c\left(t\right)$ and is the same for every evergreen contract with advance notice time τ independent of when it is signed.

The evergreen contract induces the optimal level of investment if and only if $\tau=\tau^*>0$, where

$$
\tau^*=\frac{1}{\rho+\delta}\ln\left(\frac{\lambda}{\lambda+\rho+\delta}\frac{1}{1-\frac{\rho+\delta}{s'(\sigma^*)}}\right)=\frac{1}{\rho+\delta}\ln\left(\frac{\lambda}{\lambda-R\left(\rho+\delta\right)}\right). \tag{9}
$$

As in the case of fixed-term contract, τ^* is well-defined if and only if $R\left(\rho+\delta\right)\lambda^{-1}<1$.

The price schedules $p(t)$ and $\tilde{p}(t;\xi)$ are chosen to set the seller's expected payoff to zero at each moment of time. The price $p(t)$ should give the seller zero flow utility at each moment until the arrival of outside option. In other words, $p(t)$ covers the current instantaneous costs of production and investment

$$
p\left(t\right)=c\left(t\right)+\sigma^*,
$$

where $c\left(t\right)$ is given by (1) with $\sigma(x)=\sigma^*$ for all $x\in[t_0,t]$, i.e., $c\left(t\right)=c\left(0\right)-s\left(\sigma^*\right)\left(1-e^{-\delta t}\right)/\delta$.

If the notice is sent at time ξ, the seller is protected by the contract until $\xi+\tau$. The seller's incentives to invest at time $t\in[\xi,\xi+\tau]$ are, therefore, equivalent to those under a fixed-term contract of duration $T=\xi+\tau-t$. The price $\tilde{p}(t;\xi)$ is, then, the price that gives the seller zero payoff at every moment under the fixed-term contract from ξ till $\xi+\tau$:

$$
\tilde{p}(t;\xi)=c\left(t\right)+\sigma_{\xi+\tau-x}
$$

where $\sigma_{\xi+\tau-t}$ is given by (5) for $T=\xi+\tau-t$, and $c(t)$ evolves over time according to (1) with $\sigma(x)=\sigma^*$ for all $x\in[t_0,\xi]$ and $\sigma(x)=\sigma_{\xi+\tau-x}$ for $x\in[\xi,\xi+\tau]$. The schedule $\tilde{p}(t;\xi)$ depends on the time of sending the notice, ξ, reflecting the cost $c(t)$ after ξ months of investment.

Proposition 2 *When $R(\rho+\delta)\lambda^{-1} < 1$ there exists an evergreen contract that implements the first best. The advance notice time is*

$$\tau^* = \frac{1}{\rho+\delta}\ln\left(\frac{\lambda}{\lambda - R(\rho+\delta)}\right).$$

The contract is renegotiation-proof until the outside option arrives. When the outside option arrives the contract is renegotiated to the null contract.

In equilibrium, the termination notice provision effectively serves as a termination penalty clause. In the event of the break-up the buyer has to pay τ^* month worth of the seller's payoff under the contract in place. It is important that this penalty is formulated in terms of time rather than money. Were the penalty fixed in monetary terms the contract would fail to provide correct incentives, being independent of the past investment. Contracting on time, on the other hand, essentially makes the payments contingent on the investment. The value of the penalty, defined in terms of time, endogenously reflects the seller's past investment. The advance notice time provides the seller with marginal returns $\tau^* s'(\sigma)$. Thus, each additional unit of time rewards the seller's investment through the lower production costs.

Our formulation of the evergreen contract does not include any termination fee; once the notice is sent the parties switch to a new price schedule $\tilde{p}(t;\xi)$. Alternatively, the price schedule $p(t)$ may be left intact but the termination notice is accompanied by a lump-sum termination fee. Such a fee should keep the seller's participation constraint binding and, thus, is equal to the discounted present value of the difference between $p(t)$ and $\tilde{p}(t;\xi)$ over the interval $[\xi, \xi+\tau]$. Although this fee depends on the moment when the notice sent, it is lump-sum and cannot be made contingent on the past investment. Hence the advance notice time τ remains the only instrument to adjust incentives.

The next Corollary focuses on the case when $\rho+\delta$ is small compared to λ.

Corollary 2 *When $\lambda^{-1}(\rho+\delta)$ tends to zero the optimal advance notice time is*

$$\tau^* = \lambda^{-1}R.$$

This result reflects renegotiation-proofness of evergreen contract. A fixed-term contract has to provide protection to the seller both during the relationship and at its termination while an evergreen contract has to do so only at the moment of termination. When $\rho+\delta \ll \lambda$ the difference between T^* and τ^* is exactly λ^{-1}, the expected duration of the relationship.

In the general case, $T^* - \tau^* = (\rho+\delta)^{-1} \ln \left[(\lambda+\rho+\delta)\, \lambda^{-1} \right]$, but the intuition is the same. The present discounted value of receiving \$1 for T^* months is equal to the present discounted value of receiving \$1 for the expected duration of the relationship, λ^{-1} on average, plus the notice period, τ^*. Both are equal to the social marginal return of investing \$1:

$$\int_0^{T^*} e^{-(\delta+\rho)t} dt = \int_0^{\infty} \lambda e^{-\lambda\xi} \left(\int_0^{\xi+\tau^*} e^{-(\delta+\rho)t} dt \right) d\xi = \frac{1+R}{\lambda+\rho+\delta}.$$

D Discussion

Empirical implications. The analysis above generates several testable predictions. First, both the fixed-term contract duration T^* and the advance notice time τ^* in the evergreen contract are increasing in the expected duration of the relationship, λ^{-1}, and the cooperativeness of investment, R. Parameters λ^{-1} and R can be viewed as proxies for the investment specificity. As the expected duration of the relationship increases it becomes less likely for the buyer's outside option to arrive, that is, it becomes more difficult for the buyer to find an alternative provider of the service. As the cooperativeness of investment increases the impact of the investment on the buyer's value becomes more pronounced. In other words, the value of the investment *within* the relationship is more substantial when λ^{-1} or R are large. Our results are consistent with the empirical analysis of coal contracts by Joskow (1987).[4]The thickness of market for alternative partners, λ^{-1}, is captured in Joskow's study by controlling for site specificity ("mine-mouth" location) and the cooperativeness of investment is proxied by physical asset specificity (investing in coal-burning plants for specific type of coal) and dedicated assets (investing in plants dedicated for trade with a specific partner).

Second, the optimal advance notice in the evergreen contract is always shorter than the duration of the optimal fixed-term contract, $\tau^* < T^*$. This prediction is harder to test empirically, as one need to control for the nature of uncertainty and the degree of specificity, yet it also seems to be consistent with observed characteristics of real world contracts. For instance, the length of contracts for sales of petroleum coke or aluminum usually varies from

8 to 20 years while the termination notice time is much shorter and varies from 6 months to 2 years (Goldberg and Erickson, 1987, John A. Stuckey, 1983). Labor contracts have a similar structure: the fixed-term contracts last for at least a year while the termination notices vary from 2 weeks to 3 months (William Mercer, 2001). The model suggests that the employment-at-will contracts with a short termination notice may provide as much protection for the employee's firm-specific investment as the fixed-term contracts of long duration.

Contracts on time vs. other mechanisms. Contracts on time are *contractual* solutions radically different from mechanisms exploiting repeated play. Contracts on time rely heavily on third party enforcement: both the provision of a service and its timing must be verifiable. The stationarity of the game, on the other hand, is not essential. Contracts on time attain the first best even in nonstationary environments, including those with nonstochastic end dates (see below). The performance of contracts is also not affected by continuous renegotiation while the mechanisms based on repeated play may become suboptimal. The presence of renegotiation takes away harsh punishments and therefore may destroy efficient subgame perfect equilibria (see Jean-Pierre Benoît and Vijay Krishna, 1993).

Contracts on time are also not driven by 'the endgame effect' present in models with sequential investment, e.g., Joel S. Demski and David E. Sappington (1991) and Noldeke and Schmidt (1998). As Aaron S. Edlin and Benjamin E. Hermalin (2000) argue, option contracts work in such environment if parties cannot renegotiate after the option expires. In other words, the specific investment is short-lived; the value of investment is destroyed if parties do not exercise the option. The model in this paper is different. If the buyer does not use her option to terminate, the seller's investment remains in place and parties continue to trade. The seller's investment loses value only when termination actually occurs. Moreover, one can consider a model where the outside option may also break down at some point after the arrival and the buyer would have to return to the original seller. In this case, the seller's investment retains value even after termination and it can be shown that the contracts on time still perform well.

Fixed-term contracts are related to contracts on a continuous quantity (Edlin and Reichelstein, 1996, Che and Hausch, 1999). Consider the following modification of the fixed-term contract. Instead of trading one unit for T days the parties agree on trading $q = T$ units.

Since trading more than one unit a day is infeasible, the contracts are formally equivalent. There is, however, a significant difference. While contracts on a continuous quantity allow renegotiation only before the production begins, in our setting renegotiation can occur after the production of any unit. In other words, in Edlin and Reichelstein (1996) no new information arrives after the production has started. In this paper new information can arrive during the contract term and the parties can adjust their contracts in response. Moreover, since time goes only in one direction, the contract can be only renegotiated upwards—after, say, a half of q is produced parties cannot agree on producing only a quarter of it.

Evergreen contracts are related to mechanisms in Noldeke and Schmidt (1995), where option contracts are used to partition the (continuous) state space. Our original state space is binary (the outside option has/has not arrived); however, we can also introduce a continuous state space by including the time at which the outside option arrives. In our case, the value of the buyer's option to terminate depends on the length of termination notice. As in Noldeke and Schmidt (1995), the option is exercised in some states but not in others; the buyer only sends the termination notice when the outside option has arrived. Unlike Noldeke and Schmidt, however, exercising the option is not a function of the option contract's parameters (in their case, price, in ours—the notice time τ). In our case, the fine tuning of incentives is based on adjusting the value of the option, rather than its probability to be exercised. Contracts on time work because they make the value of the option directly conditional on investment.

IV Extensions

A Non-Poisson uncertainty

In this Section we consider a general nonstationary process of the outside option arrival. The arrival time ξ is a random variable distributed on $[0, \bar{\xi}]$ with an arbitrary p.d.f. $dF(\xi)$. For expositional clarity we neglect depreciation and discounting, $\rho = \delta = 0$. The assumptions $A2$ and $A3$ are restated as

$A2'$. Boundedness: $\bar{\xi} < \infty$, $b(\infty) < \infty$, $s(\infty) < c(0)/\bar{\xi} < \infty$.

$A3'$. Superiority of the outside option: $V > v(0) - c(0) + (b(\infty) + s(\infty))\,\bar{\xi}$.

In contrast to the Poisson case, the expected duration of the relationship is no longer stationary. At time t, the expected duration of the relationship

$$E(\xi - t|t) = \frac{\int_t^{\bar{\xi}} (\xi - t) dF(\xi)}{1 - F(t)},$$

is not constant over time (for Poisson, $E(\xi - t|t) = \lambda^{-1}$).

At each moment t the first-best level of investment solves

$$E(\xi - t|t) w'(\sigma^*(t)) = 1.$$

The first-best investment is also not stationary. The shorter the remaining duration of the relationship, the lower the optimal investment.

The first best can be implemented through a sequence of constantly renegotiated fixed-term contracts. The optimal duration of the contract signed at time t is

$$T^*(t) = \frac{1}{s'(\sigma^*(t))} = (1 + R(t))E(\xi - t|t),$$

where $R(t) = b'(\sigma^*(t))/s'(\sigma^*(t))$.

The optimal level of investment can also be induced through renegotiation-proof evergreen contract with advance notice time

$$\tau^*(t) = \frac{1}{s'(\sigma^*(t))} - E(\xi - t|t) = R(t)E(\xi - t|t).$$

Both $T^*(t)$ and $\tau^*(t)$ decrease over time reflecting the shorter expected duration of the relationship. For instance, if the buyer wants to break up in the early stages of the relationship she must provide a notice well in advance; at the later stages a shorter notice is required.

Example 1 *No uncertainty. The outside option arrives at* $\xi = 1$ *with probability* 1. *Then,* $E(\xi - t|t) = 1 - t$, $T^*(t) = (1 + R)(1 - t)$, *and* $\tau^*(t) = R(1 - t)$. *At time* t *the fixed-term contract obliges parties to trade until* $t + T^*(t) = 1 + R(1 - t) = 1 + \tau^*(t)$.

Example 2 *Uniform distribution. Arrival time* ξ *is distributed uniformly on* $[0, 1]$. *Then* $E(\xi - t|t) = (1 - t)/2$, $T^*(t) = (1 + R)(1 - t)/2$, $\tau^*(t) = R(1 - t)/2$.

The main results are virtually identical to those obtained for the Poisson case. First, both fixed-term and evergreen contracts implement the first best. Second, the evergreen contract is renegotiation-proof while the fixed-term contract is constantly renegotiated. Third, at any given moment in time the difference between the term of the fixed-term contract and the advance notice time is exactly the expected duration of the relationship at that time. The only difference is that the contracts' parameters $T^*(t)$ and $\tau^*(t)$ are no longer time-independent. The newly renegotiated fixed-term contract may be shorter than the previous one. The termination notice in the optimal evergreen contract also depends on the period when the notice is sent, the contract may require longer advance notice if the termination occurs during the initial stages of the relationship.

In many real world situations, the first best requires a large capital investment in the beginning of the relationship and only marginal investments in maintenance and repairs later. The optimal contract has to provide significant protection for earlier investments, while later investments require less protection. While this can be done either through a chain of fixed-term contracts or a time-dependent termination notice (as described above), most real life contracts simply include both: relatively long finite contract duration and rather short advance notice clauses. Then at each moment the seller will use whichever clause provides her with greater protection. In the beginning of the relationship the incentives to make capital investment are driven by the long contract duration, while later marginal investments are protected by the advance notice clause.

B Bargaining power and two-sided investment

In this Section we consider a more general allocation of bargaining power and investments by both parties. For expositional clarity we keep the assumptions of Poisson arrival process and negligible discounting and depreciation $(\rho+\delta)\lambda^{-1} \ll 1$.

Bargaining power. Let $\alpha \in [0,1]$ be the seller's bargaining power, that is, the seller's share of *ex post* gains from any negotiation. Consider the seller's incentives to invest under a fixed-term contract. In addition to $U_{t_0}^S(T)$ (see (4)) the seller also receives α percent of any increase in the total surplus in each renegotiation.

First, the seller receives benefits from continuous renegotiations until the arrival of the outside option. An increase in the total surplus at time $t_0 + dt$, when the parties renegotiate the contract signed at time t_0, comes from the two sources: (i) the higher gains of trade: $[v(t_0 + dt) - c(t_0 + dt)] - [v(t_0) - c(t_0)] = w(\sigma(t_0))\,dt$, and (ii) the restoration of the sub-optimal contract duration $T - dt$ to the optimal one, T. The former effect has an additive impact on all future v and c and has to be integrated over the expected duration of the relationship

$$\alpha \int_{t_0}^{\infty} \lambda e^{-\lambda(\xi - t_0)} \left(\int_{t_0}^{\xi} \left[e^{-(\delta+\rho)(t-t_0)} w'(\sigma(t_0)) dt \right] \right) d\xi = \frac{\alpha}{\lambda + \delta + \rho} w'(\sigma(t_0)).$$

To calculate the latter effect, let $W(T)$ denote the expected total welfare at time t modulo $v(t) - c(t)$, after a fixed-term contract of duration T has been just signed. The seller's share is

$$\alpha \int_{t_0}^{\infty} \lambda e^{-\lambda(\xi - t_0)} \left(\int_{t_0}^{\xi} [W(T) - W(T - dt)] \right) d\xi = \alpha \lambda^{-1} W'(T).$$

Since, by definition, renegotiation always results in a choice of T that maximizes $W(T)$ we obtain that $W'(T) = 0$; there are no constraints on the choice of T because investment is continuous, $W(\cdot)$ is differentiable, and $W'(\cdot)$ is continuous.

Second, the seller also receives α per cent of an increase in the total welfare once the outside option arrives

$$\alpha \int_{\xi}^{\xi+T} e^{-(\delta+\rho)(t-t_0)} [V - v(t) - c(t)]\, dt.$$

This term has a negative effect on the seller's incentives, the more the seller invests the lower is the benefit of a termination. The marginal disincentive is

$$-\alpha \int_{t_0}^{\infty} \lambda e^{-\lambda(\xi - t_0)} \left(\int_{\xi}^{\xi+T} e^{-(\delta+\rho)(t-t_0)} w'(\sigma(t_0)) dt \right) d\xi = -\frac{\alpha\lambda}{\lambda + \delta + \rho} \frac{1 - e^{-(\delta+\rho)T}}{\delta + \rho} w'(\sigma(t_0)).$$

Summing up, the first order conditions for the investment σ_T at the start of a fixed-term contract of the duration T are

$$\frac{1 - e^{-(\rho+\delta)T}}{\rho + \delta} s'(\sigma_T) + \frac{\alpha}{\lambda + \delta + \rho} \left[1 - \lambda \frac{1 - e^{-(\delta+\rho)T}}{\delta + \rho} \right] w'(\sigma_T) = 1.$$

Assuming $(\rho + \delta)\lambda^{-1} \ll 1$ the optimal contract duration is

$$T^* = \frac{\lambda^{-1}(1+R)(1-\alpha)}{1-\alpha(1+R)}. \tag{10}$$

Similar analysis derives incentives under an evergreen contract. The renegotiation-proofness requires choosing optimal τ at each moment and setting the price at the level that provides both parties with shares of the surplus proportional to their bargaining powers. As a result, the bargaining power influences the seller's incentives only through the division of the total surplus when the outside option arrives. The optimal advance notice time is

$$\tau^* = \frac{\lambda^{-1}R}{1-\alpha(1+R)}. \tag{11}$$

Again, the difference $T^* - \tau^*$ is equal to the expected duration of the relationship λ^{-1}.

Both the duration T^* and advance notice time τ^* increase in the seller's bargaining power α. The total surplus at the termination of the relationship decreases in the seller's past investment, thus the allocation of bargaining power towards the seller adversely affects her investment incentives. In order to restore incentives the parties need to sign a longer (compared to $\alpha = 0$) contract. This effect is also present in two-period models (see Edlin and Reichelstein's analysis of outcomes when contract quantity is above the efficient quantity).

The first best cannot be implemented if $\alpha > 1/(1+R)$. Moreover, in this case *any* contract on time performs worse than a null contract. This failure of contracts on time is closely connected to the way in which the buyer's outside option is modelled and, ultimately, to the exact nature of the seller's investment.

If the seller invests in human capital, then after switching the buyer's utility is no longer related to seller's past investment and the value of the outside option, V, is exogenous. If, however, the investment is embodied in an asset controlled by the buyer then the buyer carries the asset over into the new relationship and the value V does depend on past investment. Specifically, consider the following setting. An alternative provider offers the service at a competitive price $\widehat{p}$, which is below any price that the original seller could offer. The buyer's valuation of the outside option at the moment ξ is $V = v(\xi) - \widehat{p}$. Then an increase in the total surplus from breaking up is $v - \widehat{p} - (v - c) = c - \widehat{p}$. As a result, optimal T^* and τ^*

become

$$T^* = \lambda^{-1}\left(1 + \frac{R}{1-\alpha}\right); \qquad \tau^* = \lambda^{-1}\frac{R}{1-\alpha}.$$

and the first best can be implemented either through a fixed-term contract or an evergreen contract. Note that the distinction between the two interpretations is irrelevant when the seller has no bargaining power, $\alpha = 0$.

Two-sided investment. Suppose that the buyer also can invest $\beta \geq 0$ at any moment of time. The investment increases the buyer's value and reduces the seller's cost, the instantaneous social returns to the investment are $w(\beta, \sigma) = b(\beta, \sigma) + s(\beta, \sigma)$. The first best solves $\lambda^{-1}\partial w/\partial \sigma = \lambda^{-1}\partial w/\partial \beta = 1$. When both parties invest and their investments have cross-effects then the first best outcome is almost impossible to implement (see Che and Hausch, 1999 and Sergei Guriev, 2003). But contracts on time remain a valuable instrument and can be used to reallocate the incentives in order to maximize the welfare in the second best.

Assuming $(\rho + \delta)\lambda^{-1} \ll 1$, a fixed-term contract of duration T provides the following investment incentives

$$(12) \qquad \frac{\partial w}{\partial \beta} = \frac{(1 + R_B)}{T\left[1 - (1-\alpha)(1 + R_B)\right] + (1-\alpha)\lambda^{-1}}, \quad \frac{\partial w}{\partial \sigma} = \frac{(1 + R_S)}{T\left[1 - \alpha(1 + R_S)\right] + \alpha\lambda^{-1}}.$$

where $R_B = \frac{\partial s}{\partial \beta}/\frac{\partial b}{\partial \beta}$ and $R_S = \frac{\partial b}{\partial \sigma}/\frac{\partial s}{\partial \sigma}$ are the cross-effects of the buyer's and the seller's investment. Suppose that the bargaining power is allocated symmetrically, $\alpha = 1/2$. The first best is implemented if and only if the cross-effects are symmetric and weaker than the self-effects, $R_B = R_S < 1$ (similar to Guriev, 2003). If the cross-effects are not symmetric then the contract duration can be used only to improve the second best. For example, if the seller's cross-effects are stronger, $R_S > R_B$, then the seller's incentives are always weaker than the buyer's. In particular, if a contract induces optimal investment by the buyer, the seller underinvests; if the contract is longer so that the seller invests optimally, then the buyer overinvests. Thus, the parties choose the contract duration to balance inefficiencies maximizing total surplus subject to (12).

Similarly, the incentives under a renegotiation-proof evergreen contract with advance notice time τ are

$$\frac{\partial w}{\partial \beta} = \frac{1 + R_B}{\tau\left[1 - (1-\alpha)(1 + R_B)\right] + \lambda^{-1}}, \quad \frac{\partial w}{\partial \sigma} = \frac{1 + R_S}{\tau\left[1 - \alpha(1 + R_S)\right] + \lambda^{-1}}. \tag{13}$$

Note that the fixed-term contract and the evergreen contract are no longer equivalent. For instance, let $\alpha = 1/2$ and $R_B < R_S < 1$, and suppose that both a fixed-term contract and an evergreen contract provide the buyer with the same incentives to invest. Then the seller has weaker incentives under the fixed-term contract T than under the evergreen contract τ. The ratio $\frac{\partial w}{\partial \sigma}/\frac{\partial w}{\partial \beta}$ under the fixed-term contract (12) is always higher than the one under the evergreen contract (13); since w is concave in β the underinvestment is the more severe, the less $\partial w/\partial \sigma$. This observation suggests that in the general case using both fixed-term and advance notice clauses can improve welfare.

Other extensions. If the cross-effects of investment are negative, $b'(\sigma) < 0$, then the optimal duration of the fixed-term contract is shorter than the expected duration of the relationship. No evergreen contract can implement the first best because such contract would have to stipulate a negative advance notice time τ. If, however, a public randomization device is available then a stochastic evergreen contract with $\tau = 0$ that reads 'provide a service with probability $1 - |R|$' achieves the first best. If the self-effects of investment are negative, $s'(\sigma) < 0$, then contracts on time does not work and are dominated by a null contract.

Similarly, if investment is multi-dimensional then the contracts on time are also inefficient. If investment in cost reduction is separate from investment in buyer's utility, then contracts on time would provide incentives for the former but not for the latter. Even if each investment activity had a non-trivial effect on both cost and utility, the first best could not be implemented due to a multi-tasking problem (one instrument, time, cannot achieve several targets).

V Conclusion

In this paper we show how time clauses–either contract duration or advance notice period for unilateral termination–help to provide incentives for specific investments even if other contractual instruments fail to overcome the hold-up problem. We find that the structure of contracts on time is driven by the duration of relationship, cooperativeness of investment, and renegotiation-proofness of the contract. Our model is the first step in describing the incentive effects of various contracts on time and understanding the rich structure of real-world contracts.

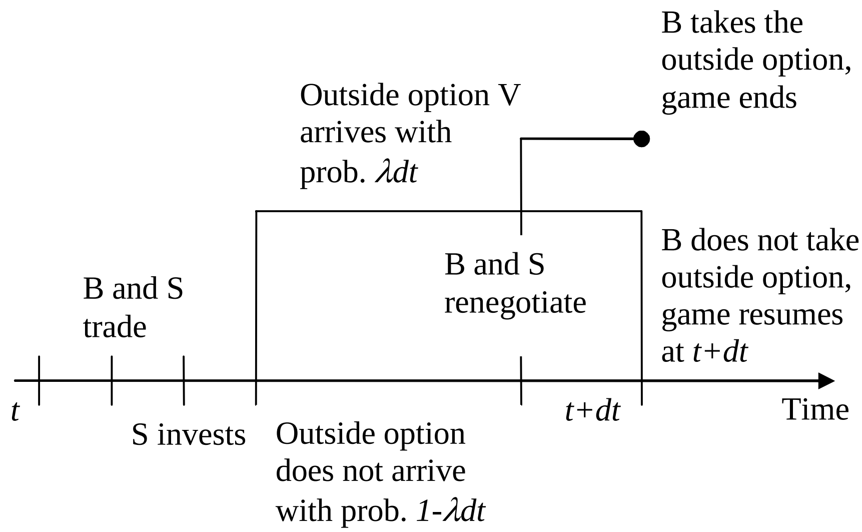

Figure 1: Timing

Industry	**Parties**	**Duration, yrs**
Wholesale supply	RiteAid & McKesson	3
Intermediate goods and materials	RMI Titanium & Osaka Titanium	8
Building and maintenance of sewer systems	Kaiser Ventures & Speedway Development	20

Table 1: Examples of contract duration. Source: Contracting and Organizations Research Institute (CORI) K-Base. http://cori.missouri.edu.

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References

Aghion, Philippe and Bolton, Patrick. "Contracts as a Barrier to Entry." *American Economic Review*, June 1987, 77(3), pp. 388-401.

________; **Dewatripont, Mathias and Rey, Patrick.** "Renegotiation Design with Unverifiable Information." *Econometrica*, March 1994, 62(2), pp. 257-82.

Benoît, Jean-Pierre and Krishna, Vijay. "Renegotiation in Finitely Repeated Games." *Econometrica*, March 1993, 61(2), pp. 303-23.

Che, Yeon-Koo and Hausch, Donald B. "Cooperative Investments and the Value of Contracting." *American Economic Review*, March 1999, 89(1), pp. 125-47.

________ **and Sákovics, József.** "A Dynamic Theory of Hold-up." *Econometrica*, July 2004, 72(4), pp. 1063-103.

Chiappori, Pierre-Andre and Salanie, Bernard. "Testing Contract Theory: A Survey of Some Recent Work," in M. Dewatripont, L. Hansen and S. Turnovsky, eds., *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress, Volume 1*, Cambridge: Cambridge University Press, 2003, pp. 115-49.

Coase, Ronald. "The Acquisition of Fisher Body by General Motors." *Journal of Law and Economics*, April 2000, 43(1), pp. 15-31.

Crawford, Vincent P. "Long-term Relationships Governed by Short-term Contracts." *American Economic Review*, June 1988, 78(3), pp. 485-99.

Crocker, Keith J. and Masten, Scott E. "Mitigating Contractual Hazards: Unilateral Options and Contract Length." *Rand Journal of Economics*, Autumn 1988, 19(3), pp. 327-43.

Demski, Joel S. and Sappington, David E. M. "Resolving Double Moral Hazard Problems with Buyout Agreements." *Rand Journal of Economics*, Spring 1991, 22(2), pp. 232-40.

Edlin, Aaron S. and Reichelstein, Stefan. "Holdups, Standard Breach Remedies and Optimal Investment." *American Economic Review*, June 1996, 86(3), pp. 478-501.

________ **and Hermalin, Benjamin E.** "Contract renegotiation and options in agency problems." Journal of Law Economics and Organization, October 2000, 16(2), pp.

395-423.

Goldberg, Victor P. and Erickson, John R. "Quantity and Price Adjustment in Long-term Contracts: A Case Study of Petroleum Coke." *Journal of Law and Economics*, October 1987, 30(2), pp. 369-98.

Guriev, Sergei. "Incomplete Contracts with Cross-Investments." *Contributions to Theoretical Economics,* 2003, 3 (1), Article 5.

Harris, Milton and Holmstrom, Bengt. "On the Duration of Agreements." *International Economic Review*, June 1987, 28(2), pp. 389-406.

Hart, Oliver. *Firms, Contracts, and Financial Structure.* Oxford: Clarendon Press, 1995.

________ and Moore, John. "Incomplete Contracts and Renegotiation." *Econometrica*, July 1988, 56(4), pp. 755-85.

________ and Tirole, Jean. "Contract Renegotiation and Coasian Dynamics." *Review of Economic Studies*, October 1988, 55(4), pp. 509-40.

Joskow, Paul L. "Contract Duration and Relationship-Specific Investments: Empirical Evidence from Coal Markets." *American Economic Review*, March 1987, 77(1), pp. 168-85.

Llobet, Gerard; Hopenhayn, Hugo and Mitchell, Matthew. "Rewarding Sequential Innovators: Prizes, Patents, and Buyouts." Mimeo*,* CEMFI, 2002.

Lockwood, Ben and Thomas, Jonathan P. "Gradualism and Irreversibility." *Review of Economic Studies*, April 2002, 69(2), pp. 339-56.

MacLeod, W. Bentley and Malcolmson, James M. "Investments, Holdup, and the Form of Market Contracts." *American Economic Review*, September 1993, 83(4), pp. 811-37.

________ and ________. "Contract Bargaining with Symmetric Information." *Canadian Journal of Economics*, May 1995, 28(2), pp. 336-67.

Masten, Scott E. and Crocker, Keith J. "Efficient Adaptation in Long-Term Contracts: Take-or-Pay Provisions for Natural Gas." *American Economic Review*, December 1985, 75(5), pp. 1083-93.

Noldeke, Georg and Schmidt, Klaus M. "Option Contracts and Renegotiation: A Solution to the Hold-up Problem." *Rand Journal of Economics*, Summer 1995, 26(2), pp.

163-79.

Pirrong, Stephen Craig. "Contracting Practices in Bulk Shipping Markets: A Transactions Cost Explanation." *Journal of Law and Economics*, October 1993, 36(2), pp. 937-76.

Pitchford, Rohan and Snyder, Christopher M. "A Solution to the Hold-up Problem Involving Gradual Investment." *Journal of Economic Theory*, January 2003, 114(1), pp. 88-103.

Rey, Patrick and Salanie, Bernard. "Long-term, Short-term, and Renegotiation: On the Value of Commitment in Contracting." *Econometrica*, May 1990, 58(3), pp. 597-619.

Segal, Ilya and Whinston, Michael D. "The Mirrlees Approach to Mechanism Design with Renegotiation (with Applications to Hold-up and Risk-Sharing)." *Econometrica*, January 2002, 70(1), pp. 1-45.

Stuckey, John A. *Vertical Integration and Joint Ventures in the Aluminum Industry.* Cambridge: Harvard University Press, 1983.

William M. Mercer Companies. "2001/2002 Worldwide Benefit and Employment Guidelines." London, 2001.

Notes

1. The distinction between *nominal* and *actual* duration of the relationship was introduced by Philippe Aghion and Patrick Bolton (1987) who provide a rationale for nominal contract duration being *shorter* than the actual one. In their model contract duration is a signal of the seller's type, shorter contracts credibly reveal that the seller is not afraid of ex post competition.

2. Ben Lockwood and Jonathan P. Thomas (2002) consider a related issue in a different context. They study repeated prisoners' dilemma in which players cannot decrease their level of cooperation over time. The level of cooperation, therefore, is similar to specific irreversible investment in incomplete contract models.

3. A discrete time version of the model, available upon request from the authors, yields similar results. The optimal contracts are second best and converge to the first best in the limit, as a period length tends to zero.

4. Pierre-Andre Chiappori and Bernard Salanie (2003) survey other empirical papers that use related methodologies and produce similar results.