HUNTING FOR THE DISCOURAGEMENT EFFECT IN CONTESTS

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Hunting for the discouragement effect in contests

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Abstract

The "discouragement effect" (DE) is mentioned routinely as a reason for why heterogeneity is detrimental for incentives in contests. It serves as a theoretical argument for various policies aimed at homogenizing contestants. We show that, at least in static contests, the DE has no robust theoretical foundation. We divide widely used contest models into two classes. In the first class, heterogeneity either decreases or increases aggregate effort. In the second class, the effect of heterogeneity depends crucially on how it is defined. Hence, the DE cannot serve as a go-to argument for why heterogeneity in contests is undesirable.

Keywords: discouragement effect, contest, heterogeneity
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1 Introduction

The contest literature routinely discusses the so-called "discouragement effect." The story, for two-player static contests, usually goes as follows: If one player—the underdog—is weaker than the other—the favorite—the underdog's probability of winning, and hence her marginal benefit of effort, is reduced, which means her effort is reduced in equilibrium. In response, the favorite's effort is also reduced, and the overall effort in the contest goes down. This theoretical argument is then used as a basis for a broad claim that heterogeneity is detrimental for aggregate output in contests, which in turn serves as a justification for various policies aimed at "leveling the playing field" or reducing heterogeneity in environments with competitive incentives.

Yet, as an example, consider a Lazear and Rosen (1981) type tournament model with two risk-neutral players $i \in \{1, 2\}$ in which player i's output (y_i) is her effort (e_i) distorted by a zero-mean additive shock (X_i): $y_i = e_i + X_i$. Player i's cost of effort is $\frac{1}{2}e_i^2$. The player with the highest output wins and receives a unit prize, while the other player receives zero. For simplicity assume, similar to, e.g., Meyer (1991), Konrad (2009) and Brown and Minor (2014), that $X_1 - X_2$ is uniform on $[-\frac{1}{2}, \frac{1}{2}]$. Type $t_i \geq 1$ favors player i by multiplying her effort (at no cost) and proportionally decreasing the effort of the other player so that the probability of player 1 winning is $p(e_1, e_2; t_1, t_2) = \frac{1}{2} + \frac{t_1}{t_2}e_1 - \frac{t_2}{t_1}e_2$, with the variables restricted to ensure $p \in [0, 1]$. Assuming an interior equilibrium (e_1^*, e_2^*), the aggregate equilibrium effort, $e_1^* + e_2^* = \frac{t_1}{t_2} + \frac{t_2}{t_1}$, *increases* when the players become more heterogeneous.

In this paper, we systematically investigate the impact of players' heterogeneity on aggregate effort in a general static contest setting. We propose a simple taxonomy of popular contest models, dividing them into two classes we call *relative types* and *absolute types*. The example above is a model with relative types, in which the players' types matter only relative to each other. In other words, the direction of the effect of each type on aggregate effort depends crucially on its position relative to the other type. For such models we show that, under mild assumptions, an increase in heterogeneity, defined in an intuitive way, has a monotone effect on aggregate effort. Models with relative types are, therefore, suitable to study the impact of heterogeneity. While the effect is negative in some popular models, there is no fundamental reason for that. Indeed, as the example above shows, it may well be positive.

In models with absolute types, instead, aggregate effort is monotone in both players' types. This holds, for instance, when type is the player's prize valuation or the marginal cost of effort. We show that in this case aggregate effort can always be written as a function of some (generalized) mean of the types. Then, for example, increasing heterogeneity while keeping the arithmetic mean of the types constant may either increase or decrease aggregate effort, depending on how the arithmetic mean compares to the generalized mean embedded in the aggregate effort function. In other words, the existence of discouragement or encouragement effects hinges on the way heterogeneity is introduced. When the generalized mean belongs to the power mean family, whose special cases include the arithmetic, geometric and harmonic means, both discouragement and encouragement effects can always be generated by an appropriate version of higher heterogeneity.

The main message of this paper is that the prevailing common wisdom about the (static) discouragement effect does not have a strong theoretical foundation. Among frequently used contest models, there is only one (up to an isomorphism) that follows the discouragement story to the letter. Other models either produce the opposite effect—encouragement—or can have either effect depending on how an increase in heterogeneity is introduced. Generically, even in models with relative types most prone to the discouragement effect, the mechanism behind it is different from the standard story.

Relation to the existing literature This paper has far-reaching implications for the vast literature on static contests and their applications as incentive provision mechanisms (for a recent review see, e.g., Vojnović, 2016; Fu and Wu, 2019). To highlight the pervasiveness of the "common wisdom" of the discouragement effect, consider the following quotes from recent comprehensive reviews of, respectively, the experimental and theoretical literature on contests.

> The theoretical literature on contests has recognized that greater heterogeneity between players (appropriately normalized depending on the contest) leads to lower aggregate effort (Baye, Kovenock and De Vries, 1993, 1996; Gradstein, 1995; Baik, 1994; Stein, 2002). The reason for this is the so called "discouragement effect." Although the technical details underlying the discouragement effect differ from model to model, they basically arise because a weaker player, either with higher unit costs of effort or a lower value of winning, finds it relatively unprofitable to try to beat the stronger player and, consequently, cuts back on his costly expenditure. This, in turn, may allow the stronger player to bid more passively as well when compared to a contest in which he faces a player of similar strength.
>
> (Dechenaux, Kovenock and Sheremeta (2015))

> It is well known in the literature that the performance of a contest crucially depends on the competitive balance among contenders, and a more level playing field tends to fuel competition. (Fu and Wu (2019))

Similar ideas are expressed in many studies and surveys (e.g., Hillman and Riley, 1989; Schotter and Weigelt, 1992; Nitzan, 1994; Rapoport and Amaldoss, 2000; Szymanski, 2003; Runkel, 2006; Hart et al., 2015; Imhof and Kräkel, 2016). While some of the more recent surveys are more nuanced and present theoretical arguments and empirical evidence pointing in the other direction, those are still interpreted mainly as aberrations (Mealem and Nitzan, 2016; Chowdhury, Esteve-Gonzalez and Mukherjee, 2020).

There is a relatively small but growing literature questioning the universality of these claims. Ryvkin (2013) shows that in a widely used contest model small heterogeneity can lead to an increase in aggregate effort. Drugov and Ryvkin (2017) study biased contests and find a variety of contest settings where *ex ante* or even *ex post* symmetric players subjected to an arbitrary asymmetry in the winner determination process produce more in equilibrium. Bastani, Giebe and Gürtler (2020) develop a general approach to modeling contests with symmetric equilibria and show that aggregate effort can increase with heterogeneity.

Empirical studies of the discouragement effect focus mainly on a "superstar effect" the presence of strong athletes may have on peers' performance, producing mixed results. A negative superstar effect was found, for example, by Brown (2011) in golf and Bilen and Matros (2020) in chess; while a positive effect was identified by Hill (2014*b*) in sprint, Hill (2014*a*) in track and field, and Yamane and Hayashi (2015) in swimming.

The rest of the paper is organized as follows. Section 2 sets up a generic contest model and introduces the classification of relative and absolute types. Examples of models with relative and absolute types are collected in Tables 1 and 2, respectively, and referenced throughout the text. Section 3 provides results for the models with relative types while Section 4 does so for absolute types. Section 5 concludes. Proofs are collected in the Appendix.

2 The model

2.1 Preliminaries

For clarity, we consider two-player, complete information contests admitting a pure strategy equilibrium characterized by first-order conditions.[1]

[1]This covers the vast majority of models used in the contest literature, including the variants of Tullock (1980) and Lazear and Rosen (1981) models. Alternative models of static contests are all-pay auctions of complete (Baye, Kovenock and De Vries, 1993; Siegel, 2009) and incomplete (Hillman and Riley, 1989; Moldovanu and Sela, 2001) information, as well as imperfectly discriminating contests of incomplete information (Fey, 2008; Ryvkin, 2010; Wasser, 2013; Ewerhart, 2014).

The game Two players indexed by $i = 1, 2$ simultaneously and independently choose efforts $e_i \in \mathbb{R}_+$. The type of player i is $t_i \in \mathbb{R}_{++}$. Player 1's utility is given by function $f(e_1, e_2; t_1, t_2)$ such that that $f_2 < 0$, $f_{t_1} > 0$, and $f_{t_2} \leq 0$.[2] The game is symmetric; that is, the utility of player 2 is $f(e_2, e_1; t_2, t_1)$.

The most common form of function f in the contest literature is $f(e_1, e_2; t_1, t_2) = v(t_1)p(e_1, e_2; t_1, t_2) - c(e_1; t_1)$, where $v(t_i)$ is player i's prize valuation, $p(e_1, e_2; t_1, t_2)$ is a *contest success function* (CSF) that defines the probability of player 1 winning given the players' efforts and types, and $c(e_i; t_i)$ is player i's cost of effort. Our symmetry assumption is then equivalent to the contest being *unbiased*, in the sense defined by Drugov and Ryvkin (2017), or *anonymous* (Skaperdas, 1996), or *perfectly symmetric* (Dixit, 1987). While these prior definitions pertain to the CSF p, the model here is more general and does not require separability or even risk-neutrality.

Equilibrium Suppose f is sufficiently differentiable and consider an equilibrium characterized by the system of first-order conditions,

$$f_1(e_1, e_2; t_1, t_2) = 0, \quad f_1(e_2, e_1; t_2, t_1) = 0. \tag{1}$$

We will use $\tilde{f}$ to denote function f and its derivatives evaluated at $(e_2, e_1; t_2, t_1)$ where the players' identities have been swapped. We assume there exist open intervals $Y, T \subseteq \mathbb{R}_{++}$ such that the following holds.

Assumption 1 *For all* $(e_1, e_2; t_1, t_2) \in Y^2 \times T^2$, $f_{11}\tilde{f}_{11} - f_{12}\tilde{f}_{12} \neq 0$.

Equations (1) then define well-behaved implicit functions $e_i^* : T^2 \to Y$,[3] which we assume constitute the equilibrium for any $(t_1, t_2) \in T^2$. From this point on, we restrict attention to the values of (t_1, t_2) in this domain. We will use $E^*(t_1, t_2) = e_1^*(t_1, t_2) + e_2^*(t_1, t_2)$ to denote aggregate equilibrium effort.

Symmetric equilibrium Of special interest for our purposes is the symmetric equilibrium $(\bar{e}(t), \bar{e}(t))$ obtained for $t_1 = t_2 = t \in T$ and characterized by the equation

$$f_1(\bar{e}, \bar{e}; t, t) = 0. \tag{2}$$

[2] We use subscript i to denote partial derivatives of f with respect to e_i, and subscript t_i for partial derivatives with respect to t_i.

[3] From the implicit function theorem, if f_1 is a C^1 function and Assumption 1 holds, the implicit functions $e_i^*(t_1, t_2)$ are C^1.

#	$f(e_1, e_2; t_1, t_2)$	$e_1^*(t_1, t_2)$	$E^*(t_1, t_2)$	$\tau(t_1, t_2)$
1	$\frac{1}{2} + \frac{t_1}{t_2} e_1 - \frac{t_2}{t_1} e_2 - \frac{1}{2} e_1^2$	τ	$\tau + \frac{1}{\tau}$	t_1/t_2
2	$\frac{t_1 e_1}{t_1 e_1 + t_2 e_2} - e_1$	$\frac{\tau}{(1+\tau)^2}^\sharp$	$\frac{2\tau}{(1+\tau)^2}$	t_1/t_2
3	$G(e_1 + t_1 - e_2 - t_2) - c(e_1)$	$c'^{-1}(g(\tau))^\sharp$	$2c'^{-1}(g(\tau))$	$t_1 - t_2$
4	$\frac{1}{2} + (1 + t_1 - t_2)e_1 - (1 + t_2 - t_1)e_2 - \frac{1}{3} e_1^3$	$\sqrt{1+\tau}$	$\sqrt{1+\tau} + \sqrt{1-\tau}$	$t_1 - t_2$

Table 1: Examples of contest models with relative types. In examples marked with $\sharp$, $e_1^*(t_1, t_2) = e_2^*(t_1, t_2)$. In Example 3, $G(\cdot)$ is an absolutely continuous cdf, $g(\cdot)$ is the corresponding pdf, and $c(\cdot)$ is a strictly convex cost function. Example 1 is from the Introduction.

The resulting symmetric equilibrium effort $e_1^*(t, t) = e_2^*(t, t) \equiv \bar{e}(t)$ exists and is C^1 in t provided $f_{11} + f_{12} \neq 0$, with

$$\bar{e}'(t) = -\frac{f_{1t_1} + f_{1t_2}}{f_{11} + f_{12}}, \tag{3}$$

where all derivatives are evaluated at $(\bar{e}, \bar{e}; t, t)$. Aggregate effort in the symmetric equilibrium is $E^*(t, t) = 2\bar{e}(t)$. The following regularity condition ensures that individual (and aggregate) effort in the symmetric equilibrium is either constant or monotone in the type.[4]

Assumption 2 (Diagonal monotonicity) *For all $(e_1, e_2, t_1, t_2) = (\bar{e}, \bar{e}, t, t)$ such that $t \in T$ and (2) holds, $f_{1t_1} + f_{1t_2}$ does not change sign.*

2.2 Relative and absolute types

As we show below, the effect of heterogeneity on aggregate equilibrium effort depends critically on how the types (t_1, t_2) enter function f. For illustration, consider two popular models of contests of heterogeneous players used in the literature: Example 2 in Table 1 and Example 5 in Table 2. Both models are variations of the Tullock contest with linear costs, and only differ by how the types are introduced.

These two models motivate the following definition.

Definition 1 (Relative and absolute types) *The contest (or function f) has relative types if $\bar{e}(t)$ is independent of t, $\bar{e}'(t) = 0$ for all $t \in T$, and absolute types otherwise.*

[4] In Section 4, we will impose a stronger condition to have aggregate effort $E^*(t_1, t_2)$ globally monotone in both types.

#	$f(e_1,e_2;t_1,t_2)$	$e_1^*(t_1,t_2)$	$E^*(t_1,t_2)$	$m(t_1,t_2)$
5	$\frac{e_1}{e_1+e_2}-\frac{e_1}{t_1}$	$\frac{t_1^2 t_2}{(t_1+t_2)^2}$	$\frac{1}{2}m$	$\frac{2}{1/t_1+1/t_2}$
6	$\frac{(1+t_1)e_1+(1-t_2)e_2}{2(e_1+e_2)}-e_1$	$\frac{1}{8}(t_1+t_2)^\sharp$	$\frac{1}{2}m$	$\frac{1}{2}(t_1+t_2)$
7	$\frac{e_1+t_1}{e_1+t_1+e_2+t_2}-e_1$	$\frac{1}{4}-t_1$	$\frac{1}{2}-2m$	$\frac{1}{2}(t_1+t_2)$

Table 2: Examples of contest models with absolute types. In examples with $\sharp$, $e_1^*(t_1,t_2)=e_2^*(t_1,t_2)$. Example 5 is equivalent to having heterogeneous prize valuations $v(t_i)=t_i$.

Models with relative types are more suitable to study the effects of heterogeneity because the benchmark—aggregate effort in the symmetric equilibrium—is fixed. In contrast, in models with absolute types, individual and aggregate effort in the symmetric equilibrium is either increasing or decreasing in the type.[5] In such models, the impact of heterogeneity on aggregate effort needs to be carefully defined because, as we show in the following section, arbitrary changes in types can generate a combination of absolute and relative effects. We consider relative types in detail in Section 3, and absolute types in Section 4.

2.3 Local effects of heterogeneity

Before turning to our main results, we consider *local* effects of changes in heterogeneity around the symmetric equilibrium point with types (t,t). This analysis helps highlight a fundamental difference between models with relative and absolute types in how they accommodate variation in types.

It is convenient to parameterize type changes around (t,t) by differentiable curves $(t_1(d),t_2(d))\subset T^2$, with d in some interval including zero such that $(t_1(0),t_2(0))=(t,t)$. For example, a situation where player 1's ability is fixed and player 2's ability is changing can be modeled as $t_1(d)=t$, $t_2(d)=t+d$, where $d>(<)0$ corresponds to player 2 becoming stronger (weaker). A *mean-preserving* scenario where both player's abilities are changing with the (arithmetic) average ability staying fixed can be modeled as $t_1(d)=t+d$, $t_2(d)=t-d$. Abusing notation somewhat, define

$$E^*(d)=e_1^*(t_1(d),t_2(d))+e_2^*(t_1(d),t_2(d)), \tag{4}$$

[5] Nonmonotonicity is excluded by Assumption 2. It is easy, however, to construct "hybrid" models where $\bar{e}(t)$ is nonmonotone.

to be aggregate equilibrium effort along such a curve. Of special interest are conditions under which $E^*(d)$ is locally maximized or minimized at the symmetric equilibrium point, i.e., for $d = 0$.

Proposition 1 *(a) Under relative types, $E^*(d)$ has a critical point at $d = 0$ along any curve going through (t, t).*
(b) Under absolute types, $E^*(d)$ has a critical point at $d = 0$ if and only if $t_1'(0) = -t_2'(0)$.

Part (a) of Proposition 1 says that, under relative types, homogeneous types can produce a local maximum or minimum of aggregate effort along any curve going through (t, t). In other words, there is a possibility for *pure* discouragement or encouragement when the introduction of *any* heterogeneity leads to a lower or higher aggregate effort as compared to the symmetric case. Example 2 has pure discouragement, and Example 1 is that of pure encouragement.

In contrast, part (b) of Proposition 1 implies that, for absolute types, heterogeneity-driven discouragement or encouragement are possible only for a very restricted set of type changes. Locally, when the condition $t_1'(0) = -t_2'(0)$ holds, the types change in opposite directions in an arithmetic mean-preserving way: One player becomes weaker, and the other player becomes stronger by the same amount, to the first order in d. Only such type changes can be meaningfully characterized as (isolated) changes in heterogeneity for absolute types. Indeed, any type change that does not satisfy this condition brings about a homogeneous type shift which changes aggregate effort but does not alter heterogeneity.

For illustration of the latter point, consider an arbitrary type change $(t, t) \to (\hat{t}_1, \hat{t}_2)$. Defining $\hat{t} = \frac{1}{2}(\hat{t}_1 + \hat{t}_2)$, we can decompose it into two steps: (i) a homogeneous type shift $(t, t) \to (\hat{t}, \hat{t})$ that, under absolute types, leads to a change in aggregate effort that is not related to heterogeneity; and (ii) a mean-preserving change $(\hat{t}, \hat{t}) \to (\hat{t}_1, \hat{t}_2)$ around the new symmetric point $(\hat{t}, \hat{t})$. Locally, when the condition in Proposition 1(b) is satisfied, the first effect is zero and hence the type change represents a clean increase in heterogeneity. Otherwise, the impact of heterogeneity is confounded.

3 Relative types

In this Section we consider models with relative types, see Definition 1. We start by defining what it means for one type profile to be more heterogeneous than another.

Definition 2 (Surely more heterogeneous) *Profile $(\hat{t}_1, \hat{t}_2)$ is surely more heterogeneous than profile (t_1, t_2) if $\max\{\hat{t}_1, \hat{t}_2\} \geq \max\{t_1, t_2\}$ and $\min\{\hat{t}_1, \hat{t}_2\} \leq \min\{t_1, t_2\}$, with at least one inequality strict.*

In words, Definition 2 says that heterogeneity is for sure increased when a stronger type becomes stronger while a weaker one becomes weaker. This is a partial order, in the sense that some type profiles are not comparable. The advantage of this definition, apart from simplicity, is its model-independence.[6]

We say that f has a *relative types structure* if $f_1(e_1, e_2; t_1, t_2) = r(e_1, e_2; \tau(t_1, t_2))$, where $r_\tau > 0$, and τ is some function of the types such that $\tau_{t_1} > 0$ and $\tau_{t_2} < 0$. Definition 1 then implies that $\bar{\tau} = \tau(t, t)$ is independent of t. While this structure is not necessary for relative types, it is very common and applies to all examples of contests with relative types used in the literature, most of which are listed in Table 1.

Under this structure, aggregate effort is a function of τ; that is, it has the form $E^*(t_1, t_2) = \phi(\tau(t_1, t_2))$. The following proposition then provides simple sufficient conditions for heterogeneity to have a definite effect.

Proposition 2 *Suppose f has a relative types structure and $E^*(t_1, t_2) = \phi(\tau(t_1, t_2))$, where $\phi(\cdot)$ is unimodal (U-shaped) and maximized (minimized) at $\tau = \bar{\tau}$. Then aggregate effort decreases (increases) as types become surely more heterogeneous.*

In Example 2, $\tau(t_1, t_2) = \frac{t_1}{t_2}$, and $\phi(\tau) = \frac{2\tau}{(1+\tau)^2}$ is unimodal and maximized at $\bar{\tau} = \tau(t, t) = 1$, producing a discouragement effect. Namely, any sure increase in heterogeneity leads to a reduction in aggregate effort. In Example 3 based on the seminal Lazear and Rosen (1981) paper, $\tau(t_1, t_2) = t_1 - t_2$ and $\phi(\tau) = 2c'^{-1}(g(\tau))$. If $g(\cdot)$ is unimodal,[7] aggregate effort has a global maximum at $\bar{\tau} = 0$, also producing a discouragement effect.

Finally, in Example 1, $\tau(t_1, t_2) = \frac{t_1}{t_2}$, and $\phi(\tau) = \tau + \frac{1}{\tau}$ is convex and minimized at $\bar{\tau} = 1$, leading to the encouragement effect while in Example 4 $\tau(t_1, t_2) = t_1 - t_2$, and $\phi(\tau) = \sqrt{1+\tau} + \sqrt{1-\tau}$ is concave and maximized at $\bar{\tau} = 0$, leading to the discouragement effect.

Individual efforts As mentioned in the Introduction, the standard explanation for the discouragement effect is that the weaker player decreases her effort and the stronger player follows suit. Yet, in Examples 1 and 4 the individual equilibrium efforts change in opposite directions following any increase in heterogeneity. Moreover, defining $e_i^*(d) = e_i^*(t_1(d), t_2(d))$, Proposition 1(a) implies that $e_1^{*\prime}(0) = -e_2^{*\prime}(0)$ along any curve going though (t, t), i.e., locally this behavior is generic. Then, regular models will have the same behavior globally as well.

[6] For relative types, it is possible to define a complete heterogeneity order based on the function $\tau(t_1, t_2)$ introduced below. That order would be model-dependent.

[7] $g(\cdot)$ is the pdf of the difference of shocks, $X_1 - X_2$. When shocks are i.i.d., a sufficient condition for the unimodality of $g(\cdot)$ is that the pdf of X_i is unimodal (Hodges and Lehmann, 1954).

Examples 2 and 3 are special in that *both* individual efforts decrease with heterogeneity. Note first that the two examples are essentially variations of the same model, since the Tullock contest can be obtained from the Lazear-Rosen tournament when the additive noise follows the Gumbel (or extreme value type-I) distribution (Jia, Skaperdas and Vaidya, 2013; Ryvkin and Drugov, 2020). Additive types in the Lazear-Rosen tournament become multiplicative types in the Tullock CSF. Second, this is a knife-edge case in which the two efforts are always equal to each other, and not just aggregate effort but also individual efforts are maximized at (t,t), and hence $e_1^{*\prime}(0) = e_2^{*\prime}(0) = 0$. We are not aware of any other widely used model with the same property.

Connection to biased contests In models with a relative types structure, one may think about the equivalence of a contest with heterogeneous players and a biased contest with homogeneous players, where $\beta = \tau$ is the bias parameter in the sense introduced by Drugov and Ryvkin (2017). The contest is unbiased when $\beta = \bar{\tau}$. They show that a property of biased contests called *locally symmetric bias* is both necessary and sufficient for a wide class of objective functions (including aggregate effort) to have a critical point at zero bias. When players are homogeneous, the locally symmetric bias condition states that $p_{1\beta}(\bar{e},\bar{e},\bar{\beta}) = p_{2\beta}(\bar{e},\bar{e},\bar{\beta})$, where $p(e_1,e_2,\beta)$ is a biased CSF with a bias parameter β, and the contest is unbiased at $\beta = \bar{\beta}$. Consider the model in the present paper with $f(e_1,e_2;t_1,t_2) = p(e_1,e_2;t_1,t_2) - c(e_1)$, where the contest is unbiased and the CSF is a function of both players' types. This model can be re-interpreted as a biased contest if we set $t_1 = t$ and treat t_2 as a bias, with $t_2 = t$ corresponding to zero bias. Then it can be shown that the locally symmetric bias condition is equivalent to there being relative types.

4 Absolute types

In this section, we consider models with absolute types in which aggregate effort $E^*(t,t)$ is nonconstant in t, cf. Definition 1. While in general the behavior of $E^*(t,t)$ can be arbitrary, we will focus on the most important cases, covered by Assumption 2, where $E^*(t,t)$ is strictly monotone in t. Moreover, it is convenient to impose an even stronger, albeit natural, assumption under which $E^*(t_1,t_2)$ is monotone in both types.[8]

Assumption 3 (Global monotonicity) *For all (e_1,e_2,t_1,t_2) such that $(t_1,t_2) \in T^2$ and (1) holds, $f_{1t_1}(\tilde{f}_{11} - \tilde{f}_{12}) + \tilde{f}_{1t_2}(f_{11} - f_{12})$ does not change sign.*

[8]This assumption is obtained by differentiating the first-order conditions (1) and calculating $E^*_{t_i}$. As expected, at (t,t), a combination of Assumptions 1 and 3 produces Assumption 2.

For concreteness, we perform the analysis for $E^*(t_1, t_2)$ increasing, as in Examples 5 and 6. However, all the results in this section also hold if $E^*(t_1, t_2)$ is decreasing as in Example 7.

In Example 5, aggregate effort $E^*(t_1, t_2) = \frac{1}{1/t_1 + 1/t_2}$ is half of the harmonic mean of the types, while in Example 6 $E^*(t_1, t_2) = \frac{1}{4}(t_1 + t_2)$ is half of their arithmetic mean. As we show in this section, this property holds generally: Aggregate effort can be written as a function of some (generalized) mean of the types.

We will use the following definition of generalized mean.[9]

Definition 3 (m-mean) *Function $m : T^2 \to \mathbb{R}$ is an m-mean if it has the following properties:*
(a) Monotonicity: $m(t_1, t_2)$ is strictly increasing in both arguments.
(b) Symmetry: $m(t_1, t_2) = m(t_2, t_1)$ for all $(t_1, t_2) \in T^2$.
(c) Reflexivity: $m(t, t) = t$ for all $t \in T$.

The following proposition provides a key observation for the results of this section.

Proposition 3 *Under absolute types, there exists a unique m-mean such that aggregate equilibrium effort can be written in the form $E^*(t_1, t_2) = 2\bar{e}(m(t_1, t_2))$.*

In words, Proposition 3 says that for any contest model with absolute types, aggregate equilibrium effort does not change if the two players are homogenized with a type equal to a unique m-mean of the original types. The proof is based on the fact that aggregate effort already satisfies properties (a) and (b) of Definition 3, and will satisfy reflexivity as well after it is appropriately transformed via function $\bar{e}^{-1}(\cdot)$.

Motivated by Proposition 3, we say that a contest model is *characterized* by an m-mean if $E^*(t_1, t_2) = 2\bar{e}^*(m(t_1, t_2))$. The contest model in Example 5 is characterized by the harmonic mean. In other words, if types change in a way that keeps their harmonic mean constant, aggregate effort will be preserved. In Example 6 the model is characterized by the arithmetic mean and hence, changing types keeping their sum constant will not affect aggregate effort. The following definition uses this idea.

Definition 4 (m-heterogeneity) *For some mean m, a type profile $(\hat{t}_1, \hat{t}_2)$ is more m-heterogeneous than (t_1, t_2) if it is surely more heterogeneous and $m(\hat{t}_1, \hat{t}_2) = m(t_1, t_2)$.*

[9] This is the minimum set of properties usually required from a mean (see, e.g., Bullen, 2003). Another standard property is *decomposability*, i.e., that the mean is preserved when a subset of arguments is replaced with their mean value (Kolmogorov, 1930; for a reprint of the original article see Kolmogorov, 1991). For $n = 2$, the decomposability property is not a restriction.

Due to the monotonicity property of the m-mean (see Definition 3), the curve $m(t_1, t_2) =$ const has a negative slope and hence, any two points on it are ranked by sure heterogeneity. Thus, m-heterogeneity simply applies the sure heterogeneity order to those profiles that have the same m-mean.

In what follows we will consider parameterized means, $m(t_1, t_2; \rho)$, increasing in parameter ρ so that $m(t_1, t_2; \rho') \geq m(t_1, t_2; \rho)$ for $\rho' > \rho$ and any $(t_1, t_2) \in T^2$. Arguably the most popular such family is the power mean (also known as Hölder mean),

$$M_\rho(t_1, t_2) = \left(\tfrac{1}{2}t_1^\rho + \tfrac{1}{2}t_2^\rho\right)^{\frac{1}{\rho}}, \quad -\infty < \rho < \infty, \tag{5}$$

of which the harmonic (M_{-1}), geometric (M_0) and arithmetic (M_1) means are special cases. Another example is a family introduced by Beckenbach (1950), $L_\rho = \frac{t_1^\rho + t_2^\rho}{t_1^{\rho-1} + t_2^{\rho-1}}$, with $-\infty < \rho < \infty$, which also includes the harmonic (L_0), geometric ($L_{1/2}$) and arithmetic (L_1) means.[10]

We are now ready to state the main result of this section.

Proposition 4 *Consider a contest model characterized by mean $m(t_1, t_2; \rho_0)$ belonging to a family $m(t_1, t_2; \rho)$. Suppose that this family has the Spence-Mirrlees property: $\frac{m_{t_1}}{m_{t_2}}$ is increasing in ρ for $t_1 \geq t_2$.[11] Then a higher $m(t_1, t_2; \rho)$-heterogeneity leads to a decrease (increase) in aggregate equilibrium effort if $\rho > (<)\rho_0$.*

Proposition 4 shows that increasing heterogeneity in an m-mean-preserving manner can lead to either higher or lower aggregate effort. In other words, there might be either a discouragement effect or an encouragement effect, depending on how this mean m compares to the mean characterizing the model. For example, increasing heterogeneity in the form $(t, t) \to (t + d, t - d)$, i.e., keeping the arithmetic mean constant, reduces aggregate effort in Example 5 since that model is characterized by the harmonic mean, which is smaller than the arithmetic mean. In Example 6, however, which is characterized by the arithmetic mean, increasing heterogeneity in the form $(t, t) \to (\frac{t}{1-d}, \frac{t}{1+d})$, i.e., keeping the harmonic mean constant, increases aggregate effort. This is illustrated in Figure 1.

A simple intuition for why neither the discouragement nor encouragement effects can be assured is the following. Take a "standard" model with absolute types where they either increase the valuation of the prize or decrease the marginal costs, as in Example 5. The direct effect of a higher type is then to increase the player's own effort. When

[10] This mean is sometimes attributed to Lehmer (1971), even though that article appeared much later. It is monotone in t_1 and t_2 only for $\rho \in [0, 1]$.

[11] By symmetry, this is equivalent to $\frac{m_{t_1}}{m_{t_2}}$ decreasing in ρ for $t_1 \leq t_2$.

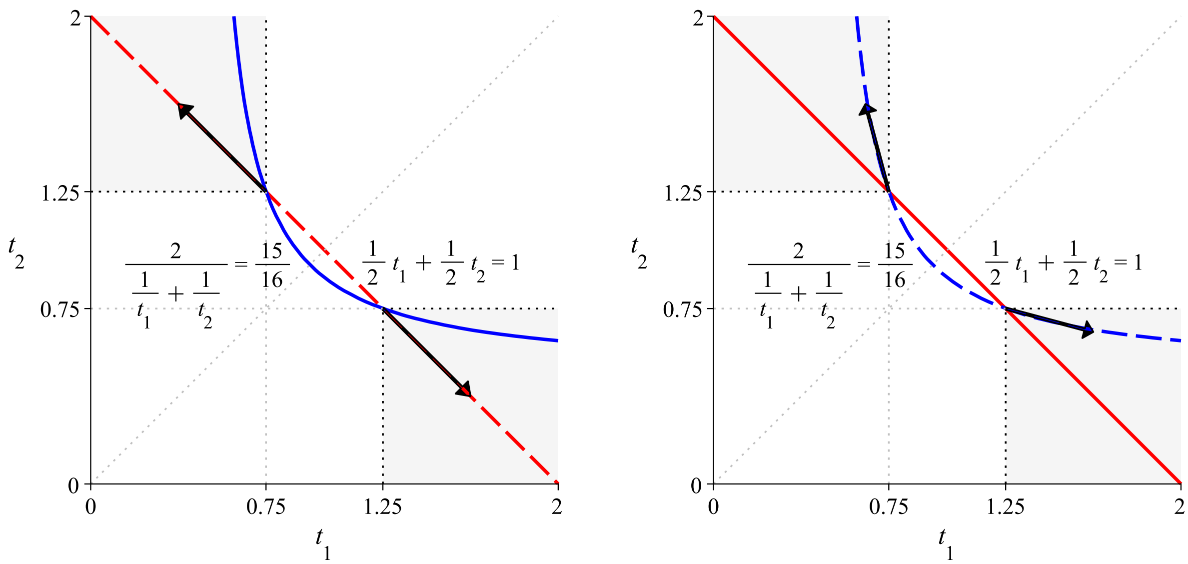

Figure 1: Example 5 characterized by the harmonic mean (left) and Example 6 characterized by the arithmetic mean (right). Shaded areas show higher sure heterogeneity for profiles $(1.25, 0.75)$ and $(0.75, 1.25)$. Arrows show higher arithmetic mean heterogeneity (left) and higher harmonic mean heterogeneity (right). Along the arrows, aggregate equilibrium effort decreases in the former case and increases in the latter one.

moving along some constant m-mean curve, types change in opposite directions. Hence, the direct effect on total effort is ambiguous and depends, in particular, on the slope of this constant m-mean curve, i.e., on how much one type has to increase to compensate for a unit decrease in the other one. Since the size and direction of the indirect (equilibrium) effects are hard to characterize *a priori*, the case for a particular direction of the change in total effort becomes really weak.

The Spence-Mirrlees property in Proposition 4, known from the contract theory (see e.g., Laffont and Martimort, 2002), ensures that any two iso-mean curves in T^2 corresponding to different means intersect at most once, and the distance between them increases from the point of their intersection on each side of the 45-degree line. Then, a higher $m(t_1, t_2; \rho)$-heterogeneity always leads to a larger change in $m(t_1, t_2; \rho_0)$. The power mean family (5) has the Spence-Mirrlees property for any ρ while the Beckenbach family has it for $\rho \leq 1$.

The range of ρ in the power mean family (5) is unbounded both from above and below, leading to the following corollary of Proposition 4.[12]

Corollary 1 *Consider a contest model characterized by an M_{ρ_0}-mean belonging to the power mean family (5). For any ρ_0 there exist ρ' and ρ'' such that a higher $M_{\rho'}$-heterogeneity*

[12]Recall that $\lim_{\rho\to-\infty} M_\rho(t_1, t_2) = \min\{t_1, t_2\}$ and $\lim_{\rho\to\infty} M_\rho(t_1, t_2) = \max\{t_1, t_2\}$.

decreases aggregate equilibrium effort and a higher $M_{\rho''}$-heterogeneity increases aggregate equilibrium effort.

Since essentially all commonly used means belong to the power mean family, Corollary 1 implies that there is always a place for both discouragement and encouragement effects, depending on how heterogeneity is increased.

5 Conclusion

The message of this paper is that the so-called "discouragement effect" in static contests—the broad claim that players' heterogeneity tends to reduce aggregate effort—does not have a robust theoretical foundation. Two remarks are due. First, these results do not imply in any way that policies aimed at restoring competitive balance in the real world should not be pursued. It is possible that the model that does predict a discouragement effect is the one that approximates the reality well. It is also possible that the discouragement effect emerges due to psychological factors or bounded rationality ignored by the standard models. Our results, however, call into question the "neoclassical" argument for leveling the playing field, namely, that the utilitarian efficiency is never harmed by such policies and hence, the efficiency and equity goals are aligned. Yet, there are certainly other powerful arguments for competitive balance, and many other situations where a trade-off between efficiency and another objective is resolved in favor of the other objective.

Second, we have focused on static contests. In dynamics contests, even if the players are symmetric at the outset, discouragement can arise endogenously due to the advantage acquired by the winner of earlier stages. Termed dynamic discouragement effect or "momentum effect," this phenomenon is predicted by a variety of models (for a review see, e.g., Konrad, 2012). For example, in "multi-battle contests" where the winner is the player who wins a certain number of stage contests first (e.g., Konrad and Kovenock, 2009; Ryvkin, 2011), the leader values another win more than the follower because it brings the leader even closer to winning the entire contest, whereas for the follower another win just means they are tied at best and have to compete again.[13] The stage game is, therefore, similar to a static model with absolute types equal to the players' prize valuations, such as Example 5. The effect of heterogeneity in such models depends on the details of the model and on how heterogeneity is defined. More generally, our results suggest that the

[13]When the stage game is an all-pay auction without intermediate prizes, this leads to players completely giving up as soon as they are trailing (Konrad and Kovenock, 2009). It is possible, however, to generate a "last stand" type behavior by the trailing player if there is a cost of losing the contest and future payoffs are discounted (Gelder, 2014).

dynamic discouragement effect is also not robust and depends on exactly how the stage contest and the dynamics are modeled.

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A Proofs

Proof of Proposition 1

Part (a) Differentiating (4), we obtain

$$\begin{aligned} E^{*\prime}(d) &= e^*_{1t_1}t_1'(d) + e^*_{1t_2}t_2'(d) + e^*_{2t_1}t_1'(d) + e^*_{2t_2}t_2'(d) \\ &= (e^*_{1t_1} + e^*_{2t_1})t_1'(d) + (e^*_{1t_2} + e^*_{2t_2})t_2'(d). \end{aligned} \tag{6}$$

By symmetry, $e_1^*(t_1, t_2) = e_2^*(t_2, t_1)$; therefore, at $d = 0$ we have $e^*_{1t_1} = e^*_{2t_2}$ and $e^*_{1t_2} = e^*_{2t_1}$, producing

$$E^{*\prime}(0) = (e^*_{1t_1} + e^*_{1t_2})(t_1'(0) + t_2'(0)) = \bar{e}'(t)(t_1'(0) + t_2'(0)). \tag{7}$$

The second equality follows by differentiating $\bar{e}(t) = e_1^*(t,t)$. Under relative types $\bar{e}'(t) = 0$.

Part (b) Consider Eq. (7). Under absolute types $\bar{e}'(t) \neq 0$ and hence, $E^{*\prime}(0) = 0 \Leftrightarrow t_1'(0) + t_2'(0) = 0$. ■

Proof of Proposition 3

Recall that $E^*(t_1, t_2) = e_1^*(t_1, t_2) + e_2^*(t_1, t_2)$. Due to symmetry, we have $e_1^*(t_1, t_2) = e_2^*(t_2, t_1)$; therefore, $E^*(t_1, t_2)$ is symmetric. Function $\bar{e}(t) = \frac{1}{2}E^*(t,t)$ is strictly monotone and hence has a unique inverse $\bar{e}^{-1}(\cdot)$. Then it is easy to see that $m(t_1, t_2) = \bar{e}^{-1}(\frac{1}{2}E^*(t_1, t_2))$ is a generalized mean. Indeed, $m(t,t) = \bar{e}^{-1}(\frac{1}{2}E^*(t,t)) = t$, and all other properties are satisfied. Thus, there exists a generalized mean $m(t_1, t_2)$ such that $E^*(t_1, t_2) = 2\bar{e}(m(t_1, t_2))$. Any other mean $\tilde{m}(t_1, t_2)$ satisfying the same has to coincide with $m(t_1, t_2)$ because $\bar{e}$ is a strictly increasing function. ■

Proof of Proposition 4

Suppose $E^*(t_1, t_2) = 2\bar{e}(m(t_1, t_2; \rho_0))$, where $m(t_1, t_2; \rho)$ satisfies the Spence-Mirrlees property. Let $\rho > \rho_0$ and consider two type profiles (θ_1, θ_2) and $(\hat{\theta}_1, \hat{\theta}_2)$ such that $(\hat{\theta}_1, \hat{\theta}_2)$ is surely more heterogeneous and $m(\theta_1, \theta_2; \rho) = m(\hat{\theta}_1, \hat{\theta}_2; \rho)$. From the symmetry and strict monotonicity of m, we can assume without loss that $\hat{\theta}_1 > \theta_1 \geq \theta_2 > \hat{\theta}_2$.

Let $t_2 = \chi_0(t_1)$ and $t_2 = \chi(t_1)$ be the implicit functions defined by the iso-mean curves $m(t_1, t_2; \rho_0) = m(\theta_1, \theta_2; \rho_0)$ and $m(t_1, t_2; \rho) = m(\theta_1, \theta_2; \rho)$, respectively. By construction, the two curves intersect at (θ_1, θ_2). Moreover, $\chi_0'(t_1) = -\frac{m_{t_1}(t_1,t_2;\rho_0)}{m_{t_2}(t_1,t_2;\rho_0)}$ and $\chi_1'(t_1) = -\frac{m_{t_1}(t_1,t_2;\rho)}{m_{t_2}(t_1,t_2;\rho)}$. The Spence-Mirrlees property then implies $\chi_1'(t_1) < \chi_0'(t_1)$ for $t_1 > \theta_1$ and hence $\chi_1(\hat{\theta}_1) < \chi_0(\hat{\theta}_1)$. This implies, by the monotonicity of m,

$$m(\hat{\theta}_1, \hat{\theta}_2; \rho_0) = m(\hat{\theta}_1, \chi_1(\hat{\theta}_1); \rho_0) < m(\hat{\theta}_1, \chi_0(\hat{\theta}_1); \rho_0) = m(\theta_1, \theta_2; \rho_0),$$

which gives $E^*(\hat{\theta}_1, \hat{\theta}_2) < E^*(\theta_1, \theta_2)$. ■