Centre for Economic and Financial Research at New Economic School

January 2019

Reconsideration of a simple approach to quantile regression for panel data: a comment on the Canay (2011) fixed effects estimator

Galina Besstremyannaya
Sergei Golovan

Working Paper No 249

CEFIR /NES Working Paper series

Reconsideration of a simple approach to quantile regression for panel data: a comment on the Canay (2011) fixed effects estimator

Galina Besstremyannaya* Sergei Golovan†

January 29, 2019

Abstract

Estimation of individual effects in quantile regression can be difficult in large panel datasets, but a solution is apparently offered by a computationally simple estimator by Ivan Canay (2011, *The Econometrics Journal*) for quantile-independent individual effects. The Canay estimator is widely used by practitioners and is often cited in the theoretical literature. However, our paper discusses two fallacies in Canay's approach. We formally prove that Canay's assumptions can entail severe bias or even non-existence of the limiting distribution for the estimator of the vector of coefficients, leading to incorrect inference. A second problem is incorrect asymptotic standard error of the estimator of the constant term. In an attempt to improve Canay's estimator, we propose a simple correction which may reduce the bias. Regarding the constant term, we focus on the fact that finding a $\sqrt{nT}$ consistent first step estimator may be problematic. Finally, we give recommendations to practitioners in terms of different values of n/T, and conduct a meta-review of applied papers, which use Canay's estimator.

Keywords: Quantile regression, Panel data, Fixed effects, Inference.

*Centre for Economic and Financial Research at New Economic School, Moscow, Nakhimovsky pr.47, gbesstre@cefir.ru

†New Economic School, Moscow, Skolkovskoe shosse, 45, sgolovan@nes.ru

1 Introduction

Use of panel data quantile regression models dates back to Koenker (2004), who considers the equation

$$Q_{y_{it}}(\tau \mid x_{ij}) = \alpha_i + x'_{it}\beta(\tau), \quad t = 1, \ldots, T_i, \quad i = 1, \ldots, n,$$

where $Q_{y_{it}}(\tau \mid x_{ij})$ denotes the value of a given quantile for conditional distribution of the continuous dependent variable y for observation i at period t. The equation specifies the individual effects α_i as n additional unknown parameters, but their estimation is difficult since n can be very large in panel datasets.

A solution is apparently offered by a computationally simple estimator by Ivan Canay (2011, *The Econometrics Journal*) for quantile-independent individual effects. Canay (2011) proposes a two-step procedure, which first gives a consistent estimation of individual effects using the within estimator and then applies the pooled version of the panel data quantile regression to the dependent variable cleared of the estimated individual effects. The Canay estimator is widely used by practitioners and is often cited in the theoretical literature. According to the Wiley online library, there are 120 citations in Web of Science journals (as of December 31, 2018), while Google Scholar gives 389 citations. The empirical applications include papers in *The Journal of the European Economic Association*, *The Economic Journal* and *Empirical Economics*, while theoretical references appear in *Econometrica* and *The Journal of Econometrics*.

However, as we show in this note, Canay's approach causes two types of incorrect inference. Firstly, the statistical tests based on the asymptotic distribution of the estimator of the coefficients may be wrong. Indeed, the main result in Canay (2011) claims the existence of a limiting distribution for the estimators of the model coefficients under the requirement of $n/T^s \to 0$ for some $s > 1$, which admits panels for which n grows faster than T (so-called wide panels). Yet, we demonstrate that the limiting distribution does not exist for wide panels. Secondly, the inference based on the asymptotic distribution of the estimator of the constant term is incorrect for another reason, owing to violation of the required assumption of additive expansion of the first step estimator into a sum of independent terms. We prove that the terms are mutually dependent for different time periods and, as a result, the derivation of the asymptotics of the second step estimator of the constant term fails. Our simulations demonstrate these two issues and their consequences for the inference.

In an attempt to improve the estimator, we propose a simple correction which may sometimes reduce the bias, although it does not alter the asymptotic behavior. As for the standard errors of the intercept estimator, we consider a simple example of a panel data model with individual effects for which we formally prove the impossibility of constructing a $\sqrt{nT}$ consistent estimator of the error term. This shows the imperfection in Canay's approach which relies on such an estimator to prove the asymptotic properties of the two-step estimator.

The remainder of this note is structured as follows. Section 2 provides a theorectical critique of the estimator, and Section 3 gives the results of simulations. The approaches to improve the estimator are discussed in Section 4. Section 5 suggests recommendations to practitioners and outlines key data issues in literature that applies the estimator. An Appendix gives a meta-review of 81 papers using the estimator, which have appeared in journals indexed by the Web of Science.

2 Theoretical critique

The approach proposed in Canay's article uses a two-step estimator for the following model

$$Y_{it} = X_{it}'\theta(U_{it}) + \alpha_i, \quad i = 1, \dots, n, \; t = 1, \dots, T,$$

where U_{it} is uniformly distributed on $(0, 1)$ and does not depend on (X_{it}, α_i). Here X_{it} includes the constant term, and the identification condition $E[\alpha_i] = 0$ is assumed.

At the first stage, a $\sqrt{nT}$ consistent estimator $\hat{\theta}_\mu$ of $\theta_\mu = E[\theta(U_{it})]$ is used to compute

$$\hat{\alpha}_i \equiv \frac{1}{T}\sum_{t=1}^{T}(Y_{it} - X_{it}'\hat{\theta}_\mu)$$

The second stage defines $\hat{Y}_{it} \equiv Y_{it} - \hat{\alpha}_i$ and the estimator $\hat{\theta}(\tau)$ as

$$\hat{\theta}(\tau) = \underset{\theta}{\operatorname{argmin}} \frac{1}{nT}\sum_{i=1}^{n}\sum_{t=1}^{T}\rho_\tau(\hat{Y}_{it} - X_{it}'\theta)v_{it} \tag{2.1}$$

where $\rho_\tau(u) = uI(\tau - I(u < 0))$ and v_{it} are positive weights, commonly set to one in estimations.

The asymptotic properties of the two-step estimator are derived using the key assumption described below, the most important part of which is an additive expansion of $\hat{\theta}_\mu$ with the independence of its terms ψ_{it}.

Assumption 4.2, Canay (2011) *The first-step estimator $\hat{\theta}_\mu$ admits the expansion*

$$\sqrt{nT}(\hat{\theta}_\mu - \theta_\mu) = \frac{1}{\sqrt{nT}} \sum_{t=1}^{T} \sum_{i=1}^{n} \psi_{it} + o_p(1), \tag{2.2}$$

where ψ_{it} is an i.i.d. sequence of random variables with $E[\psi_{it}] = 0$ and finite $\Omega_{\psi\psi} = E[\psi_{it}\psi_{it}']$.

Assumption 4.2 is then used for the derivation of the asymptotic normality of the second step estimator.[1] Note that the assumption is roughly equivalent to a $\sqrt{nT}$ consistency of the first step estimator, where $\sqrt{nT}(\hat{\theta}_\mu - \theta_\mu)$ converges to a finite distribution.

Theorem 4.1, Canay (2011) *Let $n/T^s \to 0$ for some $s \in (1, +\infty)$. Under Assumptions 3.2, 4.1 and 4.2*

$$\sup_{\tau \in \mathcal{T}} \|\hat{\theta}(\tau) - \theta(\tau)\| \xrightarrow{p} 0,$$

and

$$\sqrt{nT}(\hat{\theta}(\cdot) - \theta(\cdot)) = [-J_1(\cdot)]^{-1} \frac{1}{\sqrt{nT}} \sum_{i=1}^{n} \sum_{t=1}^{T} \{\phi_\tau(\varepsilon_{it}(\tau))X_{it} + J_2(\cdot)\xi_{it}\} + o_p(1) \tag{2.3}$$

$$\Rightarrow \mathbb{G}(\cdot) \quad in\ \ell^\infty(\mathcal{T}), \tag{2.4}$$

where $\epsilon_{it}(\tau) \equiv Y_{it}^* - X_{it}'\theta(\tau)$, $Y_{it}^* = Y_{it} - \alpha_i$, $\xi_{it} \equiv \mu_X'\psi_{it} - u_{it}$, $u_{it} \equiv Y_{it}^* - X_{it}'\theta_\mu$, $\mu_X = E[X_{it}]$, $J_1(\tau) \equiv J_1(\theta(\tau), \tau, 0)$, $J_2(\tau) \equiv J_2(\theta(\tau), \tau, 0)$, $\mathbb{G}(\cdot)$ is a mean zero Gaussian process with the covariance function $E[\mathbb{G}(\tau)\mathbb{G}(\tau')'] = J_1(\tau)^{-1}\Psi(\tau, \tau')[J_1(\tau')^{-1}]'$, $\Psi(\tau, \tau')$ is defined in the equation below, and $\ell^\infty(\mathcal{T})$ is the set of uniformly bounded functions on $\mathcal{T}$. The matrix $\Psi(\tau, \tau')$ is given by

$$\Psi(\tau, \tau') = S(\tau, \tau') + J_2(\tau)\Omega_{\xi g}(\tau') + \Omega_{g\xi}(\tau)J_2(\tau')' + J_2(\tau)\Omega_{\xi\xi}J_2(\tau')',$$

where $S(\tau, \tau') \equiv (\min\{\tau, \tau'\} - \tau\tau')E[XX']$, $\Omega_{g\xi}(\tau) \equiv E[g_\tau(W, \theta(\tau))\xi]$, and $\Omega_{\xi\xi} \equiv E[\xi^2]$.

[1] Along with Assumption 4.2, which is discussed in this note, Theorem 4.1 in Canay (2011) uses Assumption 3.2 and Assumption 4.1. The former defines fixed effects as time-independent ("location shifters") and the latter gives the expressions for the terms J_1 and J_2 in the covariance matrix of the first-step estimator.

Next, the within estimator is taken to satisfy Assumption 4.2 (see the lemma below) and therefore supposed to be an appropriate first step estimator. It is then used to construct the asymptotic covariance matrix of the two-step estimator.

Lemma A.4, Canay (2011) *Assume $\Omega_{XX} \equiv E[(X^s_{it} - \mu^s_X)(X^s_{it} - \mu^s_X)']$ is non-singular with finite norm, $n/T^a \to 0$ for some $a \in (0, \infty)$ and let Assumptions 3.2 and 4.1 hold. The within estimator of θ_μ satisfies Assumption 4.2 with the influence function*

$$\psi_{it} = \begin{pmatrix} \psi^0_{it} \\ \psi^s_{it} \end{pmatrix} \equiv \begin{pmatrix} Y_{it} - \mu_Y - \mu^{s\prime}_X \Omega^{-1}_{XX}(X^s_{it} - \mu^s_X)u_{it} \\ \Omega^{-1}_{XX}(X^s_{it} - \mu^s_X)u_{it} \end{pmatrix},$$

where $X'_{it} = (1, X^{s\prime}_{it})$, $\mu^s_X \equiv E[X^s_{it}]$, $\mu_Y \equiv E[Y_{it}]$, u_{it} is i.i.d. with $E[u_{it} \mid X_i] = 0$ and $E[u^2_{it} \mid X_i] = X'_{it}\Omega_{UU}X_{it}$, and Ω_{UU} non-singular with finite norm.

There are two errors in Canay's conclusions. Firstly, Theorem 4.1, Canay (2011) claims that the asymptotic distribution of the limiting process $\mathbb{G}(\cdot)$ has the zero mean under the condition $n/T^s \to 0$ for $s \in (1, +\infty)$. This condition holds for wide panels, for which n grows faster than T. However, as we show in Proposition 2.1 below, the bias in $\hat{\theta}(\tau)$ goes to zero with rate $1/T$, so the asymptotic property requires the condition $n/T \to 0$ (if $n/T \to \infty$, the limiting distribution does not even exist). As a result, the asymptotic inference becomes incorrect for wide panels. Secondly, there is a fallacy in Lemma A.4, Canay (2011), which states that the within estimator satisfies Assumption 4.2, Canay (2011), and can be used as a first-step estimator in Theorem 4.1, Canay (2011). Namely, the assumption of independence of the first components ψ^0_{it} is unjustified. So the within estimator does not satisfy Assumption 4.2, Canay (2011) and the asymptotic standard errors are incorrect for the constant term.

Proposition 2.1 *Given the conditions of Theorem 4.1, Canay (2011)* $T \cdot \text{bias}\,\hat{\theta}(\tau)$ *generally does not converge to zero. So when $n/T \not\to 0$, the limiting process $\mathbb{G}(\cdot)$ either has a non-zero mean* $\lim_{n,T\to\infty} \sqrt{nT}\,\text{bias}\,\hat{\theta}(\tau)$ *or or the limiting process does not exist.*

Proof Consider the model

$$Y_{it} = X'_{it}\theta(U_{it}) + \alpha_i, \quad i = 1, \dots, n, \ t = 1, \dots, T.$$

Under the definition of $u_{it} = X'_{it}(\theta(U_{it}) - \theta_\mu)$ from the proof of Lemma A.4, Canay (2011), the model can be expressed as

$$Y_{it} = X'_{it}\theta_\mu + \alpha_i + u_{it} = \theta^0_\mu + X^{s\prime}_{it}\theta^s_\mu + \alpha_i + u_{it}, \quad i = 1, \dots, n, \ t = 1, \dots, T, \quad (2.5)$$

where u_{it} are i.i.d. across i and t (and uncorrelated with X_{it}), but α_i are constant for different t under fixed i. Denote $\varepsilon_{it} = \theta^0_\mu + \alpha_i + u_{it}$. The within estimator of θ^s_μ is $\sqrt{nT}$ consistent, so

$$\hat{\varepsilon}_{it} = Y_{it} - X^{s\prime}_{it}\hat{\theta}^s_\mu = \varepsilon_{it} + o_p(1/\sqrt{nT}) \tag{2.6}$$

and

$$\hat{\theta}^0_\mu + \hat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_{it} = \frac{1}{T}\sum_{t=1}^{T}\varepsilon_{it} + o_p(1/\sqrt{nT}).$$

Therefore,

$$\begin{aligned}
\hat{Y}_{it} &= Y_{it} - \hat{\alpha}_i = Y_{it} + \hat{\theta}^0_\mu - \frac{1}{T}\sum_{t=1}^{T}\varepsilon_{it} + o_p(1/\sqrt{nT}) \\
&= X'_{it}\theta(U_{it}) + \alpha_i + \hat{\theta}^0_\mu - \frac{1}{T}\sum_{t=1}^{T}\varepsilon_{it} + o_p(1/\sqrt{nT}) \\
&= X'_{it}\theta(U_{it}) + \alpha_i + \hat{\theta}^0_\mu - \frac{1}{T}\sum_{t=1}^{T}(\theta^0_\mu + \alpha_i + u_{it}) + o_p(1/\sqrt{nT}) \\
&= X'_{it}\theta(U_{it}) + \hat{\theta}^0_\mu - \theta^0_\mu - \frac{1}{T}\sum_{t=1}^{T}(X'_{it}(\theta(U_{it}) - \theta_\mu)) + o_p(1/\sqrt{nT}) \\
&= X^{s\prime}_{it}\big((1-1/T)\theta^s(U_{it}) + (1/T)\theta^s_\mu\big) + \big((1-1/T)\theta^0(U_{it}) + (1/T)\theta^0_\mu + (\hat{\theta}^0_\mu - \theta^0_\mu)\big) \\
&\qquad - \frac{1}{T}\sum_{\substack{r=1 \\ r\neq t}}^{T}(X'_{ir}(\theta(U_{ir}) - \theta_\mu)) + o_p(1/\sqrt{nT}).
\end{aligned} \tag{2.7}$$

The third term in the last expression of (2.7) is independent of the first and the second terms, and generally cannot offset the bias in the first term, which tends to $-(1/T)\big(\theta^s(\tau) - \theta^s_\mu\big)$ when $n \to \infty$. The same argument applies to $\hat{\theta}^0(\tau)$. This proves that $T \cdot \text{bias}(\hat{\theta}(\tau))$ generally does not converge to zero. Consequently, $\sqrt{nT} \cdot \text{bias}(\hat{\theta}(\tau))$ also does not converge to zero if $n/T \not\to 0$. Since weak convergence in Theorem 4.1, Canay (2011) implies convergence of expected values, we can conclude that if $n/T \not\to 0$, then generally the limiting process is either biased from zero or does not exist. This completed the proof.

Next, we summarize the second issue in the proposition below.

Proposition 2.2 *Given the conditions of Lemma A.4, Canay (2011) the first components ψ^0_{it} of the influence vectors ψ_{it} are not generally independent across time periods if $i = 1, \ldots, n$ is fixed. Therefore, Assumption 4.2, Canay (2011) is not satisfied.*

Proof Similarly to the proof of Proposition 2.1, we start with expressing the model by equation (2.5). Then, taking expectations, we obtain

$$\mu_Y = E[Y_{it}] = E[\theta^0_\mu + X^{s\prime}_{it}\theta^s_\mu + \alpha_i + u_{it}] = \theta^0_\mu + \mu^{s\prime}_X\theta^s_\mu.$$

(Here we use the assumption $E[\alpha_i] = 0$, otherwise θ^0_μ is not identifiable.) This implies

$$Y_{it} - \mu_Y = (X^s_{it} - \mu^s_X)'\theta^s_\mu + \alpha_i + u_{it},$$

and

$$\psi^0_{it} = Y_{it} - \mu_Y - \mu^{s\prime}_X\Omega^{-1}_{XX}(X^s_{it} - \mu^s_X)u_{it} = \alpha_i + u_{it} + (X^s_{it} - \mu^s_X)'\theta^s_\mu - \mu^{s\prime}_X\Omega^{-1}_{XX}(X^s_{it} - \mu^s_X)u_{it}.$$

The last three terms in the expression for ψ^0_{it} are i.i.d. across all i and t.

Consider $t \neq t'$. Since ψ^0_{it} and $\psi^0_{it'}$ contain the same term α_i, they are generally correlated. This completes the proof. **Remark 2.1** Looking at the three terms in the last line of (2.7), we can see that the problematic estimator $\hat{\theta}^0_\mu$ enters the expression of $\hat{Y}_{it}$ as a constant shift. This implies that it does not affect the estimates of the slope $\theta^s(\tau)$ and their variance. Hence, we can conjecture that only the asymptotic standard error of the constant term is incorrectly computed in Canay (2011).

Remark 2.2 Canay (2011) provides a bootstrap procedure, which is based on sampling individuals, and is in line with Galvao and Montes-Rojas (2015). The simulations analyses in both of the above papers show that the standard errors are correct for the estimators of all model parameters. Note that Canay's results give the standard errors of the estimators of the slope only. So in our simulations, we report the bootstrap standard errors for the estimator of the whole vector of coefficients. Our results demonstrate that the bootstrap provides for correct standard errors of the estimator of the slope and the constant term.

Nonetheless, the bootstrap does not enable correct inference for wide panels. Indeed, the bootstrap distribution converges to a limiting distribution, and the limiting distribution has a large bias for such panels.

3 Simulations that demonstrate incorrect inference

3.1 Simulation details

We simulate the following data generating process:

$$\begin{aligned} Y_{it} &= \theta_0(U_{it}) + \theta_1(U_{it})X_{it} + \alpha_i = (2 + X_{it})\sqrt{U_{it}} + \alpha_i, \\ \alpha_i &= (X_{i1} + \cdots + X_{iT})/\sqrt{T} + \eta_i - E\big[(X_{i1} + \cdots + X_{iT})/\sqrt{T} + \eta_i\big], \end{aligned} \tag{3.1}$$

where U_{it} is uniformly distributed over $[0,1]$, X_{it} follow gamma distributions with shape α and scale β, and η_i is $N(0,\sigma^2)$ (all are mutually independent).

For all experiments we set $\alpha = 1$, $\beta = 1$, $\sigma = 1$, and generate $B = 1000$ samples. The maximal n is 4000 and the maximal T equals 320.

The process in (3.1) involves all $X_{i1}, \ldots, X_{iT}$ in constuction of α_i. To make the results comparable, we always simulate the panel of the longest length ($T = 320$), trim it to the desired size, and then make estimates. Accordingly, the joint distribution of X_{it}, Y_{it} is the same in all experiments.

We compute both asymptotic standard errors and bootstrap standard errors, using Canay's methodology. The bootstrap standard errors are obtained by taking $R = 500$ pseudosamples of individuals, estimating the model coefficients R times, and taking the standard deviation of the R calculated values. See the formulae in Appendix B in Canay (2011).

3.2 Bias of the limiting distribution

Firstly, we examine the bias of $\hat{\theta}(\tau)$. Note that the behavior of the bias of $\hat{\theta}_1(\tau)$ and $\hat{\theta}_0(\tau)$ is similar, so here we focus on the more important issue of the bias for the slope estimator. The estimator of the intercept is of lesser importance as it can only be used for calculating the conditional quantile forecasts, which rarely happens in empirical applications. Practitioners are primarily focused on interpreting the impact of individual factors X^s_{it}.

Tables 1 and 2 summarize our findings. Proposition 2.1 shows that the bias tends to zero with rate $1/T$, so $T \cdot \text{bias}(\hat{\theta}_1(\tau))$ does not converge to zero when $n, T \to \infty$. We calculate the estimates for a range of panel sizes in n and T to demonstrate this issue with simulations. The results, which are shown in Table 1, reveal that $T \cdot \hat{\theta}_1(\tau)$ does not tend to zero with increase in n or T. Table 1 shows that the bias may have different signs. In particular, the bias is positive for small τ, negative for large τ, and close to zero for $\tau = 0.5$.

The fact that $\sqrt{nT}\,\text{bias}(\hat{\theta}_1(\tau))$ does not converge to zero when $n/T \not\to 0$ can have a serious impact on the distribution of the z-statistics of the coefficients. To demonstrate this, we consider a set of panels with different values of n/T and calculate the z-statistic based on true value of $\theta(\tau)$:

$$z_{\hat{\theta}_1(\tau)} = \frac{\hat{\theta}_1(\tau) - \theta(\tau)}{\text{se}(\hat{\theta}_1(\tau))}.$$

Table 2 reveals that similarly to $\sqrt{nT}\,\text{bias}(\hat{\theta}_1(\tau))$, the absolute value of the bias of the z-statistic $E[z_{\hat{\theta}_1(\tau)}]$ grows considerably with increase in n/T. Yet, $E[z_{\hat{\theta}_1(\tau)}]$ should be centered around zero. Note that the problem with the

Table 1: $T \cdot \text{bias}(\hat{\theta}_1(\tau))$ for different panel sizes

	$n = 125$	$n = 250$	$n = 500$	$n = 1000$	$n = 2000$	$n = 4000$
	$\tau = 0.2$, $\theta(\tau) = (0.8944, 0.4472)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	0.302	0.309	0.304	0.299	0.305	0.304
$T = 10$	0.287	0.313	0.297	0.302	0.305	0.303
$T = 20$	0.312	0.312	0.284	0.295	0.310	0.299
$T = 40$	0.268	0.320	0.279	0.302	0.292	0.302
	$\tau = 0.5$, $\theta(\tau) = (1.4142, 0.7071)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	−0.010	−0.002	−0.003	−0.009	−0.006	−0.005
$T = 10$	−0.004	0.018	0.007	−0.002	0.008	0.007
$T = 20$	0.018	0.028	0.001	0.004	0.019	0.010
$T = 40$	0.000	0.039	0.005	−0.004	0.015	0.009
	$\tau = 0.8$, $\theta(\tau) = (1.7889, 0.8944)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	−0.334	−0.333	−0.333	−0.333	−0.331	−0.331
$T = 10$	−0.330	−0.317	−0.319	−0.325	−0.319	−0.321
$T = 20$	−0.274	−0.270	−0.286	−0.286	−0.278	−0.284
$T = 40$	−0.228	−0.224	−0.249	−0.239	−0.236	−0.238

shifted distribution of the z-statistic is most evident in low and high quantiles. For instance, the probabilites $P(|z_{\hat{\theta}_1(\tau)}| > z_{0.975})$, where $z_{0.975}$ is the 0.975 quantile of the standard normal distribution, become large for quantiles $\tau = 0.2$ and $\tau = 0.8$. This does not correspond to the asymptotic property derived in Theorem 4.1, Canay (2011), which implies that the distribution of $z_{\hat{\theta}_1(\tau)}$ should be close to the standard normal.

Figure 1 provides a graphic representation of the probability density of z-statistics. We observe large shifts of z-statistics for $\tau = 0.2$ and for $\tau = 0.8$, while the bias manifests itself only modestly at median value of $\tau = 0.5$.

At the same time, we can say that Canay's estimator performs well in terms of the asymptotic standard errors of the slope coefficients. This can be inferred from the second row in all three panels of Table 2. The expected value of the ratio $\text{se}(\hat{\theta}_1(\tau))/\sigma(\hat{\theta}_1(\tau))$ is close to one. The bootstrapped standard errors are also close to one. (See the fifth rows of the panels in Table 2.) Accordingly, the bias appears to be the only problematic issue with Canay's estimator, and it can be severe for panels with high values of n/T.

Table 2: Distribution of $\hat{\theta}_1(\tau)$ for different panel sizes

	$n = 125$ $T = 320$	$n = 250$ $T = 160$	$n = 500$ $T = 80$	$n = 1000$ $T = 40$	$n = 2000$ $T = 20$	$n = 4000$ $T = 10$
	$\tau = 0.2$, $\theta(\tau) = (0.8944, 0.4472)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$\sqrt{nT}\,\text{bias}(\hat{\theta}_1(\tau))$	0.148	0.321	0.711	1.508	3.102	6.052
$E\left[\frac{\text{se}(\hat{\theta}_1(\tau))}{\sigma(\hat{\theta}_1(\tau))}\right]$	1.021	0.996	0.998	0.995	0.975	0.916
$E[z_{\hat{\theta}_1(\tau)}]$	0.095	0.203	0.455	0.979	2.057	4.195
$P(\lvert z_{\hat{\theta}_1(\tau)}\rvert > z_{0.975})$	0.050	0.050	0.075	0.170	0.528	0.983
$E\left[\frac{\text{se}^*(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	1.031	1.020	1.044	1.010	1.020	0.975
	$\tau = 0.5$, $\theta(\tau) = (1.4142, 0.7071)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$\sqrt{nT}\,\text{bias}(\hat{\theta}_1(\tau))$	−0.018	0.042	−0.006	−0.021	0.188	0.141
$E\left[\frac{\text{se}(\hat{\theta}_1(\tau))}{\sigma(\hat{\theta}_1(\tau))}\right]$	1.022	0.998	0.998	0.981	0.956	0.899
$E[z_{\hat{\theta}_1(\tau)}]$	−0.013	0.035	−0.003	−0.014	0.159	0.128
$P(\lvert z_{\hat{\theta}_1(\tau)}\rvert > z_{0.975})$	0.046	0.042	0.049	0.050	0.062	0.086
$E\left[\frac{\text{se}^*(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	1.035	1.021	1.052	1.016	1.025	0.956
	$\tau = 0.8$, $\theta(\tau) = (1.7889, 0.8944)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$\sqrt{nT}\,\text{bias}(\hat{\theta}_1(\tau))$	−0.141	−0.269	−0.549	−1.197	−2.783	−6.424
$E\left[\frac{\text{se}(\hat{\theta}_1(\tau))}{\sigma(\hat{\theta}_1(\tau))}\right]$	1.005	1.003	0.956	1.000	0.922	0.879
$E[z_{\hat{\theta}_1(\tau)}]$	−0.173	−0.336	−0.690	−1.505	−3.481	−7.946
$P(\lvert z_{\hat{\theta}_1(\tau)}\rvert > z_{0.975})$	0.047	0.070	0.107	0.309	0.917	1.000
$E\left[\frac{\text{se}^*(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	1.031	1.020	1.043	1.020	1.030	0.945

3.3 Incorrect asymptotic standard error of the intercept

The second set of simulations focuses on asymptotic standard errors of the estimator of the constant term. We take the ratio of the standard error of $\hat{\theta}_0(\tau)$ to the true standard deviation and examine its expected value $E\left[\frac{\text{se}(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$. Similarly to the first set of experiments, we consider n in range 125 to 4000 and T in range 5 to 40.

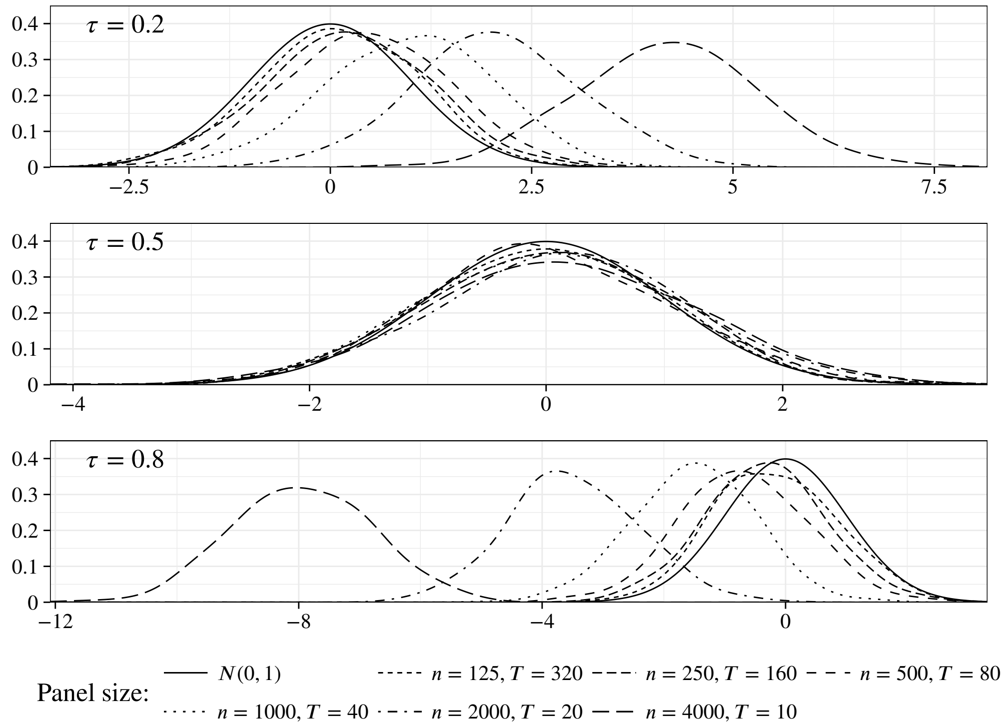

Figure 1: Kernel density estimates for the z-statistics for $\hat{\theta}_1(\tau)$ for different panel sizes

As is shown on Table 3, the value of the ratio falls with an increase in T. Accordingly, the estimator of the standard error is inconsistent, which leads to incorrect inference. We may also note that for each T the value of the ratio does not change with growth in n. Note that the decrease of the expected value of the ratio in T also reveals the incorrect rate of convergence of the asymptotic standard error. Indeed, the rate should be equal to $1/\sqrt{n}$, but the estimated rate in Theorem 4.1, Canay (2011) is $1/\sqrt{nT}$.

Finally, we conduct simulations to focus on the distribution of $\hat{\theta}_0(\tau)$ in terms of the indicators examined in Table 2. Here our analysis concentrates on standard errors, so we do not consider panels with different values of n/T, as z-statistics would have different biases in such cases. Instead, we focus on panels with a constant ratio n/T, which would be expected to produce approximately unchanging bias. The results reported in Table 4 indicate that the distribution of the z-statistics becomes wider when T grows, and the probability $P(|z_{\hat{\theta}_0(\tau)}| > z_{0.975})$ overwhelmingly exceeds 0.05. The fact is observed at all analyzed quantiles ($\tau = 0.2$, $\tau = 0.5$, $\tau = 0.8$).

On the other hand, the bootstrap standard errors $\mathrm{se}^*(\hat{\theta}_0(\tau))$ seem to be

Table 3: $E\left[\frac{\text{se}(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$ for different panel sizes

	$n = 125$	$n = 250$	$n = 500$	$n = 1000$	$n = 2000$	$n = 4000$
	$\tau = 0.2$, $\theta(\tau) = (0.8944, 0.4472)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	0.726	0.718	0.720	0.723	0.761	0.708
$T = 10$	0.619	0.621	0.643	0.620	0.647	0.610
$T = 20$	0.496	0.490	0.501	0.512	0.517	0.492
$T = 40$	0.364	0.376	0.382	0.385	0.389	0.371
	$\tau = 0.5$, $\theta(\tau) = (1.4142, 0.7071)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	0.680	0.659	0.678	0.660	0.688	0.654
$T = 10$	0.550	0.569	0.576	0.562	0.584	0.553
$T = 20$	0.428	0.435	0.438	0.437	0.455	0.428
$T = 40$	0.313	0.322	0.325	0.327	0.335	0.320
	$\tau = 0.8$, $\theta(\tau) = (1.7889, 0.8944)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	0.646	0.624	0.650	0.645	0.668	0.626
$T = 10$	0.497	0.506	0.513	0.517	0.517	0.496
$T = 20$	0.361	0.367	0.368	0.373	0.374	0.358
$T = 40$	0.255	0.261	0.265	0.263	0.265	0.256

correct, as shown in the last lines for all the panels of Table 4.

Figure 2 shows the probability distribution of the z-statistics for $\hat{\theta}_0(\tau)$. The distribution of z-statistics stretches to infinity when T grows. This means that the p-values of tests, based on these statistics, will be severely underestimated.

4 Can the estimator be improved?

4.1 Limiting distribution of coefficients

To ensure existence of the limiting distribution for the two-step estimator introduced in Canay (2011) it should be sufficient to change the requirement for the rates of convergence of n and T: $n/T^s \to 0$ for some $s \in (0, 1]$. This means that the inference is possible for long panels only, similarly to the other estimators of quantile regressions for panel data (see Kato et al. (2012)).

Is there a way to reduce the bias, so that the asymptotics would work without the requirement of $n/T \to 0$? Here we discuss an approach to eliminate the parametric term $-(1/T)(\theta(\tau) - \theta_\mu)$ from the bias of $\hat{\theta}(\tau)$. If

Table 4: Distribution of $\hat{\theta}_0(\tau)$ for different panel sizes

	$n = 125$ $T = 5$	$n = 250$ $T = 10$	$n = 375$ $T = 15$	$n = 500$ $T = 20$	$n = 1000$ $T = 40$	$n = 2000$ $T = 80$
	$\tau = 0.2$, $\theta(\tau) = (0.8944, 0.4472)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$\sqrt{nT}\,\text{bias}(\hat{\theta}_0(\tau))$	−1.884	−1.875	−2.000	−1.685	−1.600	−1.482
$E\left[\frac{\text{se}(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	0.720	0.639	0.580	0.518	0.385	0.280
$E[z_{\hat{\theta}_0(\tau)}]$	−0.347	−0.330	−0.351	−0.293	−0.276	−0.257
$P(\lvert z_{\hat{\theta}_0(\tau)}\rvert > z_{0.975})$	0.179	0.218	0.254	0.327	0.433	0.574
$E\left[\frac{\text{se}^*(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	0.977	1.023	1.030	1.062	0.989	0.984
	$\tau = 0.5$, $\theta(\tau) = (1.4142, 0.7071)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$\sqrt{nT}\,\text{bias}(\hat{\theta}_0(\tau))$	−2.428	−2.767	−3.174	−2.902	−2.688	−2.724
$E\left[\frac{\text{se}(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	0.689	0.584	0.505	0.449	0.330	0.238
$E[z_{\hat{\theta}_0(\tau)}]$	−0.539	−0.592	−0.673	−0.610	−0.560	−0.561
$P(\lvert z_{\hat{\theta}_0(\tau)}\rvert > z_{0.975})$	0.201	0.277	0.345	0.392	0.505	0.652
$E\left[\frac{\text{se}^*(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	1.004	1.002	1.027	1.044	0.988	0.977
	$\tau = 0.8$, $\theta(\tau) = (1.7889, 0.8944)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$\sqrt{nT}\,\text{bias}(\hat{\theta}_0(\tau))$	1.094	0.005	−1.041	−1.365	−2.615	−3.702
$E\left[\frac{\text{se}(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	0.657	0.529	0.436	0.379	0.264	0.184
$E[z_{\hat{\theta}_0(\tau)}]$	0.241	−0.009	−0.273	−0.363	−0.697	−0.994
$P(\lvert z_{\hat{\theta}_0(\tau)}\rvert > z_{0.975})$	0.212	0.304	0.385	0.458	0.595	0.728
$E\left[\frac{\text{se}^*(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	0.986	1.024	1.016	1.037	0.995	0.971

we use the following expression for the fixed effect estimate:

$$\hat{\alpha}_{it} = \frac{1}{T-1} \sum_{\substack{r=1 \\ r \neq t}}^{T} \hat{\varepsilon}_{ir} - \hat{\theta}_\mu^0,$$

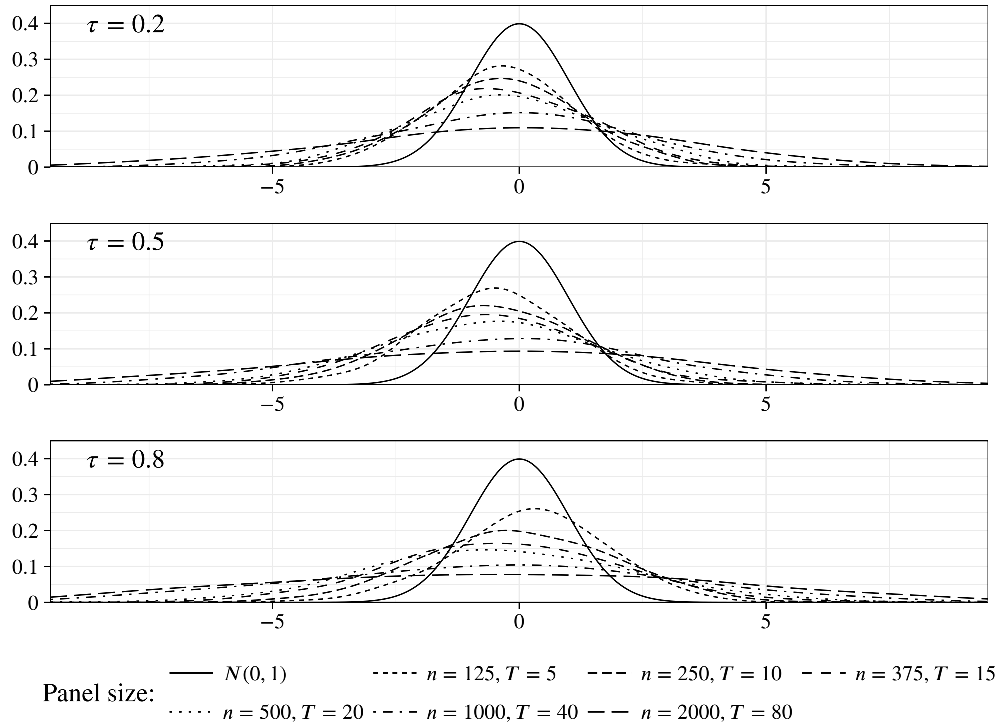

Figure 2: Probability distribution of the z-statistics for $\hat{\theta}_0(\tau)$ for different panel sizes

where $\hat{\varepsilon}_{ir}$ is defined by (2.6), then equation (2.7) changes to

$$\hat{Y}_{it} = Y_{it} - \hat{\alpha}_{it} = X_{it}^{s\prime}\theta^s(U_{it}) + \big(\theta^0(U_{it}) + (\hat{\theta}_\mu^0 - \theta_\mu^0)\big) \\ - \frac{1}{T-1}\sum_{\substack{r=1 \\ r\neq t}}^{T}(X_{ir}'(\theta(U_{ir}) - \theta_\mu)) + o_p(1/\sqrt{nT}).$$

As a result, the parametric part of the bias is removed. Note that the other part of the bias, which is caused by the last two terms in (2.7) increases (due to the fact that $T-1$ now appears in the denominator instead of T), but asymptotically it does not change. Unfortunately, the second term

$$-\frac{1}{T-1}\sum_{\substack{r=1 \\ r\neq t}}^{T}(X_{ir}'(\theta(U_{ir}) - \theta_\mu))$$

still makes the bias tend to zero with the rate of $1/T$, so the new estimator is still unsuitable for wide panels. Intuitively, elimination of the additive individual effect requires an additive transformation similar to that in Canay (2011) or the use of the general quantile regression technique for panel data. But both ways may not work for wide panels. Indeed, the former leads to bias equivalent to $1/T$ or worse, since α_i can be estimated by using at most T observations. The latter requires large values of T, as is shown in Kato et al. (2012).

Table 5 demonstrates the bias for the corrected estimator. The correction significantly reduces the absolute value of the bias for low and high quantiles. However, the bias goes up for $\tau = 0.5$. This can be explained by the fact that the correction becomes minuscule for τ so that $\theta(\tau)$ is close to θ_μ.

Table 5: $T \cdot \text{bias}(\hat{\theta}_1(\tau))$ for different panel sizes (corrected estimator)

	$n = 125$	$n = 250$	$n = 500$	$n = 1000$	$n = 2000$	$n = 4000$
	$\tau = 0.2$, $\theta(\tau) = (0.8944, 0.4472)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	0.103	0.111	0.105	0.100	0.107	0.106
$T = 10$	0.077	0.104	0.086	0.092	0.094	0.092
$T = 20$	0.098	0.096	0.068	0.080	0.095	0.084
$T = 40$	0.050	0.103	0.061	0.084	0.074	0.085
	$\tau = 0.5$, $\theta(\tau) = (1.4142, 0.7071)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	0.038	0.047	0.047	0.040	0.043	0.045
$T = 10$	0.042	0.065	0.052	0.043	0.053	0.053
$T = 20$	0.062	0.071	0.044	0.048	0.062	0.053
$T = 40$	0.042	0.081	0.047	0.038	0.057	0.051
	$\tau = 0.8$, $\theta(\tau) = (1.7889, 0.8944)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	−0.133	−0.132	−0.131	−0.132	−0.129	−0.129
$T = 10$	−0.112	−0.100	−0.101	−0.107	−0.102	−0.104
$T = 20$	−0.049	−0.045	−0.061	−0.061	−0.054	−0.059
$T = 40$	0.000	0.004	−0.021	−0.012	−0.008	−0.010

4.2 Asymptotic standard error of the intercept

Finding a $\sqrt{nT}$ consistent estimator of the constant term, as is required by Canay's procedure for the correct inference, is problematic in the model with individual effects α_i. Indeed, a new observation significantly improves the accuracy of the estimator of the constant term only if it contains information about a new individual (hence, about new α_i). Here we provide a simple example of a panel data model with individual effects, for which we strictly prove that such an estimator does not exist.

Proposition 4.3 *Let* $Y_{it} = \mu + \alpha_i + \varepsilon_{it}$*,* $i = 1, \dots, n$*,* $t = 1, \dots, T$*, where* α_i *are i.i.d.* $N(0, \sigma_\alpha^2)$*,* ε_{it} *are i.i.d.* $N(0, \sigma_\varepsilon^2)$ *and* α_i *are independent of* ε_{jt} *for all* i, j, t *(*$j = 1, \dots, n$*). Suppose* σ_α *and* σ_ε *are known. Then, the following inequality holds for any unbiased estimator* $\hat{\mu}$ *of* μ

$$\mathrm{Var}(\hat{\mu}) \geq \frac{\sigma_\alpha^2 + \sigma_\varepsilon^2/T}{n}.$$

So $\hat{\mu}$ *can be only* $\sqrt{n}$ *consistent, and not* $\sqrt{nT}$ *consistent.*

Proof The joint distribution of $Y = (Y_{11}, \dots, Y_{1T}, \dots, Y_{n1}, \dots, Y_{nT})'$ is Gaussian with the mean $\boldsymbol{\mu} = (\mu, \dots, \mu)'$ and the covariance matrix $I \otimes \Sigma$, where

$$\Sigma = \begin{pmatrix} \sigma_\alpha^2 + \sigma_\varepsilon^2 & \sigma_\alpha^2 & \dots & \sigma_\alpha^2 \\ \sigma_\alpha^2 & \sigma_\alpha^2 + \sigma_\varepsilon^2 & \dots & \sigma_\alpha^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_\alpha^2 & \sigma_\alpha^2 & \dots & \sigma_\alpha^2 + \sigma_\varepsilon^2 \end{pmatrix}.$$

This implies that the Fisher information for μ is

$$I(\mu) = \iota'(I \otimes \Sigma)^{-1}\iota = \iota'(I \otimes \Sigma^{-1})\iota,$$

where $\iota = (1, \dots, 1)'$ is a unity vector of length nT.

$$\Sigma^{-1} = \frac{1}{\sigma_\varepsilon^2(T\sigma_\alpha^2 + \sigma_\varepsilon^2)} \begin{pmatrix} (T-1)\sigma_\alpha^2 + \sigma_\varepsilon^2 & -\sigma_\alpha^2 & \dots & -\sigma_\alpha^2 \\ -\sigma_\alpha^2 & (T-1)\sigma_\alpha^2 + \sigma_\varepsilon^2 & \dots & -\sigma_\alpha^2 \\ \vdots & \vdots & \ddots & \vdots \\ -\sigma_\alpha^2 & -\sigma_\alpha^2 & \dots & (T-1)\sigma_\alpha^2 + \sigma_\varepsilon^2 \end{pmatrix}.$$

Hence, $I(\mu) = \dfrac{nT\sigma_\varepsilon^2}{\sigma_\varepsilon^2(T\sigma_\alpha^2 + \sigma_\varepsilon^2)} = \dfrac{nT}{T\sigma_\alpha^2 + \sigma_\varepsilon^2}$.

An application of the Cramér–Rao bound (see Amemiya (1985), Theorem 1.3.1) completes the proof.

As a result, we can conclude that the rate of convergence of $\hat{\theta}_0(\tau)$ cannot be $1/\sqrt{nT}$ in general, and the estimator of the intercept cannot be included in the same process as $\hat{\theta}^s(\tau)$ in Theorem 4.1, Canay (2011).

Can the properties of the estimator be improved by removing the constant term from regressors X_{it}? This implies a modification of the Canay (2011) model by imposing a restriction concerning the independence of constant terms across quantiles. Note that the original formulation is essentially:

$$Y_{it} = (\theta_0(U_{it}) + \alpha_i) + X_{it}^{s\prime}\theta^s(U_{it}),$$

which provides for different constants at different quantiles τ. Only individual effects are quantile-independent. Removing the constant term leads to the following modified equation:

$$Y_{it} = (\theta_0 + \alpha_i) + X_{it}^{s\prime}\theta^s(U_{it})$$

(for convenience, we keep θ_0 and its identifation condition $E[\alpha_i] = 0$).

The modified model does not contain $\theta_0(\tau)$, which eliminates the issue of different rates of convergence for different components of the vector $\hat{\theta}(\tau)$. Nonetheless, the estimator of θ_0 is only $\sqrt{n}$ consistent. This slow rate of convergence of $\hat{\theta}_0$ should be taken into consideration in constructing confidence intervals for the conditional quantile predictions.

Note, however, that the problem with the bias under $n/T \not\to 0$ still persists in the modified model.

5 On the applicability of the estimator

5.1 Assumption concerning mutually independent regressors

One of the applicability conditions for the Canay (2011) estimator requires the independence of regressors X_{it} both across i and across t. Yet, it is hard to satisfy this condition in empirical work. To assess the bias of the estimate, when X_{it} are correlated in t, we conduct the following experiment. $X_{it} \sim \Gamma(4, 1)$ as in the previous rounds of simulations, but X_{it} are dependent across time through a moving average process:

$$X_{it} = x_{it} + x_{it-1} + \cdots + x_{it-7},$$

where x_{it} are iid with $\Gamma(1/2, 1)$.

Table 6 shows the bias for Canay's estimator applied to data with regressors that are dependent across time.

The results of this simulation are directly comparable with Table 1, and we see that the bias differs across models with dependent and independent regressors. Note that the difference between the values of $T\cdot\text{bias}(\hat{\theta}_1(\tau))$ in the models with independent and dependent regressors is particularly noticeable for small T. At the same time, dependent regressors introduce only a minor additional bias under large T. So we may conjecture that if the process for X_{it} is ergodic in t, then there is no reason to expect the estimator to be inconsistent. The asymptotic standard errors presented in Table 7 are worse than standard errors, computed under the assumption of independent

Table 6: $T \cdot \text{bias}(\hat{\theta}_1(\tau))$ for different panel sizes (correlated regressors)

	$n = 125$	$n = 250$	$n = 500$	$n = 1000$	$n = 2000$	$n = 4000$
	$\tau = 0.2$, $\theta(\tau) = (0.8944, 0.4472)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	0.210	0.211	0.192	0.182	0.192	0.187
$T = 10$	0.253	0.227	0.225	0.208	0.225	0.218
$T = 20$	0.278	0.251	0.258	0.244	0.254	0.253
$T = 40$	0.289	0.280	0.293	0.276	0.277	0.272
	$\tau = 0.5$, $\theta(\tau) = (1.4142, 0.7071)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	−0.042	−0.045	−0.063	−0.070	−0.058	−0.064
$T = 10$	−0.031	−0.039	−0.039	−0.054	−0.037	−0.044
$T = 20$	−0.012	−0.028	−0.012	−0.021	−0.014	−0.022
$T = 40$	−0.021	−0.009	−0.002	0.001	−0.001	−0.009
	$\tau = 0.8$, $\theta(\tau) = (1.7889, 0.8944)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$T = 5$	−0.228	−0.225	−0.241	−0.241	−0.235	−0.237
$T = 10$	−0.248	−0.272	−0.266	−0.278	−0.267	−0.270
$T = 20$	−0.268	−0.280	−0.263	−0.265	−0.271	−0.270
$T = 40$	−0.233	−0.244	−0.240	−0.237	−0.237	−0.238

regressors in Table 2, especially for panels with relatively low T. On the other hand, the bootstrapped standard errors seem to work for regressors that are dependent across time, as can be seen in the last lines of each panel in Table 7.

5.2 Implications for practitioners

In this note we have touched on several problematic issues with the Canay (2011) estimator. We will now outline major concerns, relating to applicability of the estimator for purposes of empirical analysis. Firstly, the use of the estimator may cause incorrect inference, owing to the bias in the limiting distribution in wide panels. Secondly, the estimator may lead to wrong inference due to incorrect asymptotic standard error of the constant term. Finally, the assumption of independence of the predictors across time may be unlikely to hold in practice.

Note that the second issue is the least important among the three problems. Indeed, practitioners focus on the intercept only for the purposes of forecasting or computing residuals, and this task is rarely the purpose of panel data analysis. Indeed, none of the 81 papers in our meta-review of the applied literature carried out such an exercise or interpreted the significance

Table 7: Distribution of $\hat{\theta}_1(\tau)$ for different panel sizes (correlated regressors)

	$n = 125$ $T = 320$	$n = 250$ $T = 160$	$n = 500$ $T = 80$	$n = 1000$ $T = 40$	$n = 2000$ $T = 20$	$n = 4000$ $T = 10$
	$\tau = 0.2$, $\theta(\tau) = (0.8944, 0.4472)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$\sqrt{nT}\,\text{bias}(\hat{\theta}_1(\tau))$	0.141	0.400	0.740	1.382	2.542	4.359
$E\left[\frac{\text{se}(\hat{\theta}_1(\tau))}{\sigma(\hat{\theta}_1(\tau))}\right]$	0.982	0.987	0.976	0.978	0.884	0.794
$E[z_{\hat{\theta}_1(\tau)}]$	0.087	0.253	0.472	0.897	1.682	3.021
$P(\lvert z_{\hat{\theta}_1(\tau)}\rvert > z_{0.975})$	0.065	0.067	0.071	0.163	0.400	0.799
$E\left[\frac{\text{se}^*(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	0.968	0.961	0.999	1.027	1.079	1.025
	$\tau = 0.5$, $\theta(\tau) = (1.4142, 0.7071)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$\sqrt{nT}\,\text{bias}(\hat{\theta}_1(\tau))$	−0.028	0.029	0.009	0.003	−0.138	−0.880
$E\left[\frac{\text{se}(\hat{\theta}_1(\tau))}{\sigma(\hat{\theta}_1(\tau))}\right]$	1.014	0.993	0.974	0.957	0.873	0.713
$E[z_{\hat{\theta}_1(\tau)}]$	−0.021	0.027	0.008	0.004	−0.113	−0.775
$P(\lvert z_{\hat{\theta}_1(\tau)}\rvert > z_{0.975})$	0.044	0.051	0.063	0.053	0.087	0.213
$E\left[\frac{\text{se}^*(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	0.965	0.953	0.997	1.023	1.073	1.028
	$\tau = 0.8$, $\theta(\tau) = (1.7889, 0.8944)'$, $\theta_\mu = (1.3333, 0.6667)'$					
$\sqrt{nT}\,\text{bias}(\hat{\theta}_1(\tau))$	−0.093	−0.239	−0.506	−1.186	−2.708	−5.399
$E\left[\frac{\text{se}(\hat{\theta}_1(\tau))}{\sigma(\hat{\theta}_1(\tau))}\right]$	0.996	0.985	0.970	0.878	0.778	0.604
$E[z_{\hat{\theta}_1(\tau)}]$	−0.111	−0.297	−0.636	−1.500	−3.436	−6.771
$P(\lvert z_{\hat{\theta}_1(\tau)}\rvert > z_{0.975})$	0.058	0.064	0.106	0.343	0.885	0.999
$E\left[\frac{\text{se}^*(\hat{\theta}_0(\tau))}{\sigma(\hat{\theta}_0(\tau))}\right]$	0.967	0.957	0.994	1.024	1.062	1.028

of the intercept.

Our theoretical and simulational analysis suggests that the applicability of Canay's estimator is particularly problematic with panels, where n/T is large. Panels with small n/T may not suffer from the bias in the limiting distribution of the estimator of coefficients. Nonetheless, regressors that are dependent across time lead to incorrect asymptotic standard errors in such panels. The use of bootstrap methodology, especially where there is large T, could offer a solution.

Table 8 presents a summary of the caveats regarding use of the estimator in applications.

Table 8: Cautiousness with use of the Canay (2011) estimator

Panel size	Major problems	Potential solutions
large n/T	The distribution of the estimates for the vector of coefficients significantly differs from the asymptotic distribution, given in Theorem 4.1, Canay (2011). It is hard to make inference and conduct tests on coefficients. The asymptotic standard error of the intercept is incorrect.	No solution
small n/T, independent regressors	The distribution of the estimates of slope coefficients is close to the asymptotic distribution, given in Theorem 4.1, Canay (2011). However, the asymptotic standard error of the intercept is incorrect.	Bootstrap may help to solve the problems with standard errors of the intercept.
small n/T, regressors correlated across time periods	The distribution of the estimates of slope coefficients differs from the asymptotic distribution, given in Theorem 4.1, Canay (2011). The asymptotic standard error of the intercept is incorrect.	Bootstrap may help to solve the problems with standard errors of the estimates for the vector of coefficients.

5.3 A meta-review of applications in the literature

To assess to what extent practical applications may be affected by the problems of the estimator, we examined all citations to the Canay (2011) paper from the Wiley Publishers webpage of *The Econometrics Journal* (as of December 31, 2018). Of the 120 papers in Web of Science journals, which gave cited the paper, 81 employed the estimator, while others mentioned it among other theoretical approaches for analysis using panel data and quantile regression.

Literature in numerous fields of macroeconomics, microeconomics, and finance makes use of the Canay (2011) estimator. Empirical papers most often study heterogeneity of firm behavior in terms of various issues in industrial organization or corporate finance. Another frequently occuring research question in these papers is differences in the behavior of individuals and households on markets for labor, education, or energy. The Canay (2011) estimator is also applied for the analysis of longitudinal data on the development or trade in various countries or regions. Striking and rare examples of empirical work using the estimator include the economics of sovereign ranking, traffic accidents, languages spoken in the EU, and political parties.

We focus primarily on panel size (n, T, and n/T) and the use of bootstrap methodology for standard errors. As shown in the summary Table 9 and the full review in the Online Appendix, the majority of papers work with large sample sizes and relatively short time periods, which leads to higher values for n/T.

Only 6 papers have n/T below 1 and 17 papers use n/T from 1 to 10. These are mainly long macro panels with annual data on a number of countries. Large value of T (and hence relatively low n/T) can be achieved here by using quarterly data on regions or firms.

The value of T is most often rather low, and only 24 papers estimate panels with $T \geq 20$. A few papers attempt to increase the length of the panel by using monthly data, but the sample size in these papers is still large to enable low values of n/T.

It should be noted that 60% of papers report the use of bootstrap methodology for standard errors. The coefficient for the intercept and its standard error is given in about 35% of papers (roughly half of them do not use the bootstrap approach). Yet, none of these analyses interprets the value or the significance of the intercept.

To summarize, a small share of applied works use data with low values of n/T and large T. Arguably, these works provide correct inference on the coefficients (and on the standard error of the intercept under the bootstrap procedure). High values of n/T and low values of T may cause a problem in applied estimates, owing to the issue of regressors that are dependent across time and to the asymptotic bias of the coefficients.

Table 9: Summary table on applied papers

	$n/T < 1$	$1 \leq n/T < 10$	$10 \leq n/T < 100$	$100 \leq n/T < 1000$	$n/T \geq 1000$
Number of papers	6	17	18	20	16
	$T < 5$	$5 \leq T < 10$	$10 \leq T < 20$	$20 \leq T < 50$	$T \geq 50$
Number of papers	15	22	16	17	7

Note: 4 papers did not report the value of n or T.

Appendix: A meta-review of applied literature on the Canay (2011) estimator

To assess to what extent practical applications may be affected by the problems of the estimator, which have been described in our Comment, we examined all citations of the Canay (2011) paper from the Wiley Publishers webpage of *The Econometrics Journal* (as of December 31, 2018). Of the 120 papers in Web of Science journals, 81 works employed the Canay (2011) estimator, while others mentioned it among other theoretical approaches for analysis using panel data and/or quantile regression.

Following the suggestions to researchers on applicability of the estimator, which were outlined in the main text of our Comment, we focus on panel size (n, T, and n/T) and the use of bootstrap methodology for standard errors in the analyzed papers. A number of caveats apply to the review table below. Firstly, papers sometimes reported only the total number of observations (i.e., the product of nT), so we inferred the sample size n by dividing the number of observations by the length of panel. This should be regarded as an approximation, as the real-world panels are often unbalanced. The resulting value may not be a whole number. Secondly, various specifications in the same paper could employ different number of observations (for instance, due to missing values for key variables in each specification). Since we argue that applicability of the Canay (2011) estimator requires the lowest possible value of n/T, the review table reports the minimal value of n for each paper. Finally, no use of bootstrap methodology was assumed, unless otherwise explicitly stated in each paper.

Table 10: Applications of the Canay (2011) estimator

	Paper	**Field**	**Data**	**Observation**	n	T	**Bootstrap**	n/T
1	Bampinas et al. (2017)	Public/ Inequality	quarterly data for US states in Q1 1975 – Q2 2012	region	48	146	yes	0.3
2	Dufrénot and Ehrhart (2015)	Growth	data for 15 countries in 1980–2008	country	15	29	yes	0.5
3	Behera and Dash (2018)	Public/ Health	data on 21 regions in a country in 1980–2014	region	21	35	no	0.6
4	Keho (2016)	Banking/ Development	financial data on 19 countries in 1987–2013	country	19	27	no	0.7
5	Chen and Lei (2018)	Energy	cross-country panel for 1980–2014	country	30	35	no	0.9
6	Fuchs and Gehring (2017)	Finance/ Culture	143 country ranking by 9 agencies in 6 countries in Jan 1990–Jun 2013	country	143	144	no	1.0
7	McKee et al. (2015)	Education	students in 79 schools	individual	79	74	no	1.1
8	Bouthevillain and Dufrénot (2016)	Growth	data for 22 countries in 1995–2013	country	22	19	no	1.2
9	van Leeuwen et al. (2018)	Culture	cross-country data for 60 countries in 1950–1989 or 1994–2014	country	60	40	no	1.5
10	Andini and Andini (2018)	Labor	US states in 1980–2010	region	51	31	yes	1.6

Table 10: Applications of the Canay (2011) estimator

	Paper	**Field**	**Data**	**Observation**	n	T	**Bootstrap**	n/T
11	Daniels et al. (2015)	Corporate finance	US foreign direct investment to 53 countries in 1982–2007	country	53	26	yes	2.0
12	Söderlund and Tingvall (2017)	Growth/ Development	macrodata on 30 regions in 2001–2009	region	30	10	yes	3.0
13	Lacalle-Calderon et al. (2017)	Health/ Development	151 country data for 1970–2010	country	151	41	yes	3.7
14	Fink (2017)	Political	annual financial reports of 5 political parties in 16 regions in 1994–2014	regional level party	80	21	no	3.8
15	You et al. (2016)	Environment	country data in 1985–2005	country	87	21	yes	4.1
16	Imai et al. (2017)	Agriculture/ Development	data for 129 countries in 1980–2010, 3-year averages	country	44	10	no	4.4
17	Gnangnon (2016)	Trade/ Development	104 developing countries in 1990–2010	country	104	21	yes	5.0
18	Uğur and Özocakli (2018)	Health/ Agriculture	data for 80 countries in 2000-2015	country	80	16	no	5.0
19	Akkermans (2017)	Finance/ Growth	country data in 1980–2009	country	213	30	no	7.1
20	Gnangnon (2017a)	Trade/ Development	country data for 1995–2014	country	155	20	yes	7.8
21	Álvarez-Ayuso et al. (2016)	Transportation	47 provinces in 1980–2007	region	47	6	no	7.8

Table 10: Applications of the Canay (2011) estimator

	Paper	Field	Data	Observation	n	T	Bootstrap	n/T
22	You et al. (2015)	Environment	country data in 1995–2005	country	87	11	yes	7.9
23	Cooper et al. (2017)	Agriculture	county data in 1975–2013	region	367	39	yes	9.4
24	López-Espinosa et al. (2015)	Finance	quarterly financial statement of US banks in Q1 1990– Q4 2010	bank	856	84	yes	10.2
25	Martínez-Zarzoso et al. (2017)	Trade/ Development	country data for 2000–2011	country	124	12	yes	10.3
26	Jiang and Zhang (2017)	Finance/ Banking	financial data on commercial banks in 2004–2015	bank	141	12	yes	11.8
27	Andini and Andini (2014)	Finance/ Growth	78 countries in 1960–1995, 7 waves of 5 years	country	78	5	yes	15.6
28	Kostov and Le Gallo (2018)	Growth	cross-country panel for 1980–2010, 5 year averages	country	124	7	no	17.7
29	Lacalle-Calderon et al. (2018)	Development	cross-country panel for 2005, 2008 and 2011	country	57	3	yes	19.0
30	Alberini et al. (2013)	Energy	consumer survey in 2011 and 2012, matched to monthly energy consumption in Dec 2007–Apr2012	individual	1153	53	yes	21.8

Table 10: Applications of the Canay (2011) estimator

	Paper	**Field**	**Data**	**Observation**	n	T	**Bootstrap**	n/T
31	Choi and Pyun (2017)	IO/ Productivity	data on firms in 461 industry for 1990–2007	industry	461	18	yes	25.6
32	Šarić et al. (2018)	Health/ Transportation	traffic accidents on 130 roads in 2012–2016	road	130	5	no	26.0
33	Fidrmuc and Fidrmuc (2016)	Trade/ Culture	32 languages spoken in 29 countries and bilateral trade data in 2001–2007	country pairs	210	7	yes	30.0
34	Egger et al. (2016)	Patenting/ Innovation	patent data on 17 industries in 34 countriesmin 1995–2005	industry	320	8	no	40.0
35	Bykova and Lopez-Iturriaga (2018)	Trade/IO	firm data for 2004–2014	firm	518	11	no	47.1
36	Gnangnon (2017b)	Trade/ Development	cross-country data for 1995–2014 (4 year periods)	country	155	3	yes	51.7
37	Iimi et al. (2017)	Energy/ Development	household energy consumption, monthly data Jan 2007 to Jul 2010	household	4000	67	no	59.7
38	Burdín (2016)	Labor	employee-employer data in Jan 1997–Apr 2010, monthly	individual	10500	160	yes	65.6
39	Tsai et al. (2016)	Health/ Development	surveys of 1238 women, 3 waves	individual	214	3	yes	71.3
40	Mathur et al. (2016)	Finance	firms in 1964–2011	firm	3545	48	yes	73.9

Table 10: Applications of the Canay (2011) estimator

	Paper	**Field**	**Data**	**Observation**	n	T	**Bootstrap**	n/T
41	Foster-McGregor et al. (2015b)	Trade/ Development	firms in 19 countries in 2010–2011	firm	456	5	yes	91.2
42	Ohinata and Van Ours (2016)	Labor/ Education	600 primary schools, 5 waves in 1996–2005	school	600	5	yes	120.0
43	Procher et al. (2018)	Labor/ Family	household panel data for 1992–2011	individual	4935	20	no	246.8
44	Ku and Yen (2016)	Corporate finance	finanical data of firms in 2008–2012	firm	1326	5	yes	265.2
45	Oxholm et al. (2018)	Health/IO	23 performance indicators (2 groups of 19 or 4 indicators) for 764–780 general practitioners in 2004/05–2010/11	practice-indicator	1998	7	yes	285.4
46	Nordman et al. (2013)	Labor	household survey in 2002–2006	individual	948	3	yes	316.0
47	Pompei et al. (2018)	Labor	4336–4476 family firms in 2007 and 2010, 787–2661 firms in quantile regressions	firm	782	2	yes	391.0
48	Javdani (2015)	Labor/ Gender	employee-employer data pooled across 1999–2008	individual	6584	16	yes	411.5
49	Coad et al. (2016)	IO/ Innovation	firm data in 2004–2012	firm	3762	9	yes	418.0

Table 10: Applications of the Canay (2011) estimator

	Paper	Field	Data	Observation	n	T	Bootstrap	n/T
50	Trinh and Doan (2018)	Trade/IO	biannual financial data for 8318–8353 firms in 2005–2013	firm	8318	18	yes	462.1
51	Leoncini et al. (2017)	Environment/ IO	firm data for 2000–2008	firm	5498	9	yes	610.9
52	Castagnetti and Giorgetti (2018)	Labor/ Gender	4 waves of worker survey in 2005–2010	individual	2489	4	yes	622.3
53	Fitzenberger and Fuchs (2017)	Urban	housing-tenant data in 1984–2011	tenant	18601	28	yes	664.3
54	Bartelsman et al. (2014)	Innovation/ Productivity	two country dataset: 6634 and 14841 firms in 2000–2008	firm	6634	9	no	737.1
55	Fang and Niimi (2017)	Health/ Happiness	representative sample of all adults in 2009–2013	individual	3695	5	no	739.0
56	Cingano et al. (2016)	Labor	employee-employer data in 1986–1989 and 1991–1994	firm	6656	8	yes	832.0
57	Asfaw et al. (2018)	Development/ Climate	household data for 2011/2012 and 2014	household	1672	2	no	836.0
58	Cooke and Fuller (2018)	Labor/ Family	employee-employer panel for 1999–2005	establishment	5020	6	yes	836.7
59	Binder and Coad (2015)	Health	household panel data in 1996–2008	individual	9740	11	yes	885.5
60	Vu et al. (2014)	Trade	firm data in 2005–2009, 3 wave	firm	2821	3	yes	940.3

Table 10: Applications of the Canay (2011) estimator

	Paper	**Field**	**Data**	**Observation**	n	T	**Bootstrap**	n/T
61	Fuller (2017)	Labor/ Gender	employee-employer data for 1999–2005	establishment	5805	6	yes	967.5
62	Mahuteau et al. (2017)	Labor	household data for 2001–2014	individual	16624	14	no	1187.4
63	Gerdtham et al. (2016)	Health	twins born in 1933–1958 and their health, education and income in 1998–2002	individual	6656	5	no	1331.2
64	Binder (2015)	Health	individuals, surveyed in 1996–2008	individual	9341	7	yes	1334.4
65	Fang (2017)	Health/ Happiness	household survey in 2008 and 2009	individual	2709	2	no	1354.5
66	Nordman et al. (2016)	Labor/ Development	employee-employer data, 4 waves in 2000–2004	individual	5982	4	yes	1495.5
67	Mahuteau and Zhu (2016)	Health/ Happiness	household survey in 2002–2012	individual	18460	11	no	1678.2
68	Van den Berg et al. (2014)	Health	immigrants in 1949 who were subject to the mandatory enlistment test in 1984–1997	individual	4016	2.24	no	1792.9
69	Binder (2016)	Health/ Labor	household panel survey in 1996–2008	individual	25068	12	yes	2089.0
70	Damiani et al. (2018)	Labor	firm data in 2007 and 2010	firm	4400	2	yes	2200.0

Table 10: Applications of the Canay (2011) estimator

	Paper	**Field**	**Data**	**Observation**	n	T	**Bootstrap**	n/T
71	Vial and Hanoteau (2015)	Labor/ Development	4 wave panel of 9157 households in 1993–2007	household	9157	4	yes	2289.3
72	Matano and Naticchioni (2016)	Labor	employee-employer data for 1985–2003	individual	49526	19	no	2606.6
73	Zhu and Chen (2016)	Health	panel data on individuals in 2001–2003	individual	12051	4	no	3012.8
74	Edwards (2012)	Education	students in a county in 1996–2006	individual	24827	7	no	3546.7
75	Matano and Naticchioni (2017)	Labor	employee-employer panel for 1996–2003	individual	38810	8	no	4851.3
76	Garsaa and Levratto (2015)	IO	firms in 2004–2011	firm	41400	8	yes	5175.0
77	Håkansson and Isacsson (2018)	Labor	25% random sample of a entire population of a country in 2003–2008	individual	273333	6	yes	45555.5
78	Foster-McGregor et al. (2015a)	Labor/ Development	firm-level data for 19 countries in 24 sectors in 2010, industry-sector as an analogue of time dimension	firm	5029		yes	NA
79	Foster-McGregor et al. (2014b)	Corporate finance	firms in 19 countries and 24 sectors	firm	3254		yes	NA
80	Foster-McGregor et al. (2014a)	Trade/ Development	firms in 19 countries and 24 sectors	firm	2870		yes	NA

Table 10: Applications of the Canay (2011) estimator

	Paper	**Field**	**Data**	**Observation**	n	T	**Bootstrap**	n/T
81	Antecol et al. (2013)	Education	randomized experiment with primary school students in 6 US regions in 2001–2003	individual			yes	NA

References

Akkermans, D. H. (2017). Net profit flow per country from 1980 to 2009: The long-term effects of foreign direct investment. *PloS one*, 12(6):e0179244. https://doi.org/10.1371/journal.pone.0179244.

Alberini, A., Gans, W., and Towe, C. (2013). Free riding, upsizing, and energy efficiency incentives in Maryland homes. *The Energy Journal*, 37:259–90.

Álvarez-Ayuso, I. C., Condeço-Melhorado, A. M., Gutiérrez, J., and Zofío, J. L. (2016). Integrating network analysis with the production function approach to study the spillover effects of transport infrastructure. *Regional Studies*, 50:996–1015.

Amemiya, T. (1985). *Advanced Econometrics*. Harvard University Press, Cambridge, MA.

Andini, C. and Andini, M. (2018). Unemployment persistence and quantile parameter heterogeneity. *Macroeconomic Dynamics*, 22:1298–320.

Andini, M. and Andini, C. (2014). Finance, growth and quantile parameter heterogeneity. *Journal of Macroeconomics*, 40:308–22.

Antecol, H., Eren, O., and Ozbeklik, S. (2013). The effect of Teach for America on the distribution of student achievement in primary school: Evidence from a randomized experiment. *Economics of Education Review*, 37:113–25.

Asfaw, S., Pallante, G., and Palma, A. (2018). Diversification strategies and adaptation deficit: Evidence from rural communities in Niger. *World Development*, 101:219–34.

Bampinas, G., Konstantinou, P., and Panagiotidis, T. (2017). Inequality, demographics and the housing wealth effect: Panel quantile regression evidence for the US. *Finance Research Letters*, 23:19–22.

Bartelsman, E., Dobbelaere, S., and Peters, B. (2014). Allocation of human capital and innovation at the frontier: firm-level evidence on Germany and the Netherlands. *Industrial and Corporate Change*, 24:875–949.

Behera, D. K. and Dash, U. (2018). Prioritization of government expenditure on health in India: A fiscal space perspective. *Socio-Economic Planning Sciences*. https://doi.org/10.1016/j.seps.2018.11.004.

Binder, M. (2015). Volunteering and life satisfaction: a closer look at the hypothesis that volunteering more strongly benefits the unhappy. *Applied Economics Letters*, 22:874–85.

Binder, M. (2016). "...Do it with joy!" – Subjective well-being outcomes of working in non-profit organizations. *Journal of Economic Psychology*, 54:64–84.

Binder, M. and Coad, A. (2015). Heterogeneity in the relationship between unemployment and subjective wellbeing: A quantile approach. *Economica*, 82:865–91.

Bouthevillain, C. and Dufrénot, G. (2016). Fiscal policies enhancing growth in Europe: does one size fit all? *Oxford Economic Papers*, 68:1146–65.

Burdín, G. (2016). Equality under threat by the talented: Evidence from worker-managed firms. *The Economic Journal*, 126:1372–403.

Bykova, A. and Lopez-Iturriaga, F. (2018). Exports-performance relationship in Russian manufacturing companies: Does foreign ownership play an enhancing role? *Baltic Journal of Management*, 13:20–40.

Canay, I. (2011). A simple approach to quantile regression for panel data. *The Econometrics Journal*, 14:368–86.

Castagnetti, C. and Giorgetti, M. L. (2018). Understanding the gender wage-gap differential between the public and private sectors in Italy: A quantile approach. *Economic Modelling*. https://doi.org/10.1016/j.econmod.2018.09.025.

Chen, W. and Lei, Y. (2018). The impacts of renewable energy and technological innovation on environment-energy-growth nexus: New evidence from a panel quantile regression. *Renewable Energy*, 123:1–14.

Choi, B.-Y. and Pyun, J. H. (2017). Industry FDI and the distribution of plant productivity: Analysis using Korean plant-level data. *The Developing Economies*, 55:105–29.

Cingano, F., Leonardi, M., Messina, J., and Pica, G. (2016). Employment protection legislation, capital investment and access to credit: evidence from Italy. *The Economic Journal*, 126:1798–822.

Coad, A., Segarra, A., and Teruel, M. (2016). Innovation and firm growth: Does firm age play a role? *Research Policy*, 45:387–400.

Cooke, L. P. and Fuller, S. (2018). Class differences in establishment pathways to fatherhood wage premiums. *Journal of Marriage and Family*, 80:737–51.

Cooper, J., Nam Tran, A., and Wallander, S. (2017). Testing for specification bias with a flexible fourier transform model for crop yields. *American Journal of Agricultural Economics*, 99:800–17.

Damiani, M., Pompei, F., and Ricci, A. (2018). The role of employee incentive pay in the competitiveness of family and non-family firms. *Economia Politica*. https://doi.org/10.1007/s40888-018-0135-1.

Daniels, J. P., O'Brien, P., and Marc, B. (2015). Bilateral tax treaties and US foreign direct investment financing modes. *International Tax and Public Finance*, 22:999–1027.

Dufrénot, G. and Ehrhart, H. (2015). The ECOWAS countries' growth rates: what makes them similar and what makes them different? A quantile regression analysis. *Canadian Journal of Development Studies/Revue canadienne d'études du développement*, 36:345–65.

Edwards, F. (2012). Early to rise? The effect of daily start times on academic performance. *Economics of Education Review*, 31:970–83.

Egger, P. H., Seliger, F., and Woerter, M. (2016). On the distribution of patent citations and its fundamentals. *Economics Letters*, 147:72–77.

Fang, Z. (2017). Panel quantile regressions and the subjective well-being in urban China: Evidence from RUMiC data. *Social Indicators Research*, 132:11–24.

Fang, Z. and Niimi, Y. (2017). Does everyone exhibit loss aversion? Evidence from a panel quantile regression analysis of subjective well-being in Japan. *Journal of the Japanese and International Economies*, 46:79–90.

Fidrmuc, J. and Fidrmuc, J. (2016). Foreign languages and trade: evidence from a natural experiment. *Empirical Economics*, 50:31–49.

Fink, A. (2017). Donations to political parties: Investing corporations and consuming individuals? *Kyklos*, 70:220–55.

Fitzenberger, B. and Fuchs, B. (2017). The residency discount for rents in germany and the tenancy law reform act 2001: Evidence from quantile regressions. *German Economic Review*, 18:212–36.

Foster-McGregor, N., Isaksson, A., and Kaulich, F. (2014a). Importing, exporting and performance in Sub-Saharan African manufacturing firms. *Review of World Economics*, 150:309–36.

Foster-McGregor, N., Isaksson, A., and Kaulich, F. (2014b). Outward foreign direct investment, exporting and firm-level performance in Sub-Saharan Africa. *Journal of Development Studies*, 50:244–57.

Foster-McGregor, N., Isaksson, A., and Kaulich, F. (2015a). Foreign ownership and labour in Sub-Saharan African firms. *African Development Review*, 27:130–44.

Foster-McGregor, N., Isaksson, A., and Kaulich, F. (2015b). Importing, exporting and the productivity of services firms in Sub-Saharan Africa. *The Journal of International Trade & Economic Development*, 24:499–522.

Fuchs, A. and Gehring, K. (2017). The home bias in sovereign ratings. *Journal of the European Economic Association*, 15:1386–423.

Fuller, S. (2017). Segregation across workplaces and the motherhood wage gap: Why do mothers work in low-wage establishments? *Social Forces*, 96:1443–76.

Galvao, A. F. and Montes-Rojas, G. (2015). On bootstrap inference for quantile regression panel data: A Monte Carlo study. *Econometrics*, 3:654–66.

Garsaa, A. and Levratto, N. (2015). Do labor tax rebates facilitate firm growth? An empirical study on French establishments in the manufacturing industry, 2004–2011. *Small Business Economics*, 45:613–41.

Gerdtham, U.-G., Lundborg, P., Lyttkens, C. H., and Nystedt, P. (2016). Do education and income really explain inequalities in health? Applying a twin design. *The Scandinavian Journal of Economics*, 118:25–48.

Gnangnon, S. K. (2016). Aid for trade and trade tax revenues in developing countries. *Economic Analysis and Policy*, 50:9–22.

Gnangnon, S. K. (2017a). Empirical evidence on the impact of multilateral trade liberalization on domestic trade policy. *Global Economy Journal*, 17(3):1–14. https://doi.org/10.1515/gej-2017-0047.

Gnangnon, S. K. (2017b). The impact of multilateral trade liberalisation on economic development: Some empirical evidence. *Economic Affairs*, 37:397–410.

Håkansson, J. and Isacsson, G. (2018). The spatial extent of agglomeration economies across the wage earnings distribution. *Journal of Regional Science*. https://doi.org/10.1111/jors.12411.

Iimi, A., Elahi, R., Kitchlu, R., and Costolanski, P. (2017). Energy-saving effects of progressive pricing and free CFL bulb distribution program: Evidence from Ethiopia. *The World Bank Economic Review*, page lhw068. https://doi.org/10.1093/wber/lhw068.

Imai, K. S., Gaiha, R., and Garbero, A. (2017). Poverty reduction during the rural–urban transformation: Rural development is still more important than urbanisation. *Journal of Policy Modeling*, 39:963–82.

Javdani, M. (2015). Glass ceilings or glass doors? The role of firms in male-female wage disparities. *Canadian Journal of Economics/Revue canadienne d'économique*, 48:529–60.

Jiang, H. and Zhang, J. (2017). Bank capital buffer, franchise value, and risk heterogeneity in China. *Research in International Business and Finance*, 42:1455–66.

Kato, K., Galvao Jr., A. F., and Montes-Rojas, G. V. (2012). Asymptotics for panel quantile regression models with individual effects. *Journal of Econometrics*, 170:76–91.

Keho, Y. (2016). Non-linear effect of remittances on banking sector development: Panel evidence from developing countries. *Theoretical Economics Letters*, 6:1096.

Koenker, R. (2004). Quantile regression for longitudinal data. *Journal of Multivariate Analysis*, 91:74–89.

Kostov, P. and Le Gallo, J. (2018). What role for human capital in the growth process: new evidence from endogenous latent factor panel quantile regressions. *Scottish Journal of Political Economy*, 65:501–27.

Ku, Y.-Y. and Yen, T.-Y. (2016). Heterogeneous effect of financial leverage on corporate performance: A quantile regression analysis of Taiwanese companies. *Review of Pacific Basin Financial Markets and Policies*, 19(3):1650015–1–34. https://doi.org/10.1142/S0219091516500156.

Lacalle-Calderon, M., Perez-Trujillo, M., and Neira, I. (2017). Fertility and economic development: Quantile regression evidence on the inverse J-shaped pattern. *European Journal of Population*, 33:1–31.

Lacalle-Calderon, M., Perez-Trujillo, M., and Neira, I. (2018). Does microfinance reduce poverty among the poorest? A macro quantile regression approach. *The Developing Economies*, 56:51–65.

Leoncini, R., Marzucchi, A., Montresor, S., Rentocchini, F., and Rizzo, U. (2017). 'Better late than never': The interplay between green technology and age for firm growth. *Small Business Economics*. https://doi.org/10.1007/s11187-017-9939-6.

López-Espinosa, G., Moreno, A., Rubia, A., and Valderrama, L. (2015). Systemic risk and asymmetric responses in the financial industry. *Journal of Banking & Finance*, 58:471–85.

Mahuteau, S., Mavromaras, K., Richardson, S., and Zhu, R. (2017). Public–private sector wage differentials in Australia. *Economic Record*, 93:105–21.

Mahuteau, S. and Zhu, R. (2016). Crime victimisation and subjective well-being: panel evidence from Australia. *Health Economics*, 25:1448–63.

Martínez-Zarzoso, I., Nowak-Lehmann D, F., and Rehwald, K. (2017). Is aid for trade effective? A panel quantile regression approach. *Review of Development Economics*, 21:e175–e203.

Matano, A. and Naticchioni, P. (2016). What drives the urban wage premium? Evidence along the wage distribution. *Journal of Regional Science*, 56:191–209.

Matano, A. and Naticchioni, P. (2017). The extent of rent sharing along the wage distribution. *British Journal of Industrial Relations*, 55:751–77.

Mathur, A., Rao, N. S., Strain, M. R., and Veuger, S. A. (2016). Dividends and investment: Evidence of heterogeneous firm behavior. *Public Finance Review*, 44:769–87.

McKee, G., Sims, K. R. E., and Rivkin, S. G. (2015). Disruption, learning, and the heterogeneous benefits of smaller classes. *Empirical Economics*, 48:1267–86.

Nordman, C. J., Nguyen, H. C., and Roubaud, F. (2013). Who suffers the penalty?: A panel data analysis of earnings gaps in Vietnam. *Journal of Development Studies*, 49:1694–710.

Nordman, C. J., Rakotomanana, F., and Roubaud, F. (2016). Informal versus formal: A panel data analysis of earnings gaps in Madagascar. *World Development*, 86:1–17.

Ohinata, A. and Van Ours, J. C. (2016). Quantile peer effects of immigrant children at primary schools. *Labour*, 30:135–57.

Oxholm, A. S., Kristensen, S. R., and Sutton, M. (2018). Uncertainty about the effort–performance relationship in threshold-based payment schemes. *Journal of Health Economics*, 62:69–83.

Pompei, F., Damiani, M., and Ricci, A. (2018). Family firms, performance-related pay, and the great crisis: Evidence from the Italian case. *Industrial and Corporate Change*. https://doi.org/10.1093/icc/dty051.

Procher, V., Ritter, N., and Vance, C. (2018). Housework allocation in Germany: The role of income and gender identity. *Social Science Quarterly*, 99:43–61.

Šarić, Ž., Xu, X., Duan, L., and Babić, D. (2018). Identifying the safety factors over traffic signs in state roads using a panel quantile regression approach. *Traffic Injury Prevention*, 19:607–14.

Söderlund, B. and Tingvall, P. G. (2017). Capital freedom, financial development and provincial economic growth in China. *The World Economy*, 40:764–87.

Trinh, L. Q. and Doan, H. T. T. (2018). Internationalization and the growth of vietnamese micro, small, and medium sized enterprises: Evidence from panel quantile regressions. *Journal of Asian Economics*, 55:71–83.

Tsai, A. C., Tomlinson, M., Comulada, W. S., and Rotheram-Borus, M. J. (2016). Intimate partner violence and depression symptom severity among South African women during pregnancy and postpartum: population-based prospective cohort study. *PLoS medicine*, 13:e1001943. https://doi.org/10.1371/journal.pmed.1001943.

Uğur, A. A. and Özocakli, D. (2018). Gıda güvencesizliğinin bazı belirleyicileri (Kantil regresyon yöntemi ve sabit etki panel yönteminin karşılaştırılması). *Sosyoekonomi*, 26(35):195–205.

Van den Berg, G. J., Lundborg, P., Nystedt, P., and Rooth, D.-O. (2014). Critical periods during childhood and adolescence. *Journal of the European Economic Association*, 12:1521–57.

van Leeuwen, B., Plopeanu, A.-P., and Foldvari, P. (2018). Publishing ideas: The factors determining the number of book titles. *Acta Oeconomica*, 68:443–66.

Vial, V. and Hanoteau, J. (2015). Returns to micro-entrepreneurship in an emerging economy: A quantile study of entrepreneurial Indonesian households' welfare. *World Development*, 74:142–57.

Vu, H., Holmes, M., Lim, S., and Tran, T. (2014). Exports and profitability: a note from quantile regression approach. *Applied Economics Letters*, 21:442–45.

You, J., Imai, K. S., and Gaiha, R. (2016). Declining nutrient intake in a growing China: Does household heterogeneity matter? *World Development*, 77:171–91.

You, W.-H., Zhu, H.-M., Yu, K., and Peng, C. (2015). Democracy, financial openness, and global carbon dioxide emissions: heterogeneity across existing emission levels. *World Development*, 66:189–207.

Zhu, R. and Chen, L. (2016). Overeducation, overskilling and mental wellbeing. *The BE Journal of Economic Analysis & Policy*, 16(4):20150187. https://doi.org/10.1515/bejeap-2015-0187.