Centre for Economic and Financial Research at New Economic School



March 2017

Self-Enforcing Trade Credit

Marta Troya-Martinez

Working Paper No 240

CEFIR / NES Working Paper series

Self-Enforcing Trade Credit

Marta Troya-Martinez*

New Economic School

March 9, 2017

Abstract

Trade credit plays a very important role in inter-firm transactions. Because formal contracts are often unavailable, it is granted within an ongoing relationship. We characterize the optimal self-enforcing contract, when the ability to repay is unknown to the supplier and the threat of trade suspension is used to discipline the buyer. The optimal contract resembles a debt contract: if the fixed repayment is met, the contract is renewed. Otherwise, the supplier demands the highest feasible repayment and suspends trade for some time. The length of the trade suspension is contingent on the repayment. We provide a novel explanation for why the quantity is undersupplied, even when a repayment is met.

JEL Classification: C73, D82, L14.

Keywords: Limited Enforcement, Trade Credit, Imperfect Monitoring, Debt Contract.

^{*}New Economic School, e-mail: mtroya@nes.ru. I would like to thank the Editor and two anonymous referees for extremely valuable comments. I am indebted to Jim Malcomson and David Myatt for their guidance. I have also benefited from helpful comments by Vicente Cuñat, Mikhail Drugov, Marcel Fafchamps, Matthias Fahn, Guido Friebel, Rachel Griffith, Lakshmi Iyer, Paul Klemperer, Meg Meyer, Ines Moreno de Barreda, Massimo Motta, Michael Powell, Patrick Rey, Andrew Rhodes, John Thanassoulis and the participants of CSAE, JEI, RES, Econometric Society NASM, RES mentoring meeting, EARIE, Barcelona GSE Winter Workshop, IIOC, Workshop on Institutions, ASSET conferences and the seminars at Oxford, CEMFI, UC3M, NES and HSE. I thank Evgenii Safonov and Alexander Tonis for excellent research assistance. I gratefully acknowledge financial support from Fundación Ramón Areces and the ESRC grant PTA-026-27-3009. Any remaining errors are my own.

1 Introduction

Supplier trade credit is the delay in the payment of goods already delivered. It accounts for about 11.5 to 17 percent of the assets for non-financial firms in the G-7 countries (Rajan and Zingales (1995)). In developing countries, limited access to capital markets makes trade credit even more important than bank credit (Fafchamps 2000). Trade credit has cushioned the effects of the global financial crisis on international trade (Chor and Manova (2012)). Its use is so widespread that the main focus of the literature has been on solving the *trade credit puzzle*, that is, why its use is so pervasive even in the presence of a competitive banking sector.

Trade credit is rarely secured on collateral and enforcing repayment through the courts can be problematic.¹ As a result, trade credit is usually granted within an ongoing relationship so that future profitable trade can be used to prevent default.² A large body of work has found evidence of the link between self-enforcing contracts and the provision of trade credit; especially (but not exclusively) in developing countries, or in international transactions, where the self-enforcing mechanism behind repeated trade can substitute for missing contract laws or differences in legal systems.³ This link is also expected to be important when transactions are not entirely legal. For example, when firms operate in the shadow economy or in black markets, such as the drug trade.

This is the first paper that takes into account the limited enforceability of trade credit in an environment where the downstream firm's ability to repay is unobservable to the supplier. Once we consider the contract self-enforceability problem, a whole new set

¹Legal costs may be too high relative to the size of the transaction, and outstanding trade credit is usually placed at the end of the debt priority queue in case of bankruptcy. Furthermore, the buyer may have been affected by a negative shock, leaving nothing for the supplier to foreclose on.

²As Cuñat and Garcia-Appendini (2012) put it: "The frequent occurrence of late payment highlights that it is hard to understand trade credit as a fully contractual, independent, one-off transaction. (...) In most cases trade credit has to be understood as a multi-period, highly non-contractual type of credit that interacts with an ongoing commercial relationship." p. 543

³See Bernstein (1992 and 1996) for the New York diamond trade and the US grain markets, Uchida *et al.* (2006) for Japanese small and mid-sized enterprises and Cuñat (2007) for UK firms. McMillan and Woodruff (1999a and 1999b), Johnson, McMillan and Woodruff (2002) and Fafchamps (1997 and 2000) provide evidence for firms in Vietnam, post-Communist and African countries. Antras and Foley (2011) and Macchiavello and Morjaria (2015) do accordingly in international trade.

of important questions arises. For instance, how does it interact with the asymmetric information problem? What happens when the quality of legal enforceability or the trust between the firms improves? What are the market outcome implications? When addressing these questions, we take the provision of trade credit as given and look at the impact on the different contract characteristics, such as non-payment penalty, quantity of the good sold and repayment.⁴

We build a model where an upstream firm repeatedly supplies a good and offers trade credit to a cashless downstream firm. For instance, the upstream firm ("she") can be a manufacturer and the downstream firm ("he") a retailer. The manufacturer's machinery is used as collateral, making her less credit constrained than the retailer.⁵ The manufacturer has all the bargaining power in dictating the terms of trade. She supplies the retailer with the quantity to be sold to the final consumer and establishes a repayment that is postponed until the sale is made. The retailer places the good in the market and obtains some stochastic revenues. Depending on the revenues, the retailer may be unable to honor the credit agreement. We characterize the optimal contract within the class of stationary contracts.

The manufacturer faces two problems. First, she cannot distinguish a genuine from a strategic default where the retailer privately diverts revenues. To induce repayment, the manufacturer ensures that it is worthwhile for the retailer to repay the credit rather than face retaliation. Informed by the trade credit literature, we restrict attention to the refusal to transact for some time as the unique form of retaliation, since it is widely accepted that the "extra enforceability power of suppliers (as compared to banks) comes from the fact that they can threaten to stop supplying intermediate goods to their customers" (Cuñat and Garcia-Appendini (2012), p. 545).⁶

⁴Instead, the main focus of the trade credit literature literature has been to explain why trade credit is granted. See Cuñat and Garcia-Appendini (2012) for a recent survey.

⁵High credit quality suppliers have a comparative advantage in securing outside finance that they can pass on small, credit-constrained buyers (Boissay and Gropp (2007)).

⁶See McMillan and Woodruff (1999a and 1999b) and Fafchamps (1997) for more evidence in the field. Fehr and Zehnder (2009) conducted a credit market experiment with a double asymmetric information problem and found that "on average lenders renew a contract with a borrower in 66% of the cases if he makes a positive repayment, but only in 18% of the cases if the does not repay." This is despite the fact

The second problem is that, even if the revenues were observable, the trade credit contract cannot be enforced. In particular, no third party can ensure that a minimum repayment is satisfied. The manufacturer needs to give away rents to make the future relationship valuable enough to the retailer so that, today, he does not "take the money and run."

We show that when both problems are present, there is a tension between preventing the retailer from lying and enforcing at least some repayment. To give incentives to reveal the truth, a larger repayment is associated with a shorter trade suspension. As a result, the incentives to take the money and run are larger for a retailer who faces a longer trade suspension as this punishment reduces the future value of the relationship. Therefore, the retailer's ability to walk away constrains the toughness of the trade suspension policy. On the other hand, the manufacturer has incentives to suspend trade for longer than is socially optimal since more retaliation reduces the need for giving away rents and thus she appropriates more surplus. Thus, the need for contract self-enforceability, by shortening the trade suppension policy, may actually be good for welfare.⁷

This result points to potential unexpected detrimental consequences of policies that improve the legal contract enforcement or increase the level of trust (for instance, by encouraging a more frequent trade).⁸ If a minimum level of contract enforcement has been reached, such policies will be followed by longer trade suspension periods that can have an overall negative impact on welfare. In these circumstances, policy efforts are best spent in relaxing firms' credit and liquidity constraints. If the retailer is enabled to borrow, he will be more capable of repaying regardless of the state, making the manufacturer's inability to observe revenues less important.

Our second result shows that the repayment schedule proposed by the manufacturer imitates a debt contract. A debt contract establishes a fixed repayment that guarantees

that a project failure yields no revenues and hence "the lack of repayment does not necessarily imply a lack of willingness to repay" p. 20.

⁷A sufficient condition for this result is that the revenue function is concave enough.

 $^{^{8}}$ We, interchangeably, interpret the value of the future as the quality of contract enforceability or the level of trust between the firms. With a more valuable future, the manufacturer trusts more the retailer to honor the contract.

the continuation of the contract. Otherwise, the manufacturer asks for the highest feasible repayment and suspends trade for a number of periods. By asking the highest possible repayment in the default states, the manufacturer can soften the trade suspension policy and reduce inefficiency. As in this paper, debt contracts have been shown to be optimal when the means to give repayment incentives entail inefficiencies such as liquidating assets (Hart and Moore (1998)), threatening to withhold the last period investment (Bolton and Scharfstein (1990)) or carrying on a costly state verification (Townsend (1979) and Gale and Hellwig (1985)). In particular, as compared to the costly state verification literature, we replace the binary state verification decision⁹ with an optimally-designed trade suspension policy which adjusts the length of suspension to the reported state in order to minimize inefficiency.

In two widely cited papers, McMillan and Woodruff (1999a and 1999b) examine the self-enforcing contracting of supplier trade credit in Vietnam. McMillan and Woodruff (1999b) find anecdotal evidence of debt-like contracts being used. These contracts are accompanied by trade suspension: "One (manager) (case #12) sent employees to visit a customer every day to ask for a late payment. 'After a few weeks of negotiation, the firm got back part of the debt and stopped selling to this customer.' Another (case #10) after some negotiation accepted 70% of the amount owed (...)" (p. 642). The authors find it surprising that "firms often try to keep the relationship going despite defection" (p. 642) whereby the supplier tries to get restitution which sometimes involves forgiving part of the debt. As a result, they conclude that retaliation is not "as forceful as in the standard repeated-game story" (p. 637). We improve on the existing literature by showing that debt contracts, together with the presence of enforceability problems, are able to rationalize such a puzzling mitigated retaliation policy.

Finally, we find that the optimal contract has important market outcome implications in that the manufacturer, inefficiently, always sells too little.¹⁰ This result is not related

⁹Mookherjee and Png (1989) show that under stochastic verification the optimal contract may not be a debt contract. See footnote 18 on how to replace the length of trade suspension by a probability of permanent termination.

¹⁰A sufficient (but not necessary) condition for this distortion to emerge is that the revenue function is log-supermodular in the quantity and the state of demand. This mild condition ensures that increasing

to well-known sources of quantity distortion occurring between vertically-related firms such as double marginalization or screening.¹¹ We identify a novel explanation for the underprovison of quantity: to ensure the retailer's good behavior, the manufacturer needs to leave him a rent, and this rent increases with the quantity supplied.

This result has important implications for the empirical industrial organization literature. It shows that having enough instruments to remove vertical externalities from market power is not enough to assume that efficient quantities are supplied when trade credit and liquidity constraints matter. Underprovision of quantity has been established before in static frameworks with credit constraints. For instance, Burkart and Ellingsen (2004) find that limiting the credit extended to the retailer is optimal in order to reduce the scope for diverting resources before those are invested into producing a (verifiable and observable) output.

The closest related paper is Cuñat (2007). He is the first to use the supplier's threat of stopping the delivery of goods as an enforcement mechanism. In his model, suppliers and buyers trade repeatedly. A successful supplier-buyer pair can update the startup technology to a more productive mature one. When this happens, the supplier becomes the only input provider. As a monopolistic supplier, she can enforce the repayment of trade credit with the threat of moving back to the startup technology. Instead, we start from the monopolistic supplier situation and consider the possibility that the supplier not only cannot legally enforce the repayment of trade credit, but also cannot observe the buyer's ability to repay.

In our analysis, we restrict attention to the class of stationary contracts (i.e. provided that the firms trade, the same contract is offered every period). In an optimal dynamic

the quantity is relatively more important when the demand state is high.

¹¹In Levin (2003) with hidden information, the agent privately observes the state of the world before the quantity is chosen. As a result, the quantity is used to screen the types. If the future value of the relationship is not large enough, the principal cannot credibly screen out all type of agents and the optimal contract pools the most efficient ones. In our model, the state of the world is privately learned by the agent after the quantity is chosen and therefore the quantity is not used to screen. Instead, due to liquidity constraints, the screening is done via a combination of trade suspension and the debt-like repayment. Pooling arises as a mean to give rents to the agent. As a result of the different pooling rationale, as the discount factor tends to 1, Levin's pooling result disappears while the debt contract continues to be optimal.

contract, the surplus is not shared with the retailer every trading period. Instead, the manufacturer should extract surplus at the beginning of the relationship and backload the reward (Thomas and Worrall (1988 and 1994)). Therefore, this contract poses credibility issues when switching retailers is easy, as pointed out by Ray (2002),¹² making it more likely for contracts to be stationary. Which type of contract is more likely to emerge in inter-firm relations is an empirical question.¹³ Brown et al. (2004 and 2012) look at the creation of relational contracts in the laboratory. They find that successful long-term relations exhibit stationary rent-sharing already from the first period (even if dynamic rent extraction is possible).

The paper proceeds as follows. Section 2 introduces the model. Section 3 characterizes the optimal contract. The role of exclusivity is studied in Section 4.1, by assuming that there is a competitive fringe of downstream firms. Section 4.2 looks at the case where revenues are observable but yet not contractible. It shows that debt contracts are nonetheless optimal. Finally, Section 5 concludes. The proofs are relegated to the Appendix.

2 The setup

A manufacturer and a retailer have the opportunity to trade over an infinite horizon of discrete periods. Both firms discount future payoffs with factor $\delta \in (0, 1)$. In each period t, the manufacturer produces q_t units of a good at marginal cost c > 0 and needs a retailer to market the product to the final consumer. The retailer sells the good at no cost and earns revenues $R(q_t; s_t)$ where s_t denotes the state of the revenues. We assume that s_t is an *iid* random variable drawn from a cumulative distribution function $H(s_t)$. The corresponding probability density function $h(s_t)$ is continuous and strictly positive for all s_t on the support $[\underline{s}, \overline{s}]$. Furthermore, we assume that the revenues are

 $^{^{12}}$ In particular, as "soon as continuation payoffs depart in an adverse way from this starting point (... [i.e. the initial rent-extraction phase]), why can't the principal simply fire this agent and replace him with another?" p 564.

¹³Note that in this model there is no permanent type of retailer (such as the level of reliability or honesty) and hence, there is nothing to learn over time.

continuous, increase in q_t and s_t and are concave in q_t . Denote the expected revenues by $R_E(q_t) = \int_{\underline{s}}^{\overline{s}} R(q_t; s_t) dH(s_t)$. Since our goal is to explore the impact that liquidity constraints have on the optimal contract, we assume that the smallest possible revenues are sufficiently small (or the value of the future sufficiently large) that the manufacturer never sets a unique repayment below $R(q; \underline{s})$.¹⁴

Assumption 1 $R(q; \underline{s}) \leq \delta R_E(q)$.

The quantity of the good is publicly observable. However, s_t is the private information of the retailer. For instance, there may be uncertainty with respect to the willingness of final consumers to pay, or the number of units that are actually sold of a perishable good.¹⁵ Other interpretations for s_t include a certain willingness to pay but either the goods or the revenues are stolen now and then (for instance, by the retailer's workers).

The retailer is credit and liquidity constrained and thus cannot pay for q_t upfront. The manufacturer sells the good on trade credit, that is, the retailer repays the manufacturer after obtaining $R(q_t; s_t)$ and within the same period (no interest rate is charged).¹⁶ The manufacturer proposes a quantity forcing contract which consists of the quantity, q_t , and a repayment, $D(\tilde{s}_t)$, for each reported state \tilde{s}_t . We consider this contract, which is equivalent to a nonlinear scheme, to remove quantity distortions coming from the double marginalization problem.

To keep the analysis simple, we assume that, provided that the firms trade, the same quantity is offered every period. Furthermore, the retailer is not able to save (i.e., any profits are consumed within the same period).

Assumption 2 We restrict attention to the class of stationary contracts that constitutes a Perfect Public Equilibria.

 $^{^{14}}$ Section 3 of Troya-Martinez (2013) uses a two state example to derive the optimal contract when this assumption is not satisfied.

¹⁵In the first case, the revenue function could take the following form: $R(q_t; s_t) = P(q_t; s_t)q_t$, where $P(q_t; s_t)$ is the inverse demand function. A potential example for the second case is: $R(q_t; s_t) = P(q_t)s_tq_t$, where $P(q_t)$ is a fixed price set by the manufacturer.

¹⁶We restrict attention to "one-part" trade credit contracts. These contracts do not have an explicit discount for early payment nor an interest rate. The National Survey for Small Business Finance (a sample of 3000 US firms in 1998) shows that when firms deal with their main supplier, 49% of the contracts are one part.

Since the same contract is offered every trading period, we can drop the time subscripts $\{q, D(\tilde{s})\}$. Once we confine ourselves to stationary contracts, trade suspension is needed to provide the retailer with incentives to repay. Informed by the trade credit literature, we restrict attention to a T-period trade suspension policy. In particular, the manufacturer commits to a length of trade suspension, $T(\tilde{s})$, for each reported state.¹⁷ Choosing the length of trade suspension is equivalent to choosing the probability of a permanent termination.¹⁸ During the trade suspension, both firms get a constant outside option which is normalized to $0.^{19}$ When $R(q; \underline{s}) > 0$, the absence of repayment is an observable breach of the agreement. Abreu (1988) shows that it is optimal to impose the maximum punishment (i.e., permanent termination) following this deviation. When $R(q; \underline{s}) = 0$, no repayment does not necessarily imply a breach of the agreement, and it is followed by $T(\underline{s})$ periods of trade suspension (which may include permanent termination). An alternative interpretation to suspending trade is to keep trading but in less profitable terms (for instance by diminishing the quality of the good).

Finally, we assume that no repayment can be legally enforced, including $D(\underline{s})$. We look for the contract $\{q, D(\tilde{s}), T(\tilde{s})\}$ that maximizes the manufacturer's profits within this class.²⁰

The timing is summarized in Figure 1. In each period, the manufacturer offers a contract to the retailer. The retailer rejects or accepts, and if he accepts, he receives q from the manufacturer and places it in the market. Then an *iid* shock is realized (and observed only by the retailer), which determines the size of the revenues. Finally, the retailer decides how much to repay, and the relationship is suspended for a number of periods if it is specified in the agreement.²¹ Note that the quantity is delivered before

¹⁷T-period trade suspension contracts are also used by credit reference agencies since they "simplify the information about each agent i with a credit report showing when the agent last 'cheated' (e.g., paid late or not at all). This information is kept on the agent's record for a set number of years T, after which time it is erased." Fafchamps (2010), p. 57.

¹⁸Denote by $t(\tilde{s})$ the probability of terminating forever with the retailer. Then $\delta^{T(\tilde{s})} = 1 - t(\tilde{s})$.

 $^{^{19}}$ We relax this assumption in Section 4.1.

²⁰We discuss in footnote 23 the implications of giving the bargaining power to the retailer instead.

²¹Termination involves inefficiencies just as in the subjective performance evaluation of Levin (2003) or Fuchs (2007). Committing to the trade suspension policy improves the manufacturer's profits and can be sustained under the belief that if the manufacturer does not retaliate, the retailer will always default.



Figure 1: Timing of the game. M stands for Manufacturer and R for Retailer.

the revenues are known and hence, it is not used to screen retailers with different revenue levels.

Let Π_R and Π_M denote, respectively, the retailer's and manufacturer's present discounted value from date t onwards:

$$\Pi_R = R_E(q) - \int_{\underline{s}}^{\overline{s}} D(s)h(s)ds + \int_{\underline{s}}^{\overline{s}} \delta^{T(s)+1} \Pi_R h(s)ds$$
(1)

$$\Pi_M = \int_{\underline{s}}^{\overline{s}} D(s)h(s)ds - cq + \int_{\underline{s}}^{\overline{s}} \delta^{T(s)+1} \Pi_M h(s)ds$$
(2)

The quantity q^{FB} that maximizes the joint surplus, $R_E(q) - cq$, is defined by: $\frac{\partial R_E(q^{FB})}{\partial q} = c$. If the manufacturer observes the revenues and can enforce any repayment, she offers q^{FB} , asks to repay all the revenues and never suspends trade.

3 Optimal contract

We first find the conditions under which the contract elicits the truth and is self-enforceable. In doing so, we also establish that the proposed repayment schedule imitates a debt contact. We then derive the features of the optimal contract in two steps. First, we characterize the contract as a function of q and then we characterize the optimal q. We finish this section with an example.

Suppose that state s has happened. By the time the retailer learns about s, q has

One simple way to make the contract credible is to give the bargaining power to the retailer. In this case, the manufacturer receives no surplus and is indifferent between terminating or continuing to trade. See footnote 23 for more details.

been supplied and hence does not depend on s nor on the report \tilde{s} . With a slight abuse of notation, denote the retailer's profits associated with \tilde{s} , for a given s and q, as:

$$\pi_R(\tilde{s};s) = R(q;s) - D(\tilde{s}) + \delta^{T(\tilde{s})+1} \Pi_R$$
(3)

where Π_R is defined in (1).

The retailer can always guarantee himself R(q; s) by stopping the contract after the sale. Therefore, for the contract to be self-enforceable, the repayment $D(\tilde{s})$ needs to be weakly smaller than the continuation value associated with the report \tilde{s} :

$$D(\tilde{s}) \le \delta^{T(\tilde{s})+1} \Pi_R \mid_{\tilde{s}=s} \forall s \tag{DE}_s$$

Note from (3) that if the retailer were not credit and liquidity constrained, his reporting incentives would not depend on s, as it is equally costly for any type s of retailer to report any \tilde{s} . Formally, there is no sorting condition: $\frac{\partial^2 \Pi_R(\tilde{s};s)}{\partial \tilde{s} \partial s} = 0$. Credit and liquidity constraints confine the repayment to the current revenues, and as a result, the limited liability condition LL_s indirectly links \tilde{s} with s.

$$D(\tilde{s}) \le R(q; s) \quad \forall s, \tilde{s} \tag{LL}_s$$

Let $u(\tilde{s})$ be the part of the retailer's payoff in (3) that does not directly depend on s: $u(\tilde{s}) = -D(\tilde{s}) + \delta^{T(\tilde{s})+1} \Pi_R$. The independence between the incentives to report a demand state and the actual demand state makes the task of inducing truth-telling simpler. In particular, truth-telling is guaranteed when $u(\tilde{s})$ is constant. For any two reports \tilde{s} and \tilde{s}' :

$$-D(\tilde{s}) + \delta^{T(\tilde{s})+1} \Pi_R \mid_{\tilde{s}=s} = -D(\tilde{s}') + \delta^{T(\tilde{s}')+1} \Pi_R \forall s, \tilde{s}'$$
 (*IC*_s)

To see why this is the case, first note that it is always possible to report less revenues: $\tilde{s} \leq s$. To prevent underreporting, $u(\tilde{s})$ must be non-decreasing. Furthermore, when the contract does not ask to repay all the revenues, then it may be possible to overreport:



Figure 2: Repayment schedule

 $\tilde{s} \geq s$. A constant $u(\tilde{s})$ prevents such deviation. In what follows, we show graphically, that even when only underreporting is feasible, in the optimal contract $u(\tilde{s})$ is constant, that is, the retailer is indifferent to reporting any state.

For a given q, Figure 2 depicts R(q; s) as a function of s. First, suppose that the retailer has earned the highest possible revenues $R(q; \bar{s})$. Obviously, the manufacturer does not suspend trade if this state is reported. Furthermore, she needs to leave the retailer a rent to have \bar{s} reported: $D(\bar{s}) < R(q; \bar{s})$. If $u(\bar{s})$ were to be increasing, a retailer with revenues $R(q; s) \in [D(\bar{s}), R(q; \bar{s}))$ would report \bar{s} . When this deviation is feasible, $u(\tilde{s})$ must be constant. This implies that the same contract, $D(\bar{s})$ and $T(\bar{s}) = 0$, is offered to the retailer when $s \in [s^*, \bar{s}]$, where s^* is the state where the retailer repays all the revenues $D(\bar{s}) = R(q; s^*)$.

Since the retailer has no money left when $s = s^*$, for $s < s^*$, the repayment must be smaller $D(s) < R(q; s^*)$. Low reported revenues can be penalized by either increasing D(s) or T(s), which decreases the retailer's continuation value. To prevent a retailer with revenues above $R(q; s^*)$ from underreporting, trade suspension is needed. How does the manufacturer choose between D(s) and T(s)? Increasing T(s) is costly for the manufacturer in terms of lost future sales but she benefits from a larger D(s). Thus, D(s) = R(s;q), when $R(q;s^*)$ cannot be paid and hence (LL_s) bind for $s \leq s^*$.

For $s < s^*$, deviations in the report can only occur downwards and can be prevented with a non-decreasing $u(\tilde{s})$. Because the payment is already at the maximum level, an increasing $u(\tilde{s})$ is the result of a harsher trade suspension policy. Since trade suspension is inefficient, trade is suspended just enough to make the retailer indifferent in his reporting strategy. In other words, in the optimal contract $u(\tilde{s})$ is constant for $s \leq s^*$. Denote $u(\tilde{s})$ by u.

Figure 2 also shows that the repayment schedule imitates a debt contract. In the good states $(s \ge s^*)$, the retailer pays a fixed amount equal to $R(q; s^*)$ and becomes the residual claimant; while in the default states $(s < s^*)$, he repays with all the revenues. This contract is optimal because it minimizes the inefficiency associated with trade suspension (by trading-off larger repayments for shorter termination periods), while inducing the retailer to report the truth.

Using the fact that $u(\tilde{s}) = u$, we can simplify (DE_s) and (IC_s) . Let us write u for s^* : $u = -R(q; s^*) + \delta \Pi_R$. Plugging this value into $\Pi_R(s; s)$ in (3) and taking the expectation over all possible states yields: $\Pi_R = R_E(q) - R(q; s^*) + \delta \Pi_R$. Rewriting this expression, we obtain:

$$\Pi_R = \frac{R_E(q) - R(q; s^*)}{1 - \delta}$$
(4)

Since the retailer earns the same u at any state, we can reduce (DE_s) to one constraint (DE). Writing again u at s^* and using (4) yields:

$$u = \frac{\delta R_E(q) - R(q; s^*)}{1 - \delta} \ge 0 \tag{DE}$$

where the inequality follows from (DE_{s^*}) . In each period, the retailer obtains at least the difference between what he can expect to walk away with next period if he stays in the relationship and the maximum repayment today.

Similarly, using the fact that $u = u(s^*) = u(s)$ for $s < s^*$, together with (4), we obtain

the new (IC):

$$\delta^{T(s)+1} = \delta - (1-\delta) \frac{R(q;s^*) - R(q;s)}{R_E(q) - R(q;s^*)}$$
(IC)

Two observations are in order. First, T(s) decreases in the current revenues R(q; s)(and hence, the repayment) and it is zero when $s = s^*$. Second, trade suspension is permanent if: *i*) the worse demand state occurs and it is such that it yields no revenues (i.e., $R(q; \underline{s}) = 0$) and *ii*) (*DE*) binds. Otherwise, trade suspension is temporary.

The last observation highlights that the choice of $T(\underline{s})$ introduces a tension between inducing truth-telling and enforcing the contract. A longer trade suspension reduces the incentives to underreport revenues. However, when $s = \underline{s}$, $R(q; \underline{s}) > 0$ and (DE) binds, the manufacturer is constrained in the choice of T(s) by the contract self-enforceability. In particular, the trade suspension's length is chosen so that the retailer is indifferent between stealing today's revenues $R(q; \underline{s})$ or the next time he has the chance to do it, that is, after $T(\underline{s}) + 1$ periods: $R(q; \underline{s}) = \delta^{T(\underline{s})+1}R_E(q)$. The dynamic enforceability problem limits how tough the manufacturer can be with the retailer, and the resulting trade suspension is temporary. Instead, when $R(q; \underline{s}) = 0$, this tension vanishes because there are no current revenues to run away with. The manufacturer can then be as tough as she wishes, and termination is permanent. When the future becomes so valuable that (DE) is slack, the same amount of trade suspension becomes more effective in punishing the retailer and a temporary trade suspension is implemented if $s = \underline{s}$.

Proposition 1 For a given quantity q, there is a threshold s^* such that the manufacturer:

(i) implements a repayment schedule that resembles a debt contract in which the retailer repays R(q; s) if $s \leq s^*$ and $R(q; s^*)$ otherwise,

(ii) adopts a contingent contract renewal policy whereby a larger repayment is associated with a (weakly) shorter trade suspension. Zero repayment triggers permanent trade suspension if: (a) a minimum repayment is always feasible, or (b) a minimum repayment is not feasible and the dynamic enforceability constraint binds, and

(iii) allows the profit of the retailer to be at least one-period expected revenues.

We are left to find the optimal pair of q and s^* that maximizes Π_M subject to (DE)

and (IC). When choosing s^* , the manufacturer faces the following trade-off: a larger s^* involves a larger expected repayment today; however, this comes at the cost of longer termination²² to keep the truth-telling incentives unchanged and hence, a tighter self-enforceability problem. An increase in the quantity leads to an increase in the total payment as well as an increase in the production costs.

Using Proposition 1, it is possible to rewrite Π_M in terms of Π_R :

$$\Pi_M = \frac{R_E(q) - cq}{1 - \int_s^{\overline{s}} \delta^{T(s) + 1} h(s) ds} - \Pi_R$$
(5)

Note that the first term contains the total surplus, therefore, the quantity is distorted downwards if: i) Π_R increases in q, and/or ii) the length of trade suspension T(s) is non-decreasing in q for all $s \leq s^*$. Both conditions are met if the revenue function is log-supermodular in q and s.

Proposition 2 In the optimal contract, a sufficient (but not necessary) condition for the quantity to be distorted downwards is $\frac{\partial^2 \ln R(q,s)}{\partial q \partial s} \ge 0.$

This property of the revenue function has been known to be crucial for comparative statics in a wide range of models (Athey (2002) and Costinot (2009)). It captures the idea that increasing the quantity is relatively more important when the demand state is high. Since the incentives to misbehave are larger for higher demand states, the profit given to the retailer as well as the punishment should increase with a larger quantity. Therefore, despite using a quantity forcing contract, the quantity sold by the manufacturer is underprovided.²³

 $[\]frac{1}{2^2} \text{The length of trade suspension increases in } s^*: \frac{\partial \delta^{T(s)}}{\partial s^*} = -\frac{1-\delta}{\delta} \frac{\partial R(q;s^*)}{\partial s^*} \frac{R_E(q) - R(q;s)}{(R_E(q) - R(q;s^*))^2} < 0 \text{ for } \forall s \leq s^*$ where the inequality follows from the need to have $\pi_R > 0$, which requires $R_E(q) > R(q;s^*)$.

²³When the retailer has the bargaining power, he chooses q and s^+ to maximize: $\pi_R = \frac{R_E(q)-cq}{1-\int_s^{s^+} \delta^{T(s)+1}h(s)ds} - \pi_M$ subject to (*IC*), (*DE*), (*LL*_s) for $s \leq s^+$ and the participation constraint of the manufacturer, $\pi_M = 0$. The retailer does not need to share surplus with the manufacturer and, since the retailer keeps all the profits, (*DE*) binds for a smaller set of parameters. Thus, we conjecture the quantity distortion to be significantly smaller. The quantity is, nonetheless undersupplied. This is because a small quantity distortion has a second order effect on the surplus but a first order effect on revenues and hence on the incentives to misbehave. As a result, retaliation can be softened and this has a first order effect on the surplus.

Next section illustrates these results with the multiplicative revenue function.

3.1 Example with a multiplicative shock

Let us consider the multiplicative revenue function, R(q;s) = sR(q), which is logsupermodular since $\frac{\partial^2 \ln sR(q)}{\partial q\partial s} = 0$. With this function, (DE) becomes $\delta E(s) - s^* \ge 0$ where E(s) is the expected state. Note that as the future becomes more valuable (δ increases), the manufacturer can always choose a high enough s^* to make this constraint bind. By (IC), increasing s^{*} triggers a tougher trade suspension. The next proposition shows that when the future becomes very valuable, it is optimal for the manufacturer to leave (DE) slack, even if this means giving away larger rents to the retailer u > 0. Furthermore, it shows that when the self-enforceability problem does not bite, the optimal s^* and q do not depend on δ . A more valuable future helps the manufacturer enforce the contract but does not help her tackle the asymmetric information problem. In contrast, when (DE) binds, s^* and q increase in δ . The repayment established by the debt contract is bounded by what the retailer expects to steal the next period if he does not repay: $R(q; s^*) = \delta E(s) R(q)$, and a more valuable future allows the manufacturer to increase this payment. Finally, it shows that the per-period welfare does not necessarily increase with a more valuable future. In particular, define the elasticity of marginal revenue as: $\sigma(q) = \frac{-R''(q)R(q)}{R'(q)^2}$. This measure tell us that an increase in revenues by 1% decreases marginal revenue by $\sigma(q)$ %. If $\sigma(q)$ is large enough, which requires the revenue function to be sufficiently concave, the per-period welfare is non-monotonic in δ . Denote by δ_{DE} the largest δ such that (DE) binds at the optimal contract.

Proposition 3 For the case of the multiplicative shock, in the optimal contract:

- (i) s^* and q are increasing in the discount factor δ if $\delta < \delta_{DE}$
- (ii) s^* and q are independent of δ if $\delta_{DE} < \delta$
- (iii) the dynamic enforceability constraint does not bind for large enough δ : $\delta_{DE} < 1$

(iv) if the elasticity of the marginal revenue, $\sigma(q)$, is such that $\sigma(q) \geq \frac{1}{\delta_{DE}} - 1$, the per-period welfare is non-monotonic in δ .



Figure 3: Optimal q and s^* when R(q) = (10 - q) q, c = 2 and $s \sim U[0, 1]$

Figure 3 depicts the optimal s^* and q as the future becomes more valuable for $s \sim U[0,1]$. Trade occurs if the value of the future or level of trust is large enough $(0.45 \leq \delta)$. To the left of the dashed line at $\delta_{DE} = 0.82$, both the contract enforceability and the asymmetric information problems bind. To the right of the dashed line, only the asymmetric information problem bites. Both s^* and q weakly increase with δ because more trust relaxes the self-enforceability problem. This enables the manufacturer to ask for a larger repayment (larger s^*) and offer a larger quantity in exchange. When $\delta > \delta_{DE}$, the choice of q and s^* is not affected by δ . Therefore, the quantity remains distorted downwards even as the firms become arbitrarily patient.

The left-hand side of Figure 4 depicts the expected trade suspension associated with s^* . Suppose first that only the asymmetric information problem binds (i.e. $\delta \geq \delta_{DE}$). The manufacturer supplies q = 2 and asks for a repayment of 40% of R(q). She chooses a trade suspension policy that allows her to implement this contract. When the future becomes more valuable, the same policy becomes more effective and she can relax it. As a result, the trade suspension policy becomes more lenient with the value of the future.

Suppose now that the value of the future is so low that enforcing the contract becomes problematic (i.e. $\delta < \delta_{DE}$). Then, a tougher retaliation is needed to be able to supply q = 2 and have a repayment equal to 40% of R(q). Such a tough trade suspension policy, together with the low value of the future, reduces the worthiness of staying within the relationship so much that the retailer prefers to walk away. Thus, the manufacturer needs to soften the trade suspension policy and as a result, decrease the repayment and quantity.



Figure 4: Optimal expected T(s) and actual T(s) if $\delta = 0.77 (\text{grey})/\delta = 0.85 (\text{black})$ when R(q) = (10 - q) q, c = 2 and $s \sim U[0, 1]$

The right-hand side of Figure 4 depicts the optimal trade suspension policy as a function of the realized state. The grey line corresponds to a discount factor below δ_{DE} , while the black line is drawn for $\delta > \delta_{DE}$. Both discount factors have the same associated expected T(s). Note that from the right-hand side of Figure 3, $s^* \leq 0.4$ for all δ . Since the worse state of the world yields no revenues, from Proposition 1, permanent termination is triggered only if (DE) binds (i.e. the grey line). Instead, if (DE) does not bind and nothing is repaid, the manufacturer only terminates for ten periods. In line with equation (IC), as the state increases, the length of trade suspension decreases, but it does so faster for the case of $\delta < \delta_{DE}$. Indeed, when the self-enforceablity problem is a constraint, the

harshness of the punishment is limited by the retailer's ability to walk away.



Figure 5: Per period profits when R(q) = (10 - q) q, c = 2 and $s \sim U[0, 1]$

Figure 5 depicts the per-period average surplus generated in the market, $(1 - \delta)(\pi_R + \pi_M)$, and how it is shared between both firms. Since the manufacturer has all the bargaining power, as δ increases, she keeps increasing s^* and q as long as her profits increase. For the retailer, an increase in s^* means keeping a smaller share of the surplus, but also a larger q and hence, larger rents. Since both s^* and q weakly increase with δ , the retailer's per period profits are not monotonic in the level of trust.

Overall, if R(q) is sufficiently concave, the total surplus is also non-monotonic in the level of trust. An increase in s^* decreases the total surplus via the inefficiency created by the trade suspension, while an increase in q increases the total surplus because the quantity is under-supplied. For a level of trust below $\delta = 0.69$, the increase in efficiency from a larger volume of trade compensates for the inefficiency from a longer trade suspension. When the revenues are concave enough, the positive effect of increasing q decreases with the quantity, and after $\delta = 0.69$, the negative effect dominates the positive one. The manufacturer, nonetheless, keeps increasing both q and s^* because this allows her to keep a larger share of the albeit smaller surplus. Thus, when the level of trust is intermediate, the self-enforceability problem is good for welfare because it limits how inefficiently tough the manufacturer can be with the retailer.

Corollary 1 When the level of trust is intermediate, policy efforts targeted at further increasing the level of trust may not be desirable.

There are policies that make the contracts more legally enforceable or increase the level of trust. Proposition 3 suggests that when there is a minimum level of contract enforceability or trust, such policies are not only ineffective at increasing the volume of trade but are also welfare detrimental in that they trigger too much trade suspension. The lack of legal enforceability protects the retailer (and the welfare) from the manufacturer's inefficient rent-extraction. The implications of this result are important for policy makers. It suggests that when the level of trust is intermediate, policy efforts should be redirected first at relaxing credit and liquidity constraints. If the retailer is enabled to borrow, he will be more capable to repay regardless of the state. As a result, the manufacturer's inability to observe revenues becomes less important.

4 Robustness checks

4.1 Manufacturer's outside option

Using the example from Section 3.1, we determine if the contract is substantially affected by allowing the manufacturer to obtain a non-negative outside option, π_O , every period that she is in the trade suspension phase. We can interpret this outside option as the manufacturer selling through an inefficient competitive fringe of sellers while she is suspending trade with the main retailer. This possibility introduces a participation constraint for the manufacturer that the contract needs to satisfy. We show that, provided that the one-period expected profits of selling to the retailer are larger than π_O , the contract is qualitatively unaffected.

Proposition 4 When the self-enforceability problem is present, the optimal q and s^{*} are unaffected by π_O . Otherwise, a larger π_O is associated with a larger s^{*} and q.

We illustrate these findings in Figures 6 and 7 where we reproduce the Figures of Section 3.1 in grey and plot the new solution for $\pi_O = 0.8$ in black.



Figure 6: Optimal q and s^* when R(q) = (10 - q) q, c = 2, $s \sim U[0, 1]$ and $\pi_0 = 0.8$

The manufacturer is more reluctant to sell to the retailer when the value of the future is low. If trade nonetheless occurs and the future is sufficiently valuable ($\delta > 0.88$), then the outside option makes retailation less onerous for the manufacturer. She claims a larger fixed repayment (larger s^*) and suspends trade for more states of the demand. By appropriating more surplus from the retailer, the manufacturer is able to offer a larger (but still distorted) quantity. Instead, when enforcing the contract is problematic ($\delta \leq 0.88$), the contract offered is identical to the one offered in the absence of an outside option. The manufacturer would like to choose a larger s^* , but cannot because that would decrease the value of the relationship, making the retailer walk away. Note as well that the range of parameters where the contract self-enforceability matters is larger. In Figure 7, we plot the expected trade suspension policy and the per period profits from the relationship (excluding the outside option that the manufacturer obtains when dealing with a third party). Up to $\delta_{DE} = 0.82$, the per period profits are the same as in the absence of an outside option. Otherwise, the per period profits are smaller. Despite the larger quantity supplied, the manufacturer suspends trade more often, and less surplus is created within the relationship.



Figure 7: Per period profits when R(q) = (10 - q) q, c = 2, $s \sim U[0, 1]$ and $\pi_O = 0.8$

In the future, it would be interesting to explore in more detail the role of retail competition by looking at how the optimal contract and trade suspension policy change when the manufacturer can sell through several competing retailers simultaneously.

4.2 Observability of the revenues

In this Section, we consider the scenario where the revenues are observable to the manufacturer. However, they are not verifiable and hence cannot be pledged. Falling to repay D(s) is now an observable deviation from the agreement. Abreu (1988) shows that it is optimal to impose the maximum punishment (i.e., permanent termination) if the revenues are misreported. Furthermore, given the inefficiency of trade suspension, it is optimal to continue trading when the revenues are truthfully reported. The manufacturer still needs to make sure that the contract is self-enforceable. In particular, the new dynamic enforceability constraint is:

$$D(s) \le \delta \Pi_R \,\forall s \tag{DE'_s}$$

The retailer can still guarantee himself R(q; s) by stopping the contract after the sale. Therefore, the repayment D(s) needs to be weakly smaller than the continuation value which is now the same for all s since there is no particular trade suspension associated with a given s. It is easy to see that the dynamic enforceability constraint is more binding for a retailer who got larger revenues since the repayment is expected to be larger. This is in stark contrast with the case of unobservable revenues. From (DE_s) , a retailer with smaller revenues has a more stringent constraint. This is due to the smaller continuation value resulting from the impending trade suspension used to screen retailers.

Proposition 5 For a given quantity q, there is a threshold $s^{*'}$ such that the manufacturer: (i) implements a repayment schedule that resembles a debt contract in which the retailer repays R(q; s) if $s \leq s^{*'}$ and $R(q; s^{*'})$ otherwise,

(ii) renews the contract every period if the agreed repayment is met, and

(iii) supplies a downwards distorted quantity.

The previous proposition shows that a debt contract is nonetheless optimal albeit for a different reason. With asymmetric information, a debt contract is optimal because it minimizes the inefficiency associated with trade suspension. Instead, with symmetric information, a debt contract, by asking for the maximum repayment in lower states can decrease the repayment in higher states, thereby relaxing the dynamic enforceability constraint. In a static framework, this result is reminiscent of the findings by Innes (1990). Under some conditions, Innes (1990) finds that the debt contract is optimal in an environment with moral hazard, limited liability and an observable output. The desirability of the contract is associated with making the agent the residual claimant in the good times and extracting all the surplus in the bad ones.

Finally, the quantity is also undersupplied. The retailer should be given rents so he does not walk away from the contract, and these rents should increase with the quantity provided.

5 Conclusions

The goal of this paper has not been to explain why trade credit is offered by a supplier. Taking this decision as given, we explore how trade credit affects the different contract characteristics. We find that the optimal contract resembles a debt contract. Furthermore, trade credit bundled with liquidity constraints has an important impact on the market outcome in that the quantity sold is lower than the efficient one.²⁴

This model delivers a set of testable predictions. For instance, partial repayment is likely to be followed by temporary trade suspension when the asymmetry of information between the manufacturer and the retailer is important. Instead, when there is symmetry of information between firms, partial repayment is followed by continued trade.

The variability in the value of the future or level of trust can also be used to test the model's predictions. Good proxies of the level of trust are frequency of trade or improvement in the legal contract enforcement. The quantity is expected to increase with the level of trust, while the expected length of trade suspension has a non-monotonic pattern. In particular, when the trust level is intermediate, a further increase in trust increases the harshness of the trade suspension policy. Instead, when the level of trust is high, further increases in trust lead to more lenient trade suspension policies.

6 Appendix

Proof of Proposition 1. To prove part (*i*), let us first rewrite the payoffs as follows: $\Pi_{R} = \frac{R_{E}(q) - E(D)}{1 - E(\delta^{T+1})} \text{ and } \Pi_{M} = \frac{E(D) - cq}{1 - E(\delta^{T+1})} \text{ respectively, where } E(D) = \int_{\underline{s}}^{\overline{s}} D(s)h(s)ds \text{ and } E(\delta^{T+1}) = \int_{\underline{s}}^{\overline{s}} \delta^{T(s)+1}h(s)ds.$

Assume by contradiction that there is a subset $C \subset S$ with non-zero probability measure such that for $s \in C$ both T(s) > 0 and D(s) < R(q, s) at the optimal contract. Consider

 $^{^{24}}$ Note that this result is not in contradiction with the view of trade credit as a way to foster sales as without trade credit no sale is possible.

an alternative contract:

$$\{\widehat{D}(s),\widehat{T}(s)\} = \begin{cases} \{D(s),T(s)\} & \text{if } s \notin C\\ \{D(s)+\rho(s),T(s)-\gamma(s)\} & \text{if } s \in C \end{cases}$$

It is possible to find $\rho(s), \gamma(s) > 0$ defined on a compact set C such that $\widehat{D}(s) \leq R(q, s)$, $\widehat{T}(s) \geq 0, \ \widehat{u} = u$ and $\widehat{\Pi}_R = \Pi_R$. If $s \notin C$ then $\rho(s), \gamma(s) = 0$. Take some $\gamma(s) > 0$, satisfying $\widehat{T}(s) \geq 0$, and set $\rho(s)$ such that: $\rho(s) = (\delta^{T(s)+1-\gamma(s)} - \delta^{T(s)+1})\Pi_R > 0$. Then

$$\widehat{\Pi}_{R} = \frac{R_{E}(q) - E(D) - E(\rho)}{1 - E(\delta^{\widehat{T}+1})} = \frac{R_{E}(q) - E(D) - (E(\delta^{\widehat{T}+1}) - E(\delta^{T+1}))\frac{(R_{E}(q) - E(D))}{1 - E(\delta^{T+1})}}{1 - E(\delta^{\widehat{T}+1})} = \frac{(R_{E}(q) - E(D))(1 - E(\delta^{T+1}) - E(\delta^{\widehat{T}+1}) + E(\delta^{T+1}))}{(1 - E(\delta^{\widehat{T}+1}))(1 - E(\delta^{T+1}))} = \Pi_{R}$$

and

$$\widehat{u}(s) = -D(s) - \rho(s) + \delta^{\widehat{T}(s)+1} \widehat{\Pi}_R = -D(s) + \delta^{T(s)+1} \Pi_R = u(s)$$

Considering the functions $\epsilon \rho(s)$ and $\epsilon \gamma(s)$ and taking an arbitrary small $\epsilon > 0$, by the Weierstrass theorem, it is possible to find ϵ for which both $\widehat{D}(s) \leq R(q; s)$ and $\widehat{T}(s) \geq 0$. Consider now the manufacturer's profit: $\widehat{\Pi}_M = \frac{E(D) + E(\epsilon \rho) - cq}{1 - E(\delta^{T+1}) - (E(\delta^{T+1}) - E(\delta^{T+1}))} > \frac{E(D) - cq}{1 - E(\delta^{T+1})} = \Pi_M$. Then, our new contract is feasible and strictly better for the manufacturer than the old one which leads to a contradiction.

Therefore, with probability 1 at optimal contract either, D(s) = R(q; s) or T(s) = 0. Since h(s) is well behaved and R(q; s) continuous, it is possible to find an optimal contract, for which sets $\{s \mid D(s) = R(q; s)\}$ and $\{s \mid T(s) = 0\}$ are closed. If these sets are not empty and given that $[\underline{s}, \overline{s}]$ is a connected set, there is a point s^* at their intersection, i.e. $T(s^*) = 0$ and $D(s^*) = R(q; s^*)$. For all $s > s^*$ we could not have D(s) = R(q; s), otherwise u being a constant gives us negative values of T(s); hence, T(s) = 0. For all $s < s^*$ we could not have T(s) = 0, otherwise u being a constant gives us D(s) > R(q; s); hence, D(s) = R(q; s).

Part (ii) follows directly from (IC) and Abreu (1988). For Part (iii) note that $\Pi_R =$

 $R_E(q) + u > 0. \quad \blacksquare$

Proof of Proposition 2. Take Π_M in (5). To show that q is distorted downwards, we need to show that $1 - \int_{\underline{s}}^{\overline{s}} \delta^{T(s)+1} h(s) ds$ and Π_R are non-decreasing in q.

We will first undertake a change of variables, whereby the manufacturer chooses q and u instead of q and s^* . Then we derive the first order conditions and show that q is distorted downwards whenever $\frac{\partial^2 \ln R(q,s)}{\partial q \partial s} \geq 0$.

The manufacturer maximizes Π_M subject to (IC) and (DE). Note that the retailer's participation constraint does not bind, as he can always walk away with R(q; s). Using (DE), we can rewrite (IC) as follows:

$$\delta^{T(s)+1} = \frac{u + R(q, s)}{u + R_E(q)} \text{ for } s \le s^*$$

Then,

$$\int_{\underline{s}}^{\overline{s}} \delta^{T(s)+1} h\left(s\right) ds = \int_{\underline{s}}^{s^*} \frac{u + R\left(q, s\right)}{u + R_E\left(q\right)} h\left(s\right) ds + \delta\left(1 - H\left(s^*\right)\right)$$

Since the asymmetric information problem always binds, we plug (IC) into Π_M . The problem of the manufacturer can then be written as:

$$\max_{q,u} L = \frac{R_E(q) - cq}{1 - \left[\int_{\underline{s}}^{s^*} \frac{u + R(q,s)}{u + R_E(q)} h(s) \, ds + \delta \left(1 - H(s^*)\right)\right]} - \left(u + R_E(q)\right) + \lambda u,$$

where $\lambda \ge 0$ is the Lagrange multiplier of the (DE) constraint $u \ge 0$ and s^* is a function of q and u given implicitly by

$$R(q, s^*) = \delta R_E(q) - (1 - \delta) u \tag{6}$$

Before taking the first-order conditions, note that s^* only appears in the denominator of

L and that:

$$\frac{\partial}{\partial s^*} \left[\int_{\underline{s}}^{s^*} \frac{u + R(q, s)}{u + R_E(q)} h(s) \, ds + \delta \left(1 - H(s^*) \right) \right] = \frac{u + R(q, s^*)}{u + R_E(q)} h(s^*) - \delta h(s^*) \\ = \frac{\delta \pi_R}{\pi_R} h(s^*) - \delta h(s^*) = 0.$$

Hence, we can ignore the effect of q and u through s^* .

Now take the first-order conditions:

$$\frac{\partial L}{\partial q} = \frac{\left[R'_{E}(q) - c\right]P + \left[R_{E}(q) - cq\right]\int_{\underline{s}}^{s^{*}} \frac{R'(q,s)\left[u + R_{E}(q)\right] - \left[u + R(q,s)\right]R'_{E}(q)}{\left[u + R_{E}(q)\right]^{2}}h\left(s\right)ds}{P^{2}} - R'_{E}(q) = 0$$

$$\frac{\partial L}{\partial u} = \frac{\left[R_{E}(q) - cq\right]\int_{\underline{s}}^{s^{*}} \frac{R_{E}(q) - R(q,s)}{\left[u + R_{E}(q)\right]^{2}}h\left(s\right)ds}{P^{2}} - 1 + \lambda = 0$$

where $P = 1 - \left[\int_{\underline{s}}^{s^*} \frac{u + R(q,s)}{u + R_E(q)} h(s) \, ds + \delta \left(1 - H(s^*) \right) \right].$

At the first-best quantity $R'_E(q) = c$, $\frac{\partial L}{\partial q}$ becomes

$$\frac{\partial L}{\partial q} = \frac{\left[R_E\left(q\right) - cq\right]\int_{\underline{s}}^{s^*} \frac{R'(q,s)[u + R_E(q)] - [u + R(q,s)]R'_E(q)}{[u + R_E(q)]^2} h\left(s\right) ds}{P^2} - R'_E\left(q\right)$$

Hence, a sufficient condition for the quantity being distorted downwards is

$$\int_{\underline{s}}^{s^{*}} \left[R'(q,s) \left[u + R_{E}(q) \right] - \left[u + R(q,s) \right] R'_{E}(q) \right] h(s) \, ds \le 0 \tag{7}$$

A sufficient condition for this is

$$R(q,s) R'_{E}(q) - R'(q,s) R_{E}(q) + u [R'_{E}(q) - R'(q,s)] \ge 0 \text{ for } s \le s^{*}$$

Sufficient conditions for this are

$$(C1): \frac{R'_{E}(q)}{R_{E}(q)} \ge \frac{R'(q,s)}{R(q,s)} \text{ and } (C2): R'_{E}(q) \ge R'(q,s) \text{ for } s \le s^{*}$$

Since $R_E(q) \ge \frac{R(q,s^*)}{\delta} > R(q,s)$ for $s \le s^*$, (C1) implies (C2).

By the mean value theorem, there exists $\tilde{s} \in [\underline{s}, \overline{s}]$ such that $R_E(q) = R(q, \tilde{s})$. Since $R_E(q) > R(q, s^*), \ \tilde{s} > s^*$. Thus, if $\frac{\partial^2 \ln R(q,s)}{\partial q \partial s} \ge 0$, then

$$\frac{\partial}{\partial q} \ln R\left(q,\tilde{s}\right) \ge \frac{\partial}{\partial q} \ln R\left(q,s^*\right) \ge \frac{\partial}{\partial q} \ln R\left(q,s\right) \text{ for } s \le s^*$$

which completes the proof. \blacksquare

Proof of Proposition 3. The manufacturer solves:

$$\max_{q,s^*} \Pi_M = \frac{\widehat{E}(s,s^*)R(q) - cq}{(1-\delta)\frac{E(s)-\widehat{E}(s,s^*)}{E(s)-s^*}}$$

subject to (DE): $\frac{\delta E(s)-s^*}{1-\delta}R(q) \ge 0$, where $\widehat{E}(s,s^*) = H(s^*)E(s \mid s \le s^*) + (1 - H(s^*))s^*$. The optimal s^* and q are determined by these first order conditions:

$$q: R'(q) = \frac{c}{\widehat{E}(s, s^*)} > \widetilde{c}$$
(8)

$$s^* : \min\left\{\delta E(s), s^{**}\right\} \tag{9}$$

where s^{**} is the solution to

$$G(s^*) = (E(s)R(q) - cq) \left(s^* - \widehat{E}(s, s^*) + H(s^*)(E(s) - s^*)\right) - R(q) \left(E(s) - \widehat{E}(s, s^*)\right)^2 = 0$$

Note that $G(s^*)$ is negative for $s^* = \underline{s}$ and positive for $s^* = E(s)$, so the solution to $G(s^*) = 0$, $s^*(\underline{s}, E(s))$, exists. Whether it is unique, depends on the shape of $R(\cdot)$. Let us postpone the proof of uniqueness until part *(iv)* of this proof.

To prove part (i), note that the optimal trade suspension is $\delta^{T(s)+1} = \frac{(1-\delta)s+\delta E(s)-s^*}{E(s)-s^*}$, and hence $\partial \delta^{T(s)+1}/\partial s^* < 0$. The cross derivatives are $\frac{\partial^2 \Pi_M}{\partial q \partial \delta} = 0$ after using $\frac{\partial \Pi_M}{\partial q} = 0$, $\frac{\partial^2 \Pi_M}{\partial s^* \partial \delta} = 0$ after using $\frac{\partial \Pi_M}{\partial s^*} = 0$ and $\frac{\partial^2 \Pi_M}{\partial s^* \partial q} > 0$ using $\frac{\partial \Pi_M}{\partial q} = 0$.

To prove parts (ii) and (iii), we show that $\frac{\partial \Pi_M}{\partial s^*} < 0$ at $s^* = \delta E(s)$ in the limit when

 $\delta \longrightarrow 1$. After some algebra:

$$\begin{aligned} \frac{\partial \Pi_M}{\partial s^*} \Big|_{s^* = \delta E(s)} &= \frac{\left[1 - H\left(\delta E(s)\right)\right] R(q) E(s)}{E(s) - \hat{E}(s, \delta E(s))} \\ &- \frac{\left[\hat{E}(s, \delta E(s)) R(q) - cq\right] H\left(\delta E(s)\right) \left[E(s) - E\left(s \mid s \le \delta E(s)\right)\right]}{\left(1 - \delta\right) \left(E(s) - \hat{E}(s, \delta E(s))\right)^2} \end{aligned}$$

Note that $E(s) - \widehat{E}(s, \delta E(s)) = [1 - (1 - H(\delta E(s)))\delta] E(s) - H(\delta E(s)) E(s | s \le \delta E(s))$ and hence becomes $E(s) - \widehat{E}(s, E(s)) = H(E(s)) [E(s) - E(s | s \le E(s))]$ in the limit when $\delta \to 1$. The first order condition simplifies to:

$$\lim_{\delta \to 1} \left. \frac{\partial \Pi_M}{\partial s^*} \right|_{s^* = \delta E(s)} = E(s) \left[\frac{\left[1 - H\left(\delta E(s)\right) \right] R(q) - \frac{\widehat{E}(s, \delta E(s))R(q) - cq}{(1 - \delta)E(s)}}{H\left(\delta E(s)\right) \left[E(s) - E\left(s \mid s \le \delta E(s)\right) \right]} \right]$$

which is negative. Noting that (8) and $G(s^*)$ do not depend on δ completes the proof.

To prove part *(iv)*, note that if *(DE)* does not bind, by part *(ii)*, q and s^* do not depend on δ and neither does $(1 - \delta) (\Pi_R + \Pi_M)$. Denote by δ_{DE} the discount factor at which *(DE)* stops binding. Hence, it is enough to show that $\frac{\partial (1-\delta)(\Pi_R + \Pi_M)}{\partial \delta^-} < 0$ at δ_{DE} . Using (8) and (9), we obtain that $\frac{\partial (1-\delta)(\Pi_R + \Pi_M)}{\partial \delta}$ is equal to

$$\left(\frac{c^2(1-\delta)(1-H(s^*))E(s)}{-R''(q)\widehat{E}(s,s^*)^3} - \frac{(E(s)R(q)-cq)\left((\delta E(s)+(1-\delta)H(s^*))E(s)-\widehat{E}(s,s^*)\right)}{\left(E(s)-\widehat{E}(s,s^*)\right)^2}\right)E(s)$$

Take $\delta = \delta_{DE}$ so that both $s^* = \delta E(s)$ and $G(s^*) = 0$ can be applied. Then

$$\frac{\partial(1-\delta)(\Pi_R+\Pi_M)}{\partial\delta} = \left(\frac{c^2(1-\delta)(1-H(s^*))E(s)}{-R''(q)\widehat{E}(s,s^*)^3} - R(q)\right)E(s)$$

which negative if -R''(q) is large enough. This can be rewritten using the "elasticity of

marginal revenue" $\sigma(q)$ as follows:

$$\sigma(q) = \frac{-R''(q)R(q)}{R'(q)^2} \ge \frac{E(s)(1-\delta_{DE})(1-H(s^{**}))}{\widehat{E}(s,s^{**})}.$$
(10)

Taking into account that $\widehat{E}(s, s^{**}) \ge (1 - H(s^{**}))s^{**}$ and $\delta_{DE} = s^{**}/E(s)$, give us the following sufficient condition: $\sigma(q) \ge \frac{1}{\delta_{DE}} - 1$ for $q = q(\delta_{DE})$.

Finally, we prove that the solution s^* is unique. If $\delta < \delta_{DE}$, then s^* is uniquely determined: $s^* = \delta E(s)$. Consequently, it is enough to prove the uniqueness for $\delta \ge \delta_{DE}$ when $s^* \in (\underline{s}, E(s))$ is given by $G(s^*) = 0$. Uniqueness requires $G(s^*)$ to be increasing in s^* . So let us take the derivative of $G(s^*)$:

$$\begin{aligned} G'(s^*) &= (E(s)R(q) - cq)(E(s) - s^*)h(s^*) \\ &+ (1 - H(s^*))\left(E(s) - \widehat{E}(s, s^*)\right)\left(2R(q) - \frac{c^2(1 - H(s^*))(E(s) - s^*)}{-R''(q)\widehat{E}(s, s^*)^3}\right) \\ &= (E(s)R(q) - cq)(E(s) - s^*)h(s^*) \\ &+ (1 - H(s^*))R(q)\left(E(s) - \widehat{E}(s, s^*)\right)\left(2 - \frac{(1 - H(s^*))(E(s) - s^*)}{\sigma(q)\widehat{E}(s, s^*)}\right). \end{aligned}$$

The first term is positive, but the second one may be positive or negative depending on $\sigma(q)$. However, if (10) holds then

$$G'(s^*) > (1 - H(s^*))R(q) \left(E(s) - \widehat{E}(s, s^*) \right) \frac{(3 - 2\delta)E(s) - s^*}{(1 - \delta)E(s)} > 0.$$

So the solution to $G(s^*) = 0$ is unique.

Proof of Proposition 4. The manufacturer maximizes:

$$\Pi_M = \frac{\widehat{E}(s, s^*)R(q) + H(s^*) \pi_O \frac{s^* - E(s|s \le s^*)}{E(s) - s^*} - cq}{(1 - \delta) \frac{E(s) - \widehat{E}(s, s^*)}{E(s) - s^*}}$$

subject to (DE) and the manufacturer's participation constraint, $\Pi_M \geq \frac{\pi_O}{1-\delta}$, that can be

simplified, after some algebra, to:

$$\widehat{E}(s,s^*)R(q) - cq \ge \pi_O \tag{PC_M}$$

If no constraint binds, the first order conditions are (8) and:

$$\frac{\partial \Pi_M}{\partial s^*} = (1 - H(s^*)) \frac{E(s) - s^*}{E(s) - \hat{E}(s, s^*)} R(q) -H(s^*) \frac{E(s) - E(s \mid s \le s^*)}{\left(E(s) - \hat{E}(s, s^*)\right)^2} \left[\hat{E}(s, s^*) R(q) - cq - \pi_O\right] = 0$$

When (DE) binds, (8) remains unaffected and s^* is uniquely determined by (DE). Note that $\frac{\partial^2 \Pi_M}{\partial s^* \partial \pi_O} > 0$, $\frac{\partial^2 \Pi_M}{\partial q \partial \pi_O} = 0$ and $\frac{\partial^2 \Pi_M}{\partial s^* \partial q} > 0$ after using $\frac{\partial \Pi_M}{\partial q} = 0$. Hence a larger π_O has a larger associated s^* and q. Because Π_M is larger when (DE) does not bind, for a given π_O , (PC_M) is more likely to bind when (DE) also binds. Because there is downward quantity distortion, if the optimal quantity without considering (PC_M) does not satisfy (PC_M) , no other quantity will and q = 0.

Proof of Proposition 5. To prove part (i), note that the manufacturer's contract needs to respect (DE'_s) and (LL_s) . For a given q, suppose that the optimal contract has a state \hat{s} with an associated repayment $D(\hat{s})$ such that $(DE'_{\hat{s}})$ does not bind. If $(LL_{\hat{s}})$ does not bind either, then the manufacturer can increase the repayment by ε and this new contract still respects $(DE'_{\hat{s}})$ and $(LL_{\hat{s}})$. Since this deviation is profitable for the manufacturer, the optimal repayment schedule must satisfy:

$$D(s) = \max\left\{R(q, s), \delta\Pi_R\right\}$$
(11)

Let $s^{*'}$ be the state such that: $R(q, s^{*'}) = \delta \Pi_R$. The left hand side of the equation is increasing in $s^{*'}$. Moreover, the right hand side is decreasing in $s^{*'}$, since:

$$\Pi_{R} = \frac{\int_{s^{*\prime}}^{\bar{s}} \left[R(q,s) - R(q,s^{*\prime}) \right] h(s) \, ds}{1 - \delta}$$

As a result, $s^{*'}$ exists and is unique.

Part (ii) follows from Abreu (1988).

Finally, we proceed to prove part *(iii)*. Given (11), the manufacturer solves:

$$\max_{q} \Pi_{M} = \frac{\int_{\underline{s}}^{s^{*'}} R(q,s) h(s) \, ds + (1 - H(s^{*'})) R(q,s^{*'}) - cq}{1 - \delta}$$

subject to $(DE'_{s^{*\prime}})$: $R(q, s^{*\prime}) = \delta \Pi_R$. Let us rewrite $\Pi_M = \frac{R_E(q) - cq}{1 - \delta} - \Pi_R$ and plug $(DE'_{s^{*\prime}})$ into Π_R which gives: $\Pi_R = \frac{\int_{s^{*\prime}}^{\overline{s}} R(q,s)h(s)ds}{1 - \delta H(s^{*\prime})}$. We first show that we can ignore the effect of q on $s^{*\prime}$ because $\frac{\partial \Pi_M}{\partial s^{*\prime}} = 0$. First, note that $\frac{\partial \Pi_M}{\partial s^{*\prime}} = -\frac{\partial \Pi_R}{\partial s^{*\prime}}$. Let us then compute $\frac{\partial \Pi_R}{\partial s^{*\prime}}$:

$$\frac{\partial \Pi_R}{\partial s^{*\prime}} = \frac{\delta \int_{s^{*\prime}}^{\bar{s}} R(q,s) h(s) ds - R(q,s^{*\prime}) (1 - \delta H(s^{*\prime}))}{(1 - \delta H(s^{*\prime}))^2} h(s^{*\prime}) \\
= \frac{\delta \int_{s^{*\prime}}^{\bar{s}} [R(q,s) - R(q,s^{*\prime})] h(s) ds - (1 - \delta) R(q,s^{*\prime})}{(1 - \delta H(s^{*\prime}))^2} h(s^{*\prime}) \\
= \frac{\delta \Pi_R - R(q,s^{*\prime})}{(1 - \delta H(s^{*\prime}))^2} (1 - \delta) h(s^{*\prime}) = 0$$

where the last step follows from using $(DE'_{s^{*'}})$. So we can now focus on the direct effect of q:

$$\frac{\partial \Pi_M}{\partial q} = \frac{\frac{\partial R_E(q)}{\partial q} - c}{1 - \delta} - \frac{\int_{s^{*\prime}}^{\overline{s}} \frac{\partial R(q,s)}{\partial q} h(s) \, ds}{1 - \delta H(s^{*\prime})}$$

At the first best quantity $\frac{\partial R_E(q)}{\partial q} = c$, $\frac{\partial \Pi_M}{\partial q} < 0$ which completes the proof.

References

- ABREU, D., (1988), "On the Theory of Infinitely Repeated Games with Discounting," *Econometrica*, Vol 56(6), pp 383-96.
- [2] ANTRAS, P., and FOLEY, C.F., (2011), "Poultry in Motion: A Study of International Trade Finance Practices," NBER Working Paper Series, No. 17091.
- [3] ATHEY, S., (2002), "Monotone Comparative StaticsUnder Uncertainty," Quarterly Journal of Economics, Vol 117, pp 187–223.

- [4] BERNSTEIN, L., (1992). "Opting Out of the Legal System: Extralegal Contractual Relations in the Diamond Industry.", *Journal of Legal Studies*, Vol 21, pp 115–157.
- [5] BERNSTEIN, L., (1996). "Merchant Law in a Merchant Court: Rethinking the Code's Search for Immanent Business Norms.", University of Pennsylvania Law Review, Vol 144(5), pp 1765–1821.
- [6] BOLTON, P. and SCHARFSTEIN, D.S., (1990), "A Theory of Predation Based on Agency Problems in Financial Contracting", *American Economic Review*, Vol 80, pp 93-106.
- [7] BROWN, M., A. FALK, and E. FEHR., (2004), "Relational contracts and the nature of market interactions." *Econometrica*, Vol. 72(3), pp. 747-780.
- [8] BROWN, M., A. FALK, and E. FEHR., (2012), "Competition and relational contracts: the role of unemployment as a disciplinary device." *Journal of the European Economic Association*, Vol. 10(4), pp. 887-907.
- [9] BURKART, M., and T. ELLINGSEN., (2004), "In-Kind Finance: A Theory of Trade Credit." American Economic Review, Vol. 94, pp. 569-590.
- [10] COSTINOT, A., (2009), "An elementary theory of comparative advantage", Econometrica, Vol 77(4), pp 1165–1192.
- [11] CUÑAT, V. M., (2007), "Trade Credit: Suppliers as Debt Collectors and Insurance Providers", *Review of Financial Studies*, Vol 20, pp 491-527.
- [12] CUÑAT, V. M., and GARCIA-APPENDINI, E., (2012), "Trade credit and its role in entrepreneurial finance", Chapter for Oxford Handbook of Entrepreneurial Finance (edited by Douglas Cumming), Oxford University Press.
- [13] FAFCHAMPS, M., (1997), "Trade credit in Zimbabwean manufacturing", World Development, Vol 24(3), pp 795-815.

- [14] FAFCHAMPS, M., (2000), "Ethnicity and credit in African manufacturing", Journal of Development Economics, Vol 61, pp 205-235.
- [15] FAFCHAMPS, M., (2010), "Spontaneous Markets, Networks and Social Capital: Lessons from Africa" Chapter in *Institutional Microeconomics of Development*, (edited by Timothy Besley and Rajshri Jayaraman), MIT Press, Cambridge Mass.
- [16] FEHR, E., and ZEHNDER, C., (2009), "Reputation and Credit Market Formation: How Relational Incentives and Legal Contract Enforcement Interact", IZA Discussion Paper No. 4351.
- [17] FUCHS, W., (2007), "Contracting with Repeated Moral Hazard and Private Evaluations." American Economic Review, Vol 97(4), pp 1432-1448.
- [18] GALE, D. and HELLWIG, M., (1985), "Incentive-Compatible Debt Contracts: The One-Period Problem." *Review of Economic Studies*, Vol 52, pp 647-663.
- [19] HART, O. and MOORE, J., (1998), "Default and Renegotiation: A Dynamic Model of Debt." Quarterly Journal of Economics, Vol 113, pp1-41.
- [20] INNES, R.D., (1990), "Limited Liability and Incentive Contracting with Ex-ante Action Choices." *Journal of Economic Theory*, Vol 52, pp 45-67.
- [21] JOHNSON, S., MCMILLAN, J., and WOODRUFF, C., (2002), "Courts and Relational Contracts", Journal of Law, Economics and Organization, Vol 18, pp 221-77.
- [22] MACCHIAVELLO, R., and MORJARIA, A., (2015), "The Value of Relationships: Evidence from a Supply Shock to Kenyan Flower Exports", American Economic Review, Vol. 105, pp 2911-45.
- [23] MCMILLAN, J. and WOODRUFF, C., (1999a), "Interfirm Relationships and Informal Credit in Vietnam," *Quarterly Journal of Economics*, Vol 114, pp 1285-1320.
- [24] MCMILLAN, J. and WOODRUFF, C., (1999b), "Dispute Prevention Without Courts in Vietnam," Journal of Law, Economics, and Organization, No15, pp 637-58.

- [25] MOOKHERJEE, D. and PNG, I., (1989), "Optimal auditing insurance and redistribution," Quarterly Journal of Economics, Vol 104, pp 399-415.
- [26] RAJAN, R. and ZINGALES, L., (1995), "What Do We Know about Capital Structure? Some Evidence From International Data" *Journal of Finance*, Vol 50, pp 1421-1460.
- [27] RAY, D., (2002), "The time structure of self-enforcing agreements", *Econometrica*, Vol 70, pp 547-582.
- [28] THOMAS, J., and WORRALL, T., (1988), "Self-Enforcing Wage Contracts," *Review of Economic Studies*, Vol 55, pp 541-554.
- [29] THOMAS, J., and WORRALL, T., (1994), "Foreign Direct Investment and the Risk of Expropiation," *Review of Economic Studies*, Vol 61, pp 81-108.
- [30] TOWNSEND, R.M., (1979), "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory*, Vol 21, pp 265-293.
- [31] TROYA-MARTINEZ, M., (2013), "Vertical Relational Contracts and Trade Credit," Economics Series Working Papers 648, University of Oxford, Department of Economics.
- [32] UCHIDA, H., UDELL, G.F., and WATANABE, W., (2006), "Are Trade Creditors Relationship Lenders?" http://www.rieti.go.jp/jp/publications/dp/06e026.pdf.