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For five stock market crashes, we compare price declines with predictions from market microstructure invariance. During the 1987 crash and the sales by Société Générale in 2008, prices fell by magnitudes similar to predictions from invariance. Larger-than-predicted temporary price declines during two flash crashes suggest rapid selling exacerbates transitory price impact. Smaller-than-predicted price declines for the 1929 crash suggest slower selling stabilized prices and less integration made markets more resilient. Quantities sold in the three largest crashes suggest fatter tails or larger variance than the log-normal distribution estimated from portfolio transitions data.

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After stock market crashes, rattled market participants, frustrated policymakers, and puzzled economists are typically unable to explain what happened. In the aftermath, studies have documented unusually heavy selling pressure during these episodes. According to conventional wisdom, however, stock markets have such great liquidity that the dollar magnitudes of large sales are far too small to explain observed declines in prices.

We question the conventional wisdom from the perspective of market microstructure invariance, a conceptual framework developed by Kyle and Obizhaeva (2016). Combined with a linear price impact model, invariance can explain why order flow imbalances—expressed as a fraction of average daily volume—result in greater price impact in large markets than in small markets. As a result, when market impact estimates based on applying invariance to individual stocks are extrapolated to the entire stock market, these estimates become large enough to explain stock market crashes.

We study five crash events, chosen because data on the magnitude of selling pressure became publicly available in the aftermath:

- After the stock market crash of October 1929, the report by the Senate Committee on Banking and Currency (1934) (the "Pecora Report") attributed the dramatic plunge in broker loans to forced margin selling during the crash.
- After the October 1987 stock market crash, the U.S. Presidential Task Force on Market Mechanisms (1988) (the "Brady Report") documented quantities of stock index futures contracts and baskets of stocks sold by portfolio insurers during the crash.
- After the futures market dropped by 20% at the open of trading three days after the 1987 crash, the Commodity Futures Trading Commission (1988) documented large sell orders executed at the open of trading; the press identified the seller as George Soros.
- After the Fed cut interest rates by 75 basis points in response to a worldwide stock market plunge on January 21, 2008, Société Générale revealed that it had quietly been liquidating billions of Euros in stock index future positions accumulated by rogue trader Jérôme Kerviel.
- After the flash crash of May 6, 2010, the Staffs of the CFTC and SEC (2010b,a) cited as a trigger large sales of futures contracts by one entity, identified in the press as Waddell & Reed.

We call these large sales "bets." A bet is an "intended order" or "meta-order" whose size is known in advance of trading. Large bets can result either from trading by one large entity or from correlated trades of multiple entities based on the same underlying motivation. In a speculative market, price fluctuations occur when some investors place bets which move prices while other traders attempt to profit by intermediating among the bets being placed. According to market microstructure invariance, execution of very large bets can lead to significant market dislocations.

While accurate estimates of the size of potential selling pressure were published and widely discussed months before the crashes of 1929 and 1987, market participants had different opinions concerning whether the selling pressure would have a significant effect on prices. Before the three crash events associated with the Soros trades, the Société Générale trades, and the flash crash trades, the sellers knew precisely the quantities they intended to sell, but they either estimated inaccurately or were willing to incur large transaction costs. For all five crash events, policymakers or stock market participants had in hand the information market microstructure invariance requires to quantify the price impact and thus the systemic risks resulting from sudden liquidations of large stock market exposures.

Market microstructure invariance is based on the intuition that "business time" passes more quickly in active markets than in inactive markets. When appropriate adjustment is made for the rate at which business time passes, market properties related to the dollar rate at which mark-to-market gains and losses are generated do not vary across markets. Since business time operates at a much faster pace in equity markets as a whole than in less active markets for individual stocks, one calendar day in equity index markets is equivalent to many calendar days in markets for individual stocks. Sales of a given percentage of daily volume in equity indices are effectively equivalent to the same percentage of volume executed over many consecutive days in individual stocks and therefore lead to much greater price impacts than trades of a similar magnitude in individual stocks. In this way, the invariance principle implies far greater price impacts for large sales of equity indices than conventional wisdom suggests.

Two features of microstructure invariance make practical predictions possible. First, the invariance principle generates precise formulas for market impact, the implementation of which requires only a small number of parameters to be estimated. These parameter values are the same for active markets and inactive markets, liquidations of large positions and liquidations of small positions. We therefore extrapolate to the entire market the parameters Kyle and Obizhaeva (2016) obtain from a database of more than 400,000 portfolio transition trades in individual stocks. Second, given parameter estimates, price impact estimates are functions of expected dollar volume, expected returns volatility, and the dollar size of amounts traded. It is not necessary to have additional information about other market characteristics, such as dealer market structure, information asymmetries, the motivation of traders, or order shredding algorithms.

Our estimates of price impact are much greater than implied by conventional wisdom. We define "conventional wisdom" as the idea that the demand for individual stocks or for the entire stock market is "elastic" in the sense that selling one percent of market capitalization has a price impact of less than one percent. This definition is similar to the related idea that the price impact of selling 5-10 percent of average daily volume is modest, either for an individual stock or stock index futures.

Table 1 summarizes our results for each of the five crash events. Using volume and volatility estimated from daily data over the month before the crash event, the table shows the estimated size of the dollar amounts liquidated (percent of daily volume and GDP), actual price declines (percent), and price declines predicted by invariance and conventional wisdom (percent).

Table 1—Summary of Five Crash Events: Actual and Predicted Price Declines.

	Actual	Predicted Invariance	Predicted Conventional	%ADV	%GDP
1929 Market Crash	25%	46.43%	1.36%	265.41%	1.136%
1987 Market Crash	32%	16.77%	0.63%	66.84%	0.280%
1987 Soros's Trades	22%	6.27%	0.01%	2.29%	0.007%
2008 SocGén Trades	9.44%	10.79%	0.43%	27.70%	0.401%
2010 Flash Crash	5.12%	0.61%	0.03%	1.49%	0.030%

Table 1 shows that three of the crash events involve much larger selling pressure than the other two. The 1929 crash, the 1987 crash, and the Société Générale trades of 2008 all involve sales of more than 25% of average daily volume or more than 0.25% of GDP. By contrast, the sales by Soros in 1987 and the flash crash of 2010 both involve sales of only 2.29% and 1.49% of average daily volume the previous month.

Overall, the price declines predicted by invariance are much closer in magnitude to actual price declines than predictions based on conventional wisdom, but there is substantial variation across crash events. Compared with actual price declines, the declines predicted by invariance are about twice as large for the 1929 crash, about the same for the Société Générale liquidation, about half as large for the 1987 crash, and much smaller for the 1987 Soros trades and the 2010 flash crash trades. By contrast, the price impact estimates based on conventional wisdom are minuscule in comparison to actual price changes.

Even though Kyle and Obizhaeva (2016) do not explicitly model execution speed because the effects of speed of execution are difficult to detect empirically using portfolio transitions data, we hypothesize that the speed of order execution may have influenced the temporary and permanent price impact of order flow imbalances during the five crash events. Kyle, Obizhaeva and Wang (2016) discuss a theoretical model of smooth trading which explains how price declines are affected by the speed of selling pressure and why unusually rapid, suboptimal execution of large bets can result in flash crashes.

In addition to the size and speed of selling, the various stock market crashes also may have been affected by the degree of market integration across assets, the amounts of capital available to take the other sides of large bets, and disruptions to the market mechanism which resulted from extreme price movements.

For the 1929 stock market crash, the actual price decline of 25% was much smaller than the predicted decline of 46.43%. This may be explained by efforts financial markets made to spread the impact of margin selling out over several weeks rather than several days, by greater resiliency of markets resulting from less financial integration, or by potential buyers keeping capital on the sidelines to profit from price declines widely expected to occur if margin purchases were liquidated.

For the 1987 stock market crash, the actual decline of 32% was approximately double the predicted decline of 16.77%. Perhaps price declines were exacerbated by breakdowns in the market mechanism documented in the Brady Report. For the 2008 Société Générale trades, the actual decline of 9.44% was similar to the predicted price decline of broad European indices by 10.79%. From a theoretical perspective, it is puzzling that the unanticipated trades by Société Générale appear to have had smaller impact, relative to the invariants benchmark, than the anticipated trades by the portfolio insurers.

The remaining two crashes are both "flash-crash" events in which the trades were executed in minutes, not hours or days. The actual plunges in prices associated with Soros's 1987 trades and the 2010 flash crash, 22% and 5.12% respectively, are much larger than the declines predicted by invariance, 6.27% and 0.61% respectively. Both flash crashes were followed minutes later by rapid rebounds in prices. We hypothesize that the rapid speed, large size, and immediate reversals of the large price declines resulted from the unusually rapid rate at which these trades were executed.

Since invariance generates predictions about the frequency and the size distribution of bets, it also has implications for the frequency and magnitude of crashes. Under the identifying assumption that portfolio transition orders have the same size distribution as bets, Kyle and Obizhaeva (2016) find that the distribution of unsigned bet sizes closely resembles a log-normal with variance of 2.53; its significant kurtosis suggests occasional crashes. The two flash-crashes represent approximately 4.5-standard-deviation bet events, which are expected to occur several times per year. We do not include in this study flash crash events in 1961 and 1989, as well as others, due to lack of data on selling pressure. The three largest crashes are approximately 6-standard-deviation bet events. Based on the log-normal distribution, they are expected to occur less frequently, only once in hundreds or thousands of years. Obviously, the actual frequency of crashes is far higher than implied by fitting a log-normal distribution to portfolio transition data. Either the tails of the distribution are fatter than a log-normal—consistent with a power law—or the variance is larger than estimated from portfolio transition data.

In what follows are sections discussing conventional wisdom and the animal spirits hypothesis, market microstructure invariance, particulars of each of the five crash events, the frequency of crashes, and lessons learned.

I. Conventional Wisdom, Animal Spirits, and Banking Crises

The debate about what causes market crashes started before the 1929 crash. Since then, economists and market participants have been divided into two camps which differ in their views concerning whether crashes result from rational or irrational behavior. We call explanations based on rationality "conventional wisdom" and explanations based on irrationality "animal spirits." After more than 80 years, the debate has made little progress. Neither of these two camps has offered a compelling explanation for crashes.

Conventional Wisdom.

Conventional wisdom holds that large price changes result from arrival of new fundamental information into the market, not from the price pressure resulting from buying and selling. In the 1960s and 1970s, this conventional wisdom became associated with the capital asset pricing model and efficient markets hypothesis. Conventional wisdom based on the capital asset pricing model implies that the demand for market indices is elastic and that demand for individual stocks is even more so; the quantities observed changing hands in the market are too small to explain dramatic plunges in market prices. Empirical studies based on the analysis of secondary distributions (e.g., Scholes (1972)), index inclusions and deletions (e.g., Harris and Gurel (1986) and Wurgler and Zhuravskaya (2002)), and other events usually find that selling one percent of shares outstanding has a price impact of less than one percent. By extrapolating this conventional wisdom to equity indices, researchers and regulators have concluded that stock market crashes do not result from selling pressure.

Views consistent with the conventional wisdom based on the capital asset pricing model are shared by many prominent economists.

Miller (1991), for example, states the following about the 1987 crash: "Putting a major share of the blame on portfolio insurance for creating and overinflating a liquidity bubble in 1987 is fashionable, but not easy to square with all relevant facts. . . . No study of price-quantity responses of stock prices to date supports the notion that so large a price decrease (about 30%) would be required to absorb so modest (1-2%) a net addition to the demand for shares."

As the academics most associated with portfolio insurance, Leland and Rubinstein (1988) echo this argument: "To place systematic portfolio insurance in perspective, on October 19, portfolio insurance sales represented only 0.2% of total U.S. stock market capitalization. Could sales of 1 in every 500 shares lead to a decline of 20% in the market? This would imply a demand elasticity of 0.01—virtually zero—for a market often claimed to be one of the most liquid in the world."

Brennan and Schwartz (1989) note that portfolio insurance would have a minimal effect on prices, because most portfolio-consumption models imply elasticities of demand for stock more than 100 times the elasticities necessary to explain the 1987 crash.

Conventional wisdom based on the efficient markets hypothesis can be interpreted as implying that, since many investors compete for information, it would be highly unusual for investors to have private information of sufficient value so that the information content of their trades would move the entire stock market significantly. Restricting trading to be a given percent of daily trading volume, for example five or ten percent, will have price impact close to zero. Since turnover is usually about 100% per year, the intuition about the costs of execution a small percentage of daily trading volume is closely related to the intuition about execution of a tiny percentage of market capitalization.

Based on this intuition, the Brady Report, comparing the 1929 and 1987 crashes, comes to the following conclusion about the 1929 crash: "To account for the contemporaneous 28% decline in price, this implies a price elasticity of 0.9 with respect to trading volume which seems unreasonably high. As a percentage of total shares outstanding, margin-related selling would have been much smaller. Viewed as a shift in the overall demand for stocks, margin-related selling could have accounted realistically for no more than 8% of the value of outstanding stock. On this basis, the implied elasticity of demand is 0.3 which is beyond the bound of reasonable estimates."

Many observers of the 1987 stock market crash, including Miller (1988, p. 477) and Roll (1988), looked therefore to explanations other than the price pressure of the large quantities traded to explain the large changes in prices.

We disagree with the conventional wisdom. For all five crash events, it is difficult to find new fundamental information shocks to which market prices would have reacted with the magnitude of price declines actually observed. We are left with the puzzling fact that large sales—even when known to have no information content, such as the margin sales of 1929 or the portfolio insurance sales in 1987—do appear to have large effects of prices. Our examination of five historical episodes through the lens of invariance shows that actual price changes are indeed similar in magnitude to those predicted by extrapolating estimates from data on portfolio transitions for individual stocks in normal market conditions to unusually large bets on market indices.

Animal Spirits Hypothesis.

Animal spirits holds that price fluctuations occur as a result of random changes in psychology, which may not be based on economically relevant information or rationality. The term "animal spirits" is associated with Keynes (1936), who says that financial decisions can be taken only as the result of "animal spirits—a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities." Akerlof and Shiller (2009) echo Keynes: "To understand how economies work and how we can manage them and prosper, we must pay attention to the thought patterns that animate people's ideas and feelings, their animal spirits." According to animal spirits theory, market crashes occur when decisions are driven by changes in mind-set based on emotions and social psychology instead of rational calculations. Promptly after the 1987 crash, for example, Shiller (1987) surveyed traders and found that "most investors interpreted the crash as due to the psychology of other investors."

We disagree with animal spirits theory. Although the timing of crash events may be unpredictable, market participants had mundane pre-crash explanations for why both the 1929 and 1987 crashes might occur. These explanations were remarkably accurate and did not involve animal spirits. In the months prior to the 1929 stock market crash, brokers were raising margin requirements to protect themselves from a widely discussed collapse in prices which might be induced by rapid unwinding of stock investments financed with margin loans. In the months prior to the 1987 stock market crash, the SEC—responding to worries that portfolio insurance made the market fragile—published a study describing a cascade scenario induced by portfolio insurance sales. On the day the 1987 crash occurred, academics were holding a conference on a potential "market meltdown" induced by portfolio insurance sales. It would be implausible to argue that a sudden change in animal spirits occurred coincidentally on the same day Société Générale liquidated Kerviel's rogue trades. While the sales of George Soros in 1987 may reflect the animal spirits of this one person, the rapid recoveries of prices after both flash crash events do not suggest market-wide irrationality or psychological contagion; they suggest the opposite. The animal spirits is likely relevant over longer horizons.

We chose conventional wisdom and animal spirits as alternative hypotheses to invariance because these are the explanations prominent academics have actually proposed to explain crash events. The "straw man" nature of these explanations results not from their inherent implausibility but rather from the fact that these alternative explanations do not match the facts associated with crash events.

Stock Market Crashes and Banking Crises.

The five stock market crashes differ from the long-lasting financial crises catalogued by Reinhart and Rogoff (2009), who examine sovereign defaults, banking crises associated with collapse of the banking system, exchange rate crises associated with currency collapse, and bouts of high inflation. Reinhart and Rogoff (2009) document that it usually takes many years and significant changes in macroeconomic policies and market regulations for the affected economies to recover from these fundamental problems associated with insolvency of financial institutions underlying the economy.

In contrast, stock market crashes or panics triggered by large bets are likely to be short-lived if followed by appropriate government policy. For example, the Federal Reserve System implemented an appropriately loose monetary policy immediately after the 1929 crash, which calmed down the market by the end of 1929. The great depression of the 1930s resulted from a subsequent shift towards a deflationary policy, not the 1929 crash. After the liquidation of Jérôme Kerviel's rogue trades in 2008, an immediate 75-basis point interest rate cut by the Fed may have prevented this event from immediately spiraling into a deeper financial crisis, but it did not prevent the collapse of Bear Stearns a few weeks later. It was the bursting of the real estate credit bubble, not the unwinding of Jérôme Kerviel's fraud, that led to the deep and long-lasting recession which unfolded in 2008-2009.

II. Market Microstructure Invariance

Market microstructure invariance is based on the simple intuition that stock market trading is a game played by professional traders, the "rules" of the trading game are the same across stocks and across time, but the speed with which the game is played varies across stocks based on levels of trading activity. The trading game is played faster if securities have higher levels of trading volume and volatility.

Trading in speculative markets transfers risks among traders. We refer to these risk transfers as "bets." Bets are sometimes called "meta-orders." Asset specific "business time" passes at a rate proportional to the rate at which bets arrive. Microstructure invariance conjectures that the distribution of standard deviations of gains and losses on bets is the same across markets, *when measured in units of business time*. In this sense, the rate at which speculative markets transfer risk is invariant, when adjusted for the speed with which trading occurs. Invariance also conjectures that the expected dollar transactions cost of executing a bet is constant across markets, *when the size of the bet is measured as the dollar risk transferred per unit of business time*. These scaling principles place strong restrictions on both the size distribution of bets and their market impact as a function of observable dollar trading volume and volatility.

We examine the implications of a log-linear version of the linear price impact model from Kyle and Obizhaeva (2016). The expected percentage price impact from buying or selling X shares of a stock with a current stock price P dollars, expected trading volume V shares per calendar day, and expected daily percentage standard deviation of returns σ is given by

$$\text{(1)} \qquad \ln\left(1+\frac{\Delta P(X)}{P}\right) = \frac{\bar{\lambda}}{10^4}\cdot\left(\frac{P\cdot V}{40\cdot 10^6}\right)^{1/3}\cdot\left(\frac{\sigma}{0.02}\right)^{4/3}\cdot\frac{X}{(0.01)V}.$$

Invariance imposes the restriction that the price impact of a bet of fraction X/V of expected daily volume is proportional to the product of the cube root of dollar volume $P\cdot V$ and the 4/3 power of volatility σ. Invariance implies that the proportionality factor is the same for all markets, large and small. In equation (1), this proportionality factor is expressed in terms of a price impact parameter $\bar{\lambda}$, which is scaled so that it measures in basis points the market impact of trading $X = 1\%$ of expected daily volume V of a hypothetical "benchmark stock" with stock price of $P = \$40$ per share, expected volume $V = 1$ million shares per day, and volatility $\sigma = 2\%$ per day.

We infer a value of $\bar{\lambda}$ from price impact costs that Kyle and Obizhaeva (2016) estimate by applying the concept of implementation shortfall to a dataset of more than 400,000 portfolio transition orders. A portfolio transition occurs when assets under the management of one institutional asset manager are transferred to another another. Trades converting the legacy portfolio into the new portfolio are typically handled by a professional third-party transition manager. Implementation shortfall, as discussed by Perold (1988), is the difference between actual execution prices and prices based on transactions-cost-free "paper trading" at prices observed in the market just before the order is placed. Portfolio transition trades are ideal for using implementation shortfall to estimate transactions costs because the exogeneity of the order sizes eliminates selection bias.

Kyle and Obizhaeva (2016) estimate an *average* impact cost parameter $\bar{\kappa}_I = 2.50$ basis points (standard error 0.19 basis points). Large orders are broken into pieces and executed at prices which tend to increase due to movement along an upward sloping demand schedule, which we assume to be approximately linear. Thus, we assume that total price impact is twice the average impact cost, i.e., $\bar{\lambda} = 2 \cdot \bar{\kappa}_I = 5.00$ basis points. Although invariance also has implications for bid-ask spread costs, these costs are negligible for large bets; hence, we ignore them.

The connection between short-term impact and long-term impact is not straightforward. The long-term impact of a bet depends on the information content of the bet itself. Equation (1) describes the short-term price impact. The short-term impact is likely to be related to the speed of execution. While the invariance hypothesis conjectures that expected price impact is a function of bet size, market volume, and market volatility, it leaves open the possibility that unusually rapid execution of very large orders may increase their temporary price impact. It is difficult to estimate the effect of speed of execution from portfolio transitions data because execution horizons of portfolio transitions are confined to narrow time ranges probably chosen to make temporary price impact small. Furthermore, speed of execution may affect temporary impact only for the very largest portfolio transition orders.

Prices mostly recovered within minutes after both flash crashes, within a few day's after the liquidation of Kerviel's rogue positions in 2008, and over weeks and months after market crashes of 1929 and 1987. Financial crises eventually followed the crash events of 1929 and 2008. Whether margin sales in 1929 or liquidation of Kerviel's rogue trades in 2008 had information content is a difficult question to frame in a meaningful manner. For example, perhaps Kerviel traded against informed traders who correctly foresaw the impending financial crisis, delaying the incorporation of this information into prices until his own trades were liquidated.

Kyle and Obizhaeva (2016) calibrate both linear and non-linear specifications for market impact consistent with the invariance hypothesis. From a theoretical perspective, the linear model is attractive because it is broadly consistent with the market microstructure literature, e.g., the linear impact model of Kyle (1985). From an empirical perspective, the square root specification explains price impact better than the linear model, a finding consistent with the empirical econophysics literature (Bouchaud, Farmer and Lillo, 2009). The linear model explains the price impact of the largest one percent of bets in the most active stocks slightly better than the square root model.

Due to its concavity, the square root model predicts much smaller price declines during crash events than the linear model. Invariance alone does not explain crash events; instead, crash events are explained by applying invariance to a linear model. To make this point, "invariance" assumes a linear impact function in the rest of this paper.

We choose to consider log-linear version of the market impact model rather than the simple linear model because our analysis deals with very large orders, sometimes equal in magnitude to trading volume of several trading days. In contrast, Kyle and Obizhaeva (2016) consider relatively smaller portfolio transition orders with an average size of about 4.20% of daily volume and median size of 0.57% of daily volume. For these orders, the distinction between continuous compounding and simple compounding is immaterial.

Equation (1) describes market impact during both normal times and times of crash or panic, for individual stocks and market indices. The five crashes occurred in markets with high trading volume, typically during times of significant volatility. Because of high trading volume and volatility, the market impact implied by equation (1) is greater than the impact obtained from conventional wisdom.

The difference between the linear price impact function implied by invariance and a linear price impact function consistent with conventional wisdom can be illustrated using the model of Kyle (1985). Equation (1) can be thought of as an implementation of the formula $\lambda = \sigma_V/\sigma_U$ in units of business time. The conventional wisdom about market impact arises from a naive implementation of the the same formula in units of calendar time. Under the assumptions that the standard deviation of fundamentals σ_V is proportional to price volatility $\sigma \cdot P$ and the standard deviation of order imbalances σ_U is proportional to dollar volume V, price impact can be written

$$\text{(2)} \qquad \ln\left(1 + \frac{\Delta P(X)}{P}\right) = \frac{\bar{\lambda}}{10^4} \cdot \left(\frac{\sigma}{0.02}\right) \cdot \frac{X}{(0.01)V}.$$

This equation is consistent with our definition of conventional wisdom if neither turnover rates nor volatility vary much across stocks: Price impact of a fixed fraction of volume is similar across markets regardless of their dollar volume.

A comparison of impact implied by invariance in equation (1) with the impact implied by conventional wisdom in equation (2) reveals that the price impact of a bet differs by a factor proportional to $(P \cdot V \cdot \sigma)^{1/3}$. Thus, when dollar volume $P{\cdot}V$ is increased by a factor of $1,000$—approximately consistent with dollar volume differences between a benchmark stock and stock index futures—invariance implies that market impact changes by a factor $(1000)^{1/3} = 10$ times greater than implied by conventional wisdom. Furthermore, according to conventional wisdom, doubling volatility doubles the market impact of trading a given percentage of expected daily volume. According to invariance, the price impact increases by a factor of $2^{4/3} \approx 2.52$. When these effects of volume and volatility are taken into account, we conclude that the observed market dislocations may have resulted from the selling pressure observed in active and volatile markets.

Market participants often execute large orders in individual stocks by restricting quantities traded to be not more than five or ten percent of average daily volume over a period of several days. Conventional wisdom implies that the same strategy will also be reasonable in more active markets such as markets for stock index futures. In contrast, invariance predicts that this heuristic strategy may incur much larger transaction costs when implemented in active markets.

Trading activity W is defined as the product of dollar volume and volatility, $W := V \cdot P \cdot \sigma$. Conceptually, trading activity is the standard deviation of one calendar day's dollar mark-to-market gains or losses on trading volume expected to be executed during a calendar day. For the benchmark stock, trading activity is defined s $W^* := 40 \cdot 10^6 \cdot 0.02 = 800,000$. Let $\tilde{Q}$ denote the number of shares in a bet, positive for buys and negative for sells. Invariance implies that the distribution of bet size as a fraction of volume is such that distribution $|\tilde{Q}/V| \cdot (W/W^*)^{-2/3}$ is the same for all stocks. Empirically, Kyle and Obizhaeva (2016) find that distribution of portfolio transition orders is approximately symmetric about zero, with unsigned order size close to the log-normal $\ln(|\tilde{Q}/V|) \sim N(-5.71 - 2/3 \cdot \ln(W/W^*), 2.53)$.

Figure 1 compares the five crash events with large portfolio transition orders. The horizontal axis is the log of scaled trading activity $\ln(W/W^*)$. The vertical axis is the log of order size as a fraction of expected daily volume. The horizontal line $|Q/V| = 5\%$ represents an order equal to 5% of daily volume. The lowest (red) diagonal line, which has a slope of $-2/3$ represents a median bet $|Q/V|$ as implied by invariance; it intersects the vertical axis at -5.71, for the benchmark stock, implying a median bet in the benchmark stock equal to $exp(-5.71) \approx 0.33\%$ of daily volume. The six parallel diagonal lines above represent orders whose log-size is one through six standard deviations above the median size, respectively, as implied by invariance; the vertical distance between each line is the log-standard deviation of $2.53^{1/2} \approx 1.60$. Each log standard deviation represents a factor of $exp(2.53^{1/2}) \approx 4.90$ in unsigned order sizes.

For each of the 60 months from January 2001 through December 2005, each of the 400,000+ portfolio transition orders is placed into a volume bin based on thresholds corresponding to the 30th, 50th, 60th, 70th, 75th, 80th, 85th, 90th, and 95th percentiles of the dollar volume for NYSE-listed common stocks. The 600 blue diamonds in figure 1 represent the largest order in each of 10 bins for 60 months. Since each bin contains on average about 650 points, invariance and log-normality of order size suggest that these largest portfolio transition orders should lie slightly below the 3-standard-deviation diagonal with predicted slope of $-2/3$. As it can be seen visually from the figure, this is approximately the case. Trading activity varies by a factor of about one million ($= exp(14)$) from

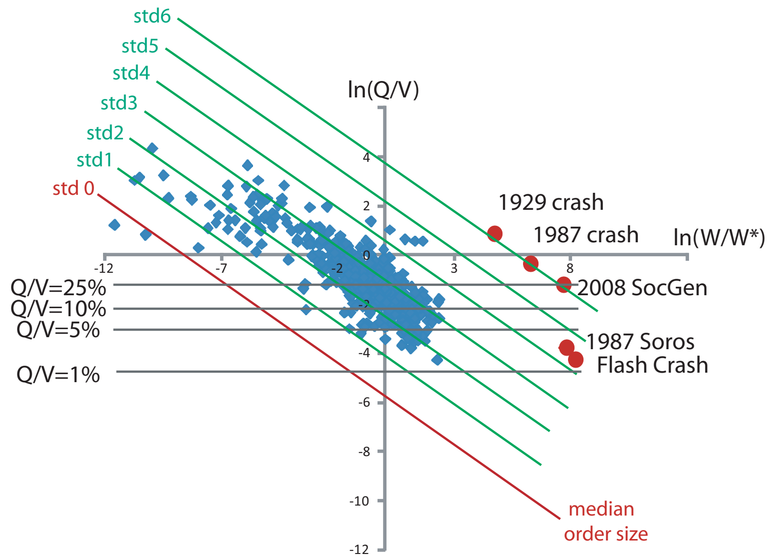

Figure 1. Largest Portfolio Transition Orders and Market Crashes.

Figure shows the largest portfolio transition orders for each month from January 2001 to December 2005 and for each of ten volume groups (blue points) as well as the bets during five market crashes (red points). Volume groups are based on thresholds corresponding to 30th, 50th, 60th, 70th, 75th, 80th, 85th, 90th, and 95th percentiles of the dollar volume for common NYSE-listed stocks. The vertical axis is $|\ln(Q/V)|$. The horizontal axis is $\ln(W/W^*)$, where $W^* = 40 \cdot 10^6 \cdot 0.02$ and $W = V \cdot P \cdot \sigma$. The median order is $-5.71 - 2/3 \cdot \ln(W/W^*)$ (red line). The x-standard deviation events are $-5.71 - 2/3 \cdot \ln(W/W^*) + x \cdot \sqrt{2.53}$ (green lines).

the least actively trade stocks with $\ln(W/W^*) = -12$ to the most actively traded stocks $\ln(W/W^*) = 2$. There is some noise introduced by the fact that bins do not contain the same number of observations; in particular, some of the bins for low volume stocks contain a small number of observations for some months, implying a somewhat smaller-than-predicted largest order sizes (60 points on left side of the graph).

The invariance hypothesis shows how to extrapolate from individual stocks to the aggregate stock and index futures market, where trading activity is larger than by a factor of about 500, i.e., from $exp(2.00)$ for the largest stocks to $exp(8.20)$ for the aggregate market. Invariance implies that extrapolation should be done along the diagonal lines, not along the horizontal lines as conventional wisdom would suggest. Invariance implies that orders of five percent of daily volume are much smaller than the median for inactive stocks, about two-standard-deviation events for active stocks, and five-standard-deviation events for the market as a whole.

Relative to the largest transition orders, the five market crashes are clearly outliers. The two flash crashes of 2.29% and 1.49% of daily volume correspond to about 4.5-standard-deviation events in log bet size. The 1929 crash of 241.52%, the 1987 crash of 66.84%, and the liquidation of Jérôme Kerviel's positions of

27.70% of daily volume correspond to about 6-standard-deviation events. Figure 1 shows that even though crash events are small as a percentage of daily volume, these are extremely large bets in the context of invariance; they are expected to trigger significant price impact.

The intuition can be further illustrated with a numerical example. For the benchmark stock, define a "large bet" as a bet equal to 30% of average daily volume, about a 3-standard-deviation event (since the median bet size equals 0.33% and one standard deviation in log bet size is a factor of 4.90). This bet has an expected price impact of 150 basis points ($= 5.00 \cdot 30$), close to one day's standard deviation of 200 basis points for the benchmark stock.

Consider the entire stock market as one single market, consisting of the stock index futures market and underlying stock market. The overall market has about $6,750$ ($= 15^3 \cdot 2$) times the volume and half the volatility of a typical stock. Invariance suggests that business time passes at a rate proportional to $W^{2/3}$. Thus, business time passes 225 times ($= (6,750 \cdot 1/2)^{2/3}$) faster in the overall market than in the market for the typical stock. Invariance implies that the distribution of bet size as a fraction of average daily volume in the market for the typical stock should be the same as the distribution of bet size as a fraction of 1/225 of average daily volume in the overall market. In the entire stock market, a "large bet" with the same bet-time frequency as a bet of 30% of daily volume in a typical stock will represent 30% of 1/225 of average daily volume, i.e., only 0.13%. Invariance implies that both large bets will have the same price impact in units of standard deviation of returns in business time. The large bet in the overall market will have price impact of 5 basis points ($= 150/200 \cdot (100 \cdot 225^{-1/2})$), not 150 basis points.

Now consider a very rare gigantic bet in the futures market equal to 30% of average daily volume, or 225 times the "large bet." Assuming linear price impact of bets, this gigantic bet has a price impact of 11%, 225 times greater than the large bet or 11 times one day's standard deviation of returns. The price impact is therefore large enough to generate a crash. Similar price impact of 11% can be obtained directly from equation (1), i.e. $5.00 \cdot 30 \cdot 3375^{1/3}/2$. In subsequent sections, we compare calculations of this nature—calibrated to the volumes and volatilities observed in actual panics and crashes—with the price dislocations observed.

Implementation Issues.

In order to apply market microstructure invariance to data on the five crash events, several implementation issues need to be addressed. When we need to make a choice, we make conservative assumptions that result in lower predictions for price impact.

First, it is difficult to identify the boundaries of the market. The volume and volatility inputs in our formulas should not be thought of as parameters of narrowly defined markets of a particular security in which a bet is placed, but rather as parameters based on the market as a whole. Securities and futures contracts, although traded on different exchanges, may share the same fundamentals and have a common factor structure. For example, when a large order is placed in the S&P 500 futures market, index arbitrage normally insures that the order moves prices for the underlying basket of stocks by about the same amount as it moves prices in the futures market. Consistent with the spirit of the Brady Report, we take the admittedly simplified approach of adding together cash and futures volume for three of the four crash events in which stock index futures markets existed. In our analysis of the Soros trades, we ignore cash market volume because the trades were executed so quickly that price pressure in the futures market was not transferred to cash markets.

Second, it is likely that the price impact of an order—especially its transitory component— is related to the speed with which the order is executed. The market impact equation (1) assumes that orders are executed at a typical speed in the relevant units of business time. For example, a very large trade in a small stock may be executed over several weeks or even months, while a large trade in the stock index futures market may be executed over several hours. Invariance does not assert that the speed of trading has no effect on price impact. Equation (1), which predicts expected price impact conditional on the size of a bet but not speed of execution, is consistent with the idea that some bets are executed quickly and have large transitory impact while other bets are executed slowly and have low transitory price impact.

Third, the spirit of the invariance hypothesis is that volume and volatility inputs into the price impact equation (1) are market expectations prevailing before the bet is placed. Expected volume and expected volatility determine the size of bets investors are willing to make and the degree of market depth intermediaries are willing to provide. We estimate volume and volatility based on historical data for different windows prior to the crash event. Execution of large bets may itself lead to temporary increases in both volume and volatility as markets digest the bet. Whether unusually high volume or volatility during the time of order execution is associated with higher price impact is not well-understood. This is an interesting issue for future research. Dramatically different price impact estimates are possible, depending on whether volatility estimates are based on implied volatilities before the crash, implied volatilities during the crash, historical volatilities based on the crash period itself, or historical volatilities based on months of data before the crash.

Fourth, there have been numerous changes in market mechanisms between 1929 and 2010, including better communications technologies, electronic handling of orders, changes in order handling rules in 1998 affecting NASDAQ stocks, a reduction in tick size from 12.5 cents to one cent in 2001, and the migration of trading volume from face-to-face trading floors to anonymous electronic platforms. While these changes may have lowered bid-ask spreads, we assume—in the spirit of Black (1971)—that such changes have had little effect on market depth. This assumption makes it possible to apply market depth estimates based on portfolio transitions during 2001-2005 to the entire period from 1929 to 2010.

Fifth, while our market impact formula predicts price impact resulting from bets, the actual price changes reflect not only sales by particular groups of traders placing large bets but also many other events occurring at the same time, including arrival of news and trading by other traders. We provide a brief discussion of how other factors may have influenced prices during the five crash events.

We next apply microstructure invariance to each of the five crash events.

III. The Stock Market Crash of October 1929

The stock market crash of October 1929 is the most infamous crash in the history of the United States. It became seared in the memories of many after it was followed by even larger stock price declines from 1930 to 1932, bank runs, and the Great Depression.

In the late 1920s, many Americans became heavily invested in a stock market boom. A significant portion of stock investments was made in leveraged margin accounts. Between 1926 and 1929, both the level of margin debt and the level of the Dow Jones average doubled in value. Both the stock market boom and the boom in margin lending came to an abrupt end during the last week of October 1929. The Dow Jones average fell 9% from 336.13 to 305.85 the week before Black Thursday, October 24, 1929, including a drop of 6% on Wednesday, October 23, 1929. This led to liquidations of stocks in margin accounts on the morning of Black Thursday; the Dow Jones average fell 11% to 272.32 during the first few hours of trading.

Immediately after the initial stock market break on Black Thursday, a group of prominent New York bankers put together an informal fund of about $750 million to support to the market. The group appears to have supported the market by allowing the positions of large under-margined stock investors to be liquidated gradually. Although prices rose to 301.22 on Friday, October 25, 1929, confidence was badly shaken.

Market conditions worsened the following week, with more heavy margin selling. The Dow plummeted 13% to 260.64 on Black Monday, October 28, 1929, followed by another 12% decline the next day, Black Tuesday, October 30, 1929, closing at 230.07. Over one week, the Dow thus fell by about 25%. The slide continued for three more weeks, with the Dow Jones average reaching a temporary low point of 198.69 on November 13, 1929, about 48% below the high of 381.17 on September 3, 1929.

In the 1920s, there was a rapid growth in credit used to finance ownership of equity securities. Similarly to the late 1990s, investor preference for equities over bonds pushed stock prices up and bond prices down. This put upward pressure on interest rates. Unlike the late 1990s, however, demand for leverage to finance stock investment increased demand for credit in the late 1920s. To finance these leveraged purchases of stocks, individuals and non-financial corporations relied either on bank loans collateralized by securities or on margin account loans at brokerage firms.

When individuals and non-financial corporations borrowed through margin accounts at brokerage firms, the brokerage firms financed only a modest portion of the loans with credit balances from other customers. To finance the rest, brokerage firms pooled securities pledged as collateral by customers under the name of the brokerage firm (i.e., in “street name”) and then “re-hypothecated” these pools by using them as collateral for broker loans. The broker loan market of the late 1920s resembled the shadow banking system of the early 2000s in its lack of regulation, perceived safety, and the large fraction of overnight or very short maturity loans.

The broker loan market was controversial during the 1920s, just as the shadow banking system was controversial during the period surrounding the financial crisis of 2008-2009. Some thought the broker loan market should be tightly controlled to limit speculative trading in the stock market on the grounds that lending to finance stock market speculation diverted capital away from more productive uses in the real economy. Others thought it was impractical to control lending in the market, because the shadow bank lenders would find ways around restrictions and lend money anyway. The New York Fed chose to discourage New York banks from lending money against stock market collateral. As a result, loans to brokers by New York banks declined after reaching a peak in 1927.

Attracted by the resulting high interest rates on broker loans—typically 300 basis points or more higher than loans on otherwise similar money market instruments—non-New York banks and non-bank lenders continued to supply capital to the broker loan market. Many of these loans were arranged by the New York banks; sometimes, non-bank lenders bypassed the banking system entirely, making loans directly to brokerage firms.

Investment trusts (similar to closed end mutual funds) placed a large fraction of the newly raised equity into the broker loan market rather than buying expensive common stocks. Corporations, flush with cash from growing earnings and proceeds of securities issuance, invested a large portion of these funds in the broker loan market rather than in new plant and equipment.

Market participants watched statistics on broker loans carefully, noting the tendency for total lending in the broker loan market to increase as the stock market rose. Markets were aware that margin account investors were buyers with “weak hands,” likely to be flushed out of their positions by margin calls if prices fell significantly. Discussions about who would buy if a collapse in stock prices forced margin account investors out of their positions resembled similar discussions in 1987 concerning who would take the opposite side of portfolio insurance trades.

While there was panic in the stock market during the 1929 crash, there was no observable panic in the money markets. The stock market panic of 1929 led to money market conditions entirely different from money market panics predating the establishment of the Fed in 1913 and the money market panic surrounding the collapse of Lehman Brothers in 2008. In these panics, fearful lenders suddenly withdrew money from the money markets, short term interest rates spiked upwards, credit standards became more stringent, and weak borrowers were forced to liquidate collateral at distressed prices. In the last week of October 1929, by contrast, interest rates actually fell and credit standards were relaxed by major banks, which cut margin requirements for stock positions. The New York Fed encouraged easy credit by purchasing government securities, by cutting its discount rate twice, and by encouraging banks to expand loans on securities to support an orderly market. The unprecedented increase in demand deposits at New York banks gave the banks plenty of cash to use to finance increased loans on securities. At the same time, some non-bank lenders abandoned the broker loan market, because falling interest rates made it less attractive.

The large increase in bank loans on securities is consistent with the interpretation that bankers took the financing of some under-margined accounts out of the hands of brokerage firms and temporarily brought the broker loans onto their own balance sheets. The gradual reduction in these loans over several weeks suggests that the bankers were liquidating those positions gradually in order to avoid excessive price impact. Instead of fire sale prices resulting from a credit squeeze, the picture was one of a sudden, brutal bursting of a stock market bubble financed by prudent margin lending to imprudent borrowers, with a rapid return to "normal" price levels in the stock market.

The Broker Loan Market.

To quantify the margin selling which occurred during the last week of October 1929, we follow the previous literature and contemporary market participants by estimating margin selling indirectly from data on broker loans and bank loans collateralized by securities.[1]

In the 1920s, data on broker loans came from two sources. First, the Fed collected weekly broker loan data from reporting member banks in New York City supplying the funds or arranging loans for others. Second, the New York Stock Exchange collected monthly broker loan data based on demand for loans by NYSE member firms. The broker loan data reported by the New York Stock Exchange include broker loans which non-banks made directly to brokerage firms without using banks as intermediaries; such loans bypassed the Fed's reporting system. Since loans unreported to the Fed fluctuated significantly around the 1929 stock market crash, we rely relatively heavily on the NYSE numbers in our analysis below, but also pay careful attention to the weekly dynamics of the Fed series for measuring selling pressure during the last week of October 1929.

Figure 2 shows the weekly levels of the Fed's broker loan series and the monthly levels of the NYSE broker loan series. Two versions of each series are plotted, one with bank loans collateralized by securities added and one without ("Fed Broker Loans," "Fed Broker Loans + Bank Loans," "NYSE Broker Loans," "NYSE Broker Loans + Bank Loans"). In addition, the figure shows the level of the Dow Jones

[1] Our analysis is based on several documents: Board of Governors of the Federal Reserve System (1929, 1927-1931); Galbraith (1954); Senate Committee on Banking and Currency (1934); Friedman and Schwartz (1963); Smiley and Keehn (1988); Haney (1932).

Industrial Average from 1926 to 1930. The time series on both broker loans and stock prices follow similar patterns, rising steadily from 1926 to October 1929 and then suddenly collapsing. According to Fed data, broker loans rose from $3.141 billion at the beginning of 1926 to $6.804 billion at the beginning of October 1929. According to NYSE data, the broker loan market rose from $3.513 billion to $8.549 billion during the same period. As more and more non-banks were getting involved in the broker loan market, the difference between NYSE broker loans and Fed broker loans steadily increased until the last week of October 1929, when non-bank firms pulled their money out of the broker loan market and the difference suddenly shrank.

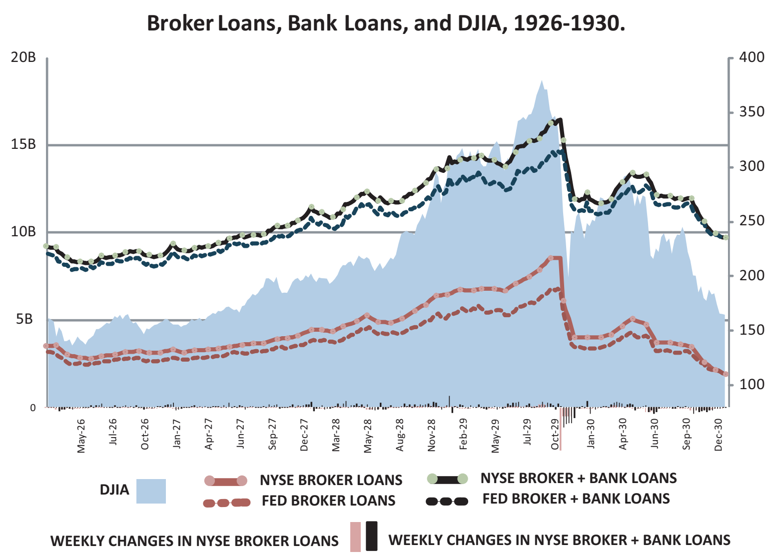

Figure 2. Broker Loans and 1929 Market Crash.

Figure shows the weekly dynamics of seven variables during January 1926 to December 1930: NYSE broker loans (red solid line), Fed broker loans (red dashed line), the sum of NYSE broker loans and bank loans (black solid line), the sum of Fed broker loans and bank loans (black dashed line), changes in NYSE broker loans (red bars), changes in the sum of NYSE broker loans and bank loans (black bars), and the Dow Jones average (in blue). Monthly levels of NYSE broker loans are marked with markers. Weekly levels of NYSE broker loans are obtained using a linear interpolation from monthly data; except for October 1929, when all changes in NYSE broker loans are assumed to occur during the last week.

During the period 1926 to 1930, weekly changes in broker loans were typically small and often changed sign, as shown in the tiny bars at the bottom of figure 2. Starting with the last week of October 1929, there were five consecutive weeks of large negative changes, almost twenty times larger than changes during preceding weeks. This de-leveraging erased the increase in broker loans which had occurred during the first nine months of the year.

Figure 3 shows how we estimate weekly margin sales from broker loan and bank

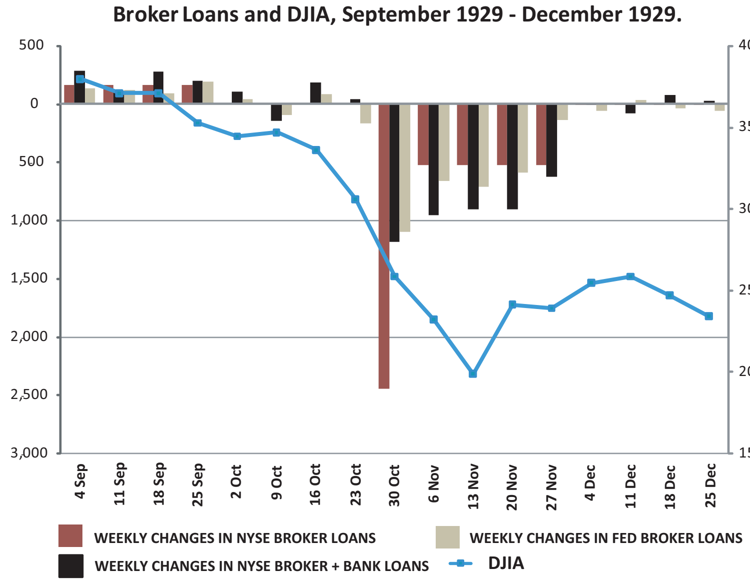

Figure 3. Broker Loans during September 1929 to December 1929.

Figure shows the dynamics during September 1929 to December 1929 of the Dow Jones average (blue line), weekly changes in NYSE broker loans (red bars), weekly changes in the sum of NYSE broker loans and bank loans (black bars), and weekly changes in Fed broker loans (grey bars). Weekly levels of NYSE broker loans are obtained using a linear interpolation from monthly data; except for October 1929, when all changes in NYSE broker loans are assumed to occur during the last week.

loan data between September 4, 1929, and December 31, 1929. (1) We difference the weekly Fed series to construct weekly changes labeled "Weekly Changes in Fed Broker Loans." (2) We interpolate the monthly NYSE series to construct a weekly series labeled "Weekly Changes in NYSE Broker Loans" by assuming that these loans changed at a constant rate within each month, except for October 1929. For October 1929, the Fed series shows little change, except for the last week. For October 1929, we therefore assume that the entire monthly change in the NYSE series represents unreported changes in broker loans which occurred during the last week of October 1929. (3) Finally, we add changes in bank loans collateralized by securities to take into account the fact that some broker loans do not represent margin sales because they were converted into bank loans collateralized by securities. The last adjustment also has a significant effect because there was an unprecedented increase in banks loans collateralized by securities during the last week of October 1929, followed by offsetting reductions during November. The results, plotted as "Weekly Changes in NYSE Broker Loans + Bank Loans," are our estimates of weekly margin selling.

For the last week of October 1929, we estimate margin selling as \$1.181 billion (the difference between the estimated reduction in broker loans of \$2.340 billion and increase in bank loans on securities of \$1.259 billion). For the three months from

September 30, 1929, to December 31, 1929, we estimate margin selling as \$4.348 billion (the difference between the reduction in NYSE broker loans of \$4.559 billion from \$8.549 billion to \$3.990 billion and an increase in bank loans on securities of \$0.211 billion from \$7.720 billion to \$7.931 billion).

We assume that every dollar in reduced margin lending or bank lending collateralized by securities represents a dollar of margin selling. We doubt that sales of bonds financed in margin accounts or transfers from bank accounts were significant during the last week of October 1929 because the high interest rate spread between broker loan rates and interest rates on bonds and bank accounts would have made it non-economical for investors to finance bonds in margin accounts or to maintain extra cash balances at banks while simultaneously holding significant margin debt.

Market Impact of Margin Selling.

Liquidation of \$1.181 billion during the last week of October and \$4.348 billion over the last three months of 1929 exerted downward price pressure on the stock market. To estimate its magnitude, we plug estimates of expected dollar volume and volatility for the entire stock market into equation (1). We treat the 1929 stock market as one market, rather than numerous markets for different stocks. We compare the price decline implied by invariance with the historical price decline of 25% during the last week of October 1929 (from 305.85 on October 23 to 230.07 on October 29) and 34% during the last three months of 1929 (from 352.57 on September 25 to 234.07 on December 25).

Our estimates are based on several specific assumptions. To convert 1929 dollars to 2005 dollars, we use the GDP deflator of 9.42. We use the year 2005 as a benchmark, because the estimates in Kyle and Obizhaeva (2016) are based on the sample period 2001-2005, with more observations occurring in the latter part of that sample. In the month prior to the market crash, typical trading volume was reported to be \$342.29 million per day in 1929 dollars, or almost \$3.22 billion in 2005 dollars. Prior to 1935, the volume reported on the ticker did not include "odd-lot" transactions and "stopped-stock" transactions, which have been estimated to be equal about 30% of "reported" volume (Board of Governors of the Federal Reserve System, 1943, p. 431). We therefore multiply reported volume by 13/10, obtaining an estimate of \$444.97 million per day. Historical volatility the month prior to October 1929 was about 2.00% per day. The total estimated margin sales of \$1.181 billion during the last week of October were approximately equal to 265% of average daily volume in the previous month.

With these assumptions, equation (1) implies that margin-related sales of \$1.181 billion triggered a price decline of 46.43%:

$$1-\exp\left[-\frac{5.00}{10^4}\cdot\left(\frac{444.97\cdot 10^6\cdot 9.42}{(40)(10^6)}\right)^{1/3}\cdot\left(\frac{0.0200}{0.02}\right)^{4/3}\cdot\frac{1.181\cdot 10^9}{(0.01)(444.97\cdot 10^6)}\right].$$

As a robustness check, table 2 reports other estimates using historical trading volume and volatility calculated over the preceding N months, with $N=1,2,3,4,6,12$.

Invariance predicts price declines ranging from 26.79% to 46.43%.

Since the reduction of broker loans of $1.181 billion was only a very small fraction of the $87.1 billion market capitalization of NYSE issues at the end of September 1929 (Brady Report, p. VIII-13), conventional wisdom implies a price change of only 1.36%. Compared with the observed price drop of 25% during the last week of October 1929, conventional wisdom predicts a much smaller decline than invariance.

Table 2—1929 Stock Market Crash: Implied Price Impact of Margin Sales.

	Months Preceding 24 October 1929					
N:	1	2	3	4	6	12
ADV (in 1929-$M)	444.97	461.45	436.49	427.20	387.18	390.45
Daily Volatility	0.0200	0.0159	0.0145	0.0128	0.0119	0.0111
Sales 10/24-10/30 (%ADV)	265%	256%	271%	276%	305%	302%
Price Impact 10/24-10/29	46.43%	36.26%	33.75%	29.93%	29.00%	26.79%
Sales 9/25-12/25 (%ADV)	977%	942%	996%	1,018%	1,123%	1,114%
Price Impact 9/25-12/25	89.95%	80.95%	78.04%	73.01%	71.66%	68.28%

Table 2 shows the implied price impact of $1.181 billion of margin sales during the week of 10/24-1/30 and $4.343 billion of margin sales during 9/25-12/25 given a GDP deflator adjustment which equates $1 in 1929 to $9.42 in 2005, along with average daily 1929 dollar volume and average daily volatility for $N = 1, 2, 3, 4, 6, 12$ months preceding October 24, 1929, based on a sample of all CRSP stocks with share codes of 10 and 11. The conventional wisdom predicts price decline of 1.36% during 10/24-10/29 and 4.99% during 9/25-12/25. The actual price decline was 25% during 10/24-10/29 and 34% during 9/25-12/25.

We also make price impact calculations for estimated margin sales of $4.348 billion during the last three months of 1929. Conventional wisdom implies a price drop of 4.99%. Invariance implies a much larger price decline, ranging from 68.28% to 89.95%, far more than the actual price decline of 34% during the last three months of 1929 and the price decline of 44% from high point in late September 1929 to low point in mid November 1929.

To many, the 1929 crash reveals a puzzling instability in financial markets. To us, the 1929 crash reveals the opposite. Compared with the four other crashes, the amount of margin selling during the 1929 crash was truly gigantic. Given 1929 GDP of $104 billion, the one week sales represent 1.14% of GDP and the three month sales represent 4.18% of GDP ($104 billion in 1929 dollars). Viewed from the perspective of market microstructure invariance, the stock market of 1929 appears to have been surprisingly resilient.

IV. The Market Crash in October 1987

On "Black Monday," October 19, 1987, the Dow Jones average fell 23%, and the S&P 500 futures market dropped 29%. From Wednesday, October 14, 1987, to Tuesday, October 20, 1987, the U.S. equity market suffered the most severe one-week decline in its history. The Dow Jones index dropped 32% from 2,500 to 1,700; as of noon Tuesday, the S&P 500 futures prices had dropped about 40% from 312 to 185.

It has long been debated whether this dramatic decrease in prices resulted from the price impact of sales by institutions implementing portfolio insurance. Portfolio insurance was a trading strategy that replicated put option protection for portfolios by dynamically adjusting stock market exposure in response to market fluctuations. Since portfolio insurers sell stocks when prices fall, the strategy amplifies downward pressure on prices in falling markets. We use estimates of portfolio insurance sales, market volume, and market volatility to calculate the price impact of portfolio insurance sales implied by invariance.

We construct estimates of sales by portfolio insurers from tables in the Brady Report, figures 13–16, pp. 197–198, obtaining results similar to Gammill and Marsh (1988). To convert 1987 dollars to 2005 dollars, we use the GDP deflator of 1.54. Over the four days October 15, 16, 19, 20, 1987, portfolio insurers sold S&P 500 futures contracts representing $10.48 billion in underlying stocks and $3.27 billion in NYSE stocks. Over the same period, portfolio insurers also bought smaller quantities of futures contracts and stocks. As a result, net sales of futures contracts and stocks combined were $9.51 billion in futures and $1.60 billion in stocks. The combined net sales were equal to $11.11 billion ($17.11 billion in 2005 dollars). Some of the market participants classified as portfolio insurers in the Brady Report abandoned their portfolio insurance strategies as prices crashed. Instead of selling the amounts dictated by portfolio insurance strategies, they switched to buying these securities. For the purpose of analyzing the price impact of portfolio insurance sales, we believe it is better to use the gross sales amount of $13.75 billion in futures and stocks combined ($21.18 billion in 2005 dollars).

In the month prior to the market crash, the average daily volume in the S&P 500 futures market was equal to $10.37 billion ($15.97 billion in 2005 dollars). The NYSE average daily volume was $10.20 billion ($15.71 billion in 2005 dollars).

To implement estimates based on invariance, we consider the entire stock market to be one market; this is consistent with the Brady Report. Some portfolio insurers abandoned their reliance on the futures markets and switched to selling stocks directly when futures contracts became unusually cheap relative to the cash market. Accordingly, we estimate sales as the sum of portfolio insurance sales in the futures market and the NYSE and expected daily volume as the sum of average daily volume in the futures market and the NYSE for the previous month. Portfolio insurance gross sales are equal to about 67% of one day's combined volume.

In the month prior to the crash, the historical volatility of S&P 500 futures returns was about 1.35% per day, similar to estimates in the Brady Report.

Plugging portfolio insurance gross sales, expected market volume, and expected market volatility into equation (1) yields a predicted price decline of 16.77%,

$$1 - \exp\left[-\frac{5.78}{10^4}\cdot\left(\frac{(10.37+10.20)\cdot 10^9\cdot 1.54}{40\cdot 10^6}\right)^{1/3}\cdot\left(\frac{0.0135}{0.02}\right)^{4/3}\cdot\frac{(10.48+3.27)}{(0.01)(10.37+10.20)}\right].$$

Table 3 reports, for robustness, other estimates based on historical trading volume and volatility calculated over the preceding N months, with $N = 1, 2, 3, 4, 6, 12$. We also report separately price impact based on portfolio insurers' gross sales and net sales. The estimated price impact of portfolio insurers' net sales ranges from 9.71% to 13.78%. The estimated price impact of portfolio insurers' gross sales ranges from 11.87% to 16.77%.

Estimates based on conventional wisdom are much smaller. According to the Brady Report there were 2,257 issues of stocks listed on the NYSE, with a value of \$2.2 trillion on December 31, 1986. Conventional wisdom implies that the portfolio insurers' gross sales of \$10.48 billion in futures and \$3.27 billion in individual stocks would have a price impact of only 0.63%. Citing similar estimates, many have rejected the idea that sales of portfolio insurers caused the 1987 market crash.

Table 3—1987 Stock Market Crash: Effect of Portfolio Insurance Sales.

	Months Preceding 14 October 1987					
N:	1	2	3	4	6	12
S&P 500 Fut ADV (1987-$B)	10.37	11.29	11.13	10.12	10.62	9.85
NYSE ADV (1987-$B)	10.20	10.44	10.48	10.16	10.04	9.70
Daily Volatility	0.0135	0.0121	0.0107	0.0102	0.0112	0.0111
Sell Orders (% ADV)	66.84%	63.28%	63.65%	67.82%	66.53%	70.33%
Price Impact of Net Sales	13.78%	11.62%	10.00%	9.71%	10.81%	11.17%
Price Impact of Gross Sales	16.77%	14.18%	12.23%	11.87%	13.20%	13.64%
Price Impact of S&P 500 Sales	14.11%	11.67%	10.10%	10.00%	10.93%	11.45%
Price Impact of NYSE Sales	13.00%	11.18%	9.56%	9.09%	10.32%	10.53%

PK: Table 3 shows the implied price impact triggered by portfolio insurers' net sales of S&P 500 futures contracts (\$9.51 billion) and NYSE stocks (\$1.60 billion), portfolio insurers' gross sales of S&P 500 futures contracts (\$10.48 billion) and NYSE stocks (\$3.27 billion), portfolio insurers' sales of S&P 500 futures adjusted for purchases of index arbitrageurs (\$10.48 billion minus \$3.27 billion), and portfolio insurers' sales of NYSE stocks adjusted for sales of index arbitrageurs (\$3.27 billion plus \$3.27 billion) in 1987 dollars. An inflation factor of 1.54 converts 1987 dollars to 2005 dollars. Average daily dollar volume and average daily volatility are based on N months preceding October 14, 1987, with $N = 1, 2, 3, 4, 6, 12$, both for the S&P 500 futures and all CRSP stocks with share codes of 10 and 11. Conventional wisdom predicts price declines of 0.51% for portfolio insurers' net sells and 0.63% for their gross sells. The actual price decline was 32% for the Dow Jones average and 40% for S&P 500 futures.

What happens if we treat the stock market as two separate markets, one for futures contracts and one for NYSE stocks? To avoid radically different price impacts in two markets connected by an index arbitrage relationship, we adjust quantities sold in these markets by the net trade imbalances of index arbitrageurs, who spread out the effects of portfolio insurers' sales across both markets. We add the NYSE's estimate of net NYSE index-arbitrage sales of \$3.27 billion (Brady Report, figures 13–14) to portfolio insurance sales in NYSE stocks and subtract the same amount from portfolio insurance sales in the futures market. This results in net sales of \$7.21 billion in the future market and \$6.54 billion in NYSE stocks. Implied price impact estimates range from 10.00% to 14.11% in the futures market and from 9.09% to 13.00% in the market for NYSE stocks. The fact that NYSE index arbitrage sales of about \$3 billion make the price impact estimates similar in both markets is consistent with the interpretation that portfolio insurance sales were driving price dynamics in both markets; this also supports the idea of treating the futures market and the market for NYSE stocks as one market connected by an arbitrage relationship.

Our implied price impact is somewhat smaller than the astonishing price drops of 32% in the cash equity market and 40% in the S&P 500 futures market observed during the 1987 market crash. The price declines may have been triggered by negative news about anti-takeover legislation and trade deficit statistic on October 14, 1987. The declines may have been aggravated by break-downs in the market mechanism which disrupted index arbitrage relationships, as documented in the Brady Report. It is therefore not surprising that actual price declines are somewhat larger than our estimates of the price impact of portfolio insurance sales. The similarity between predicted and observed price declines is consistent with our hypothesis that heavy selling by portfolio insurers played a dominant role in the crash of October 1987.

V. Trades of George Soros on October 22, 1987

People know George Soros as a philanthropist and speculator who made more than a billion dollars "breaking the Bank of England" by shorting the British pound in 1992. On Thursday, October 22, 1987, just three days after the historic market crash of 1987, George Soros had a bad day. He lost \$60 million in minutes by selling large numbers of S&P 500 futures contracts as prices spiked down 22% at the opening of trading. The rationale for the sales has been attributed to pessimistic predictions Robert Prechter made based on "Elliott Wave Theory" and similarities between the 1929 crash and the 1987 crash. In the two years following this humbling experience, Soros withdrew from active management of the Quantum Fund.

The Commodity Futures Trading Commission (1988) issued a report describing the events of October 22, 1987, without mentioning Soros by name. At 8:28 a.m. CT, October 22, approximately two minutes before the opening bell at the NYSE, a customer of a clearing member submitted a 1,200-contract sell order at a limit price of 200, more than 20% below the previous day's close of

258. Over the first minutes of trading, the price plummeted to 200, at which point the sell order was executed. At 8:34 a.m., a second identical limit order for 1,200 contracts from the same customer was executed by the same floor broker. These transactions liquidated a long position acquired on the previous day at a loss of about 22%, or about \$60 million in 1987 dollars. Within minutes, S&P 500 futures prices rebounded and, over the next two hours, recovered to the levels of the previous day's close. Within days, Soros's Quantum Fund sued the brokerage firm which handled the order, alleging a conspiracy among traders to keep prices artificially low while the sell orders were executed.

Two other events may have also exacerbated the decline in prices in the morning of October 22. First, when the broker executed the second order, he mistakenly sold 651 more contracts than the order called for. The oversold contracts were taken into the clearing firm's error account and liquidated at a significant loss to the broker. Second, the Commodity Futures Trading Commission (1988) reports that the same clearing firm also entered and filled four large sell orders for a pension fund customer between 9:34 a.m. and 10:45 a.m., with a total of 2,478 contracts sold at prices ranging from 230 to 241. Remarkably, these additional orders are for almost exactly the same size as Soros's orders, a fact which suggests information leakage or coordination regarding the size of these unusually large orders.

We compare the actual price decline of 22% with predictions based on the invariance. During the previous month, average daily volatility was 8.63%, and average daily volume in the S&P 500 futures market was \$13.52 billion (\$20.82 in 2005 dollars). The very high volatility estimate based on crash data is reasonable because market participants expected this volatility to persist. In contrast to our analysis of the 1987 crash, we assume S&P 500 futures market to be separate from the market for NYSE stocks. Since Soros's sales started just before the opening of NYSE trading, the arbitrage mechanism which connects stock and futures markets did not have time to work; futures contracts traded at levels about 20% cheaper than stocks.

Since each S&P 500 contract had a notional value of 500 times the S&P 500 index, one contract represented ownership of about \$129,000 with an S&P 500 level of 258. Equation (1) predicts that the sale of 2,400 contracts—corresponding to about \$309.60 million in 1987 dollars and equal to 2.29% of average daily volume during the previous month—would trigger price impact of 6.27%,

$$1 - \exp\Big[-\frac{5.00}{10^4} \cdot \left(\frac{13.52 \cdot 10^9 \cdot 1.54}{40 \cdot 10^6}\right)^{1/3} \cdot \left(\frac{0.0863}{0.02}\right)^{4/3} \cdot \frac{309.60 \cdot 10^6}{(0.01)(13.52 \cdot 10^9)}\Big].$$

Table 4 presents three sets of price impact estimates based on the historical trading volume and volatility of S&P 500 futures contracts calculated over the preceding N months, with $N = 1, 2, 3, 4, 6$, and 12. Invariance implies (A) price impact of 1.67% to 6.27% based on 2,400 contracts alone; (B) price impact of 2.12% to 7.90% including the 2,400 contracts and 651 error contracts (3,051 contracts in total); and (C) price impact of 3.81% to 13.85% including the 2,400 contract,

TABLE 4—OCTOBER 22, 1987: EFFECT OF SOROS'S TRADES.

	Months Preceding 22 October 1987					
N:	1	2	3	4	6	12
S&P 500 Fut ADV (1987-$B)	13.52	11.72	11.70	10.99	10.75	10.04
Daily Volatility	0.0863	0.0622	0.0502	0.0438	0.0365	0.0271
2,400 contracts as %ADV	2.29%	2.64%	2.65%	2.82%	2.88%	3.08%
Price Impact A	6.27%	4.50%	3.40%	2.96%	2.36%	1.67%
Price Impact B	7.90%	5.68%	4.30%	3.75%	2.99%	2.12%
Price Impact C	13.85%	10.06%	7.66%	6.69%	5.36%	3.81%

Table 4 shows the implied price impact of (A) Soros's sell order of 2,400 contracts; (B) Soros's sell order of 2,400 contracts plus 651 contracts of error trades (3,051 contracts in total); and (C) Soros's sell order of 2,400 contracts, plus 651 contracts of error trades, plus the sell order of 2,478 contracts by the pension fund (5,529 contracts in total). The calculations assume a GDP deflator which equates $1 in 1987 to $1.54 in 2005, average daily 1987 dollar volume and average daily volatility for $N = 1, 2, 3, 4, 6, 12$ months preceding October 22, 1987 for the S&P 500 futures contracts. Conventional wisdom predicts price declines of 0.01%, 0.02%, and 0.03%, respectively. The actual price decline in the S&P 500 futures market was 22%.

the 651 error contracts, and the 2,478 contracts sold by the pension fund (5,529 contracts in total). The actual price decline of 22% is somewhat larger than our estimate. Factors which could have led to large impact include expectations of volatility greater than our estimate based on the previous month of daily data, front-running based on leakage of information about the size of the order, and the peculiarly aggressive execution strategy of placing two limit orders with a limit price of 200, more than 20% below the previous day's close.

Conventional wisdom implies minuscule price changes for these transactions. Given the total value of $2.2 trillion of issues listed on the NYSE at the end of 1986, the Soros's sell order, the erroneous sales, and the sales by the pension fund would be expected to have a combined price impact of only 0.03%.

VI. The Liquidation of Jérôme Kerviel's Rogue Trades by Société Générale during January 21-23, 2008

On January 24, 2008, Société Générale issued a press release stating that the bank had "uncovered an exceptional fraud." Subsequent reports by Société Générale (2008a,b,c) revealed that rogue trader Jérôme Kerviel had used "unauthorized" trading to place large bets on European stock indices.

Kerviel had established long positions in equity index futures contracts with underlying values of €50 billion: €30 billion on the Euro STOXX 50, €18 billion on DAX, and €2 billion on the FTSE 100. He acquired these naked long positions mostly between January 2 and January 18, 2008, concealing them using fictitious short positions, forged documents, and emails suggesting his positions were hedged. The fall in index values in the first half of January led to losses on these hidden directional bets. The nature of the positions was uncovered on Friday, January 18. After liquidating the positions between Monday, January 21, and Wednesday, January 23, the bank had sustained losses of €6.4 billion which—after subtracting out €1.5 billion profit as of December 31, 2007—were reported as a net loss of €4.9 billion.

The Financial Markets Authority (AMF), which regulates French stock market disclosure, allowed Société Générale to delay announcing the fraud publicly for three days, so that Kerviel's positions could be liquidated quietly. The head of the central bank also delayed informing the government. As Société Générale liquidated the positions, prices fell all across Europe. The Stoxx Europe Total Market Index (TMI)—which represents all of Western Europe—fell by 9.44% from the market close on January 18, 2008, to its lowest level on January 21, 2008. The Fed unexpectedly announced an unprecedented 75-basis point cut in interest rates on January 22, 2008, several days before its regularly scheduled meeting. We do not know whether Fed officials were aware of Société Générale's sales when the decision was made to cut interest rates. This announcement had a positive effect on stock markets around the world and should have helped Société Générale to obtain more favorable execution prices on some portion of its trades. January 21, 2008, was a bank holiday in the United States. In the previous year, the futures markets had only one third of the typical volume on days when U.S. markets were closed. Lower trading volume on January 21 could have reduced market liquidity, making the unwinding of Kerviel's positions more expensive.

In explaining the costs of liquidating the positions to disgruntled shareholders already concerned about the bank's losses on subprime mortgages, bank officials blamed unfavorable market conditions, not the market impact associated with liquidating the trades themselves. Expressing conventional wisdom, the bank announced that the trades accounted for not more than 8% of turnover on any one of the futures exchanges in which they were conducted and did not have a serious market impact. We examine whether the losses associated with price impact predicted by microstructure invariance are consistent with actual reported losses and observed declines in prices.

Due to significant correlations among European markets, we perform our analysis under the assumption that all European stock and futures markets are one market. Based on data from the World Federation of Exchanges, the seven largest European exchanges by market capitalization (NYSE Euronext, London Stock Exchange, Deutsche Börse, BME Spanish Exchanges, SIX Swiss Exchange, NASDAQ OMX Nordic Exchange, Borsa Italiana) had total market capitalization in 2008 equal to $7.97 billion and average daily volume for the month ending January 18, 2008 equal to €69.51 billion.

We also sum average daily trading volume across the ten most actively traded

European equity index futures markets (Euro Stoxx 50, DAX, CAC, IBEX, AEX, Swiss Market Index SMI, FTSE MIB, OMX Stockholm 30, Stoxx 50 Euro) and find average daily futures volume of €110.98 billion. The total daily volume in both European stock and equity futures markets was equal to €180.49.

Our estimate of expected volatility is 1.10%, the previous month's daily standard deviation of returns for the Stoxx Europe Total Market Index (TMI).

According to equation (1), the liquidation of a €50 billion Kerviel's position—equal to about 27.70% of the average daily volume in aggregated stock and futures markets—is expected to trigger a price decline of 10.79% in European markets,

$$1 - \exp\Big[- \frac{5.00}{10^4} \cdot \Big(\frac{180.49 \cdot 1.4690 \cdot 0.92 \cdot 10^9}{40 \cdot 10^6} \Big)^{1/3} \Big(\frac{0.0011}{0.02} \Big)^{4/3} \frac{50}{(0.01)180.49} \Big].$$

In this equation, we use an exchange rate of \$1.4690 per Euro to convert Euro volume into U.S. dollar volume and a GDP deflator of 0.92 to convert 2008 dollars into 2005 dollars.

Table 5 shows the estimates of price impact based on historical trading volume and volatility of futures contracts on European indices calculated over the preceding N months, with $N = 1, 2, 3, 4, 6$, and 12. Invariance predicts price changes ranging from 10.59% to 12.93%. The Stoxx TMI index actually fell by 9.44% from the market close of 316.73 on January 18, 2008, to its lowest level of 286.82 on January 21, 2008.

In contrast, conventional wisdom predicts that sales of €50 billion would have a much smaller price impact of 0.43%, given that it represents less than one percent of the total capitalization of European markets, which was about €11.752 trillion in December 2007, as reported by Federation of European Securities Exchanges.

We also examine whether implied cost estimates are consistent with officially reported losses of €6.30 billion. We assume that average impact cost is equal to half of predicted price impact since—assuming no leakage of information about the trades—a trader can theoretically walk the demand curve, trading only the last contracts at the worst expected prices. Accounting for compounding, invariance predicts that the total cost of unwinding Kerviel's position is equal to 5.55% of the initial €50 billion position, i.e., €2.77 billion.

Officially reported losses also include mark-to-market losses sustained by hidden naked long positions as markets fell from the end of the previous reporting period on December 31, 2007, to the decision to liquidate the positions when the market reopened after January 18, 2008. From December 28, 2007, to January 18, 2008, the Euro STOXX 50 fell by 9.18%, DAX futures fell by 9.40%, and FTSE futures fell by 8.68%. If we assume that Kerviel held a constant long position from December 31, 2007, to January 18, 2008, then these positions would have sustained €4.62 billion in mark-to-market losses during that period. Société Générale reported, however, that Kerviel acquired his hidden long position gradually over the month of January. If we assume that Kerviel acquired his position gradually by purchasing equal quantities of futures contracts at each lower tick level from the end-of-year

Table 5—January 2008: Effect of Liquidating Kerviel's Positions.

	Months Preceding January 18, 2008					
N:	1	2	3	4	6	12
Stk Mkt ADV (2008-€B)	69.51	66.51	67.37	67.01	66.73	66.32
Fut Mkt ADV (2008-€B)	110.98	114.39	118.05	117.46	127.17	121.26
Daily Volatility	0.0110	0.0125	0.0121	0.0117	0.0132	0.0111
Order as %ADV	27.70%	27.64%	26.97%	27.11%	25.79%	26.66%
Price Impact	10.79%	12.66%	11.94%	11.53%	12.93%	10.59%
Total Losses (2008-€B)	2.77	3.27	3.08	2.97	3.34	2.72
Losses: Adj A (2008-€B)	5.08	5.58	5.39	5.28	5.65	5.03
Losses: Adj B (2008-€B)	7.39	7.89	7.70	7.59	7.96	7.34

Table 5 shows the predicted losses of liquidating Kerviel's positions of €50 billion under the assumption that the major European cash and futures markets are integrated, one Euro is worth \$1.4690, given an inflation adjustment of 0.92 to convert 2008 dollars to 2005 dollars. Results are provided based on average daily volume of the major European stock exchanges and index futures as well as daily volatilities of Stoxx Europe TMI, based on N months preceding January 18, 2008, with $N = 1, 2, 3, 4, 6, 12$. Conventional wisdom predicts price decline of 0.43%. The actual price decline in the Stoxx Europe TMI was 9.44%.

2007 close to January 18 close, we estimate that such positions would be under water by only half as much, i.e., €2.31 billion, at the close of January 18.

Table 5 reports that the sum of estimated market impact costs of liquidating rogue position range from €2.72 billion to €3.34 billion under different assumptions about expected volume and volatility. Adding mark-to-market losses sustained prior to liquidation leads to estimated losses ranging (A) from €5.03 billion to €5.65 billion if positions were acquired gradually and (B) from €7.34 billion to €7.96 billion if positions were held from the end of 2007. These estimates are similar in magnitude to reported losses of €6.30 billion.

As a robustness check, we also estimate market impact under the assumption that the Euro STOXX 50, the DAX, and the FTSE 100 futures markets are distinct markets, not components of one bigger market. In the month preceding January 18, 2008, historical volatility per day was 98 basis points for futures on the Euro STOXX 50, 100 basis points for futures on the DAX, and 109 basis points for futures on the FTSE 100. Average daily volume was €55.19 billion for Euro STOXX 50 futures, €32.40 billion for DAX futures, and £7.34 billion for FTSE 100 futures. Kerviel's positions of €30 billion in Euro STOXX 50 futures, €18 billion in DAX futures, and €2 billion in FTSE 100 futures represented about 54%, 56%, and 20% of daily trading volume in these contracts, respectively. We use an exchange rate of €1.3440 for £1 on January 17, 2008. Our calculations estimate a price impact of 12.08% for liquidation Kerviel's position in Euro STOXX 50 futures, a price impact of 10.77% for liquidation of his DAX futures position, and a price impact of 4.12% for liquidation of his FTSE futures position. Indeed, from the close on January 18 to the close on January 23, Euro STOXX 50 futures fell by 10.50%, DAX futures fell by 11.91%, and FTSE 100 futures fell by 4.65%. Note that from the close on January 18 to the lowest point during January 21 through January 23, Euro STOXX 50 futures fell by 11.67%, DAX futures fell by 12.71%, and FTSE 100 futures fell by 9.54%. The similarity of actual price declines for the STOXX 50, DAX and FTSE suggests substantial integration of European markets, consistent with our strategy of thinking of these markets as one market.

Large price declines in markets where Kerviel did not hold positions suggest that the markets are well integrated as well. From the close on January 18 to low points on January 22, the Spanish IBEX 35, the Italian FTSE MIB, the Swedish OMX, the French CAC 40, the Dutch AEX and the Swiss Market Index fell by 12.99%, 10.11%, 8.63%, 11.53%, 10.80%, and 9.63%, respectively. By January 24, all of these markets had reversed these losses substantially. Euro Stoxx 50 and FTSE reversed losses as well, but DAX recovered only partially.

VII. The Flash Crash of May 6, 2010

Not all market crashes happen in the United States in October, and not all of them last for a long time. The flash crash of 2010 occurred on May 6 and lasted for only twenty minutes.

During the morning of May 6, 2010, the S&P 500 declined by 3%. Rumors of a default by Greece had made markets nervous in a context where there was also uncertainty about elections in the U.K. and an upcoming jobs report in the U.S. In the afternoon, something bizarre happened. During the five minute interval from 2:40 p.m. to 2:45 p.m. ET, the E-mini S&P 500 futures contract plummeted 5.12% from 1,113 to 1,056. After a pre-programmed circuit breaker built into the CME's Globex electronic trading platform halted trading for five seconds, prices rose 5% over the next ten minutes, recovering previous losses.

Shaken market participants began a search for guilty culprits. "Fat finger" errors and a cyber-attack theories were quickly discarded. Many accused high frequency traders of failing to provide liquidity as prices collapsed.

After the flash crash, the Staffs of the CFTC and SEC (2010a,b) issued a joint report. The report highlighted the fact that an automated execution algorithm sold 75,000 S&P 500 E-mini futures contracts between 2:32 p.m. and 2:51 p.m. on the CME's Globex platform. The period of execution corresponded precisely to the V-shaped flash crash. The E-mini contract represents exposure of 50 times the S&P 500 index, one tenth the multiple of 500 for the older but otherwise similar contract sold by portfolio insurers in 1987. Given the S&P 500 index values, the program sold S&P 500 exposure of approximately $4.37 billion. The joint report did not mention the name of the seller, but newspapers identified the seller as Waddell & Reed.

Many people did not believe that selling 75,000 contracts could have triggered a price drop of 5%. Indeed, the $4.37 billion in sales represented only 3.75% of the daily trading volume of about 2,000,000 contracts per day in the S&P 500 E-mini futures market. Could the execution of such an order could have resulted in a flash crash?

To examine this question in the context of invariance, we make assumptions about expected trading volume and volatility. During the preceding month, the average trading volume in E-mini contracts was about \$132 billion per day. The average volume in the stock market was about \$161 billion per day. Thus, volume in the futures and stock market combined was \$292 billion. Not surprisingly, trading volume was much higher on May 6, 2010. During the previous month, average daily price volatility was about 1.07% per day. Since the 3% price drop in the morning may have reset market expectations about future volatility, we also use a rough estimate of expected volatility equal to 2.00% per day as a robustness check. Given a GDP deflator of 0.90 between 2005 and 2010, equation (1) implies that the sales of \$4.37 billion—equal to about 3.31% of average daily volume in S&P 500 E-mini futures market in the previous month or 1.49% for futures and stock market combined—is expected to trigger a price decline of 0.61%,

$$1-\exp\left[-\frac{5.00}{10^4}\cdot\left(\frac{(132+161)\cdot 0.90\cdot 10^9}{40\cdot 10^6}\right)^{1/3}\cdot\left(\frac{0.0107}{0.02}\right)^{4/3}\cdot\frac{75,000\cdot 50\cdot 1,164}{0.01\cdot(132+161)\cdot 10^9}\right].$$

Table 6 shows additional estimates based on historical volume and volatility of S&P 500 E-mini futures contracts calculated over the preceding N months, with $N = 1, 2, 3, 4, 6$, and 12, using both historical volatility and volatility of 2% per day. Conventional wisdom predicts a tiny price decline of 0.03%, given that the capitalization of U.S. market was about \$15.077 trillion at the end of 2009. Invariance predicts much larger price changes. Estimates based on historical volatility range from 0.44% to 0.73%. Estimates based on 2% volatility range from 1.39% to 1.65%. If we do not treat the cash market and the futures market as one market but focus only on the futures market, then the estimates range from 0.76% to 1.29% for historical volatility and from 2.35% to 2.91% for volatility of 2% (not reported).

The predicted price impact is smaller than the actual decline of 5.12%. We believe that unusually fast execution significantly increased the temporary price impact of these trades. This is consistent with the observed rapid rebound in prices.

VIII. The Frequency of Market Crashes

Market microstructure invariance can be used to quantify the frequency of crash events, including both the size of selling pressure and the resulting price impact.

Using portfolio transitions orders as proxies for bets, Kyle and Obizhaeva (2016) find that the invariant distributions of buy and sell bet sizes can be closely approximated by a log-normal. The distribution of bet size $\tilde{X}$ of a stock with expected daily volume of $P \cdot V$ dollars and expected daily returns volatility σ can be ap-

Table 6—Flash Crash of May 6, 2010: Effect of 75,000 Contract Futures Sale.

	Months Preceding 6 May 2010					
N:	1	2	3	4	6	12
S&P 500 Fut ADV (2010 $B)	132.00	107.49	109.54	112.67	100.65	95.49
Stk Mkt ADV (2010 $B)	161.41	146.50	142.09	143.03	132.58	129.30
Daily Volatility	0.0107	0.0085	0.0078	0.0090	0.0089	0.0108
Order as %ADV	1.49%	1.72%	1.73%	1.71%	1.87%	1.94%
Price Impact (hist σ)	0.61%	0.49%	0.44%	0.53%	0.55%	0.73%
Price Impact ($\sigma = 2\%$)	1.39%	1.52%	1.53%	1.52%	1.61%	1.65%

Table 6 shows the predicted price impact of sales of 75,000 S&P 500 E-mini futures contracts. The GDP deflator of 0.90 converts 2010 dollars to 2005 dollars. Calculations are based on average daily volume and volatility of the S&P 500 E-mini futures for the N months preceding January 18, 2008, with $N = 1, 2, 3, 4, 6, 12$. Conventional wisdom predicts a price decline of 0.03%. The actual price decline in the S&P 500 E-mini futures market was 5.12%.

proximated as a log-normal

$$\text{(3)} \qquad \ln\left(\frac{|\tilde{X}|}{V}\right) = -5.71 - 2/3 \cdot \ln\left(\frac{\sigma \cdot P \cdot V}{(0.02)(40)(10^6)}\right) + \sqrt{2.53} \cdot \tilde{Z},$$

where $\tilde{Z} \sim N(0, 1)$. Under the assumption that there is one unit of intermediation trade volume for every bet, the bet arrival rate γ per day is given by

$$\text{(4)} \qquad \ln(\gamma) = \ln(85) + 2/3 \cdot \ln\left(\frac{\sigma \cdot P \cdot V}{(0.02)(40)(10^6)}\right),$$

These equations have the following implications for a "benchmark stock" with volume of $40 million per day and volatility 2% per day. The estimated mean of -5.71 implies a median bet size of approximately $\$132,500$, or 0.33% of daily volume. The estimated log-variance of 2.53 implies that a one-standard-deviation increase in bet size is a factor of about 4.91. The implied average bet size is $469,500 and a four-standard-deviation bet is about $77 million, or 1.17% and 192% of daily volume, respectively ($0.33\% \cdot exp(2.53/2)$ and $0.33\% \cdot exp(2.53 \cdot 4)$). There are 85 bets per day. The standard deviation of daily order imbalances is equal to 38% of daily volume ($85^{1/2} exp(-5.71 + 2.53)$). Half the variance in returns results from fewer than 0.10% of bets and suggests significant kurtosis in returns.

Now let us extrapolate these estimates to the entire market, where volume is the sum of the volume of CME S&P 500 futures contracts and all individual stocks. Using convenient round numbers based on the 2010 flash crash, the volume for the entire market is about $270 billion per day, or 6,750 times the volume of a benchmark stock. The volatility of the index is about 1% per day, or half of 2% volatility of a benchmark stock. With 6,750 conveniently equal to $15^3 \cdot 2$, invariance implies that market volume consists of 19,125 bets ($85 \cdot 15^2$) with the median bet of about \$4 million ($\$132,500 \cdot 15 \cdot 2$), or 0.0014% of daily volume. The implied average bet size is \$14 million, or 0.0052% of daily volume, and a four-standard-deviation bet is \$2.310 billion ($\$469,500 \cdot 15 \cdot 2$ and $\$77 \cdot 10^6 \cdot 15 \cdot 2$), or 0.86% of daily volume. The implied standard deviation of cumulative order imbalances is 2.55% of daily volume (38%/15).

Equations (3) and (4) can be used to predict how frequently crash events occur. The three large crash events—the 1929 crash, the 1987 crash, and the 2008 Société Générale trades—are much rarer events than the two smaller crashes—the 1987 Soros trades and the 2010 flash crash.

We estimate the 1929 crash, the 1987 crash, and the 2008 liquidation of Kerviel's positions to be 6.15, 5.97, and 6.19 standard deviation bet events, respectively. Given corresponding estimated bet arrival rates of 1,887 bets, 5,606 bets, and 19,059 bets per day, such events would be expected to occur only once every 5,516 years, 597 years, and 674 years, respectively. Obviously, either the far right tail of the distribution estimated from portfolio transitions is fatter than a log-normal or the log-variance estimated from portfolio transition data is too small. In the far right tail of the distribution of the log-size of portfolio transition orders in the most actively traded stocks, Kyle and Obizhaeva (2016) do observe a larger number observations than implied by a normal distribution. It is also possible that portfolio transition orders are not representative of bets in general. If the true standard deviation of log bet size is 10% larger than implied by portfolio transition orders, then 6.0 standard deviation events become 5.4 standard deviation events, which are expected to occur about 34 times more frequently.

We estimate the 1987 Soros trades and the 2010 flash crash trades to be 4.45 and 4.63 standard deviation bet events, respectively. Given estimated bet arrival rates of 14,579 bets and 29,012 bets per day, respectively, bets of this size are expected to occur multiple times per year. We believe it likely that large bets of this magnitude do indeed occur multiple times per year, but execution of such large bets typically does not lead to flash crashes because such large bets would normally be executed more slowly and therefore have less transitory price impact.

IX. Conclusion: Lessons Learned

It is, of course, impossible to infer from only five data points definitive conclusions about the ability of microstructure invariance to predict the price impact of liquidations of large quantities. Each of the crash events has event-specific features which make it difficult to estimate the size of the positions liquidated, market expectations about long-term volume and volatility, and the effects of other contemporaneous events. Application of microstructure invariance concepts to intrinsically infrequent historical episodes therefore requires an exercise in judgement to extract appropriate lessons learned. Nevertheless, the five cases we have examined suggest important lessons, both for policymakers interested in measuring and predicting crash events of a systemic nature and for asset managers interested in managing market impact costs associated with execution of large trades that might potentially disrupt markets.

Price Impact is Large in Liquid Markets.

For the five crash events, the price declines predicted by invariance are large and much more similar in magnitude to actual price declines than predictions based on conventional wisdom. The predicted decline for the 1987 crash is about half the decline which actually occurred, and the predicted decline of the 2008 liquidation of Jérôme Kerviel's positions match the actual decline closely. This is consistent with the interpretation that microstructure invariance may apply not only to individual stocks but also to stock index futures markets or a combination of futures and cash markets. The large predicted price impacts result from the assumption that price impact is linear in trade size and the assumption that the price impact of trading a given fraction of average daily volume (measured in volatility units) is proportional to the cube root of trading activity, define as a product of dollar volume and volatility. These assumptions contrast with empirical literature suggesting that price impact is concave in trade size and the conventional wisdom that execution of a given percentage of average daily volume has similar price impact regardless of the level of trading activity in the market.

Rapid Execution Magnifies Transitory Price Impact.

The speed of execution influences how orders affect prices. Unusually rapid execution is likely to generate large temporary price impact associated with V-shaped price paths, in which prices plunge sharply and then rapidly recover. For V-shaped price paths not to make it easy for others to profit from "front-running" the trades, it is also necessary for the execution of such trades to be accompanied by a transitory increase in price volatility. In contrast, slowing down execution of bets may lessen transitory price impact by signalling that the trades are not based on private information with a short half-life.

Invariance provides a perspective for thinking about the speed of execution of bets in markets with different levels of trading activity. A good example is the 2010 flash crash. The Staffs of the CFTC and SEC (2010b) state that the order was executed extremely rapidly in just 20 minutes, while two orders of similar size had been executed before over periods of 5 or 6 hours. The selling algorithm was programmed to execute the sales at a rate equal to 9% of volume over the period of execution. Since volume increased dramatically while the sales were occurring, the order was executed in only 20 minutes. Given a difference of 15^3 in trading activity between the entire market and the market for a typical stock, invariance implies that compressing selling of about 1.50% of expected daily volume in the overall market into 1/20 of a day (about 20 minutes) would be similar to selling about 30% of expected trading volume ($= 1.50\% \cdot 20$) each day for 12 ($= 15^2/20$) consecutive days in a typical stock!

Our estimates are based on parameters calibrated using the transaction costs of portfolio transition orders. Transitions are usually executed over a period of a few days, with complex transitions sometimes implemented over a period of a few weeks. These time horizons are consistent with a prudent pace designed to keep price impact costs low. In interpreting our results, we make an identifying assumption that transition orders and bets during crash events have been executed at a "natural" speed.

In both the 1987 Soros episode and the 2010 flash crash, the trades were executed unusually rapidly, over a few minutes. There was price impact larger than predicted by invariance, unusually high volatility, and rapid price recovery. In contrast, the 1987 portfolio insurance trades and the 2008 Société Générale trades were over a few days, not a few minutes. While they induced smaller transitory price impact, there was nevertheless elevated volatility. Measures implemented by bankers and the Fed in the last week of October 1929 smoothed the margin selling out over a period of five weeks rather than a few days. This appears to have lessened temporary price impact, with price declines substantially smaller than predicted by invariance.

Kyle, Obizhaeva and Wang (2016) discuss a smooth trading model which explains theoretically how flash crashes can result from unusually rapid execution of large bets. On the one hand, markets interpret extremely rapid, heavy selling as an indication that extremely negative information is about to flow into the market. Markets collapse immediately when a heavy rate of selling is detected. When the expected negative information does not quickly materialize, prices quickly rebound. As prices spike down and then recover, much of the heavy selling takes place while prices are rebounding. On the other hand, markets interpret relatively slow selling as coincidentally correlated negative information shocks, which spread price declines out over time. Prices continue to fall as long as the slow selling takes place. The predictions of the model are broadly consistent with the empirical patterns observed during the two flash crashes.

The selling pressure during the flash crash of 2010 occurred about fifteen times faster than such large numbers of contracts are typically sold in the market. The smooth trading model predicts that such fast selling will lead to transitory price impact fifteen times greater than selling at a typical, expected rate. This is reasonably consistent with what actually happened. Invariance predicts that a price decline of 0.61% would have occurred if the bet were executed at a normal speed. The actual decline was 5.12%, about 9 times greater than predicted, and the price quickly recovered.

The Financial System in 1929 Was Remarkably Resilient.

The price declines which occurred during the 1929 stock market crash were remarkably small given the gigantic levels of selling pressure associated with liquidation of margin loans.

The 1987 portfolio insurance trades of $13 billion were equal to about 0.28% of

GDP in that year (1987 GDP was $4.7 trillion); stock prices fell 32%. During the last week of October 1929, we estimate margin related sales to be about 1% of GDP (1929 GDP was $104 billion), approximately four times the levels of the 1987 crash; yet stock prices fell only 25%. Including additional sales equal to about 3% of GDP in subsequent weeks, we estimate margin selling over several weeks to be more than 15 times greater than the 1987 crash, as a percentage of GDP.

We believe that there are three reasons that the market crash of 1929 may have been so well contained.

First, the remarkable resilience of the financial system in 1929 can be attributed to stabilizing activities undertaken by Wall Street bankers. As market prices fell during the last week of October 1929, several large bankers quickly assembled a fund of $750 million to buy securities in order to support prices. When their decisions were publicized, the sense of panic subsided. These meetings were not unprecedented. Similar actions, for example, were undertaken by J.P. Morgan and other bankers after a crash in 1907.

Second, the New York Fed also acted prudently in 1929. In the 1920s, bankers and their regulators were aware that if non-bank lenders suddenly withdrew funds from the broker loan market, there would be pressure on the banking system to make up the difference. By discouraging banks from lending into the broker loan market prior to the 1929 crash, the New York Fed increased the ability of banks to support the broker loan market after the stock market crashed. During the last week of October 1929, the New York Fed wisely reversed its course and encouraged banks to provide bank loans on securities to their clients as a substitute for broker loans. Some brokers cut margins from 40% to 20%; this slowed liquidations of stock positions. Stock market prices stabilized by the end of 1929. There were no major failures of banks or brokerage firms. The bankers were lenders into the shadow banking system, not borrowers. By spreading margin-related sales out over five weeks, rather than compressing them into several days, these stabilizing activities appear to have reduced temporary price impact.

Third, a complimentary explanation for the resilience of the financial markets in 1929 may come from market microstructure invariance itself. It may be inappropriate to assume that the stock market in 1929 was one integrated market. There were no futures markets or ETFs which allowed investors to trade large baskets of stocks. Speculative trading and intermediation associated with underwriting of new stock issues often took place in "pools," which played a role similar to hedge funds today in that they traded actively. Pools used leverage, took short position, and arbitraged stocks against options, particularly when facilitating distribution of newly issued equity. The stock pools of the 1920s were typically dedicated to trading only one stock, and investors in the pools often had close connections to the company whose stock the pool traded. There were no prohibitions against insider trading and no SEC requiring firms to disclose material information to the market. This institutional structure may have compartmentalized speculative capital into numerous separate silos, as a result of which more capital was required to sustain orderly trading. When faced with massive liquidations of margin loans, the market may have therefore found that it had more speculative capital available to stabilize markets than in a more "efficiently" leveraged system in which hedge funds can trade hundreds of stocks simultaneously. In this sense, the 1929 market appears to be more resilient than the 1987 market. The apparent lack of resilience in the 1987 market may result from greater financial integration in 1987 than 1929.

From the perspective of the invariance, the way to interpret this compartmentalization is to think of the 1929 stock market not as one large market but as many smaller markets for different individual stocks. As a hypothetical illustration of the concept, suppose that the 1929 stock market is considered to be 125 markets for 125 different stocks. For simplicity, assume all of the individual markets have the same size. Then "business time" in each stock passes at a rate $125^{2/3} = 25$ times slower than it would pass in an integrated market. Margin liquidations which would take place in one day in an integrated market would be spread out over 25 business days in compartmentalized markets. The market impact from liquidating margin loans would occur in each stock separately, reducing price impact by a factor of $125^{1/3} = 5$. As a result of being less integrated, markets absorb shocks more slowly and the impact of an aggregate shock of a given size is lessened by being absorbed as many small shocks.

Effect of Large Bets Propagates Across Integrated Markets.

Financial markets are integrated. Heavy selling in one market will affect correlated markets. During liquidation of Jérôme Kerviel's positions in January 2008, other markets where Société Générale did not intervene had very similar performance. The bank argued that its own market impact was therefore limited. Similar patterns were documented during the 1987 crash, when not only U.S. markets but also many major world markets experienced severe declines, despite the fact that the portfolio insurance selling was focused on U.S. stocks. According to Roll (1988), this indicates that portfolio insurance did not trigger the 1987 crash.

We disagree. Roll's argument does suggest that market impact estimates should take into account how market liquidity is shared across markets in different continents and markets of related assets, an issue we leave for future research. It also supports our preferred strategy for the analysis of Société Générale's trades of looking at the price effects on markets aggregated across Europe rather than focusing on isolated pools of liquidity in the market for one country's equities. It also supports our preferred strategy for the analysis of the 1987 market crash of looking at aggregated stock and futures markets.

Early Warning Systems May Be Useful and Practical.

Two crash events—the 1929 margin sales and the 1987 portfolio insurance sales—involved summing trades across numerous sellers. In both cases, data was publicly available before the crash event. Data on broker loans was published by the Federal

Reserve System and the NYSE. Estimates of assets under management by portfolio insurers were available before the 1987 crash. In both cases, potential sizes and price impact of liquidations were topics of public discussion among policy makers and market participants.

A good example is the 1987 market crash. The debate about the extent to which portfolio insurance trading contributed to the 1987 market crash started before the crash itself occurred. The term "market meltdown," popularized by then NYSE chairman John Phelan, was used in the year or so before the stock market crash to describe a scenario of cascading portfolio insurers' sell orders resulting in severe price declines and posing systemic risks to the economy. Months before the 1987 crash itself, the SEC's Division of Market Regulation (1987) published a study of a cascading meltdown scenario. After describing in some detail a potential meltdown scenario which closely resembled the subsequent crash in October 1987, the study dismissed the risk of such an event as a remote possibility, in agreement with conventional Wall Street wisdom at the time.

Many market participants were firmly convinced that, given the substantial trading volume in the U.S. equity markets—and especially the index futures market—there was enough liquidity available to accommodate sales of portfolio insurers without any major downward adjustment in stock prices. During hearings before the House Committee on Energy and Commerce (1987), Hayne E. Leland defended portfolio insurance prior to the crash: "We indicated that average trading will amount to less than 2% of total stocks and derivatives trading. On some days, however, portfolio insurance trades may be a greater fraction... In the event of a major one-day fall (e.g., 100 points on the Dow Jones Industrial Average), required portfolio insurance trades could amount to $4 billion. Almost surely this would be spread over 2-3 day period. In such a circumstance, portfolio insurance trades might approximate 9-12% of futures trading, and 3-4% of stock plus derivatives trading."

If regulators had applied simple principles of market microstructure invariance prior to the market crash of 1987, they would have been alarmed by Hayne Leland's projection of potential sales of 4% of stock-plus-futures volume over three days in response to a decline in stock prices of about 4% (100 points on the Dow Jones average). At that time, the stock market was already close to a tipping point. Historical volume and volatility in July 1987 implied that sales of $4 billion in response to a 4% price decline would lead to another drop in prices, just slightly smaller than 4%. Absent stabilizing trades by investors trading in a direction opposite from portfolio insurance, invariance implies that potential portfolio insurance sales were on the verge of triggering precisely the cascade meltdown scenario.

A similar analysis may be warranted to assess the effect of leveraged ETFs on financial markets. For example, Tuzun (2012) finds that short ETFs and leveraged long ETFs in financial stocks were close to the tipping point in 2008 and 2009. A price decline of 1% would induce leveraged ETFs to sell about $1 billion; invariance implies that this would lead to further price declines of about 1% and potentially trigger a downward spiral.

X. Conclusion

Our examination of five case studies should not be interpreted as a regression with five data points. Instead, we think that examining them leads to useful insights about why stock market crashes occur, how to prevent them if possible, and how to respond to them appropriately if not.

Market crashes continue to occur. The events in the U.S. Treasury market on October 15, 2014, described in Staffs of the Fed, the CFTC, and SEC (2015), may have been caused by execution of a large buy bet; unlike the selling of futures contracts on May 6, 2010, this led to a "flash rally," not a flash crash. The sharp V-shaped devaluation of Russian currency on December 16, 2014, was likely to be caused by a large multi-billion-dollar bet, as discussed in Obizhaeva (2015). The collapse of the Chinese stock market in the summer of 2015 was likely caused by liquidations of margin accounts, similar in many ways to the crash of 1929 in the U.S. market; in both crashes, extraordinary steps were taken to stabilize the market afterwards.

REFERENCES

Akerlof, George A., and Robert J. Shiller. 2009. *Animal Spirits: How Human Psychology Drives the Economy, and Why It Matters for Global Capitalism.* Princeton University Press.

Black, Fischer. 1971. "Towards a Fully Automated Exchange, Part I." *Financial Analysts Journal*, 27: 29–34.

Board of Governors of the Federal Reserve System. 1927-1931. *Annual Report of the Board of Governors of the Federal Reserve System.* 13th through 17th Annual Reports Covering Operations for the Years 1926 through 1930.

Board of Governors of the Federal Reserve System. 1929. *Federal Reserve Bulletin.* multiple issues.

Board of Governors of the Federal Reserve System. 1943. *Banking and Monetary Statistics, 1914-1941.*

Bouchaud, Jean-Philippe, Doyne J. Farmer, and Fabrizio Lillo. 2009. "How Markets Slowly Digest Changes in Supply and Demand." In *Handbook of Financial Markets: Dynamics and Evolution.* 47–160. North Holland.

Brennan, Michael J., and Eduardo S. Schwartz. 1989. "Portfolio Insurance and Financial Market Equilibrium." *Journal of Busines*, 62: 455–472.

Commodity Futures Trading Commission, Division of Economic Analysis and Division of Trading and Markets. 1988. *Final Report on Stock Index*

Futures and Cash Market Activity During October 1987 to the U.S. Commodity Futures Trading Commission.

Division of Market Regulation, Securitie and Exchange Commission. 1987. *The Role of Index-Related Trading in the Market Decline of September 11 and 12, 1986.*

Friedman, Milton, and Anna Jacobson Schwartz. 1963. *A Monetary History of the United States, 1867-1960. NBER Book Series Studies in Business Cycles*, Princeton University Press.

Galbraith, John Kenneth. 1954. *The Great Crash, 1929.* Boston:Houghton Mifflin Co.

Gammill, James F., and Terry A. Marsh. 1988. "Trading Activity and Price Behavior in the Stock and Stock Index Futures Markets in October1987." *The Journal of Economic Perspectives*, 2(3): 25–44.

Haney, H. Lewis. 1932. "Brokers' Loans." *Journal of the American Statistical Association*, 27: 65–72.

Harris, Lawrence, and Eitan Gurel. 1986. "Price and volume effects associated with changes in the S&P 500 list: New evidence for the existence of price pressures." *The Journal of Finance*, 41(4): 815–829.

House Committee on Energy and Commerce, U.S. Congress. 1987. *Program Trading: Hearing before the Subcommittee on Telecommunications and Finance of the Committee on Energy and Commerce.* Vol. 100th Cong., 1st Sess. Testimony of Hayne E. Leland.

Keynes, John M. 1936. *The General Theory of Employment, Interest and Money.* Palgrave Macmillan, United Kingdom.

Kyle, Albert S. 1985. "Continuous Auctions and Insider Trading." *Econometrica*, 53(6): 1315–1335.

Kyle, Albert S., and Anna A. Obizhaeva. 2016. "Market Microstructure Invariants: Empirical Hypotheses." Working Paper, University of Maryland.

Kyle, Albert S., Anna A. Obizhaeva, and Yajun Wang. 2016. "Smooth Trading with Overconfidence and Market Power." Working Paper, University of Maryland.

Leland, Hayne E., and Mark Rubinstein. 1988. "Comments on the Market Crash: Six Months After." *The Journal of Economic Perspectives*, 2: 45–50.

Miller, Merton H. 1988. "Explaining the Events of October 1987." In *After the Crash: Linkages between Stocks and Futures. AEI Studies*, , ed. Robert J. Mackay. American Enterprise Institute for Public Policy Research, University Press of America.

Miller, Merton H. 1991. *Financial Innovations and Market Volatility.* Cambridge, MA:Blackwell.

Obizhaeva, Anna A. 2015. "An Analysis of the Russian Currency Market on December 15 and 16, 2014." Working Paper, New Economic School.

Perold, André. 1988. "The Implementation Shortfall: Paper vs. Reality." *Journal of Portfolio Management*, 14(3): 4–9.

Reinhart, Carmen M., and Kenneth Rogoff. 2009. *This Time Is Different: Eight Centuries of Financial Folly.* Princeton University Press.

Roll, Richard. 1988. "The International Crash of October 1987." *Financial Analysts Journal*, 44(5): 19–35.

Scholes, Myron S. 1972. "The Market for Securities: Substitution Versus Price Pressure and the Effects of Information on Share Prices." *The Journal of Business*, 45(2): 179–211.

Senate Committee on Banking and Currency, U.S. Congress. 1934. *Stock Exchange Practices: Report Of The Committee On Banking And Currency Pursuant To S.Res. 84 (72nd Congress) and S.Res. 56 and S.Res. 97 (73rd Congress).* Vol. 73rd Congress, Second Session, U.S. Government Printing Office. "The Pecora Commission Report".

Shiller, Robert J. 1987. "Investor Behavior in the October 1987 Stock Market Crash: Survey Evidence." NBER Working Paper 2446.

Smiley, Gene, and Richard H. Keehn. 1988. "Margin Purchases, Brokers' Loans and the Bull Market of the Twenties." *Business and Economic History*, 17: 129–142.

Société Générale. 2008a. "Explanatory note about the exceptional fraud." Paris, January 27, 2008.

Société Générale. 2008b. "Mission Green, Summary Report: Interim conclusions as of February 20, 2008." General Inspection Department.

Société Générale. 2008c. "Mission Green, Summary Report." General Inspection Department, May 20, 2008.

Staffs of the CFTC and SEC. 2010a. *Preliminary Findings Regarding the Market Events of May 6, 2010.* Report of the Staffs of the CFTC and SEC to the Joint Advisory Committee on Emerging Regulatory Issues. May 18, 2010. "Preliminary Report".

Staffs of the CFTC and SEC. 2010b. *Findings Regarding the Market Events of May 6, 2010.* Report of the Staffs of the CFTC and SEC to the Joint Advisory Committee on Emerging Regulatory Issues. September 30, 2010. "Final Report".

Staffs of the Fed, the CFTC, and SEC. 2015. *The U.S. Treasury Market on October 15, 2014.* Joint staff report by U.S. Department of the Treasury, Board of Governors of the Federal Reserve System, Federal Reserve Bank of New York, U.S. Securities and Exchange Commission, U.S. Commodity Futures Trading Commission. July 13, 2015.

Tuzun, Tugkan. 2012. "Are leveraged and inverse ETFs the new portfolio insurers?" *Working Paper.*

U.S. Presidential Task Force on Market Mechanisms. 1988. *Report of the Presidential Task Force on Market Mechanisms.* Washington:U.S Government Printing Office. The "Brady Report".

Wurgler, Jeffrey A., and Ekaterina Zhuravskaya. 2002. "Does arbitrage flatten demand curves for stocks?" *The Journal of Business*, 75(4): 583–608.