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Competition among Portfolio Managers and Asset Specialization

Abstract

This paper investigates the competition among portfolio managers as they attempt to outperform each other. We provide a tractable dynamic continuous-time model of competition between two risk-averse managers concerned about relative performance. To capture the managers' asset specialization, we consider two imperfectly correlated risky stocks whereby each manager trades in one of the stocks, and so faces incomplete markets. We show that a unique pure-strategy Nash equilibrium always obtains, and provide the ensuing equilibrium portfolio policies explicitly. We find that competition makes a relatively risk tolerant manager decrease, and a risk intolerant increase, her portfolio risk. Moreover, a higher own risk aversion induces a manager to take more risk when the opponent is advantaged, in that she specializes in the stock with the relatively higher Sharpe ratio. We then explore the link between our two key ingredients, competition and asset specialization, and show that competition can be conducive to asset specialization. In particular, we find that both managers, when relatively risk tolerant, can voluntarily opt for asset specialization and the corresponding loss of diversification to avoid competing on the same turf by trading in the same set of stocks. When they are risk intolerant, however, the no-specialization scenario is more likely. When we consider a client investor of a manager, we show that her preferences for or against asset specialization could well be the opposite to that of her manager. We also examine the potential costs to a client investor, arising as managerial turnover or changing stock characteristics misaligns the client manager's policy. We find that the client loses more when it is her manager who is replaced than the other manager. In contrast, the client's losses are the same for a given change in her manager's stock characteristics as for that in the competitor manager's stock.

JEL Classifications: G11, G20, D81, C73, C61.

Keywords: Competition, Portfolio Choice, Asset Specialization, Relative Performance, Cost-Benefit Analysis.

1. Introduction

There is mounting evidence of competition among portfolio managers in the mutual and hedge fund industries. Factors such as a desire to attract money flows and career concerns incentivize a manager to deliver to her client investors higher returns relative to those delivered by the competing managers.¹ Given the important role played by portfolio managers in financial markets, many researchers have called for more work on the effects of competition. For example, Hortaçsu and Syverson (2004) state: “The mutual fund literature is not typically concerned with strategic interactions among firms in this important industry. Yet competitive forces are an important determinant of the fortunes of funds and fund families.”

The goal of this paper is to study how competition among portfolio managers affects their trading behavior, and how these effects are transmitted to their client investors. Towards this, we develop a tractable continuous-time model with two risk averse portfolio managers, where each manager in addition to caring about own horizon wealth places a certain weight on maximizing her relative return, given by the ratio of her return over the opponent’s. We also consider a client investor whose money is managed by one of the competing managers. Unlike managers, the client cares only about her own wealth. The client is passive in that she subsequently neither withdraws from nor places more money with the manager. Consistent with the evidence, we assume that portfolio managers do not invest in the same set of assets but rather trade in different stocks, reflecting their different asset specialization.² Accordingly, we adopt a framework with two imperfectly correlated risky stocks, each representing a manager’s asset specialization, and so assume that a manager can only trade in that stock (and so faces incomplete markets). Given the evidence of substantial heterogeneity in fund managers’

¹The money flows argument is that portfolio managers seek to outperform their peers so as to receive a larger share of new money from retail customers, as documented by Chevalier and Ellison (1997) and Sirri and Tufano (1998) for mutual funds, Agarwal, Daniel, and Naik (2004) and Ding, Getmansky, Liang and Wermers (2007) for hedge funds, Gallaher, Kaniel, and Starks (2006) for fund families. Chevalier and Ellison (1999), Brown, Goetzmann and Park (2001), and Kempf and Ruenzi (2008) point to career motives as a possible mechanism behind relative performance concerns.

²This is in line with Merton (1987) and the empirical evidence by Coval and Moskowitz (1999), Kacperczyk, Sialm and Zheng (2005), among others. Merton’s insight is that portfolio managers are prone to invest in familiar stocks. Coval and Moskowitz (1999) document that managers have strong preferences for geographical proximate investments, with proximity acting as a proxy for familiarity. Kacperczyk, Sialm and Zheng find that portfolio managers familiarize themselves with different industry sectors, leading to industry concentration in managers’ portfolios.

preferences (Kojien (2012)), we allow the two managers to have different risk attitudes and weights they attach to relative return. Moreover, we allow the stock Sharpe ratios to be different so as to account for possible difference in ability, whereby the manager with the higher ability specializes in the stock with the higher Sharpe ratio.

First, we fully characterize the competing managers' investment behavior. We demonstrate that a unique pure-strategy Nash equilibrium always obtains in our setting and provide explicit solutions to the equilibrium portfolio policies. We find that the presence of competition affects the managers' patterns of risk taking in the following ways. If a manager is relatively risk tolerant, she decreases her risky investments relative to the no competition case, whereas a relatively risk intolerant manager increases her risk. Essentially, competition acts against a manager's intrinsic risk-taking tendency. The intuition is that a manager, who is risk averse and who cares about relative return, is negatively affected by the opponent's investment in her risky stock as it increases the manager's relative return volatility. To offset this negative effect, the manager ponders whether to focus on reducing her own wealth volatility, achieved by investing less in her risky stock, or on reducing her relative return volatility, achieved by the opposite strategy of investing more in the stock. We find that for a relatively risk tolerant (intolerant) manager the first (second) mechanism dominates, which explains the above result. Another noteworthy feature of the equilibrium portfolios is that a competing manager may *increase* her equilibrium risk taking as she becomes more risk averse, in contrast to conventional portfolio choice results. This result obtains when the manager is disadvantaged (the Sharpe ratio of her stock is relatively low), in which case she cares more about her relative return. Hence, she increases her risky investments as her risk aversion increases. Otherwise, when the manager is advantaged, she focuses on minimizing the volatility of her absolute return, and so takes on less risk as her risk aversion goes up.

While, individually, competition and asset specialization have received considerable attention in the literature, bringing these features into a unified framework enables us to examine the possible link between the two. We uncover such a link by showing that competition can be conducive to asset specialization of portfolio managers. In particular, we find that when managers are relatively risk tolerant they may prefer the asset specialization scenario, in which each trades in a single stock specific to her, to the

no-specialization scenario, in which each manager trades in both stocks. This indicates that the observed asset specialization of managers can be [!] a result of their concerted decision, whereby each obliges to restrict her investment set by not trading in the competitor’s stock [as long as; in exchange for] the other [does; doing] the same. This arrangement reduces the number of channels through which competition may operate, which benefits both managers. The associated negative effect—lower diversification—is not strong enough when managers are relatively risk tolerant, and hence are not much affected by portfolio volatility. For risk intolerant managers, on the other hand, the diversification effect is dominant, and so we obtain the opposite result that both managers would benefit from the reverse [!] arrangement enabling them to invest in the same set of stocks. Looking at a client investor whose money is managed by a competing manager, we find that she could well be worse off if her manager enters into these kinds of arrangements. This occurs when the managers concern for relative performance is relatively high, in which case the managers objective is considerably different from that of her client who cares only about own wealth. As for the economic significance, we find that the magnitudes of the above effects can be substantial.

In addition to the aforementioned scenario changes, the manager’s and her client’s policies can get misaligned when the change concerns parameters characterizing the economic environment. Accordingly, we examine the potential losses to client investors arising when some manager is replaced by a new one with different traits (risk aversion, relative bias) or when stock characteristics change. For the baseline pre-change case, we assume that a client has given her money to the “right” manager whose investment policy is optimal from the client’s perspective. After a change, however, the manager’s new equilibrium policy gets misaligned, implying a cost to the client. Upon manager turnover, we find that, all else being equal, the cost is higher when it is the client’s manager who is replaced than when it is the competitor. With changing stock characteristics, the results are more surprising. First, we find that, in the presence of competition, the client loses when the mean return or volatility of her manager’s stock changes, which is to be contrasted to the no competition case where the associated loss would be zero. Perhaps even more surprisingly, the client loses just as much when, instead of her manager’s stock, the same change happens to the other stock, the one in which her manager does not invest but the competitor does. The distinguishing mech-

anism that is present when stock characteristics change, but not present when manager turnover occurs, is that the optimal policy of the client is affected when her manager's stock characteristics vary. It turns out that this mechanism is key in making the client equally sensitive to the two stock characteristics, whereas the sensitivities are different with respect to the two managers. For a reasonable calibration of model parameters, we find that the costs described above may well exceed 5% of a client's wealth. Taken together, our findings highlight the importance of accounting for managerial competition as far as client investors are concerned because of new channels, not present without competition, through which clients' well-being can be adversely affected.

The majority of the literature investigating managerial competition typically adopt less general settings with risk-neutrality and one or two time periods (Goriaev, Palomino and Prat (2003), Taylor (2003), Palomino (2005), Li and Tiwari (2006), Loranth and Sciubba (2006)). Browne (2000), however, studies the dynamic portfolios of two strategic investors specializing in different stocks, but for when they care solely about relative returns and when their risk aversions are not set independently but sum up to a constant. Browne does not study how asset specialization can arise due to competition. Basak and Makarov (2013) analyze the dynamic policies of strategic managers but focus on partial relative concerns, whereby a manager displays such concerns only if her relative return is above a threshold. This asymmetry leads to the possibility of non-existence of equilibrium or multiplicity, the issues not present in our model where managers are affected by relative concerns at all levels of performance. Moreover, while Basak and Makarov ignore asset specialization, the current paper considers a richer incomplete market setting that accounts for this feature. Finally, Browne and Basak and Makarov do not investigate how their findings pertaining to managers are transmitted to client investors.

In settings without competition, relative considerations are shown to explain a number of styled facts, such as in the "catching-up-with-the-Joneses" literature (Abel (1990), Gomez, Priestley, and Zapatero (2009)) and in the works on financial and investment bubbles (DeMarzo, Kaniel, and Kremer (2007, 2008)). More closely to our paper, van Binsbergen, Brandt, and Koijen (2008) study the behavior of two asset managers who care about relative returns and who invest in different subsets of all available assets. However, in van Binsbergen et al. managers care only about relative

performance, which is specified with respect to an *exogenous* benchmark designed by a decision maker (CIO), precluding competition among managers which is the main focus of our paper. They also carry out cost-benefit analysis but with focus on the decision maker, while we focus on managers and the client. Other non-strategic works in the context of money management analyze the effects of non-smooth (with kinks) relative concerns arising due to compensation contracts (Carpenter (2000), Cuoco and Kaniel (2011)) or due to money flows (Basak, Pavlova, and Shapiro (2007)).

Finally, our paper contributes to the literature that seeks to rationalize asset specialization or, the closely related notion, under diversification. Existing explanations rely on features such as trading costs (Brennan (1975)), informational frictions (van Nieuwerburgh and Veldkamp (2009, 2010)), ambiguity aversion (Uppal and Wang (2003), Boyle, Garlappi, Uppal and Wang (2012)), solvency requirements (Liu (2012)), liquidation risk (Wagner (2011)), cumulative prospect theory (Barberis and Huang (2008)), preference for skewness (Mitton and Vorkink (2007)), and rank-dependent preferences (Polkovnichenko (2005)). Our novel contribution is to show that asset specialization can arise due to competition.

The rest of the paper is organized as follows. Section 2 describes the economic set-up. Section 3 characterizes the equilibrium portfolio policies and investigates their properties. Section 4 studies the cost-benefit implications of asset specialization. Section 5 investigates how a client investors is affected by a changing economic environment. Section 6 concludes. Appendix A collects all proofs, and Appendix B analyzes the case of partial asset specialization.

2. Economy with Competition

2.1. Economic Setting

We consider a continuous-time, finite horizon $[0, T]$ setting, in which the uncertainty is driven by two correlated Brownian motions ω_1 and ω_2 , with correlation ρ . Financial investment opportunities are given by a riskless money market account and two imperfectly correlated asset portfolios, henceforth referred to as stocks 1 and 2. The money

market account pays a constant interest rate r . Each stock price, S_i , $i = 1, 2$, follows a geometric Brownian motion

$$dS_{it} = (r + \mu_i)S_{it}dt + \sigma_i S_{it}d\omega_{it}, \quad (1)$$

where the stock mean excess return, μ_i , and volatility, σ_i , are constant; for brevity, we refer to μ_i as the mean return of stock i . Without loss of generality, we assume that $\mu_i > 0$, $\sigma_i > 0$, $0 < \rho < 1$.

We consider a framework with two portfolio managers, indexed by $i = 1, 2$. Manager 1 is assumed to *specialize* in stock 1 in that the risky part of her portfolio is invested in stock 1 only, driven by Brownian uncertainty ω_1 . Similarly, manager 2 specializes in stock 2, driven by uncertainty ω_2 . There are several justifications for asset specialization. First, it can be justified by the widely documented phenomenon that investors tend to trade only in familiar assets (Merton (1987)). The idea is that since analyzing stock data takes time and effort, each manager is likely to be familiar with, and hence invests in a subset of all available assets. Accordingly, we view stock i as representing a portfolio of assets familiar to manager i .³ Our assumption that stocks 1 and 2 are imperfectly correlated, $0 < \rho < 1$, reflects the fact that different managers are not likely to be completely identical in terms of their familiar assets. Second, we show in Section 4 that asset specialization may emerge even when, inherently, each manager can invest in both stocks 1 and 2. Briefly, we demonstrate that managers may find it mutually beneficial to reach an agreement, whereby manager 1 agrees to trade only in stock 1 and abstains from stock 2, and vice versa for manager 2. Hence, competition can induce portfolio managers to voluntarily sacrifice the benefits of diversification.

Given her investment opportunity, manager i in this economy dynamically chooses a portfolio policy ϕ_i , where ϕ_{it} denotes the fraction of fund assets invested in stock i at

³Our interpretation is similar to that of He and Krishnamurthy (2012) who consider a single risky stock as representing “complex assets where investment requires some sophistication.” While it is potentially of interest to adopt a multiple stock framework to see how a manager allocates risk among individual assets in her portfolio, our focus is to understand how competition affects the *overall* level of managerial risk taking. This is consistent with the methodology of the related empirical work on portfolio management where the focus is on the *total* volatility of the fund return, and not on the portfolio composition (e.g., Brown, Harlow, and Starks (1996), Chevalier and Ellison (1997)).

time t . The investment wealth process of manager i , W_i , then follows

$$dW_{it} = [r + \phi_{it}\mu_i]W_{it}dt + \phi_{it}\sigma_iW_{it}d\omega_{it}. \quad (2)$$

We note that the managers' specialization implies that the market is incomplete for each manager. Indeed, given that a manager only invests in one risky stock familiar to her and not in both stocks, a manager cannot perfectly hedge or replicate a payoff dependent on the two sources of risk (ω_1, ω_2) by dynamically trading in one risky stock and the money market. Only in the knife-edge case of perfectly correlated managers' specializations $\rho = \pm 1$, a case not so plausible in our context, a manager faces an effectively complete market.

2.2. Modeling Competition

We envision a portfolio manager whose investment objective is twofold. First, she seeks to increase the terminal value of her portfolio. This is consistent with maximizing her own compensation given the widespread use of the linear fee structure in the mutual fund industry. Second, the portfolio manager seeks to perform well relative to a certain peer group comprised of other portfolio managers. Here, we rely on a growing body of work documenting that relative performance concerns play an important role in the behavior of portfolio managers. The often mentioned reasons for why managers care about their peers include career concerns (Chevalier and Ellison (1999), Brown, Goetzmann and Park (2001)) and a desire to attract a larger share of money flows (Basak and Makarov (2013)).

Accordingly, we consider the following objective function of manager i :

$$v_{iT} = \frac{1}{1 - \gamma_i} (W_{iT}^{1-\theta_i} R_{iT}^{\theta_i})^{1-\gamma_i}, \quad (3)$$

with $\gamma_i > 0$, $0 \leq \theta_i \leq 1$, and where W_{iT} and R_{iT} denote horizon wealth and horizon relative return, respectively. Without loss of generality, we assume that each manager's initial wealth is equal to one, $W_{i0} = 1$, implying that a manager's horizon return equals her horizon wealth. Given this, the relative returns of managers 1 and 2, R_{1T} and R_{2T} ,

defined as the ratio of the two managers' horizon returns, are given by:

$$R_{1T} = \frac{W_{1T}}{W_{2T}}, \quad R_{2T} = \frac{W_{2T}}{W_{1T}}. \quad (4)$$

The parameter θ_i reflects manager i 's *relative performance bias*, the extent to which she biases her objectives towards relative performance concerns.⁴ Alternatively, the parameter θ_1 can be interpreted as the intensity of competition faced by manager 2. Indeed, if θ_1 increases then manager 1 cares more about increasing her relative return R_{1T} , or equivalently about reducing manager 2's relative return $R_{2T} = 1/R_{1T}$. As a result of increased competition intensity, manager 2 needs to more actively counteract the downward pressure on R_{2T} . Similarly, manager 2's bias θ_2 captures the competition intensity faced by manager 1. The special case of $\theta_i = 0$ corresponds to a *normal manager i* with no relative performance concerns. The parameter γ_i captures manager i 's attitude towards risk. As is common in the portfolio choice literature, some of our predictions depend on whether γ_i is lower or higher than unity. We refer to manager i with $\gamma_i < 1$ as the risk tolerant manager and with $\gamma_i > 1$ as the risk intolerant manager. Given that each manager in our model is an individual portfolio manager, and not a representative investor who stands for a group of managers, we have chosen to consider both types of managers given the evidence of substantial heterogeneity in risk aversion among portfolio managers (Kojien (2012)).

Two features of the objective function (3) are worth commenting on since they play an important role in understanding the managers' equilibrium behavior. First, manager i is negatively affected by a decrease in her relative return R_{iT} , and hence she is harmed if the opponent's horizon return rises. Second, manager i , being risk averse, is adversely affected by a higher volatility of the relative return R_{iT} . Hence, a higher volatility of the opponent's return, which translates into a higher volatility of R_{iT} , has a detrimental effect on manager i .

⁴Basak and Makarov (2013) formally establish that such a relative return component arises endogenously in a manager's indirect utility function due to the presence of performance-sensitive money flows. In the same spirit, it can be shown here that the behavior of a manager, who is driven by standard CRRA preferences over own wealth and who is subjected to money flows at a rate R_{iT}^α , would be the same as the behavior when flows are absent but the objective function is given by (3). The relative bias θ_i would be uniquely determined from the money flows sensitivity parameter α , $\theta_i = \alpha/(1 + \alpha)$.

2.3. Nash Equilibrium Policies

To characterize the managers' behavior in the presence of competition, we look for managers' strategies that constitute a pure-strategy Nash equilibrium. Before formally defining the equilibrium, let us describe the nature of the game between managers. We postulate that both managers know all the fundamentals of the economic environment—the dynamic processes for stocks (1), objective functions $v_i(\cdot)$ (3), and initial wealth W_{i0} . Though it may appear questionable that a manager knows the characteristics of the opponent's stock that she does not invest in, recall that stock i represents a portfolio of assets familiar to manager i . Hence, manager 1 is assumed to know μ_2 and σ_2 as she can estimate them from manager 2's portfolio return (which is observable in reality), but does not invest in stock 2 due to not being familiar with its underlying assets. Similarly, for manager 2. We assume that each manager knows the opponent's preference parameters given that they could be estimated from the observed data (Kojien (2012)).

Each manager i ' strategy is a dynamic portfolio policy ϕ_{it} , which is a rule determining the fraction of wealth invested in stock i for all times $t \in [0, T)$ as a function of own time- t wealth and that of the opponent. Each manager chooses this portfolio policy to maximize her expected objective function taking the opponent's policy as given. A pair $(\phi_{1t}^*, \phi_{2t}^*, t \in [0, T])$ is a pure-strategy Nash equilibrium when each ϕ_{it}^* , $i = 1, 2$, is an optimal choice of manager i given that the opponent follows the equilibrium policy. Formally, the definition of equilibrium is as follows.

Definition 1. *A pair of portfolio policies $(\phi_{1t}^*, \phi_{2t}^*, t \in [0, T])$ constitutes a pure-strategy Nash equilibrium if manager 1's dynamic policy ϕ_{1t}^* is a best response to manager 2's dynamic policy ϕ_{2t}^* , and vice versa ϕ_{2t}^* is a best response to ϕ_{1t}^* . Namely, for a fixed ϕ_{2t}^* , ϕ_{1t}^* yields the maximum of manager 1's expected objective function*

$$\phi_{1t}^* = \arg \max_{\phi_{1t}} E[v_1(W_{1T}, R_{1T})] \quad (5)$$

subject to manager 1's dynamic budget constraint (2). Similarly, for a fixed ϕ_{1t}^ , ϕ_{2t}^* yields the maximum of manager 2's expected objective function.*

Before turning to equilibrium, we first characterize and discuss the managers' best responses, as reported in Lemma 1.

Lemma 1. *For a given manager 2's portfolio policy ϕ_2 , manager 1's best response is*

$$\frac{1}{\gamma_1} \frac{\mu_1}{\sigma_1^2} + \frac{\theta_1(\gamma_1 - 1)\rho\sigma_2}{\gamma_1\sigma_1} \phi_2. \quad (6)$$

Switching subscripts 1 and 2 in (6) yields manager 2's best response.

Lemma 1 establishes that if the opponent invests a positive amount in the risky stock, a relatively risk tolerant manager ($\gamma < 1$) responds by decreasing, and a risk intolerant ($\gamma > 1$) by increasing, her risky investments relative to the normal level. Indeed, looking at manager 1, her best response policy (6) differs from the normal policy absent relative performance considerations, $\mu_1/(\gamma_1\sigma_1^2)$, by the term $\theta_1(\gamma_1 - 1)\rho\sigma_2\phi_2/(\sigma_1\gamma_1)$ that can be negative or positive depending on whether γ_1 is below or above unity. To understand why, first note that manager 1 follows her normal policy when the opponent takes on a riskless strategy, i.e., when $\phi_2 = 0$. This is because manager 1's absolute and relative returns are the same in this case (up to a constant R_{2T}), and hence the presence of relative concerns does not distort her normal objectives. If now manager 2 invests more (a positive amount) in stock 2, the resulting increase in her return volatility translates into a higher volatility of manager 1's relative return, which is detrimental to the risk averse manager 1. There are two opposing channels through which manager 1 can offset this detrimental effect. First, she can decrease her investment in stock 1 so as to reduce the volatility of the absolute return component $W_{1T}^{1-\theta_1}$ in her objective function (3). Second, she can increase her position in stock 1 so that, given a positive correlation ρ between stocks 1 and 2, to decrease the volatility of the relative return component $R_{1T}^{\theta_1}$. For a risk tolerant manager, the first effect dominates, and so she decreases her risky investments; for a risk intolerant manager, the second effect prevails, and so she increases her risky investments. In the knife-edge case $\gamma_1 = 1$, the two effects have the same magnitude, and so manager 1's best response coincides with her normal policy.

3. Equilibrium Investment Policies

In this Section, we provide an explicit characterization of a Nash equilibrium, and also report the ensuing comparative statics results. Proposition 1 presents the managers' equilibrium portfolio policies.

Proposition 1. *There exists a unique pure-strategy Nash equilibrium in which the equilibrium portfolio policies (ϕ_1^*, ϕ_2^*) are given by*

$$\phi_1^* = \frac{\gamma_2 \mu_1 / \sigma_1 + \theta_1 (\gamma_1 - 1) \rho \mu_2 / \sigma_2}{[\gamma_1 \gamma_2 - \rho^2 \theta_1 \theta_2 (\gamma_1 - 1) (\gamma_2 - 1)] \sigma_1}, \quad (7)$$

$$\phi_2^* = \frac{\gamma_1 \mu_2 / \sigma_2 + \theta_2 (\gamma_2 - 1) \rho \mu_1 / \sigma_1}{[\gamma_1 \gamma_2 - \rho^2 \theta_1 \theta_2 (\gamma_1 - 1) (\gamma_2 - 1)] \sigma_2}. \quad (8)$$

In the special case of manager 2 having no relative performance concerns, $\theta_2 = 0$, $\theta_1 > 0$, the equilibrium policies are

$$\phi_1^* = \frac{\gamma_2 \mu_1 / \sigma_1 + \theta_1 (\gamma_1 - 1) \rho \mu_2 / \sigma_2}{\gamma_1 \gamma_2 \sigma_1} = \frac{1}{\gamma_1} \frac{\mu_1}{\sigma_1^2} + \frac{\theta_1 (\gamma_1 - 1) \rho \mu_2}{\gamma_1 \gamma_2 \sigma_1 \sigma_2}, \quad (9)$$

$$\phi_2^* = \frac{1}{\gamma_2} \frac{\mu_2}{\sigma_2^2}. \quad (10)$$

Proposition 1 reveals that manager 1's equilibrium policy (9) in the presence of relative performance concerns ($\theta_1 > 0$) but facing not competition from manager 2 ($\theta_2 = 0$) is comprised of the normal policy, $\mu_1 / (\gamma_1 \sigma_1^2)$, plus a relative performance component, $\theta_1 (\gamma_1 - 1) \rho \mu_2 / (\gamma_1 \gamma_2 \sigma_1 \sigma_2)$. It is straightforward to see that manager 1's behavior in this case represents a best response to the normal behavior of manager 2 (i.e., policy (9) is obtained by substituting (10) into (6)). Moving to the general case of competition when both managers have relative concerns ($\theta_1, \theta_2 > 0$), we see that the best response policy (9) is adjusted to include the denominator term $\rho^2 \theta_1 \theta_2 (\gamma_1 - 1) (\gamma_2 - 1)$ in (7). These equilibrium portfolios (7)–(8) reveal several salient features about the behavior of competitive managers, as summarized in Corollary 1.⁵

For clarity, in Corollary 1 we assume that the model parameters are such that the managers are long in stocks and that the denominators in (7)–(8) are positive. These restrictions on equilibria are reasonable because, first, we wish our analysis to be equally applicable to mutual and hedge funds and shorting stocks is rare among mutual funds. Second, we note that the denominators of (7)–(8) being positive occurs when the condition

$$\gamma_1 \gamma_2 - \rho^2 \theta_1 \theta_2 (\gamma_1 - 1) (\gamma_2 - 1) > 0 \quad (11)$$

⁵Recall that the quantities ϕ_1^* and ϕ_2^* denote wealth shares invested in stocks 1 and 2 (and not units of stocks), and so the fact that ϕ_1^* and ϕ_2^* are time invariant does not imply that managers follow buy-and-hold strategies in equilibrium—in fact, they dynamically rebalance their stock and bond investments to maintain their risky wealth shares constant.

holds. As we discuss in the Appendix A (proof of Corollary 1), in this case the equilibrium is stable with respect to best response dynamics; otherwise it is unstable in the sense that if one manager deviates from her equilibrium policy and then managers follow their best responses, the strategies would not converge to the Nash equilibrium ones.⁶ See DeMarzo, Kaniel, and Kremer (2007) and (2008) as recent examples of related works employing such a stability criterion.

Corollary 1. *Assume that the model parameters are such that the managers have long stock positions and the stability condition (11) holds. The equilibrium behavior of managers in the presence of competition ($\theta_1, \theta_2 > 0$) have the following salient properties:*

(i) *A relatively risk tolerant ($\gamma < 1$) manager decreases her equilibrium risky investments relative to the normal case of no competition ($\theta_1 = \theta_2 = 0$), while a risk intolerant manager ($\gamma > 1$) increases her risky investments relative to the normal case;*

(ii) *The relation between manager i 's risky investments and her risk aversion is positive when stock i Sharpe ratio is low relative to the other stock, and is negative otherwise. In particular, for manager 1, $\partial\phi_1^*/\partial\gamma_1 > 0$ when $(\mu_1/\sigma_1)/(\mu_2/\sigma_2) < \rho\theta_1/(\gamma_2 - \rho^2\theta_1\theta_2(\gamma_2 - 1))$, and $\partial\phi_1^*/\partial\gamma_1 < 0$ otherwise. Switching subscripts 1 and 2 yields the analogous result for manager 2.*

Other properties of managers' equilibrium portfolio policies are reported in Table 1.

$\partial\phi_1^*/\partial\mu_1$	$\partial\phi_1^*/\partial\sigma_1$	$\partial\phi_2^*/\partial\mu_1$		$\partial\phi_2^*/\partial\sigma_1$	
		$\gamma_2 < 1$	$\gamma_2 > 1$	$\gamma_2 < 1$	$\gamma_2 > 1$
+	-	-	+	+	-

Table 1: Managers' equilibrium portfolio policies and stock characteristics. The effects of stock 1 mean return, μ_1 , and volatility, σ_1 , on the managers' equilibrium policies. Switching subscripts 1 and 2 yields the corresponding sensitivities with respect to stock 2 characteristics.

⁶We note that the range of empirically plausible parameters under which the stability condition (11) holds is much wider than when it does not. For example, even picking extreme values for relative biases and stock correlation to increase the size of the region when the condition does not hold, $\theta_1 = \theta_2 = 1$, $\rho = 1$, we obtain that in the area of plausible risk attitudes $(\gamma_1, \gamma_2) \in ([0, 5], [0, 5])$ the region when the condition is satisfied occupies 86% of the total area. Under (more plausible) lower values of θ_1 , θ_2 , or ρ , the number is even higher.

Corollary 1(i) reveals that competition has the following effect on managerial risk taking: a manager, if risk tolerant, takes on less risk than in the normal case of no competition, and if risk intolerant takes on more risk than in the normal case. In a sense, competition works against a manager’s intrinsic risk-taking tendency. This finding is the equilibrium analogue of the earlier result of Lemma 1 that uncovers a similar pattern for a manager’s best response with similar intuition.

The second notable result of Corollary 1(ii) is that when a manager’s risk aversion increases she decreases or surprisingly increases her equilibrium portfolio risk, depending on the relation between the stock Sharpe ratios, μ_1/σ_1 and μ_2/σ_2 . A standard interpretation of why the two Sharpe ratios can be different is that managers have different abilities (Sharpe (1966)), whereby a higher ability of a manager translates into a higher Sharpe ratio of her portfolio return. Hereafter, we refer to the manager with a higher Sharpe ratio as the *advantaged* manager and to her opponent with the lower Sharpe ratio as the *disadvantaged* manager.

Using above terminology, Corollary 1 finds that, for a relatively disadvantaged manager, a higher risk aversion is associated with more risk, which is surprising because it is at odds with the standard insight of the portfolio choice literature. The intuition is as follows. Focusing on manager 1, an increase in her risk aversion γ_1 has two effects working through the absolute and relative return components of her objective function. The first effect—standard in portfolio choice—is that manager 1 wants to reduce the volatility of her absolute return, and so needs to decrease her investment in risky stock 1. On the other hand, manager 1 is also driven by relative return considerations, and so with increased risk aversion she is more sensitive to her relative return volatility. Consequently, to better offset the risky return of the opponent she needs to increase her risky investments. When manager 1 is considerably disadvantaged (stock 1 Sharpe ratio is relatively low), the advantaged manager 2 invests a relatively high amount into stock 2, leading to a high volatility of her return, and hence manager 1’s relative return. In this case, the desire to reduce her relative return volatility dominates and manager 1 increases her risky investments. Otherwise, if manager 1 is advantaged, she cares more about her absolute return volatility, and so reduces her risk exposure.

Turning to Table 1, we see that manager 1 takes on higher risk when stock 1 mean return μ_1 rises (Table 1, first cell), consistent with the standard prediction absent competition. On the other hand, manager 2 decreases her portfolio risk when risk tolerant ($\gamma_2 < 1$) and increases her risk when risk intolerant ($\gamma_2 > 1$) (Table 1, third cell), in line with the mechanisms discussed in the context of Lemma 1. From the viewpoint of stock 1 attractiveness, an increase in stock 1 volatility σ_1 has the opposite effect to an increase in the mean return μ_1 , and so effects of a higher σ_1 on the managers equilibrium portfolio policies are also the opposite to those of a higher μ_1 . This explains why the signs in the second and fourth cells are the opposite to those in the first and third cells, respectively. We provide another perspective on the results of Table 1 in Remark 1 of Section 5.

Besides being of independent interest, understanding the determinants of managers' policies is also important from the viewpoint of client investors who give their money to the managers—any mechanism that affects a manager's behavior also has implications for that manager's clients as their well-being depends on the manager's policy. In the next two Sections, we take a deeper look at this issue by bringing a client investor into our analysis and exploring two such mechanisms. First, we look at how a client is affected by a possible transition between the settings with and without asset specialization—this is analyzed in Section 4. Second, given that, from (7), manager 1's policy is affected by characteristics pertaining both to manager 1 herself and to her competitor, manager 2, it is also of interest to identify which manager's characteristics have a larger impact on the client of manager 1—this is examined in Section 5.

4. Costs and Benefits of Asset Specialization

Motivated by substantial evidence of asset specialization among real investors, a strand of theoretical literature seeks to rationalize this pattern (as reviewed in the Introduction). In this Section, we show that competition is a possible mechanism that can generate asset specialization among portfolio managers, which to our knowledge is a novel insight in the literature. Moreover, we explore how the effects of asset specialization are transmitted through managers to their clients, i.e., the individual investors

whose money are being managed by portfolio managers.

We first provide a brief overview. We consider two scenarios: with and without asset specialization. The *asset specialization scenario* is exactly as presented in Section 2—manager i is familiar only with stock i and so does not trade in the other stock. In the *no-specialization scenario*, on the other hand, each manager is familiar with and invests in both stocks, and otherwise the setup is as in Section 2. From a modelling perspective, the fundamental difference between these scenarios is that markets are incomplete with the specialization, as discussed in Section 2, but are complete without it as each manager can perfectly replicate payoffs depending on all sources of uncertainty in the economy. For each manager, we compare her indirect objective function in the two Nash equilibria under the two scenarios to determine which one she prefers and by how much. We find that, surprisingly, both managers may prefer specialization, implying that the standard diversification motives are dominated by the desire to avoid competing on the same turf by trading in the same set stocks. Under other circumstances, we show that both managers may prefer the no-specialization scenario. When the initial scenario is not the one both managers prefer, they can be expected to reach a mutually beneficial arrangement to move to the preferred one.⁷ We also consider a client investor who has entrusted her money to one of the managers, and examine whether she benefits or loses—whether her expected utility increases or decreases—when managers’ collusion leads to a scenario switch.

Accordingly, we consider cost-benefit measures λ_1 and λ_2 , where λ_i represents manager i ’s gain or loss in units of initial wealth due to moving from the no-specialization scenario to the asset specialization scenario:

$$J_i(W_{i0}) = J_i^{NoSp}((1 - \lambda_i)W_{i0}), \tag{12}$$

where $J_i(\cdot)$ denotes manager i ’s equilibrium indirect objective function under asset specialization as a function of her initial wealth, and $J_i^{NoSp}(\cdot)$ denotes that in the no-

⁷ Another possible scenario is that with partial specialization—one manager specializes and invests a single stock (and faces incomplete markets), while the other manager invests in both stocks (and faces complete markets). We provide an analysis of this scenario in Appendix B. Briefly, we find that both managers can never benefit due to a transition to this scenario from the no-specialization scenario, while they can benefit, under certain conditions, when the transition is from the specialization scenario. Overall, partial specialization appears to be a less likely scenario than the two scenarios analyzed in this Section.

specialization scenario. A positive λ_i means that manager i suffers from the transition to asset specialization, and its value quantifies the percentage of her initial wealth she is prepared to pay to prevent this transition. On the other hand, a negative λ_i implies that manager i benefits from the transition, with its absolute value denoting the percentage by which her wealth in the no-specialization scenario needs to be increased to make it as attractive as the scenario with specialization.

We also consider a client investor who provides an initial capital of W_{c0} to manager 1 to invest on her behalf between times 0 and T . Here and in what follows, subscript c denotes variables pertaining to the client. Given that modelling competition is complex and rich in implications as it is, we do not endogenize the client's decision to invest in the stock market indirectly via a portfolio manager rather than directly.⁸ Moreover, we leave the manager's fee outside the model, and so the client's horizon wealth W_{cT} is given by her initial wealth W_{c0} compounded by the return generated by manager 1 in equilibrium. The client has standard CRRA preferences (without relative concerns) defined over W_{cT} :

$$u_{cT} = \frac{W_{cT}^{1-\gamma_c}}{1-\gamma_c}, \quad (13)$$

where $\gamma_c > 0$ is the client's relative risk aversion. As above, we consider a cost-benefit measure λ_c representing the client's gain or loss due to transition from no-specialization to specialization:

$$J_c(W_{c0}) = J_c^{NoSp}((1-\lambda_c)W_{c0}), \quad (14)$$

where $J_c(\cdot)$ and $J_c^{NoSp}(\cdot)$ denote the client's expected utility with and without asset specialization, respectively. The interpretation of λ_c is similar to that of λ_1 and λ_2 —the client prefers no-specialization when λ_c is positive, and specialization when λ_c is negative.

Proposition 2 characterizes the managers' and the client's cost-benefit measures in closed-form.

Proposition 2. *The cost-benefit measure λ_1 , quantifying the effect on manager 1 of*

⁸Endogenizing the decision would require a model in which client investors were disadvantaged in some way relative to portfolio managers, e.g., facing higher information or transaction costs. Similarly to related works (van Binsbergen, Brandt, and Koijen (2008), Cuoco and Kaniel (2011)), we do not incorporate such costs into our analysis. An exception is Kaniel and Kondor (2013) who model the delegation decision.

a transition from the no-specialization scenario to the asset-specialization scenario, is given by

$$\lambda_1 = 1 - \exp((m - n)T), \quad (15)$$

where m and n are

$$m = \mu_1\phi_1^* - \theta_1\mu_2\phi_2^* - \gamma_1(\sigma_1\phi_1^*)^2/2 + \theta_1(1 - \theta_1(\gamma_1 - 1))(\sigma_2\phi_2^*)^2/2 + \theta_1(\gamma_1 - 1)\rho\sigma_1\sigma_2\phi_1^*\phi_2^*, \quad (16)$$

$$\begin{aligned} n = & \mu_1(\phi_{11}^* - \theta_1\phi_{21}^*) + \mu_2(\phi_{12}^* - \theta_1\phi_{22}^*) - \gamma_1((\sigma_1\phi_{11}^*)^2 + (\sigma_2\phi_{12}^*)^2 + 2\rho\sigma_1\sigma_2\phi_{11}^*\phi_{12}^*)/2 \\ & + \theta_1(\gamma_1 - 1)(\sigma_1^2\phi_{11}^*\phi_{21}^* + \sigma_2^2\phi_{12}^*\phi_{22}^* + \rho\sigma_1\sigma_2\phi_{11}^*\phi_{22}^* + \rho\sigma_1\sigma_2\phi_{12}^*\phi_{21}^*) \\ & + \theta_1(1 - \theta_1(\gamma_1 - 1))((\sigma_1\phi_{21}^*)^2 + (\sigma_2\phi_{22}^*)^2 + 2\rho\sigma_1\sigma_2\phi_{21}^*\phi_{22}^*)/2, \end{aligned} \quad (17)$$

and ϕ_i^* is manager i 's equilibrium policy under asset specialization (given in (7)–(8)), while ϕ_{ij}^* is manager i 's equilibrium investment in stock j under no specialization (given in (A20)–(A21) in the Appendix A). Switching subscripts 1 and 2 in (16)–(17) and plugging the ensuing expressions in (15) yields manager 2's cost-benefit measure λ_2 .

The client's cost-benefit measure is

$$\lambda_c = 1 - \exp \left[\mu_1\phi_1^* - \frac{1}{2}\gamma_c(\sigma_1\phi_1^*)^2 - \mu_1\phi_{11}^* - \mu_2\phi_{12}^* + \frac{1}{2}\gamma_c((\sigma_1\phi_{11}^*)^2 + (\sigma_2\phi_{12}^*)^2 + 2\rho\sigma_1\sigma_2\phi_{11}^*\phi_{12}^*) \right]. \quad (18)$$

Upon investigating the cost-benefit measures reported in Proposition 2, we identified two distinct types of predictions depending on managers' risk tolerances. Accordingly, in the graphical analysis below, we consider two calibrations, with risk intolerant and tolerant managers. Moreover, in each calibration, we set the client's risk aversion γ_c such that manager 1's policy ϕ_1^* is optimal for her:

$$\phi_1^* = \phi_c \equiv \frac{1}{\gamma_c} \frac{\mu_1}{\sigma_1^2}, \quad (19)$$

where the second equality represents the standard portfolio choice result under the CRRA utility specification (13). Condition (19) ensures that the client's cost-benefit measure λ_c is not driven by misalignment between the client's desired policy and the actual policy implemented under asset specialization.

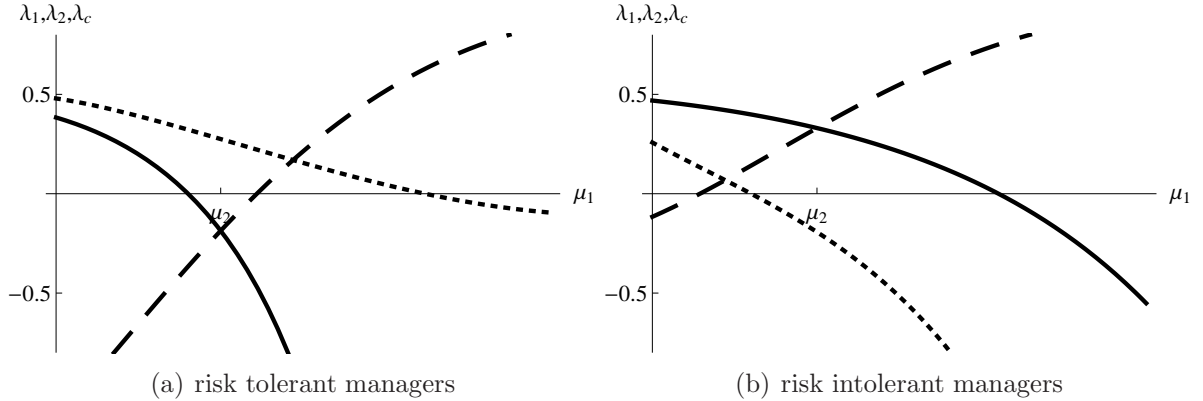


Figure 1: Costs and benefits of asset specialization with varying stock 1 mean return. The effect of stock 1 mean return μ_1 on the cost-benefit measures of manager 1 (solid line, λ_1), manager 2 (dashed line, λ_2), and the client (dotted line, λ_c). A positive λ_i indicates that manager i prefers the no specialization scenario over the asset specialization scenario, whereas a negative λ_i indicates that the opposite preference. Similarly, for λ_c . The baseline parameter values are: $r = 0.01, \mu_1 = \mu_2 = 0.2, \sigma_1 = \sigma_2 = 0.15, \rho = 0.5, \theta_1 = \theta_2 = 0.9, T = 1$. Panel (a) presents the results for risk tolerant managers $\gamma_1 = \gamma_2 = 0.8$, and panel (b) for risk intolerant managers $\gamma_1 = \gamma_2 = 3$. For each level of μ_1 , the client's risk aversion γ_c is set so that the objectives of the client and her manager 1 are aligned, i.e., condition (19) is satisfied.

We first consider the managers' and the client's cost-benefit measures as a function of stock 1 mean return, the relative attractiveness of manager 1's stock, as presented in Figure 1.⁹ Panel (a) corresponds to risk tolerant managers, while panel (b) to risk intolerant ones. Focusing first on managers, Figure 1 reveals that manager 1 (solid line) finds the no-specialization scenario more favorable than asset specialization when her stock 1 is relatively unattractive in that its mean return μ_1 is relatively low, as seen from λ_1 being positive for low μ_1 . The reason is that with no specialization manager 1 has access to the relatively more attractive stock 2, which makes manager 1 prefer no-specialization to asset specialization, where stock 2 is unavailable. On the other hand, when her stock mean return μ_1 is relatively high, manager 1 does not want to share her attractive investment opportunity with the competitor, and so prefers asset specialization ($\lambda_1 < 0$ for high μ_1). Because a higher stock 1 mean return has the opposite effect on manager 2 to that on manager 1, the behavior of manager 2's cost-benefit measure λ_2 (dashed line) is also the opposite to that of manager 1— λ_2 is negative for low μ_1 and positive for high μ_1 .

⁹Though we could study the expressions in Proposition 2 from different angles, we find the most illuminating one to be that presented in Figure 1.

A key insight from panel (a) in Figure 1 is that *both* competing managers may gain from asset specialization, as illustrated by both λ_1 and λ_2 being negative when neither manager has a considerable advantage (i.e., μ_1 and μ_2 are not too different). This underscores a possible—and novel—mechanism behind asset specialization, as managers who can in principle diversify into both available stocks are likely forgo diversification and instead opt for specialization by entering into an arrangement whereby each invests in a single stock specific to her.¹⁰ The reason is that when managers can trade in both stocks, as opposed to one stock only, there is an additional channel through which competition operates, and this increased competition negatively affects both managers. The potential positive effect of investing in both stocks—improved diversification—is not strong enough when managers are relatively risk tolerant, as seen in panel (a). When managers are risk intolerant, however, they care more about portfolio volatility, and hence about diversification. As a result, panel (b) reveals that both managers prefer the no-specialization scenario when neither has a sizeable advantage (λ_1 and λ_2 are positive when μ_1 and μ_2 are not too different). Accordingly, this scenario could well occur even if initially the managers specialize in different stocks.¹¹

Turning to the client’s cost-benefit measure λ_c (dotted line), Figure 1 reveals that the managers’ preference for a certain scenario, and a possible transition to this scenario, can well be at odds with the client’s preference. In panel (a), in the region where both managers prefer the specialization scenario ($\lambda_1, \lambda_2 < 0$ when μ_1 is around μ_2), the

¹⁰ Given our focus, we do not formally analyze a game in which such an arrangement arises as an equilibrium outcome. Briefly, this game could have the following key elements. At time 0, one manager makes an offer to the competitor with the following conditions: each manager i ought to invest only in a single stock i and follow her equilibrium policy under asset specialization (given by (7) and (8) for manager 1 and 2, respectively); if either deviates at any subsequent time $t \in (0, T)$ by investing in both stocks, the competitor also starts investing in both stocks and follows the no-specialization equilibrium policy until the horizon T (given by (A20) and (A21) in the Appendix A for manager 1 and 2, respectively). As we adopt a setting with continuous time and complete information, a manager’s deviation at time t is going to be detected in the next instant $t + dt$. Hence, neither manager will deviate because the gain from doing so accrues over the period dt and so is infinitesimal while the ensuing loss due switching to no-specialization is sizeable as it accrues over a non-trivial period of time $[t + dt, T]$.

¹¹ Even when managers prefer different scenarios, it can still be argued that one of them scenario is more likely, though the argument here is more involved. For example, suppose that managers are initially in the no-specialization scenario, and while manager 1 prefers it over asset specialization ($\lambda_1 > 0$), manager 2 would benefit from a transition to asset specialization ($\lambda_2 < 0$). If manager 2’s benefit exceeds manager 1’s loss, i.e., if $\lambda_1 + \lambda_2 > 0$, then the transition to asset specialization could occur if manager 2 finds a way to compensate manager 1 for the ensuing loss. Because the overall gain $\lambda_1 + \lambda_2$ is positive, there is a range of possible compensations such that both managers benefit in the end. If managers have different initial wealth, $W_{10} \neq W_{20}$, the condition is $\lambda_1 W_{10} + \lambda_2 W_{20} > 0$.

client finds no specialization more attractive ($\lambda_c > 0$) because of greater diversification benefits. From panel (b), the opposite pattern can occur when the managers are risk intolerant—managers prefer no specialization, but the client would rather have that her manager specialize ($\lambda_1, \lambda_2 > 0$ but $\lambda_c < 0$ when μ_1 is around μ_2). The reason is that the misalignment between manager 1’s and her client’s objective functions has less room to manifest itself in the specialization scenario given the narrower investment opportunity set, and so the client finds this scenario more attractive despite lower diversification benefits. These results vividly show that competition is a salient factor for client investors because without competition they would always prefer their managers to not specialize but invest in all available stocks.

To better clarify the role of competition behind our results, we depict in Figure 2 the cost-benefit measures λ_1 , λ_c , and λ_c for varying levels of manager 1’s relative bias θ_1 , which is a parameter also capturing competition intensity (as discussed in Section 2). From panel (a) for risk tolerant managers, we see that for manager 1 (solid line) to prefer asset specialization she needs to have a relatively high concern for competition ($\lambda_1 < 0$ when θ_1 is high). On the contrary, when risk intolerant, manager 1 prefers the no-specialization scenario more and more as her relative bias increases, as seen from the positive and increasing λ_1 in panel (b). The higher the bias is, the more the manager is affected by the opponent’s return, and so the more she values the ability to perfectly hedge against the opponent’s return risk under no-specialization. As the bias of manager 1 increases, her objective function departs more from her client’s standard CRRA objective, and so does the client’s cost measure λ_c (dotted line) relative to that of her manager λ_1 (solid line), as seen in both panels (a) and (b). We see that the two measures have the opposite signs, meaning that the client’s preference for specialization is at odds with that of her manager, when the manager’s concern for competition is sufficiently high.

Finally, looking at the scales of y -axes in Figures 1 and 2, we see that the magnitudes of the above effects can be substantial.

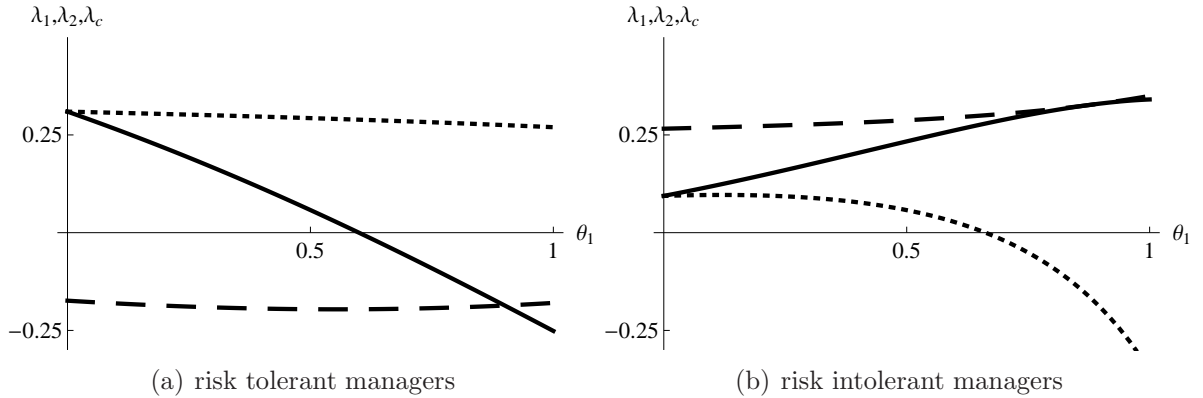


Figure 2: Costs and benefits of asset specialization for varying manager 1’s relative bias. The effect of manager 1’s relative bias θ_1 on the cost-benefit measures of manager 1 (solid line, λ_1), manager 2 (dashed line, λ_2), and the client (dotted line, λ_c). The parameter values in panels (a) and (b) are as in the respective panels of Figure 1.

5. Changing Economic Environment: Implications for Client Investors

There is a common argument that investing in mutual or hedge funds, rather than trading stocks directly, allows client investors to enjoy the benefits of stock markets without devoting much time to looking after their investments. This point is often mentioned in financial media and also advocated by mutual and hedge funds themselves as a way of attracting clients.¹² The idea is that a client needs to spend effort only at the initial stage when choosing a manager, but once the right manager has been identified and invested in, the client, as the argument goes, does not have to pay close attention to her investments. In this Section, we demonstrate that, while this argument can be justified if there were no competition, the presence of competition implies that the manager could deviate considerably from her client’s best policy when the economic environment changes due to managerial turnover or changing stocks characteristics.

Accordingly, we consider a client of manager 1, where all assumptions pertaining to the client are as in the previous Section 4. In the baseline *pre-change* period, i.e., before the environment changes, we assume that there is no misalignment of investment

¹²For example, Fidelity, one of the largest mutual fund and financial services group in the world, has the following discussion on its website: “You’ve decided your portfolio needs bonds. Now you have to decide: Will you invest in mutual bond funds or individual bonds?...Your ultimate decision will depend on how much money you have to invest, your risk appetite...Then there’s the question of whether you want to spend much time researching and monitoring your holdings...Given the complexities, bond funds probably are the right choice for many investors.”

strategies between the client and her manager 1, and so condition (19) is satisfied. We then consider the *post-change* period occurring after a manager i trait (θ_i, γ_i) or a stock i characteristic (μ_i, σ_i) changes. The former scenario corresponds to manager turnover, whereby manager i is replaced by a new manager with different traits, and the latter could occur upon the release of new information affecting the future dividends of stock i .¹³ In the post-change period, manager 1's new equilibrium policy deviates from the client's optimal policy, implying a cost for the client. To quantify this cost, we again consider a cost measure λ_c , now representing the client's loss in units of her initial wealth when her manager deviates from the client's optimal policy $\hat{\phi}_c$ to a new policy $\hat{\phi}_1^*$ due to a change in the economic environment:

$$J_c((1 - \lambda_c)W_{c0}, \hat{\phi}_c) = J_c(W_{c0}, \hat{\phi}_1^*), \quad (20)$$

where $J_c(\cdot)$ is the client's indirect objective function as a function of her initial wealth (first argument) and manager 1's policy (second argument). In equation (20) and below, we use a hat $\hat{\cdot}$ to denote the post-change value of the variable. The value of λ_c quantifies the percentage of wealth the client is prepared to pay to induce manager 1 to switch from her equilibrium policy $\hat{\phi}_1^*$ to the client's optimal policy $\hat{\phi}_c$. Since by definition $\hat{\phi}_c$ maximizes $J_c(\cdot)$, we have that λ_c cannot be negative as the client prefers $\hat{\phi}_c$ over any other policy.

Consider first the case when there is no competition, $\theta_1 = 0$. Then, manager 1 follows the normal policy $\phi_1^* = (\mu_1 - r)/(\gamma_1\sigma^2)$. Combining this with condition (19), we have that manager 1 and the client have the same risk attitude, $\gamma_1 = \gamma_c$, and hence the same objective function to start with. Given this, when stock 1 mean return or volatility vary, the post-change policy of manager 1 is still going to be optimal for the client, implying that the cost λ_c is zero in the absence of competition. The only way the client can be harmed is if manager 1 is replaced by another manager with a different risk attitude $\hat{\gamma}_1 \neq \gamma_1$. Hence, if the competition is absent, our analysis, even if a very simple one, provides some formal support for the idea that portfolio management saves clients

¹³We assume that when managers choose their policies in the pre-change period they do not account for the possibility of these changes. While it is potentially interesting to extend the model by introducing stochastic processes governing the stock parameters and manager turnover, such an extension would significantly complicate the analysis, and so is beyond the scope of this paper.

time and effort. Indeed, the above implies that within our setup, clients do not need to analyze stock data, which is presumably the most costly and time-consuming activity pertaining to stock investments. As we now show, our findings when competition is present are less comforting to client investors.

Proposition 3 characterizes the cost measure λ_c analytically and reports some of its salient properties in the general case of competition.

Proposition 3. *The cost measure λ_c , quantifying the effect of a changing economic environment on a client of manager 1, is given by*

$$\lambda_c = 1 - \exp\left(\hat{\mu}_1(\hat{\phi}_1^* - \hat{\phi}_c) - \frac{1}{2}\gamma_c\hat{\sigma}_1^2((\hat{\phi}_1^*)^2 - (\hat{\phi}_c)^2)\right), \quad (21)$$

where $\hat{\phi}_1^*$ and $\hat{\phi}_c$ are given by (7) and (19), respectively, with post-change parameter values substituted in. Moreover, λ_c has the following properties:

(i) A change in stock 1 mean return to a new value $\hat{\mu}$ has the same cost as the change in stock 2 mean return to the same value, $\lambda_c|_{\hat{\mu}_1=\hat{\mu}} = \lambda_c|_{\hat{\mu}_2=\hat{\mu}}$, assuming that the pre-change mean returns μ_1 and μ_2 are equal.

(ii) A change in stock 1 volatility to a new value $\hat{\sigma}$ has the same cost as the change in stock 2 volatility to the same value, $\lambda_c|_{\hat{\sigma}_1=\hat{\sigma}} = \lambda_c|_{\hat{\sigma}_2=\hat{\sigma}}$, assuming that the pre-change volatilities σ_1 and σ_2 are equal.

Figure 3 presents the main implications of Proposition 3 for a reasonable calibration of model parameters.¹⁴ Panels (a) and (b) of Figure 3 plot the client's cost measure λ_c (21) for varying levels of managers' relative biases (panel (a)) and attitudes towards risk (panel (b)). In each panel, the minimum cost of zero occurs at the point where the post-change value of the variable on the x -axis is equal to the initial value, i.e., when no change has occurred. The larger the change, the higher is the cost. We see that a change in the relative bias or risk attitude of manager 1—the one investing the

¹⁴The properties of λ_c stated in Proposition 3 (i) and (ii) do not rely on a particular calibration. However, while the managers behavior can be qualitatively affected by their risk tolerances, the client's cost measure turns out to only be affected in magnitude but not in its overall behavior by managers' risk attitudes. Given this, in the interest of space we employ only one of the two calibrations used in Figures 1–2, namely the one with relatively risk intolerant managers $\gamma_i = 3$. Accordingly, we set $\gamma_c = 2.1$ so that condition (19) is satisfied and manager 1's and her client's policies are initially aligned. For other managers' risk aversions, a general pattern is that the more risk averse the managers are, the larger is the cost λ_c (other things equal).

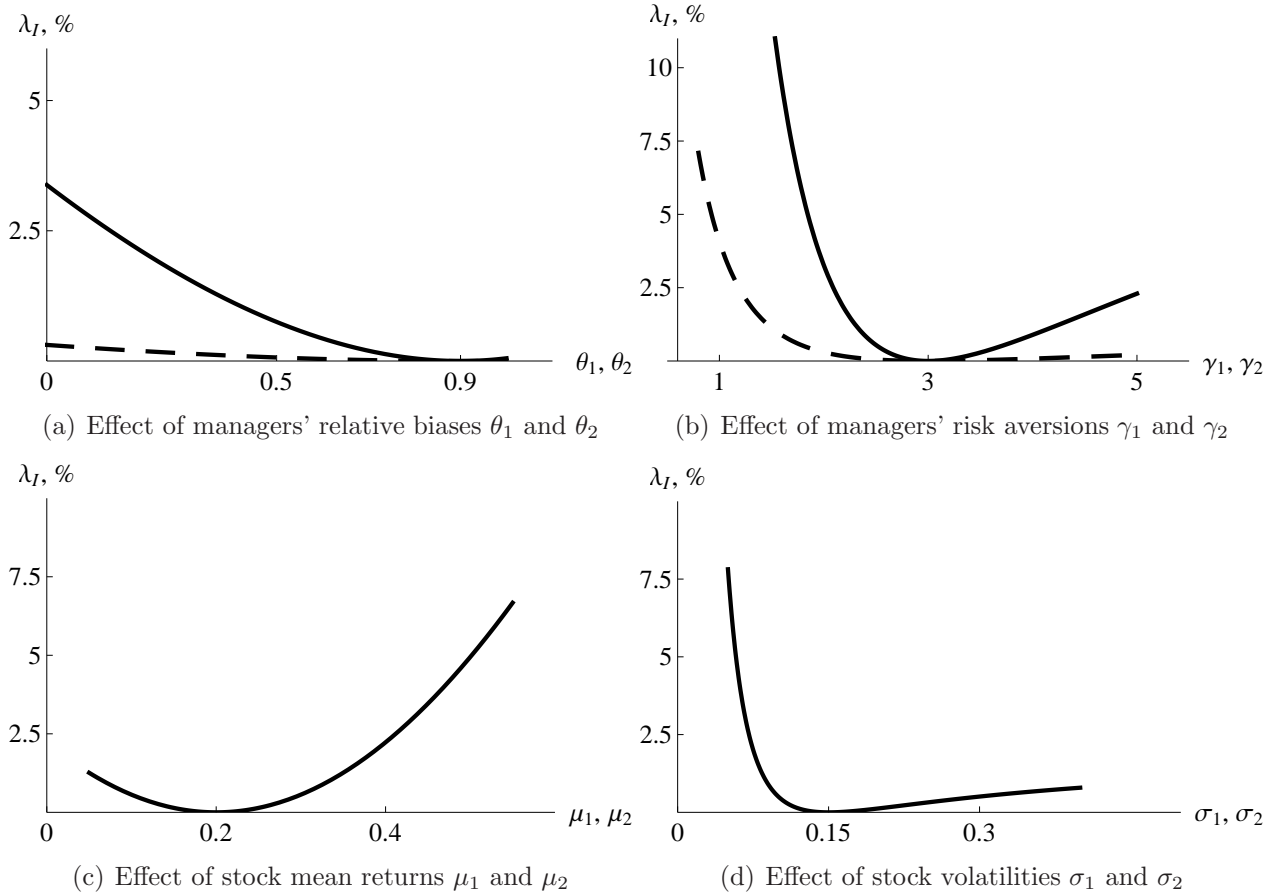


Figure 3: Client investor's costs under changing economic environment.

The client investor's cost measure λ_c for varying levels of manager 1's (solid plot) and manager 2's (dashed plot) relative biases (panel (a)) and risk aversions (panel (b)), and stocks mean returns and volatilities (panels (c) and (d), respectively). Panels (c) and (d) have one plot because the client is equally sensitive to stock 1 and stock 2 parameters (Proposition 3 (i) and (ii)). The parameter values are: $\gamma_1 = \gamma_2 = 3$, $\gamma_c = 2.1$, and the remaining parameters are as in Figure 1.

client's money—leads to a higher cost than the same change for manager 2 (the solid line is above the dashed line in each panel). The reason is that manager 1's own traits directly affect her equilibrium policy ϕ_1^* , whereas the same traits of the competitor have an indirect, and hence a smaller effect on ϕ_1^* . The client's optimal policy ϕ_c^* , on the other hand, is not sensitive to managers' traits. Hence, the difference between the client's desired policy, $\hat{\phi}_c^*$, and the actual policy implemented, $\hat{\phi}_1^*$, grows faster in response to manager 1's traits, and so does the cost λ_c . From Figure 3, the client's cost due to changing managers' traits can be considerable, potentially exceeding 5% of the client's wealth.

Following the above reasoning, one may anticipate a similar pattern when looking

at the client’s cost due to varying stock characteristics—namely, that the client would lose more for a given change in stock 1 mean return or volatility than for the same change in stock 2. Surprisingly, this is not the case and the loss is the same, as stated in Proposition 3 (i) and (ii). To understand why, let us consider two scenarios 1 and 2, where scenario i , $i = 1, 2$, refers to the case when stock i mean return increases from $\mu_i = \mu$ to $\hat{\mu}_i = \hat{\mu} > \mu$. Assuming for ease of argument that manager 1 is risk intolerant ($\gamma_1 > 1$), we have from Corollary 1 that her equilibrium policy ϕ_1^* increases in both scenarios, but more so in scenario 1 because, as above, the direct effect dominates. However, unlike the previous case in which the client’s best policy ϕ_c is unaffected by managers’ traits, now ϕ_c rises in scenario 1 while staying the same in scenario 2. In our formulation, this rise exactly offsets the larger increase of ϕ_1^* , and so scenarios 1 and 2 are equally harmful to the client, as stated in Proposition 3 (i). The result of Proposition 3 (ii) follows analogously.¹⁵

Accordingly, Figure 3(c) depicting the cost measure λ_c for varying levels of stock 1 and 2 mean returns has only one plot as the two lines corresponding to each individual stock coincide; similarly for Figure 3(d) depicting the cost for varying levels of stock volatilities. Again, zero cost ($\lambda_c = 0$) corresponds to the situation when the relevant parameter does not change, and the larger the change, the higher is the cost. Figure 3(c)–(d) reveals that changes in stock characteristics can cost the client over 5% of her wealth.

Summarizing the above analysis in terms of implications for client investors, we find that it is relatively more important to pay attention to manager turnover within a client’s chosen fund than in competing funds, though in absolute terms the latter can also lead to non-trivial costs. Moreover, a client should consider tracking news about companies in her manager’s portfolio and, *equally important*, in the competitors’ portfolios because, unlike the no-competition case, the ensuing changes in portfolios characteristics can drive a wedge between the client’s desired policy and the policy followed by her manager.

Remark 1. *Interaction of competition and asset specialization.* Having presented

¹⁵ The main message of Proposition 3(i) remains valid when the pre-change mean returns are different, $\mu_1 \neq \mu_2$. In this case, the sensitivities of the cost measure λ_c with respect to μ_1 and μ_2 , though not identical, have similar magnitudes. The same goes for Proposition 3(ii).

all results, we can discuss a methodological point concerning the importance of studying competition and asset specialization in a unified manner, as we do in this paper, as opposed to studying them individually. That is, the question is whether these two economic mechanisms interact in non-trivial ways or not. If not, a unified analysis is not likely to generate novel insights beyond those that can be obtained from the individual analyses. We find that there is in fact a profound interaction between the two mechanisms, as several of our key insights that can only be obtained in the general version of our model that features both competition and asset specialization, and are not present when one of these features is absent. Let us give one example from each Section. Considering the equilibrium policies reported in Section 3 with asset specialization, their properties with respect to stock mean returns and volatilities are qualitatively different from the corresponding properties with no specialization (more details are at the end of proof of Proposition 2 in the Appendix A). The finding that each portfolio manager could find the transition from no-specialization (complete markets) to specialization (incomplete markets) beneficial, which is one of the key results of Section 4, rests on the presence of competition, as without competition each manager always prefers no-specialization due to better diversification opportunities. Finally, as we have just established in this Section, whether competition is present or not leads to very different implications for client investors' costs arising when the economic environment changes.

6. Conclusion

This article provides an analysis of competition among portfolio managers emerging as the managers seek to outperform each other. We fully characterize the managers' equilibrium portfolio policies under incomplete markets caused by asset specialization, and investigate their properties. We establish that both managers can prefer the asset specialization when risk tolerant, in which case this trading pattern is likely to occur. When they are risk intolerant, however, the no-specialization outcome is more likely. We uncover that the managers' preference for specialization or no-specialization scenario could well go against the preference of a client investor. Finally, we investigate the potential costs to client investors due to a changing economic environment, and show that the costs due to managerial turnover have notably different properties than the

costs due to changing stock characteristics.

Appendix A: Proofs

Proof of Lemma 1. Denote $J(W_{1t}, W_{2t}, t; \phi_2)$ to be manager 1's indirect objective function as a function of her wealth W_{1t} , opponent's wealth W_{2t} , and time t , for a given opponent's portfolio ϕ_2 . The Hamilton-Jacobi-Bellman (HJB) equation corresponding to manager 1's maximization problem (5) is

$$\begin{aligned} 0 &= \max_{\phi_1} \left[J_t + (r + \mu_1 \phi_1) W_{1t} J_{W_1} + (r + \mu_2 \phi_2) W_{2t} J_{W_2} + \sigma_1^2 \phi_1^2 W_{1t}^2 J_{W_1 W_1} / 2 \right. \\ &\quad \left. + \sigma_2^2 \phi_2^2 W_{2t}^2 J_{W_2 W_2} / 2 + \rho \sigma_1 \sigma_2 \phi_1 \phi_2 W_{1t} W_{2t} J_{W_1 W_2} \right], \end{aligned} \quad (\text{A1})$$

with the boundary condition

$$J(W_{1T}, W_{2T}, T; \phi_2) = \frac{1}{1 - \gamma_1} (W_{1T}^{1-\theta_1} (W_{1T}/W_{2T})^{\theta_1})^{1-\gamma_1}. \quad (\text{A2})$$

The first order condition for problem (A1) is

$$\mu_1 W_{1t} J_{W_1} + \sigma_1^2 \phi_1 W_{1t}^2 J_{W_1 W_1} + \rho \sigma_1 \sigma_2 \phi_2 W_{1t} W_{2t} J_{W_1 W_2} = 0,$$

leading to manager 1's best response portfolio policy

$$-\frac{\kappa_1}{\sigma_1 W_{1t}} \frac{J_{W_1}}{J_{W_1 W_1}} - \frac{\rho \sigma_2 W_{2t}}{\sigma_1 W_{1t}} \frac{J_{W_1 W_2}}{J_{W_1 W_1}} \phi_2. \quad (\text{A3})$$

We conjecture that

$$J(W_{1t}, W_{2t}, t; \phi_2) = \frac{k(t)}{1 - \gamma_1} (W_{1t}^{1-\theta_1} (W_{1t}/W_{2t})^{\theta_1})^{1-\gamma_1}. \quad (\text{A4})$$

Computing the partial derivatives:

$$J_{W_1} = k(t) W_{1t}^{-\gamma_1} W_{2t}^{\theta_1(\gamma_1-1)}, \quad (\text{A5})$$

$$J_{W_1 W_1} = -k(t) \gamma_1 W_{1t}^{-\gamma_1-1} W_{2t}^{\theta_1(\gamma_1-1)}, \quad (\text{A6})$$

$$J_{W_1 W_2} = k(t) \theta_1 (\gamma_1 - 1) W_{1t}^{-\gamma_1} W_{2t}^{\theta_1(\gamma_1-1)-1}, \quad (\text{A7})$$

and plugging (A5)–(A7) into (A3) yields manager 1's best response portfolio policy (6).

We now verify the conjectured function (A4) indeed satisfies the HJB equation (A1),

and identify $k(\phi_2, t)$ explicitly. Computing J_t , J_{W_2} , and $J_{W_2 W_2}$ from (A4) and along with (A5)–(A7) substituting into (A1), we obtain that $k(t)$ satisfies

$$\frac{k'(t)}{1 - \gamma_1} + k(t)q = 0, \quad (\text{A8})$$

where q is given by

$$\begin{aligned} q &= r - \theta_1 r + \mu_1 \phi_1(\phi_2) - \theta_1 \mu_2 \phi_2 - \gamma_1 (\sigma_1 \phi_1(\phi_2))^2 / 2 \\ &+ \theta_1 (1 - \theta_1 (\gamma_1 - 1)) (\sigma_2 \phi_2)^2 / 2 + \theta_1 (\gamma_1 - 1) \rho \sigma_1 \sigma_2 \phi_1(\phi_2) \phi_2, \end{aligned} \quad (\text{A9})$$

with $\phi_1(\phi_2)$ denoting manager 1's best response to ϕ_2 (given by (6)). From (A2), the boundary condition is $k(T)=1$, and hence the solution of (A8) is

$$k(t) = e^{q(1-\gamma_1)(T-t)}. \quad (\text{A10})$$

Q.E.D.

Proof of Proposition 1. We look for a pair of mutually consistent policies (ϕ_1^*, ϕ_2^*) such that ϕ_1^* constitutes a best response to ϕ_2^* , and so ϕ_1^* equals the expression (6) evaluated at $\phi_2 = \phi_2^*$, and similarly ϕ_2^* constitutes a best response to ϕ_1^* . This leads to a system of two linear equations and two unknowns:

$$\begin{aligned} \phi_1^* &= \frac{1}{\gamma_1} \frac{\kappa_1}{\sigma_1} + \frac{\theta_1 (\gamma_1 - 1) \rho \sigma_2}{\gamma_1 \sigma_1} \phi_2^*, \\ \phi_2^* &= \frac{1}{\gamma_2} \frac{\kappa_2}{\sigma_2} + \frac{\theta_2 (\gamma_2 - 1) \rho \sigma_1}{\gamma_2 \sigma_2} \phi_1^*, \end{aligned}$$

solving which yields the equilibrium portfolio policies (7)–(8). The policies (9)–(10) are obtained by setting $\theta_2 = 0$ in (7)–(8).

In the proof of Lemma 1, we have characterized manager 1's indirect objective function for an arbitrary manager 2's portfolio policy ϕ_2 in (A4). Substituting (7)–(8) and $t = 0$ into this function yields manager 1's time-0 equilibrium indirect objective function $J_1(W_{10}, W_{20})$:

$$J_1(W_{10}, W_{20}) = \frac{e^{mT}}{1 - \gamma_1} (W_{10}^{1-\theta_1} (W_{10}/W_{20})^{\theta_1})^{1-\gamma_1}, \quad (\text{A11})$$

where m is as given by (16).

Q.E.D.

Proof of Corollary 1. As discussed in the main text, we assume that the model parameters are such that the equilibrium stability condition (11) is satisfied, and that the managers are long in the stocks. We first derive all the results reported in Corollary 1 under these conditions, and then elaborate on equilibrium stability.

From the definition of a Nash equilibrium, manager 1's equilibrium policy ϕ_1^* is a best response to manager 2's equilibrium policy ϕ_2^* , and hence the two policies are related through equation (6). Given no stock shorting, $\phi_2^* > 0$, and so from (6) we see that when manager 1 is risk tolerant ($\gamma_1 < 1$) she invests a lower wealth share into the risky stock than in the absence of competition, $\phi_1^* < \kappa_1/(\gamma_1\sigma_1)$, and for a risk intolerant manager 1 ($\gamma_1 > 1$) the opposite result obtains, $\phi_1^* > \kappa_1/(\gamma_1\sigma_1)$. Similarly, for manager 2, proving Corollary 1(i).

Differentiating (7) with respect to γ_1 and manipulating yields

$$\frac{\partial \phi_1^*}{\partial \gamma_1} = -\frac{\gamma_2((\gamma_2 - \rho^2\theta_1\theta_2(\gamma_2 - 1))\kappa_1 - \rho\theta_1\kappa_2)}{(\gamma_1\gamma_2 - \rho^2\theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1))^2\sigma_1}.$$

Switching subscripts 1 and 2 in the above yields the expression for $\partial\phi_2^*/\partial\gamma_2$. Corollary 1(ii) is then immediate.

Computing the derivatives of equilibrium policies ϕ_1^* and ϕ_2^* reported in Table 1, we get

$$\begin{aligned} \frac{\partial \phi_1^*}{\partial \mu_1} &= \frac{\gamma_2}{(\gamma_1\gamma_2 - \rho^2\theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1))\sigma_1^2}, & \frac{\partial \phi_1^*}{\partial \sigma_1} &= -\frac{\gamma_2\kappa_1 + (\gamma_2\kappa_1 + (\gamma_1 - 1)\theta_1\rho\kappa_2)}{(\gamma_1\gamma_2 - \rho^2\theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1))\sigma_1^2}, \\ \frac{\partial \phi_2^*}{\partial \mu_1} &= \frac{(\gamma_2 - 1)\theta_2\rho}{(\gamma_1\gamma_2 - \rho^2\theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1))\sigma_1\sigma_2}, & \frac{\partial \phi_2^*}{\partial \sigma_1} &= -\frac{(\gamma_2 - 1)\theta_2\mu_1\rho}{(\gamma_1\gamma_2 - \rho^2\theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1))\sigma_1^2\sigma_2}. \end{aligned}$$

The results of Table 1 are obtained from these derivatives once we take into account condition (11) and no stock shorting.

We now discuss the issue of potential instability of equilibrium. In particular, we examine when our equilibrium is stable with respect to best response dynamics. See Vives (2001) for a detailed discussion of such a stability criterion, and DeMarzo, Kaniel,

and Kremer (2007, 2008) for recent related works studying, like us, the effects of relative concerns. Before proceeding, we note that we do not take a stand on whether this stability criterion is the most appropriate in our portfolio management context. Our discussion here should not be viewed as making some definitive conclusions on equilibrium instability but rather as indicating that instability is a potential concern when (11) is violated. Exploring this issue further is beyond the scope of the current paper.

The idea underlying this notion of instability is as follows. Starting with the equilibrium outcome (ϕ_1^*, ϕ_2^*) , where ϕ_1^* and ϕ_2^* are given in (7)–(8), we consider a scenario when one manager, say manager 2, deviates from her equilibrium policy by ε , i.e., switches to the (non-equilibrium) policy $\phi_2^* + \varepsilon$. Manager 1 observes this and changes her policy from the equilibrium policy ϕ_1^* to the new policy that constitutes her best response to $\phi_2^* + \varepsilon$, as computed by substituting $\phi_2^* + \varepsilon$ into the right-hand side of (6). Then, manager 2 observes manager 1’s policy and chooses the best response policy, and so on. If the outcome of this step-by-step adjustment mechanism is the initial pair of equilibrium policies (ϕ_1^*, ϕ_2^*) , then the equilibrium is stable with respect to best response dynamics. If, however, the mechanism does not converge, the equilibrium is not stable.

Formalizing this discussion, after manager 2 deviates from ϕ_2^* by ε , manager 1’s best response is to deviate from ϕ_1^* by $\varepsilon\theta_1(\gamma_1 - 1)\rho\sigma_2/(\gamma_1\sigma_1)$, as seen by plugging $\phi_2^* + \varepsilon$ into (6). In the next step, the deviation of manager 2 from ϕ_2^* is εk , where

$$k \equiv \frac{\rho^2\theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1)}{\gamma_1\gamma_2}. \quad (\text{A12})$$

Iterating the above, we obtain that after $2n$ adjustment steps manager 2 will deviate from ϕ_2^* by εk^n , where n is an integer. Hence, the above best response dynamics converge, implying equilibrium stability, when $k < 1$. The condition $k < 1$ is equivalent to requiring

$$\gamma_1\gamma_2 - \rho^2\theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1) > 0. \quad (\text{A13})$$

Q.E.D.

Proof of Proposition 2. First, we derive the equilibrium portfolios under the no-specialization scenario when each manager can invest in both stocks. In this case, the investment wealth process of manager i follows

$$dW_{it} = [r + \phi_{i1}(\mu_1 - r) + \phi_{i2}(\mu_2 - r)]W_{it}dt + \phi_{i1}\sigma_1W_{it}d\omega_{1t} + \phi_{i2}\sigma_2W_{it}d\omega_{2t}.$$

Fixing manager 2's (yet unknown) equilibrium portfolio policies ϕ_{21}^* and ϕ_{22}^* and denoting $J^{NoSp}(W_{1t}, W_{2t}, t)$ to be manager 1's equilibrium indirect objective function under complete markets, the HJB equation is

$$\begin{aligned} 0 = & \max_{\phi_{11}, \phi_{12}} \left[J_t^{NoSp} + (r + \phi_{11}\mu_1 + \phi_{12}\mu_2)W_{1t}J_{W_1}^{NoSp} \right. \\ & + (r + \phi_{21}^*\mu_1 + \phi_{22}^*\mu_2)W_{2t}J_{W_2}^{NoSp} + (\sigma_1^2\phi_{11}^2 + \sigma_2^2\phi_{12}^2 + 2\rho\sigma_1\sigma_2\phi_{11}\phi_{12})W_{1t}^2J_{W_1W_1}^{NoSp}/2 \\ & + ((\sigma_1\phi_{21}^*)^2 + (\sigma_2\phi_{22}^*)^2 + 2\rho\sigma_1\sigma_2\phi_{21}^*\phi_{22}^*)W_{2t}^2J_{W_2W_2}^{NoSp}/2 \\ & \left. + (\sigma_1^2\phi_{11}\phi_{21}^* + \sigma_2^2\phi_{12}\phi_{22}^* + \rho\sigma_1\sigma_2(\phi_{11}\phi_{22}^* + \phi_{12}\phi_{21}^*))W_{1t}W_{2t}J_{W_1W_2}^{NoSp} \right], \end{aligned} \quad (A14)$$

with the boundary condition

$$J^{NoSp}(W_{1T}, W_{2T}, T) = \frac{1}{1 - \gamma_1} (W_{1T}^{1-\theta_1} (W_{1T}/W_{2T})^{\theta_1})^{1-\gamma_1}.$$

The first-order conditions for problem (A14) are

$$\begin{aligned} 0 = & \mu_1W_{1t}J_{W_1}^{NoSp} + \sigma_1^2\phi_{11}^*W_{1t}^2J_{W_1W_1}^{NoSp} + \rho\sigma_1\sigma_2\phi_{12}^*W_{1t}^2J_{W_1W_1}^{NoSp} \\ & + (\sigma_1^2\phi_{21}^* + \rho\sigma_1\sigma_2\phi_{22}^*)W_{1t}W_{2t}J_{W_1W_2}^{NoSp}, \end{aligned} \quad (A15)$$

$$\begin{aligned} 0 = & \mu_2W_{2t}J_{W_2}^{NoSp} + \sigma_2^2\phi_{12}^*W_{2t}^2J_{W_2W_2}^{NoSp} + \rho\sigma_1\sigma_2\phi_{11}^*W_{2t}^2J_{W_2W_2}^{NoSp} \\ & + (\sigma_2^2\phi_{22}^* + \rho\sigma_1\sigma_2\phi_{21}^*)W_{1t}W_{2t}J_{W_1W_2}^{NoSp}. \end{aligned} \quad (A16)$$

We conjecture that $J^{NoSp}(W_{1t}, W_{2t}, t)$ has the form

$$J^{NoSp}(W_{1t}, W_{2t}, t) = \frac{k^{NoSp}(t)}{1 - \gamma_1} (W_{1t}^{1-\theta_1} (W_{1t}/W_{2t})^{\theta_1})^{1-\gamma_1}, \quad (A17)$$

and substitute the required derivatives of J^{NoSp} into (A15)–(A16) to obtain the equations for manager 1's best response policies ϕ_{11}^* and ϕ_{12}^* (we add * because these are also equilibrium policies, being best responses to manager 2's equilibrium policies ϕ_{21}^*

and ϕ_{22}^*):

$$\mu_1 - \gamma_1 \sigma_1^2 \phi_{11}^* - \rho \gamma_1 \sigma_1 \sigma_2 \phi_{12}^* - (1 - \gamma_1) \theta_1 \sigma_1^2 \phi_{21}^* - \rho (1 - \gamma_1) \theta_1 \sigma_1 \sigma_2 \phi_{22}^* = 0, \quad (\text{A18})$$

$$\mu_2 - \rho \gamma_1 \sigma_1 \sigma_2 \phi_{11}^* - \gamma_1 \sigma_2^2 \phi_{12}^* - \rho (1 - \gamma_1) \theta_1 \sigma_1 \sigma_2 \phi_{21}^* - (1 - \gamma_1) \theta_1 \sigma_2^2 \phi_{22}^* = 0. \quad (\text{A19})$$

Switching subscripts 1 and 2 in (A18)–(A19) yields the two equations for manager 2's best response. Solving the resulting system of four equations with four unknowns, we obtain the Nash equilibrium policies under complete markets:

$$\phi_{11}^* = \frac{[\gamma_2 + \theta_1(\gamma_1 - 1)](\mu_1/\sigma_1 - \rho\mu_2/\sigma_2)}{(1 - \rho^2)(\gamma_1\gamma_2 - \theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1))\sigma_1}, \quad \phi_{12}^* = \frac{[\gamma_2 + \theta_1(\gamma_1 - 1)](\mu_2/\sigma_2 - \rho\mu_1/\sigma_1)}{(1 - \rho^2)(\gamma_1\gamma_2 - \theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1))\sigma_2}, \quad (\text{A20})$$

$$\phi_{21}^* = \frac{[\gamma_1 + \theta_2(\gamma_2 - 1)](\mu_1/\sigma_1 - \rho\mu_2/\sigma_2)}{(1 - \rho^2)(\gamma_1\gamma_2 - \theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1))\sigma_1}, \quad \phi_{22}^* = \frac{[\gamma_1 + \theta_2(\gamma_2 - 1)](\mu_2/\sigma_2 - \rho\mu_1/\sigma_1)}{(1 - \rho^2)(\gamma_1\gamma_2 - \theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1))\sigma_2}. \quad (\text{A21})$$

We now compute the term $k^{NoSp}(t)$ in (A17) to completely characterize $J^{NoSp}(W_{1t}, W_{2t}, t)$. Substituting the equilibrium portfolios (A20)–(A21), along with the required partial derivatives of J^{NoSp} computed from (A17), into (A14), we obtain a differential equation:

$$\frac{k^{NoSp}(t)}{1 - \gamma_1} + k^{NoSp}(t)n = 0, \quad (\text{A22})$$

where n is given by (17). Solving (A22), we get

$$k^{NoSp}(t) = e^{n(1-\gamma_1)(T-t)}. \quad (\text{A23})$$

Finally, substituting (A23) into (A17) gives manager 1's time-0 equilibrium indirect objective function under complete markets

$$J_1^{NoSp}(W_{10}, W_{20}) = \frac{e^{nT}}{1 - \gamma_1} (W_{10}^{1-\theta_1} (W_{10}/W_{20})^{\theta_1})^{1-\gamma_1}, \quad (\text{A24})$$

where n is as given by (17).

Substituting (A11) and (A24) into (14) yields

$$\frac{e^{mT}}{1 - \gamma_1} (W_{10}^{1-\theta_1} ((1 - \lambda_1)W_{10}/W_{20})^{\theta_1})^{1-\gamma_1} = \frac{e^{nT}}{1 - \gamma_1} (((1 - \lambda_1)W_{10})^{1-\theta_1} ((1 - \lambda_1)W_{10}/W_{20})^{\theta_1})^{1-\gamma_1},$$

and solving for λ_1 gives (15).

Finally, we clarify the point made in Remark 1 about different properties of equilibrium policies with complete and incomplete markets. Looking at manager 2 under incomplete markets (and manager-specific investment opportunities), from Table 1, her equilibrium investment in stock 2 ϕ_2^* decreases (increases) in stock 1 mean return μ_1 when $\gamma_2 < 1$ ($\gamma_2 > 1$). With complete markets, however, inspecting (A21) we see that the behavior of ϕ_{22}^* with respect to μ_1 is the opposite. (As in Corollary 1, we assume here that the denominator in (A21) is positive to ensure equilibrium stability.) Similarly, changing stock 1 volatility σ_1 has the opposite effects on manager 2's stock 2 investments with incomplete and complete markets. For manager 1, the argument is analogous.

Q.E.D.

Proof of Proposition 3. In our model, a client differs from a manager in that she has no relative concerns, and so a client's time-0 indirect objective function $J_c(W_{c0}, \phi_1)$ is obtained from that of manager 1 given in (A11) by setting $\theta_1 = 0$ in (A11), which gives

$$J_c(W_{c0}, \phi_1) = \frac{W_{c0}^{1-\gamma_c}}{1-\gamma_c} \exp\left((1-\gamma_c)\left(r + \mu_1\phi_1 - \frac{1}{2}\gamma_c\phi_1^2\sigma_1^2\right)T\right). \quad (\text{A25})$$

Substituting (A25) into (20) and rearranging yields (21), where all variables (apart from γ_c) have a hat $\hat{\cdot}$ because the cost λ_c is measured in the post-change period (and γ_c cannot change, and hence no hat).

To prove Proposition 3(i), we compute the argument of the exponent in (21) in two scenarios 1 and 2, where scenario i is when stock i mean return changes to $\hat{\mu}_i = \hat{\mu}$, and subtract the argument in scenario 2 from that in scenario 1. This yields after some algebra

$$\frac{\rho^2\theta_1^2\hat{\mu}(\mu_1 - \mu_2)(\hat{\mu}(\mu_1 + \mu_2) - 2\mu_1\mu_2)(\gamma_1 - 1)^2}{2\mu_1\sigma_2(\gamma_1\gamma_2 - \rho^2\theta_1\theta_2(\gamma_1 - 1)(\gamma_2 - 1))((\gamma_1 - 1)\theta_1\mu_2\rho\sigma_1 + \gamma_2\mu_1\sigma_2)},$$

which is zero when $\mu_1 = \mu_2$, and so the result in Proposition 3(i) obtains.

Following the same steps as above but for stocks volatilities instead of mean returns, we obtain that the difference between the arguments of the exponent in (21) evaluated

at $\hat{\sigma}_1 = \hat{\sigma}$ and $\hat{\sigma}_2 = \hat{\sigma}$ is

$$\frac{\theta_1^2 \rho^2 \mu_1 \mu_2^2 (\sigma_1 - \sigma_2) (2\hat{\sigma} - \sigma_1 - \sigma_2) (\gamma_1 - 1)^2}{2\sigma_2 \hat{\sigma}^2 (\gamma_1 \gamma_2 - \rho^2 \theta_1 \theta_2 (\gamma_1 - 1) (\gamma_2 - 1)) (\theta_1 (\gamma_1 - 1) \rho \mu_2 \sigma_1 + \gamma_2 \mu_1 \sigma_2)},$$

from which the result in Proposition 3(ii) follows.

Q.E.D.

Appendix B: Partial Asset Specialization

In Section 4, we have looked at two scenarios, the specialization scenario where each manager trades in a single stock and the no-specialization scenario where each trades in both available stocks. We have described economic conditions under which: (i) both managers prefer specialization over no specialization and so have incentives to induce transition from no specialization to specialization, and (ii) both favor no specialization over specialization and so are likely to move to the no-specialization scenario. In this appendix, we analyze whether both managers can benefit from a move to the “in-between” scenario, the *partial specialization scenario* where only one manager specializes and invests in a single stock while the other manager invests in both available stocks. Unlike the specialization and no-specialization scenarios where the degree of market completeness is the same for both managers, the partial specialization scenario features asymmetric market completeness—the specializing manager faces incomplete markets while markets are complete for the non-specializing manager. However, we are still able to obtain closed-form solutions for all pertinent quantities.

Let us first summarize our main findings. We show that a transition from no specialization to partial specialization cannot be beneficial to both managers. In particular, the manager who switches to specialization—whose investment opportunity set shrinks [after the transition]—is harmed while the other manager who retains the ability to invest in both stocks benefits. On the other hand, a transition from specialization to partial specialization may benefit both managers, but so may a more “sweeping” move to no specialization which, moreover, implies a higher overall gain. All in all, our analysis below suggests that partial specialization is a less likely scenario, and so in the interest of space we do not report results on how client investors are affected by a move to this scenario.

Analogously to the cost-benefit measure λ_i , $i = 1, 2$, defined in the main paper (see equation () and the following discussion), we consider measures λ_i^{Sp} and λ_i^{NoSp} quantifying the effect on manager i of a transition from the specialization (λ_i^{Sp}) and no specialization (λ_i^{NoSp}) scenarios to the partial specialization scenario. A positive λ_i^{Sp} indicates that manager i prefers specialization over partial specialization, and hence is harmed by a transition from specialization to partial specialization; a negative λ_i^{Sp}

indicates the opposite. The measure λ_i^{NoSp} is similar but applies to a move from no specialization to partial specialization. In the process of deriving these measures, we obtain analytically the Nash equilibrium policies under partial specialization. For manager 1 who specializes in stock 1, the equilibrium policy is a scalar ϕ_1^* representing a share of wealth invested in stock 1. For manager 2 who trades in both stocks, the equilibrium policy is a 2-tuple $(\phi_{21}^*, \phi_{22}^*)$ where ϕ_{2i}^* is a wealth share invested in stock i . Proposition B1 reports the equilibrium policies and the cost-benefit measures in closed form.

Proposition B1. *The unique Nash equilibrium portfolio policies under partial specialization are given by*

$$\phi_1^* = \frac{(\gamma_2 + (\gamma_1 - 1)\theta_1)\mu_1}{(\gamma_1\gamma_2 - (1 - \gamma_1)(1 - \gamma_2)\theta_1\theta_2)\sigma_1^2}, \quad (B1)$$

$$\phi_{21}^* = \frac{(\gamma_2 - 1)(\gamma_2 + (\gamma_1 - 1)\theta_1)\theta_2\mu_1}{(\gamma_1\gamma_2 - (1 - \gamma_1)(1 - \gamma_2)\theta_1\theta_2)\gamma_2\sigma_1^2} - \frac{\rho\mu_2\sigma_1 - \mu_1\sigma_2}{(1 - \rho^2)\sigma_2\gamma_2\sigma_1^2}, \quad \phi_{22}^* = \frac{\mu_2\sigma_1 - \rho\mu_1\sigma_2}{(1 - \rho^2)\gamma_2\sigma_1\sigma_2^2}. \quad (B2)$$

The cost-benefit measures λ_1^{NoSp} and λ_2^{NoSp} , quantifying the effect on manager 1 and 2, respectively, of a transition from the no-specialization scenario to the partial specialization scenario, are

$$\lambda_1^{NoSp} = 1 - \exp((\ell_1 - n)T), \quad \lambda_2^{NoSp} = 1 - \exp((\ell_2 - n)T),$$

where n is given by (17) and ℓ_1 and ℓ_2 are

$$\begin{aligned} \ell_1 &= \mu_1(\phi_1^* - \theta_1\phi_{21}^*) - \mu_2\theta_1\phi_{22}^* - \gamma_1(\sigma_1\phi_1^*)^2/2 + \theta_1(\gamma_1 - 1)(\sigma_1^2\phi_1^*\phi_{21}^* + \rho\sigma_1\sigma_2\phi_1^*\phi_{22}^*) \\ &\quad + \theta_1(1 - \theta_1(\gamma_1 - 1))((\sigma_1\phi_{21}^*)^2 + (\sigma_2\phi_{22}^*)^2 + 2\rho\sigma_1\sigma_2\phi_{21}^*\phi_{22}^*)/2, \\ \ell_2 &= \mu_2\phi_{22}^* + \mu_1(\phi_{21}^* - \theta_2\phi_1^*) - \gamma_2((\sigma_2\phi_{22}^*)^2 + (\sigma_1\phi_{21}^*)^2 + 2\rho\sigma_2\sigma_1\phi_{22}^*\phi_{21}^*)/2 \\ &\quad + \theta_2(\gamma_2 - 1)(\sigma_1^2\phi_{21}^*\phi_1^* + \rho\sigma_2\sigma_1\phi_{22}^*\phi_1^*) + \theta_2(1 - \theta_2(\gamma_2 - 1))(\sigma_1\phi_1^*)^2/2. \end{aligned}$$

The cost-benefit measures λ_1^{Sp} and λ_2^{Sp} , quantifying the effect on manager 1 and 2, respectively, of a transition from specialization to partial specialization, are given by

$$\lambda_1^{Sp} = 1 - \exp((\ell_1 - m)T), \quad \lambda_2^{Sp} = 1 - \exp((\ell_2 - m)T),$$

where m is given by (16).

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