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Exchange rate pass-through, monetary policy, and variability of exchange rates

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Abstract

We document that contribution of identified US monetary shock to exchange rate variability differs across currencies and is inversely related to the degree of a country's US dollar exchange rate pass-through into import prices. We explore this empirical pattern under the assumption that each central bank, when choosing its monetary policy, takes into account in which currency its country's exports and imports are denominated. The choice of imports invoicing currency will affect both the degree of exchange rate pass-through and the monetary policy response. Different shape of monetary policy reaction function will result in different contribution of monetary shocks to the exchange rate dynamics. We illustrate this mechanism using a simple general equilibrium model.

JEL Classification: F41, F42

Key words: Exchange rate; pass-through; invoicing currency; monetary policy; monetary shocks; variance decomposition.

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1 Introduction

Monetary policy shocks have been viewed as a major source of exchange rate fluctuations (Eichenbaum and Evans 1995). In this study we focus on the contribution of monetary shocks originating in the US to short-term variability of exchange rates for currencies that free-float against the US dollar. We observe that the importance of the US monetary shocks as a source of short-term fluctuations varies considerably across currencies. Furthermore, we document that a higher degree of exchange rate pass-through (ERPT) into a country's import prices is associated with a lower fraction of its exchange rate explained by US monetary shocks identified within a respective structural (factor augmented) vector autoregression (VAR), and the correlation coefficient of -0.9 is significant statistically.

We explain the observed empirical pattern using a simple two-country general equilibrium model. We make two assumptions. First, the degree of ERPT is mostly determined by the currency in which its imports are denominated, with producing currency pricing resulting in complete pass-through and local currency pricing in zero pass-through. This assumption is supported by the empirical evidence in Gopinath, Itskhoki, and Rigobon (2010) that a choice of currency for invoicing imports is the most important determinant of pass-through, as Figures 1 and 2 in their paper suggest. Second, when setting its monetary policy, the central bank in each country takes into account in which currency its country's imports and exports are invoiced. The choice of a currency for invoicing imports is assumed exogenous.

In a nutshell, the intuition behind our theoretical results is as follows. Each central bank responds to country-specific productivity shocks originating at home and/or abroad. Under the producer currency pricing (PCP), a regime when all imports prices are predetermined in units of producer's currency, the optimal policy is inward-looking (Corsetti and Pesenti 2008). That means that the optimal policy rule requires that each central bank respond only to domestic productivity shocks ignoring shocks originating abroad. The reason is the following. When prices of goods are predetermined in the producer's currency, a favorable domestic productivity shock will tend to widen domestic firms's markups and thus throw the economy away from the potential. A money injection by the domestic central bank will raise nominal domestic wages and stabilize markups from domestic sales. Another effect of the monetary response at home will be nominal depreciation that will work towards stabilizing markups from foreign sales. It follows that if the central bank focuses on "keeping its own house in order" (stabilizing markups from domestic sales), the rest of the job (stabilizing markups from foreign sales) is done by endogenous responses of the exchange rate. It follows that there is no need for the foreign central bank to respond to shocks originating at home and vice versa.

The situation is remarkably different under the local currency pricing (LCP), a regime when all imports prices are predetermined in units of a destination country's currency. A favorable productivity shock at home will widen domestic firms' markups from domestic sales only. A monetary expansion at home will tend to stabilize markups from domestic sales through an endogenous increase in domestic wages. The effect on the markups from foreign sales, however, will be destabilizing: endogenous nominal depreciation induced by the monetary response and higher nominal wages will result in a shrinkage of firms' markups.

It turns out that, under LCP, the foreign central bank will be interested in responding to shocks originating at home. Indeed, if the central bank does nothing, then foreign currency appreciation caused by monetary expansion at home will reduce the foreign firms' markups from sales abroad (i.e. in the home country). An optimal reaction of the foreign central bank would be to raise money supply to partially offset this distortion at the expense of some destabilization of foreign country's firms' markups from domestic sales (i.e. in the foreign country). It follows that, even when central banks in both countries do not cooperate, their optimal policies under LCP will tend to make the exchange rates less volatile, all other things being equal.

The optimal monetary policy results discussed above are well known and long established in the New Open Economy Macro (NOEM) literature (Corsetti and Pesenti 2008). The novelty of our approach is that we apply these normative findings to explain empirical observations, specifically, a particular empirical fact about short-term behavior of exchange rates. If we believe that central banks have incentives to condition their policies on prevailing invoicing practices, then these incentives should have implications for exchange rate behavior across different currency invoicing regimes. In particular, the contribution of foreign monetary policy shocks to exchange rate variability will be different. Monetary policy shocks are modeled in a conventional fashion, as random deviations of the policy instrument from the level prescribed by the (optimal) policy rule. Since it is optimal to make the exchange rate more volatile under PCP compared with LCP, the contribution of noise, i.e. monetary policy shocks, to the overall exchange rate volatility will be lower in the case of PCP.

Our paper makes two contributions to the literature. First, it offers a more nuanced view on the role of monetary shocks in short-term fluctuations of exchange rates compared with earlier papers in that area. This paper suggests that the importance of monetary policy shocks as sources of exchange rate fluctuations can vary across currencies and offers one arguably important determinant of such variation, specifically, the choice of imports-invoicing currency. The imports-invoicing regime affects the choice of optimal monetary policy rule by the central bank, and this, in turn, influences exchange rate dynamics.

Second, our paper contributes to the branch of the New Open Economy Macro literature that discusses optimal monetary policy design. We show that, if countries act as predicted in that literature in constructing their monetary policy rules, then this should have certain consequences for the behavior of their exchange rates.

The rest of the paper is organized as follows. Section 2 presents empirical evidence on the relationship between a contribution of monetary shocks to exchange rate variability and exchange rate pass-through, which motivates our theoretical model. Section 3 describes a two-country open economy general equilibrium model with prices predetermined for one period, which is rather standard in the NOEM literature. Section 4 works out optimal monetary rules under alternative regimes of imports invoicing. Section 5 presents our main theoretical results, specifically, the contribution of monetary shocks into short-term exchange rate variability under alternative regimes of imports invoicing. Section 6 concludes.

2 Empirical motivation

Our sample consists of nine currencies that were very close to freely floating in the post-Bretton-Woods era (i.e. after March 1973) according to Reinhart and Rogoff's (2004) classification. Our choice of currencies was limited by OECD countries and was subject to two constraints. First, we did not include countries that adopted the euro in January 1999 or later. Second, we did not include such countries as Mexico or Korea that switched multiple times from float to fix and back over the covered time interval.

[TABLE 1 HERE]

For each country, Table 1 reports three statistics related to its bilateral nominal exchange rate with the US. The first one is the contribution of the US monetary policy (MP) shock to the exchange rate variability. It represents a fraction of the identified US monetary shock in the forecast error variance decomposition of the exchange rate averaged out for horizons up to 12 months. The structural US MP shock is identified within a structural factor-augmented vector autoregression (FAVAR) using a block-recursive identification scheme suggested by Stock and Watson (2005), which is similar in spirit to Christiano, Eichenbaum, and Evans (1999).

Factor-augmented VARs were introduced into the literature by Bernanke, Boivin, and Eliasz (2005) and since then have become a popular method of analyzing the behavior of macroeconomic variables. The particular identifying assumptions exploited in our study are the ones suggested by Stock and Watson and are the following. All variables are grouped into three groups, slow-moving (output, employment, consumer prices, labor market indicators), the monetary policy instrument (the federal funds rate), and fast-moving variables (producer prices, orders, survey indicators, asset prices). All structural shocks are assumed to belong to one of the three groups: slow, monetary policy, and fast shocks. Slow-moving variables are assumed to respond contemporaneously (more precisely, within a month) only to the slow shocks while to all other shocks with a lag. The federal funds rate responds contemporaneously to the slow and the MP shocks, and with a lag to the rest of them. Fast-moving variables respond contemporaneously to all kinds of shocks. Although this procedure identifies only one structural shock of interest, the MP shock, one conceivable interpretation for slow shocks would be exogenous shifts in aggregate demand and/or relative demand for goods produced by different sectors while fast shocks can reflect news and changes in sentiment of consumers and investors. The identification procedure used by Bernanke, Boivin, and Eliasz (2005) is similar to Stock and Watson's (2005).

Once the monetary policy shock is identified, one can employ such a standard tool of SVAR/SFAVAR analysis as forecast error variance decomposition (FEVD). For a given horizon, FEVD shows relative contributions of different kinds of structural shocks into variability of an indicator of interest. The fraction of exchange rate variability explained by the MP shock, average for horizons up to 12 months, is exactly what is reported in the second column of Table 1.

The original Stock and Watson's (2005) data set contains 132 monthly time series for the US: real activity and labor market indicators, producer and consumer prices, interest rates and asset prices, survey indicators, etc. It also includes four bilateral nominal exchange rates (Canadian dollar, Japanese yen, Swiss franc, and British pound of sterling). We added five more bilateral exchange rates with the US dollar (Australian dollar, Danish krone, Norwegian krone, New Zealand dollar, and Swedish krona) taken from the Global Financial Data database. The time period, originally covering January 1959 through December 2003, was shortened to January 1973 through December 2003 so that only post-Bretton-Woods time observations were used in the analysis. We exploited Stock and Watson's (2005) program code, originally in GAUSS and translated to MATLAB. The data set with detailed description of variables and the program code in GAUSS are available from Mark Watson's home webpage at Princeton University.

The second statistic reported in Table 1 (the third column) is the correlation between the price index of the country's non-commodity imports and its nominal exchange rate against the US dollar, both variables being log-differenced. The data are at quarterly frequencies, seasonally-adjusted, and cover the period from 1974:II through 2007:IV. They come from the World Economic Outlook database provided by OECD. This correlation is a proxy for the degree of exchange rate pass-through into import prices. A high value is likely to indicate that most of a country's imports are denominated in US dollars.

Finally, the last column of Table 1 shows volatility for each currency as measured by the standard deviation of the respective exchange rate in log-differences on the time interval covering January 1973 through December 2003. The monthly exchange rate data are taken from the Global Financial Data online database.

Although the number of observations is low (only nine), the negative correlation between the two statistics, the fraction of exchange rate variability due to US monetary shocks, on the one hand, and the correlation between changes in non-commodity imports prices and bilateral exchange rates, on the other hand, is remarkably strong with correlation coefficient of -0.9, which is statistically significant. Figure 1 shows the scatter plot for these two statistics. For countries that experience a low degree of passthrough, US monetary shocks tend to explain higher fraction of its exchange rate variability.

[FIGURE 1 HERE]

Ideally, we would strongly prefer to have (i) more countries in our sample, and (ii) a more direct proxy for the choice of a currency, in which country's imports are invoiced. The first limitation, as already mentioned, is due to the fact that there are not too many countries that can be characterized by relatively free floating exchange rate regime according to Reinhart and Rogoff's (2004) classification for prolonged periods of time so that the estimation of FEVDs would be feasible. In particular, we had to exclude all countries that adopted the euro after 1999. This bounded our sample to only nine countries. Clearly, it does not make sense to extend the sample by including countries with less flexible exchange rate regimes. In that case shocks get partially absorbed by a change in the central bank's foreign reserves.

The second above-mentioned problem, our inability to better approximate the prevailing import invoicing practice, is due to the scarcity of relevant data. Such data are used, for example, in Goldberg and Tille (2008). For a sample of countries, this paper reports shares of imports and exports denominated in the US dollars (vehicle currency) and in the exporter/importer's currency.

Regretfully, the available data cover only three countries in our sample, Australia, UK, and Japan, which prevents us from switching to using these data.

Interestingly, our data contain five countries that are also present in the above-mentioned study by Gopinath, Itskhoki, and Rigobon (2010). The subset of countries looks like the following: Switzerland (0.38), Japan (0.21), UK (0.19), Sweden (0.12), Canada (0.04), as shown in Table 1 in the cited paper. The numbers in the parentheses following country names show the fraction of non-dollar priced goods imported from a respective country. It is straightforward to see that the ordering of countries by the share of non-dollar imports to the US exactly coincides with the ordering of countries by the fraction of the US monetary shocks in the variance of the bilateral nominal exchange rate of a respective country against the US dollar, as shown in the second column of Table 2. The share of non-dollar priced imported goods for those five countries is also negatively correlated with the degree of exchange rate passthrough (third column in Table 1). Assuming that imports invoicing practices are approximately symmetric between countries within each country pair (i.e., the fraction of non-dollar priced goods imported from Switzerland to the US roughly equals the fraction of non-Swiss-franc priced goods imported from the US to Switzerland), this pattern might be viewed as suggestive evidence in favor of robustness of the empirical association shown on Figure 1, which motivates our study. The incidence of local currency pricing of imports is positively correlated with the contribution of the US monetary shocks to the exchange rate volatility.

3 Model

Our setup is similar to Corsetti and Pesenti (2008) and Goldberg and Tille (2009). The global economy consists of two countries, Home and Foreign. The population size in both countries equals one. Each country produces a unit continuum of differentiated product varieties using homogeneous labor as the only input. Goods are freely traded across countries. For simplicity, there is no trade in assets, so that trade balance is zero in each period for both countries.

3.1 Preferences and intratemporal consumption choice

A representative consumer in Home has preferences over consumption, hours worked, and real money balances

$$\log C + \chi \log \frac{M}{P} - \kappa L$$

She consumes goods both produced domestically and imported from abroad:

$$C = \left[(1-\alpha)^{\frac{1}{\phi}} C_H^{\frac{\phi-1}{\phi}} + \alpha^{\frac{1}{\phi}} C_F^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}$$

where C_i denotes consumption of goods produced in country $i \in \{H, F\}$. It contains quantities of each variety produced in a respective country. Parameter $0 \leq \alpha \leq \frac{1}{2}$ characterizes intensity of home bias in consumption and strength of trade links between the two countries. Special case $\alpha = \frac{1}{2}$ corresponds to no home bias in consumption whereas $\alpha = 0$ means that countries live in autarky and do not trade with each other.

As a convention, we will assume that all individual goods produced in the global economy are indexed as $j \in [0, 2]$, of which $j \in [0, 1)$ are produced in Home, $j \in [1, 2]$ in Foreign. Then

$$C_H = \left(\int_0^1 C_H(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$$

$$C_F = \left(\int_1^2 C_F(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$$

where $C_i(j)$ is the volume of Home consumption of variety j that is produced in country i, $j \in [0, 1)$ if $i = H$ and $j \in [1, 2]$ if $i = F$.

Given C and respective prices, demand in Home for composite good produced in countries H and F is derived from respective intratemporal optimization problems

$$C_H = (1-\alpha) C \left(\frac{P_H}{P} \right)^{-\phi}$$

$$C_F = \alpha C \left(\frac{P_F}{P}\right)^{-\phi}$$

Given C_H and C_F and respective prices, demands for variety j produced in country H or F equal

$$C_H(j) = C_H \left[\frac{P_H(j)}{P_H}\right]^{-\theta}$$

$$C_F(j) = C_F \left[\frac{P_F(j)}{P_F}\right]^{-\theta}$$

respectively where $P_i(j)$ is price that consumer in Home pays for variety j of good produced in country $i \in \{H, F\}$,

$$P = \left[(1-\alpha)P_H^{1-\phi} + \alpha P_F^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{1}$$

is the unit price of Home consumption,

$$P_H = \left(\int_0^1 P_H(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}} \tag{2}$$

and

$$P_F = \left(\int_1^2 P_F(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}} \tag{3}$$

are Home unit prices of composite goods produced in Home and Foreign respectively. Similar expressions apply to Foreign consumption and price indices.

3.2 Intertemporal consumption allocation

A representative consumer in country $i \in \{H, F\}$ maximizes her life-time expected utility subject to an intertemporal budget constraint. For simplicity, assume that there is no trade in assets between the two countries. Provided that there is no capital involved into production, prices are fixed only for one period, and shocks are serially uncorrelated, the multiperiod optimization for a representative household in Home splits into a sequence of one-period problems:

$$\max U = \mathbb{E}\left[\log C + \chi \log \frac{M}{P} - \kappa L\right] \tag{4}$$

$$\text{s.t.} \qquad PC + M = \Pi + WL - T \tag{5}$$

where W is the wage rate, Π firms' profits, and T lump-sum taxes. Without loss of generality, money balances as of beginning of the period are set equal to zero.

Use budget constraint (5) to eliminate C from (4). The first-order conditions with respect to money balances and labor hours supplied are respectively

$$M = \chi PC \tag{6}$$

$$W = \kappa PC = \frac{\kappa}{\chi} M \tag{7}$$

The optimization problem and the first-order conditions for a representative consumer in Foreign look similarly.

3.3 Timing of events

At the beginning of each period, firms set prices that will be in effect for that period. Then productivity shocks in both countries are materialized. Central banks respond to shocks by changing money supply. Finally, production and consumption occur.

3.4 Price setting and production

The price for domestic sales is set one period in advance and denominated in national currency. The price for foreign sales can be predetermined either in the currency of producer or consumer. Following the literature (e.g., as summarized in Corsetti and Pesenti 2008) we consider three cases:

1. *Producer Currency Pricing (PCP)*: In both countries, each firm sets its price for foreign sales

in terms of national currency. The price paid by foreign consumers equals the predetermined producer price multiplied by the bilateral exchange rate.

2. *Local Currency Pricing (LCP)*: In both countries, each firm sets its price for exports in terms of the consumer's country currency.

3. *Vehicle Currency Pricing (VCP)*: All prices for foreign sales are predetermined in terms of the currency of Foreign, which serves as an invoicing currency for international deals. This means that the price paid by Home consumers for goods produced in Foreign is obtained as the predetermined price in Foreign currency multiplied by the bilateral exchange rate while the price paid by Foreign consumers for goods produced in Home is predetermined in terms of the Foreign currency.

Firm $j \in [0, 1)$ in Home produces its output using labor as the only production factor

$$Y(j) = ZL(j)$$

where $L(j)$ is the amount of (homogeneous) labor employed by firm j, Z is country-specific productivity shock, which is serially uncorrelated with $\mathbb{E} \log Z = 0$. This is one of two shocks in our model. The other shock is the monetary policy shock, which will be introduced later as a deviation of the monetary policy from the optimal plan derived within the model.

Given prices, the volume of production by firm j in the Home country is determined by the world demand for its product:

$$\begin{aligned} Y(j) &= ZL(j) = C_H(j) + C_H^*(j) = C_H \left[\frac{P_H(j)}{P_H} \right]^{-\theta} + C_H^* \left[\frac{P_H^*(j)}{P_H^*} \right]^{-\theta} \\ &= (1-\alpha) C \left(\frac{P_H}{P} \right)^{-\phi} \left[\frac{P_H(j)}{P_H} \right]^{-\theta} + \alpha C^* \left(\frac{P_H^*}{P^*} \right)^{-\phi} \left[\frac{P_H^*(j)}{P_H^*} \right]^{-\theta} \end{aligned}$$

The volume of production by firm $j \in [1, 2]$ in the Foreign country is determined in a similar way.

In a symmetric equilibrium with $P_H(j) = P_H$, $P_H^*(j) = P_H^*$, $C_H(j) = C_H$, $L(j) = L$, etc. we obtain

$$Y = ZL = (1-\alpha)C\left(\frac{P_H}{P}\right)^{-\phi} + \alpha C^*\left(\frac{P_H^*}{P^*}\right)^{-\phi} \tag{8}$$

$$Y^* = Z^*L^* = \alpha C\left(\frac{P_F}{P}\right)^{-\phi} + (1-\alpha)C^*\left(\frac{P_F^*}{P^*}\right)^{-\phi} \tag{9}$$

3.5 Money and exchange rates

In the Home country, the government issues new money by handing out nominal lump-sum transfers:

$$M = -T.$$

The amount of money issued is determined by the optimal monetary policy followed by the central bank and a random shock, as will be shown below. Again, without loss of generality, nominal money balances inherited from the previous period are set equal to zero.

In the absence of capital flows between the countries (by assumption), the balanced trade condition applies. The total value of the Home country imports must be equal to the value of its exports expressed in a common currency:

$$P_F C_F = \mathcal{E} P_H^* C_H^* \tag{10}$$

or

$$\alpha PC\left(\frac{P_F}{P}\right)^{1-\phi} = \mathcal{E}\alpha P^*C^*\left(\frac{P_H^*}{P^*}\right)^{1-\phi}$$

Use (6) to substitute for PC and P^*C^*:

$$M\left(\frac{P_F}{P}\right)^{1-\phi} = \mathcal{E}\alpha M^*\left(\frac{P_H^*}{P^*}\right)^{1-\phi}$$

It follows that

$$\mathcal{E} = \frac{M}{M^*}\left(\frac{P_F}{P}\frac{P^*}{P_H^*}\right)^{1-\phi} \tag{11}$$

Use expressions for the aggregate price level to show that

$$\mathcal{E} = \frac{M}{M^*} \frac{\alpha + (1-\alpha)\left(\frac{P_F^*}{P_H^*}\right)^{1-\phi}}{\alpha + (1-\alpha)\left(\frac{P_H}{P_F}\right)^{1-\phi}} \tag{12}$$

3.6 Goods markets clearing

Use the balanced trade condition (10) to exclude C^* from (8):

$$ZL = (1-\alpha)\frac{PC}{P_H}\left(\frac{P_H}{P}\right)^{1-\phi} + \alpha\frac{P^*C^*}{P_H^*}\left(\frac{P_H^*}{P^*}\right)^{1-\phi} = PC\left[\frac{1-\alpha}{P_H}\left(\frac{P_H}{P}\right)^{1-\phi} + \frac{\alpha}{\mathcal{E}P_H^*}\left(\frac{P_F}{P}\right)^{1-\phi}\right]$$

or, equivalently,

$$C = ZL\tau \tag{13}$$

where

$$\tau \equiv \left[(1-\alpha)\left(\frac{P_H}{P}\right)^{-\phi} + \alpha\left(\frac{P_F}{P}\right)^{-\phi}\frac{P_F}{\mathcal{E}P_H^*}\right]^{-1} \tag{14}$$

The term τ in (13) captures the effect of the terms of trade between the two countries on the consumption possibility frontier of a Home consumer. According to (14), a deterioration in the terms of trade, which is equivalent to an increase in term $\frac{P_F}{\mathcal{E}P_{H*}}$ will reduce τ and, by (13), lower C for given L and Z. Equation (13) can be interpreted as an aggregate supply relationship. Its closed-economy analog would be $C = ZL$.

The aggregate supply equation for Foreign is

$$C^* = Z^*L^*\tau^* \tag{15}$$

where

$$\tau^* \equiv \left[(1-\alpha)\left(\frac{P_F^*}{P^*}\right)^{-\phi} + \alpha\left(\frac{P_H^*}{P^*}\right)^{-\phi}\frac{\mathcal{E}P_H^*}{P_F}\right]^{-1} \tag{16}$$

3.7 Flexible-price equilibrium

If prices are fully flexible then it does not matter in what currency a firm's foreign sales are denominated. It is equivalent to assuming that the firm will set its prices conditional on realizations of productivity shocks at home and abroad. Given Z, firm j in the Home country will choose $P_H(j)$ and $P_H^*(j)$ to maximize

$$\Pi(j) = \left(P_H(j) - \frac{W}{Z}\right)(1-\alpha)C\left[\frac{P_H(j)}{P_H}\right]^{-\theta}\left(\frac{P_H}{P}\right)^{-\phi} + \left(\mathcal{E}P_H^*(j) - \frac{W}{Z}\right)\alpha C^*\left[\frac{P_H^*(j)}{P_H^*}\right]^{-\theta}\left(\frac{P_H^*}{P^*}\right)^{-\phi}$$

In a symmetric equilibrium, all Home firms will charge the same price: $P_H(j) = \mathcal{E}P_H^* = P_H$. The optimally chosen price is characterized by a constant markup over the marginal cost:

$$P_H = \mathcal{E}P_H^* = \frac{\theta}{\theta - 1}\frac{W}{Z}$$

Similarly, for the Foreign country

$$P_F^* = \frac{P_F}{\mathcal{E}} = \frac{\theta}{\theta - 1}\frac{W^*}{Z^*}$$

Use FOC (7) for the consumer optimization problem to substitute for W and obtain

$$P_H = \mathcal{E}P_H^* = \frac{\kappa\theta}{\chi(\theta - 1)}\frac{M}{Z} \tag{17}$$

Similarly,

$$P_F^* = \frac{P_F}{\mathcal{E}} = \frac{\kappa\theta}{\chi(\theta - 1)}\frac{M^*}{Z^*} \tag{18}$$

Use the expression (1) for the aggregate price level in the Home country to obtain

$$\left(\frac{P}{P_F}\right)^{1-\phi} = \alpha + (1-\alpha)\left(\frac{P_H}{P_F}\right)^{1-\phi} = \alpha + (1-\alpha)\left(\frac{M}{Z}\frac{Z^*}{M^*}\frac{1}{\mathcal{E}}\right)^{1-\phi} \tag{19}$$

Similarly,

$$\left(\frac{P^*}{P_H^*}\right)^{1-\phi} = \alpha + (1-\alpha)\left(\frac{P_F^*}{P_H^*}\right)^{1-\phi} = \alpha + (1-\alpha)\left(\frac{M^*}{Z^*}\frac{Z}{M}\mathcal{E}\right)^{1-\phi} \tag{20}$$

Plugging (19) and (20) into expression (11) gives a non-linear equation that determines the equilibrium nominal exchange rate under flexible prices:

$$\mathcal{E} = \frac{M}{M^*}\frac{\alpha + (1-\alpha)\left(\frac{M^*}{Z^*}\frac{Z}{M}\mathcal{E}\right)^{1-\phi}}{\alpha + (1-\alpha)\left(\frac{M}{Z}\frac{Z^*}{M^*}\frac{1}{\mathcal{E}}\right)^{1-\phi}} \tag{21}$$

In a deterministic case with constant money supplies

$$Z = \bar{Z}, \qquad Z^* = \bar{Z}^*, \qquad \frac{\bar{M}}{\bar{Z}} = \frac{\bar{M}^*}{\bar{Z}^*} = 1$$

the nominal exchange rate should satisfy

$$\bar{\mathcal{E}} = \frac{\bar{M}}{\bar{M}^*}\frac{\alpha + (1-\alpha)\left(\bar{\mathcal{E}}\right)^{1-\phi}}{\alpha + (1-\alpha)\left(\frac{1}{\bar{\mathcal{E}}}\right)^{1-\phi}} \tag{22}$$

Assuming that deviations of productivity levels Z and Z^* from $\bar{Z}$ and $\bar{Z}^*$ respectively,

$$z = \frac{Z-\bar{Z}}{\bar{Z}}, \qquad z^* = \frac{Z^*-\bar{Z}^*}{\bar{Z}^*}, \tag{23}$$

are small in percentage terms, we will log-linearize equation (21) to find an approximate solution for the exchange rate. In what follows we will use small letters to denote relative deviation of a respective variable from its deterministic equilibrium value, for example:

$$\varepsilon = \frac{\mathcal{E}-\bar{\mathcal{E}}}{\bar{\mathcal{E}}}, \qquad \text{etc.}$$

Solving the log-linearized version of (21) yields

$$\varepsilon = m - m^* - \frac{2(1-\alpha)(\phi-1)}{1+2(1-\alpha)(\phi-1)}(z - z^*) \tag{24}$$

Now find the rest of the flexible-price equilibrium. The log-linearized prices set by firms, equation (17) and a similar expression for the Foreign firm, are

$$p_H = m - z, \qquad p_H^* = m - z - \varepsilon, \qquad p_F = m^* - z^* + \varepsilon, \qquad p_F^* = m^* - z^* \tag{25}$$

Aggregate price levels are

$$p = (1-\alpha)p_H + \alpha p_F, \qquad p^* = (1-\alpha)p_F^* + \alpha p_H^* \tag{26}$$

FOCs w.r.t. M for the consumer optimization (6) become

$$p + c = m, \qquad p^* + c^* = m^* \tag{27}$$

Log-linearized versions of the goods market clearing conditions (8) and (9) are

$$z + l = (1-\alpha)[c - \phi(p_H - p)] + \alpha[c^* - \phi(p_H^* - p^*)] \tag{28}$$

$$z^* + l^* = (1-\alpha)[c^* - \phi(p_F^* - p^*)] + \alpha[c - \phi(p_F - p)] \tag{29}$$

The flexible-price equilibrium prices – besides the nominal exchange rate that is already determined by (24) – and allocations can be found by solving the system of 10 equations (25) – (29) for 10 unknowns l, l^*, c, c^*, p, p^*, p_H, p_H^*, p_F, and p_F^*. In particular, the equilibrium values of employment, aggregate consumption and aggregate price levels turn out to be

$$l = l^* = 0 \tag{30}$$

$$c = z - \frac{\alpha(z - z^*)}{1 + 2(1-\alpha)(\phi - 1)}, \qquad c^* = z^* - \frac{\alpha(z^* - z)}{1 + 2(1-\alpha)(\phi - 1)} \tag{31}$$

$$p = m - z + \frac{\alpha(z - z^*)}{1 + 2(1-\alpha)(\phi - 1)}, \qquad p^* = m^* - z^* + \frac{\alpha(z^* - z)}{1 + 2(1-\alpha)(\phi - 1)} \tag{32}$$

3.8 Equilibrium under Producer Currency Pricing

When producer prices are predetermined one period in advance in terms of national currency, firm j in the Home country solves the profit maximization problem by picking $P_H(j)$:

$$\begin{aligned} \mathbb{E}\frac{\Pi(j)}{PC} &= \mathbb{E}\frac{1}{PC}\left[P_H(j) - \frac{W}{Z}\right] C_H(j) + \mathbb{E}\frac{1}{PC}\left[P_H(j) - \frac{W}{Z}\right] C_H^*(j) \\ &= \mathbb{E}\frac{1}{PC}\left(P_H(j) - \frac{W}{Z}\right)(1-\alpha)C\left[\frac{P_H(j)}{P_H}\right]^{-\theta}\left(\frac{P_H}{P}\right)^{-\phi} \\ &+ \mathbb{E}\frac{1}{PC}\left(P_H(j) - \frac{W}{Z}\right)\alpha C^*\left[\frac{P_H(j)/\mathcal{E}}{P_H^*}\right]^{-\theta}\left(\frac{P_H^*}{P^*}\right)^{-\phi} \end{aligned} \tag{33}$$

In the above expression, the marginal utility of consumption $\frac{1}{C}$ serves as the appropriate stochastic discount factor for the firm j's real profits $\frac{\Pi(j)}{P}$.

Note that because the price elasticity of demand on the Home and Foreign market is the same and equals θ, there will be no third-degree price discrimination and the law of one price will hold as an equilibrium outcome. To make things simpler, we take this into account directly on the profit optimization stage.

Then optimally chosen preset prices are:

$$P_H = P_H^*\mathcal{E} = \frac{\theta\kappa}{\chi(\theta-1)}\mathbb{E}\frac{M}{Z}, \qquad P_F^* = \frac{P_F}{\mathcal{E}} = \frac{\theta\kappa}{\chi(\theta-1)}\mathbb{E}\frac{M^*}{Z^*} \tag{34}$$

The only difference between (34) and analogous flexible-price expressions (17) and (18) is that now $\frac{M}{Z}$ and $\frac{M^*}{Z^*}$ enter in expectation. Log-linearization of (34) yields

$$p_H = p_H^* + \varepsilon = \mathbb{E}(m - z), \qquad p_F^* = p_F - \varepsilon = \mathbb{E}(m^* - z^*) \tag{35}$$

Plug (35) into the log-linearized version of (12) to obtain

$$\varepsilon = \frac{m - m^* + 2(1-\alpha)(\phi-1)(p_H - p_F^*)}{1 + 2(1-\alpha)(\phi-1)}$$

Given that prices P_H and P_F^* are predetermined one period in advance it must be the case that $p_H = 0$ and $p_F^* = 0$ (recall that small letters denote relative deviations of respective variables from their deterministic equilibrium values). It follows that

$$\varepsilon = \frac{m - m^*}{1 + 2(1-\alpha)(\phi-1)} \tag{36}$$

The equilibrium values of aggregate price levels, consumption and employment turn out to be

$$p = \alpha\varepsilon, \qquad p^* = -\alpha\varepsilon \tag{37}$$

$$c = m - \frac{\alpha(m-m^*)}{1+2(1-\alpha)(\phi-1)}, \qquad c^* = m^* + \frac{\alpha(m-m^*)}{1+2(1-\alpha)(\phi-1)} \tag{38}$$

$$l = m - z, \qquad l^* = m^* - z^* \tag{39}$$

3.9 Equilibrium under Local Currency Pricing

When export prices are predetermined one period in advance in terms of the destination currency, firm j in the Home country solves the profit maximization problem by picking $P_H(j)$ and $P_H^*(j)$:

$$\begin{aligned} \mathbb{E}\frac{\Pi(j)}{PC} &= \mathbb{E}\frac{1}{PC}\left[P_H(j) - \frac{W}{Z}\right]C_H(j) + \mathbb{E}\frac{1}{PC}\left[P_H^*(j)\mathcal{E} - \frac{W}{Z}\right]C_H^*(j) \\ &= \mathbb{E}\frac{1}{PC}\left(P_H(j) - \frac{W}{Z}\right)(1-\alpha)C\left[\frac{P_H(j)}{P_H}\right]^{-\theta}\left(\frac{P_H}{P}\right)^{-\phi} \\ &+ \mathbb{E}\frac{1}{PC}\left(P_H^*(j)\mathcal{E} - \frac{W}{Z}\right)\alpha C^*\left[\frac{P_H^*(j)}{P_H^*}\right]^{-\theta}\left(\frac{P_H^*}{P^*}\right)^{-\phi} \end{aligned} \tag{40}$$

Then optimally chosen preset prices are:

$$P_H = \frac{\theta\kappa}{\chi(\theta-1)}\mathbb{E}\frac{M}{Z}, \quad P_H^* = \frac{\theta\kappa}{\chi(\theta-1)}\mathbb{E}\frac{M}{\mathcal{E}Z}, \quad P_F^* = \frac{\theta\kappa}{\chi(\theta-1)}\mathbb{E}\frac{M^*}{Z^*}, \quad P_F = \frac{\theta\kappa}{\chi(\theta-1)}\mathbb{E}\frac{\mathcal{E}M^*}{Z^*} \tag{41}$$

or in log-deviations

$$p_H = \mathbb{E}(m-z), \qquad p_H^* = \mathbb{E}(m-z-\varepsilon), \qquad p_F^* = \mathbb{E}(m^*-z^*), \qquad p_F = \mathbb{E}(m^*-z^*+\varepsilon) \tag{42}$$

Log-linearize the exchange rate equation (12) to obtain

$$\varepsilon = m - m^* + (1-\alpha)(\phi-1)(p_H + p_H^* - p_F - p_F^*)$$

Given that all prices P_H, P_F, P_H^*, and P_F^* are predetermined one period in advance it must be the case that their log-deviations from steady-state levels are zeros:

$$p_H = 0, \qquad p_H^* = 0, \qquad p_F = 0, \qquad p_F^* = 0.$$

It follows that

$$\varepsilon = m - m^* \tag{43}$$

The equilibrium values of aggregate price levels, consumption and employment turn out to be

$$p = 0, \qquad p^* = 0 \tag{44}$$

$$c = m, \qquad c^* = m^* \tag{45}$$

$$l = -z + (1-\alpha)m + \alpha m^*, \qquad l^* = -z^* + (1-\alpha)m^* + \alpha m \tag{46}$$

3.10 Equilibrium under Vehicle Currency Pricing

Suppose now that all international trade deals are invoiced in units of the Foreign country's currency and that the exports prices are predetermined one period in advance. Thus the currency of the Foreign country plays the role of the vehicle currency in international trade. Firm j in the Home country solves the profit maximization problem (40) by picking $P_H(j)$ and $P_H^*(j)$ while firm j' in the Foreign country will solve the profit maximization problem similar to (33) by picking $P_F^*(j')$. In the latter case the Home consumer will pay the price $P_F(j') = \frac{P_F^*(j')}{\mathcal{E}}$ for each unit of good j'.

Then optimally chosen preset prices are:

$$P_H = \frac{\theta\kappa}{\chi(\theta-1)}\mathbb{E}\frac{M}{Z}, \qquad P_H^* = \frac{\theta\kappa}{\chi(\theta-1)}\mathbb{E}\frac{M}{\mathcal{E}Z}, \qquad P_F^* = \frac{P_F}{\mathcal{E}} = \frac{\theta\kappa}{\chi(\theta-1)}\mathbb{E}\frac{M^*}{Z^*}$$

or in log-deviations

$$p_H = \mathbb{E}(m-z), \qquad p_H^* = \mathbb{E}(m-z-\varepsilon), \qquad p_F^* = \mathbb{E}(m^*-z^*), \qquad p_F = \mathbb{E}(m^*-z^*)+\varepsilon \quad (47)$$

Plug (47) into the log-linearized version of (12) to obtain

$$\varepsilon = \frac{m - m^* + (1-\alpha)(\phi-1)(p_H + p_H^* - 2p_F^*)}{1 + (1-\alpha)(\phi-1)}$$

Given that all prices P_H, P_H^*, and P_F^* are predetermined one period in advance it must be the case that $p_H = 0$, $p_H^* = 0$, and $p_F^* = 0$. It follows that

$$\varepsilon = \frac{m - m^*}{1 + (1-\alpha)(\phi-1)} \quad (48)$$

The equilibrium values of aggregate price levels, consumption and employment turn out to be

$$p = \alpha\varepsilon, \qquad p^* = 0 \quad (49)$$

$$c = m - \frac{\alpha(m - m^*)}{1 + (1-\alpha)(\phi - 1)}, \qquad c^* = m^* \tag{50}$$

$$l = -z + m, \qquad l^* = -z^* + (1-\alpha)m^* + \alpha m \tag{51}$$

4 Optimal monetary policy with no cooperation

We assume that, in each country, the central bank responds to productivity shocks at home and abroad by adjusting money supply, given a policy reaction function of the central bank in the other country. We assume that the central banks of the two countries *do not cooperate*. As is standard in the literature, the objective function of the central bank in country $i \in \{H, F\}$ coincides with the expected utility of a representative consumer in that country with the real balances term ignored:

$$\tilde{U} = \mathbb{E}\left[\log C - \kappa L\right], \qquad \tilde{U}^* = \mathbb{E}\left[\log C^* - \kappa L^*\right]$$

Now we show that, no matter in which currency prices are predetermined, the expected level of employment is constant and equal to that under flexible prices:

$$\mathbb{E}L = \mathbb{E}L^* = \frac{\theta - 1}{\kappa\theta}$$

Suppose first that firms in Home predetermine their export prices in units of Home currency. That implies that P_H is fixed and $P_H^* = \frac{P_H}{\mathcal{E}}$. Equation (14) can be simplified as

$$\tau = \frac{P_H}{P}\left[\frac{(1-\alpha)P_H^{1-\phi} + \alpha P_F^{1-\phi}}{P^{1-\phi}}\right]^{-1} = \frac{P_H}{P} \tag{52}$$

where the second equality follows from (1). FOC (6) from the Home consumer's optimization problem and the aggregate supply relationship (13) imply

$$L = \frac{1}{\chi}\frac{M}{ZP\tau} \tag{53}$$

Plugging (52) into the above formula and taking expectation gives

$$\mathbb{E}L = \frac{1}{\chi}\mathbb{E}\left(\frac{1}{P_H}\frac{M}{Z}\right) = \frac{1}{P_H}\frac{1}{\chi}\mathbb{E}\left(\frac{M}{Z}\right) = \frac{\theta-1}{\kappa\theta}\frac{1}{P_H}P_H = \frac{\theta-1}{\kappa\theta}$$

where the second equality is due to the fact that P_H is predetermined and the third one holds by (34).

Now suppose that all firms at home and abroad preset their export prices one period in advance in terms of the destination country's currency. That means that P_H^* and P_F are fixed. It follows that aggregate price levels P and P^* are fixed as well: recall that prices for domestic sales are predetermined in terms of national currency. Plugging (14) into equation (53) yields

$$\begin{aligned}
\mathbb{E}L &= \frac{1}{\chi}\left[(1-\alpha)\left(\frac{P_H}{P}\right)^{-\phi}\frac{1}{P}\mathbb{E}\left(\frac{M}{Z}\right) + \alpha\left(\frac{P_F}{P}\right)^{1-\phi}\frac{1}{P_H^*}\mathbb{E}\left(\frac{M}{\mathcal{E}Z}\right)\right] \\
&= \frac{\theta-1}{\kappa\theta}\left[(1-\alpha)\left(\frac{P_H}{P}\right)^{-\phi}\frac{P_H}{P} + \alpha\left(\frac{P_F}{P}\right)^{1-\phi}\frac{1}{P_H^*}P_H^*\right] = \frac{\theta-1}{\kappa\theta}
\end{aligned}$$

where the second equality follows from (41) and the third one from (1).

The VCP case can be analyzed in a similar fashion with no changes in the conclusion.

The finding that predetermined prices optimally chosen by firms leave the expected employment at the natural level is a well known result in the monetary economics literature (e.g., ch. 3 in Woodford 2003). The above discussion suggests that the central bank needs to pay attention only to the consumption term in the expected utility $\mathbb{E}\log C$ of a representative consumer in a respective country.

In general, the central bank in the Home country will set the monetary policy rule, i.e. a mapping from (Z_s, Z_s^*) to M_s where s is a state of nature, that maximizes the expected utility of the Home representative household given the monetary policy rule chosen by the central bank in the Foreign country, and vice versa:

$$\frac{\partial\tilde{U}}{\partial M_s} = \frac{\partial\mathbb{E}\log C}{\partial M_s} = \frac{\partial\mathbb{E}\log(M/P)}{\partial M_s} = \frac{\pi_s}{M_s} - \frac{\partial\mathbb{E}\log P}{\partial M_s} = 0 \tag{54}$$

$$\frac{\partial \tilde{U}^*}{\partial M_s^*} = \frac{\pi_s}{M_s^*} - \frac{\partial \mathbb{E} \log P^*}{\partial M_s^*} = 0 \tag{55}$$

where P and P^* are functions of actual or expected values of money supplies M and M^*, productivity levels Z and Z^*, and the nominal exchange rate $\mathcal{E}$, the latter being a function of fundamentals itself according to (12).

The idea behind (54) and (55) is very simple. The central bank in each country designs the optimal plan of its future actions in any possible state of the global economy as indexed by the pair (Z_s, Z_s^*) taking as given the reaction of the Foreign central bank to shocks. When solving its optimal policy problem, each country's monetary authority admits/understands that rational economic agents in both countries take into account both policy functions in their pricing decisions. Maximization of the expected utility of the national representative individual (net of the real-money-balances term) serves as a natural/standard criterion for optimal policy.

To rephrase, optimal non-cooperative monetary policy rules are obtained as the Nash equilibrium in a game played by the two central banks. Equations (54) and (55) determine a pair of Nash equilibrium strategies M_s and M_s^* for any given state of nature (Z_s, Z_s^*). The exact functional forms that govern mappings of fundamentals M_s, M_s^*, Z_s, and Z_s^* into P_s and P_s^* depend on the choice of the export-invoicing currency. Now we turn to the analysis of the optimal monetary policy under three alternative regimes of export invoicing, one at a time: PCP, LCP, and VCP.

4.1 Producer Currency Pricing

By inspecting equations (30), (31), (38), and (39), one can notice that if the central banks set $M_s = Z_s$ and $M_s^* = Z_s^*$, or

$$m_s = z_s, \qquad m_s^* = z_s^* \tag{56}$$

respectively, then this policy choice will yield the flexible-price (the second-best) allocation (the first-best allocation can be achieved by subsidizing firms to eliminate the distortion caused by monopolistic pricing with the subsidy financed via lump-sum taxes). It turns out that these particular monetary policy rules arise as the Nash equilibrium in the game played by the two central banks.

In a sense, the optimal policy of the central bank is purely "inward-looking" as it responds only to domestic productivity shocks and ignores those happening abroad. Exactly the same policy would be optimal if the economy were closed (Corsetti and Pesenti 2008).

The reason why the inward-looking monetary policy is optimal is very intuitive. Under predetermined prices, a favorable productivity shock in the Home country will create a gap between actual and desired markups of Home producers. The monetary policy should be seeking to close this gap and stabilize markups both from domestic and foreign sales. The job of the central bank should be to focus on the former, stabilizing markups from domestic sales. The rest will be taken care of by the endogenous exchange rate response. Indeed, a monetary expansion in Home in response to the domestic productivity improvement will push real wages and work towards eliminating the wedge between actual and desired markups from domestic sales. Higher real wages will make consumers willing to accommodate a part of the extra supply of domestic goods. At the same time, according to (12), the monetary stimulus will induce a nominal depreciation in Home. This will make Foreign customers willing to consume more Home-produced goods. As a result, the level of employment in Home will remain at the natural rate.

4.2 Local Currency Pricing

Under LCP, the non-cooperative optimal policy rules happen to be

$$m_s = (1-\alpha)z_s + \alpha z_s^*, \qquad m_s^* = (1-\alpha)z_s^* + \alpha z_s \tag{57}$$

Each central bank responds to both domestic and foreign shocks in productivity. The rationale for this is the intention of the central bank to (imperfectly) stabilize markups of domestic firms applied to domestic and export prices.

Under LCP, the export prices of Home producers are predetermined in terms of the Foreign country currency. That means that fluctuations in the bilateral exchange rate will not affect the price paid by Foreign customers and, hence, affect their consumption decisions: the exchange rate passthrough into import prices is zero. A monetary expansion in Home will stimulate domestic consumption and lead. The resulting nominal depreciation of the Home currency will deteriorate the terms of trade for Foreign and lead to overemployment of labor. This provides a rationale for an expansionary monetary response in Foreign. The latter, in turn, will stimulate domestic demand and help absorb a part of extra supply from Home. The optimal monetary policy will thus react to productivity shocks originating both at home and abroad. This result is also well familiar from the NOEM literature (Corsetti and Pesenti 2008).

One can note that weight in the rule (57) assigned to the shock abroad equals α, the parameter characterizing the intensity of the home bias in consumption. In the extreme case $\alpha = 0$ when domestic consumers do not have any interest in imported goods, the economy, in effect, gets closed to foreign trade. It is natural that the optimal monetary policy becomes absolutely inward-looking: $m_s = z_s, m_s^* = z_s^*$.

4.3 Vehicle Currency Pricing

Under VCP, assuming the currency of the Foreign country serves as the vehicle currency in international trade, the non-cooperative optimal policy rules turn out to be

$$m_s = z_s \qquad m_s^* = (1-\alpha)z_s^* + \alpha z_s \tag{58}$$

The optimal policy of the Foreign country whose money serves as the vehicle currency in international trade will react to shocks originating in both countries. This makes perfect sense given that prices of all goods its residents consume are predetermined in the Foreign currency. The reason why monetary policy in Foreign should react to shocks in Home is similar to that in the LCP case. A productivity improvement in Home will induce a monetary expansion there, which will lead to nominal appreciation of the Foreign currency and a resulting deterioration in Foreign's terms of trade and overemployment of labor. A monetary expansion in Foreign appears desirable as it helps partially restore the internal balance. Home, on the other hand, does not need to react to shocks originating in Foreign since monetary expansion in Foreign will increase Home consumption through improved terms of trade without creating distortions on the labor market.

5 Contribution of Foreign monetary policy shocks into Home exchange rate variability under alternative choices of export-invoicing currency

5.1 Results

Here we introduce monetary policy shocks. We follow the literature (e.g., Christiano, Eichenbaum, and Evans 1999) interpreting them as random unintended deviations from the monetary policy rule. That means that, instead of "ideal" monetary policy rules (56), (57), and (58), the central banks will actually follow monetary policy rules that are characterized by a certain degree of noise:

$$\text{PCP:} \qquad m = z + \eta, \qquad m^* = z^* + \eta^* \tag{59}$$

$$\text{LCP:} \qquad m = (1-\alpha)z + \alpha z^* + \eta, \qquad m^* = (1-\alpha)z^* + \alpha z + \eta^* \tag{60}$$

$$\text{VCP:} \qquad m = z + \eta, \qquad m^* = (1-\alpha)z^* + \alpha z + \eta^* \tag{61}$$

Note that from now on we will omit the subscript s for the state of nature and write m instead of m_s, which should not result in any confusion.

Under PCP, provided that both central banks follow their optimal monetary policy rules, the nominal exchange rate behavior can be obtained by substituting (59) in (36):

$$\varepsilon = \frac{z - z^* + \eta - \eta^*}{1 + 2(1-\alpha)(\phi - 1)} \tag{62}$$

It follows that

$$\left.\frac{\text{Var}(\varepsilon|\eta^*)}{\text{Var}(\varepsilon)}\right|_{PCP} = \frac{\sigma_\eta^2}{2(\sigma_\eta^2 + \sigma_z^2)} \tag{63}$$

where $\sigma_\eta^2 = \text{Var}(\eta) = \text{Var}(\eta^*)$, $\sigma_z^2 = \text{Var}(z) = \text{Var}(z^*)$.

Under LCP, provided that both central banks follow their optimal monetary policy rules, the nominal exchange rate behavior can be obtained by substituting (60) in (43):

$$\varepsilon = (1 - 2\alpha)(z - z^*) + \eta - \eta^* \tag{64}$$

It follows that

$$\left.\frac{\text{Var}(\varepsilon|\eta^*)}{\text{Var}(\varepsilon)}\right|_{LCP} = \frac{\sigma_\eta^2}{2(\sigma_\eta^2 + (1-2\alpha)^2\sigma_z^2)} \tag{65}$$

Under VCP, provided that both central banks follow their optimal monetary policy rules, the nominal exchange rate behavior can be obtained by substituting (61) in (48):

$$\varepsilon = \frac{(1-\alpha)(z - z^*) + \eta - \eta^*}{1 + (1-\alpha)(\phi - 1)} \tag{66}$$

It follows that

$$\left.\frac{\text{Var}(\varepsilon|\eta^*)}{\text{Var}(\varepsilon)}\right|_{VCP} = \frac{\sigma_\eta^2}{2(\sigma_\eta^2 + (1-\alpha)^2\sigma_z^2)} \tag{67}$$

By comparing the right-hand sides of (63) – (67), it is straightforward to conclude that

$$\left.\frac{\text{Var}(\varepsilon|\eta^*)}{\text{Var}(\varepsilon)}\right|_{PCP} < \left.\frac{\text{Var}(\varepsilon|\eta^*)}{\text{Var}(\varepsilon)}\right|_{VCP} < \left.\frac{\text{Var}(\varepsilon|\eta^*)}{\text{Var}(\varepsilon)}\right|_{LCP} \tag{68}$$

Note that despite the tendency for the contribution of Foreign MP shocks to be larger under LCP compared with PCP, nothing prevents the variance of the nominal exchange rate to remain roughly constant between the regimes. To be more precise, let us assume for a moment that the variance of monetary shocks is negligibly small compared with that of productivity shocks. Given that, if parameters ϕ and α satisfy

$$\alpha = (1 - 2\alpha)(1 - \alpha)(\phi - 1) \tag{69}$$

then the variance of the nominal exchange rate Var(ε) is exactly the same in all three regimes. If we set the share of imports in consumption $\alpha = 0.2$, which is close to what we observe in the data, then we need $\phi \simeq 1.4$ to satisfy condition (69). The value of 1.4 of the cross-elasticity of substitution between imported and domestic goods lies well within the conventional range for this parameter $\phi \in [0.5, 2]$.

More generally, to guarantee that the variance of the exchange rate under PCP and LCP be equal, parameters of the model should satisfy condition:

$$\frac{\sigma_z^2}{\sigma_\eta^2} = \frac{[1 + 2(1-\alpha)(\phi - 1)]^2 - 1}{1 - (1 - 2\alpha)^2[1 + 2(1-\alpha)(\phi - 1)]^2}$$

It follows directly from equating variances of (62) and (64).

It should be mentioned that the conventional range for the cross-elasticity of substitution between domestic and imported goods ϕ is fairly wide. There is an even deeper disagreement in the literature on whether ϕ is greater or less than one. Obstfeld and Rogoff (2001) in their Six Major Puzzles paper cite a number of studies that often find the estimated ϕ (θ in their notation) to be around 5 – 6 (page 345). The fact that estimated ϕ is well above one serves as an important element in their explanation of major puzzles in international macro. For example, moderate or even relatively small transportation costs in combination with high ϕ will make domestic consumers reallocate their consumption away from imports and towards domestically produced goods thus giving rise to a home bias in consumption.

Corsetti, Dedola, and Leduc (2008) argue that the cross-elasticity of substitution parameter is likely to be less than one and this is a key assumption in their theoretical model that aims to explain the Backus – Smith puzzle. A standard model with complete markets predicts that a country whose consumption grows faster compared with the rest of the world should experience a real depreciation. The data show the opposite. Corsetti, Dedola, and Leduc (2008) drop the complete markets assumption and introduce very persistent productivity shocks, domestic and foreign, as the only source of fluctuations. In order to shut off the expenditure switching between domestically produced goods and imports they need to make the two kinds of goods complementary, which requires low ϕ. Their empirical estimates suggest that ϕ indeed appears to be safely less than one, which makes their explanation of the Backus – Smith puzzle viable.

It is worth noting that whether imports are substitutes or complements with respect to domestically produced goods does not affect our conclusions about contribution of Foreign monetary shocks into exchange rate variability. Indeed, neither of expressions (63), (65), and (67) contains ϕ. No matter if imports and domestic goods are substitutes or complements, the contribution of monetary shocks abroad is higher under LCP and lower under PCP with VCP being in between. The value of ϕ, however, does affect the size of exchange rate variability. In particular, our model is able to produce roughly equal size of exchange rate volatility across different regimes (recall the last column of Table 1) only if ϕ is above one (substitutability between domestically produced and imported goods).

5.2 Discussion

Here we offer an intuitive explanation to our findings. We focus on two polar cases, PCP and LCP. Let us start with expressions (36) and (43) that relate exchange rates to the fundamental $m - m^*$, money supply differential. If the monetary policy were the same in the two regimes then the exchange rate would be more volatile under LCP unless $\phi \leq 1$. The value $\phi = 1$ corresponds to the unit cross-elasticity of substitution between Home and Foreign goods. In this case, consumers in each country spend fixed fractions $1 - \alpha$ and α on goods produced domestically and abroad accordingly. Values in the range $\phi < 1$ look unrealistically as they suggest that domestic and imported goods look more like complements. It is more plausible to assume that $\phi > 1$ meaning that domestic and imported goods are considered by consumers more like substitutes. In that respect, we follow Obstfeld and Rogoff (2001) who summarize the evidence and conclude that setting $\phi > 1$ is more reasonable and safer. Comparing (36) and (43) one can verify that a unit increase in $m - m^*$ will induce greater exchange rate response under LCP than under PCP. The reason is that, given $\phi > 1$, the expenditure switching effect is present under PCP. An increase in m, holding m^* fixed will raise aggregate demand in Home and, hence, will raise demand for Home and Foreign goods. Under LCP, the prices are held constant within the period when the shock arrives and, in order to keep the balanced trade condition (10), the exchange rate must depreciate one to one, which leads to equation (36). If the expenditure switching effect is in place as under PCP, then endogenous depreciation will deteriorate the Home country's terms of trade and force its residents to consume less Foreign goods. This will reduce the size of the exchange rate response required to balance international trade.

One can argue that the contribution of monetary shocks to the exchange rate variability under LCP being higher than under PCP is just a trivial consequence of the expenditure switching effect discussed above (compare formulas (36) and (43)). That reasoning would imply, however, that exchange rates should be more volatile under LCP than under PCP. Inspecting the third column of Table 1 suggests that volatility does not decrease as we move from countries with low ERPT to those with high ERPT (i.e. from LCP to PCP), with Canada being a remarkable exception. We believe instead that the monetary policy design matters. Specifically, if the central bank takes into account imports-invoicing practices established in its country (as suggested by numerous studies in the NOEM literature), then this would affect the shape of monetary policy reaction function. Indeed, under PCP, central banks should leave more flexibility to the exchange rate that will act as a shock absorber. Under LCP, when the expenditure switching effect is not operational, the noncooperative optimal policy of the central banks in both countries will tend to limit exchange rate fluctuations as a way to stabilize markups of resident firms servicing both markets. Thus the fact that the monetary policy is a function of the existing imports-invoicing practices will countervail the consequences of the expenditure switching effect that make the exchange rate look more volatile under LCP than under PCP (formulas (36) and (43)). On balance, as it was already mentioned, the exchange rate volatility may well be similar under both regimes (compare formulas

(62) and (64)).

5.3 Calibration

Now, let us try calibrate (68) to see if we are able to match the data. Set $\alpha = 0.2$ as above and try two options: (i) $\sigma_\eta^2 = \sigma_z^2$ (MP shocks are as volatile as productivity shocks); (ii) $\sigma_\eta^2 = 0.5\sigma_z^2$ (MP shocks are half as volatile as productivity shocks). The calibration results are shown in Table 2.

[TABLE 2 HERE]

Comparison of Tables 1 and 2 suggests that our simple model does quite a decent job explaining the effect of the export-invoicing currency choice on the contribution of foreign monetary policy shocks to variability of the nominal exchange rate.

A typical VCP country in our sample is Australia. The correlation between the price of its non-commodity imports and the exchange rate of the Australian dollar against the US dollar is 0.76, which reflects high pass-through of exchange rate changes into import prices. The contribution of the US MP shocks to the variability of the Australian dollar is quite low – 3.9%.

A typical LCP country in our sample is Switzerland. The correlation between the price of its non-commodity imports and the exchange rate of the Swiss frank against the US dollar is 0.18, which reflects low pass-through of exchange rate changes into import prices. The contribution of the US MP shocks to the variability of the Swiss frank is remarkably high – 44.3%.

6 Conclusion

Although monetary shocks have been considered as a major determinant of exchange rate behavior, the literature did not pay much attention to the fact that their contribution to exchange rate variability differs remarkably across currencies. Our study documents this pattern and offers a theoretical explanation to it.

We show empirically that the contribution of the US monetary policy shocks to forecast error variance decomposition of bilateral exchange rates to the US dollar is negatively correlated with the degree of exchange rate pass-through for a respective country. This suggests that monetary shocks in the US tend to explain a higher fraction of exchange rate variability for those countries whose imports prices are less correlated with the US dollar.

The key element in our theoretical explanation is what currency is used for invoicing imports. We do not model such a choice in our model but simply treat it as exogenous as in Goldberg and Tille (2009). We look at various alternatives, one at a time, such as producer, consumer and vehicle currency pricing. To explain the observed empirical regularity, we employ a known result from the literature that optimal monetary policy depends on the choice of imports-invoicing currency. This choice will thus affect both the degree of exchange rate pass-through and the monetary policy response. Different shape of monetary policy reaction function will result in different contribution of monetary shocks to the exchange rate dynamics.

We deliberately keep our model very simple as Goldberg and Tille (2009). This allows us to obtain closed-form solutions and tractability of results. It comes at a price however. In particular, we are bounded to deal with rather stylized monetary policy rules and cannot analyze more conventional Taylor rules and alike. This could be done within a fully-fledged dynamic stochastic general equilibrium (DSGE) framework, which we leave for future research.

We see three directions for future research that follow from our study. First, it looks worth connecting the role of monetary shocks in exchange rate variability with primitives of imports-invoicing currency choice (e.g., as discussed in Goldberg and Tille 2008). Second, replicating the exercise in a fully-fledged DSGE framework with explicit intertemporal links and trade in financial assets should also be helpful both as a robustness check and a way to see if the new features will reshape our findings obtained within the simplest possible framework. Finally, an extension of the model to the case of multiple shocks (e.g., preference shocks in addition to the productivity shocks) also looks as a natural further step.

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Table 1: Exchange rate passthrough into import prices and contribution of US monetary policy shocks to exchange rate variability

Country	$\frac{\text{Var}(\Delta \ln \mathcal{E} \mid \eta^{USMP})}{\text{Var}(\Delta \ln \mathcal{E})}$	corr$(\Delta \ln P^{IMP}, \Delta \ln \mathcal{E})$	Std.Dev.$(\Delta \ln \mathcal{E})$
Australia	0.039	0.76	0.024
New Zealand	0.075	0.65	0.026
Canada	0.126	0.81	0.011
Sweden	0.262	0.24	0.025
United Kingdom	0.287	0.28	0.025
Japan	0.305	0.32	0.028
Norway	0.318	0.07	0.024
Denmark	0.415	-0.18	0.027
Switzerland	0.443	0.18	0.031

Note: For each country listed in the first column, the second column of the table shows the average fraction of US monetary shocks in the forecast error variance decomposition (FEVD) of its bilateral nominal exchange rate to the US dollar for horizons up to 12 months. The third column reports the estimated elasticity of the aggregate price of non-commodity imports with respect to nominal exchange rate to the US dollar. The fourth column contains estimated volatilities of respective bilateral exchange rates. Sample period is January 1973 through December 2003. FEVDs are estimated using Structural Factor Augmented Vector Autoregression approach (SFAVAR). See section "Empirical Motivation" in the main text for more details.

Table 2: Calibration results: Contribution of Foreign monetary policy shocks to variability of Home nominal exchange rate under alternative regimes

α	$\frac{\sigma_z^2}{\sigma_\eta^2}$	PCP	VCP	LCP
0.2	1	0.25	0.30	0.37
0.2	2	0.17	0.22	0.29
0.2	3	0.13	0.17	0.24
0.2	4	0.10	0.14	0.20
0.3	1	0.25	0.34	0.43
0.3	2	0.17	0.25	0.38
0.3	3	0.13	0.20	0.34
0.3	4	0.10	0.17	0.30

Note: Entries of columns 3 to 5 show the fraction of Foreign monetary policy shock in the variance of the bilateral nominal exchange rate for three different regimes of exports invoicing: producer currency pricing (PCP), vehicle currency pricing (VCP), and local currency pricing (LCP). Model parameter α characterizes the degree of home bias in consumption with smaller values indicating a stronger bias. Parameter $\frac{\sigma_z^2}{\sigma_\eta^2}$ is the relative variance of shocks in productivity (z) and monetary policy (η).

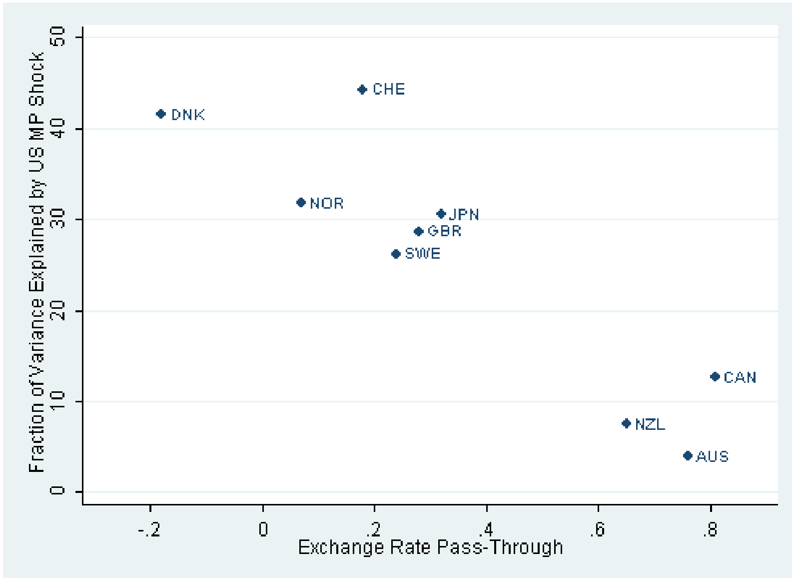

Figure 1: Exchange rate passthrough into import prices and contribution of US monetary policy shocks to exchange rate variability

Note: The variable on the horizontal axis is exchange rate pass-through estimated as the elasticity of a country's non-commodity imports to its bilateral nominal exchange rate to the US dollar. The variable on the vertical axis is the average fraction of US monetary shocks in the forecast error variance decomposition (FEVD) of its bilateral nominal exchange rate to the US dollar for horizons up to 12 months estimated using Structural Factor Augmented Vector Autoregression approach (SFAVAR) (see section "Empirical Motivation" in the main text for more details). Sample period is January 1973 through December 2003. Each observation on the scatterplot corresponds to a country.