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Abstract

Warm-glow refers to other-serving behavior that is valuable for the actor *per se*, apart from its social implications. We provide axiomatic foundations for warm-glow by viewing it as a form of preference for larger choice sets, in the sense of the literature on freedom of choice. Specifically, an individual who experiences warm-glow prefers the freedom to be selfish: she values the availability of selfish options even if she plans to act unselfishly. Our theory also provides foundations for empirically distinguishing between warm-glow and other motivations for prosocial behavior. The implied choice behavior subsumes Riker and Ordeshook (1968) and Andreoni (1990).

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1 Introduction

In the last decades, there has been a surge of interest in models of prosocial behavior which depart from the traditional approach that explains such behavior with the classical notion of altruism. It has been argued, for instance, that charitable donations may be motivated by a desire for status, acclaim or self-satisfaction (e.g., Arrow, 1972; Becker, 1974; Andreoni 1989, 1990; Glazer and Konrad, 1996; Bénabou and Tirole, 2006). In turn, in the voting literature, a remarkable example is the seminal paper of Riker and Ordeshook (1968), which maintains that citizens may be perceiving the act of voting as a civic duty, independently of the social consequences of their individual decisions. More recently, Coate and Conlin (2004), and Feddersen and Sandroni (2006) envision citizens who deem voting as an ethical duty whenever this is justified from a rule utilitarian perspective.

Conceptually, the non-altruistic decision makers considered in these alternative models can be further classified into at least two types: those who are motivated by intrinsic pleasure associated with a particular form of prosocial behavior, which is often referred to as *warm-glow* motivation; and those who perceive such behavior as an unpleasant obligation because acting selfishly might lead to even more unpleasant experiences such as losing self-respect, feeling guilty or ashamed.[1]

As noted by Andreoni (2006) and Diamond (2006), understanding the real motivation behind a given form of prosocial behavior may have profound implications in welfare analysis. For instance, an agent who enjoys the act of giving *per se* would typically be worse off upon an increase in taxes for the (public) provision of a public good if government spending happens to fully crowd out private contributions, and thereby, leave unaltered the final allocation. In turn, non-altruistic agents of the second type that we mentioned above might be better off if paying taxes as a le-

[1] Dillenberger and Sadowski (forthcoming) suggest that there is a further distinction between guilt and shame, as the former is a private experience while the latter requires publicly observable behavior. Glazer and Konrad (1996) note that an analogous distinction exists between warm-glow motivation and one's desire to signal her status. In the present paper we abstract from this observability issue. However, we shall discuss Dillenberger and Sadowski in more depth momentarily, as it has important reflections on the place of the present paper in the literature.

gal obligation is preferable to "voluntarily" (but reluctantly) contributing to the public good because of the unpleasant feelings associated with not doing so. Finally, in stark contrast with both of these types, a purely altruistic agent, who solely cares about the implied allocation, would certainly be neutral against such a policy.

Putting aside the difficulties regarding the two non-altruistic types, even well-established, behavioral distinctions between pure altruism and warm-glow motivation are often quite involved and demand some assumptions on the form of utility functions. Notably, in his seminal papers, Andreoni (1989, 1990) proposes a warm-glow model of public good provision that can improve upon the predictions of the classical model of altruism (which seem to be distant from empirical evidence).[2] However, Andreoni's analysis relies on the assumption that agents' private consumption as well as their contribution to public good are strictly increasing functions of their wealth. This assumption, in turn, rules out quasi-linear utility functions, among others.

Just as in Andreoni (1989, 1990), in the present paper we focus on warm-glow driven behavior. Our starting point is that a typical person would attach an intrinsic value to an other-serving action only when this is an act of free will, but not if she is somehow forced to act unselfishly.[3] This observation, which would seem self-evident on many occasions, points to a connection with a separate line of research, pioneered by Sen (1985, 1988), that is concerned with the measurement of freedom of choice associated with menus (i.e., choice sets). Taking as primitive a preference relation over menus, in a nutshell, this literature maintains that a large set of alternatives, which offers a certain degree of freedom of choice, may be more valuable for the decision maker than *the* alternative that she would select from that set (Sen, 1985, 1988; Puppe, 1996; Sugden, 1998, among others).[4]

Inspired by this literature, we propose a theory of preference relations

[2] We elaborate more on Andreoni's findings in Section 4.1.

[3] Indeed, in the aforementioned literature on prosocial behavior, the word "giving" often refers to a voluntary act.

[4] This contention contrasts with the "preference for flexibility" approach that focuses on the *instrumental* value of larger menus driven by choice uncertainty (Kreps, 1979; Dekel, Lipman, and Rustichini, 2001). In Section 6, we relate the present paper to Kreps (1979).

over *menus* of social alternatives that provides foundations for a class of warm-glow models, by relating the intrinsic value of an other-serving action to the presence of *freedom to be selfish*, the option of "not giving." As we shall clarify momentarily, a major advantage of the menu choice approach is that it arms us with clear-cut distinctions between warm-glow giving, pure altruism and giving as an unpleasant obligation.

For a generic social allocation x in $\mathbb{R}^k_+$, let the first component x_1 stand for the private consumption of the decision maker in question. In its simplified form, our main finding is a characterization of preference relations over menus of efficient allocations A that can be represented with an (indirect) utility function V which takes the form

$$V(A) = \max_{x \in A} U(x, \max_{y \in A} y_1 - x_1)$$

for a function $U : \mathbb{R}^k_+ \times \mathbb{R}_+ \to \mathbb{R}$. The term $\max_{y \in A} y_1 - x_1$ is the private cost that the decision maker incurs if she decides to select the allocation x from the menu A. In turn, we interpret $U(x, \max_{y \in A} y_1 - x_1)$ as the utility of selecting x from A. If the function U is constant in its last argument, the representation reduces to standard utility maximization. In general, however, U is weakly increasing in its last argument, implying that the utility of selecting x from A increases with the difference $\max_{y \in A} y_1 - x_1$. Hence, the act of selecting a particular allocation x becomes more enjoyable as the maximum possible private consumption increases, and the least satisfying menu that admits the choice of x is the one that offers no option but x.

It seems reasonable to interpret the maximum possible private consumption offered by a menu as a measure of how selfishly the decision maker could have acted if she were not to select a given allocation. When viewed in this way, our representation theorem establishes a tight relation between the notions of warm-glow giving and freedom to be selfish, as we suggested earlier.

The representation implies that given a menu A as above, the decision maker would select the allocation that maximizes $U(x, \max_{y \in A} y_1 - x_1)$ over A. In Section 4.1, we show that this choice behavior subsumes Andreoni's (1989, 1990) warm-glow model. In Section 4.2, we reinterpret the notion of an allocation as a vector that lists the (expected) material payoffs of the individuals in the society, which significantly extends the scope of our representation. In Section 4.3, we apply this extended representation to the problem of voter turnout in large elections, and show that the associated behavior subsumes the civic-duty model of Riker and Ordeshook (1968). Thereby, we lay foundations for two prominent models from different subfields of social choice theory.

Why does the menu choice approach help us distinguish the aforementioned motivations? The answer is simple enough to be explained here: Firstly, those agents who enjoy giving as an act of free will exhibit preference for larger menus. In particular, as we model here, they tend to enjoy menus which facilitate a stronger perception of freedom to be selfish. By contrast, purely altruistic agents are neutral against the size of the choice set they face, because they solely care about the final outcome. In turn, an agent who perceives giving as an unpleasant obligation would exhibit preference for smaller menus that restrict other-serving options, for upon removal of such options the agent could select self-serving options as she actually wishes, without experiencing any negative feelings.

Economic agents' preferences over menus can be detected in consumption-saving problems that we routinely encounter. In Section 4.1.1, we provide an example of a bequest giving problem which illustrates this point. The temporal utility functions in this example are quasi-linear, which is at odds with Andreoni's analysis as we noted earlier. Consequently, for any given amount of saving, the giving behavior of the purely altruistic agents in period 2 coincides with that of the agents motivated by warm-glow (despite the presence of a policy variable that influences intergenerational income distribution). Yet, since the two types evaluate differently budget sets which become available in period 2, their saving behavior in period 1 is also distinct, pointing to added descriptive power of the menu choice approach.

Another important issue that has attracted attention in the literature is that providing extrinsic incentives for prosocial behavior, say by means of a government policy, may actually crowd out intrinsic motivations. Bénabou and Tirole (2006) cite mounting evidence that supports this observation, and provide a suitable game-theoretic model. The present paper offers an alternative perspective on this phenomenon within the framework of individual decision making problems. Specifically, our representation implies that a policy that decreases the payoff associated with the most selfish option, such as a fine on selfish behavior, may crowd out intrinsic motivations by effectively reducing the value of giving as an act of free will. In Section 4.3, we shall readdress this issue in relation to fines on abstention in elections.

In the decision theory literature, the closest paper to ours is due to Dillenberger and Sadowski (forthcoming), which provides a dual theory of preference relations over menus of social allocations. Their focus is a negative form of prosocial behavior driven by shame associated with selfish acts.[5] Accordingly, their main representation result describes an agent who exhibits preference for *smaller* menus, which provides a formal basis for our related discussion above. The present paper has further differences in terms of the implied choice among social allocations for a fixed menu. In fact, a most ethical option acts as a reference point in the calculus of shame proposed by Dillenberger and Sadowski. Specifically, the utility associated with the choice of an allocation is modeled as a decreasing function of the distance between that allocation and the most ethical option. However, selfish and ethical modes of behavior may well coincide because the ethical behavior is determined by maximization of a welfare function (that strictly increases in the decision maker's material payoff). For instance, in a large election, the corresponding agent may think that abstention is the ethical option, because voting is unlikely to influence others' welfare but it incurs significant private costs. By contrast, our model includes those agents who always attach an intrinsic value to costly actions that might help others, as in Riker and Ordeshook (1968) and Andreoni (1989, 1990).

Another related paper is Cherepanov, Feddersen, and Sandroni (2011), who propose an abstract model of choice among alternatives, holding fixed the menu that the decision maker faces. The main point of Cherepanov

[5] Dillenberger and Sadowski focus on a two person set-up. In order to facilitate the "shame" interpretation, they also assume that the recipient can observe the decision maker's behavior. However, their key ideas appear to be applicable in a multi-person set-up, and general enough to capture unselfish behavior driven by other forms of negative emotions (such as guilt).

et al. (2011) is that, unlike pure altruism, the relevant forms of non-altruistic behavior may lead to violations of the Weak Axiom of Revealed Preferences. In concert with this observation, in our model the utility function that governs the choice of allocations is menu dependent.[6] Yet, the choice behavior that corresponds to our model is not within the scope of Cherepanov et al. (2011), because that paper models "warm-glow payoff" as a fixed number that does not depend on the menu that the agent faces or the allocation that leads to the warm-glow experience. By contrast, a crucial feature of our approach is that the warm-glow payoff increases with the private cost associated with the allocation in question. As we noted earlier, this, in turn, implies a trade-off between extrinsic and intrinsic motivations. Moreover, and more important, the framework of Cherepanov et al. (2011) is not suitable for distinguishing the negative and positive types of non-altruistic agents that we discussed above. (More on this point in Section 7.)

In the next section we introduce our model, while Section 3 contains a behavioral characterization of our warm-glow representation. In Section 4, we discuss the applications of our representation. Section 5 is devoted to a choice theoretic study of implied second stage behavior for a fixed menu. Section 6 relates our representation to that of Kreps (1979), while Section 7 concludes. All proofs and some other supplementary material are relegated to appendices.

2 The Model

We consider a decision maker in a society. There is one private good and at most one public good.[7] Set $X := \mathbb{R}^k_+$ where $k \geq 2$ is an integer. We refer to an element $x := (x_1, ..., x_k)$ of X as an **allocation.** The first component x_1 stands for the private consumption of our decision maker. In turn, any other component x_i represents either the private consumption of another agent i or the amount of the public good (if it exists). Thus, k equals the number of consumption variables related to the decision problem in question and it can exceed the cardinality of the society at most by one.

[6] Our model is, however, *rational* as a theory of preference relations over *menus.*

[7] In Section 4.2, we discuss an extension that allows for multiple private and public goods.

In what follows, **the agent** refers to our decision maker.

The agent's preferences are described by a binary relation $\succsim$ over a collection of subsets of X. Let $\mathcal{A}$ denote this collection of sets, which will be specified momentarily. Each set in $\mathcal{A}$ represents a **menu**, that is, a set of allocations from which the agent will make a choice in a subsequent stage. Our analysis of $\succsim$ will build upon a suitable interpretation of how the agent might be planning to behave in the second stage. Then, in Section 5.2, we will explicitly model the associated second stage choice behavior.

As usual, for a set $A \subseteq X$, we say that an element x of A is **efficient** (**in** A) if there does not exist a $y \in A$ such that $y_i \geq x_i$ for $i = 1, ..., k$ with strict inequality for some i. The **Pareto frontier of** A, denoted as $\mathcal{P}(A)$, consists of all efficient allocations in A. In turn, a pair of distinct allocations x, y are **Pareto incomparable** if $\mathcal{P}(\{x, y\}) = \{x, y\}$.

Next, we define $\mathcal{A}$ as the collection of all sets $A \subseteq X$ which satisfy the following two properties:

(i) $\mathcal{P}(A)$ is a nonempty, compact set.

(ii) There exists a $y^* \in \mathcal{P}(A)$ such that $x_i \geq y_i^*$ for every $x \in \mathcal{P}(A)$ and $i = 2, ..., k$.

Recall that the Pareto frontier of a nonempty, compact subset of X is nonempty (and bounded). Property (i) rules out the cases in which the Pareto frontier of such a set is not closed.[8] In a slightly more general fashion, a bounded subset of X that is not closed also qualifies if its Pareto frontier is closed. Note that when k equals 2, property (ii) trivially follows from (i). In this case, there is a unique efficient allocation that maximizes x_1, which is at the same time the unique minimizer of x_2 among efficient allocations. Property (ii) filters higher dimensional sets that have an analogous feature: *For each $A \in \mathcal{A}$, there exists a unique allocation $y^*(A)$ in $\mathcal{P}(A)$ such that $y_1^*(A) \geq x_1$ for every $x \in \mathcal{P}(A)$ (or equivalently, for every $x \in A$). Moreover, $y^*(A)$ is also the unique allocation in $\mathcal{P}(A)$ that satisfies (ii).* (We omit the proof of this simple observation.) Naturally, we view $y^*(A)$ as the **most selfish** option in the menu A, as it maximizes the agent's private consumption. The crucial implication of (ii) is that

[8] See Arrow, Barankin, and Blackwell (1953) for an example of a compact, convex subset of an Euclidean space that has a non-closed Pareto frontier.

in the second stage, if the agent decides to select an efficient allocation x with $x_1 < y_1^*(A)$, the private consumption that she thereby gives up is converted into public good or private consumption of some other agents, without reducing the goods available to any other agent. Thus, $y^*(A)$ can also be seen as the *least generous* option available to the agent, in terms of her influence on others' consumption.

In applied models of charity, the agent often has an initial endowment of the private good, and the choice set A in question consists of all allocations that the agent can obtain by distributing her endowment among the k consumption variables, given other factors such as government transfers and subsidies, other agents' behavior, prices, and the technology that transforms the private good into public good. Such choice sets are within the scope of our analysis, for by privately consuming all her endowment, typically, the agent can maximize her private consumption while minimizing her contributions to all other variables.

In passing, we define a subcollection of $\mathcal{A}$ that is of particular importance:

$$\mathcal{A}_{\mathcal{P}} := \{A \in \mathcal{A} : \mathcal{P}(A) = A\}.$$

This is the collection of all sets in $\mathcal{A}$ which consist of efficient allocations. It is worth noting that if a set A belongs to $\mathcal{A}_{\mathcal{P}}$, then any nonempty, closed subset of A that contains $y^*(A)$ also belongs to $\mathcal{A}_{\mathcal{P}}$. Another useful observation is that $\mathcal{A}_{\mathcal{P}}$ contains $\{x\}$ for any allocation x.

3 The Representation Theorem

In this section, we formally introduce our representation and its behavioral characterization. We start with a standard rationality requirement:

Weak Order (A1). $\succsim$ is a complete and transitive binary relation on $\mathcal{A}$.

The next axiom states that increasing the size of the Pareto frontier of a menu cannot harm the agent.

Pareto Monotonicity (A2). For any $A, B \in \mathcal{A}$, if $\mathcal{P}(A) \supseteq \mathcal{P}(B)$, then $A \succsim B$.

Notice that $\mathcal{P}(A) \in \mathcal{A}_{\mathcal{P}}$, that is, $\mathcal{P}(\mathcal{P}(A)) = \mathcal{P}(A)$ for any $A \in \mathcal{A}$. Hence, (A2) immediately implies $\mathcal{P}(A) \sim A$ for any menu A. Therefore, our remaining axioms focus on those sets in $\mathcal{A}_{\mathcal{P}}$.

A crucial assumption in the standard model of menu choice is the following:

$$A \cup B \sim A \quad \text{or} \quad A \cup B \sim B. \tag{1}$$

The underlying idea is that if the agent can perfectly anticipate which alternative she will select from $A \cup B$, she would evaluate $A \cup B$ solely with that particular alternative (which must belong to A or B). Thus, property (1) describes a purely *instrumentalist* decision maker who views a menu solely as a means toward her final choice. On the other hand, as the literature on freedom of choice maintains, a menu may be valuable *per se,* independently of the alternative that will eventually be selected. We shall therefore allow a menu to be strictly better than any of its subsets. While this is a starting point of all models on freedom of choice, our representation requires an axiom that relaxes (1) in a special way:

Weak Instrumentalism (A3). Let A, B be nonempty, compact sets such that $A \cup B \in \mathcal{A}_{\mathcal{P}}$. If $y^*(A \cup B) \in A \cap B$, then $A \cup B \sim A$ or $A \cup B \sim B$.

To gain insight, consider a set of three allocations $C := \{x, y, z\}$ that belongs to $\mathcal{A}_P$, and let y be the most selfish option in C. Then (A3) implies $C \sim \{x, y\}$ or $C \sim \{y, z\}$. If only the former equivalence holds, in violation of (1) we may still have $\{y, z\} \prec C \succ \{x\}$. We interpret such preference as follows: The agent will select x from the set C. This, in itself, is a reasonable explanation of the pattern $\{y, z\} \prec C$. Moreover, by construction, x is not the most selfish option in C. Hence, by selecting x from C the agent experiences warm-glow. If, however, x were the only available option, selecting x would merely be a necessity which would not cause a warm-glow experience. In other words, selecting x from C with her free will is more valuable for the agent than the mere consumption of x, which explains $C \succ \{x\}$.

On the other hand, the above interpretation is also compatible with $C \succ \{x, y\}$. After all, the menu C provides a higher degree of freedom compared to $\{x, y\}$. Therefore, in principle, selecting x from C could be more enjoyable than selecting x from $\{x, y\}$. Property (A3) rules out such cases. Intuitively, this axiom requires that the strength of warm-glow experience should only depend on the most selfish option and the alternative that the agent will eventually select.[9] In line with this, Claim 2 in Appendix C shows that every menu A in $\mathcal{A}_{\mathcal{P}}$ contains an allocation x such that $A \sim \{x, y^*(A)\}$. Of course, here, x is interpreted as the allocation that the agent will select from A.

While (A3) gives a special role to the most selfish option as a determinant of the strength of warm-glow experience, the axiom is silent about the nature of this relation. How does the agent's welfare depend on the most selfish option, holding fixed the alternative which will be selected? Our next axiom answers this question.

Monotone Warm-Glow (A4). Let $\{x, y\} \in \mathcal{A}_{\mathcal{P}}$ and $y_1 \geq x_1$. If $\{x\} \prec \{x, y\} \succ \{y\}$, then $\{x, z\} \succsim \{x, y\}$ for all $z \in X$ such that $z_1 \geq y_1$ and $\{x, z\} \in \mathcal{A}_{\mathcal{P}}$.

Given a pair of allocations x, y as in this axiom, $\{x\} \prec \{x, y\} \succ \{y\}$ tells us that the agent would select x from $\{x, y\}$, as we discussed earlier. Suppose that holding the final choice fixed, the strength of warm-glow experience depends only on the maximum possible private consumption and is an increasing function of it. Then, replacing y with an allocation z as in (A4) could only make the agent better off. Indeed, the agent can always select x from $\{x, z\}$, which is better than selecting x from $\{x, y\}$ by assumption. This is the content of the axiom.

Remarkably, (A4) implies a notion of warm-glow that does not depend on how the agent's choice of an allocation compares with other available allocations in terms of other agents' welfare. For instance, if $x_i - y_i$ is substantially larger than $x_i - z_i$ for $i = 2, ..., k$, selecting x over y can be viewed as a much more generous act, in terms of what the others receive, compared to selecting x over z. This, in turn, could reduce the appeal of $\{x, z\}$ relative to $\{x, y\}$, contrary to (A4). In Section 7, we discuss an extension of the present model that relates the strength of warm-glow experience to others' welfare. It should be noted, however, that the (more)

[9] In this regard, our approach is akin to several other papers, including Gul and Pesendorfer (2001), Noor and Takeoka (2011) and Dillenberger and Sadowski (forthcoming), albeit these papers are concerned with modeling preference for smaller menus. It is also worth noting that, to the best of our knowledge, (A3) is a novel axiom, but in the papers that we just mentioned an analogous property can be deduced from other axioms. Needless to say, our axioms are independent of each other (see Appendix A).

egoistic notion of warm-glow which corresponds to (A4) has found some important applications in the literature (see Sections 4.1 and 4.3), perhaps because the classical notion of altruism is already based on one's concern for others.

The following axiom rules out negatively interdependent preferences over singletons. Throughout the remainder of the paper, $\geq$ stands for the usual partial order on a Euclidean space.

Nonnegative Interdependence (A5). $\{x\} \succsim \{y\}$ for any $x, y \in X$ with $x \geq y$.

It is worth noting that (A5) also allows for a purely selfish attitude over singletons as would be represented by the function $\{x\} \rightarrow x_1$.

Finally, we assume that $\succsim$ is continuous on $\mathcal{A}_{\mathcal{P}}$ with respect to the Hausdorff metric[10] (induced by the Euclidean norm).

Continuity (A6). The sets $\{B \in \mathcal{A}_{\mathcal{P}} : B \succsim A\}$ and $\{B \in \mathcal{A}_{\mathcal{P}} : A \succsim B\}$ are closed in $\mathcal{A}_{\mathcal{P}}$, for each $A \in \mathcal{A}_{\mathcal{P}}$.

The next definition formalizes our representation notion.

Definition 1. A binary relation $\succsim$ on $\mathcal{A}$ admits a **warm-glow representation** if there exists a function $U : X \times \mathbb{R}_+ \rightarrow \mathbb{R}$ that satisfies the following two properties:

(i) $U(x, \cdot)$ is weakly increasing on $\mathbb{R}_+$ for each $x \in X$, and $U(\cdot, 0)$ is weakly increasing on X.

(ii) For each $A, B \in \mathcal{A}$,

$$A \succsim B \quad \text{iff} \quad \max_{x \in \mathcal{P}(A)} U\left(x, y_1^*(A) - x_1\right) \geq \max_{x \in \mathcal{P}(B)} U\left(x, y_1^*(B) - x_1\right).$$

We say that such a function U is a **utility index** for $\succsim$.[11]

The representation suggests that, when faced with a menu A in the second stage, the agent will follow a two step choice procedure. First, she will

[10] The Hausdorff distance between two nonempty, compact sets $A, B \subseteq \mathbb{R}^k$ equals the maximum of $\max_{x \in A} \min_{y \in B} \|x - y\|$ and $\max_{y \in B} \min_{x \in A} \|x - y\|$, where $\|\cdot\|$ stands for the Euclidean norm.

[11] On a technical note, let us emphasize that (when $\succsim$ is reflexive) this definition requires the existence of $\max_{x \in A} U\left(x, y_1^*(A) - x_1\right)$ for any $A \in \mathcal{A}_{\mathcal{P}}$. Of course, in practice, this forces one to demand some continuity properties from U, as we shall do momentarily.

eliminate those alternatives in A which are not efficient. Then, she will select an efficient allocation which maximizes the function $U(x, y_1^*(A) - x_1)$ over $\mathcal{P}(A)$. The additional utility of selecting x from $\mathcal{P}(A)$ relative to the mere consumption of x is given by $U(x, y_1^*(A) - x_1) - U(x, 0)$. We view this difference as the **warm-glow payoff** associated with the former act. A crucial feature of the representation is that the warm-glow payoff is a weakly increasing function of the difference between the maximum possible private consumption that the agent can attain and her actual choice of private consumption. We denote by λ the last argument of U, which corresponds to this difference.

Remark 1. Our representation restricts the calculus of warm-glow to efficient allocations, so that, perforce, the agent does not get a warm-glow payoff from "burning" her endowment of private good or reducing the consumption of everyone. By contrast, given a menu $A \in \mathcal{A}$, the allocation that solves the problem $\max_{x \in A} U(x, y_1^*(A) - x_1)$ may well be inefficient, unless U is constant in λ (see Appendix A, Example A3).

Let us now define $X_0 := \{y \in X : y_i = 0 \text{ for } i = 2, ..., k\}$. If it belongs to X_0, an efficient allocation in a given menu can only be the most selfish option. Since, according to our representation, the agent does not experience warm-glow by selecting such allocations, the behavior of $U(y, \lambda)$ for $y \in X_0$ and $\lambda > 0$ has no implications on the preference relation $\succsim$. The existence of a utility index which is continuous over this irrelevant part of its domain is a technically challenging issue, which seems to be of limited interest. We shall address this matter in Appendix B and focus here on utility indices that are continuous over the relevant set, $(X \times \mathbb{R}_+) \backslash (X_0 \times \mathbb{R}_{++})$.

We are now ready to state our representation theorem, which is the main finding of the paper.

Theorem 1. *A binary relation* $\succsim$ *on* $\mathcal{A}$ *satisfies* (A1)-(A6) *if, and only if, it admits a warm-glow representation with a utility index that is continuous over* $(X \times \mathbb{R}_+) \backslash (X_0 \times \mathbb{R}_{++})$.

Some particular forms of utility indices are of special interest. First, if the utility index U is constant in λ for each x, then the agent never experiences warm-glow. In this case, the representation reduces to classical utility maximization: $A \succsim B$ iff $\max_{x \in A} U(x, 0) \geq \max_{x \in B} U(x, 0)$ for every $A, B \in \mathcal{A}$. Of course, $\succsim$ admits such a utility index if and only if (1) holds for any pair of menus. While this is a special case of Theorem 1, curiously, the particular utility index that we construct in the proof of Theorem 1 is not necessarily constant in λ, even if property (1) holds for any pair of menus. The difficulty stems from the fact that when $\{x\} \prec \{x, y\} \sim \{y\}$ and $y_1 > x_1$, the representation does not impose a tight restriction on the value $U(x, y_1 - x_1)$. Roughly speaking, in such instances all we know is that $U(x, y_1 - x_1)$ must be between $U(x, 0)$ and $U(y, 0)$. From this perspective, the classical representation corresponds to setting $U(x, y_1 - x_1) := U(x, 0)$, whenever "one can."

In the next proposition, we establish the existence of a general utility index of this sort. While it provides a tight characterization of the role that property (1) plays, this approach leads to additional continuity issues. Specifically, the obtained utility index is only upper semi-continuous if the agent violates (1) at some instances. In what follows, we say that an allocation x is **critical** if there exists an allocation y such that $\{x, y\} \in \mathcal{A}$, $y_1 > x_1$ and $\{x\} \prec \{x, y\} \succ \{y\}$.

Proposition 1. *Let $\succsim$ be a binary relation on $\mathcal{A}$ that satisfies* (A1)-(A6). *Then, $\succsim$ admits a utility index $U : X \times \mathbb{R}_+ \to \mathbb{R}$ such that:*

(i) *$U(x, \cdot)$ is constant unless x is the limit of a sequence of critical allocations.*

(ii) *$U(x, \cdot)$ is not constant whenever x is a critical allocation.*

(iii) *U is upper semi-continuous over $(X \times \mathbb{R}_+) \backslash (X_0 \times \mathbb{R}_{++})$.*

Of course, if $\succsim$ admits a utility index that is constant in λ, the agent can be considered as *purely altruistic* unless the utility index is merely a function of x_1. Another case of special interest is when the utility index depends only on x_1 and λ. This corresponds to a *purely egoistic* agent who is solely motivated by warm-glow and her private consumption. Part (i) of the next proposition characterizes this case, while part (ii) clarifies when we can find a utility index that is weakly increasing on $X \times \mathbb{R}_+$.

Proposition 2. *Let $\succsim$ be a binary relation on $\mathcal{A}$ that satisfies* (A1)-(A6). *Then:*

(i) *$\succsim$ admits a utility index that solely depends on x_1 and λ if, and only* *if, for any* $\{x, y\} \in \mathcal{A}$ *and* $\{x', y'\} \in \mathcal{A}$ *such that* $x_1 = x'_1$ *and* $y_1 = y'_1$*, we have* $\{x, y\} \sim \{x', y'\}$*.*

(ii) $\succsim$ *admits a utility index that is weakly increasing on* $X \times \mathbb{R}_+$ *if, and only if, for any* $\{x, y\} \in \mathcal{A}$ *and* $\{x', y'\} \in \mathcal{A}$ *such that* $x \geq x'$*,* $y \geq y'$ *and* $y_1 - x_1 \geq y'_1 - x'_1 \geq 0$*, we have* $\{x, y\} \succsim \{x', y'\}$*.*

As we noted earlier, a given preference relation $\succsim$ may admit a multitude of utility indices. The particular utility indices that we construct in the proofs of Theorem 1 and Proposition 1 have quite simple forms and they appear to be relatively well-behaved. For instance, they are both compatible with both parts of Proposition 2 (see Claims 6 and 7 in Appendix C). But this is not to mean that these particular utility indices are guaranteed to be compatible with all properties that one might demand in a given application. In Appendix C (Claim 5), therefore, we provide a full characterization of the class of all utility indices which are compatible with a given utility function over menus. We hope that this may facilitate alternative (and perhaps, more sophisticated) constructions that might be useful in future research. It should be emphasized, however, that given the preference relation $\succsim$, the choice of a utility index does not have a strong influence on the implied second stage behavior, and the identification of second stage choices becomes perfect for those utility indices which satisfy a mild regularity condition (see Section 5.1 below).

We conclude this section with a technical note which proves useful in what follows.

Lemma 1. *Let* $\succsim$ *be a binary relation on* $\mathcal{A}$ *that satisfies* (A1)-(A6)*. Then, for any function* $V : \mathcal{A} \to \mathbb{R}$ *that represents* $\succsim$*, there exists a utility index* U *such that* $V(A) = \max_{x \in \mathcal{P}(A)} U(x, y_1^*(A) - x_1)$ *for every* $A \in \mathcal{A}$*.*

4 Applications

4.1 Andreoni's Model

Andreoni (1989, 1990) studies a game on public good provision between a set of individuals $\{1, ..., I\}$. He assumes that there is one public good and one private good, and that one unit of the private good can be converted into one unit of the public good with a linear technology. Each individual

i is endowed with an amount w_i of the private good (or, equivalently, w_i units of dollars) that she can allocate between her private consumption, x_i, and her gift to the public good, g_i. In turn, the government subsidies private giving at a rate $s_i < 1$ (for the individual i) and levies lump sum taxes τ_i. So, $G := \sum_{i=1}^{I} g_i$ is the total private contributions to the public good, and $T := \sum_{i=1}^{I} \tau_i - s_i g_i$ is the net tax receipts which is fully used for the provision of public good. A generic agent, say agent 1, takes as given the private consumption and gifts of others, $(\overline{x}_2, \overline{g}_2), ..., (\overline{x}_I, \overline{g}_I)$, and chooses a consumption-gift pair (x_1, g_1) that solves a problem of the following form:

$$\max \mathcal{U}(x_1, G+T, g_1) \quad \text{subject to} \quad \begin{aligned} x_1 + (1-s_1)g_1 + \tau_1 &= w_1 \\ \text{and } 0 \leq x_1 &\leq w_1 - \tau_1. \end{aligned} \tag{2}$$

Here, $\mathcal{U}$ is a weakly increasing function on $\mathbb{R}^3_+$, which captures altruistic concerns[12] and warm-glow experience by its second and third arguments, respectively.

In our terminology, then, agent 1 faces the menu

$$A := \{(x_1, \overline{x}_2, ..., \overline{x}_I, G+T) : 0 \leq x_1 \leq w_1 - \tau_1,\ x_1 + (1-s_1)g_1 + \tau_1 = w_1, \\ G + T = \tau_1 + (1-s_1)g_1 + \textstyle\sum_{i=2}^{I} \tau_i + (1-s_i)\overline{g}_i\}.$$

Clearly, with $X := \mathbb{R}^{I+1}_+$, this menu belongs to $\mathcal{A}_{\mathcal{P}}$ and the most selfish allocation, $y^*(A)$, equals $(w_1 - \tau_1, \overline{x}_2, ..., \overline{x}_I, \tau_1 + \sum_{i=2}^{I} \tau_i + (1-s_i)\overline{g}_i)$. Thus, upon solving for g_1 in the budget constraint, we see that $g_1 = (y_1^*(A) - x_1) \,/\, (1-s_1)$. That is, g_1 is simply proportional to $y_1^*(A) - x_1$, the last argument of a utility index in our terminology. So, the function $U(x, \lambda) := \mathcal{U}(x_1, x_{I+1}, \frac{\lambda}{1-s_1})$, defined on $X \times \mathbb{R}_+$, would qualify as a utility index for a preference relation as in our theory, and the allocations that solve the problem $\max_{x \in A} U\left(x, y_1^*(A) - x_1\right)$ would coincide with those

[12] As Andreoni (1989, 1990) points out, the private consumption of a given individual would act as if it is a public good from others' perspective when they are altruistic in the classical sense. Therefore, in the literature on philanthropy, it is customary to view one's concern for the public good as a form of altruism. By the same token, often the models either take into account one's concern for others' private consumption, as in Roberts (1984), or one's concern for the public good, as in Andreoni (1989, 1990), but not both. For conceptual clarity, in the present paper we have chosen to refer to a public good separately.

solving (2).[13] To summarize, second stage behavior implied by our theory subsumes Andreoni's model upon a minor adjustment for subsidies.

Remark 2. When $s_1 > 0$, the gift g_1 does not precisely coincide with $y_1^*(A) - x_1$, because x_1 is the subsidized consumption. Put formally, at the moment of giving, the agent can actually consume $\widetilde{x}_1 := x_1 - s_1 g_1$, which satisfies $g_1 = y_1^*(A) - \widetilde{x}_1$, but subsequently the government subsidy increases the corresponding consumption to x_1.

The main contribution of Andreoni's model is that, under suitable assumptions, it makes the equilibrium amount of the public good sensitive to fiscal policies and income distribution, unlike the corresponding models of pure altruism which predict that government grants and subsidies should crowd out voluntary contributions dollar-for-dollar and that the total supply of the public good should be independent of income distribution.[14] Andreoni's approach is supported by substantial empirical evidence on incomplete crowding out (Abrams and Schmitz, 1978, 1984; Clotfelter, 1985; Steinberg, 1989) and non-neutrality of income distribution (Hochman and Rodgers, 1973).

While Andreoni's findings are based on some reasonable assumptions on the form of the utility indices, from a foundational point of view these assumptions might be restrictive. For instance, Andreoni assumes that the private consumption and gift of an agent are both strictly increasing functions of her wealth, which rules out quasi-linear utility indices. Indeed, it can easily be seen that the allocation choice implied by the purely *altruistic* utility index $u(x_1) + G + T$ would simply coincide with that induced by the purely *egoistic* utility index $u(x_1) + g_1$ if there are no subsidies. On a related note, Bergstrom, Blume, and Varian (1986, Section 2) em-

[13] Although we have set $k := I + 1$ for the domain of the preference relation, upon an obvious transformation one could also let $k = 2$, since the agents cannot influence the private consumption of others.

[14] For theoretical findings on crowding out under pure altruism, see Warr (1982), Roberts (1984), Bernheim (1986), and Andreoni (1988), among others. In turn, neutrality of income distribution under pure altruism has been demonstrated by Warr (1983) and Bergstrom et al. (1986). However, these findings are subject to some exceptions: if only a subset of the agents make donations, government spending as well as income distribution may influence the equilibrium amount of the public good (Bergstrom et al., 1986). Moreover, under alternative tax schemes (as opposed to lump-sum taxes that we discussed above), government subsidies may also be effective (Andreoni and Bergstrom, 1996).

phasize that with quasi-homothetic utility indices, income transfers would be neutral even in a model of impure altruism, as that of Andreoni (for a related finding, see also Proposition 2 of Andreoni (1990)). Finally, we should recall that taking into account the boundary solutions complicates further the task of distinguishing pure and impure altruism (see footnote 14).

In view of these remarks, our menu choice approach does not only provide foundations for Andreoni's model, but it also arms us with a clear-cut distinction between purely altruistic agents and those motivated by warm-glow. Indeed, to test the hypothesis of pure altruism, in a suitable experiment one can simply check whether the subjects violate property (1) systematically. In turn, one can detect economic agents' preferences over menus in consumption-saving problems that we routinely encounter. We provide such an example below that demonstrates the added descriptive power of our approach, which might be important for the purposes of welfare analysis as we noted in the introduction.

4.1.1 Bequest Giving with Quasi-Linear Utility Indices

Consider two generations within a family, parents and a heir. In period 1, the parents allocate their wealth, w_0, between their private consumption, x_0, and saving, $w_1 = w_0 - x_0$. At the beginning of period 2, they receive an income support $\rho(w_1)$ which is financed by a tax on the heir. We assume that $\rho : \mathbb{R}_+ \to [0, w_0]$ is a differentiable function. The parents allocate their adjusted income between their period 2 consumption, x_1, and a bequest, $g_1 = w_1 + \rho(w_1) - x_1$. The heir's initial wealth also equals w_0. She moves last and consumes all of her adjusted income, $x_2^* = w_0 + g_1 - \rho(w_1) = w_0 + w_1 - x_1$. We now examine parents' behavior in a subgame perfect equilibrium.

First of all, the menu that the parents face in period 2 takes the form

$$A(x_0, w_0) := \{(x_1, x_2^*) : 0 \leq x_1 \leq w_1 + \rho(w_1),\ x_2^* = w_0 + w_1 - x_1\}.$$

This menu belongs to $\mathcal{A}_{\mathcal{P}}$ with $X := \mathbb{R}_+^2$, and the most selfish allocation is given by $(w_1 + \rho(w_1), w_0 - \rho(w_1))$. Thus, we also see that $g_1 = y_1^*(A(x_0, w_0)) - x_1$.

In turn, the parents' problem in period 1 is to make a choice among the pairs of the form $(x_0, A(x_0, w_0))$. Let W be a utility function over $\{(x_0, A) : x_0 \in \mathbb{R}_+,\ A \in \mathcal{A}\}$ that represents the parents' preferences. In view of Lemma 1, if these preferences restricted to $\{(x_0, A) : A \in \mathcal{A}\}$ satisfy the properties (A1)-(A6) for each x_0, we can find a utility index $U_{x_0} : \mathbb{R}^3_+ \to \mathbb{R}$ such that

$$W(x_0, A(x_0, w_0)) = \max\{U_{x_0}(x_1, x_2, g_1) : (x_1, x_2) \in A(x_0, w_0)\}. \qquad (3)$$

Let us now consider a purely egoistic utility index $U^e_{x_0} = u(x_0) + u(x_1) + g_1$, and a purely altruistic one $U^a_{x_0} = u(x_0) + u(x_1) + x_2$, where $u : \mathbb{R}_+ \to \mathbb{R}_+$ is a function that satisfies the Inada conditions. Just as in the corresponding model of Andreoni, after substituting for g_1 and x^*_2, we immediately see that for any fixed (x_0, w_0), the maximizers of $U^a_{x_0}$ and $U^e_{x_0}$ over the set $A(x_0, w_0)$ coincide. That is, in this setup, we cannot distinguish the two types of parents based on period 2 behavior.

On the other hand, the saving behavior of the two types are typically different, because the income support influences the marginal value of saving for egoistic parents by altering their perception of freedom in period 2. Indeed, among the interior solutions of period 2 (which correspond to large values of adjusted income $w_1 + \rho(w_1)$), the value on the right side of (3) takes the form $u(x_0) + u(\overline{x}_1) + w_1 + \rho(w_1) - \overline{x}_1$ for egoistic parents, while it takes the form $u(x_0) + u(\overline{x}_1) + w_0 + w_1 - \overline{x}_1$ for altruistic parents (here, $\overline{x}_1$ is the number that satisfies $u'(\overline{x}_1) = 1$). Thus, the marginal value of saving equals 1 for altruistic parents while it equals $1 + \rho'(w_1)$ for egoistic parents. In particular, if ρ is a decreasing function of w_1 (which corresponds to a progressive income support), the marginal value of saving for egoistic parents is smaller, and hence, they save less than altruistic parents. If $w_1 + \rho(w_1)$ is increasing in w_1, this also implies that egoistic parents leave a smaller bequest. Moreover, while ρ is neutral in the case of altruistic parents, the saving of egoistic parents increases with an upward shift in $\rho'(\cdot)$.

Of course, the comparative statics of this narrowly tailored model does not point to a serious economic finding. Rather, the exercise demonstrates how the menu choice approach provides additional means of distinguishing purely altruistic agents from those motivated by warm-glow, which directly build upon the fact that government policies may influence agents' welfare, even when they are neutral in terms of second stage behavior.

4.2 Alternative Sets of Social Outcomes

Before discussing another application, we need to clarify how our theory can be extended to alternative sets of social outcomes. To this end, suppose that the set of allocations X is of the form $X = X_1 \times \cdots \times X_k$, where X_i is a separable metric space for each i. Then, under suitable assumptions on the behavior of $\succsim$ over the collection of singletons $\{\{x\} : x \in X\}$, we can find an aggregator $\varphi : \mathbb{R}^k \to \mathbb{R}$ and functions $\pi_i : X_i \to \mathbb{R}$ for $i = 1, ..., k$, such that $\{x\} \succsim \{y\}$ if and only if $\varphi(\pi_1(x_1), ..., \pi_k(x_k)) \geq \varphi(\pi_1(y_1), ..., \pi_k(y_k))$.[15] If we abstract from public goods so that x_i corresponds to the private consumption of individual i (which may also be a random variable), just as in Harsanyi's (1953, 1955) theory of utilitarianism, on occasion it may be appropriate to interpret π_i as a measure of well-being of individual i from the perspective of the decision maker in question, who acts as a social planner. In fact, that π_i depends solely on x_i would suggest one to view this function as the material payoff of individual i.

Once we agree on this interpretation, we could restate properties (i) and (ii) that define the collection of relevant menus and the axioms (A1)-(A6), in terms of the payoff vectors $(\pi_1(x_1), ..., \pi_k(x_k))$ and utility possibility sets of the form $\{(\pi_1(x_1), ..., \pi_k(x_k)) : x \in A\} \subseteq \mathbb{R}^k$. In particular, we could let $y^{*\pi}(A)$ be an allocation that maximizes the function π_1 over a qualifying menu A, and give the role of $y_1^*(A) - x_1$ in the basic theory to the difference $\pi_1(y_1^{*\pi}(A)) - \pi_1(x_1)$. By pursuing this approach, it is a straightforward exercise to obtain an extension of Theorem 1 that delivers a utility representation of the form

$$V_\pi(A) := \max_{x \in \mathcal{P}(A)} U\left(\pi_1(x_1), ..., \pi_k(x_k), \pi_1\left(y_1^{*\pi}(A)\right) - \pi_1(x_1)\right)$$

for a function $U : \mathbb{R}^k \times \mathbb{R}_+ \to \mathbb{R}$ (we omit the details of this derivation).

[15] A large body of literature is devoted to the study of axiomatic foundations of such representations that also demand the aggregator to be additive (see Wakker (1989, Chapter 3) and references therein). In turn, a nonadditive form of the representation can be derived by imposing a weak separability property along the lines of Mak (1984).

Remark 3. When individuals' utility from private and public goods can be separated from each other, the above argument can also be applied in a framework with a finite number of public goods.

4.3 Voting as a Civic Duty

Explaining voter turnout in large elections has been a major challenge for political economists. The difficulty stems from the fact that when many people vote, the probability of being decisive (pivotal) for a single voter is close to zero, whereas voting incurs significant costs. In an earlier attempt to resolve this paradox, Riker and Ordeshook (1968) suggested that the act of voting may be valuable *per se*, as the citizens may perceive it as a civic duty.

Suppose there are two candidates, ℓ and r, and that the agent in question prefers candidate ℓ. Specifically, let us assume that the victory of ℓ will bring a material payoff $\mathfrak{u} > 0$ to our agent, whereas victory of r is worth 0. Given other voters' behavior, let $p_j > 0$ be the probability of being pivotal for the agent if she votes for candidate j, and let P be the probability of winning for candidate ℓ if she abstains. Finally, let c denote the cost of voting, and d the payoff associated with the act of voting, as posited by Riker and Ordeshook.

The implied expected payoff scheme reads as follows:

$$\begin{array}{ll} (P + p_\ell)\,\mathfrak{u} - c + d & \text{if the agent votes for } \ell, \\ P\mathfrak{u} & \text{if the agent abstains,} \\ (P - p_r)\,\mathfrak{u} - c + d & \text{if the agent votes for } r. \end{array}$$

Thus, the agent would never vote for r, while the decision between abstaining and voting for ℓ is determined by the following simple rule:

$$\text{vote for } \ell \quad \text{if and only if} \quad p_\ell \mathfrak{u} + d \geq c.$$

In particular, no matter how small p_ℓ might be, our agent would vote if $d \geq c$.

While that voters may be motivated by a sense of duty is a widely accepted view, recently scholars proposed some extensions which can explain several other aspects of voters' behavior as well as the high turnout rates themselves (see, e.g., Coate and Conlin, 2004; Feddersen and Sandroni, 2006). These alternative models are sensitive to the specification of voters' statistical distribution, for they relate the turnout rate of a group of individuals to their likelihood of influencing the election outcome.[16] Riker-Ordeshook approach, on the other hand, is compatible with high turnout rates irrespective of how an individual or a group of individuals might influence the election outcome. We shall now show how our representation can reproduce the calculus of voting suggested by Riker and Ordeshook.

Following Section 4.2, let X_i be the space of lotteries over the real line, and π_i be the expectation operator over X_i. Each action a available to the agent in question, individual 1, induces a vector of lotteries $x(a) \in X_1 \times \cdots \times X_k$, given the behavior of other $k-1$ voters. So, the agent evaluates action a with the associated expected payoff vector $(\pi_1(x_1(a)), ..., \pi_k(x_k(a)))$.

The agent believes that the victory of ℓ will contribute to the (material) payoff of everyone in the society, implying that $\pi_i(x_i(\text{vote for } \ell)) > \pi_i(x_i(\text{vote for } r))$ for every i. Moreover, as before, the victory of candidate ℓ is worth $\mathfrak{u} > 0$ for the agent herself, so that

$$\pi_1(x_1(\text{vote for } \ell)) = (P + p_\ell)\,\mathfrak{u} - c \quad \text{and} \quad \pi_1(x_1(\text{abstain})) = P\mathfrak{u}.$$

It follows that when p_ℓ is small, as would be the case in a large election, and if $c > 0$, the agent's expected payoff would be higher if she abstains. On the other hand, the agent believes that if she were to vote for ℓ, she would be contributing to the expected payoff of everyone else. Thus, the menu of lottery vectors $\{x(\text{vote for } \ell),\ x(\text{abstain})\}$ belongs to $\mathcal{A}_{\mathcal{P}}$ in our extended theory, and $x(\text{abstain})$ is the most selfish option. In turn, the corresponding warm-glow component is given by $\pi_1(x_1(\text{abstain})) - \pi_1(x_1(\text{vote for } \ell)) = c - p_\ell \mathfrak{u} > 0$.

As a final step, let us suppose that the utility index of the agent is of the form $U = \pi_1 + f(\lambda)$, so that we have a purely egoistic agent at hand. Then, according to our extended theory, the agent should solve the

[16] Evren (2010) provides a discussion of the role of voters' distribution in these recent models.

following problem:

$$\max\{\pi_1(x_1(\text{vote for } \ell)) + f(c - p_\ell \mathfrak{u}),\ \pi_1(x_1(\text{abstain})) + f(0)\}.$$

That is, the agent should vote if, and only if, $p_\ell \mathfrak{u} + f(c - p_\ell \mathfrak{u}) - f(0) \geq c$. Also note that if f is continuous, $f(c - p_\ell \mathfrak{u})$ will be approximately equal to $f(c)$ for small values of p_ℓ. Thus, the parameter d in the Riker-Ordeshook model simply corresponds to the warm-glow payoff $f(c - p_\ell \mathfrak{u}) - f(0) \approx f(c) - f(0)$.

Beyond the technical details, our theory endogenizes the parameter d of Riker and Ordeshook by viewing the act of voting as a selfless action taken by *free will*. Indeed, if citizens were forced to vote, say by a prohibitively high fine on abstention, it would seem reasonable to assume that they would not attribute an intrinsic value to the act of voting. This is precisely what our model predicts: Given a fine ϕ on abstention, the difference $\pi_1(x_1(\text{abstain})) - \pi_1(x_1(\text{vote for } \ell))$ reduces to $c - p_\ell \mathfrak{u} - \phi$, leading to a smaller warm-glow payoff $f(c - p_\ell \mathfrak{u} - \phi) - f(0)$. Put differently, in line with our earlier remark, our model implies that a fine on abstention may crowd out voters' intrinsic motivation. In a dual fashion, a policy that aims to reduce voting costs may crowd out intrinsic motivations through the same mechanism. As Bénabou and Tirole (2006) also point out, this phenomenon seems to underlie Funk's (2010) findings which show that the introduction of mail voting in Switzerland failed to raise the turnout rates in some communities.[17]

5 On Second Stage Choice Behavior

As we have seen in the previous section, in applied warm-glow models, the focus is often the social consequences of individuals' behavior. Thus, in view of the multiplicity of the utility indices that we noted in Section 3, it is of major importance to determine the extent of uniqueness of the implied second stage choice behavior. In this section, we first address this issue and then provide an axiomatic characterization of second stage

[17]Studying reflections of these observations on voters' welfare may be an interesting venue for future research, for, to our knowledge, the existing models on voter welfare simply focus on extrinsic motivations (see Börgers, 2004; Krasa and Polborn, 2009).

choices associated with a given preference relation over menus.

5.1 Uniqueness

As in Section 3, let $\succsim$ be a binary relation on $\mathcal{A}$ that satisfies properties (A1)-(A6), and let U be a utility index for $\succsim$. Our representation suggests that when faced with a menu $A \in \mathcal{A}$, in the second stage the agent's potential choices would coincide with the following set:

$$\mathbf{C}_U(A) := \left\{ \widehat{x} \in \mathcal{P}(A) : U\left(\widehat{x}, y_1^*(A) - \widehat{x}_1\right) = \max_{x \in \mathcal{P}(A)} U\left(x, y_1^*(A) - x_1\right) \right\}.$$

As we noted earlier, when $\{x\} \prec \{x, y\} \sim \{y\}$ and $y_1 > x_1$, we cannot pin down how $U(x, y_1 - x_1)$ compares with $U(x, 0)$ and $U(y, 0)$. In particular, depending on the choice of the utility index, we may either have $U(x, y_1 - x_1) = U(y, 0)$ or $U(x, y_1 - x_1) < U(y, 0)$. In both cases, it would follow that the agent may select y from $\{x, y\}$, but whether x could *also* be selected depends on the choice of the utility index. On the other hand, when $\{x, y\} \succ \{y\}$ and $y_1 > x_1$, we must certainly have $U(x, y_1 - x_1) > U(y, 0)$, so that x can be identified as the unique choice from $\{x, y\}$.

These observations readily extend to arbitrary menus. That is, for any $A \in \mathcal{A}$, if the most selfish option does not belong to $\mathbf{C}_U(A)$, then we have $\mathbf{C}_U(A) = \mathbf{C}_{\widetilde{U}}(A)$ for any other utility index $\widetilde{U}$. In particular, $\mathbf{C}_U(A)$ contains the most selfish option if and only if this is the case for any other utility index. What remains undetermined is if (and which) other allocations can be selected along with the most selfish option when the latter belongs to the choice correspondence:

Proposition 3. *Let U and $\widetilde{U}$ be a pair of utility indices for $\succsim$. Then, for any $A \in \mathcal{A}$,*

(i) *$y^*(A) \notin \mathbf{C}_U(A)$ implies $\mathbf{C}_U(A) = \mathbf{C}_{\widetilde{U}}(A)$;*

(ii) *$y^*(A) \in \mathbf{C}_U(A)$ if, and only if, $y^*(A) \in \mathbf{C}_{\widetilde{U}}(A)$.*

The level of identification determined by Proposition 3 seems to be quite satisfactory. In particular, the intersection of all compatible choice correspondences is always nonempty. Put formally, for any $A \in \mathcal{A}$, the set

$$\bigcap \{\mathbf{C}_U(A) : U \text{ is a utility index for } \succsim\}$$

either contains $y^*(A)$, or it equals $\mathbf{C}_U(A)$ for an arbitrary utility index U. It also follows that for any pair of utility indices U and $\widetilde{U}$, whenever both $\mathbf{C}_U(A)$ and $\mathbf{C}_{\widetilde{U}}(A)$ consist of single allocations, we must, in fact, have $\mathbf{C}_U(A) = \mathbf{C}_{\widetilde{U}}(A)$.

Yet, it may be of interest to note that we can obtain perfect identification for utility indices which satisfy the following additional property.

Regularity. Let $\{x, y\} \in \mathcal{A}_{\mathcal{P}}$ be such that $U(x, y_1 - x_1) = U(y, 0)$ and $y_1 > x_1$. Then, any neighborhood of $\{x, y\}$ contains a pair of allocations $\{x', y'\} \in \mathcal{A}_{\mathcal{P}}$ such that $U(x', y'_1 - x'_1) > U(y', 0)$ and $y'_1 > x'_1$.

In what follows, we say that a utility index is **regular** if it satisfies the above property. The notion of regularity is a variant of the local non-satiation property familiar from the classical consumer theory. On a related note, the next example shows that in the classical model, monotonicity of a utility index implies its regularity.

Example 1 (Classical Case). Let U be a utility index that is constant in λ and suppose that $U(\widehat{x}, 0) > U(x, 0)$ whenever $\widehat{x}_i > x_i$ for $i = 1, ..., k$. Given any $x \in X$ and $\varepsilon > 0$, set $x' := (x_1 + \varepsilon, x_2 + \varepsilon, ..., x_k + \varepsilon)$. Then, for any $\{x, y\}$ in $\mathcal{A}_{\mathcal{P}}$ with $y_1 > x_1$ and any $\varepsilon < y_1 - x_1$, the pair $\{x', y\}$ also belongs to $\mathcal{A}_{\mathcal{P}}$. Moreover, $U(x, y_1 - x_1) = U(y, 0)$ implies $U(x', y_1 - x'_1) = U(x', 0) > U(x, 0) = U(y, 0)$. So, with $y' := y$, as $\varepsilon \to 0$, the pair $\{x', y'\}$ satisfies the requirements for regularity of U.

As we shall see momentarily, the regularity notion proves quite general even outside the classical model. Before presenting some examples in this direction, we state our identification result for regular utility indices:

Proposition 4. *Let U and $\widetilde{U}$ be a pair of regular utility indices for $\succsim$ which are also continuous over $(X \times \mathbb{R}_+) \backslash (X_0 \times \mathbb{R}_{++})$. Then, $\mathbf{C}_U(A) = \mathbf{C}_{\widetilde{U}}(A)$ for any $A \in \mathcal{A}$.*

In view of this proposition, when $\succsim$ admits a regular utility index that is also continuous over $(X \times \mathbb{R}_+) \backslash (X_0 \times \mathbb{R}_{++})$, we denote by $\mathbf{C}_{\succsim}$ the unique choice correspondence associated with such utility indices.

5.1.1 More on Regularity

The next two examples show that (1) under pure egoism, strict quasi-concavity implies regularity; and (2) even an impure form of altruism suffices for regularity.

Example 2 (Pure Egoism). Consider a utility index U and a strictly quasi-concave function $u : \mathbb{R}^2_+ \to \mathbb{R}$ such that $U(x, \lambda) = u(x_1, \lambda)$ for every $(x, \lambda) \in X \times \mathbb{R}_+$. Let $x, y \in X$ be such that $y_1 > x_1$ and $U(x, y_1 - x_1) = U(y, 0)$, i.e., $u(x_1, y_1 - x_1) = u(y_1, 0)$. Put $\lambda^* := y_1 - x_1$, and $(x', \lambda') := \alpha(x, \lambda^*) + (1 - \alpha)(y, 0)$ for an arbitrary $\alpha \in (0, 1)$, so that $(x'_1, \lambda') = \alpha(x_1, \lambda^*) + (1-\alpha)(y_1, 0)$. By strict quasi-concavity of u, we then have $u(x'_1, \lambda') > u(y_1, 0)$; that is, $U(x', \lambda') > U(y, 0)$. Moreover, if $\{x, y\}$ belongs to $\mathcal{A}_\mathcal{P}$, so does $\{x', y\}$. Finally, note that $y_1 - x'_1 = \alpha(y_1 - x_1) = \lambda'$ and $x' \to x$ as $\alpha \to 1$. Thus, as $\alpha \to 1$, the pair $\{x', y\}$ satisfies the requirements for regularity of U.

Remark 4. Given a strictly concave function f on $\mathbb{R}_+$, both of the functions $u_1 = f(x_1) + \lambda$ and $u_2 = x_1 + f(\lambda)$ are strictly quasi-concave on $\mathbb{R}^2_+$. Hence, Example 2 also includes such quasi-linear functions. Quasi-linearity of the latter form might be especially important in an extended version of our model based on expected material payoffs, as we discussed in Section 4.3 above.

Example 3 (Impure Altruism). Let U be a utility index such that $U(\widehat{x}, \lambda) > U(x, \lambda)$ whenever $\widehat{x}_1 \geq x_1$ and $\widehat{x}_i > x_i$ for $i = 2, ..., k$. Given any $x \in X$ and $\varepsilon > 0$, set $x' := (x_1, x_2 + \varepsilon, ..., x_k + \varepsilon)$. Then, for any pair $\{x, y\}$ in $\mathcal{A}_\mathcal{P}$ with $y_1 > x_1$, the pair $\{x', y\}$ also belongs to $\mathcal{A}_\mathcal{P}$. Moreover, $U(x, y_1 - x_1) = U(y, 0)$ implies $U(x', y_1 - x'_1) = U(x', y_1 - x_1) > U(y, 0)$. Since ε can be selected arbitrarily small, it follows that U is regular.

When we combine Examples 1-3, it appears that one would rarely encounter a non-regular utility index in applications. We should note, however, that it is a nontrivial problem to obtain a characterization of preference relations which admit a regular utility index, for the definition of regularity refers to the condition $U(x, y_1 - x_1) = U(y, 0)$. In turn, this equality implies $\{x, y\} \sim \{y\}$, but the converse does not hold as we discussed earlier. We do not pursue this problem further in the present paper.

5.2 A Joint Characterization of Second Stage Behavior

Given a choice correspondence[18] $\mathbf{C}$ on $\mathcal{A}$, in this section we study some joint properties of $\mathbf{C}$ and $\succsim$ that allow us to relate $\mathbf{C}$ to choice correspondences of the form $\mathbf{C}_U$. Because of the uniqueness issue that we have just discussed, for a non-regular utility index U, we will seek only a partial relation between $\mathbf{C}$ and $\mathbf{C}_U$ that is analogous to Proposition 3. In turn, when $\succsim$ admits a regular utility index that is also continuous over $(X \times \mathbb{R}_+)\backslash(X_0 \times \mathbb{R}_{++})$, we will be able to obtain a full characterization of the case $\mathbf{C} = \mathbf{C}_{\succsim}$.

We start with two basic assumptions:

Efficiency (H1). $\mathbf{C}(A) = \mathbf{C}(\mathcal{P}(A))$ for any $A \in \mathcal{A}$.

Weak WARP (H2). Let $A \in \mathcal{A}_{\mathcal{P}}$ and pick a nonempty, compact set $B \subseteq A$. Then, $y^*(A) \in B$ and $\mathbf{C}(A) \cap B \neq \emptyset$ imply $\mathbf{C}(A) \cap B = \mathbf{C}(B)$.

Since, in our model, the most selfish option acts as a reference point that influences the agent's second stage behavior, (H2) asserts that the conclusion of classical WARP holds necessarily, only when the most selfish option in B coincides with the most selfish in the larger set A. On the other hand, for such pair of sets A and B, instances of the form $\mathbf{C}(A) \cap B = \emptyset$ must correspond to those cases in which the agent strictly prefers A to B. The next axiom formalizes this observation.

Sophistication (H3). Let $B \subseteq X$ be a nonempty, compact set such that $B \cup \{x\} \in \mathcal{A}_{\mathcal{P}}$ for an allocation $x \in X\backslash B$. Then, $y^*(B \cup \{x\}) \in B$ implies

$$B \cup \{x\} \succ B \quad \text{if and only if} \quad \mathbf{C}(B \cup \{x\}) = \{x\}.$$

By definition of a utility index U, for such B and x, the conditions $B \cup \{x\} \succ B$ and $\mathbf{C}_U(B \cup \{x\}) = \{x\}$ are equivalent to each other. So, (H3) requires the equality of $\mathbf{C}_U(B \cup \{x\})$ and $\mathbf{C}(B \cup \{x\})$ in such cases. The following result extends this equality to all instances in which the agent does not select the most selfish option.

[18] As usual, a *choice correspondence* on $\mathcal{A}$ refers to a set valued function $\mathbf{C}$ on $\mathcal{A}$ such that $\emptyset \neq \mathbf{C}(A) \subseteq A$ for every $A \in \mathcal{A}$.

Proposition 5. *Let U be a utility index for $\succsim$, and $\mathbf{C}$ be a choice correspondence on $\mathcal{A}$ that satisfies* (H1) *and* (H2)*. Then, the pair* $(\mathbf{C}, \succsim)$ *satisfies* (H3) *if, and only if, the following two properties hold for any $A \in \mathcal{A}$:*

(i) $y^*(A) \notin \mathbf{C}_U(A)$ *implies* $\mathbf{C}_U(A) = \mathbf{C}(A)$.

(ii) $y^*(A) \in \mathbf{C}_U(A)$ *if, and only if,* $y^*(A) \in \mathbf{C}(A)$.

The next item in our agenda is to obtain a characterization of $\mathbf{C}_{\succsim}$. To this end, we first restate the regularity property in terms of second stage choices:

Choice Regularity (H4). Take any $\{x, y\} \in \mathcal{A}_{\mathcal{P}}$ with $y_1 > x_1$, and suppose that there exists a neighborhood $\mathcal{N}$ of $\{x, y\}$ in $\mathcal{A}_{\mathcal{P}}$ such that $\{x', y'\} \sim \{y'\}$ for every $\{x', y'\} \in \mathcal{N}$ with $y'_1 > x'_1$. Then, we must have $\mathbf{C}(\{x, y\}) = \{y\}$.

Remark 5. It is readily verified that a given utility index U for $\succsim$ is regular if, and only if, the pair $(\mathbf{C}_U, \succsim)$ satisfies (H4). Hence the term "choice regularity."

We also assume that second stage choices are continuous in a standard sense:

Closed Graph (H5). $\{(x, A) : x \in \mathbf{C}(A),\ A \in \mathcal{A}_{\mathcal{P}}\}$ is a closed subset of $X \times \mathcal{A}_{\mathcal{P}}$.

The promised characterization of $\mathbf{C}_{\succsim}$ reads as follows.

Proposition 6. *Let $\mathbf{C}$ be a choice correspondence on $\mathcal{A}$ and suppose that $\succsim$ admits a regular utility index that is continuous over* $(X \times \mathbb{R}_+) \setminus (X_0 \times \mathbb{R}_{++})$*. Then, the pair* $(\mathbf{C}, \succsim)$ *satisfies* (H1)-(H5) *if, and only if,* $\mathbf{C} = \mathbf{C}_{\succsim}$.

It should be noted that Noor and Takeoka (2011, Section 4) present a closely related discussion of second stage behavior associated with a class of temptation driven preference relations over menus. In their model, the most tempting option acts as a reference point that influences the agent's choice of an alternative: Holding fixed the alternative x that will be selected, the agent's welfare *decreases* with the difference between the maximum possible temptation utility and that of x, giving rise to preferences for smaller sets. Thus, the two models are quite distinct, and yet, curiously, our axioms that link second period choices to preference relation $\succsim$ are very similar to those of Noor and Takeoka. In particular, they postulate conceptually equivalent versions of Weak WARP and Sophistication, under the same names. In turn, one of their axioms (Ex Post Decreasing Self Control) ensures that a functional in their representation is strictly increasing, and thereby, takes the role of regularity property in our model.[19] Thus, it appears that, rather than the linking axioms, mainly it is the properties of $\succsim$ that distinguish the two models.

In passing, let us emphasize that the main merit of this section has been the partial identification results, Propositions 3 and 5, as Noor and Takeoka do not report comparable findings. Moreover, as we will see in the next section, the Sophistication axiom provides a clear-cut distinction between our theory and a corresponding model of menu choice based on the notion of preference for flexibility.

6 Relations to Kreps' Model of Preference for Flexibility

Following Kreps' (1979) pioneering work, the literature on preference for flexibility also focuses on decision makers who prefer a menu to all of its subsets (see, e.g., Dekel et al., 2001; Epstein et al., 2007; Ahn and Sarver, 2011). This literature attributes violations of property (1) to uncertainty of future preference relations over the set of alternatives. The decision maker in question is solely concerned about her final choices just as in the case of pure altruism. Yet, she still exhibits preference for larger sets, since, on occasion, she cannot precisely predict which alternative she would select from a given menu in period 2. In particular, instances of the form $B \prec B \cup \{x\} \succ \{x\}$ correspond precisely to those cases in which the agent is unsure whether she would select x or an element of B when

[19] When discussing this axiom, Noor and Takeoka (p. 17) assert that "if he [the agent] can pick μ from $\{\mu, \eta\}$ – albeit not uniquely if he is on the margin between exerting self-control or not – then he can pick μ uniquely in $\{\mu, \mu\alpha\eta\}$." Here, α is a number in $(0, 1)$ and $\mu\alpha\eta$ stands for the mixture $\alpha\mu + (1-\alpha)\eta$ of the lotteries μ and η. When adapted to our setting, this quotation amounts to saying that $U(x, y_1 - x_1) \geq U(y, 0)$ implies $U(x, y'_1 - x_1) > U(y', 0)$ for $y' := \alpha x + (1-y)$ and $\alpha \in (0, 1)$, which is a stronger property than regularity of U.

faced with $B \cup \{x\}$.

By stark contrast, unless x is the most selfish option in $B \cup \{x\}$, whenever $B \prec B \cup \{x\}$ our model predicts that the agent would *certainly* select x from $B \cup \{x\}$. In turn, if x is the most selfish option, $B \cup \{x\} \succ \{x\}$ implies that the agent would *not* select x from $B \cup \{x\}$. These observations point to a clear-cut distinction between the two models, to the extent that in period 2, one can verify the random choice behavior that the preference for flexibility model predicts. More generally, holding fixed the most selfish option, the second stage behavior implied by our theory is within the scope of the standard choice model, whereas preference for flexibility approach predicts a stochastic behavior as in McFadden and Richter (1991) or McFadden (2005).

On the other hand, if one solely focuses on preferences over menus, the difference between the two approaches becomes less stark. In particular, a preference relation lies at the intersection of our model with that of Kreps (1979) if it admits a utility function as follows:

$$V_K(A) := \alpha \max_{x \in A} u^a(x) + (1-\alpha) u^e(y_1^*(A)),$$

where $\alpha \in (0,1)$, $u^a : \mathbb{R}_+^k \to \mathbb{R}$ is weakly increasing (and continuous), and $u^e : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing. While the corresponding preference relation satisfies the properties (A1)-(A6), this representation describes an agent who believes that in period 2, she may either wish to act unselfishly as guided by the function u^a (which will happen with probability α), or select the most selfish option in a purely egoistic manner.

It should be noted, however, that preference relations in our model do not always admit a representation à la Kreps, for such a representation requires the submodularity axiom which posits that $A \cup C \sim A \cup B \cup C$ whenever $A \sim A \cup B$. By contrast, in our model, we may well have $A \cup C \prec A \cup B \cup C$ and $A \sim A \cup B$, as the most selfish options (i.e., reference points) in $A \cup B$ and $A \cup B \cup C$ may be different. For example, we may have $\{x, y\} \sim \{y\}$ for some x, y with $y_1 > x_1$, but the agent may strictly prefer a menu of the form $\{x, y, z\}$ to $\{y, z\}$ whenever $z_1 > y_1$, as the stronger warm-glow experience associated with the sacrifice $z_1 - x_1$ may convince the agent to select x uniquely from $\{x, y, z\}$.

7 Concluding Remarks

In this paper we proposed a representation notion for preference relations over menus of social allocations. Inspired by the literature on preference for freedom of choice, the main idea of our model is that a menu that allows the decision maker to select an other-serving option by her free will may be more valuable than a menu which leaves no choice but that particular option. We have also shown that the second stage behavior implied by our representation subsumes that modeled by Riker and Ordeshook (1968) and Andreoni (1989, 1990).

Our representation describes an agent who experiences warm-glow only when her actions (might) help others. However, the strength of warm-glow experience (i.e., the warm-glow payoff) is solely a function of the agent's private cost, irrespective of how strongly the other agents are influenced. While in Andreoni's framework there is a linear relation between one's sacrifice of private consumption and her contribution to the public good, in that of Riker and Ordeshook, warm-glow payoff is driven by a sense of civic duty, independently of the social consequences of the act of voting, just as our representation predicts. This is not to mean, however, that the warm-glow experience should truly be independent of others' welfare. In particular, one can think of a more general version of our representation that models the warm-glow payoff as an increasing function of the agent's contribution to every other individual's payoff, as well as her private cost (see Appendix A, Example A1). This alternative model is consistent with all of our axioms except for (A4), but at present we do not know how the corresponding class of preference relations can be characterized. We leave this as an open problem for future research.

If one views "giving" as an act of free will, as opposed to a compulsory transfer of resources, our model can simply be seen as a theory of "preference for giving." On the other hand, social pressure or negative feelings such as shame may also motivate other-serving actions even if the decision maker in question dislikes such mode of behavior. Dillenberger and Sadowski (forthcoming) focus on this phenomenon. Their key idea is that such a decision maker would dislike the presence of other-serving options, and hence, exhibit preference for commitment, as opposed to preference for larger sets that we model in this paper.

In Dillenberger and Sadowski, the utility associated with the choice of an allocation decreases with the distance between that allocation and a most ethical option. As we have noted in the introduction, in this approach, selfish and ethical modes of behavior can coincide, whereas our model is consistent with those agents who always value the act of giving. It is also worth noting that the most ethical option in Dillenberger and Sadowski can be viewed as the maximizer of an altruistic utility function. Consequently, it appears that the implied second stage behavior has a closer relation with classical altruism.[20] In view of these remarks, we consider the present paper and that of Dillenberger and Sadowski as complements.

Following a fundamentally different approach, Cherepanov et al. (2011) focus on a standard choice theoretic framework. In their model, each menu contains a special alternative that the agent "aspires" to choose. If a given alternative x is the aspiration in the menu that the agent faces, selecting x brings a payoff $D > 0$ in addition to the utility of x. In choice situations that involve a conflict between the material well-being of the decision maker and that of the society, it may be suitable to think of the aspiration as an other-serving option. While several prominent models, including Riker and Ordeshook (1968), Coate and Conlin (2004) and Feddersen and Sandroni (2006), are within the scope of this theory, Cherepanov et al. (2011) do not address the problem of distinguishing between the negative and positive versions of non-altruistic agents that we have discussed above. Does the agent experience a penalty D upon selfish behavior because of psychological or social reasons, although normatively she has nothing against such mode of behavior? Or, does she attach an intrinsic value D to the act of selecting her aspiration with her free will? Unlike the present paper and Dillenberger and Sadowski (forthcoming), the model of Cherepanov et al. (2011) is consistent with both scenarios.

[20] In particular, second stage behavior induced by Theorem 2 of Dillenberger and Sadowski is consistent with the classical model. One can also think of an analogous refinement of our representation that implies WARP-consistent second stage behavior. However, unlike the classical model, the corresponding second stage "utility" function would not be increasing in the agent's material payoff. (The details are available upon request from the authors.) We do not pursue this modified approach here, because it could be restrictive when applied to models of public good provision.

Appendix A. Independence of the Axioms

In this appendix, we show that none of the axioms (A2)-(A5) can be dropped from the statement of Theorem 1. (Interested readers may contact the authors for further examples that demonstrate necessity of (A1) and (A6).)

Example A1 (Monotone Warm-Glow). Let $U : X \times \mathbb{R}_+^k \to \mathbb{R}$ be a weakly increasing, continuous function. Define $\mathcal{V} : \mathcal{A} \to \mathbb{R}$ as

$$\mathcal{V}(A) := \max_{x \in \mathcal{P}(\mathcal{A})} U(x, y_1^*(A) - x_1, x_2 - y_2^*(A), x_3 - y_3^*(A), ..., x_k - y_k^*(A)).$$

Denote by $\succsim$ the preference relation on $\mathcal{A}$ induced by $\mathcal{V}$. Notice that given a pair of nonempty compact sets A, B with $A \cup B \in \mathcal{A}_{\mathcal{P}}$, whenever $y^*(A \cup B) \in A \cap B$, we have $y^*(A) = y^*(A \cup B) = y^*(B)$. It obviously follows that $\succsim$ satisfies (A3). It is also easy to verify axioms (A1), (A2), (A5) and (A6). To see that (A4) may fail, let $k := 3$ and suppose $U(x, 10, 5, 5) > U(x, 11, 1, 1)$ for some $x \in X$ with $\min\{x_2, x_3\} > 5$. Put $y := (x_1 + 10, x_2 - 5, x_3 - 5)$, $z := (x_1 + 11, x_2 - 1, x_3 - 1)$ and suppose also that $U(x, 10, 5, 5) > \max\{U(x, 0, 0, 0), U(y, 0, 0, 0), U(z, 0, 0, 0)\}$.[21] Then, $V(\{x, y\}) = U(x, 10, 5, 5) > \max\{V(\{x\}), V(\{y\}), V(\{x, z\})\}$, implying that $\succsim$ violates (A4).

Example A2 (Weak Instrumentalism). Set $k := 2$ and put $y_{*1}(A) := \min\{x_1 : x \in \mathcal{P}(A)\}$ for $A \in \mathcal{A}$. Define a function $\mathcal{V} : \mathcal{A} \to \mathbb{R}$ as

$$\mathcal{V}(A) := \max_{x \in \mathcal{P}(\mathcal{A})} x_1 x_2 (y_1^*(A) - x_1)(y_1^*(A) - y_{*1}(A)).$$

That the induced preference relation $\succsim$ satisfies (A1), (A5) and (A6) is obvious. Moreover, $\mathcal{P}(A) \supseteq \mathcal{P}(B)$ implies $y_1^*(A) - y_{*1}(A) \geq y_1^*(B) - y_{*1}(B)$ and $y_1^*(A) - x_1 \geq y_1^*(B) - x_1$ for any $x \in B$. This verifies (A2). Also note that for any $\{x, y\} \in \mathcal{A}_{\mathcal{P}}$ with $y_1 \geq x_1$, we have $\mathcal{V}(\{x, y\}) = x_1 x_2 (y_1 - x_1)^2$. In turn, (A4) is readily deduced from this observation. Next, set $A := \{(2, 0), (1, 2)\}$ and $B := \{(2, 0), (0, 3)\}$ so that $A \cup B = \{(2, 0), (1, 2), (0, 3)\}$ which belongs to $\mathcal{A}_P$. Notice that $y^*(A \cup B) = (2, 0) \in A \cap B$. Yet, $\mathcal{V}(A) = 2$, $\mathcal{V}(B) = 0$ while $\mathcal{V}(A \cup B) = 4$. This contradicts (A3).

[21] For instance, with $x := (6, 6, 6)$, the function $U(x, \lambda_1, \lambda_2 . \lambda_3) := x_1 x_2 x_3 \lambda_1 \lambda_2 \lambda_3$ satisfies all of the conditions mentioned above.

Example A3 (Pareto Monotonicity). Given a weakly increasing function $U : X \times \mathbb{R}_+ \to \mathbb{R}$, for any $A \in \mathcal{A}$ set

$$\mathcal{V}(A) := \max_{x \in A} U\left(x, y_1^*(A) - x_1\right).$$

It is clear that the corresponding preference relation satisfies (A1) and (A3)-(A6). Now, set $U(x, \lambda) := x_1 \lambda$ for $(x, \lambda) \in X \times \mathbb{R}_+$. Put $y := (2, 1)$, $x := (1, 1)$ and $A := \{x, y\}$. Then $U(y, y_1^*(A) - y_1) = U(y, 0) = 0 < U(x, y_1^*(A) - x_1) = 1$. Thus, $\mathcal{V}(A) > \mathcal{V}(\{y\})$ while $\{y\} = \mathcal{P}(A)$, a contradiction to (A2).

Example A4 (Nonnegative Interdependence). In Definition 1, if we drop the condition that $U(\cdot, 0)$ is weakly increasing, the obtained representation would still satisfy (A1)-(A4) and (A6). Moreover, whenever the said condition fails, (A5) would also fail. An example of such a function is $U(x, \lambda) := x_1 - x_2 + \lambda$ for $(x, \lambda) \in \mathbb{R}^3_+$.

Appendix B. On the Existence of Continuous Utility Indices

As we discussed in Section 3, the behavior of a utility index U over the set $X_0 \times \mathbb{R}_{++}$ has no implications on the corresponding preference relation. Put differently, if U is a utility index for $\succsim$, so is any other real map on $X \times \mathbb{R}_+$ that coincides with U over $X \times \mathbb{R}_+ \backslash (X_0 \times \mathbb{R}_{++})$ and that satisfies part (i) of Definition 1. Therefore, in principle, one can obtain a utility index that is continuous over the entire set $X \times \mathbb{R}_+$ by an extension procedure. Specially, after finding a utility index that is continuous over $X \times \mathbb{R}_+ \backslash (X_0 \times \mathbb{R}_{++})$, one can hope to extend this function continuously to $X \times \mathbb{R}_+$ (without violating part (i) of Definition 1). However, this is not a straightforward issue because the set $X \times \mathbb{R}_+ \backslash (X_0 \times \mathbb{R}_{++})$ is not closed, making it impossible to appeal to known results (such as Tietze extension theorem). In fact, unless the utility index in question is uniformly continuous over a specific collection of bounded subsets of $X \times \mathbb{R}_+ \backslash (X_0 \times \mathbb{R}_{++})$, such an extension does not exist.[22] Thus, the problem of finding a continuous utility index seems to boil down to the problem of finding a utility

[22] Indeed, a continuous function over a compact set is uniformly continuous. So, if A is a bounded and relatively closed subset of $X \times \mathbb{R}_+ \backslash (X_0 \times \mathbb{R}_{++})$, we can find a continuous extension of the utility index in question only if it is uniformly continuous over A.

index that is uniformly continuous over a specific collection of sets. On the other hand, to the best of our knowledge, axiomatic foundations of uniformly continuous utility functions is an unexplored area. While this might be an interesting question for future research, it is beyond the scope of the present paper.

Appendix C. Proofs

As it is straightforward, we omit the "only if" part of the proof of Theorem 1. To prove the "if" part of Theorem 1 and Proposition 1, let $\succsim$ be a binary relation on $\mathcal{A}$ that satisfies (A1)-(A6). We start with the following claim, which is an immediate consequence of Pareto Monotonicity, as we noted earlier.

Claim 1. *$A \sim \mathcal{P}(A)$ for any $A \in \mathcal{A}$.*

Recall that if a set A belongs to $\mathcal{A}_{\mathcal{P}}$, any nonempty, closed subset of A that contains $y^*(A)$ also belongs to $\mathcal{A}_{\mathcal{P}}$. So, for each $A \in \mathcal{A}_P$, the set $\mathbf{A} := \{\{x, y^*(A)\} : x \in A\}$ is contained in $\mathcal{A}_{\mathcal{P}}$. In fact, $\mathbf{A}$ (equipped with the Hausdorff metric) is homeomorphic to A, and compact in particular. Thus, $\succsim$ admits a maximal set in $\mathbf{A}$ by Continuity axiom. That is, there exists an allocation $\overline{x}(A)$ in A such that $\{\overline{x}(A), y^*(A)\} \succsim \{x, y^*(A)\}$ for every $x \in A$. The following claim proves a related observation that we mentioned earlier.

Claim 2. *For any $A \in \mathcal{A}_{\mathcal{P}}$, we have $A \sim \{\overline{x}(A), y^*(A)\}$.*

Proof. Fix a set A that belongs to $\mathcal{A}_{\mathcal{P}}$, and let $\{x^1, ..., x^n, ...\}$ be a countable, dense subset of A. For every $n \in \mathbb{N}$, put $A^n := \{x^1, ..., x^n\} \cup \{\overline{x}(A), y^*(A)\}$. Pareto Monotonicity implies $A^1 \succsim \{\overline{x}(A), y^*(A)\}$. Moreover, $y^*(A^1) = y^*(A)$, and hence, either $A^1 \sim \{x^1, y^*(A)\}$ or $A^1 \sim \{\overline{x}(A), y^*(A)\}$ by Weak Instrumentalism. As $\{x^1, y^*(A)\} \precsim \{\overline{x}(A), y^*(A)\}$, either equivalence implies $A^1 \precsim \{\overline{x}(A), y^*(A)\}$, that is, $A^1 \sim \{\overline{x}(A), y^*(A)\}$. Similarly, either $A^2 \sim \{x^2, y^*(A)\}$ or $A^2 \sim A^1$, and in both cases, we have $A^2 \sim \{\overline{x}(A), y^*(A)\}$. Inductively, it follows that $A^n \sim \{\overline{x}(A), y^*(A)\}$ for every n. Moreover, since the sequence $A^1, A^2, ...$ is uniformly bounded in Euclidean norm and increases with respect to set inclusion, it is well known that $A^n \to \mathrm{cl}(\bigcup_{n=1}^{\infty} A^n)$ in Hausdorff metric (see, e.g., Dekel et al., 2001, Lemma 5). In turn, $\text{cl}(\bigcup_{n=1}^{\infty} A^n)$ equals A by construction. Hence, Continuity axiom implies $A \sim \{\overline{x}(A), y^*(A)\}$, as we sought. $\square$

Next, we establish the existence of a utility function over menus.

Claim 3. *There exists a function $V : \mathcal{A} \to \mathbb{R}$, which is continuous over $\mathcal{A}_{\mathcal{P}}$, such that $A \succsim B$ iff $V(A) \geq V(B)$, for every $A, B \in \mathcal{A}$.*

Proof. It is well-known that when endowed with Hausdorff metric, the space of all nonempty, compact subsets of $\mathbb{R}^k$ is separable. Therefore, as a subspace, $\mathcal{A}_{\mathcal{P}}$ is also a separable metric space. Hence, Debreu's classical theorem implies that there exists a continuous function $\widetilde{V} : \mathcal{A}_{\mathcal{P}} \to \mathbb{R}$ that represents $\succsim$ over $\mathcal{A}_{\mathcal{P}}$. In view of Claim 1, we can complete the proof by setting $V(A) := \widetilde{V}(\mathcal{P}(A))$ for every $A \in \mathcal{A}$. $\square$

In what follows, $\mathbf{0}$ stands for the origin of $\mathbb{R}^{k-1}$, and given an $x \in X$, we write x_{-1} instead of $(x_2, ..., x_k)$. It is important to note that for any $\lambda \in \mathbb{R}_+$ and $x \in X$, the set $\{x, (x_1 + \lambda, \mathbf{0})\}$ belongs to $\mathcal{A}$. Moreover, Pareto Monotonicity and Nonnegative Interdependence imply

$$B \succsim \{x\} \quad \text{for every } B \in \mathcal{A} \text{ and } x \in B.$$

We utilize these facts without further mention throughout the remainder of the proof.

The next claim proves useful.

Claim 4. *Let $x, y, y' \in X$ be such that $\{x, y\} \in \mathcal{A}_{\mathcal{P}}$, $\{x, y'\} \in \mathcal{A}$, $y \geq y'$ and $y_1 \geq x_1$. Then,*

$$V(\{x, y\}) \geq V(\{x, y'\}). \tag{4}$$

Proof. As $V(\{x, y\}) \geq \max\{V(\{x\}), V(\{y\})\} \geq \max\{V(\{x\}), V(\{y'\})\}$, (4) trivially holds if $V(\{x, y'\}) = \max\{V(\{x\}), V(\{y'\})\}$. In turn, if $V(\{x, y'\}) > \max\{V(\{x\}), V(\{y'\})\}$, the allocations x and y' must be distinct and Pareto incomparable (by Pareto Monotonicity), so that $\{x, y'\}$ belongs to $\mathcal{A}_{\mathcal{P}}$. Moreover, y'_1 must be larger than x_1, as we also have $x_{-1} \geq y_{-1} \geq y'_{-1}$. Therefore, in this case, (4) follows from Monotone Warm-Glow. $\square$

We now characterize the class of utility indices compatible with V.

Claim 5. *Let $U : X \times \mathbb{R}_+ \to \mathbb{R}$ be a function that satisfies Definition 1(i). Then, the following two conditions are equivalent:*

(i) $V(A) = \max_{x \in A} U\left(x, y_1^*(A) - x_1\right)$ *for all* $A \in \mathcal{A}_{\mathcal{P}}$.

(ii) $V\left(\{x, (x_1 + \lambda, \mathbf{0})\}\right) \geq U\left(x, \lambda\right) \geq V\left(\{x\}\right)$ *for all* $(x, \lambda) \in (X \times \mathbb{R}_+) \setminus (X_0 \times \mathbb{R}_{++})$, *and the former inequality holds with equality whenever* $\{x\} \prec \{x, (x_1 + \lambda, \mathbf{0})\} \succ \{(x_1 + \lambda, \mathbf{0})\}$.

Proof. First, suppose that (i) holds. Then, $V(\{x\}) = U\left(x, 0\right)$ for $x \in X$. Hence, the latter inequality in (ii) follows from the weak monotonicity of $U(x, \cdot)$. Now, take any $(x, \lambda) \in (X \times \mathbb{R}_+)\setminus(X_0 \times \mathbb{R}_{++})$. If $\lambda > 0$, the point x does not belong to X_0, meaning that $x_{-1} \neq \mathbf{0}$. So, we must have $\{x, (x_1 + \lambda, \mathbf{0})\} \in \mathcal{A}_{\mathcal{P}}$. Therefore, in this case, upon setting $A := \{x, (x_1 + \lambda, \mathbf{0})\}$, (i) immediately implies the first inequality in (ii), which must hold with equality whenever $V(\{x, (x_1+\lambda, \mathbf{0})\}) > U\left((x_1 + \lambda, \mathbf{0}), 0\right) = V(\{(x_1+\lambda, \mathbf{0})\})$. In turn, if $\lambda = 0$, Claim 1 implies $V\left(\{x, (x_1 + \lambda, \mathbf{0})\}\right) = V(\{x\})$, and hence, (ii) reduces to the equality $V\left(\{x\}\right) = U\left(x, 0\right)$, which we verified earlier.

We shall now show that (ii) implies (i). To this end, take any $A \in \mathcal{A}_{\mathcal{P}}$. Let us write $\overline{x}$ instead of $\overline{x}(A)$, and y^* instead of $y^*(A)$.

From Claim 2 and the definition of $\overline{x}$, it follows that

$$V(A) = V(\{\overline{x}, y^*\}) \geq V(\{x, y^*\}) \quad \text{for } x \in A. \tag{5}$$

Moreover, Claim 4 implies

$$V(\{x, y^*\}) \geq V(\{x, (y_1^*, \mathbf{0})\}) \quad \text{for } x \in A. \tag{6}$$

Next, we note that for any $x \in A$, the vector $(x, y_1^* - x_1)$ cannot belong to $X_0 \times \mathbb{R}_{++}$, as otherwise y^* would strictly Pareto dominate x, which contradicts the assumption that A consists of efficient allocations. Hence, the former inequality in (ii) implies $V(\{x, (y_1^*, \mathbf{0})\}) \geq U\left(x, y_1^* - x_1\right)$ for $x \in A$. Combining this observation with (5) and (6), we see that $V(A) \geq \sup_{x \in A} U\left(x, y_1^* - x_1\right)$.

To prove the converse inequality, obviously, it suffices to show that

$$V(\{\overline{x}, y^*\}) \leq \max_{x \in \{\overline{x}, y^*\}} U\left(x, y_1^* - x_1\right). \tag{7}$$

Clearly, $\max\left\{V(\{\overline{x}\}), V(\{y^*\})\right\} \leq \max_{x \in \{\overline{x}, y^*\}} U\left(x, y_1^* - x_1\right)$ by the latter inequality in (ii). Thus, (7) trivially holds whenever $\max\{V(\{\overline{x}\}), V(\{y^*\})\} = V(\{\overline{x}, y^*\})$. Assume therefore that $\max\{V(\{\overline{x}\}), V(\{y^*\})\} < V(\{\overline{x}, y^*\})$. Then, $\overline{x}$ and y^* must be distinct as well as Pareto incomparable, implying that $\overline{x}_{-1} \neq \mathbf{0}$ and $y_1^* > x_1$. Hence, $\{\overline{x}, (y_1^*, \mathbf{0})\}$ belongs to $\mathcal{A}_{\mathcal{P}}$. So, we can apply Monotone Warm-Glow, which implies $V(\{\overline{x}, y^*\}) \leq V(\{\overline{x}, (y_1^*, \mathbf{0})\})$. Since $V(\{(y_1^*, \mathbf{0})\}) \leq V(\{y^*\})$, it therefore follows that $\max\{V(\{\overline{x}\}), V(\{(y_1^*, \mathbf{0})\})\} < V(\{\overline{x}, (y_1^*, \mathbf{0})\})$. By the final statement in (ii), we must then have $U(\overline{x}, y_1^* - \overline{x}_1) = V(\{\overline{x}, (y_1^*, \mathbf{0})\})$. We thus conclude that $V(\{\overline{x}, y^*\}) \leq U(\overline{x}, y_1^* - \overline{x}_1)$. This proves (7).

It follows that $V(A) = \sup_{x \in A} U(x, y_1^* - x_1) = \max_{x \in \{\overline{x}, y^*\}} U(x, y_1^* - x_1)$. Finally, the latter equality implies that the function $x \to U(x, y_1^* - x_1)$ attains its maximum over A (either at $\overline{x}$ or y^*), which completes the proof. $\square$

Another useful observation related to the construction of utility indices is that whenever $\{x\} \sim \{x, (x_1 + \lambda, \mathbf{0})\}$ for some $x \in X \backslash X_0$, we must have $U(x, \lambda) = U(x, 0) = V(\{x\}) = V(\{x, (x_1 + \lambda, \mathbf{0})\})$, in view of Claim 5. On the other hand, when $\{x\} \prec \{x, (x_1 + \lambda, \mathbf{0})\} \sim \{(x_1 + \lambda, \mathbf{0})\}$, any value of $U(x, \lambda)$ between $V(\{x\})$ and $V(\{x, (x_1 + \lambda, \mathbf{0})\})$ qualifies, subject to monotonicity and continuity conditions demanded from the function U. In such cases, we shall set $U(x, \lambda) := V(\{x, (x_1 + \lambda, \mathbf{0})\})$ in the proof of Theorem 1, and $U(x, \lambda) := V(\{x\})$ in the proof of Proposition 1 upon a minor modification required for upper semi-continuity of U.

The next claim completes the proof of *Theorem 1.*

Claim 6. *Define* $U : X \times \mathbb{R}_+ \to \mathbb{R}$ *as* $U(x, \lambda) := V(\{x, (x_1 + \lambda, \mathbf{0})\})$ *for every* $(x, \lambda) \in X \times \mathbb{R}_+$. *Then, the function* U *is a utility index for* $\succsim$ *which is continuous over* $(X \times \mathbb{R}_+) \backslash (X_0 \times \mathbb{R}_{++})$.

Proof. Note that U trivially satisfies the conditions in Claim 5(ii). Moreover, $U(x, 0) = V(\{x, (x_1, \mathbf{0})\}) = V(\{x\})$ for every $x \in X$; so, $U(\cdot, 0)$ is weakly increasing on X by Nonnegative Interdependence. Now, fix an $x \in X$, and a pair of numbers λ, λ' such that $\lambda > \lambda' \geq 0$. If $x_{-1} = \mathbf{0}$, Claim 1 and Nonnegative Interdependence imply $V(\{x, (x_1 + \lambda, \mathbf{0})\}) = V(\{(x_1 + \lambda, \mathbf{0})\}) \geq V(\{(x_1 + \lambda', \mathbf{0})\}) = V(\{x, (x_1 + \lambda', \mathbf{0})\})$. On the other hand, if $x_{-1} \neq \mathbf{0}$, the set $\{x, (x_1 + \lambda, \mathbf{0})\}$ belongs to $\mathcal{A}_{\mathcal{P}}$, and Claim 4 implies $V(\{x, (x_1 + \lambda, \mathbf{0})\}) \geq V(\{x, (x_1 + \lambda', \mathbf{0})\})$. Therefore, in either case, $U(x,\lambda) \geq U(x,\lambda')$, which proves that $U(x,\cdot)$ is weakly increasing. In view of Claims 1 and 5, we conclude that U is a utility index for $\succsim$.

To verify continuity of U, take a sequence (x^n,λ^n) in $(X\times\mathbb{R}_+)\backslash(X_0\times\mathbb{R}_{++})$ that converges to a point (x,λ) which also belongs to $(X\times\mathbb{R}_+)\backslash(X_0\times\mathbb{R}_{++})$. Note that the sequence $\{x^n,(x_1^n+\lambda^n,\mathbf{0})\}$ converges to $\{x,(x_1+\lambda,\mathbf{0})\}$ in Hausdorff metric.

First, let us suppose $\lambda>0$ so that $x_{-1}\neq\mathbf{0}$ and $\{x,(x_1+\lambda,\mathbf{0})\}\in\mathcal{A}_{\mathcal{P}}$. It also follows that $\lambda^n>0$, $x_{-1}^n\neq\mathbf{0}$ and $\{x^n,(x_1^n+\lambda^n,\mathbf{0})\}\in\mathcal{A}_{\mathcal{P}}$ for all sufficiently large n. Therefore, the continuity of V over $\mathcal{A}_{\mathcal{P}}$ implies $\lim_n V(\{x^n,(x_1^n+\lambda^n,\mathbf{0})\})=V(\{x,(x_1+\lambda,\mathbf{0})\})$, that is, $\lim_n U(x^n,\lambda^n)=U(x,\lambda)$.

Suppose now $\lambda=0$ so that $U(x,\lambda)=V(\{x\})$. Recall that $U(x^n,\lambda^n)\geq V(\{x^n\})$ for every n. Moreover, V is continuous over the set $\{\{x\}:x\in X\}$ which is contained in $\mathcal{A}_{\mathcal{P}}$. Thus, it follows that $\liminf_n U(x^n,\lambda^n)\geq\lim_n V(\{x^n\})=U(x,\lambda)$. Since $\lambda^n=0$ implies $U(x^n,\lambda^n)=V(\{x^n\})$, without loss of generality we can assume $\lambda^n>0$ for every n, so that $x_{-1}^n\neq\mathbf{0}$. Then, $A^n:=\{x^n,(x_1^n+\lambda^n,\frac{n}{n+1}x_{-1}^n)\}$ belongs to $\mathcal{A}_{\mathcal{P}}$ for every n. Moreover, by construction $A^n\to\{x\}$ in Hausdorff metric, and $V(A^n)\geq V(\{x^n,(x_1^n+\lambda^n,\mathbf{0})\})=U(x^n,\lambda^n)$ by Claim 4 and the definition of U. Since V is continuous over $A_{\mathcal{P}}$, we therefore conclude that $U(x,\lambda)=\lim_n V(A^n)\geq\limsup_n U(x^n,\lambda^n)$; that is, $U(x,\lambda)=\lim_n U(x^n,\lambda^n)$. $\square$

Remark C1. In the proof of continuity above, it is important that the limit point (x,λ) does not belong to $X_0\times\mathbb{R}_{++}$. For example, suppose that $\succsim$ admits a utility index $\widetilde{U}$ which is continuous over $X\times\mathbb{R}_+$. Let V be the associated utility function over menus so that $V(A):=\max_{x\in A}\widetilde{U}(x,y_1^*(A)-x_1)$ for $A\in\mathcal{A}_{\mathcal{P}}$. Fix a vector $(x,\lambda)\in X_0\times\mathbb{R}_{++}$, and note that $V(\{x,(x_1+\lambda,\mathbf{0})\})=V(\{(x_1+\lambda,\mathbf{0})\})=\widetilde{U}((x_1+\lambda,\mathbf{0}),0)$, as $(x_1+\lambda,\mathbf{0})$ Pareto dominates x. But it may well be the case that $\widetilde{U}(x,\lambda)>\widetilde{U}((x_1+\lambda,\mathbf{0}),0)$, implying $\lim_{x'\to x}\widetilde{U}(x',\lambda)>\widetilde{U}((x_1+\lambda,\mathbf{0}),0)$. In particular, for $x'\in X$ with $x'_{-1}\neq\mathbf{0}$ and $x'_1=x_1$, we may have $\lim_{x'\to x}V(\{x',(x'_1+\lambda,\mathbf{0})\})>V(\{x,(x_1+\lambda,\mathbf{0})\})$. So, the utility index in Claim 6 need not be upper semi-continuous at a point that belongs to $X_0\times\mathbb{R}_{++}$, even if V is induced by a continuous utility index. As we discussed in Appendix B, the characterization of the existence of a utility index that is also continuous over $X_0\times\mathbb{R}_{++}$ appears to be a challenging problem, which we leave open.

We now prove *Proposition 1*:

Claim 7. *Let X_c be the set of critical allocations, and define a function $U : X \times \mathbb{R}_+ \to \mathbb{R}$ as*

$$U(x,\lambda) := \begin{cases} V(\{x,(x_1+\lambda,\mathbf{0})\}) & \text{if } x \in \text{cl}(X_c), \\ V(\{x\}) & \text{otherwise.} \end{cases}$$

Then, U is a utility index for $\succsim$ which satisfies conditions (i)-(iii) *of Proposition 1.*

Proof. It is obvious that U satisfies the conditions in Claim 5(ii) and that $U(\cdot,0)$ is weakly increasing on X. Moreover, the proof of Claim 6 shows that the function $\lambda \to V(\{x,(x_1+\lambda,\mathbf{0})\})$ is weakly increasing on $\mathbb{R}_+$, which implies that $U(x,\cdot)$ is also weakly increasing on $\mathbb{R}_+$ for each $x \in X$. It follows that U is a utility index for $\succsim$. By definitions, it is also clear that part (ii) of Proposition 1 is true for any utility index, and that the function U that we constructed satisfies part (i).

To prove that U is upper semi-continuous, take a sequence (x^n,λ^n) in $(X\times\mathbb{R}_+)\backslash(X_0\times\mathbb{R}_{++})$ that converges to a point (x,λ) which also belongs to $(X\times\mathbb{R}_+)\backslash(X_0\times\mathbb{R}_{++})$. Recall that by continuity of V over singletons, we have $\lim_n V(\{x^n\}) = V(\{x\})$.

Without loss of generality, we can assume either (a) $x^n \in X\backslash\,\text{cl}(X_c)$ for every n; or (b) $x^n \in \text{cl}(X_c)$ for every n. In case (a), we have $U(x^n,\lambda^n) = V(\{x^n\})$ for every n, and hence, $\lim_n U(x^n,\lambda^n) = V(\{x\}) \leq U(x,\lambda)$, where the weak inequality follows from the definition of U. On the other hand, in case (b), the point x also belongs to $\text{cl}(X_c)$. So, we have $U(x,\lambda) = V(\{x,(x_1+\lambda,\mathbf{0})\})$ and $U(x^n,\lambda^n) = V(\{x^n,(x_1^n+\lambda^n,\mathbf{0})\})$ for every n. Thus, in this case, Claim 6 implies that $U(x,\lambda) = \lim_n U(x^n,\lambda^n)$. □

Proposition 2 can easily be verified using the utility indices in Claims 6 or 7. In turn, *Lemma 1* holds since in the above construction, we have utilized continuity of V only to deduce the desired continuity properties of the utility indices. So, it only remains to prove Propositions 3-6.

Proof of Proposition 5. We omit the "if" part of the proof, which is a routine exercise. For the "only if" part, by Efficiency it suffices to focus on a menu A that belongs to $\mathcal{A}_{\mathcal{P}}$. Let us write y^* instead of $y^*(A)$, and suppose $y^* \notin \mathbf{C}_U(A)$. Pick any $x \in \mathbf{C}_U(A)$. Then, $U(x, y_1^* - x_1) > U(y^*, 0)$, so that $\{x, y^*\} \succ \{y^*\}$. But then, Sophistication implies $\mathbf{C}(\{x, y^*\}) = \{x\}$. So, by Weak WARP, we must have $y^* \notin \mathbf{C}(A)$. Now, pick any $x' \in \mathbf{C}(A)$, and suppose by contradiction that $x \notin \mathbf{C}(A)$. Then, from Weak WARP it follows that $\mathbf{C}(\{x', x, y^*\}) = \{x'\}$, while Sophistication implies $\{x', x, y^*\} \succ \{x, y^*\}$. In turn, the latter condition would imply $U(x', y_1^* - x_1') > U(x, y_1^* - x_1)$, which contradicts the hypothesis that x belongs to $\mathbf{C}_U(A)$. So, we conclude that $\mathbf{C}_U(A) \subseteq \mathbf{C}(A)$. To prove the converse inclusion, suppose now that $x' \notin \mathbf{C}_U(A)$. Then, $U(x, y_1^* - x_1) > \max\{U(y^*, 0),\ U(x', y_1^* - x_1')\}$. But this amounts to saying $\{x, x', y^*\} \succ \{x', y^*\}$, while Weak WARP implies $x' \in \mathbf{C}(\{x, x', y^*\})$, a contradiction to Sophistication. So, we also have $\mathbf{C}(A) \subseteq \mathbf{C}_U(A)$, as we sought. This completes the proof of (i).

It remains to show that $y^* \notin \mathbf{C}(A)$ implies $y^* \notin \mathbf{C}_U(A)$. To this end, suppose $y^* \notin \mathbf{C}(A)$ and, as before, pick an arbitrary $x' \in \mathbf{C}(A)$. Then, $\mathbf{C}(\{x', y^*\}) = \{x'\}$ by Weak WARP, and $\{x', y^*\} \succ \{y^*\}$ by Sophistication. So, it follows that $U(x', y_1^* - x_1') > U(y^*, 0)$, and hence, $y^* \notin \mathbf{C}_U(A)$. $\square$

Note that *Proposition 3* is an immediate consequence of Proposition 5, because a choice correspondence of the form $\mathbf{C}_{\widetilde{U}}$ satisfies (H1)-(H3) for any utility index $\widetilde{U}$. We proceed to:

Proof of Proposition 6. Let U be a regular utility index that is continuous over $(X \times \mathbb{R}_+) \backslash (X_0 \times \mathbb{R}_{++})$ so that $\mathbf{C}_U = \mathbf{C}_{\succsim}$. We omit the "if" part. For the "only if" part, suppose that $(\mathbf{C}, \succsim)$ satisfies (H1)-(H5). By Efficiency, without loss of generality we can focus on a set A that belongs to $\mathcal{A}_{\mathcal{P}}$. Moreover, in view of Proposition 5, we can assume $y^*(A) \in \mathbf{C}_U(A) \cap \mathbf{C}(A)$. Now, take any $x \in \mathbf{C}(A)$ that is distinct from $y^*(A)$, and suppose by contradiction that $x \notin \mathbf{C}_U(A)$. Then, $U(x, y_1^*(A) - x_1) < U(y^*(A), 0)$. By continuity of U and definition of a utility index, clearly, it then follows that there exists a neighborhood $\mathcal{N}$ of $\{x, y^*(A)\}$ in $\mathcal{A}_{\mathcal{P}}$ such that $\{x', y'\} \sim \{y'\}$ for every $\{x', y'\} \in \mathcal{N}$ with $y_1' > x_1$. But then, Choice Regularity implies $\mathbf{C}(\{x, y^*(A)\}) = \{y^*(A)\}$, whereas Weak WARP implies $\mathbf{C}(\{x, y^*(A)\}) = \{x, y^*(A)\}$, a contradiction. Conversely, take any $\widehat{x} \in \mathbf{C}_U(A)$ that is distinct from $y^*(A)$ so that $U(\widehat{x}, y_1^*(A) - \widehat{x}_1) = U(y^*(A), 0)$. From regularity of U, it then follows that there exists a sequence $\{x^n, y^n\}$ in $\mathcal{A}_{\mathcal{P}}$ that converges to $\{\widehat{x}, y^*(A)\}$ such that $U(x^n, y_1^n - x_1^n) > U(y^n, 0)$ and $y_1^n > x_1^n$ for every n. Clearly, we must also have $\lim_n x^n = \widehat{x}$. Moreover, Proposition 5(i) implies $\mathbf{C}(\{x^n, y^n\}) = \{x^n\}$ for every n. So, by Closed Graph, we see that $\widehat{x} \in \mathbf{C}(\{\widehat{x}, y^*(A)\})$. Thus, the desired conclusion follows from Weak WARP: $\widehat{x} \in \mathbf{C}(A)$. $\square$

For the sake of completeness, we also provide a proof of Proposition 4, which simply follows the second part of the proof of Proposition 6.

Proof of Proposition 4. In view of Proposition 3, it suffices to show that $\mathbf{C}_U(A) = \mathbf{C}_{\widetilde{U}}(A)$ for any $A \in \mathcal{A}_{\mathcal{P}}$ such that $y^*(A) \in \mathbf{C}_U(A) \cap \mathbf{C}_{\widetilde{U}}(A)$. Let A be such a set, and take any $\widehat{x} \in \mathbf{C}_U(A)$ that is distinct from $y^*(A)$ so that $U(\widehat{x}, y_1^*(A) - \widehat{x}_1) = U(y^*(A), 0)$. Then, as in the proof of Proposition 6, regularity of U and Proposition 3(i) imply that there exists a sequence $\{x^n, y^n\}$ in $\mathcal{A}_{\mathcal{P}}$ that converges to $\{\widehat{x}, y^*(A)\}$ such that $\lim_n x^n = \widehat{x}$ and $\mathbf{C}_{\widetilde{U}}(\{x^n, y^n\}) = \{x^n\}$ for every n. Since $\mathbf{C}_{\widetilde{U}}$ satisfies Closed Graph, it follows that $\widehat{x} \in \mathbf{C}_{\widetilde{U}}(A)$. So, $\mathbf{C}_U(A) \subseteq \mathbf{C}_{\widetilde{U}}(A)$, and symmetrically, we also have $\mathbf{C}_U(A) \supseteq \mathbf{C}_{\widetilde{U}}(A)$. $\square$

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