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Risk Premia: Short and Long-term

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Abstract

Hansen (2011) considers risks associated with cash flows at alternative horizons. He shows that in the long run many investor preference specifications do not imply risk premia substantially different from those implied by simple expected utility model. The main result of this paper is that the generalized disappointment aversion model of Routledge & Zin (2010) amplifies risk premium not only in the short run but also for assets that pay off long into the future. The reason behind this result is that this utility alters the risk-neutral distribution of future economy outcomes relative to the expected utility. The latter shares the risk-neutral distribution with more general Epstein & Zin (1989) recursive utility. I also analyze the risk premium term structure implied by the three utilities and find that its slope critically depends on the volatility of the transient payoff component with high enough volatility leading to negatively sloped term structure.

Keywords: term structure of risk premium; Epstein-Zin utility; generalized disappointment aversion; finite state economy; Markov chain
JEL Classification: G12

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1 Introduction

The financial economics literature on risk-return trade-off and risk compensation is voluminous and growing. Most of the theoretical developments in this area focus on short or even instantaneous time frames. Very little is known about risk and its compensation for agents who invest over a long time span. The main interest of this paper is the premium on the asset dividends that are paid beyond an immediate next period. In numerous attempts to resolve conflicts between theory and empirics, asset pricing models have been extended to include a variety of different preference structures. As Hansen (2011) shows, many of these preferences have only transient implications and make no difference in pricing payoffs with long maturity. Examples of these include recursive utility of Epstein & Zin (1989), habit formation models of Constantinides (1990), Campbell & Cochrane (1999), Santos & Veronesi (2010), or the model of temporal risk aversion of Van den Heuvel (2008).[1] Here I show that the generalized disappointment aversion[2] model of Routledge & Zin (2010) brings a new important dimension into analysis of risk compensation at long horizons.

The main result of this paper is that the generalized disappointment aversion (GDA) utility function amplifies risk premia at all investment horizons in comparison to the simple expected utility function. Instead, Epstein-Zin utility delivers very similar long-term premia, but different short-term premia compared to expected utility. In other words generalized disappointment aversion is a permanent transformation of the expected utility function, in that it amplifies risk premia at all investment horizons. Instead, Epstein-Zin utility is approximately transient transformation of expected utility, implying very similar long-term, but different short-term premia.

The main feature of the GDA utility that produces substantially different risk premia for any horizon is its asymmetric response to different economic outcomes through the implied skewed risk-neutral probabilities. In contrast, Epstein-Zin model, although makes investors forward looking, shares the same risk-neutral distribution with expected utility, at least in continuous time (Hansen, 2011). Investors with GDA recursive preferences, besides being forward-looking, are disappointed disproportionately more by a recession than they are pleased with an equally positive economic outcome. This attitude toward risk shifts risk-neutral probabilities toward the bad outcomes of the economy. The risk-neutral probability shift reduces asset prices, and it increases expected risky returns. Routledge & Zin (2010) and later Bonomo et al. (2011) in a different model with long run risks show that GDA preferences are able to amplify returns at the short investment horizon. Here I extend this conclusion to the long horizon. Empirical analysis supports this finding.

[1]Also see Backus et al. (2004) for other preference structures.

[2]Disappointment aversion concept was introduced by Gul (1991).

I also analyze the risk premium term structure implied by the three utilities and find that its slope critically depends on the volatility of the transient payoff component with high enough volatility leading to negatively sloped term structure. In this sense the paper is related to Daniel & Marshall (1997); Lemke & Werner (2009); Berg (2010) and Van Binsbergen et al. (2011). But at the same time the distribution of the transient component only plays a role in the short run while in the long run state dependence of the premium disappears. The long run limit of the term structure only depends on the growth characteristics of the cash flows and the risk-neutral distribution implied by a particular preference structure.

In this paper I consider only three utility functions, expected utility, Epstein-Zin, and GDA. One reason for this choice is that each preference structure in this list is a one step more general than the previous which make a very appealing comparison between the three. Besides, Epstein-Zin utility became a workhorse model for financial economics. Finally, GDA model has superior ability to reproduce financial market phenomena.

The main result of the paper is based on the methodology of Hansen & Scheinkman (2009) and Alvarez & Jermann (2005) who propose multiplicative decomposition of stochastic discount factors into exponential growth, a permanent component, and a transient component. Exponential growth determines long-term returns. The permanent component alters the probability distribution and incorporates the preference structure into risk-neutral probabilities. This component is the main factor in the pricing of the long-dated cash flows. The transient component only affects pricing at short investment horizons. Bansal & Yaron (2004) use this short-term feature of Epstein-Zin utility to build a consumption-based model and solve an equity premium puzzle at the short investment horizon. But in the long run many asset differences disappear and leave an investor's portfolio exposed to a few defining trends similar for many utility specifications.

In this paper I consider an economy with a finite number of states and construct a consumption-based asset pricing model of dividend cash flows with alternative maturities. My approach generalizes the model of Mehra & Prescott (1985) in two directions. First, the number of states is any finite number, including two. Second, it allows for pricing not only consumption flow, but also any dividend process with its unique characteristics. The last but the most important generalization is that the model allows for pricing of payoffs at any future horizon. The discrete time finite state model considered here has two tightly linked advantages. First, the implications are very robust to the number of states of the economy. Second, in the two-state model all the parameters have a clear and intuitive interpretation in terms of preference parameters, consumption persistence, means and variances of cash flows.

I document the parametric sensitivity of the model for short- and long-term risk premia. In particular, I show that the Epstein-Zin and GDA implied risk premia are not sensitive to the elasticity of intertemporal substitution (EIS). Hansen et al. (2008) price a cross-section of stock returns at long horizons and find that the EIS is not a significant factor in determining long-term portfolio returns. In this paper I generalize this finding to a framework in which the stochastic discount factor is not approximated around unit EIS. Moreover, I document that the weak dependence on EIS pertains to more general recursive GDA utility. This result is interesting in the light of active debate on the range of values for EIS.

Here I show that in a discrete time model the Epstein-Zin modification of expected utility is not exactly transient. This goes in contrast to the continuous time model of Hansen (2011) where he shows that Epstein-Zin modification only affects the transient component. In my model it modifies risk-neutral probabilities, or the permanent component, as well. Empirical analysis confirms this finding although it shows that the deviation is negligible.

The paper is organized as follows. Section 2 introduces basic assumptions on the model economy. In the same section I outline asymptotic results for pricing of long-dated payoffs. Section 3 presents theoretical analysis of the model and shows that GDA modification of the expected and Epstein-Zin utilities has a permanent effect and amplifies premia at any horizon. Section 4 gives a description of the data and estimation method. Section 5 presents the analysis of empirical implications of the model and its sensitivity analysis.[3] Section 6 concludes.

2 Asset Pricing Model

2.1 Consumption and dividend dynamics

In this section I describe the joint dynamics of consumption and asset dividends. Then I show how to price any asset given its dividend stream and probability distribution.

The aggregate consumption stream C_t grows at rate equal to x_t. The dividend stream D_t growth rate is y_t. Each state of the economy is characterized by a pair of consumption and dividend growth rates, $\mathbf{z}_t = (x_t, y_t)$, taking values in a finite set, $\{(\lambda_1, \nu_1), \ldots, (\lambda_n, \nu_n)\}$ with each state characterized by a pair $z_i = (\lambda_i, \nu_i)$. The finite state probability transition $n \times n$ matrix $\mathbb{P} = [p_{ij}]_{i,j=1}^n$ describes the evolution of a two-dimensional random variable, with elements corresponding to consumption and dividend growth rates,

$$p_{i,j} = P\left[(x_{t+1}, y_{t+1}) = (\lambda_j, \nu_j) | (x_t, y_t) = (\lambda_i, \nu_i)\right].$$

[3] The numerical study considers for simplicity of exposition only two-state economy. However, additional numerical work immediately expandable to an arbitrary number of states is available upon request. It shows that the main conclusions are robust to the number of states.

For future convenience define separately the vector of consumption growth rates $\lambda = (\lambda_1, \ldots, \lambda_n)^T$ and dividend growth rates $\nu = (\nu_1, \ldots, \nu_n)^T$. Along with that define two diagonal matrices: $\Lambda = \text{diag}(\lambda)$, and $\Theta = \text{diag}(\nu)$.

With this first order Markov structure it is rather easy to redefine conditional expectation through the matrix language. For any function $\psi(z_i)$ of a state we can define the conditional expectation operator in matrix language as

$$
\begin{aligned}
E[\psi(\mathbf{z}_{t+1}) | \mathbf{z}_t = z_i] &= \sum_{j=1}^{n} P[\mathbf{z}_{t+1} = z_j | \mathbf{z}_t = z_i] \psi(z_j) \\
&= \sum_{j=1}^{n} p_{ij} \psi(z_j) = [\mathbb{P}\psi]_i \,,
\end{aligned}
$$

where I denote the vector $\psi = [\psi(z_1), \ldots, \psi(z_n)]^T$, and $[\mathbb{P}\psi]_i$ is the ith element of n-vector $\mathbb{P}\psi$. In other words, the conditional expectation can be fully described by defining conditional expectation operator as $\mathbb{E}_t\psi = \mathbb{P}\psi$, the product of probability transition matrix and a vector of possible future outcomes. Now recall that the solution, if it exists, of the system of linear equations $\pi = \mathbb{P}^T\pi$ is the marginal probability distribution of the state vector, $\pi_i = P[\mathbf{z}_t = z_i]$. Using this definition one could also define unconditional expectation $\mathbb{E}\psi = \pi^T\psi$ which in other words is an inner product of two vectors.

Consider the cash flow decomposed into a growth component y_{t+1} and a transient component f_{t+1} so that the future time $t+1$ payoff is given by their product $(y_{t+1}f_{t+1})$. Then the current conditional expectation is computed as

$$
\sum_{j=1}^{n} p_{ij}\nu_j f_j = [\mathbb{P}\Theta f]_i = [\mathbb{G}f]_i \,.
$$

This formula provides for the definition of a new matrix $\mathbb{G}$. I would call it a conditional growth matrix. The intuition behind this definition is the following. Say we have fixed probabilities of transition between economy states and dividend growth rates ν_j that are common for several assets. At the same time these assets differ in stationary payoff component f. Then one can compute the expected value of future payoffs by simply multiplying by the conditional growth matrix $\mathbb{G}$. This approach allows separate analysis of growth component on pricing of future cash flows.

The canonical asset pricing model is

$$
P_t = E_t[S_{t,t+1} \times (y_{t+1}f_{t+1})] \,,
$$

where P_t is the current value of the future payoff $(y_{t+1}f_{t+1})$ growing at the rate y_{t+1}, and

$S_{t,t+1}$ is the stochastic discount factor. Expectation is taken with respect to the current information set. In the context of this paper, that is, in a finite state economy, the price of a future asset payoff is given by a weighted sum of the possible future outcomes:

$$P_i = \sum_{j=1}^{n} p_{ij} s_{ij} \nu_j f_j = [\mathbb{S}\Theta f]_i = [\mathbb{Q} f]_i \,, \quad \text{or} \quad P = \mathbb{Q} f.$$

The weights are given by conditional transition probabilities and possible values of the stochastic discount factor. After all each state dependent price value is an element of a vector itself represented as a product of the matrix $\mathbb{Q}$ and the vector of transient component f of the future payoff. The matrix $\mathbb{Q}$ may be understood as a conditional growth adjusted pricing matrix. Applying this transition matrix to extracted current payoff component one obtains its current price. Once again, this approach makes it possible to analyze the effect of cash flow stochastic growth rates on asset prices and returns of several assets.

The above equation may be used to price the unit riskless payoff, which is simply a vector of one, corresponding to each state of nature. Besides, such a payoff does not have a growth component, only state-independent component. Hence, the price of a riskless payoff is

$$P_i^f = \sum_{j=1}^{n} p_{ij} s_{ij} = [\mathbb{S}\mathbf{1}_n]_i \,, \quad \text{or} \quad P^f = \mathbb{S}\mathbf{1}_n.$$

This is essentially the price of a plain vanilla bond. The formula presents a third definition of a useful transition matrix $\mathbb{S}$. This matrix is also a conditional pricing matrix as the above defined $\mathbb{Q}$. But the critical difference is the absence of growth component in its definition. Simply put, this transition matrix maps future stationary payoffs to their current prices.

At this point I defined the expectation of growing payoffs, their price, and a price of a riskless bond. This information is sufficient to define expected returns of risky and riskless cash flows. Taking the element-wise reciprocal of the bond price vector, $R_1^f = (\mathbb{S}\mathbf{1}_n)^{-1}$, and transforming to log scale I obtain generic formula for the log risk-free return:

$$\log R_1^f = -\log\left(\mathbb{S}\mathbf{1}_n\right).$$

Subscript 1 is a reference to the unit horizon of the return. The above definitions make it natural to define a one period return in log scale as a ration between expected payoff and its current price:

$$\log R_1^e = \log\left(\mathbb{G} f\right) - \log\left(\mathbb{Q} f\right).$$

The natural extension of computing prices for securities paying off in one period is pricing of cash flows over different horizons. In finite space this extension reduces to taking matrix powers. With a slight abuse of notation this statement is proven below using the law of iterated expectations:

$$
\begin{aligned}
P_{t,t+h} =& E_t\left[S_{t,t+h} \times \left(\prod_{i=1}^{h} y_{t+i} f_{t+h}\right)\right] \\
=& E_t\left[S_{t,t+h-1} \prod_{i=1}^{h-1} y_{t+i} \times E_{t+h-1}\left[S_{t+h-1,t+h} \times (y_{t+h} f_{t+h})\right]\right] \\
=& E_t\left[S_{t,t+h-1} \prod_{i=1}^{h-1} y_{t+i} \times \mathbb{S}\Theta f\right] \\
=& E_t\left[S_{t,t+h-2} \prod_{i=1}^{h-2} y_{t+i} \times (\mathbb{S}\Theta)^2 f\right] \\
\ldots& \\
=& (\mathbb{S}\Theta)^h f = \mathbb{Q}^h f.
\end{aligned}
$$

The above formula is the application of canonical asset pricing model to the cash flow maturing in h periods. The payoff is decomposed into transient component f_{t+h} and stochastic multiplicative growth component $\prod_{i=1}^{h} y_{t+i}$ which is simply an accumulation of stochastic growth rates y_{t+i} over h consecutive periods. This payoff is discounted by an appropriate stochastic discount factor $S_{t,t+h}$. Finally, the conditional expectation with respect to a current information set is taken to obtain the price denoted by $P_{t,t+h}$. The second line above uses the law of iterated expectations to isolate the last period before the maturity. It also uses the multiplicative property of the stochastic discount factor. The conditional expectation with respect to information set at time $t+h-1$ of the discounted growing payoff may be written in matrix form as $\mathbb{S}\Theta f$ or simply $\mathbb{Q}f$. Repeating the cycle I obtain the final expression of $\mathbb{Q}^h f$ which matches the future stochastically growing payoff with its current price given a specific stochastic discount factor.

The above defined prices of distant cash flows or dividends may be used to define the familiar price of security paying growing dividend stream into the infinite future. The definition is simply the infinite sum of the prices indexed by payoff horizon,

$$P_t = \sum_{i=1}^{\infty} P_{t,t+i} = \sum_{i=1}^{\infty} \mathbb{Q}^i f = (\mathbb{Q} - \mathbb{I})\, f.$$

Using the above defined prices of horizon specific cash flows we can compute corresponding log return, risk-free rate, and premium indexed by horizon h, respectively:

$$\begin{aligned}
\log R_h^e &= \frac{1}{h}\left[\log\left(\mathbb{G}^h f\right) - \log\left(\mathbb{Q}^h f\right)\right], \\
\log R_h^f &= -\frac{1}{h}\log\left(\mathbb{S}^h \mathbf{1}_n\right), \\
RP_h^e &= E\left[\log R_h^e - \log R_h^f\right].
\end{aligned}$$

These values give a characterization of returns and premia for any general though finite horizon. It turns out that the limit of the risk premium expression above for the infinite payoff horizon has a precise mathematical expression. Defining $\rho(\mathbb{M})$ as a principal eigenvalue of a matrix $\mathbb{M}$, namely the eigenvalue of $\log \mathbb{M}$ with the largest real part, it can be shown that

$$\begin{aligned}
RP_\infty^e &= \lim_{h\to\infty}\frac{1}{h}\left[\log\left(\mathbb{G}^h f\right) - \log\left(\mathbb{Q}^h f\right) + \log\left(\mathbb{S}^h \mathbf{1}_n\right)\right] \\
&= \rho(\mathbb{G}) - \rho(\mathbb{Q}) + \rho(\mathbb{S}).
\end{aligned}$$

Here each term in the sum converges to its corresponding infinite horizon limit represented by the principal eigenvalue $\rho(\cdot)$. This convergence result does not exhaust the importance of principal eigenvalues for asset pricing. Following Hansen & Scheinkman (2009) I show that they play an important role in decomposition of asset returns at any payoff horizon.

2.2 Pricing with expected utility

Define intertemporal expected utility function (or simply expected utility further in the text) as

$$U_0 = E_0\left[\sum_{t=0}^{\infty}\beta^t \frac{C_t^{1-\gamma}-1}{1-\gamma}\right],$$

where β is subjective discount factor and γ is a coefficient of relative risk aversion. Stochastic discount factor (SDF) can be written as

$$S_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

Take for the purpose of illustration $\beta = 1$ and $\gamma = 0$. This combination of parameters implies linear utility function $U_0 = \sum_{t=0}^{\infty} E_0 C_t$, and a constant stochastic discount factor. This is exactly the case of a risk-neutral investor who does not discount different states of the economy differently and derives her utility from the expected aggregate consumption over infinite time.

The Euler equation that prices next period random payoff f_{t+1} for this particular SDF is

$$P_t = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\psi_{t+1}\right].$$

Given the finite state model described above, I can write the price as

$$P_i = \beta\sum_{j=1}^{n} p_{ij}\lambda_j^{-\gamma}\psi_j, \quad \text{or} \quad P = \beta\mathbb{P}\Lambda^{-\gamma}\psi.$$

This expression shows that the power utility model is characterized by the following conditional state price matrix:

$$\mathbb{S}_X = \beta\mathbb{P}\Lambda^{-\gamma}. \tag{1}$$

2.3 Pricing with the Epstein-Zin utility

In this section I obtain pricing operators implied by a particular case of the Epstein-Zin utility function.

Define the Kreps-Porteus special case of Epstein-Zin (EZ thereafter) recursive utility function (Kreps & Porteus, 1978; Epstein & Zin, 1989) as:

$$V_t = \left[C_t^{1-\frac{1}{\sigma}} + \beta\left[\mathcal{R}_t\left(V_{t+1}\right)\right]^{1-\frac{1}{\sigma}}\right]^{\frac{1}{1-\frac{1}{\sigma}}},$$

where risk adjustment or certainty equivalent is given by[4]

$$\mathcal{R}_t\left(V_{t+1}\right) = \left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}.$$

SDF implied by such a utility function can be written as

$$S_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}}\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right)^{\frac{1}{\sigma}-\gamma}.$$

Compare this SDF with the corresponding expression for the power utility function. Note that if one imposes the restriction that the risk aversion parameter is the reciprocal of the intertemporal substitution coefficient[5], $\gamma = \frac{1}{\sigma}$, then Epstein-Zin SDF reduces to the one implied by the expected utility function.

As it was shown by Epstein & Zin (1989) the EZ utility implies the following SDF in

[4]Some authors use another parametrization such as $\rho = 1 - \frac{1}{\sigma}$ and $\mathsf{a} = 1 - \gamma$. Such definitions are useful for reconciliation of results from different sources.

[5]$\sigma > 1/\gamma\,(< 1/\gamma)$ corresponds to preferences for early (late) resolution of uncertainty, respectively.

terms of the well-known and measurable quantities:

$$S_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{\frac{1-\gamma}{1-\sigma}} \left(\beta R^m_{t,t+1}\right)^{\frac{1-\gamma\sigma}{\sigma-1}},$$

where $R^m_{t,t+1}$ is the total one-period return on market portfolio. Hence the conditional state price matrix is

$$\begin{aligned}
\mathbb{S}_Z =& \beta \mathbb{P} \Lambda^{\frac{1-\gamma}{1-\sigma}} \odot (\beta R^m)^{\frac{1-\gamma\sigma}{\sigma-1}} \\
=& \mathbb{S}_X \odot \left(\beta R^m \Lambda^{-1}\right)^{\frac{1-\gamma\sigma}{\sigma-1}} \\
=& \mathbb{S}_X \odot \mathcal{K},
\end{aligned} \tag{2}$$

As it is commonly used in the literature (see, e.g., Garcia & Renault, 1998), market payoff is the total endowment of the economy, C_t. Hence, the return is expressed as $R^m_{t,t+1} = \frac{P^m_{t+1}+C_{t+1}}{P^m_t}$, where P^m_t is the price of market portfolio. In the context of the current model consumption growth rates are known but market returns necessary for asset pricing are not. I postpone the discussion on market return solution until all stochastic discount factors are introduced.

Clearly, the asset price computed using the EZ utility coincides with the one from the expected utility with the restriction that $1/\sigma = \gamma$. As a confirmation that SDF derived from the EZ utility, (2) reduces to SDF in expected utility (1).

For the sake of illustration let $\beta = 1$ and $\gamma = 0$. This extreme case implies that the SDF is not constant and equal to

$$S_{t,t+1} = \left(R^m_{t,t+1} \frac{C_t}{C_{t+1}}\right)^{\frac{1}{\sigma-1}}.$$

Note that $\gamma = 0$ corresponds to the risk-neutral agent. Even with such an agent, the SDF remains random and will certainly covary with consumption equity and even with some dividend streams. Thus, the risk premium is not expected to be zero even for a risk-neutral agent.

2.4 Pricing with GDA utility

The SDF for Generalized Disappointment Aversion (GDA) developed by Routledge & Zin (2010) is

$$S_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma}} \left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right)^{\frac{1}{\sigma}-\gamma} \frac{1 + (\alpha - 1)\, I\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} < \kappa\right)}{1 + \kappa^{1-\gamma}(\alpha - 1)\, E_t I\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})} < \kappa\right)},$$

where parameter $\alpha \geq 1$ is called disappointment aversion, and κ is called disappointment threshold. They show that the SDF may be rewritten as

$$S_{t,t+1} = h_{t+1}^{1-\gamma} \left(R_{t,t+1}^m\right)^{-1} \frac{1 + (\alpha - 1) I \left(h_{t,t+1} < \kappa\right)}{1 + \kappa^{1-\gamma} (\alpha - 1) E_t I \left(h_{t,t+1} < \kappa\right)},$$

where

$$h_{t,t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{\frac{1}{1-\sigma}} \left(\beta R_{t,t+1}^m\right)^{\frac{\sigma}{\sigma-1}}$$

depends on aggregate consumption growth rates and return on market portfolio.

In matrix notation that would look as follows:

$$\mathbb{S}_G = \mathbb{S}_Z \odot \left[\frac{1 + (\alpha - 1) I (H < \kappa)}{1 + \kappa^{1-\gamma} (\alpha - 1) \left[\mathbb{P} \odot I (H < \kappa)\right] \mathbf{1}_n \mathbf{1}_n^T}\right] = \mathbb{S}_Z \odot \mathcal{G}, \tag{3}$$

where the division in the brackets is element-wise matrix division. Also, $I (H < \kappa)$ stands as an indicator function. If an element of matrix H is less than parameter κ, then the corresponding element of the matrix $I (H < \kappa)$ is equal to one, otherwise zero. Conditional expectation $E_t I \left(h_{t,t+1} < \kappa\right)$ is represented by the $n \times 1$ vector $\left[\mathbb{P} \odot I (H < \kappa)\right] \mathbf{1}_n$. Finally,

$$H = (\beta R^m)^{\frac{\sigma}{\sigma-1}} \Lambda^{\frac{1}{1-\sigma}}.$$

Note that whenever $\alpha = 1$ we are back to the case of EZ utility.

In order to understand the inner workings of this utility function let me take $\gamma = 0$, $\sigma = \infty$, $\kappa = \beta = 1$, and conditional on the current state of the economy, where market return is equally likely to be above or below one. This combination of preference parameters reduces the SDF to the following expression:

$$S_{t,t+1} = \frac{1 + (\alpha - 1) I \left(R_{t,t+1}^m < 1\right)}{1 + (\alpha - 1) E_t I \left(R_{t,t+1}^m < 1\right)} = \frac{\begin{cases} 1, & \text{if} \quad R_{t,t+1}^m \geq 1 \\ \alpha > 1, & \text{if} \quad R_{t,t+1}^m < 1 \end{cases}}{(1 + \alpha)/2}. \tag{4}$$

Now everything depends on the market return. This implies only two possible values for the SDF: $2/(1+\alpha)$ and $2\alpha/(1+\alpha)$. The ratio between these two is α. As long as this parameter is chosen to be greater than one, the ratio between two possible SDF values is greater than one. Moreover, the larger SDF value corresponds to the negative outcome of the economy, when consumption equity loses in value. Hence, the SDF becomes amplified for bad outcomes. This creates negative covariance between SDF and a risky return. In addition, even when other utilities imply constant SDF, GDA utility implies a truly stochastic discount factor.

2.5 Solution for market price-dividend ratio

In the previous sections I have introduced three particular utility functions each one being more general than the other one. I have also shown how to represent stochastic discount factors in the context of my model. There I postponed the discussion of an important piece of the whole picture. Stochastic discount factors in two cases, EZ and GDA, depend on the total one period return on the market portfolio. In this section I shown how to find this missing piece.

According to the canonical pricing model the price of a claim to market portfolio or, equivalently, to the claim on aggregate economy endowment, can be written as the conditional expectation of a discounted future payoff (price plus dividend-consumption):

$$P_t = E_t\left[S_{t,t+1}\left(P_{t+1}+C_{t+1}\right)\right],$$

which may rewritten as

$$\frac{P_t}{C_t} = E_t\left[S_{t,t+1}\frac{C_{t+1}}{C_t}\left(\frac{P_{t+1}}{C_{t+1}}+1\right)\right].$$

Since the solution is time invariant (Mehra & Prescott, 1985), I can write the price indexed by state and conditional on initial dividend value $C_t = c$ as

$$\frac{P\left(c,i\right)}{c} = \sum_{j=1}^{n} p_{ij}s_{ij}\lambda_j\left[\frac{P\left(c\lambda_j,j\right)}{c\lambda_j}+1\right].$$

Since price is homogeneous of degree one in c, the solution can be written as $P\left(c,i\right) = w_i c$. Here w_i is a state-dependent but time-homogeneous price-dividend ratio. This implies the following n equations with n unknowns:

$$w_i = \sum_{j=1}^{n} p_{ij}s_{ij}\lambda_j\left(w_j+1\right).$$

Or with vector notation $w = \left[w_1,\ldots,w_n\right]^T$, these equations can be written as $w = \mathbb{Q}\left(w+\mathbf{1}_n\right)$, or explicitly as

$$w = \left(\mathbb{Q}^{-1}-\mathbb{I}_n\right)^{-1}\mathbf{1}_n.$$

In order to solve the model with EZ or GDA utility it is first necessary to find the expression for the market return $R^m_{t,t+1}$ or in matrix notation of the current model R^m. Provided that the solution is time-invariant and homogeneous of degree one, the market return may be written in terms of consumption growth rates λ_j and market price-dividend ratio w_j:

$$R_{ij}^m = \frac{P(\lambda_j c, j) + \lambda_j c}{P(c, i)} = \lambda_j \frac{w_j + 1}{w_i},$$

or in matrix notation

$$R^m = w^{-1} (w + \mathbf{1}_n)^T \Lambda, \tag{5}$$

where w^{-1} corresponds to element-wise division $\left[\frac{1}{w_1}, \ldots, \frac{1}{w_n}\right]^T$. Plug this matrix to the expression of stochastic discount factor, EZ or GDA depending on the purpose, and write the canonical pricing equation for the return once again:

$$\mathbf{1}_n = [\mathbb{S} \odot R^m] \mathbf{1}_n.$$

This system of n non-linear equations with n unknowns does not have a closed form solution so numerical methods provided are used in the empirical section.

The solution for the matrix of market total one-period returns R^m completes the definition of conditional state price matrices for EZ and GDA cases in (2) and (3), respectively.

3 Model Analysis

In this section I present the analytical investigation of the model. I intend to show that in discrete time the EZ utility, which is a modification of expected utility, is very close to being transient in the sense that it does not modify risk-neutral distribution implied by expected utility. The distance is minimal for conventional levels of relative risk aversion and preferences toward early resolution of uncertainty. I will also show that the GDA utility modification is permanent in the sense that it modifies risk-neutral probability distribution implied by expected utility. This will increase SDF variance and eventually the distance between the long-term risky and riskless returns, the risk premium.

It turns out that under certain regularity conditions a matrix $\mathbb{M}^h$ that may stand for any of conditional growth matrix, growth adjusted, or unadjusted state price matrices ($\mathbb{G}^h$, $\mathbb{Q}^h$, or $\mathbb{S}^h$) is decomposed as follows:

$$\mathbb{M}^h = \exp(\rho h) \, \Phi \hat{\mathbb{M}}^h \Phi^{-1},$$

where ρ is the already introduced spectral radius of the matrix $\mathbb{M}$, ϕ is the corresponding $n \times 1$ eigenvector, and $\Phi = \operatorname{diag}(\phi)$ is the diagonal matrix. More specifically, ρ and ϕ are the solution of $\mathbb{M}^h \phi = \exp(\rho h) \, \phi$.

In particular, the multiplicative decomposition of a state price matrix takes the form

$$\mathbb{S} = \exp(\rho) \times \Phi \times \hat{\mathbb{S}} \times \Phi^{-1}. \tag{6}$$

Here $\hat{\mathbb{S}}$ is interpreted as a probability transition matrix but with risk-neutral probabilities that take into account preference structure and attitudes toward risk. In the terminology of Hansen & Scheinkman (2009) this matrix is a permanent component. This applies for the following intuitive reason. Suppose we are interested in pricing dividend flow over a certain horizon h. For that we have to take the power of the state price matrix and multiply it by the vector of stationary payoffs. So the h's power of a generic transition matrix is

$$\mathbb{S}^h = \exp(\rho h) \times \Phi \times \hat{\mathbb{S}}^h \times \Phi^{-1}.$$

Notice that cumulative multiplication of the matrix cancels out Φ and Φ^{-1} in the middle so that the only accumulated components are exponential growth and $\hat{\mathbb{S}}$. Hence, any change in this matrix propagates to any horizon. At the same time, this justifies the terminology for Φ, the transient component.

Now consider that we have a modification of the SDF transition matrix that fits into the following form:

$$\tilde{\mathbb{S}} = W^{-1} \times \mathbb{S} \times W, \tag{7}$$

where W is a diagonal matrix. Using this modification multiply the original decomposition (6) from the right by W and by its inverse on the left:

$$W^{-1}\mathbb{S}W = \exp(\rho) \times W^{-1}\Phi \times \hat{\mathbb{S}} \times \Phi^{-1}W.$$

The left-hand side should immediately be recognized as the modified matrix $\tilde{\mathbb{S}}$. But does the right-hand side maintain the multiplicative decomposition? The answer is yes due to the same intuition I outlined above:

$$\tilde{\mathbb{S}}^h = \exp(\rho h) \times W^{-1}\Phi \times \hat{\mathbb{S}}^h \times \Phi^{-1}W.$$

The diagonal matrices $W^{-1}\Phi$ and $\Phi^{-1}W$ are not compounded, while exponential growth and $\hat{\mathbb{S}}$ are. Even more important, growth and permanent components are unaltered. All of the modification is gone to the transient component. Such a modification of a transition matrix may be called transient.

From the definition of the EZ stochastic discount factor matrix in (2) and the conditional market returns in (5) it is straightforward to see that the modification of the expected utility

SDF takes the following form:

$$\mathbb{S}_Z = W^{-\theta} \times \mathbb{S}_X \times (W + \mathbb{I})^{\theta},$$

where $\theta = \frac{1-\gamma\sigma}{\sigma-1}$ and W is the diagonal matrix of market price-dividend ratios. Normally in the data we see price-dividend ratios larger that one, hence we can safely assume that each element of the matrix W is larger than one, or $W \geq \mathbb{I}$. Suppose that the decomposition of $\mathbb{S}_X$ is the same as in (6).

Once again, our eventual target is pricing of the long dated payoffs. So, for a start take the second power of the state prices matrix which would relate prices of assets that pay off in two periods,

$$\mathbb{S}_Z^2 = W^{-\theta} \times \mathbb{S}_X \times \left(\mathbb{I} + W^{-1}\right)^{\theta} \times \mathbb{S}_X \times (W + \mathbb{I})^{\theta}.$$

Clearly, the term in the middle does not reduce to the identity. But it is possible to infer some limiting cases that are summarized in Table 1 on page 29. The first column divides two possible cases for the relative risk aversion γ. The second column branches two more possibilities for the relation of elasticity of intertemporal substitution σ and risk aversion. The case of $\sigma > 1/\gamma$ corresponds to preferences for the early resolution of uncertainty, while the reverse case corresponds to preferred late resolution of uncertainty. Then, there is another interesting branching out depending on the side from which the elasticity converges to one, form above or below. Each of these cases essentially leads to a particular limit for the parameter θ. It turns out that for conventional values of relative risk aversion somewhere above one and elasticity slightly above one, that is preferences toward early resolution of uncertainty, the modification matrix $\left(\mathbb{I} + W^{-1}\right)^{\theta}$ is expected to be close to identity, that is

$$\begin{aligned}\mathbb{S}_Z^2 \approx& W^{-\theta} \times \mathbb{S}_X^2 \times (W + \mathbb{I})^{\theta} \\ =& \exp\left(\rho_X h\right) \times W^{-\theta}\Phi \times \hat{\mathbb{S}}_X^h \times \Phi^{-1} (W + \mathbb{I})^{\theta}.\end{aligned}$$

In the last equation above I used multiplicative decomposition (6) with subscripts corresponding to expected utility function.

So, given that risk aversion is above one and the agent prefers early resolution of uncertainty I may write the approximate relationship for a general horizon h as

$$\mathbb{S}_Z^h \approx \exp\left(\rho_Z h\right) \times W^{-\theta}\Phi \times \hat{\mathbb{S}}_X^h \times \Phi^{-1} (W + \mathbb{I})^{\theta}.$$

I expect that the growth/decay ρ_Z of the EZ transition matrix will be different than that of expected utility since it has to compensate for the growth/decay of the matrix $\left(\mathbb{I} + W^{-1}\right)^{\theta}$. So the EZ modification is approximately transient in discrete time. Hansen et al. (2008) show that in continuous time the same modification is exactly transient. Here, the magnitude of the transient effect depends on the magnitude and variation of market price-dividend ratios, which are the entries of matrix W. The smaller and more variable they are, the closer the modification to being transient.

We see that the growth rate of the EZ discount factor may be different from the one implied by the expected utility. Hence, long-term expected returns will be different in both models. I intend to show that although long-term returns are different, the long-term risk premia are the same. Consider a stream of payoffs with corresponding growth matrix Θ and the growth adjusted state price matrix of the expected utility, $\mathbb{Q}_X = \mathbb{S}_X \Theta$. Using the decomposition of the state price matrix matrix write

$$\mathbb{Q}_X = \mathbb{S}_X \Theta = \exp(\rho_X) \times \Phi_X \times \hat{\mathbb{S}}_X \Theta \times \Phi_X^{-1}.$$

Naturally, the matrix $\hat{\mathbb{S}}_X \Theta$ does not possess the property of the probability transition matrix. So, decompose it as

$$\hat{\mathbb{S}}_X \Theta = \exp(\tilde{\rho}_X) \times \tilde{\Phi}_X \times \tilde{\mathbb{S}}_X \times \tilde{\Phi}_X^{-1},$$

and plug this decomposition back into the state price matrix:

$$\mathbb{Q}_X = \exp(\rho_X + \tilde{\rho}_X) \times \Phi_X \tilde{\Phi}_X \times \tilde{\mathbb{S}}_X \times \tilde{\Phi}_X^{-1} \Phi_X^{-1}.$$

Similarly, decompose the state price matrix of the EZ utility function:

$$\mathbb{Q}_Z \approx \exp(\rho_Z + \tilde{\rho}_X) \times \Phi_X \tilde{\Phi}_X W^{-\theta} \times \tilde{\mathbb{S}}_X \times (W + \mathbb{I})^{\theta} \tilde{\Phi}_X^{-1} \Phi_X^{-1}.$$

Notice that both pricing matrices, $\mathbb{Q}_X$ and $\mathbb{Q}_Z$, have exactly the same modification $\tilde{\rho}_X$ in the growth rate. In this notation the long-term risk-free rate is defined as $(-\rho_X)$ and $(-\rho_Z)$ for EU and EZ models, respectively. Long-term expected asset return is determined by dividend growth matrix, which is unaltered, and by pricing matrix growth rates $(\rho_X + \tilde{\rho}_X)$ and $(\rho_Z + \tilde{\rho}_X)$ for each model. This leaves the difference between risky and riskless returns equal to $\tilde{\rho}_X$ for each of the two models. In other words, this proves that the long-term risk premia are very close to each other in EU and EZ models. This conclusion is solely due to the fact that the permanent component, or risk-neutral probability transition matrix, is shared by both models.

Consider the modification of SDF implied by the GDA utility function which is represented in matrix form by (3). Clearly, this modification may not be represented in the symmetric form in (7) or even the one implied by the EZ utility which is close to being symmetric under certain combination of preference parameters. Take for example the extreme case where risk aversion is zero, $\gamma = 0$, elasticity of intertemporal substitution is infinite, $\sigma = \infty$, discount rate and disappointment threshold are one, $\kappa = \beta = 1$, and conditional on the current state of the economy market return is equally likely to be above or below one. This restriction produces SDF as in (4):

$$S^{GDA}_{t,t+1} = \mathcal{G} = \frac{\begin{cases} 1, & \text{if} \quad R^m_{t,t+1} \geq 1 \\ \alpha > 1, & \text{if} \quad R^m_{t,t+1} < 1 \end{cases}}{(1+\alpha)/2}.$$

Note that for a given current state of the economy the SDF, up to a constant $\frac{2}{1+\alpha}$, takes only two values, 1 and α. In order for this modification to be symmetric in the sense of (7), the SDF has to take at least three values, one of which should be $1/\alpha$, which is not the case here. Hence, the modification is not symmetric and does not admit the form in (7).

So, the decomposition of the SDF is

$$\mathbb{S}^h_G = \exp(\rho_G h) \times \Phi_G \times \hat{\mathbb{S}}^h_G \times \Phi_G^{-1}.$$

Each component in this decomposition is potentially different from its counterpart of other utilities. Most importantly, the permanent component, the risk-neutral probability transition matrix, is modified. So the GDA utility modification may be considered permanent. Recall that GDA is constructed in such a way that it is amplified when the future outcome is perceived to be bad for an investor. This implies that the risk-neutral probabilities will be skewed towards possible economic downturns. Inevitably such a tilt in probabilities will amplify the negative covariance of the SDF and risky returns. This in turn will lead to the increased distance between growth rates in risky and riskless returns. Hence, the long-term risk premium is expected to be amplified along with the short one.

4 Data and estimation

In this section I briefly describe the data and estimation technique that are used to provide a quantitative illustration of the theoretical result of this paper. The general idea is to estimate consumption growth rates and probability transition matrix over finite set of economy states. Then, solve for market price-dividend ratios and compute model implied one-period dividend returns.

The data for the US annual aggregate real consumption and the US population is obtained from the Federal Reserve Bank of St. Louis[6]. Dividing aggregate consumption by population and converting to growth rates, I obtain real consumption per person growth rate, which plays the role of economy endowment growth series. The data range spans almost 80 years from 1930 to 2009. The data on risk-free and market returns were obtained from the Fama-French data library[7].

In order to quantify implications of the model I have chosen an approach that differs from the one used by Mehra & Prescott (1985). They used a simple calibration to match model parameters and growth rate moments computed from historical US consumption data. There are three parameters to match in their model, μ, δ, p, and three empirical moments: mean, standard deviation, and first order autocorrelation of consumption growth. Here the model is much more flexible since it allows a general number of economy states and unrestricted probability transition matrix.

In this paper I use instead the methodology suggested by Tauchen (1986) to estimate Markov transition matrix and growth rates of consumption. Two features of their estimation procedure are decisive factors in my choice. First, the method works for any finite number of states, not just two. Second, the method works for both univariate and multivariate time series, which allows estimation of joint dynamics of consumption and dividends.

The essence of the method is to fit VAR(1) to the data and estimate model errors that are assumed to be independent. These errors are used to estimate its univariate densities, for example, by non-parametric kernel methods. I used the non-parametric density estimator with default bandwidth and Gaussian kernel provided by MatLab. An equally spaced grid is then used to assign probabilities to the finite number of intervals on the grid. Transition probabilities for the original processes are backed out from the VAR structure and its parameter estimates. For a more detailed algorithm see Tauchen (1986).

5 Results

The analysis of the model implications is performed through the sensitivity analysis of one period dividend return moments. Table 2 on page 29 shows the benchmark parameter values. Given these values I vary each parameter over the large intervals in order to demonstrate the effects of each one on asset pricing implications.

For illustrative purposes I perform sensitivity analysis for the following simple case. Let

[6] Federal Reserve Bank of St. Louis data library: http://research.stlouisfed.org/fred2/

[7] Fama-French data library: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

the probability transition matrix is

$$\mathbb{P} = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}.$$

Consumption growth rates are mean μ_c, standard deviation δ_c, that is $\lambda = (\mu_c - \delta_c, \mu_c + \delta_c)^T$. Similarly, for dividend growth rates $\nu = (\mu_e - \delta_e, \mu_e + \delta_e)^T$. The transient component of the payoff is assumed to be $(1 - \delta_p, 1 + \delta_p)^T$.

This section is organized as follows. Section 5.1 analyzes numerical decomposition of the SDF matrix. Section 5.3 breaks down market return into permanent and transitory components and shows their dependence on model parameters. Section 5.2 looks at the term structure of risk premia.

5.1 Transition matrix decomposition

In this section I discuss numerical results of the state price matrix decomposition. It will show that the permanent component, or risk-neutral probability transition matrix, is almost the same for expected and EZ utility functions. On the other hand, the subjective probability distribution implied by GDA is tilted toward bad economic outcomes.

Table 3 on page 30 reports state price matrices and its decomposition for each of the three utility functions. The first row and first column of each matrix correspond to the low consumption growth.

First of all note that the modification matrix of the EZ utility, $\mathcal{K}$, has the diagonal elements very close to one and off-diagonal elements being almost reciprocal of each other. This numerical result simply follows from construction of this matrix as a ratio of market price-dividend ratios. As it was shown in Section 3 this modification of the SDF is representable in the form

$$\mathbb{S}_Z = W^{-\theta} \times \mathbb{S}_X \times (W + \mathbb{I})^{\theta},$$

where W is the diagonal matrix with market price-dividend ratios, and $\theta = \frac{1-\gamma\sigma}{\sigma-1}$ is the function of preference parameters. In the same section I have argued that as long as relative risk aversion is above one ($\gamma > 1$), and agents prefer early resolution of uncertainty ($\sigma > 1/\gamma$), the modification is very close to being perfectly transient as in continuous time. But in discrete time it is not exactly transient as evident from the table that shows some difference in $\hat{\mathbb{S}}_Z$ and $\hat{\mathbb{S}}_X$.

Also note that the decay rate of the SDF matrix in case of EZ utility is -1.02, whereas in case of expected utility it is -6.08. This is expected since for the benchmark parameter estimates θ is negative (in fact it is -28), which makes the matrix $(\mathbb{I} + W^{-1})^{\theta}$ less than identity and that makes it contribute to state price matrix decay. Hence, in order to compensate for this decay, the growth rate of the EZ discount factor has to be larger. But at the same time, since the risk-neutral conditional distribution is negligibly different for both utility functions, and its variance is almost the same, 0.35 for EU against 0.34 for EZ, the distance between two growth rates, SDF and pricing transition, is only slightly different, 0.84 against 0.82.

To wrap up comparison between the EU and EZ utilities, notice almost twice the difference between the variances of their state price matrices, 0.14 against 0.32, respectively. This large variance contributes to the elevated risk premium in the short term implied by the EZ utility function.

Now look at the modification of the EZ utility by the matrix $\mathcal{G}$ corresponding to GDA utility. This matrix does not have diagonal elements close to one, and off-diagonal elements are far from being reciprocal of each other, which is a sign of asymmetry. This makes it impossible to decompose this matrix into the product of stationary components as it was done for EZ preferences. As it was shown in Section 3 such a modification should inevitably alter risk-neutral probabilities. This theoretical intuition is fully supported by empirical results. Notice the permanent component of the GDA discount factor is heavily skewed toward future possible recession. This implies almost a threefold increase in the variance of the permanent component, 0.99 for GDA against 0.34 for EU and EZ. Hence, the difference in growth speed between SDF and the pricing matrix is amplified, 1.89 against 0.82.

Finally, note the threefold increase in the variance of the SDF itself, 0.96 for GDA against 0.32 for EZ. This contributes another increase in the magnitude of the short-term risk premium, in addition to the EZ implied premium. So it is clear that the GDA modification of the previous two utility functions has a permanent effect and is reflected in the amplified long-term risk premium.

5.2 Term structure of risk premia

Here I analyze the term structure of market risk premium shown in Figure 1 on page 31. The time horizon goes from one month to ten years. All three utilities imply different risk premia at short horizons. With increase in time horizon all three premia increase monotonically but with decreasing speed. At horizon of 10 years the change in premia is barely noticeable and the level is very close to the asymptotic value of the long-run premia. What is important and clear from previous considerations is that the EZ implied premium converges to the one implied by expected utility. This brings up the main result of the paper again. Namely, EZ utility modifies expected utility only to a degree that short term payoffs imply different compensation. In the long run EZ utility does not imply anything different. On the other hand GDA utility modifies expected utility so that the risk-neutral distribution is even more skewed towards bad economy outcomes.

Once again, Figure 1 on page 31 shows that the term structure implied by all three utility functions is positively sloped. But this result was obtained under the assumption that transient payoff component is the same across all economy states, that is $\delta_p = 0$. It turns out, as will be seen later, term structure depends a lot on the volatility of the transient component. In fact I will show that with high enough volatility the term structure may be reversed which means that short term dividends are riskier than the long term ones.

5.3 Sensitivity analysis

In this section I concentrate on sensitivity analysis of the premia. There are two values to track, the average premium on the asset paying off a dividend next period, and on the asset that pays off a dividend indefinitely far into the future, RP_1^e and RP_∞^e, respectively. I set the default parameter values to those listed in Table 2 on page 29 and vary each in some wide but reasonable range. This gives me an almost complete picture of main factors affecting both short and long risk premia. To simplify the analysis I have chosen an aggregate endowment as a benchmark dividend process which is clear from the choice of benchmark parameters μ_e and δ_e that are equal to μ_c and δ_c. The effects of differences in the dividend process are analyzed separately.

Each figure in Section B (except for the term structure discussed above) is a panel of two graphs. Left graph represents next period market risk premium, while the right one corresponds to infinite horizon risk premium. All three utility models are included in the analysis. In essence, these two figures correspond to two opposite ends of the term structure of the risk premium.

5.3.1 Sensitivity to risk aversion

I start with the effect of the conventional risk aversion parameter depicted in Figure 2 on page 31. The parameter changes between 0 and 10 which is widely accepted to be reasonable bounds. Naturally, all implied premia increase with higher risk aversion parameter. The maximum values of short market premia are 1.5%, 2.1%, and 3.6% for EU, EZ, and GDA utilities, respectively. The maximum values of long market premia are 3.2%, 3.2%, and 4.3% for EU, EZ, and GDA utilities, respectively.

First of all note that the expected utility implied infinite horizon risk premium coincides with the one implied by the EZ utility. At the same time implied short-term premia are quite different for each model. This empirical result supports the main theoretical finding of this paper which says that GDA implied premium is amplified at all payoff horizons. On the other hand, EZ utility amplifies premium only in the short horizon relative to expected utility. The root cause of this amplification as I have shown in Section 3 is in altered risk-neutral distribution in the case of GDA. At the same time, EZ utility does not change risk-neutral distribution relative to expected utility even though it decreases the covariance between returns and stochastic discount factor.

5.3.2 Sensitivity to elasticity of intertemporal substitution

Figure 3 on page 32 shows the effects of elasticity of intertemporal substitution that varies from 0.2 to 2. First of all, expected utility is not a function of σ, or rather risk aversion parameter in this model is fixed to be the reciprocal of it. So, the implied premia are not expected to be changing with respect to this parameter in case of expected utility. This is confirmed by the flat lines corresponding to expected utility implied short and long premium.

Elasticity has some effect on next period premium for EZ and GDA cases. Especially for values smaller than one the premium somewhat decreases for both utilities. It seems that unless this parameter is very close to zero, there is not much impact on the implied premia. These results play well with similar findings of Bonomo et al. (2011).

In the long run elasticity does not have any effect on any of the three implied premia.

5.3.3 Sensitivity to generalized disappointment aversion

Disappointment aversion α shows up as an input only to the GDA utility function, so in Figure 4 on page 32 we see only one set of implied values changing. At the point where $\alpha = 1$ we know that GDA utility collapses to the special case of EZ. This is confirmed on the graph where two lines are intersected. For parameter greater than one the preferences become increasingly asymmetric and put much more weight on negative economic outcomes. This asymmetry increases negative correlation between stochastic discount factor and returns. This channel leads to increased risk premium.

In the long run similar trend is in place but the remaining channel is different. With increase in α and asymmetry in investor's preferences the risk-neutral distribution of consumption/dividend growth rates becomes more skewed relative to expected utility. Since this change is permanent it affects all future horizon returns which is illustrated on the right-hand side figure where premium increases from 3.2% to 4.2%.

5.3.4 Sensitivity to disappointment threshold

One more parameter that affects only implications of the GDA utility is the disappointment threshold. The effects are shown in Figure 5 on page 33. The broken line for GDA implied risk premia is the reflection of the fact that there are only two states of the economy in this empirical exercise. Specifically, discontinuities appear at the points where one more conditional return becomes below the threshold of disappointment κ. For the specific interval of parameter values there is enough variation in market returns to say that some are disappointing and some are not. This duality increases the risk premium both in the short and long run.

5.3.5 Sensitivity to transition probability

Now look at the effect of transition probability parameter p depicted in Figure 6 on page 33. Here I assume that $q = p$. In the left graph we see that the premium implied by expected utility is almost symmetrical U-shaped line with the maximum reached for conditional probability being 0.5. This means that the riskiest payoffs in this utility are those that are completely unpredictable. The case of EZ utility is more complicated. Here most of the premium is paid for p close to one. These values mean that the economy growth with the same rate for a long period of time consecutively and switches infrequently. For forward-looking investors this situation is clearly riskier than simple uncertainty about the next period growth rates. GDA utility premium has some discontinuities that are explained in the previous section. In the interval where the premium is amplified, the line has a U-shaped form with a maximum somewhere above $p = 0.5$. This is due to the negative market returns that correspond to future recessions. Unpredictable growth is coupled with unpredictable market returns and hence, maximum risk compensation.

In the long run all of the premia are monotonically increasing with decreasing frequency of regime switches. The intuition is the following. The risk premium of payoffs on any horizon is equal to absolute value of covariance between stochastic discount factor and stochastic growth compounded over this period. Increased persistence of economy states and compounding of growth and stochastic discount factor increases the variance of resulting variables over the long period of time, hence, increasing absolute value of covariance. In other words, the risk of investing over distant horizons is the risk of being caught in one economy state for a prolonged period of time. This conclusion applies to all three utility functions.

5.3.6 Sensitivity to consumption growth parameters

Mean consumption μ_c growth does not change the covariance between stochastic discount factor and compounded consumption growth simply by design of covariance as a statistical measure. It only affects the scale of net market return and risk-free rate, but not their difference. This observation applies to any investment horizon. This intuition is confirmed in Figure 7 on page 34.

Figure 8 on page 34 shows the effect of the second parameter affecting consumption dynamics, standard deviation δ_c. Both ends of the term structure of risk premium for all three utility functions depend positively on variation in consumption growth. Recall from Section 2 that all stochastic discount factors contain inverse power function of consumption growth. Then, the dividends so far are equal to consumption endowment. Hence, the absolute value of covariance would increase with an increase in consumption variability. EZ and GDA utility functions modify expected utility in such a way that it does not change the direction of dependence which is obvious from the plot. Still, the covariance is changed which is reflected in different premia for a given value of δ_c.

Also note the corner case, where standard deviation is equal to zero. There the consumption stream is a constant and this case becomes trivial since there is no randomness in such a model.

5.3.7 Sensitivity to dividend growth parameters

The model is general enough to price not only aggregate endowment but also dividend streams that differ from consumption in their conditional distribution. So far I looked only at market premium which corresponds to risk embedded in claims to aggregate consumption stream. In the two state model there are two degrees of freedom with respect to dividend dynamics, mean and standard deviation of dividend growth rates, μ_e and δ_e, respectively. Varying each one of them I leave the other the same as in consumption growth series.

Sensitivity with respect to mean growth is represented by Figure 9 on page 35. Note that both plots are indistinguishable from Figure 7 on page 34 where market premium was analyzed. This means that the level of growth rates does not change the riskiness of investments in this framework.

Figure 10 on page 35 shows the dependence on standard deviation of dividend growth. In this particular model and a particular choice of parameters, dividend growth series covaries positively with consumption growth. Hence, it is expected that the standard deviation of dividends which is greater than the respective value for consumption would produce a larger absolute covariation between stochastic discount factor and dividend growth over any horizon. Therefore, the premium should be greater which is confirmed by the plot. In general, both left and right panel show almost linear positive dependence on standard deviation of dividend growth rates which may be interpreted as risk-return trade-off. Again zero standard deviation means constant dividend series and no risk which results in zero risk premia for all utilities.

5.3.8 Sensitivity to the transient component of the payoff

As it is clear from the analytical expression of risk premium over alternative horizons the transient component may be normalized to have mean one. Then, the definition of this component $(1-\delta_p, 1+\delta_p)^T$ is justified. In addition, $|\delta_p|$ corresponds to the standard deviation of the transient component. Figure 11 on page 36 shows how premia depends on this parameter. In particular, if $\delta_p = 0.1$ it means that the transient payoff in the good/bad state of the economy is 10% larger/smaller than the mean, respectively. In the case of next period return (left panel) zero parameter matches the benchmark case with transient component being equal across the states. For values higher than zero the payoff is higher in good state of the economy which translates into higher premia for all utilities. On the other hand, when the relation between transient component and economy state is completely reversed, the premium is lower than in benchmark case. At some point when the payoff in good state of the economy is so low that it overshadows any growth prospects, the premium becomes negative.

For an increasingly long horizon the dependence on the state is eliminated which is reflected by the flat lines for all utility functions in case of infinite horizon. This result is expected since what matters for the long-run return is the asymptotic growth rate of stochastic components of the model which do not depend on the state dependent transient component of the payoff.

It is interesting to note that this state dependence of expected premia may generate the term structure of the risk premia that is different from what is shown in Figure 1 on page 31. In particular, if the transient component of the next period payoff deviates from the mean by more than 1%, then the term structure will be downward sloping from the GDA utility. If δ_p is greater than approximately 2%, then both GDA and EZ would imply negatively sloped term structure while the one implied by expected utility will remain positively sloped. Finally, after about 5% deviation from the mean all three term structures will be negatively sloped. This means that higher volatility of payoffs, especially when there is a positive correlation with consumption growth, generates higher excess returns on short horizon dividends. At the same time long term premium remains that same and does not depend on the economy state.

6 Conclusion

This paper builds on several strands of literature. In contrast to Mehra & Prescott (1985) I build the asset pricing model with an arbitrary finite number of economy's states. This assumption leads to convenient matrix representations of several asset pricing features. Operator methods introduced by Hansen & Scheinkman (2009) and their multiplicative decomposition result provide a way to directly compute model-implied long-term risk premium. I also consider differences in implications of the three utility preference structures: expected utility, recursive utility of Epstein & Zin (1989), and generalized disappointment aversion by Routledge & Zin (2010).

My main contribution is to demonstrate that the GDA utility is a modification of expected utility that has both permanent and transitory effects. GDA preferences put excess weight on the bad outcomes of economy, thus altering risk-neutral probabilities of the future economy outcomes. This increases the variance of SDF and consequently the risk premia at any horizon. This is in contrast with the Epstein-Zin utility, which is a transient modification of expected utility as shown by Hansen (2011). It implies different short-term risk premia but is identical in the long run.

I also show that the term structure of risk premium critically depends on the variance of the transient component of the payoff function. If this component is volatile and increases the payoff in times of high consumption growth the term structure of payoff is negatively sloped. This makes short dividends riskier than long ones. In the long run the dependence on the transient component disappears and leaves only growth characteristics and risk-neutral distribution to determine the long run risk premium.

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A Tables

γ	σ	σ	θ	$(\mathbb{I}+W^{-1})^{\theta}$
>1	$>\frac{1}{\gamma}$	$\nearrow 1$	$\to +\infty$	$\to \infty$
		$\searrow 1$	$\to -\infty$	$\to \mathbb{I}$
	$<\frac{1}{\gamma}<1$		<0	$<(\mathbb{I}+W^{-1})$
<1	$<\frac{1}{\gamma}$	$\nearrow 1$	$\to -\infty$	$\to \mathbb{I}$
		$\searrow 1$	$\to +\infty$	$\to \infty$
	$>\frac{1}{\gamma}>1$		<0	$<(\mathbb{I}+W^{-1})$

Table 1: Relation between EZ preference parameters and state price matrix modification. Here γ is relative risk aversion, σ is elasticity of intertemporal substitution, parameter θ is equal to $\frac{1-\gamma\sigma}{\sigma-1}$, $\mathbb{I}$ is an identity matrix, W is the diagonal matrix of price-dividend ratios.

Description	Notation	Value
Estimated parameters		
Transition probability (recession)	p	0.69
Transition probability (boom)	q	0.72
Mean consumption growth, %	μ_c	2.18
Standard deviation of consumption growth, %	δ_c	4.32
Non-estimated parameters		
Mean dividend growth, %	μ_e	2.18
Standard deviation of dividend growth, %	δ_e	4.32
Deviation of the transient component of the payoff, %	δ_p	0
Discounting factor	β	0.98
Risk aversion	γ	10
Elasticity of intertemporal substitution	σ	1.5
Disappointment aversion	α	5
Disappointment threshold	κ	1.05

Table 2: Benchmark model parameters.

	EU	EZ	GDA
$\mathbb{S}$	$\begin{bmatrix} 0.8435 & 0.1602 \\ 0.3452 & 0.3742 \end{bmatrix}$	$\begin{bmatrix} 0.8861 & 0.1058 \\ 0.5816 & 0.3964 \end{bmatrix}$	$\begin{bmatrix} 0.9551 & 0.0273 \\ 0.8440 & 0.1378 \end{bmatrix}$
$\mathcal{K}$		$\begin{bmatrix} 1.0505 & 0.6605 \\ 1.6851 & 1.0596 \end{bmatrix}$	
$\mathcal{G}$			$\begin{bmatrix} 1.0778 & 0.2583 \\ 1.4510 & 0.3477 \end{bmatrix}$
$\hat{\mathbb{S}}$	$\begin{bmatrix} 0.8963 & 0.1037 \\ 0.6024 & 0.3976 \end{bmatrix}$	$\begin{bmatrix} 0.8952 & 0.1048 \\ 0.5995 & 0.4005 \end{bmatrix}$	$\begin{bmatrix} 0.9722 & 0.0278 \\ 0.8597 & 0.1403 \end{bmatrix}$
ϕ	$\begin{bmatrix} 0.8541 & 0.5201 \end{bmatrix}$	$\begin{bmatrix} 0.7141 & 0.7000 \end{bmatrix}$	$\begin{bmatrix} 0.7073 & 0.7069 \end{bmatrix}$
$-\rho(\mathbb{S})$, %	6.0759	1.0212	1.7774
$-\rho(\mathbb{Q})$, %	6.9118	1.8391	3.6621
$\rho(\mathbb{S}) - \rho(\mathbb{Q})$, %	0.8359	0.8179	1.8847
$Cov(\mathbb{S}, R^m)$	-0.0015	-0.0263	-0.0420
$V[\mathbb{S}]$	0.1407	0.3203	0.9633
$V\left[\hat{\mathbb{S}}\right]$	0.3469	0.3413	0.9993

Table 3: State price matrix decomposition. Model-implied SDF transition matrix $\mathbb{S}$, modification matrices $\mathcal{K}$ and G, SDF permanent component $\hat{\mathbb{S}}$, principal eigenvector ϕ, growth rate of SDF $\rho(\mathbb{S})$, growth rate of the pricing matrix $\rho(\mathbb{G}) = 2.4415\%$, the difference between the last two, unconditional variance of SDF $V[\mathbb{S}]$ and of its permanent component $V\left[\hat{\mathbb{S}}\right]$. First row of the matrix corresponds to the low current consumption growth. First column of the matrix corresponds to the low future consumption growth.

B Figures

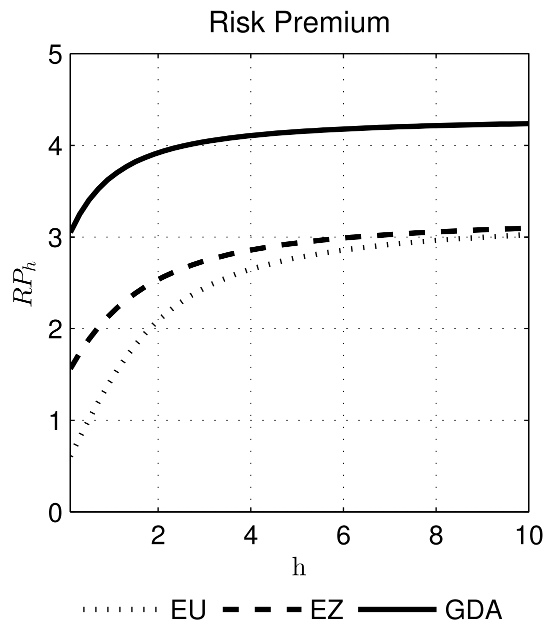

Figure 1: Term structure of the market risk premium. Horizontal axis is measured in years. On the vertical axis risk premium is annualized percent.

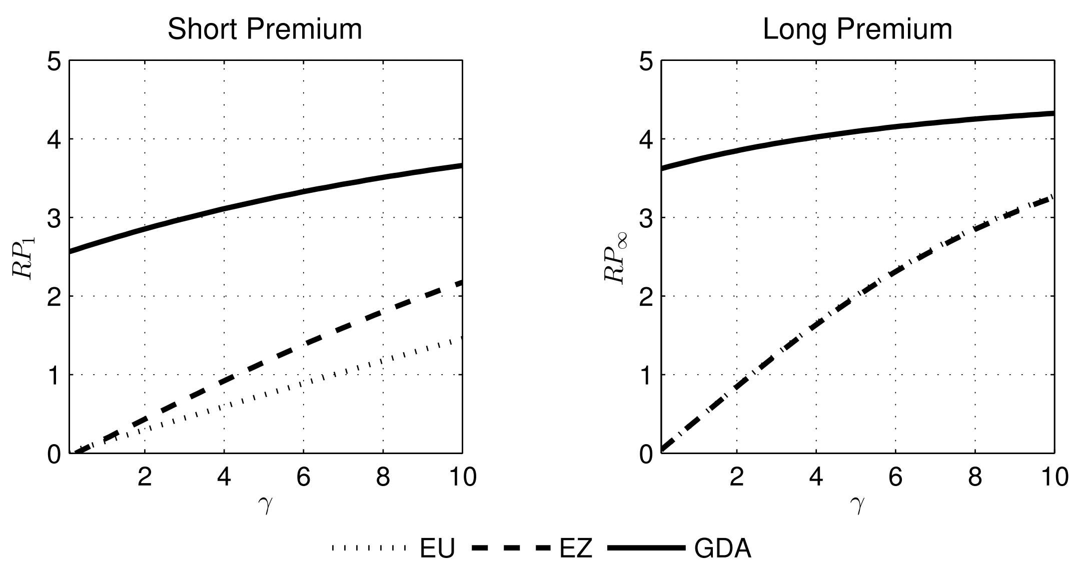

Figure 2: Sensitivity of the risk premia to the risk aversion parameter. Left panel depicts one period premia. Right panel depicts infinite horizon premia.

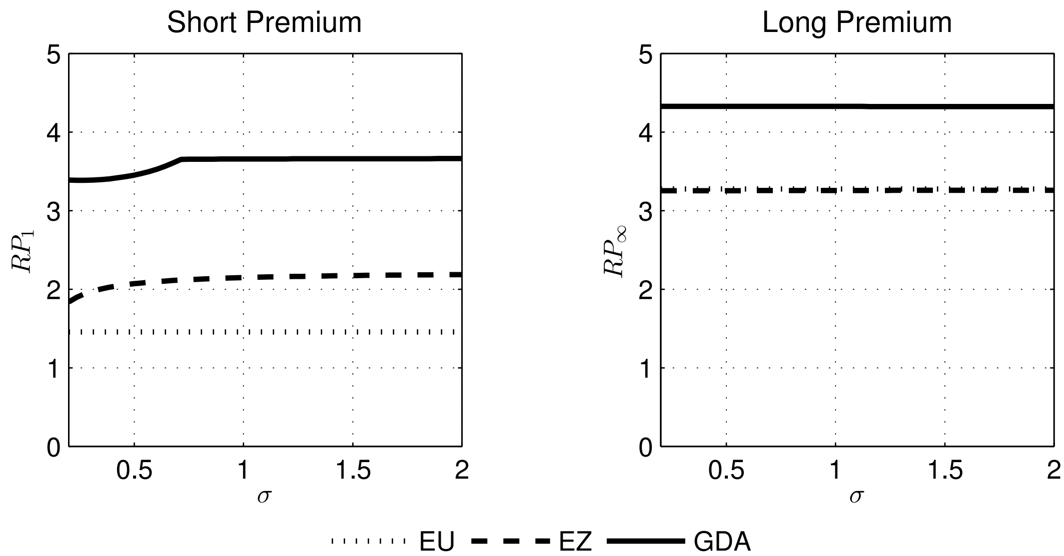

Figure 3: Sensitivity of the risk premia to the elasticity of intertemporal substitution parameter. Left panel depicts one period premia. Right panel depicts infinite horizon premia.

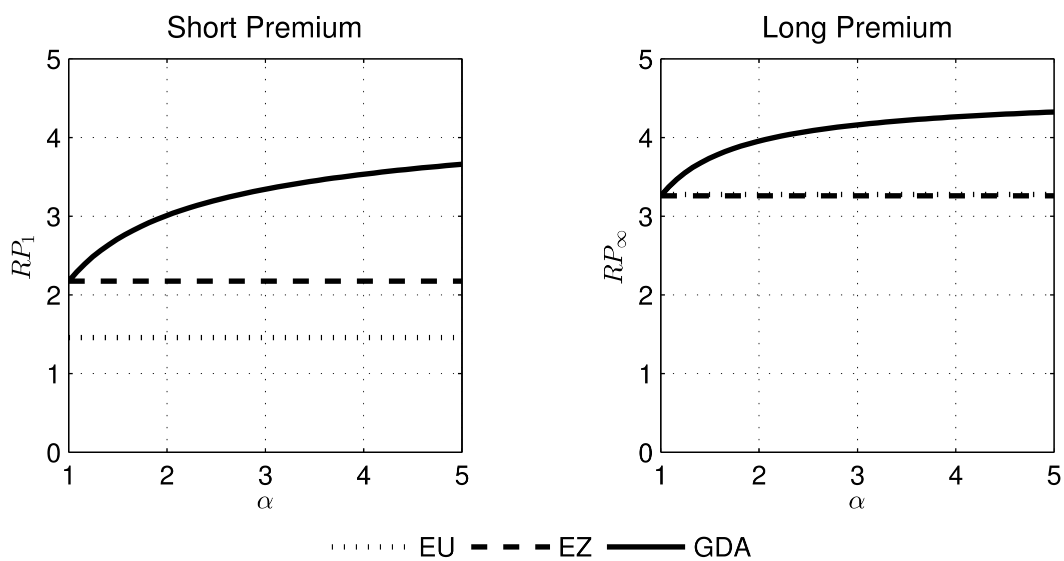

Figure 4: Sensitivity of the risk premia to the generalized risk aversion parameter. Left panel depicts one period premia. Right panel depicts infinite horizon premia.

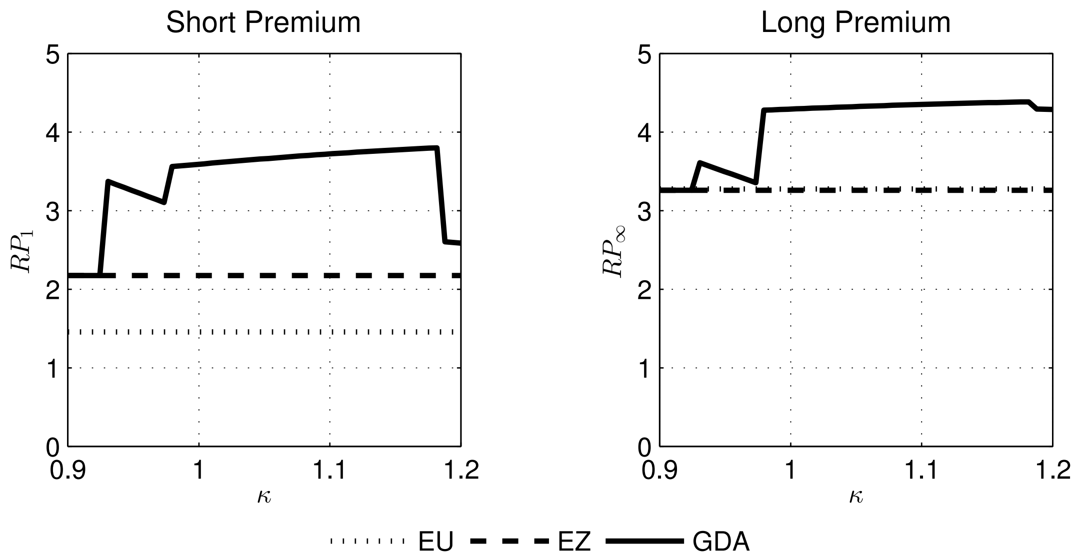

Figure 5: Sensitivity of the risk premia to the disappointment threshold parameter. Left panel depicts one period premia. Right panel depicts infinite horizon premia.

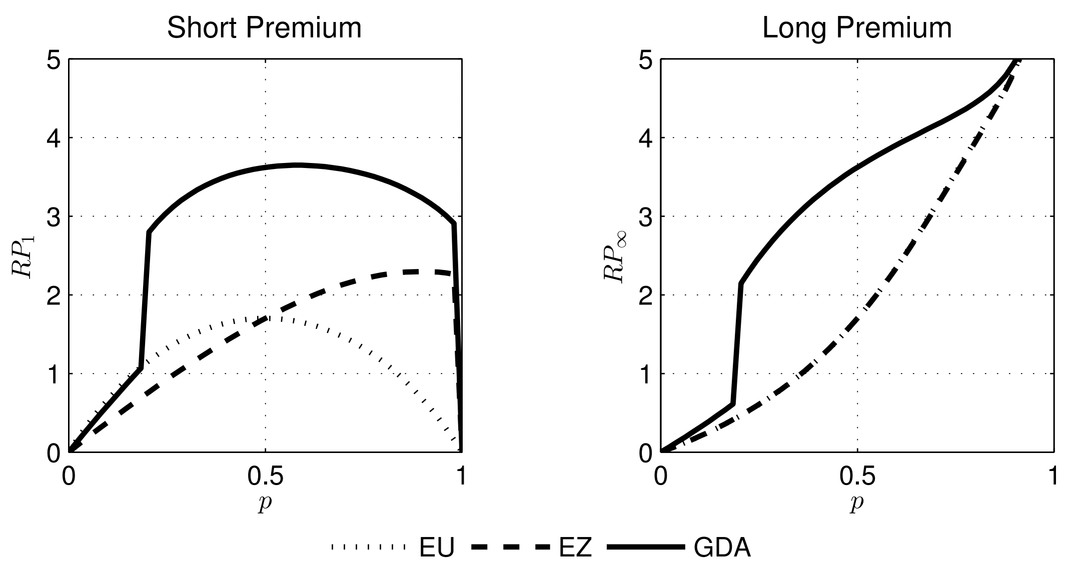

Figure 6: Sensitivity of the risk premia to the probability transition parameter. Left panel depicts one period premia. Right panel depicts infinite horizon premia.

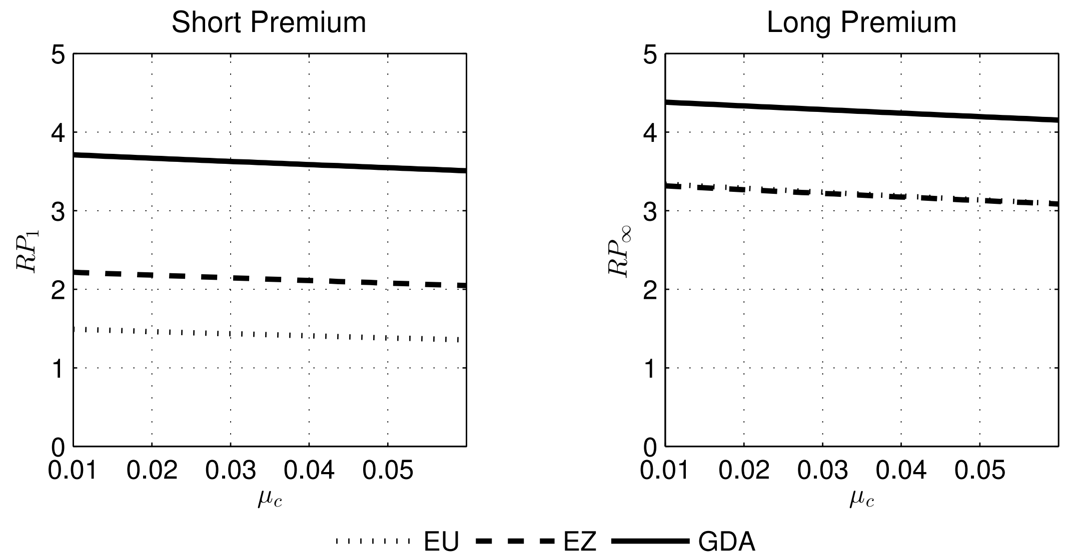

Figure 7: Sensitivity of the risk premia to the mean consumption growth rate parameter. Left panel depicts one period premia. Right panel depicts infinite horizon premia.

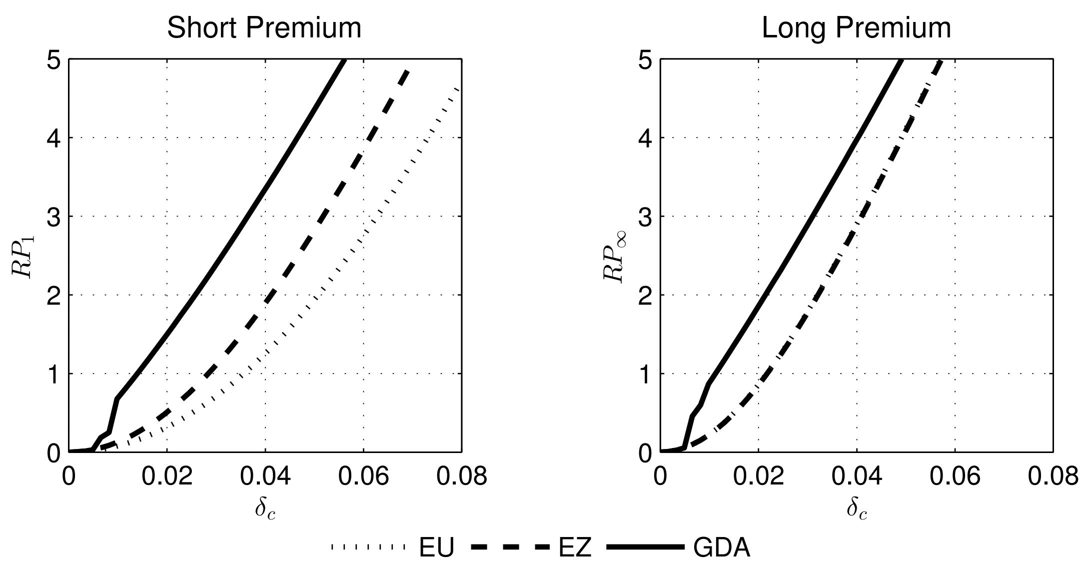

Figure 8: Sensitivity of the risk premia to the consumption growth standard deviation parameter. Left panel depicts one period premia. Right panel depicts infinite horizon premia.

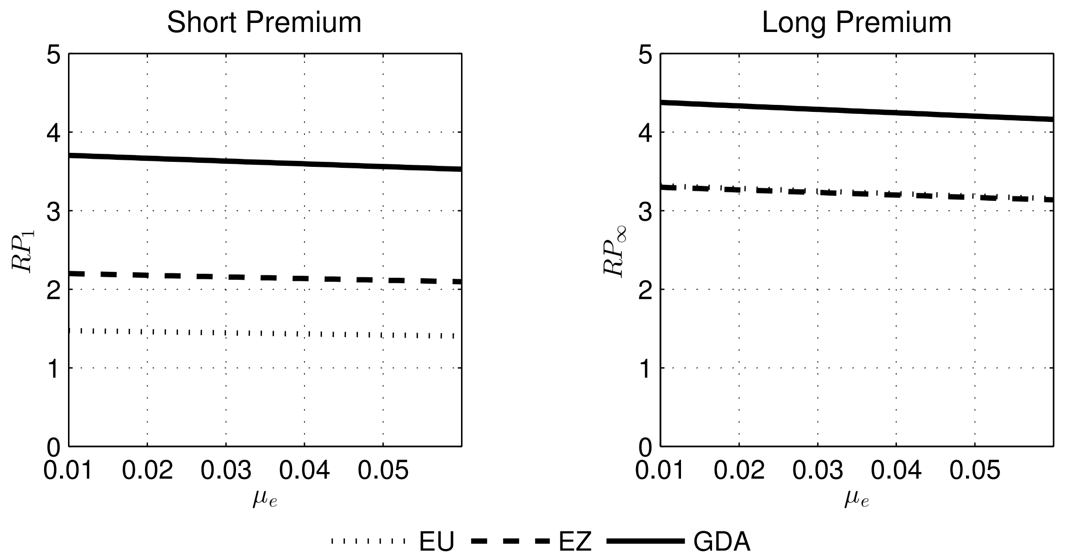

Figure 9: Sensitivity of the risk premia to the mean dividend growth rate parameter. Left panel depicts one period premia. Right panel depicts infinite horizon premia.

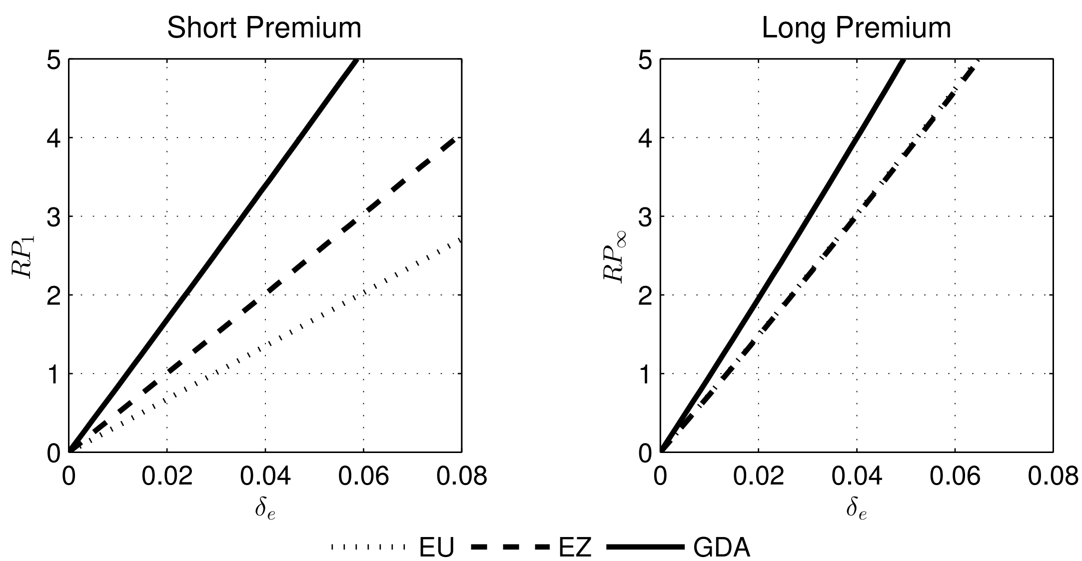

Figure 10: Sensitivity of the risk premia to the dividend growth standard deviation parameter. Left panel depicts one period premia. Right panel depicts infinite horizon premia.

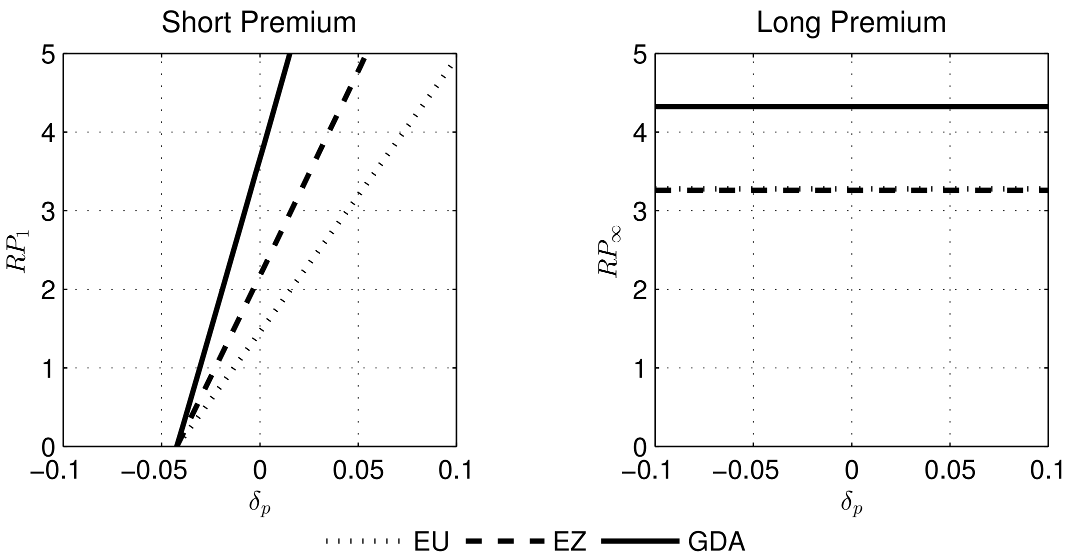

Figure 11: Sensitivity of the risk premia to the deviation δ_p of the transient payoff component from its mean. In the two state model $|\delta_p|$ coincides with standard deviation parameter of the transient payoff component. Left panel depicts one period premia. Right panel depicts infinite horizon premia.