

# Countercyclical Trade Balance and Persistent Real Exchange Rates in a Neomonetarist Model

Oleg Zamulin \*

New Economic School (NES) and  
Centre for Economic and Financial Research (CEFIR)

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## Abstract

A sticky price dynamic general equilibrium business cycle model of a small open economy achieves several powerful results. Presence of physical investment and incomplete markets helps generate initial current account deficits after money shocks. The trade balance is hence countercyclical in a simulated economy, in which the business cycle is driven by monetary or persistent technological shocks. A high degree of real price rigidity for nontradable goods produces persistent deviations from purchasing power parity. Likewise, the presence of the nontradable sector can generate overshooting of the exchange rate, which depends on non-separability of labor and consumption, as opposed to the more traditional liquidity effect based overshooting. However, the volatility of nominal and real exchange rates observed in the data can only be reproduced after the introduction of local currency pricing assumption. Lastly, the proposed model is analytically tractable, which helps obtain firm intuition before using numerical solutions.

JEL Classifications: E32, F41.

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# 1 Introduction

This paper presents a novel model of a small open economy with sticky prices, which is primarily intended for the study of the current account dynamics in the presence of monetary shocks, but also has implications for the persistence and volatility of real exchange rates. The benchmark version of the model is based on the model of Kimball (1995) and takes the framework of the small economy in Obstfeld and Rogoff (1995). Thus, the modeled economy has two types of final goods: an aggregate of nontradable goods produced by domestic monopolistic competitors, and a tradable good, which, of course, can be either produced or purchased from abroad. The price index for the nontradable aggregate is sticky and evolves gradually towards the shadow value – the price that would be prevalent in a flexible-price economy. The price of the tradable good, on the other hand, is completely flexible, and by the law of one price, is proportional to the exchange rate. If the foreign price level is normalized to unity, then the price of tradables and the exchange rate are exactly the same. Aside from the price stickiness, the model possesses all standard features of the real business cycle literature, and hence is labeled “neomonetarist” following Kimball (1995). Modifications of the benchmark model with sticky prices of tradable goods are also considered.

The proposed model is meant to complement the rapidly growing literature in the field of new open economy macroeconomics.<sup>1</sup> Due to technical complexity, all of the models proposed in this literature are greatly simplified in many important dimensions. At the same time, they are complicated enough as to prohibit an analytical analysis that could help understand the intuitive structure of the models. In this paper, I manage to proceed quite far without the reliance on computational methods. Before introducing a more realistic model with investment adjustment costs, I build a simpler version, whose dynamics can be understood in full with help of a phase diagram. Although the dynamics of the model become much richer once adjustment costs are introduced, the basic intuition obtained in the analytically tractable model remains unaltered.

The model of this paper has at least two distinct features not common in the sticky-price literature. First of all, most such models abstract from capital accumulation, while capital markets potentially can have important implications for the current account. Second, this model incorporates a sticky-price nontradable sector with an explicitly modeled real rigidity, which helps produce persistent deviations from PPP. The nontradable sector also helps achieve slight overshooting of the exchange rate without relying on the liquidity effects (fall of interest rates after a money shock).

The presence of physical capital, together with a lack of perfect insurance, is crucial in achieving an initial current account deficit in response to a money shock, or more generally, countercyclical trade balance observed in data. This finding, commonly labeled “J-curve response,” is consistent with some empirical studies (Lane 1999), but uncommon within the theoretical literature, mostly because the literature focuses on exchange rates, and abstracts from modeling investment. Likewise, counter-cyclical trade balance is a stylized fact that is not a robust conclusion in either real business cycle literature (Mendoza 1991, Backus, Kehoe and Kydland 1995) nor in sticky-price

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<sup>1</sup>See Lane (1999) for a survey.

literature (Chari, Kehoe and McGrattan 2000).

The intuition for the negative current account response is the following. After a monetary shock, the exchange rate depreciates immediately, thus making the nontradable goods relatively cheap. Hence, resources are temporarily shifted into production of nontradables, and consumption of nontradables goes up. For that, the country immediately borrows capital from abroad, as well as shifts some of the capital utilized in the tradable sector towards production of nontradables. In the long run, however, the relative price of the two types of goods evens out, and resources are shifted back. Yet, the country now has the debt accumulated from the transition period, which it has to pay back. Hence, consumption of both types of goods has to be at a lower new steady-state value. Thus, the immediate response of the current account is to go in deficit; however, as the need in extra capital reduces, resources flow back abroad.

The long-run non-neutrality of money is an interesting and robust conclusion in the open economy business cycle models, but this paper demonstrates that this non-neutrality is negligible and the real variables end up in a steady state very close to the one before the shock. This is especially true when the elasticity of intertemporal substitution for consumption is low. This quantification of the long-run real effects and demonstration of their sensitivity to utility specification is another contribution of the paper.

An interesting result of the paper is the ability of the model to generate exchange rate overshooting, which is different in nature from Dornbusch's original liquidity-effect-based overshooting. However, the degree of overshooting in the benchmark clearly falls short of the overshooting needed to explain the observed volatility of exchange rates, especially in presence of investment adjustment costs. I demonstrate that the modification of the model which incorporates the now popular assumption of local currency pricing (LCP) can increase the degree of overshooting substantially, once again without reliance on the liquidity effect. As an alternative, I demonstrate that non-separability of labor and consumption in the utility function can be a source of overshooting in itself. This finding is especially interesting in light of the evidence against the traditional Dornbusch-style overshooting (Faust and Rogers 2000).

The rest of the paper is organized as follows. Section 2 lays out the simple version of the model with log utility and no investment adjustment costs. A graphical solution demonstrates that following a permanent increase in the money supply, capital is borrowed from abroad on impact, and additional borrowing takes place along the way to the new steady state. Numerical solutions put the long run increase in debt at about 0.3% of capital stock in response to a 1% increase in money supply. The exchange rate adjusts to the new steady state value immediately. Section 3 demonstrates the sensitivity of these results to a reduction in intertemporal elasticity of substitution from the implied value of 1 to 0.35, which comes together with introduction of non-separability between consumption and labor in utility. It is demonstrated analytically that following the initial borrowing of capital (current account deficit) resources are lent back abroad, thus resulting in a J-curve response of the current account. The sign of the long-run response of the debt position is indeterminate in general, but this debt is estimated to be negligible after parametrization. Also,

it is shown that the exchange rate has to overshoot for reasons other than Dornbusch's liquidity effect based mechanism. The drawback of the simple model is the immediate infinite investment, with capital being shifted from abroad at no cost. Section 4 introduces a more realistic version with investment adjustment costs. The long-run behavior of the model does not seem to change in any significant way from the simpler version, but the short-run responses become more realistic. Section 5 introduces a careful treatment of price rigidity analogous to that of Kimball (1995). It is demonstrated that a strong degree of real rigidity is capable of generating high persistence of the real exchange rate, similarly to the argument of Bergin and Feenstra (2000). However, contrary to the model of Bergin and Feenstra, the persistent deviations from PPP are obtained by means of a nontradable goods sector, without reliance on the local currency pricing assumption. The model's drawback is the inability to replicate the volatility of the exchange rates. Section 6 endogenizes the rate of time preference, which makes it possible to perform stochastic simulations of the model. The section also reports the simulated moments for two alternative versions of the model: one with sticky prices in both sectors, and one based on local currency pricing. The counter-cyclicality of the trade balance is a robust conclusion with either money shocks or persistent technology shocks, but is not obtained with temporary technology shocks. The persistence of the real exchange rates is close to that observed in the data, but the volatility is lower. A reasonable degree of exchange rate volatility is obtained only in the model based on LCP with strong complementarity between labor and consumption. Section 7 concludes.

## 2 The Basic Model

### 2.1 The Household

A representative household is solving the problem:

$$\max_{C_N, C_T, L, I} \int_0^{\infty} e^{-\rho t} [\beta \log C_{N,t} + (1 - \beta) \log C_{T,t} - v(L_t)] dt,$$

where  $C_N$  and  $C_T$  are consumption of non-tradable goods and tradables, respectively,  $L$  is the labor supply, and  $I$  is investment in tradable capital, subject to the budget constraints

$$(1) \quad \dot{K}_t = I_t - \delta K_t,$$

$$(2) \quad \dot{B}_t = rB_t + W_t L_t + R_t K_t + \Pi_t - \frac{P_{N,t}}{P_{T,t}} C_{N,t} - C_{T,t} - I_t - T_t,$$

and the No-Ponzi-Game condition

$$\lim_{t \rightarrow \infty} e^{rt} B_t = 0,$$

where  $B$  is the stock of foreign bonds,  $W$  is the wage,  $R$  is the rental rate of capital,  $\Pi$  is the economic profits of the firms owned by households,  $P_N$  is the price of the nontradables,  $P_T$  is the price of tradables,  $I$  is investment, and  $T$  is the lump-sum taxes equal to the change of the money supply. All variables other than the nontradables are denominated in the price of tradables.

The time indices are dropped because these equations are the same at any time period. Thus, the households are choosing how much to consume of both types of product, and how much total labor to provide. Labor goes to both sectors of the economy, in proportion to the relative demand for labor. Since labor can move freely between sectors, the wage is the same, and what enters the budget constraint is the total labor supply multiplied by the common wage. Households own capital that they rent out, and own the firms, whose profits they collect. They maximize their utility taking the wage, the rental rate, and the prices as given.

Sticky prices will be introduced only in the nontradable sector. Since this is a small open economy, the exchange rate does not need to be introduced explicitly. Instead, the price of the tradable goods is equal to the exchange rate as is suggested by the law of one price with foreign price level normalized to unity.

The quantity equation for this model is

$$M_t = V(P_{T,t}C_{T,t} + P_{N,t}C_{N,t}),$$

where  $M$  is the money supply,  $V$  is a constant velocity term, and consumption is chosen instead of output on the grounds of simplicity. This equation can be viewed as the money demand equation. The money is given or taken away from the public by means of government transfers, and hence the money supply is given simply by

$$\dot{M}_t = -P_{T,t}T_t.$$

Dropping the time subscripts from here on, the current-value Hamiltonian for the problem is

$$H = \beta \log C_N + (1 - \beta) \log C_T - v(L) + \lambda(I - \delta K) + \nu(rB + WL + RK + \Pi - \frac{P_N}{P_T}C_N - C_T - I - T)$$

and the first-order conditions for  $C_N$ ,  $C_T$ ,  $L$ , and  $I$  are, respectively:

$$(3) \quad \frac{\beta}{C_N} = \nu \frac{P_N}{P_T},$$

$$(4) \quad \frac{1 - \beta}{C_T} = \nu,$$

$$(5) \quad v'(L) = \nu W,$$

$$(6) \quad \lambda = \nu.$$

From these first-order conditions, the relative consumption of the two consumption goods is a function of relative price

$$(7) \quad \frac{C_T}{C_N} = \frac{1 - \beta}{\beta} \frac{P_N}{P_T},$$

The first-order conditions also lead to a simplification of the quantity equation:

$$(8) \quad M = V \frac{P_T}{\lambda}.$$

Of course, such a simple form of the money demand is an artifact of the Cobb-Douglas utility and of the consumption-dependent quantity equation.

Further, the two Euler equations are

$$(9) \quad \dot{\lambda} = \lambda(\rho + \delta - R),$$

$$(10) \quad \dot{\nu} = \nu(\rho - r).$$

Since the interest rate is exogenous, (10) suggests that a steady state can only be achieved when  $\rho = r$ . Otherwise,  $\nu$  and  $\lambda$  have a deterministic exponential growth at all times. Hence, I restrict  $\rho = r$ , so that  $\lambda$  and  $\nu$  are constant. This is a standard assumption in small open economy models and states that the country faces the steady-state world rate of interest. Even if deviations from this parity occur due to world shocks, they can be only temporary, unless the world rate of time preference changes. From (4), consumption of tradables is constant over time as well.

## 2.2 Production side

The production side consists of two sectors, which produce the two different goods. Capital and labor can move freely between the two sectors, as long as the identities  $L_N + L_T = L$  and  $K_N + K_T = K$  hold. For this reason, the wage and the rental rate have to be the same in both sectors, as long as the amount produced of each type of good is positive. The product of the nontradable sector can only be used for consumption, and cannot be traded for investment goods. Thus,  $C_N = Y_N$ . The product in the tradable sector, on the other hand, can be freely traded for investment goods and bonds in the world market.

Production of the tradable goods is competitive and the technology is a simple constant returns to scale Cobb-Douglas  $Y_T = ZK_T^\theta L_T^{1-\theta}$ , where  $Z$  is the common level of technology.

Since prices in the tradable sector are flexible, the firms are maximizing profits at all times:

$$\max_{K_T, L_T} Y_T - WL_T - RK_T$$

Hence,

$$(11) \quad R = Z\theta \left( \frac{K_T}{L_T} \right)^{\theta-1},$$

$$(12) \quad W = (1 - \theta)Z \left( \frac{K_T}{L_T} \right)^\theta.$$

The capital/labor ratio in the tradable sector and the wage are pinned down from these conditions, since the rental rate is given exogenously by the rest of the world.

In the nontradable sector, on the other hand, prices are rigid. Price rigidity requires market power, and hence each representative firm produces a differentiated product with a generalized Cobb-Douglas technology with internal increasing returns to scale. Hence, the aggregate production function for nontradables is  $Y_N = F(ZK_N^\theta L_N^{1-\theta})$ . The particular form of increasing returns is

assumed to be fixed cost with the same steady-state factor composition as the variable costs. Having the fixed costs is valuable, because this form of increasing returns allows the model to bring long-run profits to zero, and at the same time does not distort the steady state labor and capital demand elasticities, making them identical in both sectors.

In the short run, the firms in the nontradable goods sector are not able to maximize profits: they face a given price, and the quantity produced is then determined by demand. Hence, they minimize cost of producing that pre-specified amount. Therefore, factors are not being paid their marginal product. However, the cost-minimization problem gives us first-order conditions analogous to those for profit maximization, with the one difference that the right-hand side of both equations is pre-multiplied by a Lagrange multiplier and the price ratio. Hence, the capital/labor expenditure ratio is the same in both sectors:

$$\frac{WL_i}{RK_i} = \frac{\theta}{1-\theta}, \quad i = N, T,$$

which can be summed to the aggregate

$$(13) \quad \frac{WL}{RK} = \frac{\theta}{1-\theta}.$$

Thus, the capital-labor ratio is the same in both sectors, even with sticky prices.

Since  $C_N = Y_N$  and  $\Pi = \frac{P_N}{P_T}Y_N + Y_T - WL - RK$ , (2) can be re-written as the material balance condition for the tradable sector:

$$(14) \quad Y_T + rB - \dot{B} = C_T + I,$$

which simply says that total income (production, interest and borrowing) in the tradable sector should equal the sum of tradable consumption and investment.

The assumption of perfect competition in the tradable sector and the exogenous rental rate pin down several variables. First of all, the factor price possibility frontier derived from profit maximization implies

$$(15) \quad W = (1-\theta)Z^{\frac{1}{1-\theta}} \left(\frac{R}{\theta}\right)^{\frac{\theta}{\theta-1}},$$

which pins down the wage. For any given  $\lambda$ , the total labor supply is then implicitly defined by the labor supply equation (5). Again, it is important to remember that after a shock,  $\lambda$  is itself always at the new steady state level, unless there is a temporary shock to the world interest rate. The capital/labor ratio is obtained from (13) and (15):

$$(16) \quad \frac{K_i}{L_i} = \left(\frac{R}{Z\theta}\right)^{\frac{1}{\theta-1}}, \quad i = N, T,$$

which holds true dropping the indices. Then, production in the tradable sector is

$$(17) \quad Y_T = \frac{K_T R}{\theta}.$$

### 2.3 Steady State

An interesting observation is that a number of variables in this setting immediately jump to the new steady state value and remain there following a permanent exogenous shock, such as, for example, money or technology shocks. Thus,  $\lambda, W, L, K$  are pinned down by the above equations. I will show now that  $I, Y, C_T, P_T$  also jump to the new steady state values immediately after the shock.

Equation (1) indicates that in steady state

$$\frac{I^*}{K^*} = \delta,$$

and hence  $I$  is also pinned down. Thus, the aggregate values of capital, labor, and, therefore, output, jump immediately to the steady state following an exogenous shock. However, the relative price follows a more complex path due to the stickiness of the price in the nontradable sector. Thus, the process of adjustment to the steady state consists only of movements of resources between sectors. At the same time, consumption of tradables is completely smoothed out, as can be seen from the first-order condition (4). The price of tradables  $P_T$  is a constant as well from money demand (8). However, the result that the exchange rate (price of tradables) adjusts immediately to the steady state is an artifact of the particular money demand I chose. In principle, overshooting can be obtained if the money demand is made dependent on income instead of consumption.

In the non-tradable sector, the steady-state analogue of (17) is

$$(18) \quad Y_N^* = \frac{RK_N^*}{\theta\gamma^*},$$

where  $\gamma$  is the degree of returns to scale.<sup>2</sup> The relative price in the steady state equals the desired mark-up  $\mu^*$ , since the marginal cost is the same for both firms:  $P_N^* = \mu^* P_T^*$ .

The material balance condition becomes

$$(19) \quad Y_T^* + rB^* = C_T^* + I^*.$$

This equation simply says that in the tradable sector, total income should equal total expenditure. In order to solve for the labor and capital shares in the two sectors, I need to use a variable that denotes the steady state value of the bond/capital ratio. The holdings of bonds are a historical variable here, and a steady state exists for each value of  $B$ , so the steady state is not unique. I can use (7), (17), and (18) to re-write (19) as

$$\frac{R}{\theta}K_T^* + rB^* = \frac{(1-\beta)\mu^*R}{\beta\theta\gamma^*}K_N + \delta K^*$$

and remembering that  $K_N + K_T = K$ ,

$$(20) \quad \zeta \equiv \frac{K_T^*}{K^*} = \frac{L_T^*}{L^*} = \frac{\delta\beta\theta\gamma^* + (1-\beta)\mu^*R}{R(\beta\gamma^* + (1-\beta)\mu^*)} - \frac{r\beta\theta\gamma^*}{R(\beta\gamma^* + (1-\beta)\mu^*)}b^*,$$

<sup>2</sup>This equation is derived by observing that the total revenue  $P_N^*Y_N^*$  equals  $\mu^*MC \cdot Y_N^* = \mu^*\frac{TC}{\gamma}$ , where the last equality follows from a standard observation that returns to scale equals to the ratio of average to marginal cost. The desired equality is then obtained by observing that the total cost  $TC = P_T\frac{RK_N}{\theta}$ . The mark-up does not appear in the equation because it cancels out with the relative price, in contrast to the expression in Kimball (1995).



is the share of total labor or capital utilized in the traded sector, where  $b^* = \frac{B^*}{K^*}$ . This parameter will be used extensively in the rest of the paper.<sup>3</sup> Note that the expression for  $\zeta$  simplifies considerably if one assumes that the steady state degree of returns to scale equals the mark-up, as I will assume in my calibration. In fact, in this case almost all of the log-linearized equations become identical to the case with constant returns and no mark-up, as  $\mu^*$  and  $\gamma^*$  cancel everywhere.

Now that I know the steady-state shares of capital and labor utilized in the two sectors, I can solve for steady-state values of labor and capital in individual sectors. Using (18), we find that

$$(21) \quad C_N^* = Y_N^* = \frac{K_N R}{\theta \gamma^*} = \frac{(1 - \zeta) R}{\theta \gamma^*} K^* = \frac{\beta}{\lambda^*},$$

where the last equality is the steady state version of (3). Result (21) together with the steady-state versions of (16), (5), and (15) gives us  $\lambda^*$  in terms of exogenous variables. Given  $\lambda^*$ , one can obtain the steady state values of all the other variables of interest. In order to obtain explicit solutions, however, one needs to assume a specific form of the function  $v(L)$  in the individual utility function.

## 2.4 A permanent increase in the money supply

In order to solve for the dynamics of the model following a money shock, I need to log-linearize the above equations. Similarly to Obstfeld and Rogoff (1995), I log-linearized around an *initial* steady state, with some arbitrary level of inherited debt. This steady-state is not unique, as the debt position of the country may change.<sup>4</sup> Although the model above may seem formidable, the log-linearized equations turn out to be surprisingly simple. The equations fall into four categories: utility maximization, production, quantity constraints, and dynamic equations. Since I am considering a money shock, I set  $\check{R} = 0$  and  $\check{Z} = 0$  throughout, where  $\check{X} = \frac{dX}{X^*} = d \log X$  denotes a log deviation from the steady state value of  $X$ .

*Utility maximization:* Log-linearizing the first-order conditions, we get

$$(22) \quad \check{C}_N = -\check{P}_N + \check{P}_T - \check{\lambda},$$

$$(23) \quad \check{C}_T = -\check{\lambda}.$$

$$(24) \quad \frac{\check{L}}{\eta} = \check{W} + \check{\lambda},$$

where  $\eta = \frac{v'(L^*)}{L^* v''(L^*)}$ .

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<sup>3</sup>There is a caveat for (20): for arbitrary values of  $b^*$ ,  $\zeta$  can become negative or exceed one. A  $\zeta > 1$  implies that the country has borrowed an immense amount, and needs to spend a great deal on financing the debt. Hence, production of tradables has to be high, and mathematically can exceed total production. This case is unlikely, however, and can be ruled out by setting a restriction on borrowing. A negative  $\zeta$  implies that the country has a great deal of bonds, and does not need to produce tradables, but rather can buy them using the interest income. Instead the country produces a great deal of nontradables. Again, a non-negativity constraint may be necessary to restrict the production of nontradables from exceeding total production.

<sup>4</sup>A way to address this problem is to introduce endogenous rate of time preference in order to pin down a unique steady-state level of consumption, and therefore, all other aggregate variables (Obstfeld 1981, Mendoza 1991). This will be shown in Section 6.

*Production side:* I derive the following very simple equations, which are independent of each other and the other equations considered here:

$$(25) \quad \check{W} = 0,$$

$$(26) \quad \check{K} = \check{L},$$

$$(27) \quad \check{K}_T = \check{L}_T,$$

$$(28) \quad \check{Y}_T = \check{K}_T,$$

$$(29) \quad \check{Y}_N = \gamma^* \check{K}_N,$$

$$(30) \quad \check{C}_N = \check{Y}_N.$$

These equations simply say that the wage remains pinned down by the factor price possibility frontier, capital/labor ratio remains constant in both sectors and in aggregate, tradable output is proportional to capital from (17) and consumption equals production in the nontradable sector. The equation (29) comes from the log-linearized production function  $\check{Y}_N = \gamma^*[\theta\check{K}_N + (1 - \theta)\check{L}_N]$ , and the fact that capital/labor ratio is constant in the nontradable sector as well.

*Quantity constraints:* the identities that say capital and labor in the two sectors sum up to the aggregate imply

$$(31) \quad \check{L} = \zeta\check{L}_T + (1 - \zeta)\check{L}_N$$

$$(32) \quad \check{K} = \zeta\check{K}_T + (1 - \zeta)\check{K}_N$$

and the quantity equation is written as

$$(33) \quad \check{M} = \check{P}_T - \check{\lambda}.$$

*Dynamic equations:* the capital accumulation equation (1) is log-linearized to be  $\dot{\check{K}} = \delta(\check{I} - \check{K})$ . However, since capital is always in the steady state, we get

$$(34) \quad \check{I} = \check{K}$$

everywhere other than at the instant of the jump in capital, at which point investment is infinite and is financed solely by borrowing from abroad.

It is less straight-forward to log-linearize the bond-accumulation equation because of the possibility that  $B_0^* = 0$ . Percent deviations from zero are undefined. Hence, I define  $\check{B} \equiv dB/K^*$ , which is change in bond holdings as a percentage of steady state capital stock. Then, (14) gives us

$$\dot{\check{B}} = \frac{Y_T^*}{K^*} \check{Y}_T - \frac{I^*}{K^*} \check{I} - \frac{C_T^*}{K^*} \check{C}_T + r\check{B},$$

and plugging in the steady-state values of the above shares in terms of exogenous parameters,

$$(35) \quad \dot{\check{B}} = \frac{\zeta R}{\theta} \check{Y}_T - \delta \check{I} - \frac{(1 - \beta)(1 - \zeta)\mu^* R}{\theta\beta\gamma^*} \check{C}_T + r\check{B}.$$

Since in the model, both capital and bond holdings are jump variables, but their sum is not, it makes sense to introduce a variable denoting total assets,  $A \equiv K + B$ . Since I measure deviations of bonds from steady state in percent of capital stock, the log-linearized version of this definition is  $(1 + b^*)\check{A} = \check{K} + \check{B}$ . The dynamic equation for the total assets is then

$$(36) \quad (1 + b^*)\dot{\check{A}} = \frac{\zeta R}{\theta} \check{Y}_T - \frac{(1 - \beta)(1 - \zeta)\mu^* R}{\theta\beta\gamma^*} \check{C}_T + r(1 + b^*)\check{A} - (r + \delta)\check{K}.$$

Note that investment drops out.

For now, the rigid price of non-tradables gradually approaches its shadow value:

$$(37) \quad \dot{\check{P}}_N = -\kappa(\check{P}_N - \check{P}_N^*),$$

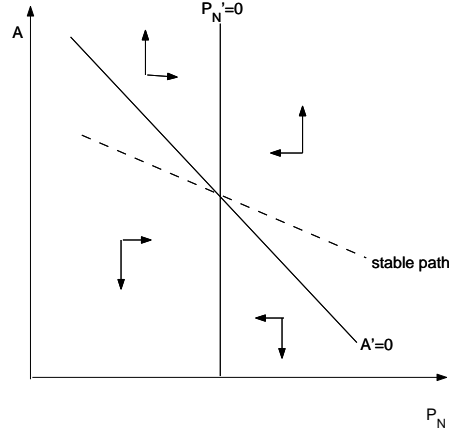
where  $\check{P}_N^*$  is the price that would prevail if the prices were fully flexible, and the economy were at a steady state. The parameter  $\kappa$  is the macroeconomic rate of price adjustment. Since at the new steady state the prices of tradables and nontradables are proportional, we have  $\check{P}_N^* = \check{P}_T$ , and therefore,  $\check{P}_N^* = \check{M} - \frac{\check{K}}{\eta}$ . Thus, the price of non-tradables is assumed to gradually approach the price of tradables, since in any steady state, they differ by a constant mark-up.

## 2.5 Describing the response to a money shock

There are three stages in the adjustment to the money shock: the original jump, the transitional dynamics, and the new steady state, to which the system will converge. In closed-economy macroeconomics, we normally think of monetary shocks as being neutral in the long run, and hence the old and the new steady states are identical in all real variables. However, this does not need to be so in an open economy. The temporary high demand stimulates a permanent increase in capital financed by borrowing from abroad. Later, a fraction of production will have to go towards the service of this acquired debt, but production overall will be higher. Thus, money shocks can have permanent real effects. Intuition suggests, of course, that these effects are likely to be small: a large increase in production means a large increase in labor supply, which will be possible only if the labor supply is elastic. Since a fraction of the newly produced goods goes towards debt payments, the change in welfare cannot be big, and hence a large increase in labor supply cannot be an optimum. Simulations provided later in the paper will show that the real effects are indeed negligible. Another thing to note is that these real effects can only be a result of unanticipated jumps, and hence cannot be used by the monetary authority systematically.

In order to characterize the dynamics, I need an additional restriction. The immediate jump in capital cannot be financed by domestic investment (or rather, only an infinitesimal part of it can), and therefore, needs to be financed by borrowing from abroad. Effectively, the economy has a stock of capital and a stock of bonds, and the sum of these stocks cannot jump; instantaneous changes can only be in the form of transferring wealth from one form to the other. For this reason, it is best to analyze the system using the dynamic equation for the total assets (36). Then, the dynamic

Figure 1: The Phase Diagram



system of equations that characterizes the response to the shock can be deduced from equations (22)-(36):

$$(38) \quad (1 + b^*)\dot{A} = \left( \frac{R(1 - \theta)}{\theta} + \frac{(1 - \beta)(1 - \zeta)\mu^*R}{\theta\beta\eta\gamma^*} \right) \check{K} - \frac{R(1 - \zeta)}{\theta\gamma^*}(\check{M} - \check{P}_N) + r(1 + b^*)\check{A}$$

$$(39) \quad \dot{\check{P}}_N = \frac{\kappa}{\eta}\check{K} + \kappa(\check{M} - \check{P}_N),$$

where both variables are historical. However,  $K$  serves as a free endogenous jump variable, a shifter, which brings the system to the stable path immediately following the shock.

In order to see the adjustment process, it is helpful to first draw the phase diagram of the system taking capital as a parameter, which may change its value at the time of the shock and stay at the new steady state value. The phase diagram is depicted in Figure 1.

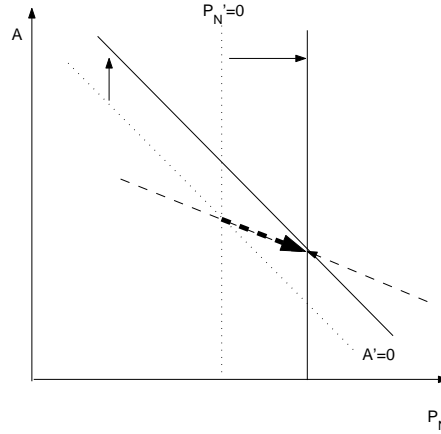
Now, we are ready to analyze the response to a permanent increase in the money supply, shown in Figure 2. The new price of nontradables will have to be higher, hence the  $\dot{P} = 0$  isocline shifts out to the right. The assets isocline then has to shift out as well, so that the new steady state remains on the old stable path, which is necessary since neither variable can jump.

As one can see from the picture, the amount of total assets decreases. However, the graph does not immediately demonstrate what happens to the stocks of capital and bonds individually. In order to see this, however, consider the new steady-state versions of (38) and (39), with the left-hand sides set to zero. Adding these equations together, and eliminating  $(\check{M} - \check{P}_N)$ , we get

$$(40) \quad r(1 + b^*)\check{A}^* = -\frac{R}{\theta} \left( (1 - \theta) + \frac{(1 - \beta)(1 - \zeta)\mu^*}{\beta\eta\gamma^*} + \frac{1 - \zeta}{\eta\gamma^*} \right) \check{K}^*.$$

This equation states that the steady state change in capital is of opposite sign from the change in total assets. Lower wealth leads to a need for more output in order to repay the debt. Therefore, capital increases. The bonds fall initially by the same percentage, and then decline even further.

**Figure 2: The Response to Money Shock**



These developments can also help us deduce what happens to other variables. Obviously, labor is permanently higher, since capital is higher. Consumption of both types are permanently lower in the long run. However, in the short run, consumption of nontradables is temporarily high, as people shift into that type of goods due to their relative cheapness. Hence, resources, such as labor and capital, have to shift towards the nontradable sector to satisfy the demand.

## 2.6 Calibration and Simulations

With intuition clear, it now makes sense to solve the model numerically, in order to get a grasp of the quantitative properties of the model. In order to accomplish this, I need to approximate the continuous-time equations by the corresponding discrete-time approximations. This can be done easily by replacing the time derivatives with period differences: for example, setting  $\dot{A} \approx \check{A}_{t+1} - \check{A}_t$ , and attaching a subscript  $t$  to all right-hand side variables.

In approximation of continuous time processes, it is logical to make the chosen time periods small, so that the obtained simulations are more in line with the underlying equations. Thus, I take one time period to be one hundredth of a year, so all annual rates (discount, interest, depreciation, and price adjustment) need to be divided by 100.

I report the values of the parameters used here and later in the paper in Table 1. For most part, I take parameters to be as in Kimball (1995). Thus, I set  $\rho = .02$  per year, and  $\delta = .08$  per year, which I divide by 100 to make time periods small,  $\theta = .3, \eta = 1, \mu^* = \gamma^* = 1.1$ . The two coefficients, which are not in Kimball (1995) are the rate of price convergence  $\kappa$  and the steady-state fraction of nontradables  $\beta$ . For now, I set  $\kappa = 0.5$  per year, which will be changed once I introduce a complete analysis of price dynamics, and  $\beta = 0.5$ . The parameter  $\kappa$  is the macroeconomic rate of price adjustment, and its chosen value implies that the half-life of the deviation of the price level from the long-run value is about one year.

Table 1: Benchmark parameter values.

Parameter	Description	Value
<i>Yearly rate parameters</i>		
$\rho$	Discount rate	2% per year
$r$	Real interest rate	2% per year
$\delta$	Depreciation rate	8% per year
<i>Utility parameters</i>		
$\beta$	Share of non-tradable goods in utility	0.5
$h$	Share of leisure in utility spent on production of nontradables	0.5
$s$	Elasticity of intertemporal substitution	0.35
$\eta$	Consumption-constant elasticity of labor supply	1
$\tau$	Ratio of steady-state wage income to consumption	1
<i>Production parameters</i>		
$\theta$	Share of capital in production	0.3
$j$	Elasticity of $Q$ w.r.t. investment/capital ratio	0.2
$\mu^*$	Steady-state mark-up	1.1
$\gamma^*$	Steady-state degree of internal returns to scale	1.1
<i>Price-adjustment parameters</i>		
$\kappa$	Macroeconomic rate of price adjustment	0.5 per year
$\alpha$	Microeconomic rate of price adjustment	1 per year
$\epsilon^*$	Steady-state elasticity of demand	11
$\omega$	Elasticity of marginal revenue w.r.t. relative output	4.37
<i>Steady state conditions</i>		
$b^*$	Original steady-state bond/capital ratio	0

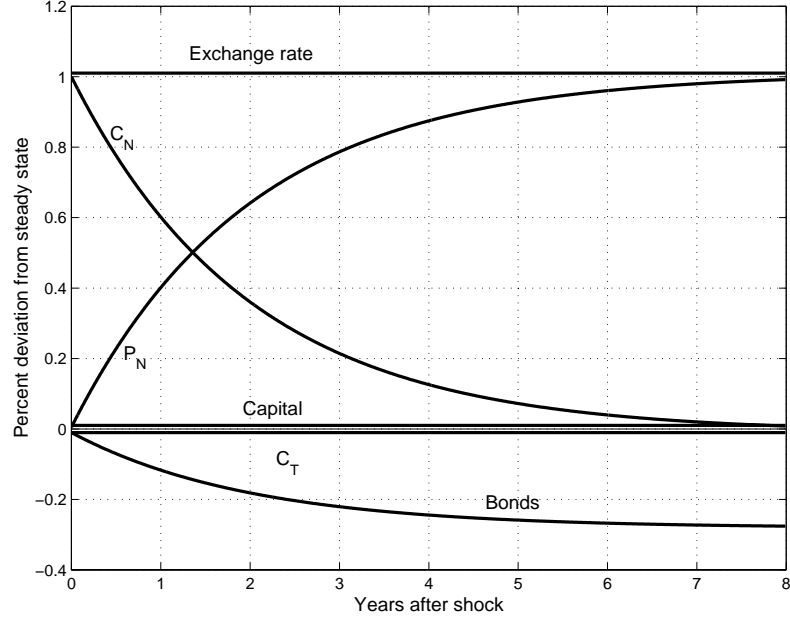
The impulse responses to a one percent shock to money is shown in Figure 3.<sup>5</sup> As one can see, the simulations support the theoretical predictions. The exchange rate jumps to the new steady state immediately and the consumption of nontradables is temporarily high. Also, the long-run effects are seen to be negligible: capital is permanently higher, and consumption of tradables is permanently lower, but the magnitudes of these deviations are miniscule. Hence, the long-run non-neutrality of money in open economy models should not be over-emphasized, as was argued originally by Obstfeld and Rogoff (1995).

### 3 Generalizing the Utility Function

In the previous analysis, I assumed a simple log utility, which implies additive separability of consumption and leisure. Such a utility function is simpler to analyze, but is highly unrealistic, as was argued by Kimball (1995) and Basu and Kimball (2001). More generally, it makes sense to

<sup>5</sup>The impulse responses were plotted using the Matlab package offered by Uhlig (1999).

**Figure 3: Responses to a Shock in Money**



assume King-Plosser-Rebelo utility

$$u(C, L) = \frac{C^{1-\frac{1}{s}} e^{(\frac{1}{s}-1)v(L)}}{1-\frac{1}{s}},$$

where  $C = C_N^\beta C_T^{1-\beta}$ . In previous analysis with log utility, I implicitly assumed that  $s = 1$ , where  $s$  represents the intertemporal elasticity of substitution. Using this utility function, we can get the following first-order conditions for utility maximization:

$$(41) \quad C^{-\frac{1}{s}} e^{(\frac{1}{s}-1)v(L)} \beta \left( \frac{C_N}{C_T} \right)^{\beta-1} = \lambda \frac{P_N}{P_T}$$

$$(42) \quad C^{-\frac{1}{s}} e^{(\frac{1}{s}-1)v(L)} (1-\beta) \left( \frac{C_N}{C_T} \right)^\beta = \lambda$$

$$(43) \quad v'(L) C_T = (1-\beta)W.$$

Note that the condition

$$\frac{C_T}{C_N} = \frac{1-\beta}{\beta} \frac{P_N}{P_T}$$

still holds. I used this condition in deriving the labor supply (43), which is also unchanged from the labor supply equation of the basic model with separable utility.

One thing is immediately apparent from these first-order conditions: capital is no longer restricted to be at its steady state value at all times. Previously, we had that result because the capital/labor ratio is fixed, and labor is proportional to  $\lambda$  by (5). That result was an artifact of the utility function being additively separable in consumption and leisure.

Further, we are no longer able to write the money demand in terms of  $\lambda$  and the price of tradables alone. However, the Cobb-Douglas aggregate of the two consumption good types still allows us to write the quantity equation in a simple enough form:

$$(44) \quad M = \frac{V}{1-\beta} P_T C_T = \frac{V}{\beta} P_N C_N.$$

The log-linearized version of the model differs by four equations from the model presented in Section 2.4. These equations are the first-order conditions for consumption, money demand, and the capital-accumulation equation. They look like this:

*First-order conditions:*

$$(45) \quad (\beta + s(1-\beta))\check{C}_N + (1-\beta)(1-s)\check{C}_T - (1-s)(1-\beta)^{1-\beta}\beta^\beta\mu^{*-\beta}\tau\check{L} = -s(\check{\lambda} + \check{P}_N - \check{P}_T)$$

$$(46) \quad \beta(1-s)\check{C}_N + (1-\beta(1-s))\check{C}_T - (1-s)(1-\beta)^{1-\beta}\beta^\beta\mu^{*-\beta}\tau\check{L} = -s\check{\lambda}$$

$$(47) \quad \frac{\check{L}}{\eta} + \check{C}_T = \check{W},$$

where  $\tau = \frac{W^*L^*}{C^*}$ . Note that the labor supply equation is the same as before, while the first two conditions simplify nicely to  $\check{C}_T + \check{P}_T = \check{C}_N + \check{P}_N$ , which is also the same as before.

*The money demand:*

$$(48) \quad \check{M} = \check{P}_T + \check{C}_T = \check{P}_N + \check{C}_N$$

*Capital accumulation* is described by the same equation as before without the result of constant capital:

$$(49) \quad \dot{\check{K}} = \delta(\check{I} - \check{K})$$

Again, the primary difference from the previous analysis is that capital can no longer be considered a shifter – capital is now moving during the adjustment process. However, the first-order condition for consumption of tradables allows us to express capital as a linear combination of  $P_N$  and  $\lambda$ . Since  $\lambda$  stays at the new steady state value, it can be used as a shifter instead of capital. Using (48) for  $\check{C}_N$ , (47) for  $\check{C}_T$ , and remembering that  $\check{K} = \check{L}$ , (46) reduces to

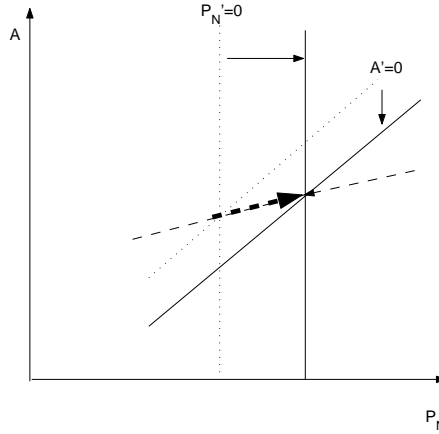
$$(50) \quad \beta(1-s)(\check{M} - \check{P}_N) - \left( \frac{1-\beta(1-s)}{\eta} + (1-s)(1-\beta)^{1-\beta}\beta^\beta\mu^{*-\beta}\tau \right) \check{K} = -s\check{\lambda}$$

In order to draw a phase diagram with two variables, I reduce all of the equations to the same two variables as before:  $A$  and  $P_N$ . Using (50) and defining for simplicity  $a \equiv \frac{1-\beta(1-s)}{\eta} + (1-s)(1-\beta)^{1-\beta}\beta^\beta\mu^{*-\beta}\tau$ , (38) and (39) become

$$\begin{aligned} (1+b^*)\dot{A} &= \left( \frac{R(1-\theta)}{\theta} + \frac{(1-\beta)(1-\zeta)\mu^*R}{\theta\beta\eta\gamma^*} \right) \frac{s}{a} \check{\lambda} \\ &+ \left[ \left( \frac{R(1-\theta)}{\theta} + \frac{(1-\beta)(1-\zeta)\mu^*R}{\theta\beta\eta\gamma^*} \right) \frac{\beta(1-s)}{a} - \frac{R(1-\zeta)}{\theta\gamma^*} \right] (\check{M} - \check{P}_N) + r\check{A} \\ \dot{P}_N &= \left( \frac{\kappa\beta(1-s)}{\eta a} + \kappa \right) (\check{M} - \check{P}_N) + \frac{\kappa s}{\eta a} \check{\lambda} \end{aligned}$$



**Figure 4: The Response to Money Shock: Case 2**



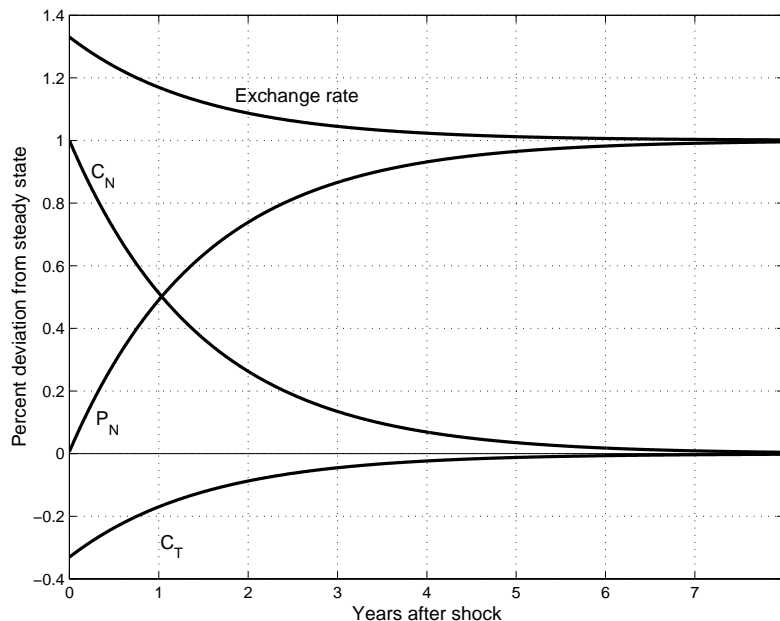
In this case, the dynamics of the model in response to a money shock can be different, depending on the sign of the coefficient in front of the money balances in the dynamic equation for assets. If this coefficient is negative, then the phase diagram looks exactly as in Figure 2, that is, isocline  $\dot{A} = 0$  is downward sloping. It is also possible, however, that this isocline is upward-sloping. Let me consider these two cases individually.

*Case 1: Isocline slopes down.* In this case, the dynamics look exactly as before. The difference is that the shifter is now  $\lambda$ . However, we still have the result from (40) that capital ends up higher than initially. We also can see from (50) that in the transition period, capital is falling, since money and  $\lambda$  stay constant, while the price of nontradables is rising. Therefore, initially, capital jumps up, while bond holdings jump down to finance that increase in capital. Then, capital is allowed to slowly deteriorate, but not all the way back to the original level. Thus, labor, together with capital, ends up higher than initially, while consumption ends up lower, with a negative long-run effect on welfare.

*Case 2: Isocline slopes up.* These dynamics are symmetric to Case 1 and are depicted in Figure 4. Total assets go up in the process, and hence capital needs to end up lower than at the initial steady state. However, one can still show that the original change in capital is positive. For that, consider the dynamic equation for total assets in its original form (38) at the time of the shock. We have  $\check{P}_N = \check{A} = 0, \dot{\check{A}} > 0, \check{M} > 0$ . The only way this can hold is if  $\check{K} > 0$ . Thus, capital increases and bonds fall. However, in this case capital then goes down to a level below the original. Thus, labor goes down as well, while consumption rises.

Thus, I have shown that in both of the cases with the general utility function, the basic result is the same as before: in response to a money shock, the price of nontradable goods becomes relatively low, and people shift towards consumption of those goods. In order to increase production to satisfy the increased demand, the country borrows capital, which goes into the nontradable sector. As the price of the nontradables approaches the price of tradables, however, consumption of those

**Figure 5: Impulse Responses to a Money Shock**



goods goes down, and so does the capital employed in the sector, as well as the total capital in the country.

The behavior of the current account in the transition phase is ambiguous for general values of parameters, however. All models predict the original deficit at the time of the shock; but then, the current account, represented by the growth of bond holdings, is negative in the simple case of  $s = 1$ , positive in Case 2 with the general utility, and indeterminable in Case 1 with the general utility.

An interesting result of the last section is that  $s < 1$  generates overshooting of the exchange rate. This follows from the fact that capital, and therefore labor supply, jumps up on impact, and then gradually goes down. This suggests by the labor supply relation (47) that  $\check{C}_T$  first jumps down, and then comes gradually up. Then, by money demand (48),  $\check{P}_T$ , which equals the exchange rate in this model, jumps up and then converges downward.

### 3.1 Simulations

Again, the theoretical results can be checked by simulating the model numerically. In order to accomplish this, I need to introduce the values of two more parameters. Following Basu and Kimball (2001) and Kimball (1995), I set  $s = 0.35$  and  $\tau = 1$ .

Some of the responses to a money shock are shown in the Figure 5. As one can see, overshooting is present, as was predicted, but is not very strong. In principle, the degree of overshooting appears to be sensitive to the value of  $\beta$ , which is the share of steady state nontradable consumption. But even a very high value of  $\beta$  cannot get the immediate response of the exchange rate above two per

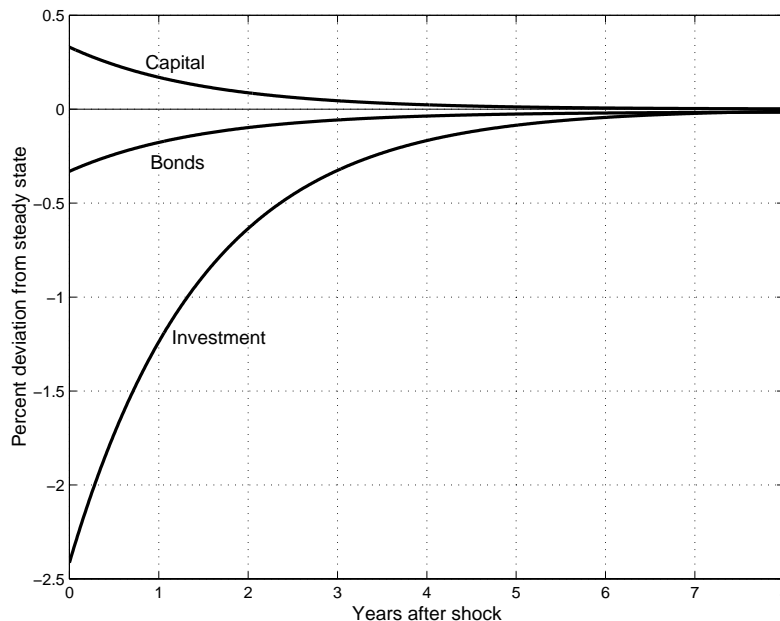
cent. Consumption of non-tradables, due to this overshooting, actually goes down for some time, and then converges to a small negative value. Consumption of nontradables behaves similarly to the previous case.

The weakness of overshooting of the exchange rate should not cause too much worry, however. The evidence of overshooting is weak. Faust and Rogers (2000) summarize the evidence and argue that overshooting, if present at all, is unlikely to be caused by the traditionally assumed interest rate parity, and cannot explain the observed difference between the volatility of monetary aggregates and the exchange rates. This marked volatility of the exchange rates relative to money is precisely what has been driving researchers to build models with a strong degree of overshooting.

The argument of Faust and Rogers (2000) that overshooting is not caused by the interest rate parity is particularly interesting, because this parity is not what drives overshooting in this model. The traditional Dornbusch argument includes the liquidity effect (fall in the nominal interest rate following a positive money shock), which forces the value of the currency to go up (and the exchange rate to go down) to compensate foreign investors for low yields. However, in the long run, the exchange rate has to end up higher than before the money shock; hence, in the short run, the exchange rate has to go above its long-run value. The same logic has been used in some modern models as well (Chari et al. 2000, Betts and Devereux 2000). In my model, however, money demand is interest-insensitive, and therefore money shocks cause no liquidity effects. Overshooting comes from a different source. The increased demand stimulates an increase in total labor, which by the labor supply equation (47) drives down the consumption of tradables. Since consumption of tradables is down, the price of nontradables has to go up by the quantity equation. However, labor supply goes up only temporarily, while the aggregate demand is high due to price stickiness, so eventually the price of tradables has to fall. Hence, in the short run, we should see overshooting. Remember, once again, that in the case of a separable utility function, overshooting does not take place, even though equation (47) still holds: consumption of tradables is perfectly smoothed, and labor has to be perfectly smoothed as well. What is different here is that labor and consumption are complements. Hence, in the first-order condition for nontradable consumption (41), the left-hand side (marginal utility of nontradable consumption) depends negatively on  $C_N$  and positively on labor. So an increase in  $C_N$  caused by a monetary expansion makes households willing to supply more labor unless  $P_T$  increases. Labor supply only needs to respond so much in order to satisfy the additional demand. Hence, in order to prevent too much of a labor supply response, the price of tradables needs to respond extra, driving down tradable consumption and hence reducing the desire for extra work. In this way, complementarity of labor and consumption by itself can cause overshooting of the exchange rates.

The impulse responses of capital, bonds, and investment are shown in Figure 6. Here, it is seen that Case 2 from the previous section seems to be prevalent for these parameter values, but only slightly, since long-run effects on both capital and bonds are small. Hence, the stable path in Figure 4 is almost flat. However, this result is sensitive to parameter value of  $s$ , and a higher  $s$  would make the long-run bond holdings significantly below zero.

**Figure 6: Impulse Responses to a Money Shock**



Another striking result is, of course, the sharp reduction in investment. Basically, immediately following the shock, investment is infinite. Following the shock, however, capital is falling at a decelerating rate. Hence, investment is low but increasing. Likewise, the current account throughout is in surplus as the quantity of bonds grows. However, this is not true on impact: the quantity of bonds drops, which implies an instantaneous deficit. Hence, the current account goes in deficit or exhibits a “J-curve response,” using the term frequently mentioned in the literature. This feature will be more apparent in the next section.

## 4 Adding Investment Adjustment Costs

The result that investment drops following a money shock is strikingly unrealistic, but should not cause too many worries. Investment does go up immediately following the shock, but for an infinitesimal period of time, during which capital is instantaneously shifted towards the home country from abroad. Thus, on impact, investment is positive and infinite, and hence, the addition of adjustment costs, however small, would break this result and cause investment to be finite and positive in the first period. Therefore, I want to extend the model to include the adjustment costs. The drawback is that from now on I lose the ability to present the model graphically, since I have more than two state variables. However, the intuition obtained previously will help interpret the numerical simulations that follow.

A complication in adding investment adjustment costs is that they have to be introduced for both sectors of the economy to get the desired effect. If the gross investment were made costly,

but capital and labor remained perfectly fluid between sectors, then the response to the money shock would not involve much investment; rather, resources would be shifted across sectors only. Hence, I need to block this perfect factor mobility by making investment in each individual sector costly. Likewise, in order to avoid the perfectly fluid labor mobility that I assumed previously, I will assume that the workers have different attitudes towards working in the two sectors. This can be interpreted loosely as a situation in which the labor force is divided into two types of workers: those who are trained to work in one sector, and those who are trained to work in the other. One can then imagine that each household has workers of both types, so hours put into both sectors appear in the representative household's utility. The need to move labor from one sector to another is then represented as reduction of one type of labor and increase of the other, which, due to diminishing marginal utility, implies costs to labor mobility which are here represented as utility costs. The reason to have this imperfect labor mobility is once again to prevent a money shock from merely causing a temporary migration of workers from one sector to another, with little total effect on the output.

The resulting model will be best described by near monetary neutrality in the tradable sector with large real effects of the money shock in the nontradable sector.

#### 4.1 Utility maximization

I change preferences in the following way. The felicity of an individual shall now be

$$U = \frac{C^{1-\frac{1}{s}}}{1-\frac{1}{s}} e^{(1-\frac{1}{s})[h \log(2h-L_N) + (1-h) \log(2(1-h)-L_T)]}.$$

This felicity function says that households have a total of two units of time that they can spend on work, of which fraction  $h$  is time available for work in the non-tradable sector, and  $1-h$  for work in the tradable sector. Since substitution between sector-specific labor is no longer perfect, this specification should limit inter-sectoral movement of labor. Another way to put it is that the production possibility frontier between the two sectors is now concave.

Further, I impose two capital accumulation equations:

$$(51) \quad \dot{K}_N = K_N J \left( \frac{I_N}{K_N} \right),$$

$$(52) \quad \dot{K}_T = K_T J \left( \frac{I_T}{K_T} \right).$$

The function  $J(\cdot)$  is similar to the one in Kimball (1995), and satisfies the following conditions:  $J(\delta) = 0$ , since  $\delta$  is the steady-state investment-capital ratio and  $J'(\delta) = 1$  by normalization. Thus, the adjustment costs are introduced as losses of capital associated with its rapid installation or removal.

With these equations defined, I write down the current-value Hamiltonian:

$$\begin{aligned}
H &= \frac{C^{1-\frac{1}{s}}}{1-\frac{1}{s}} e^{(1-\frac{1}{s})[h \log(2h-L_N)+(1-h) \log(2(1-h)-L_T)]} + \\
&\lambda_N K_N J\left(\frac{I_N}{K_N}\right) + \lambda_T K_T J\left(\frac{I_T}{K_T}\right) + \\
&\nu \left[ rB + W_T L_T + W_N L_N + R_N K_N + R_T K_T + \Pi - \frac{P_N}{P_T} C_N - C_T - I_N - I_T - T \right].
\end{aligned}$$

The first-order conditions for  $C_N, C_T, L_N, L_T, I$ , and  $X$  are, respectively,

$$(53) \quad C^{-\frac{1}{s}} e^{(1-\frac{1}{s})[h \log(2h-L_N)+(1-h) \log(2(1-h)-L_T)]} \beta \left(\frac{C_N}{C_T}\right)^{\beta-1} = \nu \frac{P_N}{P_T}$$

$$(54) \quad C^{-\frac{1}{s}} e^{(1-\frac{1}{s})[h \log(2h-L_N)+(1-h) \log(2(1-h)-L_T)]} (1-\beta) \left(\frac{C_N}{C_T}\right)^{\beta} = \nu$$

$$(55) \quad C^{1-\frac{1}{s}} e^{(1-\frac{1}{s})[h \log(2h-L_N)+(1-h) \log(2(1-h)-L_T)]} \frac{h}{2h-L_N} = \nu W_N$$

$$(56) \quad C^{1-\frac{1}{s}} e^{(1-\frac{1}{s})[h \log(2h-L_N)+(1-h) \log(2(1-h)-L_T)]} \frac{1-h}{2(1-h)-L_T} = \nu W_T$$

$$(57) \quad Q_N J' \left(\frac{I_N}{K_N}\right) = 1,$$

$$(58) \quad Q_T J' \left(\frac{I_T}{K_T}\right) = 1,$$

where  $Q_N \equiv \frac{\lambda_N}{\nu}$ , and  $Q_T \equiv \frac{\lambda_T}{\nu}$  are the marginal values of nontradable and tradable capitals in real units of currency. Wages and rental rates are no longer restricted to be the same in the two sectors, since the assumption of perfect mobility has been abandoned. Furthermore, outside of the steady state, the rental rate is no longer given by the world market, again due to inability to move capital costlessly between countries.

The first-order conditions for labor and consumption can be reduced to simpler relative equations:

$$(59) \quad \frac{C_N P_N}{C_T P_T} = \frac{\beta}{1-\beta}$$

$$(60) \quad \frac{(2(1-h)-L_T)W_T}{(2h-L_N)W_N} = \frac{1-h}{h}.$$

Note that in this setting, the steady-state relative price no longer needs to be equal to the markup, because factor prices are not identical. However, numerical simulations will show that the discrepancy from unity is negligible. The steady-state relative price and wage close to unity are desirable: we would like labor to be eventually distributed according to the needs of the economy, not according to subjective tastes of the individuals. Hence, in the long run, individuals should not care where they work and wages should be the same, as was the case in the previous model. Since the rental rate is the same for both sectors in the steady-state, it would make sense for the costs

of production to be identical, and the relative price would again be the mark-up. In principle, I could impose this logic by setting the steady-state relative wage to be equal to unity. However, I will proceed without making this restriction.

Another variable that no longer is restricted to be constant is the multiplier  $\lambda_i$  for both sectors  $i = N, T$ , which I divide by  $\nu$  following Kimball (1995) in order to get the shadow prices of capital  $Q_i$  denominated in physical consumption units.  $\nu$ , however, remains a constant, which makes it easy to convert  $\lambda_i$  into  $Q_i$ . Thus, the Euler equations for the maximization problem above are

$$(61) \quad \frac{\dot{Q}_N}{Q_N} = r - J \left( \frac{I_N}{K_N} \right) - \frac{R_N - I_N/K_N}{Q_N},$$

$$(62) \quad \frac{\dot{Q}_T}{Q_T} = r - J \left( \frac{I_T}{K_T} \right) - \frac{R_T - I_T/K_T}{Q_T},$$

$$(63) \quad r = \rho.$$

In principle, the steady state conditions are similar to those of the basic model. The only important difference is that in general, the relative wage at the steady state does not need to be unity, and hence, the relative price between sectors does not need to be equal to the mark-up. The relationship between the two is given by the following expression:

$$\frac{P_N^*}{P_T^*} = \mu^* \left( \frac{W_T^*}{W_N^*} \right)^{\theta-1}.$$

The other equation that is obtained from this is the generalization of (18):

$$\frac{P_N^*}{P_T^*} Y_N^* = \frac{R^* K_N^* \mu^*}{\theta \gamma^*},$$

in which the relative price and the mark-up no longer cancel out. The actual value of the relative wage and the relative price depends on the particular steady-state.

As alluded to earlier, one could impose equality of wages in the two sectors at the steady-state. On the basic level, such a restriction brings the model more in line with the basic version. However, this restriction also pins down a particular steady-state. To see this, set  $W_N^* = W_T^*$  in (60) and combine this equation with the steady-state condition  $L_T^*/L_N^* = \zeta/(1-\zeta)$ , which continues to hold in the model with adjustment costs. Then, we get  $h = 1 - \zeta$ , and the only free parameter in  $\zeta$ , by equation (20) is the bond-capital ratio  $b^*$ . Thus, the bond-capital ratio is pinned down by the parameter  $h$  of the utility function, and the steady-state is then determined. Of course, such a restriction does not implies a unique long-run steady-state. Following a shock, the wage rates will not converge back to the same level. In order to force them to come back to the same level, one would need to make this restriction in the optimization problem of the household from the start.

## 4.2 Log-linearized Equations

The whole model in the linearized form consists of the following 21 equations:

*Utility maximization:*

$$(64) \quad [s(1-\beta) + \beta]\check{C}_N + (1-s)(1-\beta)\check{C}_T - (1-s)(1-\theta)\beta\frac{\gamma^*}{\mu^*}\check{L}_N - \frac{(1-s)(1-\theta)\zeta\beta\gamma^*}{(1-\zeta)\mu^*}\check{L}_T \\ = -s\check{\nu} - s\check{P}_N + s\check{P}_T,$$

$$(65) \quad \check{C}_N + \check{P}_N = \check{C}_T + \check{P}_T,$$

$$(66) \quad \check{C}_T = -\frac{(1-\theta)\beta\gamma^*}{h\mu^*}\check{L}_N + \check{W}_N,$$

$$(67) \quad \frac{\zeta\beta(1-\theta)\gamma^*}{(1-h)(1-\zeta)\mu^*}\check{L}_T - \check{W}_T = \frac{(1-\theta)\beta\gamma^*}{h\mu^*}\check{L}_N - \check{W}_N,$$

$$(68) \quad \check{Q}_N = j(\check{I}_N - \check{K}_N),$$

$$(69) \quad \check{Q}_T = j(\check{I}_T - \check{K}_T).$$

where  $j = \frac{-\delta J''(\delta)}{J'(\delta)}$ . To log-linearize the first-order conditions for consumption and labor, I derive the following steady-state elasticities:  $\frac{L_N^*}{2h-L_N^*} = \frac{\beta(1-\theta)\gamma^*}{h\mu^*}$  and  $\frac{L_T^*}{2(1-h)-L_T^*} = \frac{\zeta\beta(1-\theta)\gamma^*}{(1-h)(1-\zeta)\mu^*}$ , for which profit-maximization conditions in the production sector are necessary.

*The production side* is modeled as before, except for the different wages and rental rates, which are no longer constant. Hence, we get three equations for the tradable sector (two first-order conditions for profit maximization and the production function), and two equations for the nontradable sector (the factor expenditure shares and production function):

$$(70) \quad (\theta-1)\check{K}_T - (\theta-1)\check{L}_T = \check{R}_T,$$

$$(71) \quad \theta\check{K}_T - \theta\check{L}_T = \check{W}_T,$$

$$(72) \quad \check{Y}_T = \check{K}_T + \check{R}_T,$$

$$(73) \quad \check{K}_N + \check{R}_N = \check{W}_N + \check{L}_N,$$

$$(74) \quad \check{C}_N = \gamma^*[\theta\check{K}_N + (1-\theta)\check{L}_N].$$

*Quantity Equations* define the sector-specific capital variables and the money demand:

$$(75) \quad \check{K} = \zeta\check{K}_T + (1-\zeta)\check{K}_N,$$

$$(76) \quad \check{I} = \zeta\check{I}_T + (1-\zeta)\check{I}_N,$$

$$(77) \quad \check{M} = \check{P}_T + \check{C}_T.$$



The *dynamic equations* include the three dynamic constraints, the price process, and the three Euler equations:

$$(78) \quad \dot{\check{B}} = r\check{B} + \frac{\zeta R^*}{\theta} \check{Y}_T - \delta \check{I} - \frac{(1-\beta)(1-\zeta)R^* \mu^*}{\beta\theta\gamma^*} \check{C}_T,$$

$$(79) \quad \dot{\check{K}}_N = \delta(\check{I}_N - \check{K}_N),$$

$$(80) \quad \dot{\check{K}}_T = \delta(\check{I}_T - \check{K}_T),$$

$$(81) \quad \dot{\check{P}}_N = -\kappa\check{P}_N + \kappa\check{P}_T + \kappa(1-\theta)\check{W}_N - \kappa(1-\theta)\check{W}_T,$$

$$(82) \quad \dot{\check{\nu}} = 0,$$

$$(83) \quad \dot{\check{Q}}_N = r\check{Q}_N - R^* \check{R}_N,$$

$$(84) \quad \dot{\check{Q}}_T = r\check{Q}_T - R^* \check{R}_T.$$

Here, the price equation is (81), with the shadow price determined by the steady-state condition  $\frac{P_N^*}{P_T^*} = \mu^* \left( \frac{W_T^*}{W_N^*} \right)^{\theta-1}$ , obtained from profit maximization of the firms in both sectors, without imposition of the equal wages at the steady state. If one made the restriction, then the wage terms in (81) would be dropped. The steady state rental rate is common for both sectors and is therefore denoted by  $R^* = \delta + \rho$ .

### 4.3 Simulations

In order to simulate the model above, I need to assign values to several new coefficients. Following Kimball (1995), I assign  $j = 0.2$ . The shares of labor devoted to tradable and nontradable production I make equal to each other, setting  $h = 0.5$ .

First, I consider the case without a unique steady state. Figure 7 demonstrates the responses of the stock variables: the capital stock in the two sectors, and the quantity of bonds. First of all, one can observe that the response of all of these variables is very small: neither variable changes by more than one tenth of a percent in response to a 1% money shock. The biggest response is in the non-tradable sector, of course, as capital there is accumulated in order to satisfy the additional demand for the nontradable goods. This capital is both borrowed abroad and built at home. The capital built is manifested in a slight increase of capital in the tradable sector, which implies that this sector is working hard in order to produce resources needed for the extra production of non-tradables. The fact that capital in the tradable sector flattens out at a higher level than the original steady state once again means that a certain amount of that sector's product is spent on financing the incurred debt.

The borrowed capital is manifested in the negative bond position at all points following the shock. Note that the hump-shaped response of the bond position implies a J-curve in the current account. At first, resources are borrowed from abroad in order to satisfy the extra demand. Later, as the price level adjusts to the long-run equilibrium, the need for capital is reduced, and the resources are lent to the rest of the world, with the current account turning into surplus. This surplus is slightly enhanced by the interest payments on the debt.

**Figure 7: Impulse Responses with Adjustment Costs: Stock Variables**

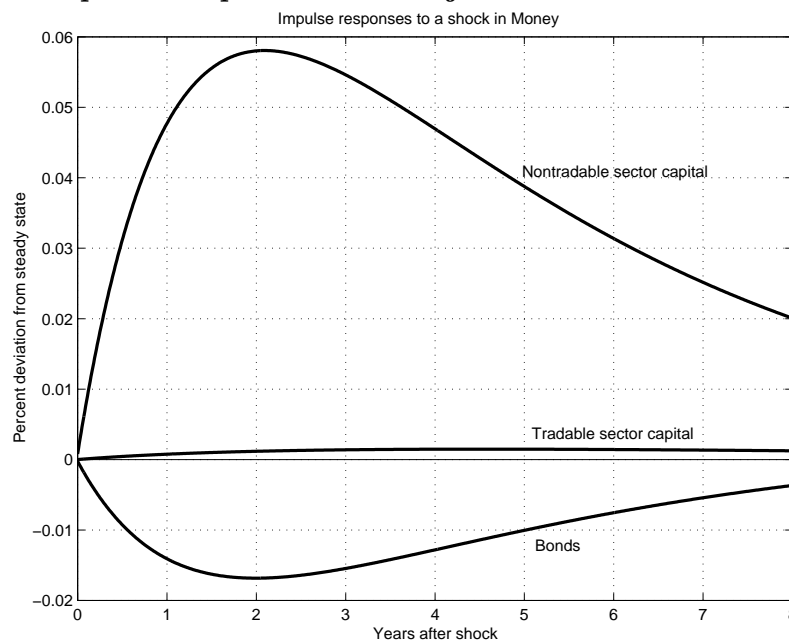
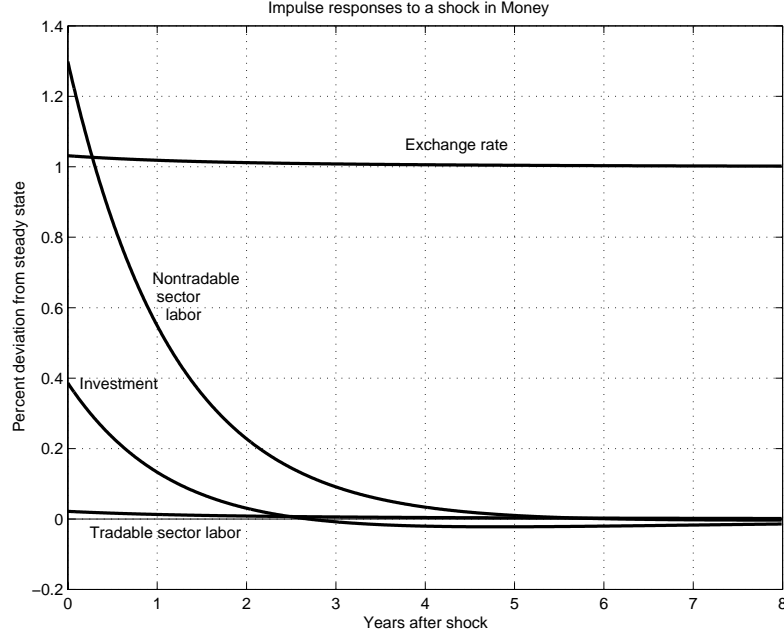


Figure 8 then shows the response of several illustrative flow variables, namely, the exchange rate, gross investment, and the labor supply in both sectors. It is clear that this set-up makes the tradable sector virtually money-neutral. The overshooting of the exchange rate is very small, and so is the change in that sector's employment. I do not report the responses of other variables specific to the tradable sector, such as investment, the real wage, and the rental rate, but all of them respond only marginally. This finding should come as no surprise: the movement of capital is equally costly between sectors and between countries. At the same time, workers have separate utilities with respect to working in the two sectors. These two restrictions make interactions between sectors small, and hence the flexible-price tradable sector behaves as in a money-neutral model. In principle, more overshooting can be achieved by allowing capital to move between sectors at lower cost than between countries, but the merits of this achievement are not clear.

#### 4.4 On the overshooting of the exchange rate

The presence of nontradable goods can help generate more overshooting than is seen in Figure 8 if one allows for a liquidity effect, that is, a money demand that depends on the nominal interest rate. Such overshooting was achieved, for example, by Obstfeld and Rogoff (1995) in the small open economy version of their model. However, I will demonstrate that overshooting so achieved cannot be strong enough to explain the high volatility of exchange rates. To see this, consider a

Figure 8: Impulse Responses with Adjustment Costs: Flow Variables



utility function as before, only augmented by a money supply term:

$$U = \frac{C^{1-\frac{1}{s}}}{1-\frac{1}{s}} e^{(1-\frac{1}{s})[h \log(2h-L_N) + (1-h) \log(2(1-h)-L_T)]} + g\left(\frac{M}{P}\right),$$

where the aggregate price index is defined as  $P = P_N^\beta P_T^{1-\beta} / \beta^\beta (1-\beta)^{(1-\beta)}$ . The corresponding budget constraint should be

$$\dot{B} = rB + W_T L_T + W_N L_N + R_N K_N + R_T K_T + \Pi - \frac{P_N}{P_T} C_N - C_T - I_N - I_T - i \frac{M}{P_T} - T,$$

where the nominal interest rate is defined  $i \equiv r + \dot{P}_T / P_T$ . Thus, the opportunity cost of holding money is the nominal interest rate, which needs to be denominated in the price of tradables to be consistent with the way the real interest rate is defined.

Such a set-up gives the following first-order condition for the money supply:

$$g'\left(\frac{M}{P}\right) \frac{1}{P} = \frac{\nu i}{P_T},$$

which can be log-linearized to obtain

$$\chi \left( \check{M} - \frac{\chi-1}{\chi} \beta \check{P}_N - \frac{(\chi-1)(1-\beta)+1}{\chi} \check{P}_T \right) = -\frac{1}{\chi} (\check{\nu} + \check{i}),$$

where  $\chi = -\frac{(M/P)g''(M/P)}{g'(M/P)} \Big|_*$  > 0. This formula for money demand is imperfect because it forces the money demand to be equally elastic with respect to consumption and the interest rate, which,

of course, is not in line with the empirical evidence, which puts the interest elasticity at very low levels. However, the expression is sufficient to see that overshooting should exist and cannot be big. To see this, suppose that  $\check{\nu} = 0$ . This is a reasonable approximation, since we have already seen that this multiplier is virtually unaffected by money shocks. Also, the price of nontradables is rigid, so on impact,  $\check{P}_N = 0$ . Hence, it is not possible that the exchange rate  $\check{P}_T$  increases proportionally with money and stays at the new level: the interest rate goes down in such a situation, which means that the exchange rate is falling, by the definition of the nominal interest rate. Hence, the exchange rate needs to overshoot. Yet, we can also see the possible maximum level of overshooting: if the exchange rate jumps by more than  $\frac{\chi}{(\chi-1)(1-\beta)+1}$ , then the nominal interest rate actually goes up, which forces the exchange rate to keep increasing resulting in a non-stable response. With the benchmark value  $\beta = 0.5$ , the possible maximum is 2, obtained when  $\chi$  approaches infinity. Furthermore, this maximum can be obtained only for extreme values of  $\chi$ . If  $\chi = 20$  so that the interest elasticity of money demand is 0.05 (Mankiw and Summers 1986), the exchange rate responds only by 1.2% to a 1% shock to money. Of course, overshooting can become larger when  $\beta$  is allowed to increase.

The degree of overshooting obtained here cannot possibly account for the high volatility of exchange rates, unlike the degree of overshooting frequently demonstrated in the local currency pricing based models (Chari et al. 2000, Bergin and Feenstra 2000). The reason why local currency pricing (LCP) can help achieve reaction of exchange rates four or five times stronger than the money shock is the lack of an expenditure switching effect. Big short-run swings in the exchange rate do not have any effect on relative consumption. In models that assume producer currency pricing, such as this one, relative consumption is tied to relative price. This ability to decouple consumption and the exchange rate is a clear advantage of LCP-type models. At the same time, as I discussed above, the achievement of strong overshooting with the use of liquidity effects is difficult to defend as a desirable outcome, since no empirical support exists for this result. Indeed, if anything, the existing evidence is against it (Faust and Rogers 2000). However, an LCP modification of this paper's model in section 6.5 demonstrates that strong overshooting is possible with LCP even without the liquidity effect: only a low value of  $s$  is necessary.

## 5 The Analysis of Price Adjustment

Up to now, I have assumed an ad-hoc process for the adjustment of nontradable goods prices. In this section, I want to extend the analysis of price adjustment in Kimball (1995) to the open-economy case, and show how the presence of the real rigidity could prolong the effects of monetary shocks. Effectively, the presence of real rigidity is capable of generating the macroeconomic rate of adjustment to the new steady state much slower than the micro-economic rate of price adjustment by each individual producer. Bergin and Feenstra (2000) later demonstrated how the same techniques can be used together with the assumption of local currency pricing to prolong deviations from PPP, and thus explain the observed persistence of the real exchange rates. Here, I demonstrate that the

same result can be achieved in this setting without the assumption of local currency pricing relying solely on presence of non-tradable goods.

## 5.1 Theoretical procedure

First, I want to briefly summarize the necessary equations from Kimball (1995), and show how they apply to my model. I stated earlier that the variables  $C_N$  and  $Y_N$  refer to aggregates of individual nontradable goods, rather than to one good, as is the case with the tradable goods sector. This differentiation of products within an aggregate is necessary to justify sticky prices, which are possible only in presence of some market power.

Define the aggregator for the nontradable goods as

$$(85) \quad 1 = \int_0^1 \Upsilon(y_l/Y_N) dl,$$

where  $y_l$  denotes the quantity of a differentiated nontradable good produced by producer  $l$ . The aggregator function has the following properties:  $\Upsilon(1) = 1$ ,  $\Upsilon'(\cdot) > 0$ ,  $\Upsilon''(\cdot) < 0$ . The aggregator is homogeneous of degree one by construction. Thus, this is a general way of aggregating a continuum of goods: normally in literature, a specific form of such an aggregator is proposed, such as a CES utility function (Obstfeld and Rogoff 1995) or a translog structure (Bergin and Feenstra 2000). However, a specific form is not necessary here, since all I need is the elasticity of demand for an individual good near the steady state. I will also need to assume that the elasticity is non-constant around this steady state, in order to get a high degree of real price rigidity.

Thus, demand for each good is easy to obtain from cost minimization with the above aggregate. The first-order condition is

$$p_l = \frac{\Lambda}{Y_N} \Upsilon' \left( \frac{y_l}{Y_N} \right),$$

where  $\Lambda$  is a Lagrange multiplier. Denoting the relative output of a firm  $\xi = y_l/Y_N$ , the elasticity of demand faced by an individual producer  $l$  is

$$\epsilon(\xi) = -\frac{\Upsilon'(\xi)}{\xi \Upsilon''(\xi)},$$

and then the log-linearized demand is

$$(86) \quad \check{y}_l - \check{Y}_N = -\epsilon^*(\check{p}_l - \check{P}_N).$$

Note that this is the relative demand with respect to other nontradable goods, rather than with respect to all goods in the economy, which also include the tradable good. Of course, the price of the tradable good also influences this demand, and this effect is implicitly shown here by the aggregate quantity of nontradables  $Y_N$ , which is determined by the first order conditions of the representative household. Thus, one can think of a household solving a sequential problem: first, making a choice between tradables and nontradables, and second, finding the optimal composition of the nontradable goods basket.

For the cost side of the model, I present the marginal cost denominated in nontradables, denoted by a general function of individual and aggregate outputs:

$$\frac{MC}{P_N} = \Phi(P_T/P_N, W_N, R_N).$$

The marginal cost here depends on three variables. Directly, the factor prices represent marginal costs, and since these prices are assumed to be denominated in tradable good prices, the relative price will also matter.

Then, it is helpful to define the “desired price”  $p^\#$ , which is the price that maximizes instantaneous profits, and thus would be chosen by a firm if its own price were fully flexible, while all other prices in the sector remained at the predetermined level (remember that in presence of sticky prices, a firm cannot maximize profits and can only minimize costs of production). The associated desired output is  $y^\#$ , and the profit maximization condition is

$$(87) \quad \frac{p^\#}{P_N} = \mu(y^\#/Y_N)\Phi(P_T/P_N, W_N, R_N),$$

where  $\mu(y^\#/Y_N) = \frac{\epsilon(y_l/Y_N)}{\epsilon(y_l/Y_N)-1}$  is the desired mark-up. This condition says that the marginal revenue  $\frac{p^\#}{\mu(y^\#/Y_N)}$  at the optimum needs to equal the marginal cost.

Log-linearizing (87), we obtain

$$(88) \quad \check{p}^\# - \check{P}_N = \frac{1}{\epsilon^*\omega} \check{\Phi},$$

where

$$\omega = \left( \frac{1}{\epsilon(y/Y_N)} + \frac{(y/Y_N)\mu'(y/Y_N)}{\mu(y/Y_N)} \right) \Big|_{(y_l, Y_N) = (Y_N^*, Y_N^*)} = \left( \frac{1}{\epsilon^*} + \frac{\mu'(1)}{\mu(1)} \right)$$

is the elasticity of the marginal revenue with respect to an expansion in an individual firm’s relative output. One can see the intuition in the expression itself:  $1/\epsilon^*$  is the rate at which the demand curve falls, while  $\mu'(1)/\mu(1)$  is the rate at which the mark-up falls. Since the mark-up is the wedge between the price and the marginal revenue at the desired price level, the sum of these two numbers is precisely the rate at which marginal revenue falls.

The expression (88) demonstrates the importance of the non-constant elasticity of demand. If the elasticity is constant and  $\mu'(1) = 0$ , then the term  $1/\epsilon^*\omega$  equals unity and drops out of the expression. Hence, the desired price moves one-for-one with the marginal cost, which increases following a money shock and the increase of production. Introduction of a smoothed-out kink in the demand curve by choosing a plausible positive value for  $\mu'(1)/\mu(1)$  can decrease the desired relative price significantly, and thus introduce real price rigidity. In the case of a CES aggregator (85), the elasticity  $1/\epsilon^*\omega$  is restricted to be unity; in the case of translog preferences, to one half.

The marginal cost is given by  $\Phi = \frac{P_T}{P_N} R_N^\theta W_N^{(1-\theta)}$ . In the basic model without the adjustment costs, the wage and the rental rate in both sectors are pinned down by the rest of the world. Hence, the marginal cost in that setting is simply the price of tradables times a constant. However, in the

model with adjustment costs, factor prices are allowed to vary. So in general, the expression for the relative desired price is

$$(89) \quad \check{p}^\# - \check{P}_N = \frac{1}{\epsilon^* \omega} (\check{P}_T - \check{P}_N + \theta \check{R}_N + (1 - \theta) \check{W}_N).$$

So far, I have described the environment faced by an individual firm, but have not talked about the actual form of price stickiness. The set-up is the following. As in Calvo (1983), each firm gets a chance to adjust its price at a stochastic rate  $\alpha$ . Of course, given that chance, the firm does not adjust to the desired price  $p_t^\#$ , since other firms still have their prices sticky. Instead, firms adjust to a reset price  $b_t$ , which depends on the current economic conditions, and expectation of what will happen before the next chance to reset the price arrives. By symmetry, the reset price is the same for all firms at any instant, and hence, the growth of the aggregate price level in the nontradable sector is approximately

$$(90) \quad \dot{\check{P}}_{N,t} = \alpha (\check{b}_t - \check{P}_{N,t}).$$

This equation says that the rate of price growth equals the fraction of firms which reset price at the instant, times the size of the change. Maximizing expected profits of a firm over the time period before the next chance to adjust the price arrives, we arrive at a standard first-order condition for the reset price:

$$b_t = \int_t^\infty \alpha e^{-(r+\alpha)(\tau-t)} p_\tau^\# d\tau.$$

This condition simply says that the reset price equals to the expected weighted average of future desired prices, discounted by the interest rate as well as probability of a new chance to reset the price. Applying Leibniz' rule to the expression, we get the following equation for the growth rate of the reset price:

$$(91) \quad \dot{\check{b}}_t = (\alpha + r) [\check{b}_t - \check{p}_t^\#].$$

Then combining with (90) and dropping time subscripts, the relative reset price growth is:

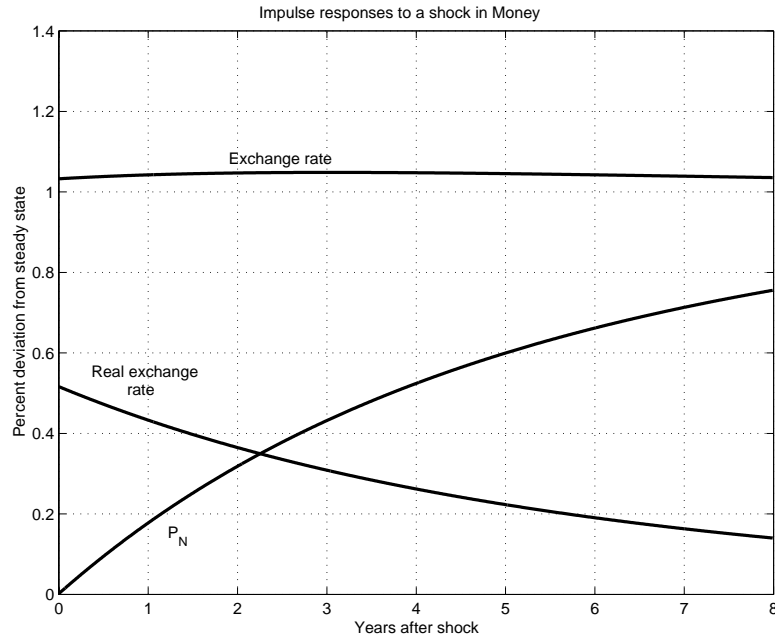
$$(92) \quad \dot{\check{b}} - \dot{\check{P}}_N = r(\check{b} - \check{P}_N) - (\alpha + r)(\check{p}^\# - \check{P}_N).$$

Since the value of  $\check{p}^\# - \check{P}_N$  is known, one can deduce the value of the reset price and, correspondingly, the growth of the price level in the nontradable goods sector.

## 5.2 Simulations

The difficulty with the above analysis is the absence of a credible empirical estimate for the parameter  $\omega$ . The other additional parameters discussed above are easier to quantify. Thus,  $\epsilon^*$  is dictated to be 11 by the steady state value for the mark-up assumed previously. The microeconomic rate of price adjustment  $\alpha$  can be borrowed from Taylor (1999), who shows that on average, firms adjust prices about once per year. Since the simulations here take one period to be one hundredth of a

**Figure 9: Impulse Responses with a Real Rigidity Introduced**



year, I set  $\alpha = 0.01$ , which corresponds to 1 per year. For  $\omega$ , Kimball (1995) proposes the value of 4.37. This value implies that  $\mu'(1)/\mu(1) = 4.28$ , and a 1% increase in the market share brings the elasticity of demand from 11 down to 8. I will take this value as the benchmark.

The impulse responses for these parameters are shown in Figure 5. Besides the price index for nontradables and the price of tradables (the exchange rate), I also include the response of the real exchange rate, defined as  $R\check{E}R \equiv \check{P}_T - \check{P}$ , where  $P$  is defined as in section 4.4. One can see that the degree of real rigidity posited by Kimball (1995) results in an extreme degree of persistence: the half-life of the price level and the real exchange rate is about four years or 16 quarters. A quarterly AR(1) process with such a half-life would have a coefficient of almost 0.96, which would also be the first autocorrelation. This autocorrelation is only slightly higher than those reported by Bergin and Feenstra (2000) for real exchange rates in several countries, which range from 0.91 for France to 0.95 for Canada and the United Kingdom. However, this correlation is significantly higher than those reported for HP-filtered data. Therefore, even a milder degree of real rigidity could generate persistence of the real exchange rate close to that observed in the data.

### 5.3 Temporary money shocks

Of course, another reason why the deviations from PPP in the previous section appear so persistent is that the money shock has been assumed to be permanent. Thus, in a certain sense, the real exchange rates are persistent by assumption. In order to make sure that the persistence obtained is not an artifact of this assumption, I estimate impulse responses for more realistic processes of



the money supply. I estimate an AR(1) process

$$m_t = \gamma_m m_{t-1} + e_t$$

for American and Canadian data, where  $m_t$  is the detrended log of either M1, M2, or M3. The detrending method used is either Hodrick-Prescott filter with smoothness parameter 1600 (the data are quarterly, 1965-2000), or a high-pass filter with highest frequency of six quarters and lowest of eight years. The estimates range between 0.88 and 0.95. Thus, the money supply is itself a persistent series.

Taking an AR(1) process for money with quarterly  $\gamma_m = 0.9$  results in less persistent deviations from PPP. Thus, the half-life of these deviations is reduced to about six quarters. Although this half-life seems drastically lower than the 16 quarters found with permanent money shocks, the difference is less striking if one considers autocorrelations. Thus, an AR(1) process which would yield such a half-life has a root of about 0.89.

Although these estimates are crude, the basic point demonstrated here is that a strong degree of real rigidity in the sticky-price nontradable sector is able to increase the persistence of real exchange rates significantly.

## 6 Stochastic simulations

Up to now, I have been concentrating on developing intuition by using graphical methods and simulated impulse responses. In this section, I want to put the model to test to see if it is capable of replicating certain moments of variables relevant for the international business cycle, such as correlations of output and the current account or nominal and real exchange rates. This analysis is performed by assuming a stochastic process for the money supply or/and technology and simulating series of the endogenous variables. In order to perform such simulations using quarterly data for the stochastic processes, I have to switch from simulations where one period represents 1/100 of a year to a more traditional quarter-long period, which potentially could alter the smoothness of price adjustment.

Technically, stochastic simulations of a log-linearized system cannot be done in the absence of a deterministic steady state, as is the case in the above model. Standard methods of creating such a steady state include the introduction of either an endogenous rate of time preference (Obstfeld 1981, Mendoza 1991) or complete markets (Chari et al. 2000, Bergin and Feenstra 2000). For purposes of this paper, I find it more useful to use the endogenous rate of time preference, because this method does not alter the short-run behavior of the model in any significant way, especially if the sensitivity of the impatience rate is assumed to be small. Complete markets, on the other hand, imply that the current account is exactly balanced at each point of time: effectively, the country's gross national product is insured to stay constant regardless of fluctuations in the gross domestic product.

## 6.1 Endogenous rate of time preference

The standard assumption to make is that the endogenous rate of time preference depends on the level of utility or certain variables that enter the utility function. Instead, I make impatience depend on the level of the total value function. Such a specification allows me to keep all of the first order conditions unchanged, and makes only minimal changes to the model.

The maximization problem of Section 4.1 is now transformed into

$$\max \mathcal{V}_0$$

subject to the same dynamic constraints for bonds and capital as before plus an additional constraint

$$(93) \quad \dot{\mathcal{V}} = \phi(\mathcal{V}) - U(C_N, C_T, L_N, L_T),$$

where  $\mathcal{V}$  is the value function, and the felicity  $U(C_N, C_T, L_N, L_T)$  is the same as before.<sup>6</sup> The function  $\phi(\mathcal{V})$  has the following properties close to steady state:  $\phi(\mathcal{V}) < 0$ ,  $\phi'(\mathcal{V}) > 0$ ,  $\phi''(\mathcal{V}) < 0$ .

With this specification, the present value Hamiltonian is

$$\begin{aligned} \mathcal{H} &= \Lambda_N K_N J(I_N/K_N) + \Lambda_T K_T J(I_T/K_T) \\ &+ \Psi(rB + W_N L_N + W_T L_T + R_N K_N + R_T K_T + \Pi - \frac{P_N}{P_T} C_N - C_T - I_N - I_T - T) \\ &+ \mu(\phi(\mathcal{V}) - U(C_N, C_T, L_N, L_T)). \end{aligned}$$

The first-order conditions with respect to consumption, leisure, and investment are similar to those in section 4.1. They become exactly the same after the transformation of variables  $Q_i \equiv \Lambda_i/\Psi$ , and  $\nu \equiv -\Psi/\mu$ . These newly defined costate variables are stationary, unlike the original costates  $\Lambda_i$ ,  $\Psi$ , and  $\mu$ . Thus, they effectively transform the first-order conditions back into the current-value form.

The Euler equations for the stationary costates  $Q_N$  and  $Q_T$  also remain unchanged. This justifies the choice of notation for  $Q$ 's and  $\nu$ : these variables have the same meaning as before, and they enter the first order conditions in the same way. The only equation that is different is the Euler equation for  $\nu$ , which is now

$$(94) \quad \dot{\nu} = r\nu - \phi'(\mathcal{V})\nu,$$

and, of course, there is an additional equation (93), which was not in the model previously. These two equations demonstrate the presence of the unique long-run steady state. The steady state version of (94) implicitly pins down the steady-state value of  $\mathcal{V}$ :  $r = \phi'(\mathcal{V}^*)$ . Thus,  $\phi'(\mathcal{V})$  is the analog of the discount rate  $\rho$ , assumed in the previous sections. Then, with the steady-state  $\mathcal{V}^*$ , (93) gives an additional constraint  $\phi(\mathcal{V}^*) = U(C_N^*, C_T^*, L_N^*, L_T^*)$ . Now there are exactly enough equations to find steady state values for all variables, including the debt/capital ratio  $b^*$ . For this

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<sup>6</sup>Note that the maximization problem in the previous sections, 2.1 and 4.1 could be re-written in an analogous form:  $\max \mathcal{V}_0$  subject to  $\dot{\mathcal{V}} = \rho\mathcal{V} - U(C_N, C_T, L_N, L_T)$ . The only difference is that now the discounting takes the form of a general function of the value.

purpose, I need to assume a particular form of  $\phi(\mathcal{V})$ , which is difficult to calibrate. Alternatively, one could set a value for  $b^*$ , as before, and then the form of  $\phi(\mathcal{V})$  is not needed. Actually, these values disappear in log-linearization, and only the second derivative of  $\phi$  needs to be calibrated. This makes sense intuitively, as the second derivative measures the sensitivity of the discount rate, and hence dictates how fast the value wants to return to the steady state level.

The log-linearized versions of the two equations are then

$$\begin{aligned}\dot{\tilde{\mathcal{V}}} &= r\tilde{\mathcal{V}} - \beta\check{C}_N - (1 - \beta)\check{C}_T + (1 - \theta)\beta\frac{\gamma^*}{\mu^*}\check{L}_N + \frac{\zeta\beta(1 - \theta)\gamma^*}{(1 - \zeta)\mu^*}\check{L}_T, \\ \dot{\tilde{\nu}} &= -\phi''(\mathcal{V}^*)\tilde{\mathcal{V}},\end{aligned}$$

where tilde denotes an absolute, rather than percent, deviation from the steady state. Note that both of these variables are allowed to jump, even though  $\mathcal{V}$  is treated here as a state variable.

## 6.2 Calibration and results

In order to simulate the model, one needs a value for  $\phi''(\mathcal{V}^*)$  and a stochastic process for the money supply. For the parameter  $\phi''(\mathcal{V}^*)$ , it makes sense to choose a small value for it, since the original goal was merely to remove the technical problem of non-stationary equilibrium. Indeed, for values smaller than about -0.0001 in absolute value, the impulse responses become visually indistinguishable from those reported before. This is precisely what we should aim for, since the original impulse responses demonstrated only negligible long-run deviations from the steady state.

Further, the stochastic process for the money supply is based on the first-order autoregressions for monetary aggregates of Canada and the United States mentioned in section 5.3. Thus, I set

$$\check{M}_t = 0.9\check{M}_{t-1} + \epsilon_t^m$$

for quarterly data where the standard deviation of  $\epsilon_t^m$  is taken to be 1, although this estimate varies strongly between the aggregates and filtering methods, ranging between 0.49 and 1.78.

It is interesting to see how the current account and trade balance behave in this model. In principle, these two variables are very close to each other: the current account is defined as  $CA \equiv \dot{B}$ , while the net exports are  $NX \equiv \dot{B} - rB$ . Backus et al. (1995) report the statistics for the net exports as share of GDP, and I will concentrate on this statistic as well. Deviations of real GDP are defined as  $\check{Y} \equiv \frac{(1-\zeta)\mu^*}{\zeta\gamma^*+(1-\zeta)\mu^*}\check{Y}_N + \frac{\zeta\gamma^*}{\zeta\gamma^*+(1-\zeta)\mu^*}\check{Y}_T$ . The variable  $nx \equiv NX/Y$  can then be backed out easily from the log deviations.

Table 2 reports the results for certain business cycle moments. These results were obtained by simulating 100-quarter long series 200 times and then averaging the results. For comparison, the table also reports the average values of the same variables estimated with actual data for eleven countries by Backus et al. (1995). The statistics for nominal and real exchange rates are averages of those for nine countries reported by Chari et al. (2000).

The biggest success of the model is the counter-cyclicality of the trade balance, which is not normally found in monetary open economy models (nor is this a robust finding in the real business

Table 2: Stochastic simulations results

	$\sigma_x^a$		$\sigma_x/\sigma_Y$			$\rho(x, Y)$			$\rho(x_t, x_{t-1})$			
	Y	$\mathcal{E}$	RER	nx	I	C	nx	I	C	Y	RER	
Data	1.46	8.55	8.06	.66	2.60	.90	-.32	.78	.73	.74	.82	.97
<i>Model with flexible prices of tradables</i>												
M shocks	0.76	2.13	.99	.25	2.17	1.22	-.79	.97	.99	.83	.84	.99
Temp Z shocks	1.71	0.21	.06	.86	.43	.13	.99	.96	.45	.86	.97	-.38
Pers Z shocks	1.50	2.31	.85	.50	1.48	1.14	-.39	.88	.96	.96	.90	.76
M and pers Z	1.81	3.23	1.31	.42	1.60	1.14	-.41	.90	.97	.93	.87	.85
<i>Model with sticky prices of tradables</i>												
M shocks	1.39	1.69	.69	.29	1.55	.92	-.51	.82	.99	.80	.90	.79
Temp Z shocks	.73	1.37	.61	2.32	5.04	1.20	.53	-.45	.04	.82	.89	.86
Pers Z shocks	.26	.87	.37	2.82	15.36	2.15	-.87	.70	.29	.94	.97	.89
M and pers Z	1.43	1.97	.79	.57	3.12	1.02	-.39	.49	.92	.80	.91	.80
<i>Model based on local currency pricing</i>												
M shocks	1.55	4.01	3.89	2.99	11.52	1.26	-.45	.62	.74	.93	.84	.99
Temp Z shocks	1.36	2.52	2.51	1.72	7.72	.14	.34	.16	.74	.97	.75	.99
Pers Z shocks	1.25	2.35	1.93	1.05	4.84	0.99	-.23	.41	.82	.97	.93	.83
M and pers Z	2.07	4.64	4.31	2.32	9.07	1.16	-.38	.54	.79	.94	.85	.95

<sup>a</sup> $\sigma_x$  refers to the standard deviation of variable  $x$ ,  $\rho(x, y)$  to the correlation coefficient between variables  $x$  and  $y$ . Money shocks are all temporary, with autocorrelation 0.9 and standard deviation of error term 1%. Temporary technology shocks have autocorrelation 0.9 and standard deviation of error term 0.85. Persistent technology shocks have autocorrelation 0.999 with standard deviation 0.4. Moments are averages across those for 200 independently simulated series, each being 100 quarters long.

cycle literature). Likewise,  $nx$  is found to be less volatile than output. At the same time, quantitatively, the model has two major shortcomings: output and net exports are not as volatile as in the data and the negative correlation between output and net exports is too strong.

The biggest shortcoming of the model is, of course, low volatility of exchange rates, which is not surprising since overshooting was not generally obtained. In principle, some overshooting can be generated by assuming interest-sensitive money demand as discussed in section 4.4. At the same time, the correlation of real and nominal exchange rates is strong, as well as the persistence of the real exchange rates. This is clearly a result of the strong degree of real rigidity assumed in the model.

### 6.3 Technology shocks

Adding technology shocks is possible as well. With the process for technology

$$\tilde{Z}_t = 0.9\tilde{Z}_{t-1} + \epsilon_t^z,$$

with a standard deviation of the error term 0.85 as in Backus et al. (1995), one gets the moments also reported in Table 2. One can see that the predictions of these shocks for the trade balance are much worse: now both output and the trade balance are too volatile, but more importantly, the two are strongly positively correlated. The intuition is the following. After a technology shock, the country becomes rich and starts lending resources abroad. These resources need to be produced in the tradable sector, so capital and labor both shift towards that sector, and tradable output jumps up strongly. Thus, the inter-sectoral volatility is entirely different than with the money shocks. In the nontradable sector, on the other hand, output reacts only by a small amount. Both capital and labor go down, but output is up slightly due to better technology. Counter-cyclicality of trade balance can be achieved, however, by a combination of making the nontradable sector small, technology shocks persistent, and almost eliminating investment adjustment costs.

In terms of the exchange rates, one can see that they barely react to technology shocks in this setting. Likewise, the correlation with the exchange rate is negative. Other implausible moments generated with technology shocks include the low volatility of investment and consumption. This result is due to the aggregation between sectors. Investment actually reacts strongly in both sectors, but in opposite directions. The tradable sector invests a lot, because firms in this sector want to sell goods abroad. The nontradable sector, however, reduces investment sharply: workers prefer to take the extra leisure instead of working harder following a technological improvement, and hence no extra capital is needed. Once again, making the nontradable sector small or getting rid of sticky prices in the nontradable sector would help increase these volatilities.

Alternatively, one could consider more persistent technology shocks with a smaller variance. The process assumed above is estimated using the Solow residual for the U.S. economy. At the same time, it has been widely recognized that the Solow residual contains a strong endogenous component, which makes the observed series much more volatile and less persistent than the actual

technology (Gali 1999). Hence, I also try a process for  $\check{Z}$  with autocorrelation 0.999 and a standard deviation of the error term of 0.4. The table shows that this specification improves the performance of technology shocks significantly. Most importantly, the correlation of the trade balance with output turns negative, which is due to the fact that the economy accumulates capital for a long time ahead, since the technological improvement dies out only slowly.

Table 2 also shows the results for the combination of monetary and persistent technology shocks. It is seen that such a combination improves most simulated moments.

#### 6.4 Sticky prices of the tradable goods

Here I relax the assumption of the perfect flexibility of prices in the tradable sector. This assumption produces a highly questionable implicit result that the tradable sector production (generally thought of as industrial manufacturing) is much smoother than production in the nontradable sector (normally believed to be represented by services).

The simplest way of switching to a model with sticky prices of tradable goods is to think of all of the domestic tradable output as being produced for export. Likewise, all of the domestic consumption of tradable goods is imported. Such an assumption is consistent with a set-up in which there is a continuum of varieties of tradable goods in the world, of which only an infinitesimal fraction is produced in the small home country. If consumers demand all varieties, the locally produced goods are only a small portion of their baskets, and hence can be ignored in the aggregate. All of the production in the sector, with the exception of an infinitesimal fraction, is consumed abroad.

Assuming that the price of the domestically produced tradables is sticky in the exporter's currency, the model becomes different by only three equations. First, the tradable firms are no longer perfectly competitive, and with sticky prices cannot maximize profits. Hence, profit-maximizing conditions for the wage and the rental rate of capital do not hold; instead, only the relative shares of labor/capital expenditures are known. Further, the domestic producers are assumed to face a downward sloping demand curve

$$(95) \quad Y_T = D\left(\frac{P_T}{\mathcal{E}}\right),$$

where  $\mathcal{E}$  is the exchange rate defined as units of home currency per one unit of foreign currency, and the standard assumption  $D'(\cdot) < 0, D''(\cdot) > 0$  holds. The material balance condition for this economy is

$$\dot{B} = rB + \frac{P_T}{\mathcal{E}}Y_T - C_T - I.$$

Lastly, one needs to put  $\mathcal{E}$  instead of  $P_T$  in all household first-order conditions.

For calibration purposes, the elasticity of foreign demand is set at 1.2 following Kollmann (1997). The speed of convergence of the tradable goods price is set at the microeconomic rate  $\alpha$ , because given the immediate adjustment of the exchange rate and hence the prices of all competitors,

domestic firms in the sector must adjust close to fully once given a chance. With these values of parameters, the simulations results are reported in the lower part of Table 2.

For money shocks, one can see that most moments do not change greatly. The volatility of exchange rates becomes even lower, however, and the correlation of nominal and real exchange rates drops as well. This is due to the fact that in this model, the nominal exchange rates actually undershoot whenever  $s < 1$ . This occurs because the extra output generates more labor,  $C_T$  rises due to complementarity with labor, and by the quantity equation  $\mathcal{E}$  needs to undershoot. This is different from the model with flexible tradable prices because here a stronger response in the exchange rate generates a stronger response in tradable output by (95), which means more labor and hence more  $C_T$  and then again a smaller response of  $\mathcal{E}$ .

Temporary technology shocks, on the other hand, perform far less impressively. The most spectacular failure is the counter-cyclical of investment. This result is due to the fact that the country, once endowed with better technology, produces more of tradable goods and lends resources abroad. Hence the strong positive reaction of the current account. At the same time, domestic factor input goes down strongly. Both labor and capital are reduced, which means investment is lower, if not negative.

Once again, persistent technology shocks have more plausible predictions, with the exception of the highly volatile investment. This high volatility of investment is a standard problem in open economy models, and can be treated by blocking movement of capital across countries, say, by increasing the adjustment costs for the aggregate investment. Total output is also not volatile enough, again due to sticky prices and the form of money demand.

Much better results are achieved with a combination of shocks where monetary disturbances dominate. Thus, investment volatility is brought down to a reasonable level, while output volatility is increased.

## 6.5 Local currency pricing

In this section, I change the nature of price stickiness yet again by making markets segmented and allowing firms to set rigid prices in the currency of the consumer. Such modeling is tricky in the small open economy setting, because local currency pricing (LCP) requires explicit modeling of both the cost and demand structure of firms, in order to deduce the optimal reset price. In the model outlined so far, these characteristics of the foreign economy are not known. Yet, a shortcut can be made following the work of Kollmann (1997) and Bergin (2001), who also built small open economy models with LCP.

Thus, I take the structure of section 6.4 where all of the domestic tradable output is exported, while all of capital and tradable consumption is imported. Then, the domestic tradable importer faces demand

$$Y_T = D(P_T^f),$$

where  $P_T^f$  is the sticky foreign-currency price of the domestic tradable, with superscript  $f$  denoting

the foreign currency. The overall price level in the rest of the world is normalized to one. Thus, tradable output is rigid together with the price at which it is sold. The firms in this sector minimize costs as usual, and choose the reset price by the rule outlined in section 5 with the marginal cost given in foreign currency by

$$\check{\Phi}_T = \theta \check{R}_T + (1 - \theta) \check{R}_N.$$

Foreign consumer goods and capital, on the other hand, are imported into the country by a continuum of importing firms. They buy the product at the world price (with perfect exchange rate pass-through), but then sell it to the final consumer at a rigid price  $P_F$ . Hence, a certain asymmetry exists between import prices and export prices: on the level of wholesale trade, exchange rate pass-through is perfect for imports, but zero for exports. On the level of the final consumption, however, exchange rate pass-through is zero in either direction. This means that a devaluation of the local currency hurts the profits of the local importers, but not the profits of the local producers of tradable goods. Therefore, such a set-up could potentially produce a downward bias on the current account.

Contemporaneously, importing firms do not maximize anything as they do not control the amount of either output or inputs. Instead, they simply satisfy the demand for tradable goods at given prices, earning profits

$$\Pi_F = (P_F - \mathcal{E})(C_T + I).$$

They maximize discounted future profits, however, by choosing a reset price with the marginal cost in terms of domestic tradable consumption

$$\check{\Phi}_F = (\check{\mathcal{E}} - \check{P}_F).$$

Everything else in this economy remains as before, with the exception of the fact that all of the first-order conditions need to be denominated in one currency. Thus, the dynamic budget constraint for households is now

$$\dot{B} = rB + W_N L_N + W_T L_T + R_N K_N + R_T K_T + \Pi - \frac{P_N}{\mathcal{E}} C_N - \frac{P_F}{\mathcal{E}} (C_T - I_N - I_T) - T,$$

and the material balance condition is

$$\dot{B} = rB + P_T^f Y_T - C_T - I.$$

Note that the tradable consumption and investment are pre-multiplied by the relative price  $P_F/\mathcal{E}$  in the household constraint, but not in the material balance condition. This is due to the fact that consumers buy the goods at sticky domestic prices from importing firms, whereas the country as a whole buys these goods at the world prices. The change in the relevant relative prices in the budget constraint is then seen in the first-order conditions for consumption and investment, as well as the Euler equations for  $Q_N$  and  $Q_T$ .

At the steady state,  $P_T^{f*} = 1$  since the world price level is unity, and  $P_F^* = \mu^* \mathcal{E}^*$ , since the price of imported goods is set as the mark-up over the marginal cost, which is here simply the exchange



rate. The steady-state mark-up is assumed to be the same for all three types of firms. With the benchmark parameter  $\mu^* = 11$  reported in Table 1, the price elasticity of demand is then equal to 11. This elasticity is necessary to justify the high degree of real rigidity, but seems too high when applied to the domestic producers of tradable goods. It implies that the elasticity of domestic exports with respect to the foreign price is 11, while empirical estimates put this value at around 1.2 as mentioned in section 6.4. This does not create big problems, however, since the foreign price itself does not move much. Alternatively, however, one could delink the internal elasticity within the aggregate of the domestic tradables sold abroad and the elasticity of the total aggregate with respect to the aggregate price level.

The results of the simulations are reported in the lower part of Table 2. One can see that the primary achievement of the model is still present: net exports continue to be negatively correlated with output in the presence of money shocks, and this correlation is in fact closer to the data than the correlation obtained in the previous versions of the model. However, the quantitative failure of the LCP-based model is the enormous volatility of investment and the trade balance. This volatility is due to the fact that capital is cheap: its price is sticky as opposed to the previous versions of the model, where capital was bought at the current exchange rate at world prices. Hence, a monetary expansion causes a huge influx of capital into the domestic country, resulting in a strong current account deficit and a corresponding positive investment response in both sectors.

Temporary technology shocks, on the other hand, continue to perform worse: the trade balance is still pro-cyclical. What happens is that following an improvement in technology, the country borrows in order to finance additional investment. Thus, the current account is in deficit. At the same time, output is fixed in physical units, since it is determined by demand at the current price. Yet, in terms of the foreign currency, in which foreign bonds are denominated, output falls due to the depreciation of the home currency, which produces the positive correlation. This correlation is reversed, once again, by making the technology shocks persistent.

The volatility of real exchange rates is greatly increased in this model, even though money demand remains insensitive to the interest rate. The reason for this is now well understood in the literature on LCP: this form of price rigidity delinks the exchange rates and relative consumption of two types of goods. Hence, the exchange rate can be quite volatile without affecting the welfare of the households. The exchange rate can only affect the profits of the firms, but these profits are outside of the producers' control in the short run.

Even though the volatility of the exchange rates is still below the one in the data for most countries (although it is high enough for some countries - for example, Canada), this volatility can be increased by reducing  $s$  from the benchmark 0.35 to a lower value. Thus,  $s = 0.2$  as in Kimball (1995), which is low but plausible, almost doubles the volatility. At the same time, this also increase the volatility of output and the current account, which are already too volatile. Very high investment adjustment costs are needed to bring down this volatility to that observed empirically, about  $j = 2$ . Such a value could be justified as transport costs or tariffs (remember that in this model all capital is imported).

## 7 Conclusion

In the paper, I build and analyze a dynamic sticky price model of a small open economy. In spite of the model's richness, I manage to describe a fair amount of intuition using analytical methods, before I proceed to numerical simulations. Further, I demonstrate several powerful results.

First of all, the current account is predicted to go immediately in deficit in response to a money shock. At first, additional resources are required for the sticky-price nontradable sector, and hence they are brought in from abroad; as prices adjust, however, the extra resources are channeled back abroad, together with the interest payments. Hence, the current account exhibits a "J-curve" response. Numerical simulations demonstrate that the trade balance is countercyclical, as in the data, in the presence of money shocks and persistent technology shocks. However, temporary technology shocks result in a strongly pro-cyclical trade balance. Such countercyclical behavior of the current account is a result of capital investment and incomplete markets.

Second, the long-run indebtedness response of the economy is sensitive to the choice of utility specification. When there is a high degree of consumption smoothing (low elasticity of intertemporal substitution), these long-run effects are negligible in the baseline model. Hence, long-run neutrality of money is shown to be a reasonable approximation.

Third, the model makes it possible to explicitly introduce real rigidity into the framework, and analyze its implications for real exchange rate persistence. It is shown that this persistence is indeed close to the one observed in the data when real rigidity is strong.

Fourth, the presence of nontradable goods makes it possible to have overshooting of exchange rates in the absence of liquidity effects. Instead, the overshooting is caused by the complementarity between labor and consumption. However, this overshooting is not sufficient to explain the observed volatility of the exchange rates, unless local currency pricing is introduced. The value of obtaining this overshooting is questionable, however, since empirical evidence points against this source of exchange rate volatility.

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