Optimal Capital - Labor Taxes under Uncertainty and Limits on Debt

Irina Yakadina*
Department of Economics, Univertsitat Pompeu Fabra†

November 22, 2001

Abstract

How to set optimal capital-labor taxes over the business cycle if markets are incomplete? I amend the baseline model of Chari, Christiano and Kehoe (1994) by limited short sales requirement and show that this is enough for the expected capital income taxes to differ from zero so that the famous Chamley (1986) result does not hold.

Under complete markets, there is just one period of transition characterized by a huge expected capital levy (the capital income tax at the rate of more than 100%) and a labor income subsidy. Immediately after, the economy jumps to the stationary distribution with zero expected capital income taxes and nearly constant labor income taxes.

Under debt limits, the transition takes longer, and the initial period expected capital levy is significantly reduced or eliminated, as well as the initial labor income subsidy. For most of my specifications of debt limits, the labor income tax affirms Barro's assertion and exhibits a random walk-like behavior.

I come out with a simple rule for the planner suggesting to pay a capital income subsidy, in order to enhance the capital accumulation, whenever the consumers run short of savings in bonds. My model predicts that average capital taxes in good states of the economy should be lower than in bad ones.

Finally, both taxes are countercyclical but they react differently to the innovations to the government spending shock and to the technological shock. In the short run, negative shocks to either budget or technology lead to an increase in capital taxes and a cut in labor taxes. Both shocks result in permanently higher long run taxes.

*I am grateful to Albert Marcet, Michael Reiter, and the participants of CREI Macro workshop at UPF.
†Ramon Trias Fargas 25-27, 08005 Barcelona (Spain); e-mail: irina.yakadina@econ.upf.es

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1 Introduction

This paper introduces a simple form of market incompleteness to study the problem of optimal capital and labor income taxation in a stochastic growth model. I consider the optimal fiscal policy of a government, acting as a benevolent Ramsey planner, in presence of exogenous limits on borrowing and savings.

We know from Zhu (1992) that the extension of Chamley-Judd result of zero long run capital income taxes should not necessarily hold for the stochastic economies. However, the numerical results of Chari, Christiano and Kehoe (1994)affirm that zero expected capital taxes are optimal in the long run[1]. My model leads to non-zero capital income taxes even for the simple baseline specification of the consumers' preferences.

Under complete markets, the government optimal taxation structure of Chari, Christiano and Kehoe (1994), means zero expected capital income taxes and labor income taxes with very low volatility. This long run result is achieved after a one-period transition characterized by a levy on all the capital income and a part of the existing capital stock. The levy is announced for the next period after the economy starts and is anticipated by a large government borrowing used to repay the initial indebtedness and for a labor income subsidy to the consumers. With enough assets in hand, the government abolishes the capital income taxation, as predicted by Chamley (1986) and Judd (1985), and lends to the consumers each period an amount close to the current GDP.

Capital levy and labor subsidy together with long run government savings are unsatisfactory policy recommendations for a government seeking to switch to an optimal fiscal policy from the current tax code. Historically, announced capital levies proved unsustainable unless they were effectively replaced by a moderate capital income taxation spread over many years (Eichengreen, 1990). Problems with labor income subsidies are discussed in Coleman II (2000). Modern governments are limited in borrowing, to prevent them from shifting the tax burden to the future generations. Last but not least, it is difficult to imagine that the consumers have the ability of issuing an unlimited amount of fully state-contingent debt, as they are supposed to under complete markets. It is reasonable to assume rather that there may be circumstances in which the government has an advantage in providing insurance against some contingencies, one of which being the insurance against the government budget constraint.

We know from Chari and Kehoe (1998) that imposing an upper limit on the capital income tax does not change the structure of the optimal solution but just lengthens the number of periods during which the capital income will be completely taxed away. Therefore, I choose to impose limits on the amount of a state-contingent bond that the Ramsey planner can issue each period. My basic model retains the Chari, Christiano and Kehoe environment while adding asset market incompleteness.

I find that putting limits on debt and assets of the planner is enough for the expected capital taxes to be different from zero even for simple utility functions. I use short run Monte Carlo simulations to solve the model numerically for various specifications of debt limits. The transition to the stochastic steady state takes longer, the initial period expected capital levy is significantly reduced or eliminated, as well as the initial labor income subsidy.

I come out with the following predictions for the optimal capital-labor income taxes: they should be both countercyclical, but they should respond differently to negative shocks to the government budget and technology. The response of capital taxes is rather neoclassical while labor taxes behave in line with Keynesian recommendations ("cut taxes in bad states"). Relative to serial correlation and volatility properties, I conclude that under incomplete markets, labor income tax rates become much more volatile and affirm Barro's random walk assertion for most of debt limits specifications.

[1] In their model with high risk aversion, expected capital income taxes have a small mean and a high standard deviation. However, consumers should be more risk averse than what is usually assumed by the RBC literature.

There is a simple policy rule for the planner suggesting to pay a capital income subsidy, in order to enhance the capital accumulation, whenever the consumers run short of savings in bonds. Simulations show that the government should announce a capital income tax cut in good states of the economy and raise expected capital taxes in bad times.

The paper is organized as follows: Section 2 reviews the main issues of the related literature on optimal taxation, Section 3 sets up the model with debt limits, Section 4 discusses the theoretical implications of market incompleteness for the optimal fiscal policy of the planner. Choices of debt limits and parameters of the model are described in Section 5, Section 6 presents findings from solving the model numerically using the Parameterized Expectations Algorithm by Marcet, and Section 7 concludes.

2 Review of the Literature

A long line of researchers after Frank Ramsey (1927) have been studying models of optimal taxation. The existing literature combines the general equilibrium set up with the long tradition of commodity taxation in public economics.

2.1 Zero Capital Taxes

Theoretically, as proved in Zhu (1992), for general utility functions in the context of stochastic growth model, the long run capital income taxes may or may not be zero. But there exists a very extensive literature suggesting that the capital income taxation should be abolished not to distort the capital accumulation. In a broad class of optimal taxation models, the government taxes capital income at a high rate in the initial periods. This is done in order to build up a surplus which is then used to set to zero the long run capital income taxes, in case of a deterministic model, or the ex ante expected capital income taxes, for the stochastic set up.

The deterministic result is due to Judd (1985) and Chamley (1986) and is intuitive from the point of view of the optimal commodity taxation principle. This finding has generated a stream of research which, on the one hand, has generalized it for many other environments, and, on the other hand, has come out with examples of the set ups for which the long run capital income taxes may be different from zero. Among the partisans of abolishing the capital income taxation in the long run we find works of Lucas (1990), followed by Jones, Manuelli and Rossi (1993), and Corsetti and Roubini (1996). They internalize the human capital accumulation as a market activity to conclude that, in the steady state, all the taxes that distort capital accumulation, i.e. both the capital income tax and the human capital income tax, should be zero. Coleman II (2000) does a similar exercise in presence of capital, labor and consumption taxes. Chari and Kehoe (1998) show that, even with heterogeneous consumers and the Ramsey planner putting no weight on the capitalists, taxing capital in the long run is still not optimal. Chari, Christiano and Kehoe (1994, 1995) prove that for the class of utility functions additively separable between consumption and leisure, the tax rate on capital income is zero from period two onwards. The numerical results of Chari, Christiano and Kehoe (1994) demonstrate that the ex ante expected capital income taxes are statistically very close to zero for the stochastic set up.

Jones, Manuelli and Rossi (1997) summarize the conditions which favor eliminating capital income taxation in the long run and construct two examples of non-zero limiting tax on capital: one when pure rents appear in the consumer's budget constraint, and the other with restrictions on tax rates when the planner is forced to tax equally two different types of labor. The latter example gives raise to 7% capital income taxes in the long run.

I choose the specifications for production and preferences for which, under the complete markets, the ex ante expected capital income taxes are identically zero starting from the first period onwards. I show that imposing tight enough exogenous limits on debt and savings of the planner, gives rise to ex ante expected capital income taxes with a positive mean and high variability. Therefore, if markets are incomplete then abolishing the capital income taxation after a small number of periods is not a policy recommendation anymore.

2.2 Smooth Labor Taxes

Should labor income taxes follow a random walk? Barro (1979) finds that tax smoothing implies that income taxes should be a martingale regardless of the stochastic process for government spending. That is, if in the model with no capital the government has the access to the risk-free debt only, the serial correlation properties of taxes are independent of the serial correlation properties of government spending. A random walk with small innovation variance appears smooth in the sense of Barro's "tax smoothing" which refers to equalizing the welfare loss from taxation across time and different states of nature and not to the constancy of labor tax rates.

Lucas and Stokey (1983) show that in a model with compete markets and no capital, tax rates inherit the serial correlation of government expenditures.

Marcet, Sargent and Seppäla (2000) try to recover a version of Barro's random walk for labor taxes in the context of Lucas and Stockey's (1983) economy but with risk-free debt only. They stress the important role of borrowing constraints and show for which type of constraints Barro's random walk result prevails. I follow their strategy of putting "time-invariant ad hoc debt limits" on the planner that I call exogenous limits on debt.

Chari and Kehoe (1998) also analyze Barro's assertion that optimal taxes should follow random walk in the model with capital accumulation. Their model of complete markets predict smooth taxes in the sense of having small variances rather than being random walks. They conjecture that if asset markets were incomplete, then the analysis would be much more complicated and would depend on the precise details of incompleteness.

Scott (2000) shows that for the model of Chari, Christiano and Kehoe (1994) with complete markets labor taxes fully inherit the serial correlation properties of employment. In the same model with risk-free debt only, the stochastic process for labor taxes does contain a unit root. However, optimal labor tax rates still depend positively on employment.

Marcet and Scott (2001) provide empirical evidence in favor of market incompleteness in the US by investigating the behavior of the fiscal deficit and government debt in Ramsey models with and without capital accumulation and labor taxes only. Without state-contingent debt, future taxes should increase in response to a higher current deficit. Therefore, the model predicts that with only risk-free debt, labor taxes should be more variable, especially at low frequencies.

I study the model of Chari, Christiano and Kehoe (1994) with fully state-contingent debt but still incomplete assets markets and with both capital and labor income taxes. I find that labor income taxes both possess a unit root component, depend on employment, and are functions of expected future solvency of the planner. They have higher persistence and volatility than in the case of complete markets. Moreover, most of the specifications of debt limits, a unit root like behavior dominates the effect of employment on labor taxes, so that their path is independent of the government expenditure process (therefore, any unanticipated shock to the government budget has a permanent effect on labor taxes).

2.3 Complete Markets and Indeterminacy of Capital Income Tax Rates

Zhu (1992), Chari, Christiano and Kehoe (1994, 1995), Bohn (1994), Chari and Kehoe (1998) have pointed out that in the complete markets stochastic environment, the capital income tax rates and the state-contingent returns on government bonds can not be uniquely determined by the first-order conditions of the planner.

This happens because the planner has access to "too many" state contingent instruments. With the full set of state-contingent returns on debt in hand and allowing the capital income taxes to vary with the states of the economy, the planner can insure against all relevant shocks to the budget in many different ways. For the purpose of financing government spending, a low return on bond in some state S_t can serve as a substitute for a high tax rate on capital in this state. Notice that both instruments are distortionary: higher capital taxes reduce capital accumulation, while higher debt will generate higher interest payments in the future that leads to higher tax rates.

Individual investment decisions are based on weighted averages of state-contingent returns. The capital tax rates in the different states can be altered without affecting investment decisions as long as the relevant expectations are left unchanged. Intuitively, individuals are not affected by shifts in capital tax rates across states of nature that satisfy the Euler condition for capital, because they have complete markets available to purchase any desired state-contingent stream of consumption.

In my model, the degree of indeterminacy can be significantly reduced if I specify exogenous limits on debt tight enough to be often binding. Formally, however, just one restriction for a large number of states of the economy is not enough to solve the problem of indeterminacy. Hence, I proceed with ex ante expected capital income tax rates used in the literature with complete markets.

3 The Model

The Ramsey planner maximizes the representative consumer's utility over the set of competitive equilibria in the economy. This set is determined by the first-order and transversality conditions from the consumer's problem, the marginal productivity factor prices set by competitive firms, the government budget constraints, and the market clearing conditions. In addition I impose restrictions on period-by-period borrowing and saving of the planner, that I refer to as limits on debt.

3.1 Set of Competitive Equilibria

The economy is decentralized with three perfectly competitive markets: the labor market, the capital market and the market of government bonds with one period maturity and state-contingent returns. Both capital and bonds market open after the technology and government spending shocks are realized. I use the convention that variables dated t are measurable with respect to the history of shocks up to t.

A **competitive equilibrium** for this economy consists of a policy $\pi = (\tau_t, \theta_t, R_{b,t})_{t=0}^{\infty}$, an allocation $x = (k_t, l_t, c_t, b_t)_{t=0}^{\infty}$, and a price system $(w_t, r_t)_{t=0}^{\infty}$ that satisfy

1. the first-order conditions of the representative consumer's problem determining the household's

consumption-leisure and consumption-investment choices for both capital and government bonds[2]:

$$1-\tau_t = -\frac{u_{l,t}}{u_{c,t}w_t} \quad (1)$$
$$u_{c,t} = \beta E_t u_{c,t+1}(1+(1-\theta_{t+1})(r_{t+1}-\delta)) \quad (2)$$
$$u_{c,t} = \beta E_t u_{c,t+1} R_{b,t+1} \quad (3)$$

2. the budget constraint of the consumer

$$c_t + k_t + b_t = (1-\tau_t)w_t l_t + (1+(1-\theta_t)(r_t-\delta))k_{t-1} + R_{b,t}b_{t-1} \quad (4)$$

3. the factor prices that the competitive firm chooses equal to the corresponding marginal productivities

$$r_t = F_{k,t}(k_{t-1}, l_t, z_t) \quad (5)$$
$$w_t = F_{l,t}(k_{t-1}, l_t, z_t) \quad (6)$$

4. the government budget constraint and the debt limits for the amount of bonds that the planner can issue each period

$$g_t + R_{b,t}b_{t-1} = \tau_t w_t l_t + \theta_t r_t k_{t-1} + b_t \quad (7)$$
$$\underline{M} \leq b_t \leq \overline{M} \quad (8)$$

Following Marcet, Sargent and Seppäla (2000), I assume that the consumer also faces debt limits but less stringent than the planner: $\underline{M}^{cons} \leq \underline{M}$ and $\overline{M} \leq \overline{M}^{cons}$. Therefore, in equilibrium the consumer's problem always has an interior solution.

5. the market clearing conditions for the goods market are satisfied because the consumer's budget constraint and the planner's budget constraint together imply the resource constraint

$$c_t + k_t + g_t = F(k_{t-1}, l_t, z_t) + (1-\delta)k_{t-1} \quad (9)$$

and bonds market clears by the Walras' law.

6. the transversality conditions

$$\lim_{t\longrightarrow\infty} \beta^t E_t u_{c,t+1} R_{k,t+1} k_t = \lim_{t\longrightarrow\infty} \beta^t E_t u_{c,t+1} R_{b,t+1} b_t = 0 \quad (10)$$

where $R_{k,t+1} = 1+(1-\theta_{t+1})(r_{t+1}-\delta)$ is the gross after-tax rate of return on capital.

[2] A representative consumer solves for the allocations taking as given the government policy (taxes and and vectors of state-contingent returns on debt) and the factor prices:

$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$ s.t.

$c_t + k_t + b_t = (1-\tau_t)w_t l_t + (1+(1-\theta_t)(r_t-\delta))k_{t-1} + R_{b,t}b_{t-1}$

3.2 Ramsey Allocations and Policies

The Ramsey planner plays a two-stage Stackelberg game with the public: in period zero, the government announces the policy π and lets the consumers and the firms choose their allocations $x(\pi)$ and factor prices $w(\pi)$ and $r(\pi)$ as the best response to π[3]. In equilibrium, the planner must satisfy his budget constraint and the limits on debt taking as given the reaction functions of the agents, i.e. the allocation rule and the pricing rules. These requirements impose the restrictions on the set of allocations that the government can achieve by varying its policies.

A **Ramsey equilibrium** for this economy is a policy π, an allocation rule $x(\cdot)$, and price rules $w(\cdot)$ and $r(\cdot)$ such that

1. the policy π maximizes $E_0 \sum \beta^t u(c_t(\pi), l_t(\pi))$ subject to the government budget constraint (7) and the limits on debt (8) and rules for allocations and prices are given by $x(\pi)$, $w(\pi)$, $r(\pi)$
2. for every π', the allocation $x(\pi')$, the price system $w(\pi')$, $r(\pi')$, and the policy π' constitute a competitive equilibrium.

I use a standard strategy of recasting the Ramsey problem in terms of a constrained choice of allocations. To do that, I use the conditions (1), (2), (4)-(6) to express τ_t, θ_{t+1}, $R_{b,t}$, r_t, w_t, respectively.

Proposition 1 *Under exogenous limits on debt, the competitive equilibrium allocations are characterized by the same resource constraint and period zero implementability constraint as in Chari, Christiano and Kehoe (Proposition 1, page 622) plus a sequence of period-by-period participation constraints of the form*

$$\underline{M} \leq E_t \sum_{j=1}^{\infty} \beta^{t+j} \frac{u_{c,t+j} c_{t+j} + u_{l,t+j} l_{t+j}}{u_{c,t}} - k_t \leq \overline{M}, \text{ for all } t \geq 0 \tag{11}$$

The proof is given in the Appendix 1.

3.2.1 Ramsey Allocations Problem

The benevolent government maximizes the representative consumer's utility

$$\max_{(c_t, l_t, k_t, b_t)_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_t, l_t)\} \tag{12}$$

- subject to the economy's resource constraint

$$c_t + k_t + g_t = F(k_{t-1}, l_t, z_t) + (1 - \delta) k_{t-1} \tag{13}$$

  to which I attach the Lagrange multiplier $\beta^t \eta_t$,

[3] We abstract here from the isues of time inconsistency assuming that the government has a commitment technology so that it will follow exactly the particular sequence of policies announced at period zero. Recent literature specifies several kinds of such commitment mechanisms, like overaccumulation of capital as in Benhabib and Rustichini (1997), or a specific term structure of debt as in Barro (1997).

- the implementability constraint (the consumer's budget constraint) at $t = 0$[4]

$$c_0 + k_0 + b_0 = -\frac{u_{l,0} l_0}{u_{c,0}} + ((1-\overline{\theta}_0)(F_{k,0} - \delta) + 1)k_{-1} + \overline{R_{b,0} b_{-1}} \tag{14}$$

  with the Lagrange multiplier λ_0 which is often called in the literature the cost of distortionary taxation,

- the sequence of participation constraints (11) that I choose to write in the form

$$u_{c,t}(b_t + k_t) = \beta E_t(u_{c,t+1} c_{t+1} + u_{l,t+1} l_{t+1}) + \beta E_t u_{c,t+1}(b_{t+1} + k_{t+1}) \tag{15}$$

  to which I attach $\beta^t \psi_t$,

- and the limits on the planner's borrowing and saving

$$\underline{M} \leq b_t \leq \overline{M} \tag{16}$$

  with $\beta^t \nu_{1,t}$ and $\beta^t \nu_{2,t}$, respectively,

- for given $\overline{R_{b,0} b_{-1}}$, $\overline{\theta}_0$, and k_{-1}.

This Ramsey allocations problem is not recursive since future control variables appear in the participation constraints facing the planner each period. Thus, the optimal choice at period t is not an invariant function of the natural state variables.

3.3 Recursive Formulation

Following the recursive contracts approach of Marcet and Marimon (1998), this problem can be made recursive by enlarging the state space: ψ_{t-1} becomes another state variable, I refer to it as to the costate Lagrange multiplier.

The Lagrangian of the new saddle point minimax problem can be rewritten as

$$\begin{aligned} \min_{(\psi_t)_{t=0}^{\infty}} \max E_0 \sum_{t=0}^{\infty} \beta^t \{ & u(c_t, l_t) + (\psi_{t-1} - \psi_t) u_{c,t}(b_t + k_t) + \psi_{t-1}(u_{c,t} c_t + u_{l,t} l_t) + \\ & (\nu_{1,t} - \nu_{2,t}) u_{c,t} b_t + (\nu_{2,t} \overline{M} - \nu_{1,t} \underline{M}) u_{c,t} + \\ & \eta_t (F(k_{t-1}, l_t, z_t) + (1-\delta) k_{t-1} - c_t - k_t - g_t) \} + \\ & \lambda_0 [(u_{c,0} c_0 + u_{l,0} l_0 + u_{c,0}(b_0 + k_0) - u_{c,0}(\varkappa_{initial} + (1 - \overline{\theta}_0) F_{k,0} k_{-1})] \end{aligned} \tag{17}$$

with the maximization variables being $(c_t, l_t, k_t, b_t, \eta_t, \nu_{1,t}, \nu_{2,t}, \psi_t)_{t=0}^{\infty}, \lambda_0$.

The Kuhn-Tucker multipliers $\nu_{1,t}, \nu_{2,t}$ are non-negative for any t.

By notation,

$$\psi_{-1} = 0 \tag{18}$$

$$\varkappa_{initial} = \overline{R_{b,0} b_{-1}} + (1 - \delta(1 - \overline{\theta}_0)) k_{-1} \tag{19}$$

where $\overline{R_{b,0} b_{-1}}$, $\overline{\theta}_0$, and k_{-1} are given.

[4]The presence of period zero implementability constraint comes from the fact that we use the period t budget constraint of the consumer to express $R_{b,t} b_{t-1}$ in terms of allocations. But $R_{b,0} b_{-1}$ is given, so the period zero budget constraint remains an additional restriction on the set of CE allocations.

3.3.1 FOC

The structure of the model is such that the first-order conditions for the period zero are different from the rest of the periods because of the nature of the implementability constraint. I assume that the utility function is separable in consumption and leisure, i.e. for all t,

$$u(c_t, l_t) = (1-\gamma)\frac{c_t^{1-\sigma_c}}{1-\sigma_c} + \gamma\frac{(1-l_t)^{1-\sigma_l}}{1-\sigma_l} \tag{assumption on utility}$$

Part I. FOC for $t > 0$

$$\psi_t = \psi_{t-1} + \nu_{1,t} - \nu_{2,t} \tag{FOC with respect to b_t}$$

This equations gives us the law of motion of the costate Lagrange multiplier attached to the period t participation constraint. Thus ψ_t can be interpreted as shadow price of government savings necessary to ensure future solvency. It increases whenever the planner runs short of savings and falls if the upper limit on debt is binding.

$$(\nu_{1,t} - \nu_{2,t})u_{c,t} + \eta_t = \beta E_t \eta_{t+1}(F_{k,t+1} + 1 - \delta) \tag{FOC with respect to k_t}$$

The above equation is the consumption-capital investment choice of the planner. Under complete markets, it reduces to the common shape

$$u_{c,t} = \beta E_t u_{c,t+1}(F_{k,t+1} + 1 - \delta) \tag{20}$$

Differentiating with respect to c_t and l_t gives

$$\eta_t = u_{c,t} + \psi_{t-1}(u_{c,t} + u_{cc,t}c_t) + [\nu_{2,t}(k_t + \overline{M}) - \nu_{1,t}(k_t + \underline{M})]u_{cc,t} \tag{FOC with respect to c_t}$$

and

$$\eta_t F_{l,t} = -u_{l,t} - \psi_{t-1}(u_{l,t} + u_{ll,t}l_t) \tag{FOC with respect to l_t}$$

Again, those are the "good old" $\eta_t = u_{c,t}$ and $u_{c,t}F_{l,t} = -u_{l,t}$ from the first-best, adjusted for distortionary taxation and inequality constraints.

$$u_{c,t}(b_t + k_t) = \beta E_t(u_{c,t+1}c_{t+1} + u_{l,t+1}l_{t+1}) + \beta E_t u_{c,t+1}(b_{t+1} + k_{t+1}) \tag{FOC with respect to ψ_t}$$

$$c_t + k_t + g_t = F(k_{t-1}, l_t, z_t) + (1-\delta)k_{t-1} \tag{resource constraint}$$

$$\nu_{1,t}(b_t - \underline{M}) = \nu_{2,t}(\overline{M} - b_t) = 0 \tag{Kuhn-Tucker conditions}$$

$$\underline{M} \leq b_t \leq \overline{M} \tag{debt limits}$$

Part II. FOC for $t = 0$

$$\psi_0 = \lambda_0 + \nu_{1,0} - \nu_{2,0} \qquad \text{(FOC with respect to } b_0\text{)}$$

$$(\nu_{1,0} - \nu_{2,0})u_{c,0} + \eta_0 = \beta E_0 \eta_1 (F_{k,1} + 1 - \delta) \qquad \text{(FOC with respect to } k_0\text{)}$$

The first-order conditions with respect to c_0 and l_0 are amended by additional terms which contain the initial conditions.

$$\eta_0 = u_{c,0} + \lambda_0(u_{c,0} + u_{cc,0}c_0) + [\nu_{2,0}(k_0 + \overline{M}) - \nu_{1,0}(k_0 + \underline{M})]u_{cc,0} - \lambda_0 u_{cc,0}(\varkappa_{initial} + (1 - \overline{\theta}_0)F_{k,0}k_{-1}) \qquad \text{(FOC with respect to } c_0\text{)}$$

$$\eta_0 F_{l,0} = -u_{l,0} - \lambda_0(u_{l,0} + u_{ll,0}l_0) + \lambda_0 u_{c,0}(1 - \overline{\theta}_0)F_{kl,0}k_{-1} \qquad \text{(FOC with respect to } l_0\text{)}$$

$$u_{c,0}(b_0 + k_0) = \beta E_0(u_{c,1}c_{1+}u_{l,1}l_1) + \beta E_0 u_{c,1}(b_1 + k_1) \qquad \text{(FOC with respect to } \psi_0\text{)}$$

$$c_0 + k_0 + g_0 = F(k_{-1}, l_0, z_0) + (1 - \delta)k_{-1} \qquad \text{(resource constraint)}$$

$$\nu_{1,0}(b_0 - \underline{M}) = \nu_{2,0}(\overline{M} - b_0) = 0 \qquad \text{(Kuhn-Tucker conditions)}$$

$$\underline{M} \leq b_0 \leq \overline{M} \qquad \text{(debt limits)}$$

$$u_{c,0}c_0 + u_{l,0}l_0 + u_{c,0}(b_0 + k_0) = u_{c,0}(\varkappa_{initial} + (1 - \overline{\theta}_0)F_{k,0}k_{-1}) \qquad \text{(implementability at } t = 0\text{)}$$

4 Theoretical Predictions for Ramsey Taxes

I link the condition of one of the debt limits binding today or tomorrow to non-zero expected capital income tax proceeds. Following Zhu (1992), I derive analytical expressions for the optimal tax rates on capital and labor income. I compare the predictions for serial correlation and volatility of labor income taxes to those of Aiyagari, Marcet, Sargent and Seppäla (2001). Both specifications of incomplete markets (debt limits and risk-free debt) add more volatility to labor taxes and there is a unit root component. However, we need simulations to verify whether optimal labor taxes are smooth in the sense of following a random walk.

Issues of Indeterminacy of Capital Income Tax Rates Even though introducing tight and often binding limits on debt should be able to significantly reduce the degree of indeterminacy of the period-by-period capital tax rates, I still continue to make use of the *ex ante expected capital income tax rate* defined as

$$\theta_t^e = \frac{E_t \theta_{t+1} u_{c,t+1}(F_{k,t+1} - \delta)}{E_t u_{c,t+1}(F_{k,t} - \delta)} \qquad (21)$$

The expected capital income tax rate θ_t^e can be interpreted as the ratio of present market value of tax revenue from capital income over the present market value of capital income. Thus, this is a kind of certainty equivalent capital income tax rate.

4.1 Taxing Capital Income?

Proposition 2 *The expected value of next period capital income tax collections is different from zero if and only if (20) does not hold.*

(See Appendix 1 for proof)

In my model, the Euler equation of the consumption-capital choice of the planner is of the form:

$$(\psi_t - \psi_{t-1})u_{c,t} + \eta_t = \beta E_t \eta_{t+1}(F_{k,t+1} + 1 - \delta) \tag{22}$$

where the *shadow* price of an additional unit of resources η_t can be expressed as

$$\eta_t = u_{c,t}(1 + \psi_{t-1}(1-\sigma_c)) + u_{c,t}(\psi_t - \psi_{t-1})\sigma_c \frac{k_t + b_t}{c_t} \tag{23}$$

where I used $\nu_{2,t}(k_t + \overline{M}) - \nu_{1,t}(k_t + \underline{M}) = (\psi_{t-1} - \psi_t)(k_t + b_t)$

Plugging (23) into (22) and rearranging terms leads to a following proposition:

Proposition 3 *For any $t > 0$, $\psi_t \neq \psi_{t-1}$ is enough to give rise to $\theta_t^e \neq 0$.*

(Proof in the Appendix 1)

Corrolary 1 *If for periods t and $t+1$ both limits on debt are slack, then $\theta_t^e = 0$.*

Looking at the sign of the difference $\psi_t - \psi_{t-1}$ in case of each of the debt limits binding, I get the following result:

Proposition 4 *Lower (upper) limit on debt binding today induces positive (negative) average capital tax collections tomorrow.*

(Proof in the Appendix 1)

My simulations results fully confirm these findings and show positive (negative) ex ante expected capital income tax rates for the periods with binding lower (upper) debt limit.

4.2 Random Walk of Labor Income Taxes

For the baseline model of Chari, Christiano and Kehoe, optimal labor income tax rates given by

$$\tau_t = \frac{\lambda_0}{1 - l_t + \lambda_0} \tag{24}$$

The tax rate is a positive function of employment and thus fully inherit its serial correlation and volatility properties. As proved by Zhu (Proposition 4, p.264), smooth leisure leads to almost constant optimal labor taxes and zero expected capital taxes.

With limits on debt, the consumer's consumption-leisure choice (1) implies

$$(1-\tau_t)\frac{1-\gamma}{c_t}F_{l,t} = \frac{\gamma}{1-l_t} \tag{25}$$

while the planner's choice leads to

$$\eta_t F_{l,t} = \frac{\gamma(1 - l_t + \psi_{t-1)}}{(1-l_t)^2} \tag{26}$$

Combining the two gives us the following expression for the tax rate on labor income

$$\tau_t = \frac{\psi_{t-1}}{1 - l_t + \psi_{t-1}} + (\psi_t - \psi_{t-1})\mu_t \frac{(1 - l_t)}{1 - l_t + \psi_{t-1}} \tag{27}$$

where I defined $\mu_t = \frac{1}{1-\gamma}\frac{k_t + b_t}{c_t}$ which has a representation in terms of the right-hand side of the participation constraint (11)

$$\mu_t = \frac{1}{1-\gamma} E_t \sum_{j=1}^{\infty} \beta^{t+j} (u_{c,t+j} c_{t+j} + u_{l,t+j} l_{t+j}) \tag{28}$$

The parameter μ_t is equal to the period's t present discounted value of all future (primary) government surpluses, $\omega_{t+j}^{gov} = \tau_{t+j} w_{t+j} l_{t+j} + \theta_{t+j-1}^{e}(r_{t+j} - \delta)k_{t+j-1} - g_{t+j}$, measured in units of current marginal utility[5].

The labor income taxes fully inherit fluctuations in employment if and only if the costate variable ψ_t converges, i.e. if after some period, the state variables converge to a stationary distribution for which none of the debt limits is binding and we are back to complete markets case.

Recall the discussion from Section 2.2 whether optimal labor taxes should follow a random walk. I take a first-order approximation of τ_{t+1} around ψ_t, l_t and μ_t to get

$$\tau_{t+1} \simeq \tau_t + \frac{\psi_{t-1}(1 - \mu_t \triangle \psi_t)}{(1 - l_t + \psi_{t-1})^2} \triangle l_{t+1} + \frac{1 - l_t}{(1 - l_t + \psi_{t-1})} (\mu_t \triangle \psi_{t+1} + \triangle \psi_t \triangle \mu_{t+1}) \tag{29}$$

The first component makes labor taxes more volatile than employment[6], while the second one is a unit root component which provides additional persistence and volatility. Notice that both terms depend upon μ_t, the solvency of the planner as expected at t. As in case of Scott (2000), there is a tradeoff between the two effects. Section 6 shows that for most of the debt limits specifications, I get a random walk like behavior of the labor income taxes.

5 Numerical Aspects

I choose the functional forms and the parameters of the model as close as possible to those of Chari, Christiano and Kehoe (1994). The utility function is separable in consumption and leisure so that the Chamley result of zero long run expected capital taxes holds for the complete markets. To find a reasonable specification for the exogenous limits on debt, I first solve three models with complete markets and then set the limits as a percent of the average long run GDP of those models.

5.1 Functional Forms

The production function is Cobb-Douglas with labor-augmented technological progress

$$F(k_{t-1}, l_t, z_t) = k_{t-1}^{\alpha}(l_t z_t)^{1-\alpha} \tag{30}$$

The instantaneous utility of the consumer is of the form

[5] It is trivial to show that $u_{c,t} b_t = E_t \sum_{j=1}^{\infty} \beta^j u_{c,t+j} \omega_{t+j}^{gov}$. Combining with the participation constraint, obtains $\mu_t = \frac{1}{1-\gamma}\{ E_t \sum_{j=1}^{\infty} \beta^j u_{c,t+j} \omega_{t+j}^{gov} + u_{c,t} k_t \}$

[6] Recall that $\psi_t - \psi_{t-1}$ is positive (negative) if the lower (upper) limit is binding and zero otherwise.

$$u(c_t, l_t) = (1-\gamma)\frac{c_t^{1-\sigma_c}}{1-\sigma_c} + \gamma\frac{(1-l_t)^{1-\sigma_l}}{1-\sigma_l} \tag{31}$$

I consider the baseline model with $\sigma_c = \sigma_l = 1$ and thus logarithmic preferences.

$$u(c_t, l_t) = (1-\gamma)\ln c_t + \gamma \ln(1-l_t) \tag{32}$$

5.2 Processes for Shocks

I assume that both shocks are lognormal and follow an AR(1) process:

$$\ln z_t = \begin{cases} \overline{z} \text{ if } \ln z_t > \overline{z} \\ \underline{z} \text{ if } \ln z_t < \underline{z} \\ \rho_z \ln z_{t-1} + \varepsilon_{z,t} \text{ otherwise} \end{cases} \tag{33}$$

$$\ln \widetilde{g}_t = \begin{cases} \overline{g} \text{ if } \ln g_t > \overline{g} \\ \underline{g} \text{ if } \ln g_t < \underline{g} \\ \rho_g \ln \widetilde{g}_{t-1} + \varepsilon_{g,t} \text{ otherwise} \end{cases} \tag{34}$$

where $\overline{z} = 2\frac{\sigma_z}{\sqrt{1-\rho_z^2}} = -\underline{z}$ and $\overline{g} = 2\frac{\sigma_g}{\sqrt{1-\rho_g^2}} = -\underline{g}$.

Government expenditures follow

$$g_t = G\exp(\ln \widetilde{g}_t) \tag{35}$$

5.3 Parameters of the Model

Table 1. Baseline Model Parameter Values

Preferences	$\beta^* = .98$	$\gamma = .75$
Technology	$\alpha = .34$	$\delta^* = .08$
Stochastic process for z_t	$\rho_z = .81$	$\sigma_z = .04$
Stochastic process for g_t ($G = .07$)	$\rho_g = .89$	$\sigma_z = .07$
Initial values	$k_{-1} = 1.0$	$\overline{\theta}_0 = .27$

Source: CCK (1994, p. 632)

Note that the initial model of CCK assumes a balanced growth path of the economy at the rate $\rho = .016$. Therefore, I have to adjust for growth β^* and δ^*[7].

5.4 Debt Limits Specifications: Models 1, 2, 3A, 3B

To impose exogenous debt limits, I first solve the model with complete markets for each of the three values of the planner's initial indebtedness.

[7]The resulting $\beta^* = \beta$ for logarithmic preferences, $\delta = 1 - \frac{1-\delta^*}{e^\rho} = .095$ (see details of the adjustment procedure in Garcia-Milá et al. (1995)).

5.4.1 Role of Initial Indebtedness of the Planner

The initial indebtedness of the planner $\overline{R_{b,0}b_{-1}}$ affects the cost of distortionary taxation λ_0, which is one of the key determinants of the Ramsey allocations and of the labor income taxes. Higher $\overline{R_{b,0}b_{-1}}$ implies higher λ_0, higher debt at $t = 0$, and lower long run savings of the planner. That is why I characterize our models' specifications by the initial indebtedness.

Model 1 has an initial indebtedness as in CCK and is amended by loose limits on debt which are of the order of 50% of the average long run output (over 70% of the long run savings of the planner) under complete markets.

Model 2 has half of the initial indebtedness of Model 1 and a pretty tight limit on the consumers' debt (about 2.5% of GDP). The upper limit (on debt of the planner) is set to be equal to $\overline{R_{b,0}b_{-1}}$ of Model 2.

Model 3 has zero initial indebtedness.

Model 3A is characterized by moderate limits on debt (still 50% of the long run output and slightly over 50% of the planner's long run savings under complete markets).

Model 3B has very tight limits on both debt and savings of the planner approximating the case of a balanced budget. The planner is allowed to borrow or save less than 3% of his desired level of savings under complete markets.

6 Findings

I find that introducing market incompleteness through limited short sales requirement completely changes the optimal tax policies of the Ramsey planner. The transition now takes longer than under complete markets and its undesirable features, such as an announced capital levy and a labor income subsidy, are less pronounced or eliminated. I get non-degenerate distributions of both taxes even for the logarithmic preferences (as compared to zero expected capital tax and almost constant labor tax of Chari, Christiano and Kehoe's baseline model). Below I discuss the implications of binding debt limits for the expected capital and labor income tax rates. The allocations are also affected. Even transitionally binding debt limits under uncertainty may have an effect on levels of the variables.

6.1 Initial Period Behavior of Ramsey Taxes

The results given in the Table 2 below show that the capital levy announced at $t = 0$ for the period 1 is reduced or eliminated whenever the upper limit on debt is binding at $t = 0$ (case of all the models except 3A). The same applies to the initial labor income subsidy.

Table 2. Initial Period Behavior of Ramsey Taxes

Table 2.1.

	Model 1	**CM$_{Model1}$**
θ_0^e (%)	373.1	906.6
τ_0 (%)	30.0	-35.2

Table 2.2.

	Model 2	**CM$_{Model2}$**
θ_0^e (%)	205.6	874.9
τ_0 (%)	27.2	-31.0

Table 2.3.

	Models 3A, 3B		$\mathbf{CM}_{Model3}$
θ_0^e (%)	547.5	65.0	846.6
τ_0 (%)	-22.9	20.7	-26.9

The intuition for a different initial period behavior under incomplete markets is the following. Under complete markets, the government accumulates all its assets through the capital levy announced at $t = 0$ and applied at $t = 1$. In period zero, the deficit is high because of the inherited indebtedness and the labor income subsidy paid to the consumers, it is financed by high initial borrowing.

In my model, a more reasonable period zero behavior comes from three factors. First, a binding upper limit prohibits excessive government borrowing in period zero and thus reduces the cost of future tax increases reflected in the costate ψ_0. Recall from the FOC that $\psi_0 = \lambda_0 - \nu_{2,0}$ replaces λ_0 of the complete markets. Second, restricting the level of the long run assets automatically reduces the amount of levy in period one. Third, future use of capital income tax instrument makes unnecessary (and impossible) to accumulate the whole amount of budget surplus in period zero. Last but not least, when markets are incomplete, the sequence of period-by-period participation constraints makes it necessary to reconsider the present value of future government surpluses each period.

6.2 Lessons from Different Models

All the Models except for Model 1 are capable of generating expected capital income tax rates with a positive mean and a high standard deviation. Table 3 below shows that capital taxes are negatively correlated with output (countercyclical) and technological shock and positively to the government spending shock.

The main characteristic of labor taxes under complete markets was their extremely low volatility precluding any serial correlation from having any predictive power. Under debt limits, labor taxes become much more volatile, are negatively correlated with output but exhibit low correlations with any of the shocks. From the graphs of impulse responses for Model 3A we can see that under debt limits, labor taxes follow a kind of random walk behavior being smooth in Barro's sense (see Figures 1 and 2 in the Appendix 3).

I look at the fundamental impulse responses (measured in units of standard deviations of the variables) of taxes in Model 3B with tight upper and lower limits. (see Figures 3 and 4 in the Appendix 3). Here the state variables are such that an unexpected innovation to any of the shocks leads to binding lower limit on debt. The graphs show that both a negative innovation to the technology or a higher government spending are followed by an immediate increase in ex ante capital taxes and a fall in the labor tax rates. Thus, I might conclude that the model with debt limits predicts a neoclassical behavior of capital taxes and gives a kind of Keynesian recommendation with respect to the labor taxes. (If I try to think of some historical examples, it looks like the US government lowered taxes when the economy was doing well.)

Histograms of conditional distribution of expected capital taxes in good and bad states of the economy are given in Figures 5 and 6 of the Appendix 3. A good state is defined as a state in which the existing capital stock is relatively high, the technological shock is relatively good and the government spending is relatively low. The two graphs suggest that the expected capital tax rates should be lower in good and higher in bad times (wars). This is related to the theoretical result of the Proposition 4: the planner should set a positive capital tax when the limit on savings is binding and pay a capital income subsidy when the consumers run short of savings in government bonds.

Model 1. Model 1 is the closest specification to the complete markets. The upper and lower limits on debt are such that only the upper limit turned to be binding and only at $t = 0$. This model emphasizes the importance of my solution method: without using the short run simulations, the matrix of the states is not invertible.

I find level effects in this model relative to the complete markets case. Under complete markets, higher initial indebtedness of the planner is reflected in higher cost of distortionary taxation which leads to higher taxes at $t = 0$ (a higher announced expected capital levy and a lower initial labor income subsidy). There is no effect on the Ramsey allocations. Only labor taxes and debt are affected. This picture changes when the costate multiplier varies in the short and medium run before converging to its long run value. The model with loose limits on debt converges to a stationary distribution equivalent to a complete markets case with a much higher initial indebtedness of the planner. Therefore, the resulting Ramsey optimal allocation is characterized by lower capital, output and consumption, and higher leisure. Thus there is a level effect on allocations of an even transitionally binding limit on the initial government debt and of never binding (in equilibrium) limit on savings. Model 1 can be used to gain an insight into the cost-benefit analysis of reducing the initial capital levy.

Models 2 and 3B These two models have the same structure of debt limits: the upper one is set equal to the corresponding initial indebtedness of the planner, the lower limit (on government savings) is very close to zero and the same for both models ($\underline{M} = -0.01$). As a result, the two models converge to the same Ramsey allocation except for the equilibrium level of debt which is slightly higher for the Model 2 with a softer upper limit (see Tables A2 and A3 of the Appendix 3).

As for the Ramsey optimal policies, a tighter and more often binding upper limit of Model 3B implies a lower mean and a higher standard deviation of the expected capital income taxes, as shown in Tables 3.2 and 3.3 below. The intuition is that binding upper limit leads to expected capital subsidies (Proposition 4). The obligation to run a balanced budget induces the government to use the capital income taxes as a shock absorber.

The labor income taxes become more than ten times more volatile than under complete markets, are both negatively correlated with output and uncorrelated with the government spending shock. The fundamental impulse responses in Figures 1 to 4, suggest that the unit root component of (29) dominates so that the labor income tax rates follow a kind of random walk.

Models 3A and 3B Case of zero initial indebtedness allows for comparisons of the Model 3 to the theoretical conclusions of Zhu for the complete markets case. My simulations confirm Zhu's Proposition 5 (p. 267) that expected capital tax rates are both positive and negative, as under complete markets when the Chamley result of zero expected capital tax rates does not hold.

The Model 3A is characterized by much looser upper and lower limits than the Model 3B. Less frequently binding debt limits lead to a reduction in absolute value of cross-correlations for θ^e but the signs are preserved. Labor income taxes are smooth in Barro's sense for both specifications of the limits.

Table 3. Cyclical Properties of Taxes for Models 1, 2, 3A, 3B

Table 3.1.

$R_{b,0}b_{-1} = 0.2$	%	θ^e	τ	%	$cor(\theta^e, \cdot)$	$cor(\tau, \cdot)$
CM	*mean*	0	24.1	$cor(\cdot, y)$	NA	48.4
	std	0	0.2	$cor(\cdot, z)$	NA	47.7
	autocor	NA	62.7	$cor(\cdot, g)$	NA	55.0
Model 1	*mean*	0	26.5	$cor(\cdot, y)$	NA	49.6
	std	0	0.2	$cor(\cdot, z)$	NA	49.6
	autocor	NA	69.1	$cor(\cdot, g)$	NA	55.7

Table 3.2.

$R_{b,0}b_{-1} = 0.1$	%	θ^e	τ	%	$cor(\theta^e, \cdot)$	$cor(\tau, \cdot)$
CM	*mean*	0	23.5	$cor(\cdot, y)$	NA	48.8
	std	0	0.1	$cor(\cdot, z)$	NA	48.5
	autocor	NA	70.7	$cor(\cdot, g)$	NA	54.7
Model 2	*mean*	8.2	26.4	$cor(\cdot, y)$	-11.4	-21.6
	std	30.7	1.4	$cor(\cdot, z)$	-25.2	7.4
	autocor	48.9	91.2	$cor(\cdot, g)$	29.1	2.0

Table 3.3.

$R_{b,0}b_{-1} = 0.0$	%	θ^e	τ	%	$cor(\theta^e, \cdot)$	$cor(\tau, \cdot)$
CM	*mean*	0	23.2	$cor(\cdot, y)$	NA	49.0
	std	0	0.1	$cor(\cdot, z)$	NA	48.7
	autocor	NA	70.6	$cor(\cdot, g)$	NA	54.7
Model 3A	*mean*	7.5	24.5	$cor(\cdot, y)$	-7.9	-16.5
	std	29.8	1.2	$cor(\cdot, z)$	-19.8	6.7
	autocor	43.5	90.4	$cor(\cdot, g)$	26.9	6.6
Model 3B	*mean*	2.7	26.3	$cor(\cdot, y)$	-31.7	-12.3
	std	33.6	1.6	$cor(\cdot, z)$	-45.0	13.6
	autocor	61.3	88.8	$cor(\cdot, g)$	50.0	-3.8

6.3 Debt and Deficit under Incomplete Markets

I also look at the behavior of debt and primary government budget deficit under exogenous limits on debt and savings of the planner.

In the Table 4 below I compare the signs of impulse responses of my model to those of Marcet and Scott (2001) who described the behavior of debt in the US data and, among others, in a Ramsey model with capital accumulation but without capital income taxes and with risk-free debt only.

Table 4. Models' Predictions for Signs of Impulse Responses

	CM	**IM debt limits**	**IM risk-free debt**	**US data**
$z \uparrow$	$b \uparrow$ $deficit \downarrow$	$b \uparrow$ $deficit \downarrow$	$b \uparrow$ $deficit \uparrow$	$b \downarrow$ $deficit \downarrow$
$g \uparrow$	$b \downarrow$ $deficit \uparrow$	$b \rightarrow, \uparrow$ $deficit \uparrow$	$b \uparrow$ $deficit \uparrow$	$b \uparrow$ $deficit \uparrow$

I conclude from this exercise that the model with debt limits fits the data better than the one with complete markets and is also capable of giving a correct sign of the impulse response deficit to a positive innovation to the technology.

As in Marcet and Scott, non-neutrality of debt implies its more volatile behavior. Debt remains negatively correlated with the government spending, though less than under complete markets.

6.3.1 A Frequently Asked Policy Question

How to finance a sudden war (or other large financial need)? Unfortunately enough for the policy makers, there are almost as many answers to this crucial question of the public finance as there are different models. The first-best recommendation of taxing the existing, inelastically supplied capital stock is inapplicable (see the discussion of Section 2 about the history of capital levies between the two world wars). Barro's model suggests to raise all the tax rates a small amount so that when held constant at that level, expected value of war is financed. Under complete markets, a temporary small increase in labor income taxes is accompanied by a reduction in outstanding debt in the short run.

The impulse responses show what happens to my model if one day, after the economy has converged to a non-stochastic steady state[8], there is an increase in government spending. The government should recur to the capital income taxation next period, cutting current labor taxes. Before the unexpected positive innovation to the government spending happened, the planner was financing constant spending by constant labor taxes and the return on the (small) assets which were at the lower limit $\underline{M}$. After the shock, the return on assets goes up a lot, as the consumers demand more bonds to insure against future shocks.

Initially high g_t boosts output for the period of fixed capital taxes. Past savings are enough to cut labor taxes, to avoid big drop in both capital and hours. In the long run, there is a usual incomplete markets effect of a recession caused by a big cut in government expenditure leading to higher labor taxes, less hours and capital and thus less output and consumption, and less government assets. However, the transitional dynamics suggest that the expected capital tax shares the role of shock absorber with debt.

7 Conclusions

Asset market incompleteness may be a possible explanation of why we do observe capital income taxation but we don't see the governments that use either capital levies or labor income subsidies. Introducing exogenous limits (possibly binding in equilibrium) on debt and savings of the planner, gives rise to possibly non-zero and very volatile expected capital income taxes and does a reasonably good job in correcting the counterfactual transitional features of the complete markets model.

Incomplete markets imply a countercyclical behavior of both capital and labor income taxes. However, the two taxes should react differently to the negative shocks to the economy: expected capital taxes should be higher in bad states while labor income taxes drop in response to bad shocks.

I had difficulties with solving the model numerically when I tried to put the upper limit on debt tighter than the initial indebtedness of the government.

It is interesting to see what happens if we allow debt limits which distort the first-order conditions of the consumers.

One of the most interesting issues about the capital income taxation, if the government is to make use of it due to market incompleteness, is the redistributive effect. Chari and Kehoe (1998) proved that under complete markets, heterogeneity of consumers is not enough to give rise to non-zero capital taxes even for the case when the planner puts no weight on the capitalists. Hence once more incomplete markets can be useful to study the redistributive effect of the capital income taxation. Yakadina (2001) casts the Ramsey problem for the environment of García-Milá et al. with heterogeneous consumers. I endogenize limits on debt by looking for an optimal collateral for the consumers' borrowing.

[8] All the variables in the non-stochastic steady state of the Ramsey model are functions of λ_0 evaluated at the Ramsey optimum of the stochastic model.

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9 Appendices

9.1 Appendix 1. Proofs of Propositions

9.1.1 Proof of Proposition 1

My model has an additional constraint on the competitive equilibrium allocations which has the form

$$\underline{M} \leq b_t \leq \overline{M} \tag{36}$$

I derive an expression for debt from the Euler equation for the returns on bonds (3) using the Euler equation for capital (2) and one-period ahead present discounted version of the consumer budget constraint:

$$u_{c,t}(b_t + k_t) = \beta E_t(u_{c,t+1}c_{t+1} + u_{l,t+1}l_{t+1}) + \beta E_t u_{c,t+1}(b_{t+1} + k_{t+1}) \tag{37}$$

Substituting forward $u_{c,t+1}(b_{t+1} + k_{t+1})$ and applying the Law of Iterated Expectations[9] gives us

$$u_{c,t}(b_t + k_t) = E_t \sum_{j=1}^{\infty} \beta^j (u_{c,t+j}c_{t+j} + u_{l,t+j}l_{t+j}) \tag{38}$$

Expressing debt and substituting it into the inequality constraints gives the participation constraints of the form (11).

9.1.2 Proof of Proposition 2

The consumer's FOC for consumption-capital choice (2) and (20) hold simultaneously if and only if

$$E_t u_{c,t+1}\theta_{t+1}(F_{k,t+1} - \delta) = 0 \tag{39}$$

so the expected value of the capital income tax collections, $E_t u_{c,t+1}\theta_{t+1}(F_{k,t+1} - \delta)k_t$ is zero. Thus, if (20) does not hold, neither does (39).

9.1.3 Proof of Proposition 3

Let $\psi_t = \psi_{t-1}$ for all $t > \widetilde{t}$ and $\psi_{\widetilde{t}} \neq \psi_{\widetilde{t}-1}$ (one of the debt limits is binding for $t = \widetilde{t}$ but never after). Then the Euler equation of the planner takes the form

$$\begin{aligned} & (\psi_{\widetilde{t}} - \psi_{\widetilde{t}-1})(1 + \sigma_c \frac{k_{\widetilde{t}} + b_{\widetilde{t}}}{c_{\widetilde{t}}})u_{c,\widetilde{t}} + (1 + \psi_{\widetilde{t}-1}(1 - \sigma_c))u_{c,\widetilde{t}} \\ = \; & (1 + \psi_{\widetilde{t}}(1 - \sigma_c))\beta E_t u_{c,\widetilde{t}+1}(F_{k,t+1} + 1 - \delta) \end{aligned} \tag{40}$$

Suppose that (20) holds. Then the above equation simplifies to

$$(\psi_{\widetilde{t}} - \psi_{\widetilde{t}-1})\sigma_c \frac{k_{\widetilde{t}} + b_{\widetilde{t}} + c_{\widetilde{t}}}{c_{\widetilde{t}}} u_{c,\widetilde{t}} = 0 \tag{41}$$

or

[9] We also use a transversality condition $\lim_{T \to \infty} \beta^T E_t u_{c,t+T} b_{t+T} = 0$.

$$k_{\tilde{t}} + b_{\tilde{t}} + c_{\tilde{t}} = 0 \tag{42}$$

which would require zero wealth of the consumer in period $\tilde{t}$. Hence, (20) does not hold, and by Proposition 2, we have that the expected value of capital income tax collections is different from zero. By definition of the ex ante expected capital tax rate, θ_t^e is different from zero.

9.1.4 Proof of Proposition 4

Using the definition of the ex ante expected capital tax rates we can rewrite the expected value of capital tax collections as

$$X_t^e = \theta_t^e E_t u_{c,t+1}(F_{k,t+1} - \delta)k_t \tag{43}$$

It is equal to

$$X_t^e = (\beta E_t u_{c,t+1}(F_{k,t+1} + 1 - \delta) - u_{c,t})k_t \tag{44}$$

Assume that none of the limits is binding at $t+1$. Then $\eta_{t+1} = u_{c,t+1}(1 + \psi_t(1 - \sigma_c))$

$$X_t^e = (\psi_t - \psi_{t-1})\sigma_c \frac{k_t + b_t + c_t}{c_t} u_{c,t} \tag{45}$$

and its sign depends only on the sign of the difference $\psi_t - \psi_{t-1} = \nu_{1,t} - \nu_{2,t}$ which is positive when the lower limit is binding, negative when the upper limit is binding and zero otherwise.

9.2 Appendix 2. Algorithm of the Numerical Solution

I use the Parameterized Expectations Algorithm by Marcet (see Marcet and Lorenzoni (1998) for a detailed description of the method) to approximate the conditional expectations at time t by

$$\beta E_t f(S_{t+1}, v_{t+1}) = \Phi(S_t \beta^{PEA_n}) \tag{46}$$

where Φ is a time-invariant smooth function of the current states $S_t = [1\ k_{t-1}\ z_t\ g_t\ \psi_{t-1}]$ and the coefficients of the approximation $\beta^{PEA_n} = (\beta_1^{PEA_n}, ..., \beta_5^{PEA_n})$. See below how to find the initial guess for β^{PEA_n}. I parameterize

- the right-hand side of the Euler equation for consumption-investment choice of the planner,
- the sum of the conditional expectations on the right-hand side of the participation constraint (15) and
- the two conditional expectations that appear in the formula of the ex ante expected capital income tax rates.

The solution strategy consists in the following. Fist, I fix the initial indebtedness of the planner, $R_{b,0}b_{-1}$, and guess a value of $\lambda_0 > 0$, the Lagrange multiplier on the period zero implementability constraint. For a given value of λ_0, I first solve the FOC at $t = 0$ assuming that the debt limits are not binding, compute the solution and check whether debt falls inside the limits. If it doesn't, then set debt equal to its corresponding limit and recompute the solution. Proceed with the FOC for the rest of the periods. With the simulated variables in hand, check the distance between the left-hand side of each approximated equation and $\Phi(S_t \beta^{PEA_n})$. Find β^{PEA_n} which minimize this distance for each $n = 1, ..., 5$. Finally, compute the initial indebtedness of the planner from the implementability constraint right-hand side, and adjust λ_0. I have to iterate on λ_0 until I get the initially fixed $R_{b,0}b_{-1}$.

9.2.1 Short Run Monte Carlo Simulations

See Marcet and Marimon (1992) for the description of the method.

I need to recur to the short run Monte Carlo simulations for the following reasons: first, I want to approximate the transition really well. The period zero behavior of the model is totally different from the rest of the periods due to different FOC for c_0 and l_0. Second, I start with a low initial capital stock, very far away from its stationary long run level. I want to look at different specifications for debt limits and to be able to analyze the models with only transitionally binding limits. This is impossible to do with just a one long series of shocks due to degenerate matrix of states in case that there are few observations with binding limits.

Finally, the models with transitionally binding limits, in the very long run converge to the complete markets solution. Therefore, to study the effects of binding debt limits on the optimal taxes, I need to stick to the short and medium run analysis.

I use 200 simulations of the two shocks, each one of the lengths of 100 periods. To compute the statistics, I construct a long vector for each series taking periods between 11 and 90 for each of the 200 Monte Carlo realizations.

9.2.2 PEA and Homotopy

The main difficulty with applying the Parameterized Expectations approach is to find the starting values for the vectors of parameters $\beta^{PEA_n} = (\beta_1^{PEA_n}, ..., \beta_5^{PEA_n})$ such that they generate non-explosive stochastic processes for the allocations and bring us to the stationary distribution.

Homotopy is an approach which allows imposing ”good” initial conditions for the Parameterized Expectations Algorithm in a systematic way (Marcet and Lorenzoni (1998, p.p. 20-24)). This approach consists in always starting with a simplified version of the model which is easy to solve numerically, and then modifying the parameters slowly, to go to the desired version of the model. As long as the model goes from the known to the desired solution in a smooth way, the initial conditions are good.

To solve the model with debt limits, I proceeded in 5 iterations, i.e. I solved 5 different models, starting from the simplest one and gradually complicating it with additional state variables and constraints. Each previous iteration gave the starting Parameterization for the next one.

Iteration 1: Solve a simple stochastic growth model with only technological shock and no labor.

Iteration 2: Introduce labor to the previous model.

Iteration 3: Add the government spending shock.

Iteration 4: Solve the complete markets model.

Iteration 5: Introduce debt limits (hence, enlarge the vector of states by introducing the costate Lagrange multiplier, ψ_{t-1}.

To pass to the next step, I had to iterate on some parameter linking the two models.

9.3 Appendix 3. Graphs and Tables

9.3.1 Tables

Table A. Moments (means and standard deviations) for Models 1, 2, 3A, 3B Table A1.

$R_{b,0}b_{-1} = 0.2$	k	l	y	c	b	$R_b b$	R_k	ψ
CM	1.213	0.234	0.411	0.225	-0.274	-0.279	1.021	0.243
$\lambda_0 = 0.243$	0.075	0.006	0.024	0.011	0.027	0.037	0.006	0
Model 1	1.191	0.229	0.403	0.219	0.002	0.002	1.021	0.278
$\lambda_0 = 2.182$	0.076	0.006	0.026	0.012	0.036	0.044	0.006	0

Table A2.

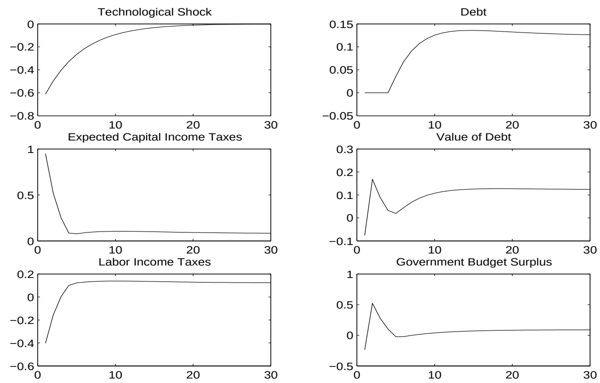
$R_{b,0}b_{-1} = 0.1$	k	l	y	c	b	$R_b b$	R_k	ψ
CM	1.221	0.235	0.413	0.226	-0.318	-0.325	1.021	0.235
$\lambda_0 = 0.235$	0.076	0.006	0.026	0.013	0.028	0.035	0.006	0
Model 2	1.178	0.228	0.400	0.219	0.011	0.012	1.020	0.281
$\lambda_0 = 1.415$	0.086	0.006	0.027	0.013	0.023	0.030	0.009	0.016

Table A3.

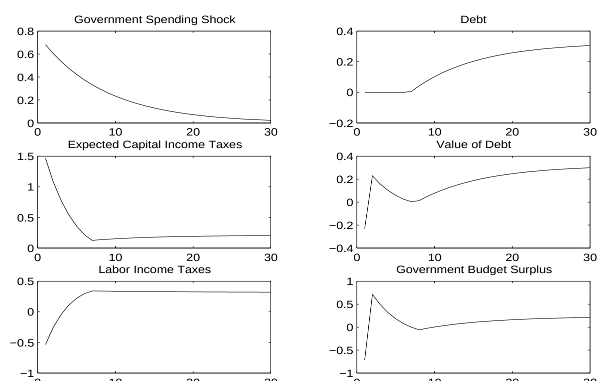
$R_{b,0}b_{-1} = 0.0$	k	l	y	c	b	$R_b b$	R_k	ψ
CM	1.221	0.236	0.413	0.227	-0.370	-0.377	1.021	0.230
$\lambda_0 = 0.230$	0.077	0.006	0.026	0.013	0.026	0.026	0.006	0
Model 3A	1.195	0.232	0.406	0.224	-0.177	-0.180	1.020	0.252
$\lambda_0 = 0.157$	0.084	0.006	0.027	0.013	0.022	0.028	0.009	0.013
Model 3B	1.178	0.229	0.401	0.219	-0.002	-0.003	1.021	0.278
$\lambda_0 = 0.777$	0.087	0.006	0.027	0.013	0.009	0.014	0.009	0.016

9.3.2 Graphs

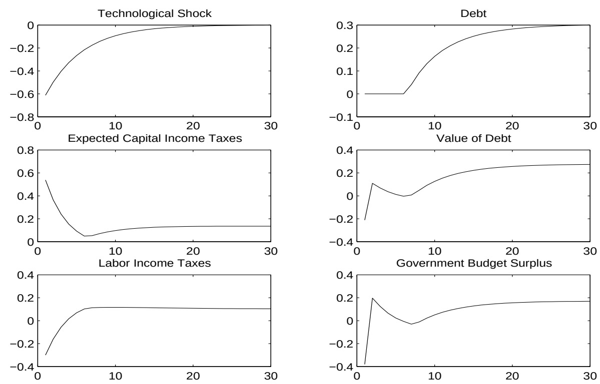
Model 3A. Fundamental Impulse Responses to z



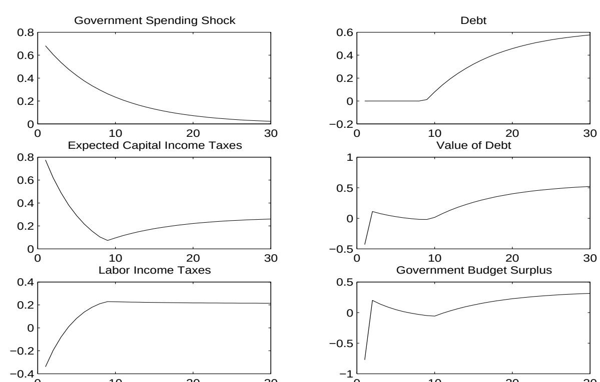
Model 3A. Fundamental Impulse Responses to g

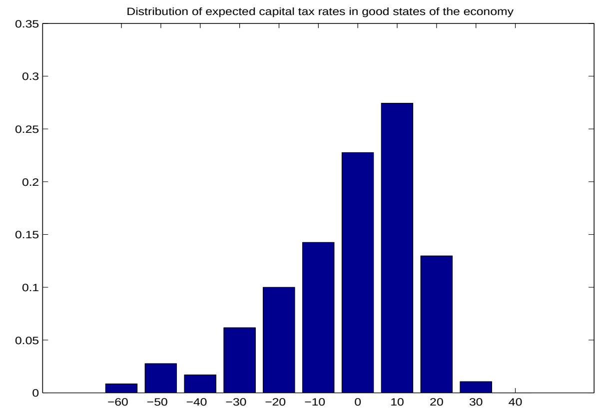


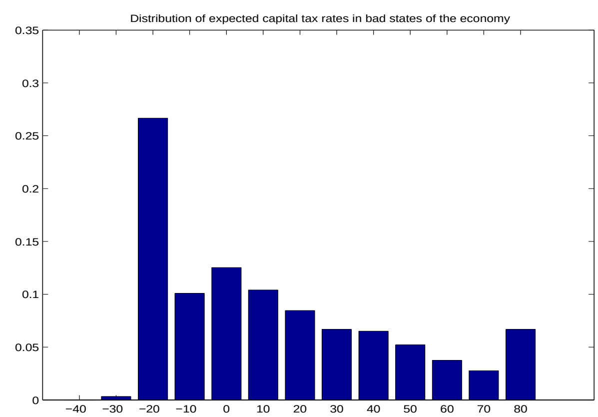
Model 3B. Fundamental Impulse Responses to z



Model 3B. Fundamental Impulse Responses to g




Distribution of expected capital tax rates in good states of the economy
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
−60
−50
−40
−30
−20
−10
0
10
20
30
40


Distribution of expected capital tax rates in bad states of the economy
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
−40
−30
−20
−10
0
10
20
30
40
50
60
70
80