Game Theory and Contract Theory in Finance

Module 3, 2017-2018

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Course information

Course Website: Course materials will be posted on my.nes

Instructor's Office Hours: by appointment

Class Times: Wednesdays, 12.00-13.30 and 13.45-15.15

Room Number: 319

Course description

This course is designed to expose the students to the basic concepts and most important issues in the modern contract theory, with special focus on financial contracts. Besides, subjects such as property rights theory, moral hazard in teams, multi-tasking, incomplete contracts and economics of organizations will be covered.

Course requirements, grading, and attendance policies

There will be two problem sets that will count for 10% of the grade and a final exam that will count for 60% of the grade. In addition, each student **must complete an independent project** which will count for the rest 30% of the grade. The project is to study a real-world contractual situation, dig out the background, find out what is specific about the arrangement, understand incentives and other properties of the contracts in question, provide a model and possibly suggest an improvement. For this students are encouraged to group in size up to five; **Successful completion of this assignment is required for passing the course.** There will be three milestones: abstract (due Sunday week 3), first version (due Sunday week 5) and final version (due Sunday week 7); you will receive feedback from me on the first two stages.

Course contents

This contents is a preliminary outline of the program, subject to adjustments as we proceed.

Week 1: Refresher in Game Theory, Nash equilibrium, SPNE in dynamic games.

Week 2: Linear compensation model, insurance-incentives tradeoff, multitasking, moral hazard in teams.

Week 3: Outside financial capacity (Tirole Ch. 3).

Week 4: Borrowing capacity (Tirole Ch. 4).

Week 5: Liquidity and risk management (Tirole Ch. 5).

Week 6: Asymmetric information and market breakdown (Tirole Ch. 6).

Week 7: Incomplete contracts.

Sample tasks for course evaluation

The following is taken from 2011 final exam when a similar class was offered at MAE program. This year more emphasis will be on financial contracts.

1. There is a principal and an agent, both risk neutral and facing no wealth constraints. The agent spends effort a, at costs $C(a) = \gamma \cdot a^2$, to come up with a project that has private value X for himself and private value Y for the principal (neither of which are contractible). The former can be either $X_H = 10$ with probability a or $X_L = -10$ with probability $1 - a$; should the former turn out to be X_H, the latter is $Y_H = 20$ with probability $p = Prob(Y_H|X_H) = 2/3$ and $Y_L < 0$ with probability $1/3$. Should the former turn out to be X_L, the latter is Y_H with probability $q = Prob(Y_H|X_L) = 1/4$. Once a project pops up, the agent immediately observes both X and Y. There is always an option not to pursue the project once it is discovered, in which case both the principal and the agent earn zero.

Your answers to what follows may depend on γ and Y_L.

a) Find first best level of a.

b) Assume that the principal, when presented with a project, can immediately observe Y; by default she has a formal right to overturn a project that she does not like, but she is thinking about selling this right to the agent (assume the authority iscontractible). Find agent's effort levels under principal's and agent's formal authority. Would it be efficient to formally delegate the authority to the agent?

c) Redo (b) assuming that the principal, when presented with a project, has to decide on approval without knowing Y.

d) Assume now that the interaction described above occurs every period forever, and the two parties face the same interest rate r. Assume that there are a few potential agents around who compete with each other, maybe to the point of having to bid for the position described above.

e) Assume that the principal, when presented with a project, can immediately observe Y; assume further that the formal delegation is not feasible (the principal cannot commit not to exercise her authority). But she can offer informal delegation, promising not to veto agent's project, under the punishment that the principal's reputation will be destroyed forever and no agent will ever want to deal with her if she defects. Under what parameters will it be (i) feasible and (ii) desirable? Give an example of such combination that it is desirable and feasible and one at which it is desirable but not feasible.

f) Assume that the principal cannot observe Y until after the project is implemented.

She may ask the agent to offer her only projects that yield Y_H but not Y_L, under the punishment that the agent who offers a Y_L project will be fired. Underwhat parameters will it be (i) feasible and (ii) desirable? Give an example of such combination that it is desirable and feasible and one at which it is desirable but not feasible.

g) Assume that, prior to everything, the principal can (publicly) invest in her ability to quickly read and assess projects, so that she can move from (e) to (d) above. How much would she be willing to invest?

h) Now change the setup slightly in such a way that principal's payoff, when not Y_H, can be either $Y_M = -5$ or $Y_L = -20$ with equal probabilities. Under what conditions will it be optimal to pursue projects yielding Y_M but not Y_L?

i) In the setup of (g), the principal is considering informal delegation (when the principal informally promises to ratify Y_H and Y_M projects) versus divestiture with relational contracting (when the authority resides with the agent, but he informally promises not to implement Y_L projects). For various values of r which of the two is feasible?

2. As in Hart, Moore (2008) there is a principal and an agent, who are considering trading at date $t = 1$. Timing is now continuous: they may choose to write a contract, specifying he range of prices $[\underline{p}; \bar{p}]$, at any point $t \imath n [0,1]$. No matter when the contract is written, it is executed at $t = 1$ if and only if there is a price within $[\underline{p}; \bar{p}]$ that is below buyer's value but above seller's costs. Each of the two parties, however, gets aggrieved when not getting the best out of the contract; an aggrieved party gets relieved by shading on the other – each dollar of aggrievement can be relieved by θ dollars of shading. There is uncertainty as to buyer's value and seller's costs at time $t = 1$. Specifically, depending on the state of the world, both the value and the costs can be high, medium or low, with $v_h = 60$, $v_m = 40$, $v_l = 20$, $c_h = 50$, $c_m = 30$, $c_l = 10$. Just prior to signing the contract, the parties obtain a single (symmetrically observable but nonverifiable) free signal $\sigma \in \{L, H\}$ about which state of the world is more likely; the longer they wait to do so, the more precise the signal. Specifically,if the signal is $\sigma = H$ at time t, then the probabilities of the three states are $\pi_h^H = (1 + t)/3$, $\pi_m = 1/3$, $\pi_l^H = (1 - t)/3$, and likewise if $\sigma = L$ at time t, then the probabilities of the three states are $\pi_h^L = (1 - t)/3$, $\pi_m = 1/3$, $\pi_l^L = (1 + t)/3$. There are costs to delaying writing a contract per unit of time: α_b for the buyer and α_s for the seller (you may think that each party has to borrow and the banks charge lower interest if they see a signed deal). Once a contract is signed, it cannot be renegotiated.

a) Once the time of signing the contract $t^* \in [0,1]$ is fixed, what is the optimal contract? What is the total surplus (net of shading costs) that it delivers? Limit your attention to contracts specifying the price range and which of the parties gets to pick the price in that range at $t = 1$.

b) What is the optimal t^*?

c) Assume now that there is only one seller who sequentially supplies two buyers with very similar items (but tailored to a specific buyer) at identical costs. The buyer who is served second observed the price that the first buyer pays and will not feel aggrieved whenever he does not have to pay more than the first buyer; the first buyer is unaware about the second buyer's existence. How would that change your analysis in (a) and (b)?

d) Discuss verbally how your analysis would change if 'no renegotiation' assumption is relaxed.

3. There is a buyer (she) and a seller (he), who are planning to trade one unit of good at date 3. At date 0, they contract on prices $(\hat{p}_0, \hat{p}_1)$ so that if the good is delivered and accepted, the buyer has to pay the seller $\hat{p}_1$ and otherwise she has to pay him $\hat{p}_0$. Denote $k = \hat{p}_1 - \hat{p}_0$. At date 1 the seller (resp., the buyer) chooses noncontractible investments σ

(resp., β) at private costs $C(\sigma) = \frac{\sigma^2}{2s}$ (resp., $C(\beta) = \frac{\beta^2}{2b}$). The costs of supplying the good for the seller (respectively, valuation of the buyer), which are realized at date 2, are then exponentially distributed with rate 1 on segment $[-\sigma, \infty)$ (resp., on $[\beta, \infty)$).

a) Find first best levels of investment σ^* and β^*.

b) Assume that the contract cannot be renegotiated. Find optimal k and corresponding levels of investments β and σ. Can there be multiple equilibria? Is first best achieved at any combination of parameter values?

c) Assume that inefficient contract will be renegotiated between dates 2 and 3. Find optimal k. Compare expected surplus with what you found in (b). Is renegotiation option desirable for the parties or not?

d) Assume now that the court can verify whether it was the seller who failed to deliver or the buyer who failed to accept the good. Find optimal 'option to sell' contract (i.e., optimal k). Is first best achieved?

e) Redo (d) for the optimal 'option to buy' contract.

Course materials

The main textbook for the course is Tirole *The Theory of Corporate Finance,* available at NES library. Other materials will be posted on my.nes.

Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.