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Another Model of Non-fundamental Exchange-Rate Uncertainty

Working Paper #

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Most of existing theories of floating exchange rates relate exchange-rate volatility to shocks in fundamentals, whereas many policy makers believe that this volatility has a significant non-fundamental component. We propose another model of exchange rates, which is consistent with policy makers view. We use a two-country two-currency version of the random matching model of Trejos and Wright (2001) to show that search frictions, uncertainty in trade within meetings and poor substitutability among currencies can produce exchange-rate dynamics consistent with empirics: both nominal and real exchange rates i) are unrelated to fundamentals, ii) are much more volatile than fundamentals, iii) show little difference in behavior, and iv) fail to satisfy conservation of volatility.

JEL: F31, C78

Keywords: exchange-rate puzzles, non-fundamental, volatility, bargaining, search.

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Большинство существующих теорий обменных курсов объясняют волатильность курсов шоками фундаментальных переменных. Большинство участников рынка, напротив, полагают, что волатильность курсов обусловлена наличием существенных нефундаментальных факторов. Мы предлагаем еще одну модель обменных курсов, соответствующую представлениям участников рынка. Мы используем версию модели со случайными встречами Трехоса и Райта (2001) с тем, чтобы показать что поиск, неопределенность в торговле между агентами и плохая взаимозаменяемость валют могут приводить к динамике обменных курсов соответствующей основным эмпирическим фактам: номинальные и реальные обменные курсы i) слабо соотносятся с текущими значениями фундаментальных переменных, ii) являются в значительно большей мере волатильными чем фундаментальные переменные, iii) демонстрируют схожую динамику и iv) нарушают теоретическую предпосылку о сохранении волатильности.

JEL: F31, C78

Ключевые слова: загадки обменных курсов, нефундаментальный, волатильность, торговля, поиск.

ISBN

Another model of non-fundamental exchange-rate uncertainty*

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Abstract

Most of existing theories of floating exchange rates relate exchange-rate volatility to shocks in fundamentals, whereas many policy makers believe that this volatility has a significant non-fundamental component. We propose another model of exchange rates, which is consistent with policy makers view. We use a two-country two-currency version of the random matching model of Trejos and Wright (2001) to show that search frictions, uncertainty in trade within meetings and poor substitutability among currencies can produce exchange-rate dynamics consistent with empirics: both nominal and real exchange rates i) are unrelated to fundamentals, ii) are much more volatile than fundamentals, iii) show little difference in behavior, and iv) fail to satisfy conservation of volatility.

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1 Introduction

Thus far economists have succeeded in putting up the list of exchange rate puzzles. Both nominal and real exchange rates i) are pretty much unrelated to fundamentals (determination puzzle), ii) are much more volatile than fundamentals (excess volatility puzzle), iii) show little difference in behavior,[1] and iv) fail to satisfy volatility conservation.[2] Related to these observations there is a belief that most of exchange-rate volatility comes from sources other than macroeconomic fundamentals. Here we present a model of non-fundamental exchange-rate uncertainty, which is consistent with these four puzzles; much of what we say about exchange rates applies to other financial markets, e.g. equity market.[3]

The existing literature on exchange-rate volatility seems to evolve along two dimensions. On one side there are models, where exchange-rate uncertainty is purely non-fundamental; in such models exchange-rate dynamics is driven by an extraneous sunspot variable.[4] These models produce exchange-rate volatility even when fundamentals remain fixed, yet they fail to address the behavior of real exchange rates. We know of only one paper, Barnett (1992), which attempts to deal with real exchange rates. The model has many dynamic equilibria consistent with disconnected and volatile real ex-

[1] See, e.g. Obstfeld and Rogoff (1996).

[2] Empirical failure of volatility conservation is documented, for example, in Flood and Rose (1995).

[3] As regards equities, the analogs of determination and excess volatility puzzles were recognized by Roll (1988) and Shiller (1981) respectively.

[4] The list of such models includes Kareken and Wallace (1981), Manuelli and Peck (1990), King, Wallace, and Weber (1992), Barnett (1992), Alonso (2004).

change rates; however, in all of such equilibria exchange-rate movements vanish over time. That point has been extensively emphasized by Alonso (2004); she presents a version of Shubik's trading post model where nominal exchange-rate movements are persistent. This paper contributes to that literature: we build a model, where real exchange rate movements are persistent, so that the model yields exchange-rate dynamics consistent with the above puzzles.

On the other side there are models, where exchange-rate volatility is a reflection of the volatility of fundamentals. These models introduce frictions into otherwise perfect markets; the list of such frictions includes search, asymmetric information about fundamentals, and noisy expectations.[5] Most of these models produce exchange-rate dynamics consistent with determination and excess volatility puzzles,[6] however our focus here is non-fundamental channel of exchange-rate uncertainty. Here the uncertainty is driven by non-fundamental sunspot fluctuations pertaining to the division of the gains from trade.

We work with a two-country two-currency version of the random-matching model of money; a similar environment is used by Trejos and Wright (2001). There are no centralized walrasian markets. In the beginning of a period individuals meet each other in pairs and at random; some of these meetings

[5] See, e.g. Alessandria (2003), Lyons (2001), and Shleifer and Summers (1990).

[6] We know of one paper in that literature, which is consistent with failure of volatility conservation: Jeanne and Rose (2002). However, Jeanne and Rose assume noisy expectations and costs of entry to forex market. In their model noise traders refrain from entering forex market in case of fixed rates because they cannot make enough profit from speculation in a low volatility environment. Then, given that a larger fraction of traders make correct expectations of returns from speculation, the volatility is low.

are between individuals from the same country, other meetings are between individuals from different countries. Given decentralized trade, standard assumptions imply that fiat money has value. We allow each country to issue its own currency and impose legal restrictions, which force producers in each country to use that country's money. The latter implies that in meetings between consumers and producers from different countries the former must exchange their currency in order to trade. Those who want to exchange currency meet in a special place, called currency exchange, where they can exchange their money. The sequence of actions in a period is the following. First, individuals observe the period division of the gains from trade (the sunspot). Then, there are meetings; consumers in meetings with foreigners visit currency exchange where they swap their money. After all trade in the currency exchange is complete, goods are produced and traded for money, consumers consume, and the next period begins.

We assume different division of the gains from trade in different kinds of meetings. We distinguish meetings by citizenship of matched individuals and by their money holdings. We require that in meetings where goods are traded for money, trades satisfy ex post individual rationality;[7] we assume a different way to divide the gains from trade in money-for-money trades in the currency exchange. In doing so we are motivated by observation that relative prices account for most of exchange rate volatility.[8] Therefore, it seems important to loose ties between fundamentals and goods prices, yet

[7] Ravikumar and Wallace (2001) and Wallace and Zhu (2003) require all trades be in pairwise core. Here, for the sake of tractability, we do not require efficiency; however all of our results apply if the core is imposed.

[8] See, e.g. Engel (1999).

to keep ties between goods prices and exchange rates. We do the latter by assuming bilateral bargaining in the currency exchange meetings.

Because goods prices comprise the value of money, uncertainty over division of the gains from trade causes fluctuations in the value of money.[9] These fluctuations affect the bargaining position of individuals in currency exchange meetings and, therefore, pass onto exchange rates. Because model fundamentals remain fixed, exchange rates are disconnected from fundamentals. Given zero volatility of fundamentals, an outside observer observes excess volatility of exchange rates even if price volatility is small. However, nominal exchange rate volatility can be much larger than underlying price volatility. In that case there is little difference in behavior of nominal and real exchange rates, so that the model is consistent with highly volatile real exchange rates. Fixing previously floating exchange rates results in a sharp decline of exchange-rate volatility. Search frictions imply that under fixed rates individuals agree to swap their money at a posted rate even if given that rate, they face an unfavorable division of the gains from trade. Provided that the uncertainty is not too big, individuals prefer to trade and consume on the spot rather than to refrain from trade and search again (which implies potentially long runs of consumption opportunities). Letting previously fixed exchange rates float has the opposite effect: when individuals trade one currency for the other, they take into account current realizations of gains from trade in goods markets, which boosts exchange rate volatility.

We think that a failure of volatility conservation displayed by our model is

[9] Uncertainty over division of the gains from trade is very similar to nominal price uncertainty, which is a standard ingredient of sunspot models of exchange-rate indeterminacy.

its important outcome; proponents of floating rates often argue that floating allows for flexible monetary policy and that having volatility in foreign exchange markets is better than having volatility in other parts of the economy, where it cannot be hedged. Such recommendations are often supported by models which display what has become known as Mundell's "incompatible trinity" — impossibility to have fixed exchange rates, free capital mobility, and independent monetary policy at the same time. Here, because volatility is not transferred elsewhere but simply vanquished, fixed exchange rates carry no impediments to the conduct of policy.[10]

Because we want to obtain closed-form solutions, we present a version with indivisible currencies and legal restrictions. As is by now standard in models with indivisible currencies, we introduce lotteries over the transfers of money as a way to approximate divisibility.[11] However, we are confident that all our findings extend into environment with perfectly divisible money and no legal restrictions. Partly, this can be guessed from the work of Molico (1997), who has computed steady-states in case of (almost) perfectly divisible money; partly, a similar conjecture comes from Trejos and Wright (2001), who have shown that local circulation of multiple currencies can be endogenous. In a model with rich set of individual money holdings there are many different kinds of meetings, so that there is a lot of room for exchange-rate uncertainty. Roughly, indivisibility makes the value function be linear and

[10] That, of course, is subject to incentive constraints; in case of a large shock to fundamentals a previously posted rate may no longer be sustainable in which case there is a need to devalue (revalue).

[11] See Berentsen, Molico, and Wright (2001) for the treatment of lotteries and further discussion.

discretizes the support of the distribution of money; because the uncertainty over division of the gains from trade shifts the entire value function, its shape (linear vs. strictly concave in case of divisible money) is not important.

The rest of the paper is organized as follows. In the next section we present the environment; section 3 provides exposition of fixed exchange rate regime; section 4 offers a discussion of floating exchange rate regime; section 5 concludes. All proofs and numerical examples are in appendix.

2 Environment

There are two countries A and B populated by a $[0,1]$-continuum of infinitely lived agents. Time is discrete and the horizon is infinite. In every period agents can produce and consume a subset of $2N$ non-storable perfectly divisible goods. An agent of type n can consume domestic good n and each of N types of foreign goods, and produce only domestic good $n+1$ (modulo N). We assume that $N \geq 3$, in which case all domestic meetings are at most single-coincidence-of-wants meetings. Each person maximizes expected discounted utility with discount parameter $\beta \in (0,1)$. Utility in a period is given by $u(x) - y$, where x denotes consumption and y denotes production of an individual. We assume that $u'(x) > 0$, $u''(x) < 0$ for all $x > 0$, that $u(0) = 0$, and that there exists $\widehat{x} > 0$ such that $u(\widehat{x}) = \widehat{x}$.

There are two indivisible fiat currencies A and B. Money A is a legal tender in country A and money B is a legal tender in country B. Initially, a fraction $m_i \in (0,1)$ of the population in the country i is endowed with one unit of money i. Both monies cannot be consumed or produced by any

private individual (they can only be used as media of exchange). Agents with money can buy consumption goods (they will be referred to as buyers), while agents without money can only produce goods (they will be referred to as sellers).

In every period each agent meets another person at random. When buyers and sellers meet in single-coincidence meetings,[12] they trade at prices, which may vary across meetings.[13] Because consumption goods are perfectly divisible, it is convenient to normalize prices such that the buyer trades the entire stock of her money (that is, given indivisibility of money, money changes hands with probability one).

We assume that all trades in goods markets satisfy ex post individual rationality. In every single coincidence meeting a pair faces a randomly drawn price to which both buyer and seller say either yes or no. If both say yes, then the seller produces consumption good in amount worth a unit of buyer's money, both good and money change hands, and the two individuals part; if at least one says no, then nothing happens in a meeting which then dissolves. Ex post individual rationality implies that the two individuals agree to trade if, given price draw, the gains from trade are nonnegative. Let p_i^z be the notation for prices in meetings of buyers and sellers from country i, and let p_{-i}^z denote prices in meetings of buyers from country i and sellers from country $-i$; z is the identity of a meeting.

Because one of our goals here is to address close similarity in behavior

[12] Notice that sometimes we shall refer single-coincidence meetings in which buyers like goods produced by sellers as trade meetings.

[13] As we already said in the introduction, one meaning of randomness is to emphasize that individuals experience various production and consumption opportunities.

of nominal and real exchange rates, we assume substantial coordination of prices across meetings; price coordination can be achieved via an extraneous sunspot variable. Specifically, we assume that at any given date sellers from country i trade with buyers (from both countries) at a uniform price p_i; thus, we omit superscript z in the notation of prices. As regards time-series price changes, we assume that pairs of prices $\mathbf{p} \equiv (p_A, p_B)$ are independently drawn from a given distribution μ; both the drawings and the distribution are common knowledge.

In every period, a measure $\gamma \in [0, \overline{\gamma}]$, $\overline{\gamma} < \min(m_A, m_B)$, of citizens of each country meet foreigners. Without any changes to substance, we assume that these are domestic buyers. Because domestic money cannot be used as payment in foreign country, those who want to trade with foreigners must obtain foreign money before making their purchase.

There are several interpretations of international trade arrangements consistent with our results; one of such interpretations is tourism: individuals (costlessly) travel abroad in order to consume foreign goods. We proceed with that interpretation for the rest of the paper.

Tourists can exchange their money in a special place on the border, which we refer to as currency exchange. As regards trade in the currency exchange, we assume that exchange rates are determined as outcomes of a bilateral trade mechanism. That is, all individuals, who want to exchange currency at a given date, are matched in pairs at random; after that, given endowments and realization $\mathbf{p}$ of prices, the mechanism specifies post-trade allocation of money holdings for each individual in every currency exchange meeting.

Because money is indivisible, post-trade money holdings of any individual are in discrete set $\mathbb{S} \equiv \{0, A, B, AB\}$; thus, any sensible mechanism should involve randomization over money, or lotteries.[14] The purpose of a lottery is to determine who of the two individuals in a meeting gets what combination of money from $\mathbb{S}$ subject to the constraint that no money is either created, or destroyed, or hoarded. Without changes to substance, we limit ourselves to lotteries, which are consistent with independent transfers of money in currency exchange meetings; all such lotteries are described by pairs of real numbers between zero and one: $\boldsymbol{\tau} \equiv (\tau_A, \tau_B)$, where τ_i is the probability that money i changes hands.

Here we concentrate on two different mechanisms, which correspond to fixed and floating exchange rate regimes. Under fixed exchange rate regime a lottery $\boldsymbol{\tau}$ is prescribed (by a third party) to all individuals in the currency exchange, under floating exchange rate regime each pair bargains about lotteries prior to allocating their money in accordance with agreed upon lottery $\boldsymbol{\tau}$. We assume that all currency swaps are ex ante voluntary; prior to drawing an allocation from a lottery, any individual can say no to that lottery and quit currency exchange with what she brought in. However, if both of the two individuals in a meeting say yes to a lottery, then they go along with any allocation provided by that lottery even if that allocation is ex post unfavorable.

Once tourists exchange their money, they enter foreign country, where they meet (foreign) sellers with probability one. Because (post-trade) allo-

[14] See Berentsen, Molico, and Wright (2002) for a treatment of lotteries in a random matching model of money.

cation of currencies in currency-exchange meetings are outcomes of lotteries over set $\mathbb{S}$, some individuals may enter foreign country having no money. Such individuals can neither consume (because they have no money to spend) nor produce (because we assume that foreigners are not allowed to work).

The timing of events is the following. The first thing an individual learns in a period is whether she travels abroad (meets foreigner) or stays in home country (meets domestic citizen) and the prices $\mathbf{p}$. Then, those who travel exchange their money, enter foreign country, buy foreign goods at price p_{-i}, consume, return to their home country, and move to the next period. Those who stay at home, meet domestic citizens, trade at price p_i (provided that their meeting is a trade meeting), consume, and move to the next period.

3 Fixed exchange rate regime: the value of money

We start with the exposition of the fixed exchange rate regime. As we already said, the distinction between fixed and floating exchange rate regimes is the distinction between who (and how often) sets up lotteries available to individuals in the currency exchange. Under fixed exchange rate regime there is a third party (the government), whose job is to pick some lottery and prevent individuals from using other lotteries in the currency exchange; under floating exchange rate regime such a party is absent, so that individuals bargain about lotteries and then follow an allocation provided by the agreed upon lottery.

Given a lottery $\boldsymbol{\tau}$ and the distribution μ of prices, the value of money

is expected discounted utility from future consumption. We assume that in the end of every period all of the stock of money i is held in country i. Then, in the next period, citizens of country i carry a measure $\gamma(1-\tau_i)$ of money i to country $-i$ as a result of currency allocation implied by the lottery $\boldsymbol{\tau}$. By the end of that period all of these people return to their home country and, because they cannot spend domestic money abroad, all of that money goes back to country i, so that all-domestic-money-in-domestic-country distribution of money is a stationary distribution. Given that end-of-period distribution, let $V_{k\,i}$ be the end-of-period value of having k, $k \in \{0, i\}$, units of money i by a citizen of country i, and let $\mathbf{V}_i \equiv (V_{0\,i}\; V_{i\,i})$. The value function $\mathbf{V}_i$, which we refer to as the *future value* of money i, is a solution to the following two-equation system of Bellman equations:

$$\mathbf{V}_i' = \beta\left[\mathbf{Q}_i' + T_i\mathbf{V}_i'\right], \tag{1}$$

where $\mathbf{Q}_i$ is the vector of next-period expected gains from trade and T_i is the transition matrix for money holdings.

Because sellers never travel abroad, the expected payoff of a seller is:

$$-\left(\psi_i + \gamma\tau_i\right)\int \frac{1}{p_i}d\mu,$$

where $\psi_i \equiv \frac{m_i-\gamma}{N}$ is the probability to meet an appropriate domestic buyer and $\gamma\tau_i$ is the probability to meet an appropriate foreign buyer. If a buyer stays at home, then her expected payoff is:

$$\varphi_i \int u\left(\frac{1}{p_i}\right)d\mu,$$

where $\varphi_i \equiv \frac{1-m_i}{N}$ is the probability to meet an appropriate seller. If a buyer goes to a foreign country, then her expected payoff is:

$$\tau_{-i} \int u\left(\frac{1}{p_{-i}}\right) d\mu,$$

which is simply the expected utility of consumption abroad times the probability of getting foreign money in the currency exchange. Then,

$$\mathbf{Q}_i' = \begin{bmatrix} -\left(\psi_i + \gamma\tau_i\right) \int \frac{1}{p_i} d\mu \\ \left(1 - \frac{\gamma}{m_i}\right) \varphi_i \int u\left(\frac{1}{p_i}\right) d\mu + \tau_{-i} \frac{\gamma}{m_i} \int u\left(\frac{1}{p_{-i}}\right) d\mu \end{bmatrix}, \tag{2}$$

where $\frac{\gamma}{m_i}$ is the probability that domestic buyer travels abroad.

We require that in equilibrium there is a trade in every trade meeting; given that requirement, the transition matrix T_i is:

$$T_i = \begin{bmatrix} 1 - \left(\psi_i + \gamma\tau_i\right) & \psi_i + \gamma\tau_i \\ \left(1 - \frac{\gamma}{m_i}\right) \varphi_i + \frac{\gamma}{m_i}\tau_i & \left(1 - \frac{\gamma}{m_i}\right)(1 - \varphi_i) + \frac{\gamma}{m_i}(1 - \tau_i) \end{bmatrix}. \tag{3}$$

Notice that because foreign money (if acquired) is spent with probability one within the same period, the probability of its acquisition, τ_{-i}, does not appear in the period-to-period transition matrix T_i; acquisition of foreign money yields an instant payoff

$$\tau_{-i} \int u\left(\frac{1}{p_{-i}}\right) d\mu,$$

which shows up in the vector of one-period expected gains from trade, $\mathbf{Q}_i$.

Because the mapping $G(\mathbf{x}') = \beta\left[\mathbf{Q}_i' + T_i \mathbf{x}_i'\right]$ is a contraction, system (1) has a unique solution,

$$\mathbf{V}_i' = \left(\frac{1}{\beta} I - T_i\right)^{-1} \mathbf{Q}_i', \tag{4}$$

where I is a 2×2 identity matrix.

The future value of money $\mathbf{V}_i$ is the continuation value of using money i as a medium of exchange. However, our sequence of events implies that agents trade knowing both whether they travel or stay at home and current prices. Consequently, their trade depends on the *current value* rather than future value of money. Given domestic price level p_i, let $v_{k\,i\,i}(p_i)$ denote the value of k, $k \in \{0, i\}$, units of money i held by domestic citizen, who stays at home. Then, the current value of a unit of money i held by such an individual, $\mathbf{v}_{i\,i}(p_i)$, satisfies:

$$\mathbf{v}'_{i\,i}(p_i) = \beta \left[\mathbf{R}'_{i\,i}(p_i) + T_{i\,i}\mathbf{V}'_i\right], \tag{5}$$

where $\mathbf{R}_{i\,i}(p_i)$ is the vector of one-period gains from trade and $T_{i\,i}$ is the transition matrix for money holdings:

$$\mathbf{R}_{i\,i}(p_i) = \left[-\left(\psi_i + \gamma\tau_i\right)\frac{1}{p_i} \quad \varphi_i u\left(\frac{1}{p_i}\right)\right] \tag{6}$$

and

$$T_{i\,i} = \begin{bmatrix} 1 - (\psi_i + \gamma\tau_i) & \psi_i + \gamma\tau_i \\ \varphi_i & 1 - \varphi_i \end{bmatrix}. \tag{7}$$

Likewise, the current value, $\mathbf{v}_{i\,-i}(p_{-i})$, of a unit of money i held by domestic citizen, who travels abroad, is the solution to:

$$\mathbf{v}'_{i\,-i}(p_{-i}) = \beta \left[\mathbf{R}'_{i\,-i}(p_{-i}) + T_{i\,-i}\mathbf{V}'_i\right],$$

where

$$\mathbf{R}_{i\,-i}(p_{-i}) = \left[0 \quad \tau_{-i} u\left(\frac{1}{p_{-i}}\right)\right] \tag{8}$$

and

$$T_{i\,-i} = \begin{bmatrix} 1 & 0 \\ \tau_i & 1-\tau_i \end{bmatrix}. \tag{9}$$

The matrix $T_{i\,-i}$ is the transition matrix associated with the transfer of money in currency exchange.

An individual is willing to trade if trading is not worse than doing nothing in a meeting. For those, who stay at home, the latter yields two incentive compatibility constraints:

$$\mathbf{v}_{i\,i}(p_i) \geq \beta \mathbf{V}_i,$$

which simplify to a familiar double inequality:

$$\frac{1}{p_i} \leq V_{i\,i} - V_{0\,i} \leq u\left(\frac{1}{p_i}\right). \tag{10}$$

The meaning of (10) is that quantity $q_i \equiv \frac{1}{p_i}$, produced in domestic meetings, must be in some middle range, so that sellers are willing to produce that quantity in exchange for money and buyers are willing to part with money in exchange for that quantity of consumption good. The incentive compatibility constraints for those who travel abroad are:

$$\mathbf{v}_{i\,-i}(p_{-i}) \geq \beta \mathbf{V}_i,$$

which yields one inequality:

$$\tau_{-i} u\left(\frac{1}{p_{-i}}\right) - \tau_i \left(V_{i\,i} - V_{0\,i}\right) \geq 0, \tag{11}$$

the meaning of which is that the expected gain from currency exchange is nonnegative.

We can now give the following definition.

Definition 1. *An allocation* $(\mu, \boldsymbol{\tau})$ *is said to be incentive feasible if, given the distribution* μ *and the lottery* $\boldsymbol{\tau}$*, the implied value function* $\mathbf{V}_i$ *satisfies all incentive compatibility constraints in (10) and (11) for all* $\mathbf{p}$ *in the support of* μ*. We say that an allocation* $(\mu, \boldsymbol{\tau})$ *is interior if it is incentive feasible and for all* $\mathbf{p}$ *in the support of* μ *all incentive compatibility constraints in (10) and (11) are slack.*

Because we require all incentive compatibility constraints hold for any price draw $\mathbf{p}$ in the support of μ, Definition 1 is consistent with ex post individual rationality in goods-for-money trade meetings, but not in currency exchange meetings. In currency exchange meetings individuals go along even with ex post unfavorable allocations of money implied by the lottery $\boldsymbol{\tau}$. The requirement we impose on incentive feasible lotteries is ex ante individual rationality; because money is indivisible, that weaker requirement is necessary for interior lotteries to emerge in equilibrium (see Berentsen, Molico, and Wright, 2002, for further details).

4 Floating exchange rate regime

Under floating exchange rate regime there is no government to set up lotteries in the currency exchange; this task is performed by individuals. We assume that before individuals "throw the dice" and follow the supplied post-trade allocation of money, they engage in bilateral bargaining about the lottery to which they set up "the dice". Once an agreement in bargaining is reached, the pair sets up the agreed upon lottery, draws an outcome, and follows

the provided allocation. The bargaining game we adopt here is a version of the alternating-offer bargaining game of Rubinstein (1982) and Trejos and Wright (1995). An important difference of our game is that waiting changes outside options of the bargaining parties. One consequence of that is that the outcome of the alternating-offer bargaining does not coincide with the (generalized) Nash bargaining solution as it does in the static game. Bargaining solution when the outside option is changing, is considered in Ennis (2001). Because here price draws $\mathbf{p}$ are *i.i.d.*, our problem is simpler than that studied by Ennis; however, that relative simplicity allows us to increase the state space of the problem and to consider uncertainty over continuous sets of prices.

4.1 The bargaining game

The game is the following. Individuals make offers about the lotteries; each of the two individuals in a meeting is equally likely to make an offer. The partner can either accept an offer, or reject an offer but stay for the next bargaining round, or quit bargaining. Acceptance of an offer ends bargaining; the two individuals draw an outcome from agreed upon lottery and follow the provided allocation of money. Rejection of an offer implies no action in a meeting; the pair simply proceeds to the next bargaining round. Quitting ends bargaining; no action is taken and the pair quits currency exchange. We introduce an option to quit into an otherwise standard bargaining model to make bargaining voluntary. Facing being locked in bargaining for a long time, individuals may accept unfavorable offers in order to quit currency exchange

and continue with their options in home country; quitting permits individuals to reject extreme prices and keep their money to buy consumption at home.

We allow for many bargaining rounds and draws of prices per period. For any pair of individuals engaged in bargaining, each round starts with a new draw of prices $\mathbf{p} \equiv (p_A, p_B)$. The waiting time between two bargaining rounds is $\Delta > 0$. Thus, a pair can bargain for a long time, which may span over many periods. Individuals, who leave currency exchange, trade with foreign sellers at current prices $\mathbf{p}$ (subject to legal restrictions). Recall, that those, who trade goods for money, do not bargain; in goods-for-money trades the pair goes along with any ex post individually rational outcome.

Because larger waiting time increases discount between two consecutive rounds, Δ affects the value of waiting and, hence, the outcome of the bargaining game. For the sake of exposition we describe the bargaining game for arbitrary Δ, but later consider the limiting case of $\Delta \to 0$, which is more convenient analytically.

We start with an infinite waiting time Δ, i.e. with a one-shot bargaining in the currency exchange. Given $\mathbf{p}$ and the future value of money $\mathbf{V}_i$, those who bargain over lotteries, seek to maximize the (net) current value of money they have:

$$\tau_{-i} u\left(\frac{1}{p_{-i}}\right) - \tau_i \left(V_{i\,i} - V_{0\,i}\right). \tag{12}$$

Because the objective is linear, for any given value of money there are many lotteries, among which individuals are indifferent. That is why making offers about pairs (τ_A, τ_B) yields a multiplicity of solutions. To circumvent the problem it is convenient to assume that agents bargain about the current

value of money implied by lotteries (as opposed to bargaining about lotteries) and accept any lottery which yields the desired payoff. Given payoff π_{-i}, which a foreigner claims for herself, the lottery, which yields the highest payoff to a citizen of country i, is a solution to:

$$\Xi_i(\pi_{-i};\mathbf{p}) \equiv \max_{\tau}\left[\tau_{-i}u\left(\frac{1}{p_{-i}}\right) - \tau_i\left(V_{i\,i} - V_{0\,i}\right)\right]$$

subject to

$$\tau_i u\left(\frac{1}{p_i}\right) - \tau_{-i}\left(V_{-i\,-i} - V_{0\,-i}\right) = \pi_{-i}.$$

The payoff to agent i is, then,

$$\Xi_i(\pi_{-i};\mathbf{p}) = \begin{cases} u\left(\frac{1}{p_{-i}}\right) - \left(V_{i\,i} - V_{0\,i}\right)\frac{\pi_{-i}+V_{-i\,-i}-V_{0\,-i}}{u\left(\frac{1}{p_i}\right)} & \text{if } 0 \leq \pi_{-i} \leq \pi^*_{-i} \\ u\left(\frac{1}{p_{-i}}\right)\frac{u\left(\frac{1}{p_i}\right)-\pi_{-i}}{V_{-i\,-i}-V_{0\,-i}} - \left(V_{i\,i} - V_{0\,i}\right) & \text{if } \pi^*_{-i} \leq \pi_{-i} \leq \overline{\pi}_{-i} \end{cases}, \tag{13}$$

where

$$\pi^*_{-i} = u\left(\frac{1}{p_i}\right) - \left(V_{-i\,-i} - V_{0\,-i}\right),$$

and $\overline{\pi}_{-i}$ is maximum payoff of the opponent agent i goes along with. The function $\Xi_i(\pi_{-i};\mathbf{p})$ is the reaction function of agent i; it is a piecewise-linear, concave, and strictly decreasing function. Because in case of an infinite waiting time Δ there is no option to continue bargaining in the next round, the best strategy of a person who makes an offer is to appropriate all of the surplus by making her partner be indifferent between acceptance and quitting. This means that such a person will claim the maximum payoff $\overline{\pi}_{-i}$, which is a unique solution to:

$$\Xi_i(\pi_{-i};\mathbf{p}) = 0. \tag{14}$$

Because we require equilibrium exchange rates be consistent with willingness to trade in all markets, (10) implies that the payoff in (14) is non-negative, so that an agreement is reached immediately. That is not surprising given the alternative of being stuck forever in the currency exchange. As waiting time decreases and the cost of waiting declines, individuals who face an unfavorable price draw may want to skip one round to gain better terms. Intuitively, if waiting is costless, then individuals should reject payoffs which are less than the average implied by the distribution μ.

Let us consider the case of a finite Δ. Given the pair of reaction functions (one for each of the two bargaining partners), one can derive the optimal strategies of players. Let $Z_i(\Delta)$ denote the continuation value of the bargaining game for a citizen of country i, and let $\delta_{-i}(\pi_i; \mathbf{p}, \Delta)$ be the probability that the opponent accepts π_i and $\phi_{-i}(\pi_i; \mathbf{p}, \Delta)$ be the probability that the opponent quits bargaining. The Bellman equation of citizen i when it is her turn to make an offer is:

$$W_i(\mathbf{p}; \Delta) = \max_{\pi_i} \left[\delta_{-i}(\pi_i + V_{ii}) + \phi_{-i} V_{ii} + (1 - \delta_{-i} - \phi_{-i}) \frac{Z_i(\Delta)}{1 + r\Delta} \right]; \quad (15)$$

the Bellman equation of the citizen of country i when it is her turn to respond to an offer is:

$$w_i(\mathbf{p}; \Delta) = \max \left[\Xi_i(\pi_{-i}; \mathbf{p}) + V_{ii},\ V_{ii},\ \frac{Z_i(\Delta)}{1 + r\Delta} \right], \quad (16)$$

where $r \equiv \frac{1}{\beta} - 1$ is a period discount rate.

Here the optimal decision of agent i is to accept offers π_{-i} below some threshold and to reject all offers above that threshold. Given that the opponent uses the same kind of strategies, the best choice of agent i in (15) is

to offer π_i, which makes the opponent indifferent between acceptance of that offer and the best of the two remaining alternatives. Thus, given $Z_i(\Delta)$, the solution (π_A, π_B) to the bargaining game satisfies the following two-equation system:

$$\Xi_i(\pi_{-i}; \mathbf{p}) + V_{i\,i} = \max\left[V_{i\,i},\ \frac{Z_i(\Delta)}{1 + r\Delta}\right].$$

Because both reaction functions $\Xi_i(\pi_{-i}; \mathbf{p})$ are strictly decreasing, the solution is unique. To find the solution, we are only left to compute $Z_i(\Delta)$, the continuation value of the bargaining game.

4.2 The continuation value of bargaining

The continuation value $Z_i(\Delta)$ is expected discounted stream of payoffs from future offers. Because individuals do not observe future prices, but only know that these prices are independently drawn from a known distribution, the continuation value $Z_i(\Delta)$ is the value of bargaining game where agents maximize expected value of money. Because future price draws are identically and independently distributed, waiting does not change outside options of the bargaining parties, so that the continuation game is a static game. A one-shot objective of an individual is:

$$\widehat{\Xi}_i(\pi_{-i}) \equiv \max_{\tau}\left[\tau_{-i}\int u\left(\frac{1}{p_{-i}}\right)d\mu - \tau_i\left(V_{i\,i} - V_{0\,i}\right)\right], \tag{17}$$

subject to:

$$\tau_i\int u\left(\frac{1}{p_i}\right)d\mu - \tau_{-i}\left(V_{-i\,-i} - V_{0\,-i}\right) = \pi_{-i},$$

which yields the reaction function $\widehat{\Xi}_i(\pi_{-i})$, which differs from (13) only in that the current prices are replaced by their expectation,

$$\widehat{\Xi}_i(\pi_{-i}) = \begin{cases} \int u\left(\frac{1}{p_{-i}}\right) d\mu - (V_{i\,i} - V_{0\,i}) \frac{\pi_{-i}+V_{-i\,-i}-V_{0\,-i}}{\int u\left(\frac{1}{p_i}\right) d\mu} & \text{if } 0 \leq \pi_{-i} \leq \widehat{\pi}^*_{-i} \\ \int u\left(\frac{1}{p_{-i}}\right) d\mu \frac{\int u\left(\frac{1}{p_i}\right) d\mu - \pi_{-i}}{V_{-i\,-i}-V_{0\,-i}} - (V_{i\,i} - V_{0\,i}) & \text{if } \widehat{\pi}^*_{-i} \leq \pi_{-i} \end{cases},$$

where $\widehat{\pi}^*_{-i} \equiv \int u\left(\frac{1}{p_i}\right) d\mu - (V_{-i\,-i} - V_{0\,-i})$.

Because each individual in a meeting is equally likely to make an offer, the continuation value $Z_i(\Delta)$ is

$$Z_i(\Delta) = \frac{1}{2}\left[W_i(\Delta) + w_i(\Delta)\right],$$

where $W_i(\Delta)$ and $w_i(\Delta)$ solve the following system of Bellman equations:

$$W_i(\Delta) = \max_{\pi_i}\left[\delta_{-i}\left(\pi_i + V_{i\,i}\right) + \phi_{-i}V_{i\,i} + (1-\delta_{-i}-\phi_{-i})\frac{Z_i(\Delta)}{1+r\Delta}\right] \tag{18}$$

and

$$w_i(\Delta) = \max\left[\widehat{\Xi}_i(\pi_{-i}) + V_{i\,i},\ V_{i\,i},\ \frac{Z_i(\Delta)}{1+r\Delta}\right]. \tag{19}$$

As above, the optimal strategy of a person whose turn is to make an offer is to make the opponent be indifferent between acceptance and the best among rejection and quitting, which yields:

$$w_i(\Delta) = \widehat{\Xi}_i(\pi_{-i}) + V_{i\,i} = \max\left[V_{i\,i},\ \frac{Z_i(\Delta)}{1+r\Delta}\right].$$

We consider equilibria in which individuals never quit bargaining.[15] No-quitting implies that the continuation value of the game must be at least as

[15] Equilibria in which individuals quit bargaining in the continuation game are equilibria of the one-shot game.

large as the value of quitting,

$$\frac{Z_i(\Delta)}{1+r\Delta} \geq V_{ii}. \tag{20}$$

Provided that conditions (20) are satisfied, the continuation game has many Nash equilibria. However, one can show (by application of the implicit function theorem) that threats to delay agreement by those who respond to offers, reduce their payoffs and, hence, are not credible. In other words, the only subgame perfect equilibrium here is the one, where agreement is immediate, which is a standard result for that class of games.[16]

Letting Δ approach zero, after straightforward manipulations we obtain:

$$\pi_i = \widehat{\Xi}_i(\pi_{-i}), \tag{21}$$

so that payoffs of the continuation game are determined as the intersection of the two reaction functions, $\widehat{\Xi}_A$ and $\widehat{\Xi}_B$. Then, explicit solution to the (two-equation) system (21) yields:

$$Z_i = \int u\left(\frac{1}{p_{-i}}\right) d\mu + V_{0\,i}, \tag{22}$$

which, as one would expect, is the expected utility from consumption abroad plus the value of being a producer in domestic country. Given (22), conditions (20) imply:

$$\int u\left(\frac{1}{p_{-i}}\right) d\mu \geq V_{ii} - V_{0\,i}, \tag{23}$$

so that individuals choose to bargain if expected utility of foreign consumption exceeds the benefit of keeping domestic money.

[16] See Rubinstein (1982) for further discussion.

4.3 Equilibria

We now turn to the description of equilibria. Suppose that in a currency exchange meeting of domestic citizen i and a foreign citizen $-i$, it is the turn of domestic citizen to make an offer. Without any changes to substance we assume for the rest of this section that in all meetings citizens of only one randomly drawn country make offers in a given bargaining round, which allows to avoid the inconvenience of having to deal with two distinct lotteries (and hence exchange rates) being offered at the same time. Given the continuation value of the foreigner and the price draw $\mathbf{p}$, the payoff domestic citizen attains for herself, $\pi_i(\mathbf{p})$, is a unique solution to:

$$\Xi_{-i}(\pi_i; \mathbf{p}) = \int u\left(\frac{1}{p_i}\right) d\mu - \left(V_{-i\,-i} - V_{0\,-i}\right). \tag{24}$$

Solving (24) for the payoff $\pi_i(\mathbf{p})$, one obtains:

$$\pi_i(\mathbf{p}) = \begin{cases} \pi_i^* - u\left(\frac{1}{p_{-i}}\right) \frac{\int u\left(\frac{1}{p_i}\right) d\mu - u\left(\frac{1}{p_i}\right)}{V_{-i\,-i} - V_{0\,-i}} & \text{if } \int u\left(\frac{1}{p_i}\right) d\mu \geq u\left(\frac{1}{p_i}\right) \\ \pi_i^* - \left(V_{i\,i} - V_{0\,i}\right) \frac{\int u\left(\frac{1}{p_i}\right) d\mu - u\left(\frac{1}{p_i}\right)}{u\left(\frac{1}{p_i}\right)} & \text{if } \int u\left(\frac{1}{p_i}\right) d\mu \leq u\left(\frac{1}{p_i}\right) \end{cases}. \tag{25}$$

Given current prices $\mathbf{p}$, the payoff $\pi_i(\mathbf{p})$ has to be at least as large as the continuation value of bargaining Z_i, otherwise an individual i will wait for the next bargaining round. If the price p_i is high, i.e.

$$\int u\left(\frac{1}{p_i}\right) d\mu \geq u\left(\frac{1}{p_i}\right), \tag{26}$$

then the current value of money i is low, which means that it is easier to part with that money. In that case provided that prices $\mathbf{p}$ satisfy:

$$1 - \frac{\int u\left(\frac{1}{p_i}\right) d\mu - u\left(\frac{1}{p_i}\right)}{V_{-i\,-i} - V_{0\,-i}} \geq \frac{\int u\left(\frac{1}{p_{-i}}\right) d\mu}{u\left(\frac{1}{p_{-i}}\right)}, \tag{27}$$

agent i demands the following lottery:

$$\tau_i = 1, \tag{28}$$

and

$$\tau_{-i} = 1 - \frac{\int u\left(\frac{1}{p_i}\right) d\mu - u\left(\frac{1}{p_i}\right)}{V_{-i\,-i} - V_{0\,-i}}, \tag{29}$$

so that she gives out her money for sure and gets money $-i$ with probability less than one. Condition (27) implies that if both prices p_A and p_B are above average, then an agreement is not reached and the bargaining parties wait for the next round which is not surprising given zero costs of haggling.

If the current price p_i is low, i.e.

$$\int u\left(\frac{1}{p_i}\right) d\mu \leq u\left(\frac{1}{p_i}\right), \tag{30}$$

then provided that prices $\mathbf{p}$ satisfy:

$$\frac{\int u\left(\frac{1}{p_i}\right) d\mu}{u\left(\frac{1}{p_i}\right)} \leq 1 - \frac{\int u\left(\frac{1}{p_{-i}}\right) d\mu - u\left(\frac{1}{p_{-i}}\right)}{V_{i\,i} - V_{0\,i}}, \tag{31}$$

the citizen of country i claims money $-i$ and agrees to surrender her money with probability less than one:

$$\tau_{-i} = 1, \tag{32}$$

and

$$\tau_i = \frac{\int u\left(\frac{1}{p_i}\right) d\mu}{u\left(\frac{1}{p_i}\right)}. \tag{33}$$

In turn, condition (31) implies that if both prices p_A and p_B are below average, then an agreement is reached immediately. If prices countermove, then given expressions (29) and (33), condition (27) means that individual i (who makes an offer) gets at least as much of foreign money as (given the same pair of prices $\mathbf{p}$) she would get had it been the turn of her opponent $-i$ to make an offer. Similarly, condition (31) means that agent i gives out no more of her money than she would give out had it been the turn of her opponent to make an offer. In both cases the meaning of inequalities (27) and (31) is that an agreement is reached if and only if there is no benefit of giving the opponent a chance to make an offer by waiting for the next bargaining round.

We say that a lottery $\boldsymbol{\tau}$ is a *solution of the bargaining problem* if, given measure μ, current prices $\mathbf{p}$, and the future value of money $\mathbf{V}$, conditions (23) hold and for some choice i of the party who makes an offer, prices $\mathbf{p}$ and lottery $\boldsymbol{\tau}$ satisfy either (26)-(28) or (30)-(33). Let S_μ be the set of prices $\mathbf{p}$ consistent with bargaining solution:

$$S_\mu = \{\mathbf{p} : \mathbf{p} \in \operatorname{supp} \mu \text{ and } (27) \text{ and } (31) \text{ hold}\}.$$

Without loss of generality we can consider measures μ which are consistent with a positive probability of an agreement being reached during one bargaining round, i.e. measures μ such that $\mu(S_\mu) > 0$.[17] We now give the

[17] That condition holds e.g. if prices p_A and p_B are independent of each other.

definition of a stationary equilibrium.

Definition 2. *A pair, price draw* $\mathbf{p}$ *and allocation* $(\mu, \boldsymbol{\tau})$ *, is said to be an equilibrium point if i) given* $(\mu, \boldsymbol{\tau})$ *the value function* $\mathbf{V}$ *is given by (4) and* $(\mu, \boldsymbol{\tau})$ *is incentive feasible, ii) given* μ*,* $\mathbf{V}$ *and* $\mathbf{p}$*, the lottery* $\boldsymbol{\tau}$ *is a solution of the bargaining problem. We say that measure* μ *is a stationary equilibrium if for every* $\mathbf{p}$ *in the set* S_μ*, there exists a lottery* $\boldsymbol{\tau}$ *such that* $\langle \mathbf{p}, (\mu, \boldsymbol{\tau}) \rangle$ *is an equilibrium point.*

Before we turn to existence, we find it useful to discuss the case of fixed prices. Fixed price assumption means that measure μ is degenerate. In that case there is no discrepancy between the current and the future value of money and between the current and the continuation value of the bargaining game. Given zero substitutability between the two currencies, that implies that those in the currency exchange swap their money holdings one-to-one. Intuitively, because foreign currency is useless in domestic country, one-to-one swap is the best deal provided that individuals do not quit, which is the case when the two currencies have sufficiently close purchasing power.

In other words, under fixed prices any incentive feasible allocation, consistent with a unit exchange rate, is an equilibrium point and, because the support of μ is a singleton, is a stationary equilibrium. That, as well as continuity of the bargaining solution and of the value of money, allows us to develop a local existence argument based on application of the Brouwer's fixed point theorem. By saying that our argument is local we mean that it asserts existence of a non-degenerate measure μ with sufficiently small support,

which satisfies our definition of stationary equilibrium.

Let δ denote a Dirac measure with support, $\operatorname{supp}\delta = \{\overline{\mathbf{p}}\}$, and let Υ be the notation for the unit lottery. The intuition behind the argument is the following. Let us fix some interior allocation (δ, Υ), called a *fixed-price allocation*, and consider a small compact convex neighborhood $\mathcal{U}$ of the unit lottery and a nondegenerate measure μ such that $\overline{\mathbf{p}}$ is the center of μ and the radius of μ is small. Then, for every lottery $\boldsymbol{\tau} \in \mathcal{U}$, an allocation $(\mu, \boldsymbol{\tau})$ is incentive feasible and no-quitting is satisfied. Next, given citizenship i of those who make offers in meetings, $i \in \{A, B\}$, define a map $\Psi^i_\mu(\boldsymbol{\tau}; \mathbf{p})$ from $\mathcal{U}$ into $[0,1]^2$ as follows. First, given price draw $\mathbf{p} \in S_\mu$, measure μ, and country i, take a lottery $\boldsymbol{\tau}$ from $\mathcal{U}$ and use (4) to compute the future value of money $\mathbf{V}$. Second, given $\mathbf{p}$, μ, and $\mathbf{V}$, use (28)-(33) to compute the solution to the bargaining problem and let that solution be the image of $\boldsymbol{\tau}$ under $\Psi^i_\mu(\boldsymbol{\tau}; \mathbf{p})$. Next, observe that if price variability is small (i.e. if the support of μ is small), then, by continuity, the image of $\boldsymbol{\tau}$ is close to Υ for every $\boldsymbol{\tau} \in \mathcal{U}$, which implies that the image of $\mathcal{U}$ under $\Psi^i_\mu(\boldsymbol{\tau}; \mathbf{p})$ is a proper subset of $\mathcal{U}$. Because $\Psi^i_\mu(\boldsymbol{\tau}; \mathbf{p})$ is continuous, Brouwer's theorem applies. Thus, for every measure μ with the center at $\overline{\mathbf{p}}$ and sufficiently small support and for every price draw $\mathbf{p}$ from the set S_μ there exists a lottery $\boldsymbol{\tau}$ such that $\langle \mathbf{p}, (\mu, \boldsymbol{\tau}) \rangle$ is an equilibrium point, that is Definition 2 is satisfied.

Proposition 1. *Let (δ, Υ) be an arbitrary interior fixed-price allocation. Then, there exists a nondegenerate measure μ with the center at $\overline{\mathbf{p}}$, where* $\operatorname{supp}\delta = \{\overline{\mathbf{p}}\}$, *which is a stationary equilibrium.*

Existence of nondegenerate equilibrium measures μ implies exchange-rate indeterminacy. Non-degeneracy of measure μ means that with some positive probability prices drawn from μ are not average prices. Then, bargaining solution (28)-(33) implies that the equilibrium lottery is not the unit lottery, so that equilibrium exchange rates are stochastic. Because all individuals face uniform draws of prices $\mathbf{p}$ at any particular date, the realized equilibrium exchange rate $\boldsymbol{\tau}$ is the same across all currency exchange meetings. That is, there is no cross-section difference in exchange rates; however, because at each date prices $\mathbf{p}$ are independently drawn from a nondegenerate distribution, there are time-series movements of exchange rates.

Although our result is local, exchange-rate indeterminacy need not be small; examples, where exchange-rate volatility exceeds CPI volatility many times are straightforward (see Appendix A). In that case the model predicts substantial volatility of nominal exchange rates vis-a-vis almost constant CPI. This means that there is little difference in time-series dynamics between nominal and real exchange rates; both of the two are disconnected from fundamentals and both are much more volatile then fundamentals, which is consistent with empirical evidence (see e.g. Obstfeld and Rogoff, 1996).

Given a non-degenerate equilibrium measure μ, replacement of that measure by the corresponding degenerate measure δ has no effects other than reduction in exchange-rate volatility (see Appendix A for discussion), which is consistent with observed empirical failure of volatility conservation (see e.g. Flood and Rose, 1995). That is, in case of exchange rate regime switch our model can display a sharp reduction in both nominal and real exchange-rate

volatility without noticeable changes to volatility of other macroeconomic variables, which may seem puzzling to an outside observer.

Because the volatility is not transferred elsewhere but simply vanquished, the latter implies that fixed exchange rates carry no impediment to the conduct of policy, in particular, monetary policy. The monetary authorities in both countries are free to adjust domestic money supplies keeping exchange rate unchanged. However, an increase (decrease) in the money supply in one country alters the future value of respective money, which provided that the shock is large, may be inconsistent with willingness of individuals to exchange currencies at a posted rate. In that case there is a need to devalue (revalue); our model is therefore consistent with conventional wisdom that fixed exchange rates are hard to maintain in face of large shocks to fundamentals. Because individuals cannot hedge exchange-rate risks, fixing previously floating exchange rate increases the amount of trade in the currency exchange and, hence, consumption and welfare. The latter, as well as (relative) policy freedom, implies that in our model fixed exchange rates perform at least as good as floating exchange rates (see King, Wallace, Weber, 1992, for a similar result), which we take more broadly as an argument in support of fixed exchange rates.

5 Concluding remarks

The message of existing models of exchange rates is that non-fundamental exchange-rate uncertainty is hard to get. Models, where such uncertainty is present, tend to assume things such as extreme substitutability among cur-

rencies, non-rational behavior of traders, or asymmetric information about fundamentals. In many models, which aim at explaining exchange-rate empirics, volatility of exchange rates is simply an amplified volatility of fundamentals.[18] Here we show that a combination of search, uncertainty in trade within meetings, and poor substitutability among currencies can explain many of empirical exchange-rate facts by non-fundamental sunspot fluctuations pertaining to the division of the gains from trade.

Our model is consistent with the following. Both nominal and real exchange rates are disconnected from fundamentals, both are more volatile than fundamentals, both can exhibit similar behavior, and both fail to satisfy conservation of volatility. Price uncertainty, which drives non-fundamental movements of exchange rates, translates to movements of both exchange rates and GDP in both countries (whereas model fundamentals remain constant); GDP is affected through what is often called the exchange rate channel. The effects are not commensurable; if the trade flow between the two countries is small, output volatility can be small vis-a-vis large exchange-rate volatility. Fixing a previously floating exchange rate decreases exchange-rate volatility with little change to output volatility and no change to volatility of model fundamentals. An outside observer may find such observations puzzling, leading to conclusions much similar to those spelled out by Flood and Rose (1995).[19]

[18] See Alessandria (2003), Shleifer and Summers (1990) and others, and Lyons (2001) and others.

[19] That paragraph (pp. 4-5) reads as follows:

> Empirically, we cannot find macroeconomic variables with volatility characteristics which mimic those of OECD exchange rates even approximately.

In the model we abstract from production and capital markets. We surmise that bringing capital into the model does not alter results provided that there are frictions in international capital markets similar to frictions in goods markets. The role of frictions in both markets is to break down arbitrage opportunities and thus disconnect exchange rates from fundamentals.

There are some exchange-rate facts, which we cannot explain here; in particular, our model cannot account for the systematic bias in currency forward rates and for observed persistence in exchange-rate time series. One way to deal with forward bias puzzle is to allow for contracts, which guarantee particular lotteries to be available at future dates. Because in our model exchange-rate volatility has a large non-fundamental component, spot prices of such contracts would include a premium for that risk, making model predictions be consistent with the evidence. The ability of individuals to hedge exchange-rate risks would also improve performance of floating exchange rates. However, we are confident that floating exchange rates cannot dominate fixed exchange rates even if individuals can fully hedge exchange-rate risks.

As regards persistence, exchange-rate uncertainty here is driven by shocks to current prices whereas the future value of money remains unchanged across meetings. A more intuitively appealing source of exchange-rate indeterminacy is uncertainty about the future value of money. Such a model would

> Intuitively, if exchange rate stability varies across regimes without corresponding variation in macroeconomic volatility, then macroeconomic variables will be unable to explain much exchange rate volatility. ... We are driven to the conclusion that the most critical determinants of exchange rate volatility are not macroeconomic.

be capable of generating sufficient persistence in exchange-rate time series (which is observed in the data), which we do not have because future prices are identically and independently distributed. Although a model, which builds upon uncertainty about future value of money, is easy to articulate, it is less easy to solve; in particular, our bargaining solution depends on future prices being *i.i.d.* Then, generalization of results in Ennis (2001) to the case of large state spaces seems to be an important step on that way.

6 Appendix A

Here we construct an example, which demonstrates that volatility of nominal exchange rates can be much larger than price volatility. Because volatility of nominal exchange rate is the sum of real exchange-rate volatility and price volatility, the latter implies that real exchange rates can be as volatile as nominal exchange rates. The example is also a useful illustration of model implications about exchange rate regime switch; we obtain that qualitative behavior of exchange rates implied by the model is very much similar to that documented by Flood and Rose (1995).

Let us assume that the two countries are identical and that trade flows between them are negligibly small, i.e. $\gamma = 0$. Assume that utility function $u(x)$ is $u(x) = x^{\alpha}$, where $0 < \alpha < 1$. Let $q_i \equiv \frac{1}{p_i}$ be the quantity produced by producers in country i given current price p_i. Assume that the cumulative distribution of quantities is a uniform distribution over a square support,

$$\operatorname{supp} \mu = \{(q_A, q_B) : q - d \leq q_A \leq q + d \text{ and } q - d \leq q_B \leq q + d\}$$

where q is the average quantity produced in both countries and d measures the range of price uncertainty. One can use (4) to show that given that distribution:

$$\int u\left(\frac{1}{p_i}\right) d\mu = q^{\alpha} + O(d), \tag{34}$$

$$\int \frac{1}{p_i} d\mu = q, \tag{35}$$

and

$$V_{ii} - V_{0i} = \frac{(1-m)q^{\alpha} + mq}{1 + rN} + O(d), \tag{36}$$

where m is money stock in both countries, r is a period discount rate, and $O(d)$ satisfies $\lim_{d \to 0} O(d) = 0$.

As a measure of relative volatility of exchange rates and prices we choose the ratio of maximum percentage deviations of the two variables from their mean values. For prices, the maximum deviation from the mean is:

$$\Delta_p = \frac{d}{q} + O(d^2).$$

To obtain the maximum deviation of exchange rates we use the bargaining solution (29), which after substitution of (34)-(36) yields:

$$\Delta_\tau = \frac{\alpha q^\alpha}{\frac{(1-m)q^\alpha + mq}{1+rN}} \frac{d}{q} + O(d^2).$$

The ratio $\frac{\Delta_\tau}{\Delta_p}$ is then:

$$\frac{\Delta_\tau}{\Delta_p} = (1 + rN) \frac{\alpha q^\alpha}{(1-m)q^\alpha + mq} + O(d). \tag{37}$$

To show that the relative volatility can be large, assume the following parameters: $u(x) = x^{0.5}$, $q = 0.002$, $m = 0.5$, and $rN = 10$. These parameters depict a world where half of the population is endowed with money and where there is a substantial specialization in production and consumption. As a consequence, individuals experience long runs of consumption opportunities, so that sustainable output is quite small. One can verify that an allocation described by these parameters is incentive feasible and satisfies (23) provided that price volatility is small enough in absolute value. Expression (37) evaluated at $d = 0$ yields $\frac{\Delta_\tau}{\Delta_p} \approx 10.5$, so that nominal exchange rate is by an order of magnitude more volatile than prices.

That is important because it implies that real exchange rate must be about ten times more volatile than prices and, hence, there is little difference in volatility between nominal and real exchange rates. High volatility of real exchange rates has been documented in many sources; a good review of such findings can be found in Obstfeld and Rogoff (1996) or in Alessandria (2003). The constructed example shows that our model is consistent with highly volatile real exchange rates.

As regards volatility conservation, assume now that authorities fix nominal exchange rate. Then, real exchange-rate volatility is simply price volatility. Because price volatility is ten times smaller, an outside observer will see a sharp decline in real exchange-rate volatility. On the other hand, there will be no dramatic change in volatility of other macroeconomic variables — money supply as well as other model fundamentals are fixed; given negligible volume of trade between the two countries, output volatility is of the same order as price volatility and the latter is unchanged. Similarly, a switch from fixed nominal exchange rates to floating, will produce large volatility of both nominal and real exchange rates without much changes to the volatility of other variables. This kind of exchange rate behavior is documented in Flood and Rose (1995), Obstfeld and Rogoff (1996), etc.; our model is thus consistent with these empirical observations.

7 Appendix B

Proposition 1. *Let* (δ, Υ) *be an arbitrary interior fixed-price allocation. Then, there exists a nondegenerate measure* μ *with the center at* $\overline{\mathbf{p}}$*, where* $\operatorname{supp}\delta = \{\overline{\mathbf{p}}\}$*, which is a stationary equilibrium.*

Proof: The proof is constructive. First, observe that, given an interior fixed-price allocation (δ, Υ), continuity of the future value of money (see 4) implies that there exists a closed neighborhood $\mathcal{U}$ of the unit lottery and a non-degenerate measure, denoted μ_{r_0}, such that both incentive compatibility constraints in (10) and (11) and no-quitting conditions (23) are satisfied for all lotteries from $\mathcal{U}$ and all price draws from (the support of) μ_{r_0}. (Notice that if (δ, Υ) is an interior fixed-price allocation, then no-quitting conditions 23 are implied by incentive compatibility constraints 11).

Let $\rho(x, y)$ be the standard distance function (metrics) in $\mathbb{R}^2$. Without loss of generality we can take a circular neighborhood $\mathcal{U}$,

$$\mathcal{U} = \{\boldsymbol{\tau} : \tau_A \leq 1,\ \tau_B \leq 1,\ \rho(\boldsymbol{\tau}, \Upsilon) \leq d_{\mathcal{U}}\},$$

where $d_{\mathcal{U}} > 0$ is the radius of $\mathcal{U}$, and an absolutely continuous measure μ_{r_0} with circular support K_{r_0},

$$K_{r_0} = \{\mathbf{p} : \rho(\mathbf{p}, \overline{\mathbf{p}}) \leq r_0\}$$

where r_0 is the radius of K_{r_0}.

Second, given some price draw $\mathbf{p}$ from $S_{\mu_{r_0}}$ and citizenship i of an individual, who makes an offer in a currency-exchange meeting, let us define a map $\Psi^i_{\mu_{r_0}}(\boldsymbol{\tau}; \mathbf{p})$, which maps $\mathcal{U}$ into $[0, 1]^2$ as follows. Given $\mathbf{p}$ and μ_{r_0}, take some

lottery $\boldsymbol{\tau} \in \mathcal{U}$ and compute the associated future value of money $\mathbf{V}$ in accordance with (4). Then, given $\mathbf{p}$, μ_{r_0} and $\mathbf{V}$, compute the bargaining solution in accordance with (28)-(33) and let that solution be the image of $\boldsymbol{\tau}$ under $\Psi^i_{\mu_{r_0}}(\boldsymbol{\tau};\mathbf{p})$. It follows from (4) and (28)-(33) that $\Psi^i_{\mu_{r_0}}(\boldsymbol{\tau};\mathbf{p})$ is single-valued and continuous.

Then consider two cases:

- i) the image of $\mathcal{U}$ under $\Psi^i_{\mu_{r_0}}(\boldsymbol{\tau};\mathbf{p})$ is a subset of $\mathcal{U}$ for all $\mathbf{p}$ in the set $S_{\mu_{r_0}}$ and
- ii) there is a pair $\mathbf{p} \in S_{\mu_{r_0}}$ and a lottery $\boldsymbol{\tau}$ such that the image of $\boldsymbol{\tau}$ under $\Psi^i_{\mu_{r_0}}(\boldsymbol{\tau};\mathbf{p})$ is not in $\mathcal{U}$.

Because $\mathcal{U}$ is circular, the latter is equivalent to saying that the distance between the unit lottery and the image of $\boldsymbol{\tau}$ under $\Psi^i_{\mu_{r_0}}(\boldsymbol{\tau};\mathbf{p})$ is greater than $d_{\mathcal{U}}$. Then, observe that case i) is straightforward because in that case Brouwer's fixed point theorem applies for any given $\mathbf{p} \in S_{\mu_{r_0}}$, so that μ_{r_0} is a stationary equilibrium. Case ii) can be worked out as follows.

Consider a family of truncated measures μ_r, each of which is defined over a circular support K_r, with radius r, $r \in [0, r_0]$, so that

$$\mu_r(A) = \frac{\mu_{r_0}(A \cap K_r)}{\mu_{r_0}(K_r)}, \tag{38}$$

where A is a dummy set in the space $\mathbb{R}^2_+$ of prices. Fix some sequence of radii $\{r_n\}_{n=1}^{\infty}$, such that $r_n \in (0, r_0)$ for all n and $\lim_{n\to\infty} r_n = 0$. Consider the first member r_1 of that sequence and the associated measure μ_{r_1} constructed from μ_{r_0} in accordance with (38). Given μ_{r_1} and a price draw $\mathbf{p}$ in the set

$S_{\mu_{r_1}}$, define a map $\Psi^i_{\mu_{r_1}}(\boldsymbol{\tau};\mathbf{p})$ in the same way as $\Psi^i_{\mu_{r_0}}(\boldsymbol{\tau};\mathbf{p})$ above. If the image of $\mathcal{U}$ under $\Psi^i_{\mu_{r_1}}(\boldsymbol{\tau};\mathbf{p})$ is a subset of $\mathcal{U}$ for all $\mathbf{p}$ in $S_{\mu_{r_1}}$, then measure μ_{r_1} is a stationary equilibrium and we are done. If not, that is if there is a pair $\mathbf{p}$ and a lottery $\boldsymbol{\tau}$ such that the image of $\boldsymbol{\tau}$ under $\Psi^i_{\mu_{r_1}}(\boldsymbol{\tau};\mathbf{p})$ is not in $\mathcal{U}$, then consider measure μ_{r_2} constructed from μ_{r_0} in accordance with (38), where r_2 is the second member of the sequence of radii $\{r_n\}_{n=1}^{\infty}$. Continue the process by induction. We need to show that for some n, measure μ_{r_n} is a stationary equilibrium.

Given an arbitrary r, $r \in [0, r_0]$, and the associated measure μ_r, let $d(r)$ be the maximum distance between the unit lottery and the image of $\mathcal{U}$ under $\Psi^i_{\mu_r}(\boldsymbol{\tau};\mathbf{p})$,

$$d(r) = \max_{(\boldsymbol{\tau},\mathbf{p})\in\Gamma(r)} \rho\left(\Psi^i_{\mu_r}(\boldsymbol{\tau};\mathbf{p}), \Upsilon\right),$$

where

$$\Gamma(r) \equiv \left\{\boldsymbol{\tau}, \mathbf{p} : \boldsymbol{\tau} \in \mathcal{U},\, \mathbf{p} \in S_{\mu_r}\right\}.$$

To show that measure μ_{r_n} is a stationary equilibrium it is sufficient to demonstrate that the distance $d(r_n)$ is less than $d_{\mathcal{U}}$ for some n. That is, it is enough to show that the limit (as n goes to infinity) of $d(r_n)$ is zero.

The latter can be done by application of the theorem of the maximum. Let us define the following three auxiliary functions:

$$G_1(r,\mathbf{p}) \equiv \max\left[0,\ \int u\left(\frac{1}{p_i}\right) d\mu_r - u\left(\frac{1}{p_i}\right)\right],$$

$$G_2(r,\mathbf{p}) \equiv \max\left[0,\ u\left(\frac{1}{p_i}\right) - \int u\left(\frac{1}{p_i}\right) d\mu_r\right],$$

$$F(r,\boldsymbol{\tau}) \equiv (-1\ 1)\mathbf{V}'_{-i},$$

where $\mathbf{V}_{-i}$ is the future value of money $-i$ computed in accordance with (4).

Because the bargaining solution implies that one of the two currencies always changes hands with probability one,

$$d(r) = \max \left[\max_{(\boldsymbol{\tau},\mathbf{p})\in\Gamma(r)} \frac{G_1(r,\mathbf{p})}{F(r,\boldsymbol{\tau})}, \max_{(\boldsymbol{\tau},\mathbf{p})\in\Gamma(r)} \frac{G_2(r,\mathbf{p})}{u\left(\frac{1}{p_i}\right)} \right] \tag{39}$$

Because μ_{r_0} is absolutely continuous, auxiliary functions G_1, G_2, and F are continuous functions of their arguments. The correspondence $\Gamma(r)$ is continuous by construction. By the theorem of the maximum, $d(r)$ is continuous function of r, so it has a limit as $r \to 0$.

Observe that

$$0 \leq d(r) \leq \max \left[\frac{\max_{\mathbf{p}\in K_r} G_1(r,\mathbf{p})}{\min_{\boldsymbol{\tau}\in\mathcal{U}} F(r,\boldsymbol{\tau})}, \frac{\max_{\mathbf{p}\in K_r} G_2(r,\mathbf{p})}{\min_{\mathbf{p}\in K_r} u\left(\frac{1}{p_i}\right)} \right].$$

Because

$$\lim_{r\to 0} \max_{\mathbf{p}\in K_r} G_1(r,\mathbf{p}) = \lim_{r\to 0} \max_{\mathbf{p}\in K_r} G_2(r,\mathbf{p}) = 0,$$

the limit of $d(r)$ as r goes to zero is zero. Given any convergent to zero sequence of radii $\{r_n\}_{n=1}^{\infty}$, that implies convergence to zero of the sequence of distances $\{d(r_n)\}_{n=1}^{\infty}$, which completes the proof.

Notice that even though the proof asserts existence of equilibrium measures, which are absolutely continuous and have circular supports, it can be generalized for the case of atomic measures and measures with arbitrary supports. However, to do this one needs to consider a more general family of measures than the family of truncated measures in (38). ■

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