Estimation of demand responses to a price change under the paucity of the data

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1999
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Данная работа посвящена анализу системы косвенного налогообложения и поиску путей ее совершенствования в условиях отсутствия регулярного мониторинга бюджетов домашних хозяйств. Главным объектом изучения является возможная реакция потребительского спроса на изменение цен. Разработано нелинейное по логарифму общего дохода обобщение классической формы кривой Энгеля, используя которое была построена полная система потребительского спроса из пяти групп товаров для восьми регионов Российской Федерации. В отсутствии панельных данных о бюджетах домашних хозяйств были получены эластичности потребительского спроса. При этом были использованы свойство симметрии Слуцкого и уравнение Эйлера.


The paper provides the analysis of indirect tax system and results of possible reforms under the paucity of adequate Panel Data. Demand responses to a price change are the main subject of consideration in the paper. The paper presents the generalization of standard Engel curve relationship. There was developed the specification of Engel curve which is non-linear in log of income. Specified Engel curves were estimated for the complete system of five commodity groups for eight regions of Russian Federation. Facing the absence of historical records, regularity conditions for utility maximization, such as Slutsky symmetry and Euler equation were used to obtain theoretically based system of demand elasticity’s.

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I. INTRODUCTION

Knowledge of demand responses to the price changes is important for designing a good Tax System. By changing the tax rate, government usually can change prices. These new prices change demand for all goods throughout the economy. So, to predict the result of a tax change a policymaker should know behavioral responses of demand to this change.

In Russia the empirical reality is often in stark contrast to the assumptions of models in which the government was seen as a benevolent dictator acting in the public interest. As a result, optimal tax models could not present the description of the social and political equilibria that actually exist. The Master Thesis uses the theory of tax reform and considers the following question. What system of prices would be desirable, at least to the extent of meeting the legitimate needs of government while minimising the cost of collection and distortion?

The theory of tax reform is concerned with small departures from an existing tax structure. The formulas for tax reform often have obvious implications for the optimum. The second part of the Master Thesis will present the theoretical description of the problem extensively treated in the monographs by Newbery and Stern (1987) and by Deaton (1997). What will mostly be concerned with the third part is an approach that specifies an Engel curve linking expenditures on individual goods to total expenditure. For many commodities there is increasing evidence that the Working-Leser form of Engel curve that was used in mentioned studies does not provide an accurate picture of individual behaviour. The Thesis suggests a generalisation of well-known Working’s Engel curve. The parts four
and five will use estimated Engel curves to identify behavioural responses of public \( \frac{\partial q_{j,h}}{\partial p_i} \) on price changes under the paucity of adequate Panel Data taking as the assumption the fact that demand functions are homogeneous of degree zero and nest the property of Slutsky symmetry. Having developed theoretical model, the part six provides the empirical analysis that was based on Russia Longitudinal Monitoring Survey for Russian Federation Fiscal Reform Project. The purpose of the Master Thesis is to reflect on the measurement of the demand responses to the price changes when adequate Panel Data is not available.

II. Tax Reform Approach

There will be used social welfare function as a function of the individual welfare levels (values of individual indirect utility function that give the maximum attainable welfare under certain prices and outlays).

\[
(2.1) \quad W = V(u_1, u_2, \ldots, u_N)
\]

where \( N \) is a number of people in the economy and the individual welfare levels are given by

\[
(2.2) \quad u_h = \psi(x_h, p)
\]

Given the indirect utility function, demand function can be derived by Roy’s identity.

\[
(2.3) \quad \frac{\partial u_h}{\partial p_i} = -q_{h,i} \frac{\partial u_h}{\partial x_h}, \text{ for } h=1..N, i=1..M, \text{ and } M - \text{quantity of goods in the economy.}
\]

\[
p_i = p_i^0 + t_i
\]
Price is the sum of a fixed price and the tax or subsidy. Government revenue in this case is:

\[(2.4) \quad R = \sum_{i=1}^{M} \sum_{h=1}^{N} t_i q_{i,h} \]

The effects of a small change in a single tax can be seen from derivatives of revenue and social welfare function.

\[(2.5) \quad \frac{\partial R}{\partial t_i} = \sum_{h=1}^{N} q_{i,h} + \sum_{h=1}^{N} \sum_{j=1}^{M} t_j \frac{\partial q_{j,h}}{\partial p_i} \]

\[(2.6) \quad \frac{\partial W}{\partial t_i} = \sum_{h=1}^{N} \frac{\partial V}{\partial u_{h_i}} \frac{\partial u_{h_i}}{\partial p_i} \]

These two equations represent the benefit and the social cost of a tax increase.

By substituting the Roy’s identity (2.3) to (2.6) we can get

\[ \frac{\partial W}{\partial t_i} = -\sum_{h=1}^{N} \frac{\partial W}{\partial x_{h_i}} q_{i,h} \]

Therefore the ratio of cost (the social cost of raising one unit of government revenue by increasing the tax on good i) can be define by

\[(2.7) \quad \lambda_i = \frac{\text{cost}}{\text{benefits}} = \frac{\sum_{h=1}^{N} \frac{\partial W}{\partial x_{h_i}} q_{i,h}}{\sum_{h=1}^{N} q_{i,h} + \sum_{h=1}^{N} \sum_{j=1}^{M} t_j \frac{\partial q_{j,h}}{\partial p_i}} \]

When all the ratios are the same, taxes are optimally set and there is no scope for beneficial reform. Goods with comparatively low \( \lambda_i \) ratios are those that are candidates for a tax increase. The numerator will be large for necessities and smaller for luxuries if the social weights favour the poor. Analysis of denominator shows that the larger the magnitude of own-price responses \( \partial q_{j,n}/\partial p_j \) of public the
less attractive it would be to raise further revenue increasing \( t_j \). Note also the potential importance of cross-price effects.

Almost all the difficulty in evaluating the cost-benefit ratio comes from the final term in the denominator, which summarises the behavioral responses to price changes. The standard data source for estimating the price responses is the historical record. Unfortunately, Russian historical records are too short for serious analysis. Thus, paucity of the data makes estimation of behavioral responses of public difficult. Even for developed countries it is not always possible to estimate demand responses with any degree of conviction for disaggregated system of commodities (Barten 1969 for an attempt using Dutch data).

**III. Estimating Engel curve**

In the presence of indirect tax changes relative prices as well as real incomes change. The approach to estimate behavioral responses begins from the specification of a standard Engel curve linking expenditures on individual goods to total expenditure and to the socio-economic and demographic characteristics of the household. The classical form of this function was introduced by Working (1943), who postulated a linear relationship between the share of the budget on each good and the logarithm of total expenditure. Such a relationship was widely used in papers of Deaton, Muellbauer, and other researchers. The regression that is usually used has the following form

\[
(3.1) \quad w_i = \alpha_i + \beta_i \ln(x) + (\text{demographic characteristics})
\]

where \( w_i \) is the share of the budget devoted to good \( i \), \( x \) is per capita total expenditure and \( n \) is household size.
(3.2) \( w_i \equiv p_i q_i / (nx) \)

This classical form of Engel curve has certain weaknesses, which are mostly due to its linear form. The point is announced linear form implies that any good is either inferior or luxury for any given income of household. That appears to be implausible for some goods. A series of recent empirical Engel curve studies indicates that it is not true for some commodities (see, for example, Atkinson (1990) Bierens and Pott-Buter (1987), Hausman (1995), Banks, Blundell, Lewbel (1997)). There is increasing evidence that slope of actual relationship between the share of the budget devoted to a good and total expenditure is positive for budgets low enough and negative for large ones. See also Clopper Almon for more criticism.

Here will be described an approach that tries to generalised Working’s Engel curve. Following evident properties of Engel curve should be held after the approximation

1) \( w(x) \) has a single peak.

That means that for the values of household income that lay to the left (right) from the peak the good has luxury (inferior) properties.

2) \( w(x)=0 \) if \( x<x_0 \)

A good is not consumed at all if household is too poor.

3) \( w(x) \xrightarrow{x \to \infty} 0 \)

The share of the budget devoted to a good is almost negligible if household’s income is large enough.
The suggested form of Engel curve that holds all the properties above is

\[
(3.3) \quad w(x) = \begin{cases} 
A \left( \frac{x_0}{x} \right)^\alpha \ln \left( \frac{x}{x_0} \right), & \text{if } x \geq x_0 \\
0, & \text{if } x < x_0 
\end{cases}
\]

Where \( A > 0, \alpha > 0, x_0 > 0 \) are coefficients of in interest. These coefficients can be estimated using household surveys data with the help of non-linear regression with \( w \) being dependent variable and \( x \) independent.

The example of shape of this suggested curve is shown here. The functional form (3.3) has a peak at point \( x_1 = x_0 \exp(1/\alpha) \). To the left of this point a good is considered to be a luxury and to the right of it is a necessity. Note the meaning of the parameter \( \alpha \). This parameter captures the nature of good. The large \( \alpha \) is for a good \( i \), the more of a necessity is good \( i \). The smaller it is the more of a luxury is a good. Note also, \( A > 0, \alpha \geq 0, x_0 \geq 0 \) should be treated as functions of price vector and show the price impact on consumption.
For estimating demand responses, we take derivative of share $w_i$ with respect to the price of a good $j$. It is shown at (3.4).

\[
(3.4) \frac{\partial w_i(p^*, x)}{\partial p_j} = \frac{\partial A_i(p^*)}{\partial p_j} L_{i,1} + \frac{\partial \alpha_i(p^*)}{\partial p_j} L_{i,2} + \frac{\partial x_0^{(i)}(p^*)}{\partial p_j} L_{i,3}
\]

where functions $L_{i,k}$ for $k=1..3$ are

\[
(3.5) \quad L_{i,1} \equiv \frac{\partial w_i}{\partial A} = \frac{w_i(p^*, x)}{A_i(p^*)};
\]

\[
(3.6) \quad L_{i,2} \equiv \frac{\partial w_i}{\partial \alpha_i} = w_i(p^*, x) \ln \left( \frac{x_0^{(i)}(p^*)}{x} \right);
\]

\[
(3.7) \quad L_{i,3} \equiv \frac{\partial w_i}{\partial x_0^{(i)}} = \frac{w_i(p^*, x)}{x_0^{(i)}(p^*)} \left( \alpha_i(p^*) - \frac{1}{\ln \left( \frac{x_0^{(i)}(p^*)}{x} \right)} \right);
\]

As it can be seen, $L_{i,k}$ for $k=1..3$ can be directly defined from household data and are known since $w_i$ is known function.

It is interesting to focus carefully on an effect of small changes of parameters $A, \alpha, x_0$. Suppose, a certain shift of a price vector changes demand on a good. Then, plausible supposition is that reaction of households with per capita total expenditure great enough should be quite moderate. The model captures this change through changes in parameters $A, \alpha, x_0$. Equation (3.5) shows that elasticity of good shares with respect to parameter $A$ is constant and equals 1. Equation (3.7) indicates that analogous elasticity with respect to $x_0$ is a bounded function and could not be greater then $\alpha$. However, partial derivative (3.6) permits elasticity of good shares with respect to $\alpha$ be an unbounded function. Namely, this elasticity declines unboundedly if income grows up. That property of elasticity
function might contradict a priory assumption of light reaction of wealthy households on a price change. Thus, that is the cause to treat parameter $\alpha_i$ of the model as an internal property of a good $i$ that should not change with prices. Obviously, $\alpha$ should depend on a quality of a good, existence of substitutes and habits of the public. In other words, parameter $\alpha$ really reflects the nature of the good and degree of its luxury and will be treated independent of price level. Having $\alpha$ constant for a given commodity, we have $L_{i,2} \equiv 0$; and price impact (3.4) can be simplified to (3.5).

$$\frac{\partial w_i(p^*, x)}{\partial p_j} = \frac{\partial A_i(p^*)}{\partial p_j} L_{i,1} + \frac{\partial x_0^{(i)}(p^*)}{\partial p_j} L_{i,3}$$

**IV. Estimating demand responses**

Since we assume that adequate data (e.g. Panel Data) is not available, we will design behavioral responses that nest the theoretical properties of the demand system.

Rewriting responses of budget shares of goods to the price change

$$\frac{\partial w_i}{\partial p_j} \bigg|_{j,i=1}$$

we get equations (4.1) and (4.2).

$$\frac{\partial q_i(p, x)}{\partial p_j} = \frac{(nx) \partial w_i(p, x)}{p_i \partial p_j} \quad \text{for} \quad j \neq i$$

$$\frac{\partial q_i(p, x)}{\partial p_i} = \frac{(nx) \partial w_i(p, x)}{p_i \partial p_i} - \frac{(nx) w_i(p, x)}{p_i^2}$$

That means the question of behavioral responses can be treated in framework of budget shares of goods in household bundle, using the notion of Engel curve.
Since we want to predict responses to tax reform, functions of a final interest are \( \frac{\partial q_i(p, x)}{\partial p_j} \) \( j, i = 1..M \). Given this, the quantity of unknown parameters we are to find is \( M^2 \). However, theory provides us with \( (M^2 + M)/2 \) restrictions. Where \( M \) of these restrictions are summarized in (4.3) and \( (M^2 - M)/2 \) are summarized in (4.4).

\[
(4.3) \quad \sum_{i=1}^{M} p_i \frac{\partial w_j(p^*, x)}{\partial p_i} = -x \frac{\partial w_j(p^*, x)}{\partial x} \quad \text{for } j=1..M \text{ - Euler equation for homogeneous budget share functions.}
\]

\[
(4.4) \quad p_j \frac{\partial w_i(p^*, x)}{\partial p_j} - p_i \frac{\partial w_j(p^*, x)}{\partial p_i} = x \left( w_i \frac{\partial w_j}{\partial x} - w_j \frac{\partial w_i}{\partial x} \right)
\]

for \( i=1,..,M, j=1,..,i-1 \).

- Slutsky symmetry which is written in terms of good shares (Since the framework uses the demand functions of single consumer)

Restrictions (4.4) can be also rewritten in terms of demand elasticities

\[
w_i \varepsilon_{i,j} - w_j \varepsilon_{j,i} = w_i w_j (\varepsilon_j - \varepsilon_i)
\]

where \( \varepsilon_{i,j} \equiv \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_i} \) and \( \varepsilon_i \equiv \frac{\partial q_i}{\partial x} \frac{x}{q_i} \).

Thus, Slutsky symmetry links properties of demand function as a function of total expenditure (right-hand term) with properties of the same demand function as a function of prices (left-hand term). For the analysis below we can denote the right-hand term of (4.4) as \( B_{i,j} \). Note that evaluation of right-hand terms of (4.3) and (4.4) should be done for a certain fixed price vector, therefore it does not require Panel Data.
\[ B_{i,j} \equiv x \left( w_i \frac{\partial w_j}{\partial x} - w_j \frac{\partial w_i}{\partial x} \right) = w_i w_j \left( -\alpha_j + \alpha_i + \frac{1}{\ln \left( \frac{x}{x_0} \right)} - \frac{1}{\ln \left( \frac{x}{x_0} \right)} \right) \]

The way to obtain theoretically based behavioral responses of public is to estimate left-hand terms by substituting estimated Engel curve to the equations (4.3) and (4.4).

By substituting the form (3.5) to the Slutsky symmetry (4.4) we can get

(4.5)

\[ p_j \left( \frac{\partial A_i(p^*)}{\partial p_j} L_{i,1} + \frac{\partial x_i^{(l)}(p^*)}{\partial p_j} L_{i,3} \right) - p_i \left( \frac{\partial A_j(p^*)}{\partial p_i} L_{j,1} + \frac{\partial x_j^{(l)}(p^*)}{\partial p_i} L_{j,3} \right) = B_{i,j}(p^*, x) \]

Since the terms on the right-hand side of (4.5) can be estimated under current price vector using the suggested Engel curve, and the functions \( L_{i,k} \) for \( k=1,3 \) and \( i=1..M \) are directly defined from household data we can use (4.5) as a regression to calculate the responses of money spend by household on each good. Dependent variables are \( B_{i,j} \), independent variables are \( p_j L_{i,k} \) and \( p_i L_{j,k} \) for \( k=1,3 \). As a result, estimated partial derivatives of coefficients of Engel curves for goods \( i \) and \( j \) give responses of budget shares of these goods to the price change according (3.4). In turn, responses of budget shares define behavioural responses of public according the equations (4.1) and (4.2).

**V. Case of three commodities**

An example of complete system of equations can be easily shown in the case of three commodities of in interest. Suppose consumer demand was divided
on 3 categories of goods \((i=1,2,3)\). The system below estimates theoretically based cross-price and own-price effects for these categories since their Engel curves of the form (3.3) have been approximated. The system consists of six equations. First three, namely equations (5.1), derive cross-price effects and the other three (equations (5.2)) are responsible for own-price effects.

\[
(5.1)
\]

\[
p_1 \left( \frac{\partial A_2(p^*)}{\partial p_1} L_{2,1} + \frac{\partial x_0^{(2)}(p^*)}{\partial p_1} L_{2,3} \right) - p_2 \left( \frac{\partial A_1(p^*)}{\partial p_2} L_{1,1} + \frac{\partial x_0^{(1)}(p^*)}{\partial p_2} L_{1,3} \right) = B_{2,1}(p^*, x)
\]

\[
p_1 \left( \frac{\partial A_2(p^*)}{\partial p_1} L_{3,1} + \frac{\partial x_0^{(3)}(p^*)}{\partial p_1} L_{3,3} \right) - p_3 \left( \frac{\partial A_1(p^*)}{\partial p_3} L_{1,1} + \frac{\partial x_0^{(1)}(p^*)}{\partial p_3} L_{1,3} \right) = B_{3,1}(p^*, x)
\]

\[
p_2 \left( \frac{\partial A_2(p^*)}{\partial p_2} L_{3,1} + \frac{\partial x_0^{(3)}(p^*)}{\partial p_2} L_{3,3} \right) - p_3 \left( \frac{\partial A_2(p^*)}{\partial p_3} L_{2,1} + \frac{\partial x_0^{(2)}(p^*)}{\partial p_3} L_{2,3} \right) = B_{3,2}(p^*, x)
\]

These three equations should be examined as regressions to find out the partial price derivatives. Then, cross-price effects are easily derived from (3.5). Notion of cross-price effects enables us to calculate sensible own-price effects with the help of the property of homogeneity (4.3) of budget share functions.

\[
(5.2) \quad p_i \frac{\partial w_i}{\partial p_i} = - \sum_{j=1,j\neq i}^M p_j \frac{\partial w_i(p^*, x)}{\partial p_j} - x \frac{\partial w_i(p^*, x)}{\partial x} \quad \text{for } i=1,2,3.
\]
As the result, complete set of price responses of consumer demand is known. Thus, when adequate data (e.g. Panel Data) is not available and standard technique can not be used the model of demand system described above still can support a policy-maker with the set of behavioral responses to price changes.

**VI. Data analysis**

Having developed theoretical model, this part provides the empirical analysis that is based on Russia Longitudinal Monitoring Survey which was complete at 1998 for Russian Federation Fiscal Reform Project. The data analysis shows the implementation of the model described above to the measurement of cross-price effects of demands. The Master Thesis considers complete demand system of five commodity groups. Examined complete system of consumer demand consists of food shares, clothing shares, vodka shares, beer shares and shares of all other goods. This choice reflects the results of previous studies. The study of Bank, Blundell and Lewbel (1997) indicates that for U.K. alcohol is one of the commodities for which the Working-Leser form of Engel curve should be strongly rejected. Nearly the same situation should be for any country, including Russia. In order to be able to compare results, there will be analysed approximately the same system of commodities as was used by Bank, Blundell and Lewbel (1997).

The fist step is to specify Engel curves. Engel curve appears to vary with region because of various reasons. However, demand on a certain good obviously has some properties that do not change greatly from region to region, for instance, income elasticity of demand for rich households. Demand function (3.3) for a good i generates income elasticity of demand in the form

\[
\varepsilon_i \equiv \frac{x}{w_i} \frac{\partial w_i}{\partial x} = -\alpha_i + \frac{1}{\ln \left( \frac{x}{x_0} \right)}
\]
As it can be seen, \( \varepsilon_i \to -\alpha_i \) if \( x \to \infty \). Since all rich households are likely to have approximately the same income elasticity of demand on a good \( i \), there is a reason to treat parameter \( \alpha_i \) as a property of the good \( i \) which does not vary with region. Thus, in the analysis below parameters \( \alpha \) and \( A \) do not depend on a region, however parameter \( x_0 \) depends since \( x_0 \) is strongly correlated with vector of relative prices. Obviously, flexibility of \( x_0 \) helps to increase homogeneity of the sample. Parameter \( x_0 \) is estimated using eight dummy variables for eight regions of Russian Federation. Table 1 presents the correspondence.

**TABLE 1**

Dummies for parameter \( x_0 \)

<table>
<thead>
<tr>
<th>Dummy</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_{0,1} )</td>
<td>Metropolitan areas: Moscow and St. Petersburg</td>
</tr>
<tr>
<td>( X_{0,2} )</td>
<td>Northern and North Western</td>
</tr>
<tr>
<td>( X_{0,3} )</td>
<td>Central and Central Black-Earth</td>
</tr>
<tr>
<td>( X_{0,4} )</td>
<td>Volga- Vaytski and Volga Basin</td>
</tr>
<tr>
<td>( X_{0,5} )</td>
<td>North Caucasian</td>
</tr>
<tr>
<td>( X_{0,6} )</td>
<td>Ural</td>
</tr>
<tr>
<td>( X_{0,7} )</td>
<td>Western Siberian</td>
</tr>
</tbody>
</table>
X_{0.8} \quad \text{Eastern Siberian and Far Eastern}

The second table shows results of non-linear approximation of Engel curve at the form (3.3) for every commodity group of in interest.

TABLE 2
Engel curves for the system of consumer demand

**FOOD SHARES**
Number of obs = 3719
R-squared = 0.9321

| food  | Coef.  | Std. Err. | t     | P>|t| | [95% Conf. Interval] |
|-------|--------|-----------|-------|-----|---------------------|
| A     | 1.397701 | 0.065349 | 21.388 | 0.000 | 1.269577 \quad 1.525825 |
| X_{0,1}| 16.05843 | 12.46255 | 1.289  | 0.198 | -8.375696 \quad 40.49255 |
| X_{0,2}| 10.18332 | 7.918221 | 1.286  | 0.199 | -5.341178 \quad 25.70781 |
| X_{0,3}| 13.62412 | 10.5685  | 1.289  | 0.197 | -7.096513 \quad 34.34476 |
| X_{0,4}| 13.15687 | 10.18153 | 1.292  | 0.196 | -6.805065 \quad 33.11881 |
| X_{0,5}| 13.73584 | 10.65597 | 1.289  | 0.197 | -7.156295 \quad 34.62798 |
| X_{0,6}| 10.34696 | 7.982567 | 1.296  | 0.195 | -5.303687 \quad 25.99761 |
| X_{0,7}| 11.94414 | 9.358641 | 1.276  | 0.202 | -6.404441 \quad 30.29273 |
| X_{0,8}| 13.8954  | 10.76304 | 1.291  | 0.197 | -7.206659 \quad 34.99746 |
| α     | 0.535474 | 0.0477564| 11.213 | 0.000 | .441843 \quad .6291058 |

**CLOTHING SHARES**
Number of obs = 2155
R-squared = 0.5999

| clothe | Coef.  | Std. Err. | t     | P>|t| | [95% Conf. Interval] |
|--------|--------|-----------|-------|-----|---------------------|
| A     | 0.100166 | 0.0216299 | 4.631 | 0.000 | 0.0577482 \quad 0.1425839 |
| X_{0,1}| 200.0913 | 55.45756 | 3.608 | 0.000 | 91.33517 \quad 308.8475 |
| X_{0,2}| 165.7518 | 49.49489 | 3.349 | 0.001 | 68.6888 \quad 262.8147 |
| X_{0,3}| 112.1232 | 35.58476 | 3.151 | 0.002 | 42.33902 \quad 181.9074 |
\[ \begin{array}{cccccc}
X_{0,4} & 121.1698 & 37.76697 & 3.208 & 0.001 & 47.10615 & 195.2335 \\
X_{0,5} & 82.65747 & 31.33927 & 2.638 & 0.008 & 21.19896 & 144.116 \\
X_{0,6} & 145.953 & 45.25149 & 3.225 & 0.001 & 57.21166 & 234.6944 \\
X_{0,7} & 73.53932 & 32.53448 & 2.260 & 0.024 & 9.736917 & 137.3417 \\
X_{0,8} & 0.0359632 & 0.1497497 & 0.240 & 0.810 & -0.2577064 & 0.3296328 \\
\alpha & 0.1984142 & 0.0610538 & 3.250 & 0.001 & 0.0786834 & 0.3181451 \\
\end{array} \]

---

### VODKA SHARES

Number of obs = 737  
R-squared = 0.6060

\[ \begin{array}{cccccc}
& \text{Vodka} & \text{Shares} & \text{Number of obs} = 737 & \text{R-squared} = 0.6060 \\
& \text{vodka} \text{ | Coef. Std. Err. t P>|t| [95% Conf. Interval]} & & & & \\
A & .3300192 & .0423563 & 7.791 & 0.000 & .2468639 & .4131746 \\
X_{0,1} & 308.7272 & 48.48675 & 6.367 & 0.000 & 213.5364 & 403.9179 \\
X_{0,2} & 113.6459 & 31.27814 & 3.633 & 0.000 & 52.23967 & 175.0522 \\
X_{0,3} & 177.3682 & 40.76349 & 4.351 & 0.000 & 97.33999 & 257.3964 \\
X_{0,4} & 144.6307 & 37.90483 & 3.816 & 0.000 & 70.21474 & 219.0467 \\
X_{0,5} & 402.2991 & 36.79441 & 10.934 & 0.000 & 330.0632 & 474.5351 \\
X_{0,6} & 189.4742 & 42.55016 & 4.453 & 0.000 & 105.9384 & 273.01 \\
X_{0,7} & 160.1961 & 46.94864 & 3.412 & 0.001 & 68.02504 & 252.3672 \\
X_{0,8} & 212.5315 & 44.57604 & 4.768 & 0.000 & 125.0183 & 300.0446 \\
\alpha & 1.225715 & 0.1354693 & 9.048 & 0.000 & 0.9597569 & 1.491672 \\
\end{array} \]

---

### BEER SHARES

Number of obs = 858  
R-squared = 0.5991

\[ \begin{array}{cccccc}
& \text{Beer} & \text{Shares} & \text{Number of obs} = 858 & \text{R-squared} = 0.5991 \\
& \text{beer} \text{ | Coef. Std. Err. t P>|t| [95% Conf. Interval]} & & & & \\
A & .0830306 & .0123735 & 6.710 & 0.000 & .0587444 & .1073168 \\
X_{0,1} & 135.6893 & 49.48885 & 2.742 & 0.006 & 38.55434 & 232.8243 \\
X_{0,2} & 308.8153 & 45.88025 & 6.731 & 0.000 & 218.7631 & 398.8675 \\
X_{0,3} & 118.3665 & 40.27624 & 2.939 & 0.003 & 39.31365 & 197.4193 \\
X_{0,4} & 88.17333 & 42.76795 & 2.048 & 0.041 & 9.247751 & 152.316 \\
X_{0,5} & 191.637 & 104.9084 & 1.841 & 0.066 & -12.6809 & 395.9549 \\
X_{0,6} & 164.103 & 55.71295 & 2.946 & 0.003 & 54.75152 & 273.4544 \\
X_{0,7} & 99.89723 & 42.90891 & 2.328 & 0.020 & 15.67534 & 184.1191 \\
X_{0,8} & 102.0989 & 49.85371 & 2.048 & 0.041 & 4.247751 & 199.95 \\
\alpha & 0.9461663 & 0.1303987 & 7.256 & 0.000 & 0.6902242 & 1.202108 \\
\end{array} \]
Comparison of values of $A$ and $\alpha$ for different commodities deserves close attention. As it can be seen, $\alpha$ takes the largest value for consumer demand on strong alcoholic beverages and the lowest value for the function of clothing shares (if do not consider the share of All Other Goods). The intuition of this fact is evident. As it was noted at part III, the large $\alpha$ is for a good $i$, the more of a necessity is good $i$. The smaller it is the more of a luxury is a good.

Another outcome is households with per capita total expenditure of $x_1 \equiv x_0 e^{\alpha} \text{roubles}$ are likely to have the greatest share of a commodity group in their bundles. This greatest possible share is $w_{\text{max}} \equiv A/(e\alpha)$. Table 3 presents the
values $w_{\text{max}}$ for goods with pronounced single peaked shape. As the model supposes, a good exhibits properties of a luxury for incomes less than $x_1$ roubles and properties of a necessity for greater incomes. Table 4 shows values of $x_1$ for commodities of interest with respect to region.

TABLE 3
Maximum possible shares in household’s bundles

<table>
<thead>
<tr>
<th>Clotthing</th>
<th>Vodka</th>
<th>Beer</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>18.6</td>
<td>10</td>
<td>3%</td>
</tr>
<tr>
<td>%</td>
<td>%</td>
<td>2%</td>
</tr>
</tbody>
</table>

TABLE 4

<table>
<thead>
<tr>
<th>Region\Commodity</th>
<th>Clothing</th>
<th>Vodka</th>
<th>Beer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metropolitan areas: Moscow and St. Petersburg</td>
<td>31300</td>
<td>700</td>
<td>390</td>
</tr>
<tr>
<td>Northern and North Western</td>
<td>25900</td>
<td>260</td>
<td>887</td>
</tr>
<tr>
<td>Central and Central Black-Earth</td>
<td>17500</td>
<td>400</td>
<td>340</td>
</tr>
</tbody>
</table>
As we can see from Bank, Blundell and Lewbel (1997), the maximum amount of alcohol shares for U.K. is about 7% and is fewer than possible share of 10% which is computed for Russia. This comparison nests perfectly with the notion that relative prices for alcohol in Russia are lower than that in Europe.

Having estimated Engel curves, the system of regressions of the type of (5.1) gives us partial derivatives of coefficients of Engel curves with respect to prices 

\[ p_i \left( \frac{\partial A_j(p^*)}{\partial p_i}, \frac{\partial x_{ij}^{*}(p^*)}{\partial p_i} \right). \]

Then, using identity (3.5), it is easy to get demand elasticities. These elasticities are calculated for each household individually, and then a weighted average is constructed, with the weights being equal to the household’s share of total sample expenditure for the relevant good. Complete system of demand elasticities is reported in Table 5.
TABLE 5
Estimated Demand Elasticities

<table>
<thead>
<tr>
<th></th>
<th>Food</th>
<th>Cloth</th>
<th>Vodka</th>
<th>Beer</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>-</td>
<td>0.089</td>
<td>0.080</td>
<td>0.349</td>
<td>0.118</td>
</tr>
<tr>
<td></td>
<td>0.0192</td>
<td>9</td>
<td>3</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Cloth</td>
<td>0.059</td>
<td>-</td>
<td>0.018</td>
<td>0.001</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.2110</td>
<td>9</td>
<td>9</td>
<td>0.0881</td>
</tr>
<tr>
<td>Vodka</td>
<td>0.723</td>
<td>0.127</td>
<td>-</td>
<td>0.044</td>
<td>0.192</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>8</td>
<td>0.6734</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Beer</td>
<td>0.891</td>
<td>0.097</td>
<td>0.126</td>
<td>-</td>
<td>0.134</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0</td>
<td>2</td>
<td>0.8846</td>
<td>3</td>
</tr>
<tr>
<td>Other</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.1939</td>
<td>0.0760</td>
<td>0.0149</td>
<td>0.0079</td>
<td>0.1388</td>
</tr>
</tbody>
</table>

Thus, there was developed the model that permits theoretically predict the demand responses to the price changes when adequate Panel Data is not available.
VI. Conclusion

The Master Thesis was motivated by the need to provide more accurate analysis of the results of indirect tax reform. Analyses of household budget surveys have shown more curvature in the Engel curve relationship for some commodities than is permitted by the standard Working form. Empirical findings indicate that rich or poor households alike may have equal expenditures or budget shares on some goods. It seems clear that we can reject the linear Working form for certain commodity groups. There was developed the specification of Engel curve, which is non-linear in log of income. One of the advantages of this specification comparing with non-linear demand models that have been already derived is that regularity constraints involving inequalities can be hold globally for such a form of Engel curve. The empirical analysis was based on Russia Longitudinal Monitoring Survey. Specified Engel curves were estimated for the complete system of five commodity groups. Regularity conditions for utility maximisation, such as Slutsky symmetry and Euler equation were used to obtain theoretically based behavioural responses of the group of the commodities. It is argued that facing the absence of historical records it is possible to involve theoretically based behavioural responses to the welfare analysis of indirect tax reform results.
REFERENCES


6. Clopper Almon. “A Perhaps Adequate Demand System with Application to France, Italy, Spain and the USA.” mimeo, *University of Maryland, College Park*, USA.
