МАГИСТЕРСКАЯ ДИССЕРТАЦИЯ

MASTER THESIS

Тема: Мотив приобретения опыта и беспокойство за репутацию при принятии рискованных решений менеджерами

Title: Acquisition of Expertise and Reputational Concerns in Risk-Taking

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Оценка/ Grade:

Подпись/ Signature:

Москва, 2011
Abstract

This paper studies how career concerns affect a manager’s choice between a risky and a safe project when the risky project allows her to acquire more expertise. Holmström (1999) analyzed how a person’s concern for a future career may influence her incentives to put in effort or make decisions on the job. It has been shown that if a manager is risk-averse, and her own perception of the talent is the same as the market’s, she prefers, all other things equal, projects that reveal least information about her ability. However, the result does not hold if the market observes whether the manager undertakes projects or not, and if it is common knowledge that undertaking projects increases expertise. A risk-averse manager’s trade-off between avoiding risk and acquiring more expertise is analyzed. The main results are as follows. The manager prefers a safest project if no expertise can be acquired, as it reduces the risk of low estimation of her talent. Nonetheless, a risky project can be chosen if it implies gaining a sufficiently wide experience. However, incentives to opt for such a project deteriorate as a higher level of the manager’s skills is originally expected. In a two-period model an attempt is made to show that in some cases the principal should not enforce the manager to shift her preferences towards a project with the highest expected payoff as costs may outweigh benefits from doing so. Moreover, there exist situations in which the principal will not be able to influence the manager’s decision by means of linear contract alone. However, option contracts may rectify the situation. A three-period model discussed separates effects of initial assessment of the talent and a learning-by-doing effect, which may work in different directions.
1 Introduction

Management compensation and project-choice moral hazard have been on the research agenda for over three decades now. One may determine two main trends: incentive models that incorporate the design of contracts, and learning models that are based on Bayes’ rule. The first type suggests possible ways of inducing unobservable or unverifiable managerial effort desirable to foster a company’s growth and fulfill its purposes by offering implicit incentives in a contract. The second draws upon the idea that a manager’s talent is originally unknown to her and to the market, and may be reevaluated when she chooses a particular level of effort or a particular type of a project. Therefore, the market’s perception of a manager’s talent evolves as the consequences of a manager’s actions are revealed to the market. Both these approaches yield a rich variety of insights into possible deviations from a company’s first-best decisions; this paper follows the second approach.

Holmström (1982, 1999) and Dewatripont et al. (1999) investigated how career concerns could affect a manager’s performance in case when the market and a manager are equally uninformed about her ability. They have suggested an idea that a manager has a concern in the market’s assessment of her talent as her wage depends on the future profitability of her actions. In particular, Holmström argues that in the absence of incentive contracts a risk-averse manager would choose investment projects that reveal least information about her ability. Many attempts to elaborate on this result have been made. Holmström included a stake on a project yield in the contract so as to induce a manager to opt for a project closer to a company’s first-best. Lambert (1986) and Milbourn et al. (2001) analyzed a manager’s choice between risky and safe projects in terms of possible investment in information before decision-making. Thus, if the signal obtained is observable for the market, it may allow a manager to shift the probabilities of the market’s perception of her abilities ex post. Scharfstein and Stein (1990) studied possible interactions between managers causing herd behavior in investment, which can also manipulate the assessment.

The idea of learning-by-doing has also emerged in several papers. A newly-hired manager can have some employment history and educational background, which give both her and
a company a signal about her skills. After working on projects for some time, a manager acquires some expertise that can shift their perception of her ability. Hu, Kale and Pagani (2008) developed a framework incorporating interactions among investors, fund companies and managers. The process of learning-by-doing is presented there as a simple evolution of a manager’s ability; the manager is fired if it falls below some predetermined level. However, the term “ability” here is a synonym to “performance indicator” rather than “talent.” In contrast, Lucas (2006) developed the model that gave an insight into possible acquiring expertise. The change of the parameters of Beta distribution was used to model this process; ability was a random variable drawn from this distribution that represented the probability that a manager would be successful in the next period. Lucas considered a learning-by-doing effect as a complement to a high effort in a sense that choosing a high level of effort in the first period automatically increases the probability of success in the second. Moreover, he concentrated on the incentive problem alone supposing that it is always optimal for the principal if the manager chooses a high level of effort.

Nonetheless, a possible influence of acquiring expertise from implementing a project on a choice between different investment projects is still of great interest. The effect of learning-by-doing in risk-taking has hardly been illuminated in the works mentioned above, although is intuitive and may emerge in a diversity of settings. It seems quite rational for students and graduates to choose a career start in strategic consulting as it suggests a wide range of various projects that can help acquire sufficient expertise to be a desired candidate in the market in future, in contrast to other business fields where candidates are expected to implement rather routine tasks with a very short period of learning. One of the common strategies is to work for a consulting firm for several years, gain wide experience and increase the level of intellectual faculty, and then become a job seeker once again but with a broader background and work experience thereby aspiring to a higher wage than originally. Thus, the influence of possible acquiring expertise may be far-reaching for people who have just launched a career. Moreover, these people are ready to work for salaries less than in industry or banking since it is not only money that determine their choice but prospects of acquiring experience as well. Learning-by-doing may affect managers as well if they have only started carving out their
career. For instance, when a person becomes a fund manager, she may be more concerned about the market’s assessment of her skills and only partly interested in earning cash. Hence, her decisions may differ from a firm’s first-best; she can try to take excessive risk but gain new knowledge. Recent empirical studies by Hayes and Hillegeist (2008) have shown that new CEOs are more likely to accept job offers with lower compensation with firms that face a high level of financial distress risk. The authors explain this result by the fact that the managers seeking a job with a high-risk firm may simply have less attractive outside options. However, the effect of acquiring expertise may also result in such evidence. The main idea is that running a high-risk firm increases opportunity and the scale of gaining new skills, which may magnify attractiveness of the job for a manager with career concerns. Indeed, the manager may accept an offer with lower compensation knowing that the expertise acquired will increase his compensation in future. An attempt is made to get an insight into the manager’s decisions over investment projects with possible acquiring expertise. This paper tries to reveal the importance of learning-by-doing in the managerial framework.

The paper consists of four parts. In the first part a model is suggested and the main result by Holmström is verified. In the second part a two-project pool is considered and incentives to acquire expertise taking risk are investigated (we will call these incentives implicit, in contrast to explicit incentives that stem from a contract with an employer). The third part deals with the contract theory and explicit incentives that can be applied in an agent-principal framework. The final part investigates implicit incentive distortion that may arise in a three-period model, in which the manager makes decision based on the outcome of the previous project taken.

Main results are as follows. If no expertise can be acquired and a risk-averse manager has only reputational concerns, she will opt for the safest project. However, the result does not hold if she can gain some experience from a risky project. Nonetheless, her incentives to prefer a risky project decline as the market a priori expects a higher level of her talent or if she gets more concerned for a project payoff. There are situations when the principal is not able to shift the manager’s preferences towards the safe projects, in which case he
can lower the manager’s wage thereby extracting her surplus from acquiring expertise even if she does not work for the principal in the next period. As for the risky project, there are cases in which the principal cannot manipulate the manager’s decision against the safe project with linear contract. Option contract on the upside alone can rectify the situation. After a successful implementation of a risky project the manager is less inclined to take risk once again. However, the more expertise she has acquired, the more likely she will choose another risky project.

2 The Model

In this section we will focus our attention on a two-period model; but it can be easily expanded on many periods and in Section V we will proceed to a three-period model. So for the next three sections consider a two-period model. In period one a manager and the market receive information on the manager’s ability \( \eta \), that is to say information on its initial distribution:

\[
\eta \sim \text{Exp}(a), \quad f_\eta(x) = \frac{1}{a} e^{-x/a}.
\]

Parameter \( a \) stands for a verifiable measure of educational background and employment history implying that a higher \( a \) refers to a higher degree in science and more work experience. Therefore, a richer history implies a higher expected level of the manager’s skill \( \mathbb{E}[\eta] = a \), but a higher dispersion \( \text{Var}[\eta] = a^2 \) as well. The ability \( \eta \) helps the manager to be successful in an investment project, which is chosen from a pool

\[
\{p_k(\eta), y_k^+, y_k^-, \mu_k\}, \quad k \in K \subseteq \mathbb{R}^+\]

where \( p_k(\eta) \) is a probability of success; \( y_k^+ \), \( y_k^- \) are project yields (losses) in cases of success and failure respectively; \( \mu_k \) is the expertise acquired while working on the project. As needed, this general setup will be simplified for tractability.

After the manager chooses a project she implements it and the result is obtained in
period 2. Knowing the outcome and the project chosen (that is $k$), the market reassesses the manager’s skills according to a simple Bayesian rule:

$$\eta^+ = \mathbb{E}[\eta|\text{success}], \quad \eta^- = \mathbb{E}[\eta|\text{failure}].$$

Let $\hat{\eta}$ be the market’s perception of the enhanced (or decreased) ability. Therefore, if there were no acquiring expertise, $\hat{\eta}$ would be a random variable that equals $\eta^+$ with probability $p_k(\eta)$ and $\eta^-$ with probability $1 - p_k(\eta)$. However, in our case the market is aware of the project difficulty, so

$$\hat{\eta} = \begin{cases} 
\eta^+ + \mu_k, & p = p_k(\eta), \\
\eta^- + \mu_k, & q = 1 - p_k(\eta).
\end{cases}$$

The fact that the probability of success, but not the outcome of a project, depends on $\eta$ makes the analysis of the problem complicated in case the manager cares about her wages which are determined in the equilibrium. Section IV deals with possible wage contracts. As for now we are interested in the manager’s incentives alone, in the absence of a contingent contract. Suppose the manager chooses a project so as to maximize the expected utility that depends not on the manager’s wages in two periods, but on the market’s assessment of her talent and the outcome of the project in the second period. This assumption may be relevant, for instance, to situations where $\eta$ represents the alternative value of the manager’s skills outside the firm. Specifically, we will take the assumption of Milbourn et al (2001) implying that the manager derives utility from $c$:

$$c = \hat{\eta} + \alpha y, \quad \alpha > 0,$$

where $\alpha$ stands for the proportion of importance the manager attaches to a project yield. The manager trying to maximize her expected utility chooses a project in the first period:

$$\mathbb{E}[u(c)] \rightarrow \max_{k \in K},$$
s.t. \( c = \hat{\eta}_k + \alpha y_k \).

Assume that utility function \( u(c) \) is strictly increasing, concave, and \( u(0) = 0 \).

Finally, we assume that the probability of success \( p \) is related to the manager’s ability \( \eta \) according to

\[
p_k(\eta) = 1 - e^{-\eta/k}.
\]

Now, \( k \) represents not only a project name but also its riskiness: the greater \( k \), the riskier the project. Let \( k = 0 \) correspond to a safe project, i.e. \( p_0(\eta) = 1 \).

**Lemma 1.** Ex post densities of \( \eta \) if project \( k \) is success and failure are as follows:

\[
f_\eta(x|s) = \frac{a + k}{a^2} \left( 1 - e^{-x/k} \right) e^{-x/a},
\]

\[
f_\eta(x|f) = \frac{a + k}{ka} e^{-x/k} e^{-x/a},
\]

Ex ante probabilities of success and failure are

\[
\mathbb{E}[P(s|\eta)] = \frac{a}{a + k},
\]

\[
\mathbb{E}[P(f|\eta)] = \frac{k}{a + k},
\]

where “s” and “f” stand for success and failure respectively.

The proofs of all lemmas are in the Appendix.

**Lemma 2.** Ex post expectations of \( \eta \) before adjustment to expertise acquired, if project \( k \) is success and failure, are as follows:

\[
\eta^+ = a + \frac{ak}{a + k},
\]

\[
\eta^- = \frac{ak}{a + k}.
\]

Now, consider a general result that was obtained in Holmström’s model and can be
transferred to our set-up.

**Proposition 1.** If \( \alpha = 0 = \mu_k, k \geq 0 \), the manager will opt for a project that reveals least information about her ability, i.e. she will pick \( k = 0 \).

**Proof.** The manager’s problem can be rewritten as

\[
\mathbb{E}[u(\hat{\eta}_k)] \longrightarrow \max_{k \in K},
\]

but

\[
\mathbb{E}[u(\hat{\eta}_k)] = \mathbb{E}[P_k(s|\eta)u(\eta^+_k) + P_k(f|\eta)u(\eta^-_k)] = \frac{a}{a + k} u \left( a + \frac{ak}{a + k} \right) + \frac{k}{a + k} u \left( \frac{ak}{a + k} \right).
\]

\[
\frac{\partial \mathbb{E}[u(\hat{\eta}_k)]}{\partial k} = \frac{a^2}{(a + k)^2} \left\{ \mathbb{E}[u'(\hat{\eta}_k)] - \frac{1}{a} \left( u \left( a + \frac{ak}{a + k} \right) - u \left( \frac{ak}{a + k} \right) \right) \right\}.
\]

\[
\left. \frac{\partial \mathbb{E}[u(\hat{\eta}_k)]}{\partial k} \right|_{k=0} = u'(a) - \frac{1}{a} u(a) < 0
\]

since \( u(c) \) is concave and \( u(0) = 0 \), which implies the fact that \( k^* = 0 \) is a local maximum.

Moreover, as the manager is risk averse, at any positive \( k \)

\[
\frac{a}{a + k} u \left( a + \frac{ak}{a + k} \right) + \frac{k}{a + k} u \left( \frac{ak}{a + k} \right) < u \left( \frac{a}{a + k} \left( a + \frac{ak}{a + k} \right) + \frac{k}{a + k} \left( \frac{ak}{a + k} \right) \right) = u(a).
\]

Q.E.D.

**Example.** Note that Proposition 1 does not hold when \( \alpha > 0 \), as concern in the profit of a project may be high enough to shift the manager’s preferences towards a risky project.

Indeed, suppose \( k = 0,1 \) and \( \alpha = 1, y_1^- = 0, y_1^+ = y \), then \( k = 1 \) will be chosen iff

\[
\frac{a}{a + 1} u \left( a + \frac{a}{a + 1} + y \right) + \frac{1}{a + 1} u \left( \frac{a}{a + 1} \right) \geq u(a),
\]

which holds for large enough \( y \).
Lemma 3. If project $k$ is chosen, ex ante expectation of the future $c$ is

$$E[c|k] = a + \left( \frac{a}{a+k}g_k + \frac{k}{a+k}b_k \right),$$

and

$$E[c^2|k] = a^2 + a^3 \frac{k}{(a+k)^2} + \left( \frac{a}{a+k}g_k^2 + \frac{k}{a+k}b_k^2 \right) + 2ag_k - \frac{2ak^2}{(a+k)^2}(g_k - b_k),$$

where

$$g_k = \mu_k + \alpha y_k^+, \quad b_k = \mu_k + \alpha y_k^-.$$

From now on we assume that there are only two projects: a safe and a risky one. Choosing a safe project will not cause any increase in the manager’s skills: $\mu_0 = 0$. Since the project is risk-free, we normalize the profit of another project to $y_0$, which is why we assume $y_0 = 0$. In contrast, a risky project with $k = 1$ allows her to acquire some expertise $\mu_1 = \mu$. Let $y_1^- = -y, \quad y_1^+ = y, \quad y > 0$. There are two possible implications. Section III deals with implicit incentives, i.e. the manager is assumed to receive a fixed wage and therefore she makes decisions based on the trade-off between an opportunity of acquiring some expertise and a risk of worsening the market’s assessment of her talent in case of failure. In section IV we add explicit incentives to the model in terms of linear compensation. Throughout these two sections we assume quadratic preferences to simplify the calculus.

3 **Implicit Incentives**

Let the utility function be quadratic:

$$u(c) = c - rc^2,$$

where $r$ is a measure of the manager’s risk-aversion: the greater it is, the less risk she is willing to take. Suppose $r$ is sufficiently small so that $c$ never goes up to $1/2r$. In this case
the safe project will bring utility to the manager:

\[ U_0 = a - ra^2. \]

We assume that \( \mu \) is bounded from above so that the best scenario when the manager chooses the risky project would yield less than the summit of the parabola:

\[ \mu : \eta^+ + \mu + \alpha y < \frac{1}{2r} \Rightarrow \mu < \frac{1}{2r} - a - \frac{a}{a+1} - \alpha y = \bar{\mu}. \]

Then, taking into account Lemma 3, utility from the risky project may be represented as

\[
U_1 = a + \mu + \alpha y \frac{a-1}{a+1} - r \left( a^2 + \alpha (a + 1) \right) + \left( \frac{a}{a+1}(\mu + \alpha y)^2 + \frac{1}{a+1}(\mu + \alpha y)^2 \right) + 2a(\mu + \alpha y) - \frac{4ya\alpha}{(a+1)^2}. \]

The risky project will be chosen iff \( U_1 \geq U_0 \).

**Lemma 4.** If \( \mu_0 = 0 \) and \( \mu_1 = \mu \), \( U_1 > U_0 \) iff

\[
r\mu^2 + (2r\alpha \bar{y} + 2ra - 1)\mu + \frac{ra^3}{(a+1)^2} + \alpha c < 0,
\]

where

\[
\bar{y} = \frac{a}{a+1}y^+ + \frac{1}{a+1}y^-,
\]

\[
c = \frac{ra}{a+1}(y^+)^2 + \frac{r}{a+1}(y^-)^2 + 2ar y^+ - \frac{2ar}{(a+1)^2}(y^+ - y^-) + (1 - 2ar)y_0 + r\alpha y_0^2 - \bar{y}.
\]

In our case, first, let \( \alpha = 0 \). Then, Lemma 4 turns into

\[ U_1 \geq U_0 \iff r\mu^2 + (2ra - 1)\mu + \frac{ra^3}{(a+1)^2} \leq 0. \tag{1} \]

If \( r \) is sufficiently small \( (r < 1/4a) \), the set of \( \mu \) such that (1) holds is an interval

\[
\mu \in [\mu^-(r,a), \mu^+(r,a)],
\]
where
\[ \mu^\pm(r, a) = \frac{1}{2r} - a \pm \frac{1}{2} \sqrt{\left(2a - \frac{1}{r}\right)^2 - \frac{4a^3}{(a + 1)^2}}. \]

However, the right boundary \( \mu^+(r, a) \) is such that
\[ \mu^+(r, a) = \frac{1}{2r} - a + \frac{1}{2} \sqrt{\left(2a - \frac{1}{r}\right)^2 - \frac{4a^3}{(a + 1)^2}} \geq \frac{1}{2r} - a \geq \bar{\mu}. \]

Thus, only \( \mu^-(r, a) \) is of any interest. This function is increasing in \( r \), which is intuitive: the more risk-averse the manager is, the less she wants to choose a risky project all other things equal. Now suppose, \( r \) is sufficiently small, so that \( \mu^-(r, a) < \bar{\mu} \). Then,
\[ U_1 \geq U_0 \iff \mu \in [\mu^-(r, a), \bar{\mu}]. \]

We can elaborate on the result: the following proposition holds.

**Proposition 2.** If \( r < \frac{1}{4a} \), the lowest possible \( \mu \) encouraging the manager to choose a risky project \( \mu^-(r, a) \) is increasing in the manager’s average ability \( a \).

**Proof.**
\[ \frac{\partial \mu^-(r, a)}{\partial a} = -1 + \left( \frac{1}{r} - 2a + \frac{3a^2}{(a + 1)^2} - \frac{2a^3}{(a + 1)^3} \right) \left( \sqrt{\frac{1}{r} - 2a} - \frac{4a^3}{(a + 1)^2} \right)^{-1} > 0 \iff \]
\[ \iff \left( \frac{1}{r} - 2a \right) + C_1 > \sqrt{\left( \frac{1}{r} - 2a \right)^2 - C_2}, \]
where
\[ C_1 = \frac{a^2(a + 3)}{(a + 1)^3}, \quad C_2 = \frac{4a^3}{(a + 1)^2}. \]

However, the last inequality is always true whenever
\[ \frac{1}{r} - 2a > \sqrt{C_2}, \]
or substituting $C_2$:

$$r < \frac{1}{2a \left(1 + \sqrt{\frac{a}{a+1}}\right)}.$$  

But easy to see that

$$\frac{1}{4a} < \frac{1}{2a \left(1 + \sqrt{\frac{a}{a+1}}\right)}.$$  

Q.E.D.

Proposition 2 states that when the market and the manager a priori expect a high level of her skill, i.e. when she has a great deal of educational background, she will be less inclined to take risk despite a chance to increase her expertise. This result makes it possible to investigate a multi-period dynamic perspective as a low initial level of the manager’s skills forces her to opt for risky projects and acquire more expertise thereby providing further incentives to pick safer projects in the future. Such analysis is performed in Section V.

Now, consider $\alpha > 0$. To simplify some calculations let $a = 1$ as now the dependency on $\alpha$ is of interest for the rest of this section. In this case, Lemma 4 is equal to

$$U_1 \geq U_0 \iff r\mu^2 + (2r - 1)\mu + \alpha y^2 + \alpha y + \frac{r}{4} < 0.$$  

Again, suppose $r$ is sufficiently small to ensure the existence of at least one $\mu$ satisfying the inequality:

$$U_1 \geq U_0 \iff \mu \in [\mu^-(r, \alpha, y), \bar{\mu}], \quad \mu^-(r, \alpha, y) = \frac{1}{2r} - 1 - \sqrt{\left(\frac{1}{2r} - 1\right)^2 - \alpha y^2 - \alpha y - 1}.$$  

**Proposition 3.** The lowest possible $\mu$ encouraging the manager to choose a risky project $\mu^-(r, \alpha, y)$ is increasing in $\alpha$ and $y$.

**Proof.** The result is straight-forward as $\alpha y^2 + \alpha y$ is increasing in $\alpha$ and $y$. Q.E.D.

Notice that $a = 1$ makes the expected profit from the risky project equal to zero. Hence, an increase in yield uncertainty $y$ will scare a risk-averse manager away from the risky project, which is quite natural. In addition to this, Proposition 3 also states that an increase
in $\alpha$ - share of a project profit in the utility - will also make the manager be less inclined to take the risk. Hence, some explicit incentives in terms of a linear contract with a principal may deeply affect the manager’s choice between the projects, especially in cases when $\alpha$ is not necessarily equal to one. We will elaborate on this issue in Section IV.

4 Endogenous Compensation Scheme

So far the manager only had concerns for her future reputation and the outcome of the project chosen. We now turn to studying a firm’s optimal contract choice. Consider the following modification of the model: the manager is hired by the principal, and in the first period the principal offers a compensation in terms of a fixed payment $\gamma$ and a share in the project $\alpha \in [0,1]$. Therefore, total compensation is $w_1 = \gamma + \alpha y_k$, where $y_k$ is the outcome of the manager’s investment decisions. Upon observing $(\gamma, \alpha)$ the manager chooses a project to implement. In the second period the outcome is revealed, the manager receives her wage $w_1$, and the principal receives $y_k - w_1$. The manager’s talent assessment $\mathbb{E}[\eta]$ is her external opportunity wage; she always has an outside option to work in the market for both periods implementing some routine, which is equivalent to safe projects, and earning reservation wage, that is her expected talent. To simplify we assume that while working for the principal, the manager does not get to make investment decisions in the second period thereby she expects the wage to be equal to $w_2 = \hat{\eta}$ in the second period. The interpretation may be such that, for example, the manager’s contract is one-period and she may not be hired by the same principal in the second period, or she implements the project and then works on it for another period without making any decisions, and, consequently, she receives the competitive wage. Thus, unlike the principal, the manager cares about her further wage and her problem may be rewritten as

$$\mathbb{E}[u(w_1) + u(w_2)] \rightarrow \max_k$$

s.t. $w_1 = \gamma + \alpha y_k, \ w_2 = \hat{\eta}(k)$,
while the principal solves the problem

\[ \mathbb{E}[y_k^* - w_1] \rightarrow \max_{\gamma \in \mathbb{R}, \alpha \in [0, 1]} \]

s.t. \( w_1 = \gamma + \alpha y_k^* \), \( k^* \) is chosen by the manager given \( \gamma, \alpha \).

The principal also has to take into consideration an outside option of the manager: she can always choose not to work for the principal at all and receive wage equal to \( \mathbb{E}[\eta] = a \) in both periods. Therefore, the following inequality must hold:

\[ \mathbb{E}[u(w_1) + u(w_2)] \geq 2u(a). \]

The project pool is the same as in the previous chapter. First, the case of the verifiable choice will be considered, in which the principal can include his desirable choice of the project into the contract. Second, the unverifiable case will be discussed. Therefore, individual rationality (IR) and incentive compatibility (IC) constraints should be taken into consideration. Finally, possible modifications of the contract that will compel the manager to bear only upside of the outcome of her decision will be analyzed.

### 4.1 First best project choice

Suppose the principal can include the desired project into the contract explicitly. Then, there is no need selecting \( \alpha \neq 0 \), as there would be losses due to the manager’s risk-aversion. Therefore, throughout this subsection optimal \( \alpha = 0 \). Moreover, IR constraints are binding. In this case, although the expected return on the risky project equals

\[ \mathbb{E}[y_1] = \frac{a - 1}{a + 1} y, \]

which is greater than zero iff \( a > 1 \), it is still questionable whether the principal will choose this project. The results of the previous chapters show that the less manager is talented ex ante, the more implicit incentives she has to opt for a risky project, which may compel the
principal to force the manager into implementing the risky project even if \( a < 1 \) as he can gain the manager’s surplus from acquiring expertise.

As the manager has a forgone opportunity of working in the market for two periods receiving a wage equal to \( \mathbb{E}[\eta] = a \), in the case of the safe project, \( IR \) constraint is as follows

\[
\mathbb{E}[u(\gamma + \alpha y_k) + u(\hat{\eta}(k))|k = 0] \geq 2u(a),
\]

or substituting \( u(x) = x - rx^2 \):

\[
\mathbb{E}[(\gamma + \alpha y_k) - r\mathbb{E}[(\gamma + \alpha y_k)^2] + \mathbb{E}[\hat{\eta}(k)|k = 0] - r\mathbb{E}[\hat{\eta}^2(k)|k = 0] \geq 2a - 2ra^2.
\]

Taking into account the fact that \( \mathbb{E}[y_k|k = 0] = \mathbb{E}[y_k^2|k = 0] = 0 \) and \( \mathbb{E}[\hat{\eta}(k)|k = 0] = a \), \( \mathbb{E}[\hat{\eta}^2(k)|k = 0] = a^2 \) we obtain

\[
\gamma - r\gamma^2 \geq a - ra^2.
\]

The smallest \( \gamma \) satisfying the above inequality is \( \gamma = a \). Thus, the principal’s profit is equal to \( \Pi_0 = -a \).

In the case of the risky project, \( IR \) constraint is given by

\[
\mathbb{E}[u(\gamma + \alpha y_k) + u(\hat{\eta}(k))|k = 1] \geq 2u(a),
\]

or substituting the utility function:

\[
(IR_1) \quad \gamma - r\gamma^2 - M \geq a - ra^2
\]

where \( M = r\mu^2 - (1-2ra)\mu + \frac{ra^3}{(a+1)^2} \). Note that according to (1) \( M = U_0 - U_1 \leq 0 \iff U_1 \geq U_0 \), i.e. the manager is going to pick a risky project if her decisions are based on implicit incentives alone. From now on we will interpret all the conditions in terms of \( M \) instead of \( \mu \) to preserve the simplicity of the result representation. We will always keep in mind that condition \( M \leq 0 \) is equivalent to the manager’s picking the risky project, while \( M > 0 \) is equal to her choosing a safe project given no explicit incentives.
Then, the minimal $\gamma$ satisfying $IR_1$ is as follows

$$\gamma = \frac{1 - \sqrt{(1 - 2ra)^2 - 4rM}}{2r}.$$ 

Therefore, the principal’s expected utility in this case is given by

$$\Pi_1 = \frac{a - 1}{a + 1}y - \frac{1 - \sqrt{(1 - 2ra)^2 - 4rM}}{2r}.$$ 

The risky project is the principal’s first best if

$$\Pi_1 \geq \Pi_0,$$

or substituting the profits and making some rearrangements

$$\frac{a - 1}{a + 1}y + \sqrt{\left(\frac{1}{2r} - a\right)^2 - \frac{M}{r}} \geq \frac{1}{2r} - a.$$  

Consequently, if $M \geq 0$ (which in terms of (1) means $\mu \leq \mu^-(r, a)$ thereby the manager is willing to choose the safe project if she has no share in the project) and $a < 1$, the risky project is never optimal. However, if one of the conditions is violated ($M < 0$ or $a > 1$), either of the projects may be selected by the principal depending on $y$ and $M$. If both are violated, the risky project is the firm’s first best.

Nonetheless, if the choice of the project cannot be written down in the contract explicitly, the principal will have to take (IC) constraints into account, which complicates the problem and may alter the firm’s choice as compared with the first best one and therefore result in efficiency losses.

4.2 Second best choice

Now suppose the manager herself chooses the project and her action cannot be stipulated in the contract explicitly. The same intuition as in the previous section holds: even if the
expected return on the risky project is greater than zero ($a > 1$), the principal’s choice in favor of that project is still ambiguous. Since the manager is more inclined to choose the safe project if the assessment of her talent is high ex ante, the principal may have to offer a substantially high bonus to shift her preferences thereby reducing the attractiveness of the risky project for the principal. Thus, the maximum utility levels will be determined given the choice of a contract, $IR$, and $IC$ constraints; they will be compared afterwards, thereby the second best solution can be found.

Nonetheless, several cases are easy to deal with. When $M \geq 0$ and $a < 1$ the manager is not willing to take risk given implicit incentives alone. Since the expectation of the payoff of the risky project is negative:

$$E[y_1] = \frac{a - 1}{a + 1} y,$$

it is never optimal for the principal to induce the manager to choose the risky project as he can achieve the first-best outcome by simply putting $\alpha = 0, \gamma = a$. Therefore, the manager will choose the safe project, which is the first-best one.

Another similar case is when $M \leq 0$ and $a > 1$. The manager is willing to tolerate risk and gain some expertise while the expectation of the profit from the risky project is $E[y_1] > 0$. Consequently, the principal will set $\alpha = 0$ and achieve the first-best outcome ($k = 1$) by extracting the manager’s surplus as was shown in the previous subsection.

Thus, we will focus our attention on the remaining two cases: $M < 0$, $a < 1$ and $M > 0$, $a > 1$ (the case $a = 1$ is trivial). These two cases will be discussed in the following way. First, we will derive the principal’s problem under the condition that the safe project is optimal. Then, we will do the same but under the condition that the risky project is optimal. Finally, we will compare the principal’s profits under these two assumptions and obtain the results as to what project is optimal unconditionally.

The safe project is desirable. If the principal wants the manager to opt for the safe project, $IC$ and $IR$ constraints should be met. Assume that $r$ is so small that (2) is equivalent to

$$(IR_0) \quad \gamma \geq a$$
for possible solutions $\gamma$.

By definition, $IC_0$ constraint can be represented as

$$E[u(\gamma + \alpha y_k) + u(\hat{\eta}(k))|k = 0] \geq E[u(\gamma + \alpha y_k) + u(\hat{\eta}(k))|k = 1].$$

After substituting the utility function and rearranging some terms, the $IC_0$ constraint may be rewritten as

$$(IC_0) \quad r\alpha^2 y^2 + M \geq \frac{a - 1}{a + 1}(1 - 2r\gamma)\alpha y.$$

Thus, the maximal profit the principal could receive when she induces choosing the safe project is

$$\Pi_0 = \max_{\alpha, \gamma} \{-\gamma\} \quad (4)$$

$$s.t. \quad r\alpha^2 y^2 + M \geq \frac{a - 1}{a + 1}(1 - 2r\gamma)\alpha y, \quad \gamma \geq a.$$  \quad (5)

If it is assumed that $M > 0$, or in terms of (1) $\mu \leq \mu^-(r, a)$, then it can easily be shown that $\alpha = 0$ and $\gamma = a$ will be solutions to the above optimization problem. This result is quite intuitive as the manager herself would be interested in safe project if she had only reputational concerns (as it was shown in the previous section). Thus, no further explicit incentives are needed.

However, the result becomes ambiguous if $\mu$ is greater than $\mu^-(r, a)$ ($M < 0$) and the manager is likely to chose a risky project given no other incentives. Examine this case in more detail. The $IC_0$ constraint may be rewritten as

$$r\alpha^2 y^2 + M \geq (1 - 2r\gamma)\alpha y \frac{a - 1}{a + 1},$$  \quad (6)

where $M < 0$ according to the assumption (the manager would be going to choose the risky project given $\alpha = 0$). If $a < 1$, i.e. the safe project is the firm’s first best a priori, the right hand side in (6) is decreasing in $\alpha$, and it follows immediately from $IR_0$ constraint and the functional form of $\Pi_0$ that we are interested in $\gamma$ that is closest to $a$. Thus, $\alpha$ should be
taken so that

\[ r\alpha^2y^2 + M = (1 - 2ar)\alpha y \frac{a - 1}{a + 1}. \]

If value of \( \alpha \) solving the above equation is greater than one, then even if \( \alpha_0 = 1 \)

\[ ry^2 + M < (1 - 2r\gamma)y \frac{a - 1}{a + 1} \bigg|_{\gamma=a}. \]

Since the right hand side is increasing in \( \gamma \), there exist no such \( \gamma \) and \( \alpha \) that \( IR_0 \) and \( IC_0 \)
are both met if \( M < 0 \) and \( a < 1 \). Thus, the restriction on \( M \) that makes \( IR_0 \) and \( IC_0 \)
incompatible is given by

\[ M < a - 1 \frac{1}{a + 1}(1 - 2ra)y - ry^2. \] (7)

**The risky project is desirable.** In this case total utility the manager would obtain choosing
\( k = 1 \) is given by

\[ \mathbb{E}[u(\gamma + \alpha y_k) + u(\hat{\eta}(k))|k = 1] = \gamma + (1 - 2r\gamma)\alpha \frac{a - 1}{a + 1} y - r(\gamma^2 + \alpha^2y^2) + a - ra^2 - M. \]

Therefore, \( IR_1 \) and \( IC_1 \) constraints are as follows

\[ \begin{align*}
(\text{IR}_1) & \quad \gamma + (1 - 2r\gamma)\alpha \frac{a - 1}{a + 1} y - r(\gamma^2 + \alpha^2y^2) - M \geq a - ra^2, \\\n(\text{IC}_1) & \quad \gamma + (1 - 2r\gamma)\alpha \frac{a - 1}{a + 1} y - r(\gamma^2 + \alpha^2y^2) - M \geq \gamma - r\gamma^2.
\end{align*} \]

Hence, if the principal wants the manager to choose the risky project, the principal’s maximum profit is

\[ \Pi_1 = \max_{\alpha, \gamma} \left\{ (1 - \alpha) \frac{a - 1}{a + 1} y - \gamma \right\} \] (8)

\[ \text{s.t. } (1 - 2r\gamma)\alpha \frac{a - 1}{a + 1} y - M \geq ra^2y^2 + \max\{a - \gamma - r(a^2 - \gamma^2); 0\}. \] (9)

As it was said above, two cases are of great interest: \( M < 0 \) and \( a < 1 \) (the manager would like to choose the risky project when it is unprofitable ex ante) and \( M > 0 \) and \( a > 1 \) (the manager would like to choose the safe project when the risky one is profitable ex ante).
Moreover, the lower $a$, the lower $\mu^-$ and the more likely $M < 0$. The higher $a$, the higher $\mu^-$ and the more likely $M > 0$.

In the first case the principal will have to reward the manager with higher $\gamma_0$ (this is optimal $\gamma$ under the condition that the safe project is the principal’s second best one) and punish her in case of failure in order to make her opt for the safe project. At the same time $\gamma_1$ (this is optimal $\gamma$ under the condition that the risky project is the principal’s second best one) may be lower since the principal can collect the manager’s surplus if she in her turn decides to take the risky project. It is straightforward, applying Lagrange multiplier method to (8), to show that $\alpha = 0$, which is quite intuitive since the manager is risk-averse and the principal is not. In order not to impose unnecessary risk on the manager, the principal can bear the risk himself but can now offer lower fixed wage to the manager. Hence, $IC_1$ constraint always holds, and the minimal $\gamma$ the principal can achieve is the smaller root in $IR_1$:

$$r\gamma^2 - \gamma + a - ra^2 + M = 0,$$

or

$$\gamma_1 = \frac{1 - \sqrt{(1 - 2ar)^2 - 4rM}}{2r}.$$

Thus, two options should be compared if there is a solution to problem (4):

$$\Pi_1(y, M) = \frac{a - 1}{a + 1}y - \frac{1}{2r}(1 - \sqrt{1 - 4ar + 4a^2r^2 - 4rM}) \text{ vs. } \Pi_0 = -a.$$

We can now obtain the following proposition.

**Proposition 4.** If the risky project is preferable for the manager given $\alpha = 0$ (that is, if $M < 0$) and $a < 1$, for any $y > 0$ there exists $M$ (that is the parameters $r, \mu$) such that it is profitable for the principal not to induce the manager to opt for the safe project, but instead to take all the surplus from the manager allowing her to choose the risky one and attaining his first best.

**Proof.** The result is straightforward as $\lim_{M \to -\infty} \Pi_1(y, M) = +\infty$ and $M$ significantly
depends on $r, \mu$ (may be made as small as necessary when $r \to 0$).

Proposition 4 states that however bad an the outcome of the project may be ex ante, the expertise acquired by the manager while engaging in it may be high enough for the principal to benefit from the project even though the manager would not work for the principal in the next period. Thus, simply be reducing the wage, the principal will gain more than he could by altering explicit incentives to shift the manager’s preferences towards the safe project.

The second case, in which $M > 0$ and $a > 1$ may be interpreted as if the manager is highly skilled a priori, and, therefore, she does not have enough implicit incentives to choose the risky project even though she is sufficiently experienced to succeed in it. The solution in this case will not be similar (implying that there is always an opportunity for the principal not to force the manager to choose the risky project) since $M$ is bounded from above for $\mu \in [0, \bar{\mu}]$. Indeed, the utility function is well-defined only if $r < \frac{1}{4a}$ and on the left boundary $\mu = 0$

$$M = \frac{ra^3}{(a + 1)^2} \leq \frac{a^2}{4(a + 1)^2} < \frac{1}{4},$$

whereas an increase in the right boundary $\mu = \bar{\mu}$ only adds more incentives for the manager to opt for the risky project thereby resulting in $M < 0$. Thus, the following proposition holds.

**Proposition 5.** If the safe project is preferable for the manager given $\alpha = 0$ (that is $M > 0$) and if $a > 1$, then there exist $y$ and $a$ such that it is profitable for the principal to induce the manager to opt for the risky project by offering her some share in the project.

**Proof.** Suppose $\alpha = 1$ and $\gamma = a$, then (9) can be rewritten as follows:

$$(1 - 2ra)\frac{a - 1}{a + 1}y - ry^2 \geq M.$$  

However, as it was shown $M$ that is bounded from above since $1 - 2ra \geq 1/2$ (which follows from assumption $r < \frac{1}{4a}$). Hence, if $a$ goes to infinity, the left hand side can be made strictly larger than the upper boundary of $M$. Then, reducing $\gamma$ by small $\varepsilon$ makes $\Pi_1 > -a = \Pi_0$

Q.E.D.
Nonetheless, this case is more complicated than the previous one: sometimes the principal will not find it optimal to induce the manager to opt for the risky project.

**Proposition 6.** If the safe project is preferable for the manager given \( \alpha = 0 \) (that is \( M > 0 \)) and \( a > 1 \), for any \( y \) and \( M \) there exists \( a \) such that it is profitable for the principal not to induce the manager to opt for the risky project.

**Proof.** Again, consider (9):

\[
(1 - 2r\gamma)\alpha \frac{a - 1}{a + 1} y \geq M + r\alpha^2y^2 + \max\{a - \gamma - r(a^2 - \gamma^2); 0\}.
\]

Since \( \alpha \) is bounded from above and \( \gamma \) is bounded from below (otherwise \( IR_1 \) constraint will never be met), it follows that for any given \( y \) and \( M \)

\[
A = (1 - 2r\gamma)\alpha y \leq \bar{A}.
\]

In this case let \( a = 1 + \varepsilon \), where \( \varepsilon = \frac{M}{2\bar{A}} \). Then

\[
(1 - 2r\gamma)\alpha \frac{a - 1}{a + 1} y \leq \frac{M}{2(a + 1)} < \frac{M}{2} \leq M + r\alpha^2y^2 + \max\{a - \gamma - r(a^2 - \gamma^2); 0\}.
\]

Q.E.D.

The proposition states that there are cases (when \( a \) is close to 1) in which the principal has no control over the manager’s decision in favor of the risky project.

Propositions 4–6 can be summarized in the following diagram, drawn for \( r = 0.01, x = 1 \).
The risky project is the principal’s second best in red areas (II–IV).

Here in area I \(M\) is positive implying that the manager is eager to choose the safe project; the optimal choice of the principal is not to induce the manager to opt for a risky project (even if \(a > 1\) as it is stated in Proposition 6). Nonetheless, in area II the result of Proposition 5 can be applied, and the risky project is optimal. Area III represents the case of negative \(M\) (the manager is willing to choose the risky project); for \(a > 1\) the result is intuitive. However, if \(a < 1\), the risky project is still optimal as the manager’s will to undertake the risky project is too strong for the principal to influence. In area IV the principal has enough power to shift the manager’s preferences toward the safe project. Nevertheless, Proposition 4 states that for small values of \(M\), the principal will be better off if he allows the manager to choose the risky project and takes all her surplus from acquiring expertise. In contrast, area V is the one, in which the second best is to induce the manager to undertake the safe project even if she is not inclined to do so.

We can now compare the first best and the second best choices. As it was noticed above, the only two relevant cases are (i) \(M < 0, a < 1\) (part of area III and areas IV,V), and (ii) \(M > 0\) and \(a > 1\) (part of area I and area II) since otherwise \(\alpha = 0\) and \(\gamma = a\) are the solutions both to the first best and the second best problems, as it was shown above.
(i) This is the case when the manager opts for the risky project if she does not have any share in the project. Thus, efficiency losses may occur only if the safe project is the firm’s first best (otherwise $\alpha = 0$ and there are no losses). Then from (3):

$$\frac{a - 1}{a + 1}y + \sqrt{\left(\frac{1}{2} - a\right)^2 - \frac{M}{r}} < \frac{1}{2} - a,$$

or

$$M > \frac{a - 1}{a + 1}y \left(1 - 2ra - \frac{a - 1}{a + 1}ry\right).$$

Now compare it with (7):

$$M < \frac{a - 1}{a + 1} \left(1 - 2ra - ry^2\right) < \frac{a - 1}{a + 1}(1 - 2ra)y - \left(\frac{a - 1}{a + 1}\right)ry^2 =$$

$$= \frac{a - 1}{a + 1}y \left(1 - 2ra - \frac{a - 1}{a + 1}ry\right) < M.$$

Hence, there is no such $M$ that the safe project is the first best choice but it is incompatible with ($IC$) constraint, and consequently the first best is attainable here (the principal can offer a share in the project to scare the manager away from a risky project). It should be noticed that in this case $\alpha > 0$ since otherwise the manager will switch to the risky project, so the principal should give the share of the project to the manager. Nevertheless, due to the fact that the manager will chose the safe project, there are now efficiency losses.

(ii) Again if the safe project is first best, the principal will set $\alpha = 0$ and the manager will choose that project given $M > 0$. Therefore, the case with the risky project being first best is of any interest. Then from (3):

$$\frac{a - 1}{a + 1}y + \sqrt{\left(\frac{1}{2} - a\right)^2 - \frac{M}{r}} > \frac{1}{2} - a,$$

or

$$M < \frac{a - 1}{a + 1}y \left(1 - 2ra - \frac{a - 1}{a + 1}ry\right).$$
Obviously, there are parameters \( r, y, a \) such that

\[
\frac{a - 1}{a + 1} y \left( 1 - 2ra - \frac{a - 1}{a + 1} ry \right) > 0,
\]

so there exists \( M \) satisfying \( M > 0 \). This observation leads to the fact that even if the principal is able to make the manager opt for a risky project, he will do this only by setting \( \alpha > 0 \) (or otherwise she will choose the safe project as \( M > 0 \)), which means that a risk neutral principal transfers some risk to a risk-averse manager, which causes welfare losses. Moreover, sometimes the safe project will be more profitable for the principal as it is stated in Proposition 6.

To sum up, in case \( a < 1 \) or \( M < 0 \) (i.e. when the manager has limited expertise at first and (or) the project is relatively productive in terms of generating additional expertise), the principal can always achieve the first best outcome without any welfare losses whereas if \( a > 1 \) and \( M > 0 \) the principal is bound to bear some extra costs. Moreover, Proposition 2 states that the former is more likely as \( a \) is growing, which is why some principals may be more willing to hire less experienced managers, and give them a smaller compensation in case they decide to take some risk and gain new experience.

4.3 Option contracts

Some possible improvement may be introduced if the risky project is desirable but \( a > 1 \) and \( M > 0 \). It may be a contract that gives the manager upside alone:

\[
w = \gamma + \alpha y^+ I_{\{\text{success}\}}.
\]

In this case (9) turns into

\[
\frac{a}{a + 1} (1 - 2r\gamma) \alpha y - M \geq \frac{a}{a + 1} r\alpha^2 y^2 + \max\{a - \gamma - r(a^2 - \gamma^2); 0\}.
\]
Thus, the left hand side increases as the manager does not have downside whereas the variance decreases, and so does the right hand side. Therefore, the result of Proposition 6 is weakened. Indeed, if, for instance, for $\gamma = a$ and $\alpha = \frac{a-1}{a} - \varepsilon$ the above inequality holds for some $y$ (which may occur if $M$ is close to zero), the principal will gain more from the risky project:

$$\Pi_1 \geq \frac{(1 - \alpha)a - 1}{a + 1} y - a > -a = \Pi_0,$$

even if $a$ is close to one. Hence, the option contract may resolve the incentive problem in this case. Nonetheless, the ultimate solution to the principal’s problem significantly depends on the parameters of the problem.

### 5 A Three-Period Model

We will now proceed to a three-period model in order to reveal how the manager’s incentives can change depending on the result of the previous period outcome. Consider a two-project pool and three periods $t = 0, 1, 2$.

Suppose the manager chooses between a safe and a risky project twice: at $t=0$ and at $t=1$. The outcomes of the projects are independent and there is no possible commitment at $t = 0$. At $t = 0$ she is aware that if she opts for a risky project ($k_0 = 1$), she will acquire expertise $\mu_1$ and therefore increase the chance of success at $t = 2$ in case another risky project is chosen at $t = 1$. If she opts for a risky project ($k_1 = 1$) for the second time at $t = 1$, she will acquire expertise $\mu_2$. As above, a risk-free project ($k_i = 0$) will not increase the manager’s skills. The information is symmetric in that the market learns the outcome of the projects taken at $t = 1$ and $t = 2$ and forms expectations of the manager’s ability. The manager does not care about the yield of the project ($\alpha = 0$), only the market’s assessment matters. Consequently, the manager solves the following problem:

$$E_0[u(E_1[\eta|k_0]) + \theta u(E_2[\eta|k_0, k_1])] \longrightarrow \max_{k_0, k_1 \in \{0,1\}}.$$
Suppose that when the manager is indifferent between the projects, she will choose a risk-free one. This assumption simply sorts out ties and makes the following proposition more concise. The proposition states that if the manager chooses not to take risk at $t = 0$, it is almost never optimal to take risk at $t = 1$.

**Proposition 7.** If $k_0 = 0$ is the optimal decision for the manager, then $k_1 = 0$.

*Proof.* Suppose $k_0 = 0$ is optimal, in which case a safe project is chosen at $t = 0$ and there is no new information at $t = 1$. Hence, at $t = 1$ the manager solves the problem

$$u(a) + \theta \mathbb{E}_1 u(\mathbb{E}_2[\eta|k_1]) \longrightarrow \max_{k_1 \in \{0,1\}} .$$

Then $k_1 = 1$ is optimal iff

$$\mathbb{E}_1 u(\mathbb{E}_2[\eta|k_1 = 1]) > u(a).$$

But in this case $k_0 = 1, k_1 = 0$ strictly dominates the current strategy as

$$\mathbb{E}_0 \{u(\mathbb{E}_1[\eta|k_0 = 1]) + \theta u(\mathbb{E}_2[\eta|k_0 = 1, k_1 = 0])\} = \mathbb{E}_0 \{u(\mathbb{E}_1[\eta|k_0 = 1])\} + \theta \mathbb{E}_0 \{u(\mathbb{E}_1[\eta|k_0 = 1])\} > u(a) + \theta \mathbb{E}_0 [u(\mathbb{E}_1[\eta|k_0 = 1])].$$

The first equality holds as $k_1 = 0$ does not give any new signal, which is why the market’s assessment of the manager’s skill remains the same. Hence, it must be that $k_1 = 0$. Q.E.D.

At this point we can concentrate on the choice $k_1$ as determined by the result of the risky project taken at $t = 0$.

First, assume that at $t = 1$ there was a failure, in which case the upgraded distribution of $\eta$ may be found using Lemma 1:

$$\eta|f = \mu_1 + \xi_f, \text{ where } \xi_f \sim f(x|f) = \frac{a + 1}{a} e^{-x} e^{-x/a}.$$
Lemma 5. If at \( t = 1 \) a risky project failed, at \( t = 2 \) the market’s assessments will be

\[
\mathbb{E}_2[\eta | ff] = \mu_1 + \mu_2 + C_{ff}(a),
\]

\[
\mathbb{E}_2[\eta | fs] = \mu_1 + \mu_2 + C_{fs}(a, \mu_1),
\]

where

\[
C_{ff}(a) = \frac{a}{2a+1},
\]

\[
C_{fs}(a, \mu_1) = \frac{a}{a+1} \frac{1 - \left(\frac{a+1}{2a+1}\right)^2 e^{-\mu_1}}{1 - \frac{a+1}{2a+1} e^{-\mu_1}},
\]

and the probability of success at \( t = 2 \) given the failure at \( t = 1 \) is given by

\[
\mathbb{E}_1 P(s \text{ at } t = 2 | \eta, f \text{ at } t = 1) = 1 - \frac{a+1}{2a+1} e^{-\mu_1}.
\]

Therefore, given a failure of the first project at \( t = 1 \) a risky project will be chosen iff

\[
\left(1 - \frac{a+1}{2a+1} e^{-\mu_1}\right) u(\mu_1 + \mu_2 + C_{fs}(a, \mu_1)) + \frac{a+1}{2a+1} e^{-\mu_1} u(\mu_1 + \mu_2 + C_{ff}(a)) \geq u \left(\mu_1 + \frac{a}{a+1}\right).
\]

(10)

The same result may be obtained for the case in which there was a success at \( t = 1 \) :

Lemma 6. If at \( t = 1 \) a risky project was successful, at \( t = 2 \) the market’s assessments will be

\[
\mathbb{E}_2[\eta | sf] = \mu_1 + \mu_2 + C_{sf}(a),
\]

\[
\mathbb{E}_2[\eta | ss] = \mu_1 + \mu_2 + C_{ss}(a, \mu_1),
\]

where

\[
C_{sf}(a) = \frac{a}{a+1} \frac{3a+2}{2a+1},
\]

\[
C_{ss}(a, \mu_1) = \frac{a}{a+1} \frac{a+2 - \frac{3a+2}{(2a+1)^2} e^{-\mu_1}}{1 - \frac{1}{2a+1} e^{-\mu_1}},
\]

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and the probability of success at $t = 2$ given success at $t = 1$ is given by

$$E_1 P(s \text{ at } t = 2|\eta, s \text{ at } t = 1) = 1 - \frac{1}{2a + 1} e^{-\mu_1}.$$  

Therefore, given a success of the first project at $t = 1$ a risky project will be chosen iff

$$\left(1 - \frac{1}{2a + 1} e^{-\mu_1}\right) u(\mu_1 + \mu_2 + C_{ss}(a, \mu_1)) + \frac{1}{2a + 1} e^{-\mu_1} u(\mu_1 + \mu_2 + C_{sf}(a)) \geq u \left( \mu_1 + a + \frac{a}{a + 1} \right).$$  

(11)

We are now interested in comparing (10) and (11) in order to find out how the implicit incentives change depending on the result of the first project. In contrast to the previous sections we assume

$$u(c) = -e^{-\gamma c}.$$  

It should be noted that the same analysis can be conducted in case of the quadratic preferences with similar results. The CARA utility function is chosen to rule out wealth effects: we would expect the decision on the new project not to depend on the outcome of the previous one, as the increment of $\mu_1$ is present from both sides of the above inequalities. However, according to Lemmas 5-6 undertaking the risky project changes the weights of each Period 2 outcome as well as the market’s assessments, which complicates the analysis.

The decision whether to choose a risky project for the second time at $t = 1$ depends on $\mu_2$: it is easy to check (as we did in Proposition 1) that due to concavity of the utility function if $\mu_2 = 0$, the left hand sides of (10) and (11) are lower than the right hand sides. Moreover, the left hand sides are increasing in $\mu_2$, hence, there is one threshold for success and one threshold for failure, below which a safe project will be chosen at $t = 1$ but above which a risky project will be chosen.

Solving (10) and (11) for $\mu_2$ we can obtain the thresholds in case of failure and success respectively:

$$\mu_2^f(a, \mu_1) = \frac{a}{a + 1} + \frac{1}{\gamma} \ln \left\{ \left(1 - \frac{a + 1}{2a + 1} e^{-\mu_1}\right) e^{-\gamma C_{ss}(a, \mu_1)} + \frac{a + 1}{2a + 1} e^{-\mu_1} e^{-\gamma C_{sf}(a)} \right\},$$

$$\mu_2^s(a, \mu_1) = \frac{a}{a + 1} + \frac{1}{\gamma} \ln \left\{ \left(1 - \frac{a + 1}{2a + 1} e^{-\mu_1}\right) e^{-\gamma C_{ss}(a, \mu_1)} + \frac{a + 1}{2a + 1} e^{-\mu_1} e^{-\gamma C_{sf}(a)} \right\},$$

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$\mu_2^s(a, \mu_1) = a + \frac{a}{a + 1} + \frac{1}{\gamma} \ln \left\{ 1 - \frac{1}{2a + 1} e^{-\mu_1} \right\} e^{-\gamma C_{ss}(a, \mu_1)} + \frac{1}{2a + 1} e^{-\mu_1} e^{-\gamma C_{sf}(a)}$,

**Proposition 8.** The following inequalities hold:

$\mu_2^s(a, \mu_1) > \mu_2^f(a, \mu_1), \quad \frac{\partial \mu_2^i(a, \mu_1)}{\partial a} > 0, \quad \frac{\partial \mu_2^i(a, \mu_1)}{\partial \mu_1} < 0, \quad i \in \{s, f\}$.

**Proof.** These properties of the thresholds that can be checked directly by taking partial derivatives, but due to complexity of the expressions we will omit them in this paper and present the results visually. The following graphs illustrate the dependency of the thresholds on $a$ and $\mu_1$.

**Figure 1:** Threshold levels of $\mu_2^s$ at $a = 0.5, a = 1, a = 2$ respectively. The blue (upper) line is for success and the red (lower) line is for failure at $t = 1$.

**Figure 2:** Threshold levels of $\mu_2^f$. The upper surface is for success at $t = 1$. 

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Properties that are visible on the graphs are as follows. First, the threshold level for success is always above the one for failure, which means that given the same parameters, if the manager decides to choose a risky project for the second time when the first one was a success, he is bound to opt for a risky project when the first one was a failure. The intuition is as follows. There are two effects that work in different directions: a learning-by-doing effect pushes the manager’s preferences towards a risky project whereas reputational concern drives them away. If the manager chooses a risky project after successful implementation of the first project, reputational concern is not very strong, which warrants implementation of a risky project in case of failure since reputational concern is even weaker in this case. Indeed, in case of failure, after the market has downgraded its assessment of the manager’s skill, she does not have much to lose in case of second failure. However, when the market comes to appreciate the manager’s talent more after a successful implementation of the first project, she can lose much in case of failure. Hence, she may not take risk for the second time.

Another observation is the fact that both curves move upward as $a$ increases. This effect may be another piece of evidence for Proposition 2. The more the manager is talented a priori the less inclined she is to take risk. However, both these thresholds are decreasing in $\mu_1$, which allows us to separate a priori expectation of her level of experience $a$ (which can be drawn from the manager’s CV for example, and, therefore, is not exact) and experience acquired from implementing a project $\mu_1$, which is well-known by all the participants. The former has a negative effect on risk-taking since the manager is afraid that the market will learn “too much” if she decides to take risk, whereas the latter shifts the whole distribution of her talent and increases the probability of success, which has a positive influence on risk-taking.

6 Conclusion

The paper explores the idea of introducing learning-by-doing into the common framework of managerial decisions on risky projects, which proves to be an intriguing one. The work deals with a model that shows some predictable results like those concerning a situation
without gaining any possible experience from risky projects; but it also supports an idea that a risk-averse manager with high initial expectation of the talent is more likely to choose a safe project than she is in case of low initial expectation. Of course the model proposed is quite general and can be studied with a project pool different from the one examined here.

As for the agent-principal framework, the contract theory implications show that sometimes it is profitable for the principal not to induce the manager to choose the safe project but to extract her surplus from acquiring expertise. Even if the choice of a project cannot be written down in the contract explicitly, the principal can achieve his first-best choice but creating certain incentives. However, when the manager is reluctant to tolerate risk, which may occur if the initial expectation of her talent is high, there may be efficiency losses as the principal will have to offer some share of the project to the manager thereby imposing some risk on her.

The result of Proposition 2 makes the multi-period case of the model quite attractive as acquiring expertise can reduce the manager’s incentives to do so in future. The three-period model gives an insight into possible effects that may account for this fact. When reputational concerns (or the fear that the market will change its assessment of the manager’s talent) are weaker after a risky project has failed, the manager has more incentives to take a risky project again. As for a learning-by-doing effect, the three-period model separates the influence of the initial expectation of the manager’s talent, which is inaccurate and therefore the manager is afraid of revealing her true level of talent in choosing the risky project, and the influence of the expertise acquired during the work on this project, which increases the possibility of success in the future and reduces the negative effect of reputational concerns on risk-taking.
References


Lemma 1.

Ex post densities of $\eta$ if project $k$ is success and failure are as follows:

\[ f_\eta(x|s) = \frac{a + k}{a^2} (1 - e^{-x/k}) e^{-x/a}, \]
\[ f_\eta(x|f) = \frac{a + k}{ka} e^{-x/k} e^{-x/a}, \]

Ex ante probabilities of success and failure are

\[ \mathbb{E}[P(s|\eta)] = \frac{a}{a + k}, \]
\[ \mathbb{E}[P(f|\eta)] = \frac{k}{a + k}, \]

where “s” and “f” stand for success and failure respectively.

Proof. From Bayes’ rule

\[ f_\eta^n(x) = f_\eta(x|s) = \frac{P(s|\eta)f_\eta(x)}{\mathbb{E}[P(s|\eta)]} = \frac{(1 - e^{-x/k}) e^{-x/a}}{a\mathbb{E}[P(s|\eta)]}, \]
\[ \mathbb{E}[P(s|\eta)] = \mathbb{E}[1 - e^{-x/k}] = 1 - \frac{1}{a} \int_0^\infty e^{-x(1/k + 1/a)} = 1 - \frac{1}{a} \frac{1}{(1/k + 1/a)} = \frac{a}{a + k} \Rightarrow \]
\[ \Rightarrow f_\eta^n(x) = \frac{a + k}{a^2} (1 - e^{-x/k}) e^{-x/a}. \]

Similarly,

\[ f_\eta^-(x) = f_\eta(x|f) = \frac{P(f|\eta)f_\eta(x)}{\mathbb{E}[P(f|\eta)]} = \frac{e^{-x/k}e^{-x/a}}{a\mathbb{E}[P(f|\eta)]} = \frac{a + k}{ka} e^{-x/k} e^{-x/a}. \]

Lemma 2.
Ex post assessments of $\eta$ if project $k$ is success and failure are as follows:

$$
\eta^+ = a + \frac{ak}{a + k},
$$

$$
\eta^- = \frac{ak}{a + k}.
$$

**Proof.**

$$
\eta^+ = \mathbb{E}[\eta|s] = \int_0^\infty x f_{\eta^+}(x)dx = \frac{a + k}{a^2} \int_0^\infty x \left(1 - e^{-x/k}\right) e^{-x/a}dx = \left\{ \int_0^\infty xe^{\lambda x}dx = \frac{1}{\lambda^2} \right\} = \\
= \frac{a + k}{a^2} \left(a^2 - \left(\frac{ak}{a + k}\right)^2\right) = \frac{a^2 + 2ak}{a + k} = a + \frac{ak}{a + k}.
$$

Similarly,

$$
\eta^- = \mathbb{E}[\eta|f] = \int_0^\infty x f_{\eta^-}(x)dx = \frac{a + k}{ak} \int_0^\infty xe^{-x/k}e^{-x/a}dx = \frac{a + k}{ak} \left(\frac{ak}{a + k}\right)^2 = \frac{ak}{a + k}.
$$

**Lemma 3.**

If project $k$ is chosen, ex ante expectation of the future $c$ is

$$
\mathbb{E}[c|k] = a + \left(\frac{a}{a + k}g_k + \frac{k}{a + k}b_k\right),
$$

and

$$
\mathbb{E}[c^2|k] = a^2 + a^3 \frac{k}{(a + k)^2} + \left(\frac{a}{a + k}g_k^2 + \frac{k}{a + k}b_k^2\right) + 2ag_k - \frac{2ak^2}{(a + k)^2}(g_k - b_k),
$$

where

$$
g_k = \mu_k + \alpha y_k^+, \ b_k = \mu_k + \alpha y_k^-.
$$
Proof.

$$\mathbb{E}[c|k] = \mathbb{E}[\mathbb{E}[\hat{\eta}|\eta]] = \mathbb{E}[(\eta^+ + g_k)P(s|\eta) + (\eta^- + b_k)P(f|\eta)] = a + \left(\frac{a}{a+k}g_k + \frac{k}{a+k}b_k\right),$$

$$\mathbb{E}[c^2|k] = \mathbb{E}[\mathbb{E}[\hat{\eta}^2|\eta]] = \mathbb{E}[(\eta^+ + g_k)^2P(s|\eta) + (\eta^- + b_k)^2P(f|\eta)] = \left(a + \frac{ak}{a+k}g_k\right)^2 \frac{a}{a+k} +$$

$$+ \left(\frac{ak}{a+k} + b_k\right) \frac{k}{a+k} = a^2 + a^3 \frac{k}{(a+k)^2} + \left(\frac{a}{a+k}g_k^2 + \frac{k}{a+k}b_k^2\right) + 2a \frac{g_k(a^2 + 2ak) + b_kk^2}{(a+k)^2} =$$

$$= a^2 + a^3 \frac{k}{(a+k)^2} + \left(\frac{a}{a+k}g_k^2 + \frac{k}{a+k}b_k^2\right) + 2ag_k - \frac{2ak^2}{(a+k)^2}(g_k - b_k).$$

Lemma 4.

If $\mu_0 = 0$ and $\mu_1 = \mu$, $U_1 \geq U_0$ iff

$$r\mu^2 + (2r\alpha\bar{y} + 2ra - 1)\mu + \frac{r\alpha^3}{(a+1)^2} + \alpha c < 0,$$

where

$$\bar{y} = \left(\frac{a}{a+1}y^+ + \frac{1}{a+1}y^-\right),$$

$$c = \frac{ra}{a+1}(y^+)^2 + \frac{r}{a+1}(y^-)^2 + 2ary^+ - \frac{2ar}{(a+1)^2}(y^+ - y^-) + (1-2ar)y_0 + r\alpha y_0^2 - \bar{y}.$$

Proof.

Taking into account Lemma 3, utility from the risky project may be represented as

$$U_1 = a + \mu + \alpha \left(\frac{a}{a+1}y^+ + \frac{1}{a+1}y^-\right) - r \left(a^2 + a^3 \frac{1}{(a+1)^2} + \right.$$

$$+ \left(\frac{a}{a+1}(\mu + \alpha y^+)^2 + \frac{1}{a+1}(\mu + \alpha y^-)^2\right) + 2a(\mu + \alpha y^+ - \frac{2a\alpha}{(a+1)^2}(y^+ - y^-)\right).$$

$$U_1 \geq U_\triangleright$$

$$\Leftrightarrow a + \mu + \alpha \left(\frac{a}{a+1}y^+ + \frac{1}{a+1}y^-\right) - r \left(a^2 + a^3 \frac{1}{(a+1)^2} + \right.$$

$$+ \left(\frac{a}{a+1}(\mu + \alpha y^+)^2 + \frac{1}{a+1}(\mu + \alpha y^-)^2\right) +$$

$$+ 2a(\mu + \alpha y^+) - \frac{2a\alpha}{(a+1)^2}(y^+ - y^-) \geq a + \alpha y_0 - r(\alpha + \alpha y_0)^2.$$
Let $\bar{y} = \left(\frac{a}{a+1}y^+ + \frac{1}{a+1}y^-\right)$, $\Delta y = y^+ - y^-$, then

$$U(1) \geq U(0) \iff \mu + \alpha(\bar{y} - y_0) - r \left(a^2 + a^3 \frac{1}{(a+1)^2} + \left(\frac{a}{a+1}(\mu + \alpha y^+)^2 + \frac{1}{a+1}(\mu + \alpha y^-)^2\right) + 2a(\mu + \alpha y^+) - \frac{2a\alpha}{(a+1)^2} \Delta y \right) + r(a + \alpha y_0)^2 \geq 0,$$

or

$$\mu + \alpha(\bar{y} - y_0) \geq r \left(a^3 \frac{1}{(a+1)^2} + \mu^2 + 2\mu\alpha\bar{y} + \alpha \left(\frac{a}{a+1}(y^+)^2 + \frac{1}{a+1}(y^-)^2\right) + 2a(\mu + \alpha y^+) - \frac{2a\alpha}{(a+1)^2} \Delta y - 2a\alpha y_0 + \alpha^2 y_0^2\right).$$

It may be represented as

$$r\mu^2 + (2r\alpha\bar{y} + 2ra - 1)\mu + \frac{ra^3}{(a+1)^2} + \alpha c < 0,$$

where

$$c = \frac{ra}{a+1}(y^+)^2 + \frac{r}{a+1}(y^-)^2 + 2ar^2 - \frac{2ar}{(a+1)^2} \Delta y + (1 - 2ar)y_0 + r\alpha y_0^2 - \bar{y}.$$ 

**Lemma 5.**

If at $t = 1$ a risky project failed, at $t = 2$ the distributions of the market’s assessments will be

$$E_2[\eta | ff] = \mu_1 + \mu_2 + \frac{a}{2a+1},$$

$$E_2[\eta | fs] = \mu_1 + \mu_2 + \frac{a}{a+1} \left(\frac{1 - (\frac{a+1}{2a+1})^2 e^{-\mu_1}}{1 - \frac{a+1}{2a+1} e^{-\mu_1}}\right),$$

and the probability of success at $t = 2$ given the failure at $t = 1$ is given by

$$E_1 P(s \ at \ t = 2 | \eta, f \ at \ t = 1) = 1 - \frac{a+1}{2a+1} e^{-\mu_1}.$$
Proof. First, calculate the distribution of $\eta$ given two failures:

$$\eta | ff = \mu_1 + \mu_2 + \xi_{ff},$$

where

$$\xi_{ff} \sim f(x|ff) = \frac{P(f \text{ at } t = 2|\eta = x, f \text{ at } t = 1)f(x|f)}{\int_0^{+\infty} P(f \text{ at } t = 2|\eta = x, f \text{ at } t = 1)f(x|f)dx}.$$

Since

$$P(f \text{ at } t = 2|\eta = x, f \text{ at } t = 1) = e^{-x-\mu_1},$$

and

$$f(x|f) = \frac{a+1}{a} e^{-x} e^{-x/a},$$

it follows that

$$\mathbb{E}_1 P(f \text{ at } t = 2|\eta, f \text{ at } t = 1) = \frac{a+1}{2a+1} e^{-\mu_1},$$

and

$$f(x|ff) = \frac{2a+1}{a} e^{-2x} e^{-x/a},$$

Similarly,

$$\eta | fs = \mu_1 + \mu_2 + \xi_{fs}, \quad \xi_{fs} \sim f(x|fs) = \frac{a+1}{a \left(1 - \frac{a+1}{2a+1} e^{-\mu_1}\right)} \left(1 - e^{-x-\mu_1}\right) e^{-x} e^{-x/a},$$

Then, taking expectations of $\eta | ff$ and $\eta | ff$ one can achieve the answer.

Lemma 6.

If at $t = 1$ a risky project was successful, at $t = 2$ the market’s assessments will be

$$\mathbb{E}_2[\eta|sf] = \mu_1 + \mu_2 + \frac{a}{a+1} \left(\frac{3a+2}{(2a+1)^2} e^{-\mu_1}\right),$$

$$\mathbb{E}_2[\eta|ss] = \mu_1 + \mu_2 + \frac{a}{a+1} \left(\frac{a+2}{2a+1} e^{-\mu_1}\right).$$
and the probability of success at $t = 2$ given success at $t = 1$ is given by

$$\mathbb{E}_t P(s \text{ at } t = 2 | \eta, s \text{ at } t = 1) = 1 - \frac{1}{2a + 1} e^{-\mu_1}.$$  

**Proof.** Since the outcome of the project do not influence ex ante probability of success (the same amount of expertise $\mu_1$ is acquired), the proof is similar to the previous one replacing $f(x|f)$ with

$$f(x|s) = \frac{a + 1}{a^2} (1 - e^{-x}) e^{-x/a}.$$  

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