COALITION-PROOF INCENTIVE CONTRACTS

Alexei Savvateev

CEMI, Moscow;
CORE, Louvain-La-Neuve, Belgium.

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“The incentive schemes optimal for Nash equilibrium implementation* < .... > *may suffer from vulnerability to collusion among agents.* < .... > *No satisfactory theory seems available* < .... > *and future research in this area will be valuable”

(Dilip Mookherjee (1984) “Optimal Incentive Schemes with Many Agents”, from *Conclusion* part.)

PLAN OF THE TALK

- **Motivating examples**
- **The model ("N-inspection problem")**
- **The main theorem**
- **Application to serial cost sharing**
- **Application to N-inspection problem**
- **Implementation**
- **Conclusions and future prospects**

MOTIVATING EXAMPLES

- **Tax evasion auditing (Savvateev 2003; Vasin 2005):**

$$u_i(z,p) = b_i(z) - pz,$$

where $b_i(z) := b(i,z)$ **is increasing, and increase in** i **with its derivative.**
Here, z **is a percentage of hidden transactions.**

- **Pollution control (thanks to H.Moulin):**

$$u_i(z,p) = b_i(z) - p \cdot \varphi(z),$$

where $\varphi(\cdot)$ **is a penalty when the pollution index is** z**.**

- **Under Nash equilibrium, it is easy to implement zero level of evasion (pollution).**

But NOT coalition-proof!

A GENERAL FORMULATION:
N-INSPECTION PROBLEM

z_i — ***cheating levels*;**

$p_i = p\left(z_i; \{z_j\}_{j \neq i}\right)$ — **a probability of being inspected.**

If there are P inspections available, then we must have that for any observed profile $z = (z_1, \ldots, z_n)$

$$\sum_{i=1}^{n} p\left(z_i; \{z_j\}_{j \neq i}\right) \leq P.$$

Perfect monitoring, costly enforcement!

HOW DOES IT RELATE TO STANDARD FRAMEWORK?

Perfect monitoring, costless enforcement ⇒

first-best is implemented via
individual piece-rate contracts

Perfect monitoring, costly enforcement,
Nash implementation

⇒ first-best through the
"first-rank contract" (as above)

AN ILLUSTRATION OF THE IMPORTANCE OF THE PROBLEM

Tax evasion; there are n agents, with benefit functions

$$\frac{z_1}{n}; \frac{z_2}{n-1}; \ldots \frac{z_{n-1}}{2}; z_n$$

To enforce $z_i \equiv 0$ under piece-rate contract, one needs

$$P = \frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{2} + 1;$$

To enforce $z_i \equiv 0$ under strong equilibrium, one needs $P = 1 + \varepsilon$! Incentive contract is *to inspect with equal probability all* $z_i > 0$.

SUFFICIENT CONDITIONS FOR STRONG EQUILIBRIUM

Consider the following "equal-treatment" mechanism design problem involving n agents:

Agents simultaneously choose levels of some "general good", $0 \leq z_i \leq 1$
(or from some finite set);

Each agent i pays for his demand, according to a "price correspondence",

$$p\left(z_i; \{z_j\}_{j \neq i}\right).$$

Agents have preference profiles $\succeq_i$ over a set of pairs $\{(z, p)\}$.

The aim is to design coalition-proof price mechanisms

MAIN THEOREM

If *preferences profile* satisfies

1. ***Spence-Mirrlees single crossing condition* in its weaker form (Milgrom, Shannon 1994); and**

   price mechanism $p\left(z_i; \{z_j\}_{j\neq i}\right)$ **satisfies**

2. ***order semi-independence:***

   $$p\left(z_i; \{z_j\}_{j\neq i}\right) = p\left(z_i; \{\max\{z_i, z_j\}\}_{j\neq i}\right), and$$

3. ***payoff complementarity* (a counter-part to *strategic complementarity*, (Milgrom, Roberts 1990):**

   $p\left(z_i; \{z_j\}_{j\neq i}\right)$ **is non-increasing in** $z_j,\ j \neq i$

THEN

There exists a strong equilibrium $z^* = (z_1^*, \ldots, z_n^*)$ ***in a game the agents play by choosing demands,*** z_i^*

****....AND THIS EQUILIBRIUM IS****

(a) *monotonic in* i, i.e. $z_1^* \le \cdots \le z_n^*$;

(b) *a maximum point* of the set of all Nash equilibria:

$$\forall i \quad z_i^* \ge \tilde{z}_i,$$

if $\tilde{z} = (\tilde{z}_1, \ldots, \tilde{z}_n)$ — another Nash equilibrium;

(c) (non-strictly) better than any other Nash equilibrium *for all* agents:

$$\forall i \quad (z_i^*, p_i^*) \succeq_i (\tilde{z}_i, \tilde{p}_i)$$

(d) can be approached by no more than n steps by a simple iterative procedure.

SERIAL SHARING OF PUBLIC COSTS

Moulin 1994 used the following formula, essentially the Shapley value, for dividing costs of a public good:

$$p\left(z_i; \{z_j\}_{j\neq i}\right) = \sum_{k=1}^{i} \frac{c(z_k) - c(z_{k-1})}{n-k+1},$$

In the convex environment, its outcome is a strong equilibrium.

***Main theorem* guarantees that it is still so for an arbitrary costs, *provided* Spence-Mirrlees conditions hold.**

VIRTUAL COST FUNCTION approach:

Replace $c(z)$ with a discrete-valued function, such that there are finite number of levels $\bar{z}_l$, $l = 1, \ldots, k$ with "incremental costs" A_l, $l = 1, \ldots, k$, and $c(\bar{z}_l) = \sum\limits_{h=1}^{l} A_h$.

MULTISTEP STRATEGIES

Multistep strategy **consists of several threshold levels**

$$0 \leq \bar{z}_1 < \cdots < \bar{z}_k < 1,$$

and probabilities

$$(A_1, \ldots, A_k)$$

attached to these thresholds.

Now,

$$p\left(z_i; \{z_j\}_{j \neq i}\right) = \sum_{\{l : \bar{z}_l < z_i\}} \frac{A_l}{\#\{j : \bar{z}_l < z_j\}}.$$

That is, agents with $z > \bar{z}_l$ divide together costs A_l for increasing the cheating degree above the threshold $\bar{z}_l$.

Every multistep strategy implements a strong equilibrium!

CONCLUSIONS

Why it is useful?

Tax evasion: relevance of multistep strategies (Vasin 2005)

Fix standard vs Fix numbers?

Best coalition-proof contracts?

Awards vs punishments

Extentions: payoff substitution