Serial Correlation and Asymptotic Variance

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Consider the stationary time series regression

$$y_t = \alpha + \beta x_t + e_t,$$

where the zero mean error e_t is uncorrelated with the regressor x_t. The object of interest is the asymptotic variance V_β of the OLS estimator of β.

(a) Construct an example where e_t is serially correlated, but nevertheless

$$V_\beta = \frac{Var(e_t)}{Var(x_t)}.$$

That is, one does *not* have to correct the standard error for serial correlation, even though serial correlation is present.

(b) Construct an example where e_t is serially uncorrelated, but nevertheless

$$V_\beta \neq \frac{Var(x_t e_t)}{[Var(x_t)]^2}.$$

That is, one *does* have to correct the standard error for serial correlation, even though there is no serial correlation in the error term.

(c) What implication do the results in (a) and (b) have regarding pre-testing for serial correlation?

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Suggested Solution

(a) Let x_t be a zero mean i.i.d. process independent of e_t, then for $j \neq 0$, even though $Cov\,(e_t, e_{t-j}) \neq 0$, we have

$$Cov\,(x_t e_t, x_{t-j} e_{t-j}) = Cov\,(x_t, x_{t-j})\,Cov\,(e_t, e_{t-j}) = 0$$

(where the first equality follows from x_t being zero mean i.i.d.), while

$$Var\,(x_t e_t) = Var\,(x_t)\,Var\,(e_t)\,,$$

so

$$V_\beta = \frac{Var\,(e_t)}{Var\,(x_t)}.$$

(b) Let x_t have mean zero and unit variance and follow a first-order autoregression driven by a strict white noise:

$$x_t = \rho x_{t-1} + \eta_t, \quad \eta_t \sim IID\left(0, 1 - \rho^2\right),$$

where $0 < \rho < 1$. Let a standard bivariate normal white noise $(v_t, w_t)'$ be independent of the process x_t. Construct e_t as

$$e_t = v_{t+1} + w_{t+1} x_{t+1} + \theta(v_t - w_t x_t)$$

for $0 < \theta < 1$. Then one can find that

$$Cov\,(x_t, e_t) = 0, \quad Cov\,(e_t, e_{t-j}) = 0 \quad \forall\ j \neq 0$$

and

$$Cov\,(x_t e_t, x_{t-1} e_{t-1}) \neq 0, \quad Cov\,(x_t e_t, x_{t-j} e_{t-j}) = 0 \quad \forall\, j > 1.$$

Thus

$$V_\beta = \frac{Var\,(x_t e_t) + 2Cov\,(x_t e_t, x_{t-1} e_{t-1})}{[Var\,(x_t)]^2} \neq \frac{Var\,(x_t e_t)}{[Var\,(x_t)]^2}.$$

(c) It follows that it is more appropriate to test for serial correlation the process $x_t e_t$ rather than e_t. By the same token, the process $z_t e_t$ should be tested if the estimator under consideration is the IV estimator with the instrument z_t.