The term structure of Russian interest rates

Stanislav Anatolyev* Sergey Korepanov

New Economic School, Moscow EvrazHolding, Moscow

*Corresponding author. Address: Stanislav Anatolyev, New Economic School, Nakhimovsky Pr., 47, Moscow, 117418 Russia. E-mail: sanatoly@nes.ru. We would like to thank Dmitry Novikov for helpful discussions, and all members of the NES research project "Dynamics and Predictability in Russian Financial Markets" for useful comments and suggestions.

The term structure of Russian interest rates

Abstract

Using the series of Moscow Interbank Offer Rates, we estimate a flexible parametrization of the diffusion process following the approach of Aït-Sahalia (1996) of matching parametric and nonparametric estimates of the marginal density. On the basis of the estimated model we compute the implied term structure using simulations.

Key words: Stochastic differential equation; Marginal density; Semi-parametric estimation; MIBOR; Term structure of interest rates.

JEL classification numbers: G12, G15, C14, C15, C22

1 Introduction

The term structure of interest rates is critical for pricing fixed-income securities. "Term-structure modeling is one of the major success stories in the application of financial models to everyday business problems" (Duffie, 2001, Chapter 7). A key element of a term structure is a model for the process followed by the instantaneous interest rate.

There exist several ways to estimate a diffusion process each having its strengths and shortcomings. In earlier years researches attempted to parsimoniously specify the drift and diffusion functions so that the difference equation could be solved analytically (e.g., Cox, Ingersoll and Ross, 1985). Lately researchers have moved to estimating the functions non-parametrically, or specifying them in more flexible ways and utilizing computer-intensive methods. For example, Chan at al. (1992) run GMM on moment conditions that are *approximately* implied by popular parametrizations of the differential equation, while Stanton (1997) performs nonparametric estimation of first-order *approximations* to the drift and diffusion functions. Hansen and Scheinkman (1995) propose to run GMM on a set of moment conditions that are an *exact* implication of a flexibly parametrized differential equation; this approach, however, leads to unpleasant concerns about a choice of the so called test function (see Hansen et al., 1997). Another possibility to employ a flexible parametrization is the approach of Aït-Sahalia (1996) of constructing an estimator and specification test based on minimization of the distance between a non-parametric and implied parametric forms of a marginal density. Even though this approach suffers from information loss due to the use of a marginal density (Pritsker, 1997), the suggestion of Aït-Sahalia (1996) to base the specification test on a transitional density leads to considerable computational difficulties in implementing the test.

In the present paper, using the series of Moscow Interbank Offer Rates (MIBOR), we explore some parametrizations of the differential equation and construct a reliable model for the term structure that may be further used for many possible applications. The analysis of Russian interest rates from the position of continuous time modeling, to our knowledge, has been encountered only in Dvorkovitch et al. (2000) who modeled the demand for Russian Government securities from non-residents. The authors employed non-parametric estimation of the drift and diffusion using the Stanton (1997) methodology, without any further investigation of properties of the data-generating process. In contrast, we follow the approach of Aït-Sahalia (1996) to arrive at point parameter estimates and a specification test statistic; in order to obtain inferences on parameter values we use a simple bootstrap procedure. Finally, on the basis of the estimated semi-parametric model we compute the implied term structure of interest rates using simulations.

The paper is organized as follows. In Section 2 we briefly describe the semi-parametric estimator and the specification test for correct parameterization of the model. In Section 3 we estimate a flexible specification for the drift and diffusion functions and test the model using the MIBOR series. In Section 4 we compute the term structure of interest rates.

2 Semi-parametric estimator and specification test

To model the behavior of instantaneous interest rate, we use a diffusion process that admits the representation by the stationary stochastic differential equation

$$dx_t = \mu(x_t)dt + \sigma(x_t)dB_t.$$

It is known (e.g., Aït-Sahalia, 1996) that for a particular parameterization of the drift and diffusion functions $\mu(x,\theta)$ and $\sigma^2(x,\theta)$, the parametric form for the marginal density can be derived as

$$\pi(x,\theta) = \frac{\xi(\theta)}{\sigma^2(x,\theta)} \exp \int_{x_0}^{x} \frac{2\mu(s,\theta)}{\sigma^2(s,\theta)} ds,$$

where

$$\xi(\theta) = \left(\int_{-\infty}^{+\infty} \left(\frac{1}{\sigma^2(x,\theta)} \exp \int_{x_0}^{x} \frac{2\mu(s,\theta)}{\sigma^2(s,\theta)} ds \right) dx \right)^{-1},$$

where x_0 is arbitrary. Aït-Sahalia (1996) proposes the following estimator of θ based on the quadratic distance from the parametric marginal density and its non-parametric estimate:

$$\hat{\theta} = \arg\min_{q \in \Theta} \int_{-\infty}^{+\infty} (\pi(x,q) - \hat{\pi}(x))^2 \pi_0(x) dx. \tag{1}$$

Here, $\pi_0(x)$ is the true marginal density, and $\hat{\pi}(x)$ is its non-parametric kernel estimator obtained from the sample $\{x_i\}_{i=1}^n$:

$$\hat{\pi}(x) = \frac{1}{nh_n} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h_n} \right),$$

where $K(\cdot)$ is a kernel function and h_n is a bandwidth parameter. By the analogy principle, we replace the integral in (1) by its sample analog

$$\frac{1}{n} \sum_{t=1}^{n} \left(\pi(x_t,\theta) - \hat{\pi}(x_t) \right)^2. \tag{2}$$

Aït-Sahalia (1996) shows that this estimator is consistent and asymptotically normal:

$$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{d} N(0, \Omega_M),$$

where Ω_M is a complicated expression that depends on the kernel and the value of θ.

The specification test is based on the idea that if the parameterization of the model is correct, the deviation of the parametric density from the non-parametric estimate should not be big. The test is based on the statistic

$$\hat{M} = h_n \sum_{i=1}^{n} (\pi(\hat{\theta}, x_i) - \hat{\pi}(x_i))^2,$$

whose null asymptotic distribution is

$$h_n^{-1/2}(\hat{M} - E_M) \xrightarrow{d} N(0, V_M),$$

where E_M and V_M may be estimated by

$$\hat{E}_M = \left(\int_{-\infty}^{+\infty} K^2(x) dx \right) \left(\frac{1}{n} \sum_{i=1}^{n} \hat{\pi}(x_i) \right)$$

and

$$\hat{V}_M = \left(\int_{-\infty}^{+\infty} \left\{ \int_{-\infty}^{+\infty} K(u)K(u+x)du \right\}^2 dx \right) \left(\frac{1}{n} \sum_{i=1}^{n} \hat{\pi}^3(x_i) \right).$$

For the Gaussian kernel which is used throughout,

$$\int_{-\infty}^{+\infty} K^2(x)dx = \frac{1}{2\sqrt{\pi}}, \quad \int_{-\infty}^{+\infty} \left\{ \int_{-\infty}^{+\infty} K(u)K(u+x)du \right\}^2 dx = \frac{1}{2\sqrt{2\pi}}.$$

To test the null hypothesis at the significance level α, the critical value is

$$\hat{c}(\alpha) = \hat{E}_M + z_{1-\alpha}\sqrt{h_n \hat{V}_M},$$

and the null hypothesis of correct specification is rejected if $\hat{M} \geq \hat{c}(\alpha)$.

3 Empirical results

The data is the series of daily observations of one-day MIBOR from January 1, 2000 to May 1, 2003, totaling to 805 observations. The data are obtained from `www.cbr.ru`, the official Central Bank of Russia site. The rate is determined daily from interbank credit trades at the Moscow Interbank Currency Exchange. The instantaneous interest rate corresponding to the observed one was computed by

$$r = 365 \ln \left(1 + \frac{r^{obs}}{365} \right).$$

As can be seen from Figure 1, the MIBOR series looks much smoother than the Eurodollar rate (cf. Aït-Sahalia, 1996, Figure 2).

The non-parametric estimator for the marginal density was computed[1] using the Gaussian kernel (other kernels produced very similar results), and shown in Figure 2. After some experimentation, h was set at 0.03. To compute the critical value for the test statistic $\hat{M}$

[1] All procedures were written in GAUSS. When numerical optimization was required, the `optmum` procedure was employed.

we employ the following characteristics of the data:

$$\frac{1}{n}\sum_{i=1}^{n}\hat{\pi}(x_i) = 6.394, \quad \frac{1}{n}\sum_{i=1}^{n}\hat{\pi}^3(x_i) = 377.97.$$

Then $\hat{E}_M = 1.79$, $\hat{V}_M = 74.6$, and the 5% critical value is $c_{0.05} = 4.26$.

First of all, fitting the popular Cox–Ingersoll–Ross (CIR), Vasicek (V) and Brennan–Schwartz (BS) models

$$\begin{aligned} \text{CIR:} \qquad & \mu(x,\theta) = \alpha_0 + \alpha_1 x, \quad \sigma(x,\theta) = \beta_0\sqrt{x}, \\ \text{V:} \qquad & \mu(x,\theta) = \alpha_0 + \alpha_1 x, \quad \sigma(x,\theta) = \beta_0, \\ \text{BS:} \qquad & \mu(x,\theta) = \alpha_0 + \alpha_1 x, \quad \sigma(x,\theta) = \beta_0 x, \end{aligned}$$

has been completely unsuccessful: the test statistics are huge compared to the critical value (e.g., it equals $\hat{M} = 72$ for the CIR model). This demonstrates that these analytically attractive models are too simple (in particular, the linearity of the drift function) for the Russian interest rate data.

In order to arrive at a flexible parametrization, we follow Stanton (1997) and estimate the first-order approximations to the drift and diffusion non-parametrically using the Nadaraya–Watson estimator with the Gaussian kernel. As the forms of emerged curves are quite close to those in Aït-Sahalia (1996), we employ his parametrization

$$\mu(x,\theta) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \frac{\alpha_3}{x}, \quad \sigma(x,\theta) = \beta_0 + \beta_1 x + \beta_2 x^{\beta_3}.$$

Since computation of asymptotic standard errors is problematic because of a complicated form of the matrix Ω_M, we employ a simple bootstrap procedure. As the proposed semi-parametric estimator does not utilize information on transitional features of the data, there is no need to preserve the temporal structure of the series, hence we adopted the basic IID bootstrap resampling. The computational difficulties are present, however, because of the need to achieve convergence for each bootstrap sample. This precludes obtaining a large number of bootstrap estimates. In our procedure, the bootstrap sample contained 50 estimates of the parameters vector. Standard errors were computed straightforwardly, taking into account that the bootstrap distribution is centered around estimated values rather than the true parameters.

The estimation results are reported in Table 1[2], and the resulting marginal density is depicted on Figure 2. The value of the test statistics is $\hat{M} = 0.069$ which is far smaller than the 5% critical value. Thus, the Aït-Sahalia (1996) flexible form fits the MIBOR data very well, and the estimated model has a strong mean reversion property. The MIBOR and Eurodollar deposit rates possess similar features, except that there is no evidence for several regimes in the MIBOR case. The two regimes for the Eurodollar are most probably due to a monetary policy shift in the beginning of 80's, while there was no structural change in Russia during the period under consideration. Note also that the value of the test statistics is appreciably below the minimal one for the Eurodollar rate, which is a consequence of a greater smoothness of the MIBOR and the absence of regime switches.

4 The term structure

One of most important applications of the diffusion model is derivation of the term structure of interest rates. If $\Lambda_{t,s}$ is a price at time t of a bond maturing time s, then it could be represented from the no-arbitrage principle as

$$\Lambda_{t,s} = E_t \left[\exp \left(\int_t^s -r_u du \right) \right].$$

The term structure is usually represented as the yield curve

$$y(\tau) = -\frac{\ln(\Lambda_{t,t+\tau})}{\tau}$$

[2]As a robustness check, we have also re-estimated the model after splitting the sample into two equal halves. All parameter estimates on the two subperiods (not shown) are quite close, and are withing two standard deviations from the estimates on Table 1.

One way to compute the term structure is to explicitly solve the Feyman–Kac differential equation. However, this option is unavailable for flexible parametrizations, hence we compute the term structure using simulations. The result is shown in Figure 3. The yield curve is monotonically increasing and slightly concave, without any humps.

5 Concluding remarks

Using the spot MIBOR series, we have estimated a flexible parametrization of the diffusion process following the approach of Aït-Sahalia (1996). The Aït-Sahalia (1996) parametrization fitted the MIBOR series very well, with decisive acceptance of the model by the specification test. On the basis of the estimated model we have computed the implied term structure using simulations. The corresponding yield curve turns out to be increasing and slightly concave.

References

1. Aït-Sahalia, Y., 1996. Testing continuous time models of the spot interest rate. Review of Financial Studies 9, 385–426.

2. Chan, K.C., Karolyi, G.A., Longstaff, F.A., Sanders A.B., 1992. An empirical comparison of alternative models of the short-term interest rate. Journal of Finance 47, 1209–1227.

3. Cox, J.C., Ingersoll, J.E. Jr., Ross, S.A., 1985. Theory of the term structure of interest rates. Econometrica 53, 385–407.

4. Dvorkovitch, A., Novikov, D., Urga, G., 2000. Risk factors influencing the demand for government securities by non-residents: The case of Russia before the August 1988 crisis. Working paper, IRMI, CUBS.

5. Duffie, D., 2001. Dynamic asset pricing theory. Princeton University Press.

6. Hansen, L.P., Scheinkman, J.A., 1995. Back to the future: Generating moment implications for continuous time Markov processes. Econometrica 63, 767–804.

7. Hansen, L.P., Conley, T.G., Luttmer, E.G.J., Scheinkman, J.A., 1997. Short-term interest rates as subordinated diffusions. Review of Financial Studies 10, 525–577.

8. Pritsker, M., 1997. Nonparametric density estimation and tests of continuous time interest rate models. Review of Financial Studies 11, 449–487.

9. Stanton, R., 1997. A nonparametric model of term structure dynamics and the market price of interest rate risk. Journal of Finance 52, 1973–2002.

Table 1

Estimates of parameters of the drift and diffusion functions

Function	$\mu(x,\theta)$				$\sigma(x,\theta)$			
Parameter	α_0	α_1	α_2	α_3	β_0	β_1	β_2	β_3
Point estimate	1.83	-0.39	-0.70	1.98	-0.03	0.57	-1.84	1.52
Standard error	(0.55)	(0.54)	(0.36)	(0.66)	(0.01)	(0.11)	(0.24)	(0.10)

Notes:

The estimates are obtained by minimizing (2), the sample distance between the parametric and non-parametric densities. The standard errors are obtained by bootstrap. For more details, see the text.

Figure 1

The MIBOR series

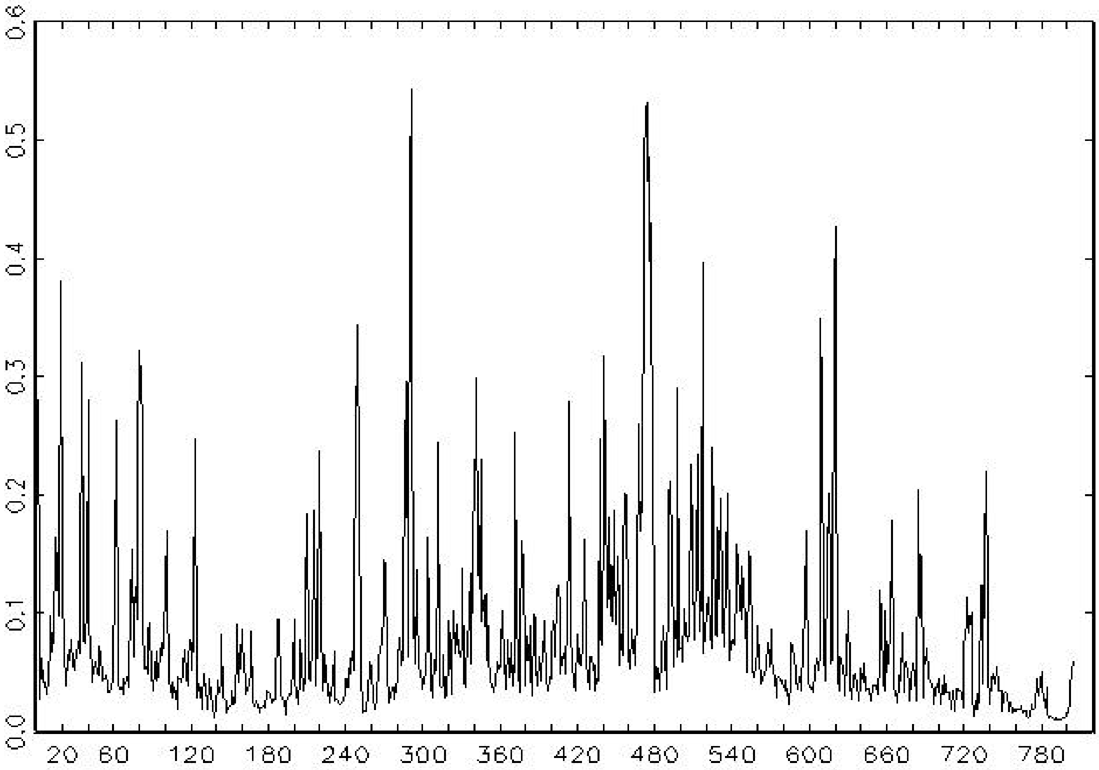

Figure 2

Parametric and non-parametric estimates of the marginal density

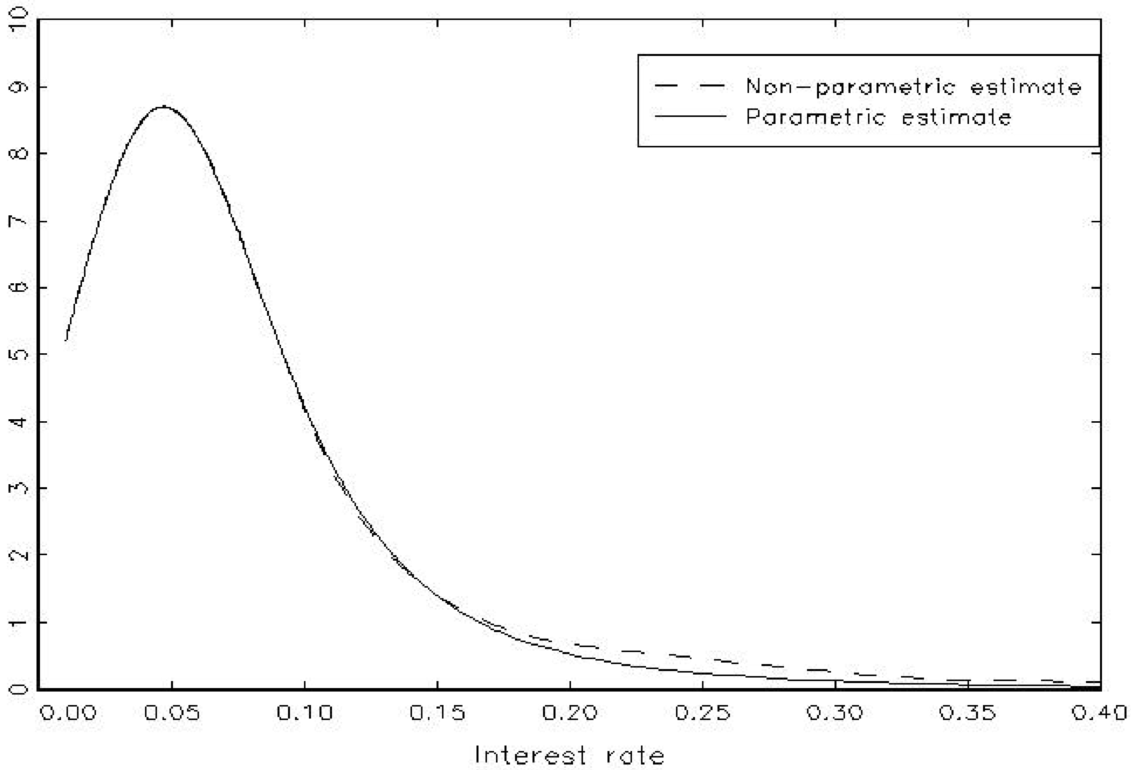

Figure 3

The term structure of interest rates