Limits of Acquisition in Price Competing Industry

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Abstract

It is well known that under differentiated product monopolistic competition any merger always increases the total profit of the merged entity. Because of this one might expect complete monopolization of a price competing industry provided that there are no (legal) barriers to acquisition (merger). In this paper we show that this is not always true. The industry may not get monopolized because the value of a fringe firm is getting higher when the concentration of the industry gets higher. This creates strategic incentives for a fringe firm to be last in the line of those who sell their businesses. Sometimes this type of incentive prevents industry monopolization.

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1 Introduction

The problem of industry monopolization is one of central questions in Industrial Organization. Farrell and Shapiro (1990), Gaudet and Salant (1992), Kamien and Zang (1990), Salant, Switzer and Reynolds (1983) studied quantity competing industries. They showed that in an industry composed of more than two firms, under a gradual merging process when at a time only two firms merge, initial acquisitions are not profitable when firms compete on quantities.[1] This may prevent industry monopolization in spite of the fact that the final outcome of the merging process benefits all parties – the monopoly collects highest possible profits in the industry, because the monopoly always has the option of mimicking the nonmonopolized market with any degree of industry concentration. Other papers such as Gowrisankaran and Holmes (2001), Lewis (1983), Krishna (1993), and Perry and Porter (1983) derived a similar result for price competing industries with shortage of capital or production capacities.

This paper contributes to the theory of industry concentration under monopolistic competition with a differentiated product in the presence of a single leading or expanding firm. To the best of our knowledge, there is no evidence of having the non-monopolization outcome in an industry where the expanding firm is allowed to make a sequence of offers for other firms businesses in order to gain more control over the market. It is known that (see Deneckere and Davidson (1983, 1985) for details) in a price competing industry if any two firms have decided to get united into a joint venture[2], they start collecting more profits than before, this being valid for an arbitrary number of firms in the market.[3] The same may be true for the quantity or Cournot competing industries provided that firms enjoy economies of scale from merger (e.g., see Kamien and Zang (1991)). Thus our analysis may be naturally extended to

[1] All these papers assume specifications of linear demand and constant marginal cost. In particular, Salant et al. (1983) showed that all mergers with market share lower than 80% are unprofitable. For other forms of demand this share takes lower values: 50% as in Cheung (1992), or even lower as in Levin (1990), Fauli-Oller (1997), and Hennessy (2000).

[2] It does not matter whether this happens by merger or acquizition.

[3] In economic theory this feature of profits is named *superadditivity*.

these situations. Hence, it may appear that by granting the right to one leading firm to buy any other firm's business, the industry will eventually become monopolized.

The main idea of this paper is to show that the acquisition process described above does not always lead to a one-firm industry. This happens for the following reason: the more industry is concentrated the higher profits are not only of the merged firm but all of other firms. Hence, if some firm is about to be offered to sell its business to an expanding firm, and it predicts that the concentration of industry will continue, this firm has an incentive to decline the offer and try to be the last in line of those who sells its business or demands more than its net current profit. This observation was also made by Stigler (1950); the following passage on pages 25 and 26 supports the same point.

> "If there are relatively few firms in the industry, the major difficulty in forming a merger is that it is more profitable to be outside a merger than to be a participant. ... Hence the promoter of a merger is likely to receive much encouragement from each firm - almost every encouragement, in fact, except participation."

Clearly, how much each firm gets paid depends on the negotiation process. We show that under certain negotiation patterns the amount the expanding firm needs to pay for the full industry capture may exceed extra profits. This happens in spite of the fact that the full monopolization gives the highest possible profits in industry. Also, it is shown that this kind of strategic behavior may stop acquisition at some point (so that the industry remains partially monopolized), or even may preclude any acquisitions. In addition, it is demonstrated that under certain circumstances the firm's acquisition does not give any net additional profits to the leading firm after full monopolization of the industry. All extra profits go to the other firms in form of payments for their businesses.

The paper is organized as follows. In Section 2 we reconcile the classical model of monopolistic competition with differentiated products. The firms are symmetric in terms of demand for their product. The only difference is that only one firm which we call an expanding or

leading firm has the right to buy the businesses of other firms, which are called fringe.[4] Then the case of two fringe firms is considered in detail. This is the smallest number when the strategic issue of order of acquisition takes place. For this framework we normalize all profits and construct a benchmark model for a game theoretical investigation. At the end of Section 2 we describe inputs acquisition problem, a special case of the benchmark model where strategic issues of being last in business selling are absent[5]. This model is contrasted with a general one for better understanding of the role of strategic issues of fringe firms. In the rest of the paper we construct games in order to analyze a wide set of bargaining processes. For any degree of bargaining power of the leading firm there are subgame perfect equilibria where some fringe firms sell their businesses at a price which is higher than the profits they collect. Moreover, at some equilibria the industry stays non-monopolized. This is demonstrated in Section 3 for shot horizon situations and in Section 4 for dynamic settings. Section 5 provides some further insight for an industry of more than two fringe firms. The paper results are summarized and future directions of research are discussed in the concluding section.

2 Benchmark model: industry with one expanding firm and two fringe firms

In this section we consider a price competing industry with three firms (one expanding and two fringe firms), and describe equilibrium situations under various ownership structures. In the second part of this section we formalize the acquisition game with normalized profits, the benchmark model. In the third part we discuss the special case of the benchmark model when strategic issues for fringe firms are absent, or fringe firm profits do not depend on industry concentration. Modelling in this section is adapted from Deneckere and Davidson (1985) to a large degree.

[4] See Gilbert and Newbery (1982) for the sources which make a firm expanding.

[5] Because there is no futher increase in profits of a fringe firm from industry concentration.

2.1 Equilibria with different ownership structure

Consider an industry with differentiated products. There are 3 firms, one expanding, or leading, firm, and two fringe firms. We index the leading firm by 0 and fringe firms by 1 and 2. Each firm has a license for production and selling of only one brand of product. Each brand is indexed according to the index of the firm producing it.

The demand for brand i $(i = 0, 1, 2)$ depends on its price p_i and average price for other brands $\bar{p}_{-i} = \frac{\sum_{j \neq i} p_j}{2}$.[6] To avoid excessive mathematics we consider the *linear demand specification* for brand i in the following form[7]:

$$D(p_i, \bar{p}_{-i}) = 1 - p_i - \gamma \left(p_i - \bar{p}_{-i}\right), \tag{1}$$

where $D_i(.,.)$ denotes the demand for brand i, and the parameter $\gamma \in (-1, +\infty)$ reflects the degree of substitutability of brands, the larger γ the more homogeneous products are. Notice that $\gamma = 0$ corresponds to the local monopoly situation, and $\gamma = +\infty$ corresponds to the homogeneous product competition. Negative values of γ correspond to complement products. Also, let us point out that only values of γ larger than -1 yield a downward sloping demand function. Other values of γ allow for collection of unlimited revenue. To demonstrate the main idea of the paper it suffices to consider the following values of γ[8]:

$$\gamma \geq -\frac{1}{2}.$$

In order to avoid the issues of economy of scale it is assumed that firms have zero costs of

[6] It is assumed that a brand ownership has no effect on its demand.

[7] This functional form is quite common in the literature, e.g. Deneckere and Davidson (1985) and Shubik (1980). Also note that the certain rescaling is made, hence, in fact, we analyze a more general linear demand specification. Also note that in the rest of the paper we use certain dependence of profit collected for each brand on the industry structure. Deneckere and Davidson (1983) showed that the same comparative results take place for a more general demand structure. Hence the conclusions of the paper are valid for a wider class of demand functions.

[8] As will be seen later, the strategic incentives of fringe firms are stronger for complements rather than for substitutes. In the two fringe firms case this implies that non-monopolization happens only for complement products. But when the number of fringe firms gets larger we get the same result for substitute products.

production[9]. The leading firm is allowed to acquire a license for any fringe firm brand provided that the fringe firm agrees to the terms of offer made by the expanding firm. Sometimes we will simply call this process the acquisition of a firm.

There are three qualitatively different industry ownership structures, each of which corresponds to the number of acquired fringe firms brands. Let us denote this number by s. Two brands being acquired implies full monopolization. Let us analyze each situaion in detail.

No fringe firm is acquired, $s = 0$. According to equation (1) the *best response pricing* for brand i to the average price for other brands $\bar{p}_{-i}$ is

$$p_i = B_i(\bar{p}_{-i}, s = 0) = \frac{1 + \gamma \bar{p}_{-i}}{2(1 + \gamma)}. \tag{2}$$

When $\gamma \geq -1/2$, there is a unique symmetric equilibrium with equal prices and profits collected from each brand[10]. They are

$$\hat{p}_i^0(\gamma) = \frac{1}{2 + \gamma} \text{ and } \hat{\Pi}_i^0(\gamma) = \frac{1 + \gamma}{(2 + \gamma)^2} \ (i = 0, 1, 2), \tag{3}$$

where the superscript indicates the number of brands captured by the expanding firm, and the subscript indexes the brand.

Notice that the equilibrium price and profit are monotonically decreasing with the degree of products substitutability γ, and converge to the case of Bertrand competition as γ tends to $+\infty$.

One fringe firm is acquired, $s = 1$. Due to the symmetry across the fringe firms it suffices to consider the situation when only brand 1 is captured by the leading firm. Clearly, the best response pricing for brand 2 is described by (2) while the optimal pricing for brands 0 and 1 no longer corresponds to formula (2) due to mutual spillover effects. Taking into

[9] Linear transformation of prices makes the situation of constant maginal costs equivalent to the zero marginal cost case.

[10] The profits collected from each brand give sufficient information for calculations of profit of any of the firms.

account this and the fact that the equilibrium prices for brands 0 and 1 should be identical, we get

$$\begin{aligned} \hat{p}_0^1(\gamma) &= \hat{p}_1^1(\gamma) = \frac{4+5\gamma}{8+12\gamma+3\gamma^2}, \qquad \hat{p}_2^1(\gamma) = \frac{4+4\gamma}{8+12\gamma+3\gamma^2}, \\ \hat{\Pi}_0^1(\gamma) &= \hat{\Pi}_1^1(\gamma) = \frac{(4+5\gamma)\,(8+14\gamma+5\gamma^2)}{2\,(8+12\gamma+3\gamma^2)^2}, \; \hat{\Pi}_2^1(\gamma) = \frac{16\,(1+\gamma)\,(1+2\gamma+\gamma^2)}{(8+12\gamma+3\gamma^2)^2}. \end{aligned} \tag{4}$$

As it is shown by Deneckere and Davidson (1985) for $\gamma > 0$, the acquisition leads to a price increase for all brands as well as to an increase in profits collected from all brands. The profit for the non-acquired brand $\hat{\Pi}_2^1(\gamma)$ is the highest, while the price is the lowest for positive values of γ, and the highest for negative values of γ.

Two fringe firms are acquired, $s = 2$. In this case the expanding firm enjoys full monopoly power and sets the monopoly pricing in order to get highest possible profits from all brands. They are

$$\hat{p}_i^2(\gamma) = \frac{1}{2} \text{ and } \hat{\Pi}_i^2(\gamma) = \frac{1}{4} \; (i = 0, 1, 2). \tag{5}$$

Compared to the previous case ($s = 1$) the prices are strictly higher for positive values of γ and strictly lower for negative values of γ. The profits are higher for brands 0 and 1 when γ is nonzero. As for brand 2 the profit is higher for positive γ, and lower for negative γ.

Having obtained the equilibrium profits under different specified ownership structures we state major results of this section in the following two Lemmas.[11] [12]

Lemma 1 *(Supermodularity Property of Total Industry Profit) i) Define* $\hat{\Pi}^s(\gamma) = \hat{\Pi}_0^s(\gamma) + \hat{\Pi}_1^s(\gamma) + \hat{\Pi}_2^s(\gamma)$ $(s = 0, 1, 2)$ *as the total industry profit. The function* $\hat{\Pi}^s(\gamma)$ *is strictly increasing in* s *for nonzero values of* $\gamma \in [-1/2, +\infty)$.

[11] More general result is obtained by Deneckere and Davidson (1985).

[12] For both lemmas, first, we need to show that profits $\hat{\Pi}_i^0(\gamma)$'s and $\hat{\Pi}_i^1(\gamma)$'s are well defined for $\gamma \in [-1/2, +\infty)$. Indeed in (3) the denominator takes positive value. As for $\hat{\Pi}_i^1(\gamma)$'s they are not specfied if the expression in (4) $\varphi_3(\gamma) = 8 + 12\gamma + 3\gamma^2$ takes zero values. This is not possible because $\varphi_3(\gamma)$ is a parabola with minimum value at $-2 < -1/2$ and $\varphi_3(-1/2) = 11/4$.

ii) For nonzero values of $\gamma \in [-1/2, +\infty)$ *the profit of the leading firm when only one brand* i *is acquired is strictly higher than the sum of profits collected from brands* 0 *and* i *before acquisition, i.e.* $\hat{\Pi}_0^1(\gamma) + \hat{\Pi}_i^1(\gamma) > \hat{\Pi}_0^0(\gamma) + \hat{\Pi}_i^0(\gamma)$.

PROOF: Part (i). From (3-5) we get the following expressions for $\hat{\Pi}^s(\gamma)$ for different levels of industry concentration s:

$$\hat{\Pi}^0(\gamma) = \frac{3(1+\gamma)}{(2+\gamma)^2},\ \hat{\Pi}^1(\gamma) = \frac{48+144\gamma+138\gamma^2+41\gamma^3}{\left(8+12\gamma+3\gamma^2\right)^2},\ \hat{\Pi}^2(\gamma) = \frac{3}{4}.$$

The increment of the total profit when only one fringe firm is acquired is

$$\hat{\Pi}^1(\gamma) - \hat{\Pi}^0(\gamma) = \frac{\gamma^2(24+68\gamma+59\gamma^2+14\gamma^3)}{\left(8+12\gamma+3\gamma^2\right)^2(2+\gamma)^2}.$$

For positive values of γ the above expression is positive. For negative values of γ the expression is negative if and only if the value of the polynomial $\varphi_1(\gamma) = 24+68\gamma+59\gamma^2+14\gamma^3$ is negative. By differentiating and checking for roots of the derivative we obtain that $\varphi_1(\gamma)$ is increasing on the interval $(-17/21, +\infty)$. Because $\varphi_1(-17/21) = 251/1323$ and $-17/21 < -1/2$, we have $\hat{\Pi}^1(\gamma) - \hat{\Pi}^0(\gamma) > 0$ for $\gamma \in [-1/2, +\infty)$.

The change of the industry structure from acquiry of one firm to full monopolization gives the following increase of the industry profits:

$$\hat{\Pi}^2(\gamma) - \hat{\Pi}^1(\gamma) = \frac{\gamma^2(24+52\gamma+27\gamma^2)}{4\left(8+12\gamma+3\gamma^2\right)^2}.$$

This expression may be negative only if the polynomial $\varphi_2(\gamma) = 24+52\gamma+27\gamma^2$ can take negative values. The function $\varphi_2(\gamma)$ is a parabola whose minimum is at $-26/27 < -1/2$. Because $\varphi_2(-1/2) = 19/4$, we get $\hat{\Pi}^2(\gamma) - \hat{\Pi}^1(\gamma) > 0$ for $\gamma \in [-1/2, +\infty)$.

Part (ii). Consider again the difference in profits defined in part (ii):

$$\hat{\Pi}_0^1(\gamma) + \hat{\Pi}_i^1(\gamma) - \hat{\Pi}_0^0(\gamma) - \hat{\Pi}_i^0(\gamma) = \frac{\gamma^2(8+28\gamma+28\gamma^2+7\gamma^3)}{\left(8+12\gamma+3\gamma^2\right)^2(2+\gamma)^2}$$

This expression may be negative only if the polynomial $\varphi_3(\gamma) = 8+28\gamma+28\gamma^2+7\gamma^3$ can take negative values. The derivative of $\varphi_3(\gamma)$ takes positive values for $\gamma > -2/3$, and $\varphi_2(-1/2) = 1/8$. Q.E.D.

Lemma 2 *(Monotonicity of Fringe Firm Profit) Suppose that fringe firm* 2 *is not acquired. Then its profit* $\hat{\Pi}_2^s(\gamma)$ $(s = 0, 1)$ *is increasing in* s *for nonzero values of* $\gamma \in [-1/2, +\infty)$.

PROOF: We have

$$\hat{\Pi}_2^1(\gamma) - \hat{\Pi}_2^0(\gamma) = \frac{\gamma^2(1+\gamma)(16+24\gamma+7\gamma^2)}{(2+\gamma)^2\left(8+12\gamma+3\gamma^2\right)^2}.$$

This expression takes negative values if and only if the polynomial $\varphi(\gamma) = 16 + 24\gamma + 7\gamma^2$ is negative. But $\varphi(\gamma)$ describes a parabola with the minimum value reached at $\gamma = -12/7 < -1/2$, and $\varphi(-1/2) = 23/4$. Q.E.D.

2.2 Preliminary discussion and the benchmark model

Lemma 1 states a well-known supermodularity property of the profit function in a price competing industry. From the first sight this seems to be undoubtedly sufficient to bring an unregulated industry to a full control by the expanding firm. Yet, from Lemma 2 it follows that a fringe firm which sells its business first of the two, and which expects complete monopolization of the industry, should raise the asking price for its brand from $\hat{\Pi}_2^0(\gamma)$ to the higher price $\hat{\Pi}_2^1(\gamma)$. Hence it is possible that due to a strategic behavior of fringe firms the expanding firm has to pay to the fringe firms $2\hat{\Pi}_2^1(\gamma)$ instead of the lower amount $\hat{\Pi}_1^0(\gamma)+\hat{\Pi}_2^1(\gamma)$. The following Lemma shows that it is possible that $2\hat{\Pi}_2^1(\gamma)$ is larger than profit gains of the expanding firm $\hat{\Pi}^2(\gamma) - \hat{\Pi}_0^0(\gamma)$ from the full industry monopolization. In the same Lemma we will also show a weaker result required below: there are values of γ for which maximum possible values for brands $2\hat{\Pi}_2^1(\gamma)$ plus extra profits of the leading firm given that one brand is acquired, $(\hat{\Pi}_0^1(\gamma) - \hat{\Pi}_0^0(\gamma)) + (\hat{\Pi}_1^1(\gamma) - \hat{\Pi}_1^0(\gamma))$, is higher than additional profits after monopolization, $\hat{\Pi}^2(\gamma) - \hat{\Pi}_0^0(\gamma)$. Using the equality of profits for all brands when $s = 0$ the above statement can be written as $2\hat{\Pi}_2^1(\gamma) + \hat{\Pi}_0^1(\gamma) + \hat{\Pi}_1^1(\gamma) > \hat{\Pi}^2(\gamma) + \hat{\Pi}_0^0(\gamma)$. Again, from expressions (3-5) we have

Lemma 3 $\exists \bar{\gamma} \in (-1/2, 0)$ *such that for all* $\gamma \in [-1/2, \bar{\gamma})$ *we have (i)* $2\hat{\Pi}_2^1(\gamma) > \hat{\Pi}^2(\gamma) - \hat{\Pi}_0^0(\gamma)$, *and (ii)* $2\hat{\Pi}_2^1(\gamma) + \hat{\Pi}_0^1(\gamma) + \hat{\Pi}_1^1(\gamma) > \hat{\Pi}^2(\gamma) + \hat{\Pi}_0^0(\gamma)$.

PROOF: Since the second inequality follows from the first we need to show Part (i). Consider the function

$$\varphi_1(\gamma) = 2\hat{\Pi}_2^1(\gamma) - \left(\hat{\Pi}^2(\gamma) - \hat{\Pi}_0^0(\gamma)\right) = -\frac{\gamma^2\left(64 + 256\gamma + 328\gamma^2 + 160\gamma^3 + 27\gamma^4\right)}{\left(8 + 12\gamma + 3\gamma^2\right)^2\left(2 + \gamma\right)^2},$$

which is continuous unless the denominator is zero. The statement of the Lemma results because $\varphi_1(-1/2) = 5/4356 > 0$. Q.E.D.

Now we construct a benchmark model which is a major building block for the rest of the analysis. First, the profits without any acquisition can be considered as firms' reservation values, so we can safely put them to zero. Second, we measure profits in terms of $\hat{\Pi}^2(\gamma) - \hat{\Pi}^0(\gamma)$, an increase in the industry profits from the full monopolization case. Given that normalization, i) if there are no acquisitions, then all firms make zero payoffs (extra profits), ii) if there is full monopolization, then the monopolist gets a unit of extra profits, iii) if there is one fringe firm, then the leading firm gets $a(\gamma)$ and the fringe firm gets $b(\gamma)$ of extra profits. These values are[13]

$$a(\gamma) = \frac{\hat{\Pi}_0^1(\gamma) - \hat{\Pi}_0^0(\gamma) + \hat{\Pi}_1^1(\gamma) - \hat{\Pi}_1^0(\gamma)}{\hat{\Pi}^2(\gamma) - \hat{\Pi}^0(\gamma)} \text{ and } b(\gamma) = \frac{\hat{\Pi}_2^1(\gamma) - \hat{\Pi}_2^0(\gamma)}{\hat{\Pi}^2(\gamma) - \hat{\Pi}^0(\gamma)}.$$

By substituting in the brands profit values from equations (3-5) we get closed form expressions for $a(\gamma)$ and $b(\gamma)$. When the specification of demand is given by (1), they have the following forms:

$$a(\gamma) = \frac{4(8 + 28\gamma + 28\gamma^2 + 7\gamma^3)}{3(8 + 12\gamma + 3\gamma^2)^2} \text{ and } b(\gamma) = \frac{4(1+\gamma)(16 + 24\gamma + 7\gamma^2)}{3(8 + 12\gamma + 3\gamma^2)^2}. \tag{6}$$

Figure 1 presents the graphs of $a(\gamma)$ and $b(\gamma)$.

[13] Note that $a(\gamma)$ and $b(\gamma)$ are not well defined when γ is zero. This happens because profits collected from each brand do not depend on the industry structure. Because only nonzero values of γ are considered, this is immaterial for our analysis.

Figure 1: Functions $a(\gamma)$ and $b(\gamma)$.

As a direct corollary of above Lemmas the following properties of $a(\gamma)$ and $b(\gamma)$ can be obtained.

Lemma 4 *For nonzero values of* $\gamma \in [-1/2, +\infty)$, *(i) The values* $a(\gamma)$ *and* $b(\gamma)$ *are positive, (ii)* $a(\gamma) + b(\gamma) < 1$, *and (iii)* $a(\gamma) < b(\gamma)$. *Moreover, (iv)* $\exists \bar{\gamma} \in (-1/2, 0)$ *such that* $b(\gamma) > \frac{1}{2}$, *and hence* $a(\gamma) + 2b(\gamma) > 1$ *for* $\gamma \in [-1/2, \bar{\gamma})$.[14]

PROOF: Part (i). For $a(\gamma)$ the statement of the Lemma follows from part (ii) of Lemma 1, and for $b(\gamma)$ it follows from Lemma 2.

[14] The location of $\bar{\gamma}$ is shown on Figure 1.

Part (ii) follows from part (i) of Lemma 1.

Part (iii). Consider the following difference:

$$b(\gamma) - a(\gamma) = \frac{4}{3(8 + 12\gamma + 3\gamma^2)},$$

which is strictly positive for $\gamma \in [-1/2, +\infty)$ because the parabola $g(\gamma) = 8 + 12\gamma + 3\gamma^2$ takes minimum value at $\gamma = -2$, and $g(-1/2) = 11/4$.

Part (iv) follows from Lemma 3 and the following:

$$b(\gamma) = \frac{\hat{\Pi}_2^1(\gamma) - \hat{\Pi}_2^0(\gamma)}{\hat{\Pi}^2(\gamma) - \hat{\Pi}^0(\gamma)} = \frac{1}{2}\frac{2\hat{\Pi}_2^1(\gamma) - 2\hat{\Pi}_2^0(\gamma)}{\hat{\Pi}^2(\gamma) - \hat{\Pi}^0(\gamma)} > \frac{1}{2}\frac{\hat{\Pi}^2(\gamma) - \hat{\Pi}_0^0(\gamma) - 2\hat{\Pi}_2^0(\gamma)}{\hat{\Pi}^2(\gamma) - \hat{\Pi}^0(\gamma)} = \frac{1}{2},$$

and

$$a(\gamma) + 2b(\gamma) = \frac{\hat{\Pi}_0^1(\gamma) + \hat{\Pi}_1^1(\gamma) + 2\hat{\Pi}_2^1(\gamma) - 4\hat{\Pi}_0^0(\gamma)}{\hat{\Pi}^2(\gamma) - \hat{\Pi}^0(\gamma)} > \frac{\hat{\Pi}^2(\gamma) + \hat{\Pi}_0^0(\gamma) - 4\hat{\Pi}_0^0(\gamma)}{\hat{\Pi}^2(\gamma) - \hat{\Pi}^0(\gamma)} = 1,$$

where the first inequality follows from part (i) of Lemma 3, and the second follows from part (ii). Q.E.D.

In order to avoid extra notation sometimes we will just use a and b instead of $a(\gamma)$ and $b(\gamma)$ in the rest of the paper. Also we assume that γ is nonzero in order to deal with positive values of a and b.

2.3 Inputs acquisition problem as special case of benchmark model

In this section we present a special case of our model where there are no strategic incentives for fringe firms. Suppose there are three firms, one expanding and two fringe ones. Each fringe firm has one unit of an irreplaceable input. Both inputs are needed for the leading firm to implement a project which results in one unit of profit. The value of any input is zero for either fringe firm. Unlike in our benchmark model, possession of only one input by the leading firm has no effect on any firm's profit. This situation is a special case of the main model when values of a and b are zero. We use this model in parallel to the main one to better understand how it is important to have an increase in the profit of a fringe firm while the industry gets more concentrated.

3 Short horizon models

Now let us investigate the problem of acquisition of fringe firms. The simplest modelling is that in the form of short horizon games. First we will construct a baseline acquisition game. As it will be shown the Nash equilibria of this game have the following features: if the industry gets monopolized each firm gets paid the value b for its business (fringe firm's extra profit when it stays an only competitor of the leading firm). Also, it is possible to have one fringe firm non-captured when the extra profit for the expanding firm from full monopolization $(1-a)$ does not cover the payments for both fringe firms $2b$. In this case the acquired firm gets zero extra payment for its business[15]. In the rest of the section the robustness of conclusions based on the baseline model will be checked by considering its various modifications.

In this paper the subgame perfection concept is applied to Nash equilibria in pure strategies.[16] Whenever it is necessary, to better understand the model, subgame perfect equilibria in mixed strategies are analyzed. From time to time, for brevity we call a subgame perfect Nash equilibrium simply an equilibrium.

3.1 A short horizon model with non monopolization

To obtain the situation when the industry does not get monopolized let us consider the following three stage short horizon game which we call

Game Γ_1 (Main game):

Stage 1: The expanding firm makes simultaneous offers (z_1, z_2) to fringe firms 1 and 2, which it is ready to pay for their businesses.

[15] Remember that a zero extra payment means that the fringe firm just gets compensated for profit that would be collected if it stays in business.

[16] One may wonder why subgame perfect equilibria in mixed strategies are not analysed. We do not do this for the following reasons. First, all games considered have at least one equilibrium in pure strategies. Second, the main purpose of the paper is to demonstrate a possibility of non-monopolization. Third, an analysis of mixed strategies is quite tedious and, as a matter of fact, does not affect the main result of the paper.

Stage 2: Each fringe firm observes all offers and simultaneously with the other fringe firm accepts $\{A\}$ or rejects $\{R\}$ the corresponding offer.

Stage 3: Price competition by firms in business (the expanding firm and fringe firm(s) who rejected the offer) takes place, and firms in business receive profits expressed in (3-5).

Generically, this game has a unique subgame perfect profit profile in pure strategies. The following Lemma formalizes this result.[17]

Lemma 5 *The game Γ_1 has the following outcome profile as subgame perfect equilibria: If $2b+a<1$ both firms get captured at price b for each. In the opposite case $2b+a>1$ only one firm gets acquired at a zero price. When $2b+a=1$, either of the above outcomes is possible.*[18]

PROOF: To find subgame perfect Nash equilibria we use the backward induction method. At stage 3 there is a unique outcome given by (3-5). Any subgame at stage 2 is characterized by offers (z_1, z_2) of the leading firm to fringe firms 1 and 2, correspondingly. The subgame perfection refinement principle requires that given these offers (z_1, z_2) the fringe firms play the Nash equilibrium. The corresponding subgame where each fringe firm has two strategies $\{A, R\}$ is shown on Figure 2. We call this subgame as $\tilde{\Gamma}_1(z_1, z_2)$. Let us point out that for any pair of nonnegative z_1 and z_2 the subgame $\tilde{\Gamma}_1(z_1, z_2)$ has at least one equilibrium in pure strategies, which makes it appropriate to work with subgame perfect equilibria in pure strategies.

Next, we calculate the minimum possible payoff of the leading firm. It should be as high as $1-2b$. Indeed, let it be, on the contrary, $1-2b-4\varepsilon$, where $\varepsilon>0$, then the expanding firm is able to offer $(z_1, z_2) = (b+\varepsilon, b+\varepsilon)$. For these offers the subgame $\tilde{\Gamma}_1(b+\varepsilon, b+\varepsilon)$ has only one equilibrium $\{A, A\}$ and at this equilibrium the expanding firm obtains $1-2b-2\varepsilon$. On the

[17] Recall that 1 unit of profit corresponds to a difference between profit in the monopolized industry and profits from all brands when no fringe firm is captured. a stands for extra profits of the merged entity when one brand is acquired, and b stands for for additional profit of the non-captured fringe firm.

[18] It can be checked that $2b(\gamma)+a(\gamma)=1$ at $\gamma \simeq -.4509$. Lower values of γ correspond to the situation of acquisition of one firm.

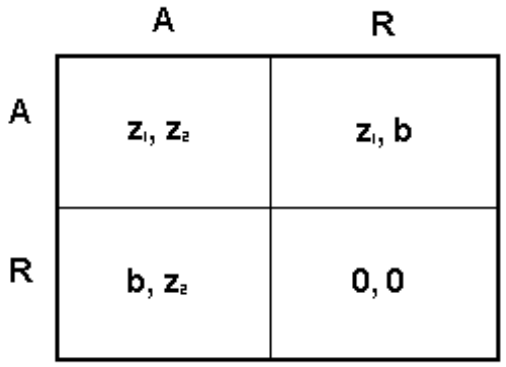

Figure 2: Normal form of subgame $\tilde{\Gamma}_1(z_1, z_2)$ of game Γ_1 at stage 2.

other hand, the subgame payoff to the leading firm should be no less than a. Indeed, for any equilibrium with the payoff of $a - 2\varepsilon$, $\varepsilon > 0$ firm 0 may offer $(z_1, z_2) = (\varepsilon, \varepsilon)$. The subgame $\tilde{\Gamma}_1(\varepsilon, \varepsilon)$ has only two equilibria $\{A, R\}$ and $\{R, A\}$. Either of these equilibria yields $a - \varepsilon$ to the expanding firm. Hence, at any subgame perfect equilibrium the leading firm receives no less than $Z = \max\{1 - 2b, a\}$.

Now let us show that Z is an upper bound for the payoff of firm 0. There are three fundamentally different equilibrium situations which differ by the number of captured fringe firms $s = 0, 1, 2$. Let us calculate the highest possible payoff $U(s)$ for each value of s. When no fringe firm is captured ($s = 0$), clearly, the leading firm receives no extra profits $U(0) = 0$. In the case of one captured fringe firm ($s = 1$), the acquired firm should receive a non-negative payoff, hence $U(1) = a$. Finally, when two fringe firms are acquired, each of them should be paid at least b, otherwise a fringe firm has a profitable deviation of rejecting the offer. This implies that $U(2) = 1 - 2b$. All this means that the leading firm cannot get more than the payoff of $\max_s U(s) = \max\{1 - 2b, a\}$. Hence, at any equilibrium the expanding firm receives $\max\{1 - 2b, a\}$. Depending on which value under the max function is higher we have two situations. Let us analyze them separately.

Case $1 - 2b > a$ or $2b + a < 1$. From the previous analysis we know that the equilibrium payoff of firm 0 is $1 - 2b$, and in this equilibrium both fringe firms must be captured at offers

$(z_1, z_2) = (b, b)$. Indeed, the subgame $\tilde{\Gamma}_1(b, b)$ has the equilibrium $\{A, A\}$. One can check that at any subgame $\tilde{\Gamma}_1(z_1, z_2)$ there is no equilibrium that yields to firm 0 a payoff which is strictly higher than $1 - 2b$. This proves the first statement of the Lemma.

Case $1 - 2b < a$ or $2b + a > 1$. Here in the equilibrium the leading firm must capture only one fringe firm at a zero price. The subgame $\tilde{\Gamma}_1(0, 0)$ has the equilibrium $\{A, R\}$ with the payoff of a to the expanding firm, and, as we already know, no subgame $\tilde{\Gamma}_1(z_1, z_2)$ has an equilibrium that provides a strictly higher payoff to firm 0. This yields the second statement of the Lemma.

One can check that in the boundary case $1 - 2b = a$ the equilibria constructed for the two situations with strict inequalities coexist together. Q.E.D.

One may wonder what happens if the fringe firms are allowed to play mixed strategies at stage 2 of the game Γ_1. The lemma in the appendix describes this in detail. This lemma shows that if $1 - 2b \geq a$ then there emerges no equilibrium payoff profiles beyond those described in Lemma 5, whereas if $1 - 2b < a$ then there is a set of new payoff profiles. At these equilibria the expected payoff of the expanding firm is strictly smaller than a. At any equilibrium there is non-monopolization with a positive probability.

The main conclusion of this subsection is the following. When there is a sufficiently high increase of the profit of a fringe firm while the industry gets more concentrated, the industry does not get completely monopolized. Part (iii) of Lemma 4 says that in the case of complimentary products this situation is plausible. In the second part of this section we consider other modifications of the basic game. This way we verify the robustness of our non-monopolization result to natural extensions of the basic bargaining process for brands purchasing.

3.2 Other variations of the short horizon model

Let us now consider three other natural modifications of the game Γ_1. In the first modification it is allowed for fringe firms to submit ask prices for their businesses. This way we will verify

how the monopolization result depends on the absence or presence of bargaining power of a fringe firm. This game will be denoted Γ_1'. In the other games we address the issue of simultaneity of offers. There, we allow for the expanding firm to make offers sequentially. In the first game the offers are coming to fringe firms in predetermined order, and in the second game the leading firm is allowed to choose a fringe firm to whom to make an offer. The extensive forms of the last two games are depicted on Figures 3 and 4. Lemmas 6–8 below describe subgame perfect Nash equilibria of these games. Also the possible equilibrium payoffs and resulting industry structures are established for these modifications of the main game Γ_1. This is done because this is cumbersome and beyond the main goal of study while the resulting industry structure and profits are of main interest. Let us proceed with the formal description of the results.

Game Γ_1': This is a three stage game.

Stage 1: The expanding firm makes simultaneous nonnegative offers (z_1, z_2) to the fringe firms, and at the same time the fringe firms submit nonnegative ask bids (r_1, r_2) for their businesses.

Stage 2: If the ask bid r_i of any fringe firm i is less or equal than the corresponding offer z_i, the leading firm pays r_i to this firm and gets control over its business.

Stage 3: Price competition by firms in business is realized and they receive payoffs according to (3-5).[19]

Lemma 6 *In the game* Γ_1' *an equilibrium where no fringe brand is captured always exists. Also, there is a continuum of equilibria where one fringe is captured, and the acquired firm receives a nonnegative payment which is less than or equal to* a*. If* $b \leq 1/2$*, then any pair of payments* z_1 *and* z_2 *from the leading firm such that* $z_1, z_2 \in [b, 1-a]$ *and* $z_1 + z_2 \leq 1$ *corresponds to a subgame perfect Nash equilibrium with full monopolization.*[20]

[19] This stage is exactly the same as stage 3 of the main game Γ_1, and it is the last stage of the other modifications of Γ_1.

[20] At any full monopolization equilibrium ask bids are equal to corresponding offers.

PROOF: An equilibrium outcome of this game may result in any number of fringe firms acquired. The offers $(0,0)$ from the expanding firm and ask bids $(1,1)$ from the fringe firms yield zero acquisition in the industry, and this constitutes a subgame perfect Nash equilibrium. This shows that the non-monopolization outcome always takes place with zero payoffs (extra profits) to all firms.

Now suppose that only one fringe firm i gets captured by the leading firm. Any ask bid r_i such that $r_i \in [0,a]$, and an offer z_i which is equal to r_i, are equilibrium strategies given that the other fringe firm j asks the unity price $r_j = 1$ and receives the zero offer[21] $z_j = 0$. Clearly, in any equilibrium where only one firm gets acquired no fringe firm can receive more than a for its business because in this case the expanding firm would have negative extra profits, while zero offers to the fringe firms ensure nonnegative extra profits.

In the situation when two firms get acquired, each fringe firm should receive at least b for its business, otherwise a fringe firm which receives less than b is able to submit an ask price which is higher than the corresponding offer, break the deal, and get b units of extra profits by staying in the industry. Now let us find the upper bound for the values that fringe firms receive. We know that the expanding firm is able to break any positive payment contract by submitting the zero offer. So if it breaks one contract with price, say r, it does not have to pay the value r but loses $1-a$ units of extra profits. If it breaks two contracts it does not have to pay equilibrium offers to the fringe firms but it loses one unit of extra profits. Hence, neither fringe firm can count on more than $1-a$ transfer from the leading firm, and the sum of brand acquisition payments cannot be more than 1. Finally, one can verify that a quadruple of ask bids r_1 and r_2 and offers z_1 and z_2 such that $r_1 = z_1 \in [b, 1-a]$, $r_2 = z_2 \in [b, 1-a]$ and $r_1 + r_2 \leq 1$, yields a subgame perfect Nash equilibrium. We can find such quadruple only if $b \leq 1/2$. Q.E.D.

In contrast to what happens in game Γ_1 both fringe firms are able to provide counteroffers in the game Γ_1'. This gives them extra bargaining power. As a result there always exists an

[21] There are other offer-ask combinations which pevent an acquisition of the other fringe firm.

equilibrium where no brand is acquired. Also, there are equilibria where only one brand is captured, and any part of the surplus a may go as a payment for the acquired brand. As for the full monopolization, unlike the game Γ_1 the complete capture seizes to exists when $2b > 1$ which is a stronger condition than the one of the main game, $2b + a > 1$. This still validates the main conclusion of the paper. When $2b \leq 1$, there are equilibria where fringe firms are captured, and the latter can receive any share of extra profits as additional compensation for being out of business (due to Lemma 4(iii)). Also, it is worth mentioning that due to the presence of an ask price for a fringe firm business there are equilibria where expanding does not benefit at all from the full industry monopolization.[22] In spite of many interesting features, in this paper the game $\Gamma_1^{'}$ is not of main interest because there is excessive multiplicity of equilibria.

Game $\Gamma_1^{''}$: There are five stages, an outcome of every stage is observable by each firm.

Stage 1: The expanding firm submits an offer $z_1 \in [0, +\infty)$ to fringe firm 1.[23]

Stage 2: The first fringe firm accepts ($\{A\}$) or rejects ($\{R\}$) this offer.

Stage 3: The expanding firm makes an offer $z_2 \in [0, +\infty)$ to fringe firm 2.

Stage 4: The second fringe firm accepts ($\{A\}$) or rejects ($\{R\}$) this offer.

Stage 5: Price competition by the firms in business is realized, and they receive payoffs according to (3-5).

Lemma 7 *In the game* $\Gamma_1^{''}$, *if* $a + 2b < 1$, *there is a unique equilibrium where both firms get acquired at the same price* b. *There is only one equilibrium where the second fringe firm is acquired at a zero price at stage 4, when* $a + 2b > 1$. *The case* $a + 2b = 1$ *yields both of the above equilibria.*

PROOF: All information sets of the game $\Gamma_1^{''}$ are singletons. Hence, any subgame perfect equilibrium can easily be found by the method of backward induction. Indeed, at stage 4 fringe

[22] As will be seen later, a similar result holds for a dynamic setting, which plays a very crucial role for the existence of non monopolization equilibria.

[23] In this game fringe firms are indexed according to the order of offers they get.

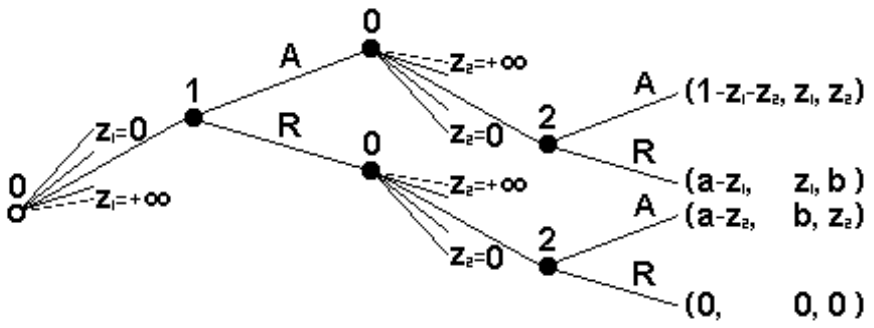

Figure 3: Extensive form of game Γ_1''.

firm 2 accepts any nonnegative offer if the other fringe firm has not got captured before, and accepts any offer which is larger than or equal to b otherwise. Then at stage 3 the optimal strategy of the leading firm is to offer b to fringe firm 2 if firm 1 is acquired, and 0 otherwise. Fringe firm 2 accepts these offers. Now let us analyze stage 2. Firm 1 "knows" that fringe firm 2 is going to be captured in any equilibrium history. Hence fringe firm 1 gets b units of extra profits if it rejects the offer at stage 2. As a result, any offer lower than b is rejected, and any offer larger than b is accepted under equilibrium play. Finally, at stage 1 by choosing an offer to firm 1 the expanding firm is able to control the number of brands which are going to be captured. In the case of two brands it gets at most $1 - 2b$ units of extra profits, and in the case of one brand it gets a. The highest gains determine the strategy of the expanding firm in equilibrium. Q.E.D.

The game Γ_1'' is a variation of the main game where the only difference is that offers are coming sequentially to the fringe firms and outcome of any bargaining process becomes public knowledge. As it can be seen, if we do not care about the identity of captured firms there is no difference in equilibrium outcomes between the games Γ_1 and Γ_1''. Hence, timing is irrelevant for the bargaining process.

Game Γ_1''': This is a variation of the game Γ_1'' which we call Γ_1''', and it is almost the same

as Γ_1'' with an exception of the possibility for the expanding firm to choose a fringe firm to which to make the offer. In particular, it is allowed to choose the same firm twice.[24] The details follow.

Stage 1: The expanding firm makes an offer $z_1 \in [0, +\infty)$ to fringe firm 1 OR an offer $z_2 \in [0, +\infty)$ to fringe firm 2.

Stage 2: A fringe firm which received the offer at previous stage accepts $\{A\}$ or rejects $\{R\}$ it.

Stage 3: The expanding firm makes an offer $\bar{z}_i \in [0, +\infty)$ to any fringe firm i which is still in business.

Stage 4: Fringe firm i accepts $\{A\}$ or rejects $\{R\}$ the offer of stage 3.

Stage 5: Price competition by firms in business is realized and they receive payoffs according to (3-5).

Lemma 8 *In the game Γ_1''' there is a continuum of equilibria where both fringe firms get captured with payments $z \in [0, \min\{b, 1-a-b\}]$ and $\bar{z} = b$, where z is the payment to the fringe firm which is acquired first, and $\bar{z}$ is that paid to the other fringe firm. When $a+2b > 1$, there are two additional equilibria where only one of two fringe firms gets captured at a zero price at stage 4.*

PROOF: Analysis of the game Γ_1''' is similar to that of the game Γ_1''. Following the same logic we conclude that at stage 3 at equilibrium play the leading firm offers b if there is a single brand left in the industry, and 0 otherwise. At an equilibrium these offers always get accepted. The equilibrium behavior of a fringe firm at stage 2 is different from that in the game Γ_1''. Acceptance of an offer depends on which brand will be targeted by the leading firm after rejection at stage 2. If this fringe firm is going to be approached again, then any nonnegative offer is going to be accepted, otherwise the fringe firm only accepts offers that are no less than b. Hence, there is a possibility to construct an equilibrium where at stage 2

[24] Note that if a fringe firm accepts an offer at stage 2 the leading firm must submit an offer to the other firm.

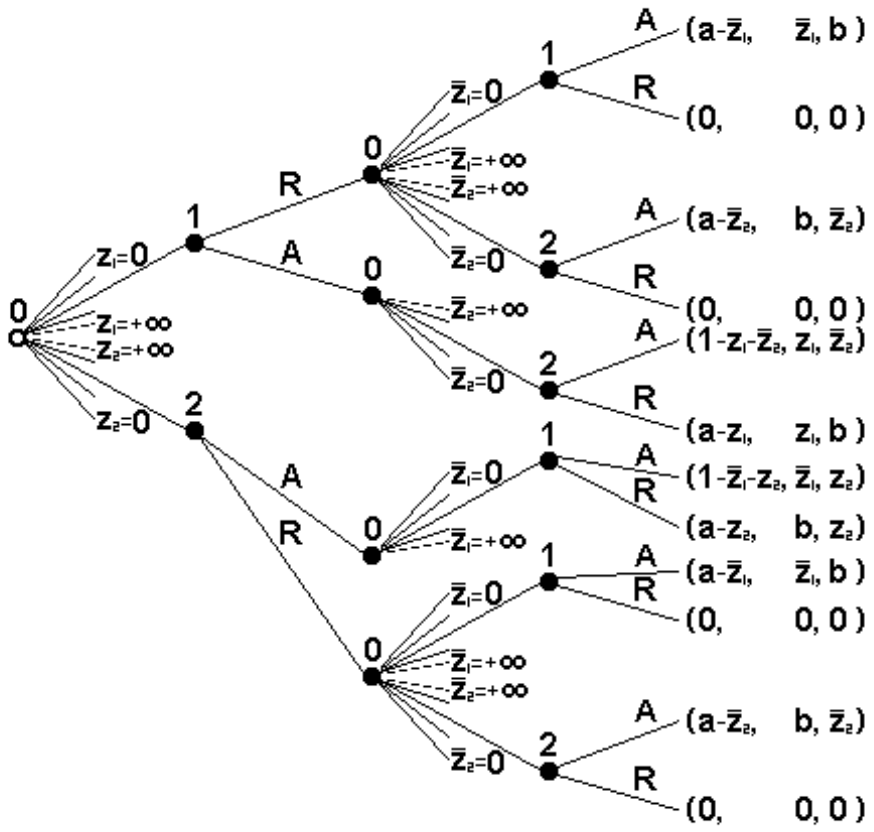

Figure 4: Extensive form of game Γ_1'''.

some fringe firm accepts an offer $z \in [0, b]$, and the other fringe firm accepts an offer of value b. Since the leading firm has net payoff of size a when at stage 4 it acquires only one fringe firm, this gives the following restriction for z net payoff from two brands acquisition: $1 - z - b$ is no less than a, or $z \leq 1 - a - b$.[25] Compared to the game Γ_1'', where for the expanding firm

[25] In the constructed equilibrium a fringe firm, say i, which is offered z, rejects any other offer which is less or equal than b, because it "expects" that the other fringe firm j will get captured later. This is why a deviation of the expanding firm if of the form: offer some $c > 0$ to firm i at stage 1, and at stage 3 offer $\varepsilon/2$ to firm i if the previous offer was rejected and offer $b + \varepsilon$ to firm j, otherwise it is not beneficial for the expanding firm. Indeed, given this deviation, if ε is sufficiently small under the equilibrium strategy, firm i rejects the offer and accepts the next offer which brings the payoff $a - \varepsilon/2$ to firm 0.

there is no freedom of choice of a target firm, this yields a larger equilibrium set.[26] Q.E.D.

Compared to the game Γ_1'', in the game Γ_1''' additional bargaining freedom is provided to the expanding firm. During a sequential bargaining process the leading firm is allowed to select a fringe firm to which to make an offer. Hence, when a fringe firm says "No" to the very first offer, the expanding firm has a right to approach again the same fringe firm with a new offer. This freedom to choose always assures the existence of an equilibrium where both fringe firms get captured. Here the firm which is acquired last always gets b for its business, while the other fringe firm is compensated with a value which is no greater than the minimum of $1-a-b$ (additional extra surplus from this fringe firm acquisition given that the other fringe firm is captured with the payment b) and b (the payoff that this fringe firm collects given that the other fringe firm is out of the industry).[27] Notice that in the game Γ_1''' we have a possibility of having zero extra benefits for the expanding firm in the fully monopolized industry. When extra profits of all firms from acquisition of just one brand increase dramatically (more exactly, when $a+2b>1$) there exists an equilibrium where only one fringe firm is acquired at a zero price. This happens because it is possible to support this type of equilibrium by a set of "beliefs" where the fringe firm staying in the industry "believes" that the other fringe firm is going to be approached by the leading firm. No deviation of the leading firm from the equilibrium strategy can affect these "beliefs". To summarize, there are two types of equilibria: an additional bargaining power (the ability to select a fringe firm to which to make an offer) always creates an equilibrium with full monopolization but it does not completely eliminate the non-monopolization equilibrium where the profits from capture

[26] If one allows for the expanding firm to mix over fringe firms this does not change the set of equilibrium payoff profiles. This happens for the following reason: at any equilibrium in a subgame which starts at stage 4 and with two fringe firms "survived" the leading firm makes a zero offer with some probability α to firm 1 and with complementary probability to firm 2. Any fringe firm accepts her offer. This implies that at stage 2 firm 1 has a secured payoff of $(1-\alpha)b$ while firm 2 has αb. Because in order to construct any subgame perfect Nash equilibrium there is a freedom to choose any equilibrium of stage 4 we have the same equilibrium oucomes as with pure startegies.

[27] Note that in any equilibrium any fringe firm accepts any offer that is stricly larger than b.

of one brand rise dramatically.

3.3 Application to the inputs acquisition problem

Let us remind the reader that in the inputs acquisition problem input possession does not have any effect on any firm's profits. Now we apply the four games constructed in the previous subsection to this problem. This will provide a better picture of importance of the order of acquisition. As stated before our model encompasses the inputs acquisition problem as a special case with zero a and b. An application of Lemmas 5–8 shows that in any game but Γ_1' there is a unique equilibrium outcome with complete acquisition of inputs at a zero price. Since in the game Γ_1' a fringe firm has a right to provide a counteroffer for its business we have a continuum of equilibria with any degree of monopolization. More specifically, when only one input gets acquired, zero price is paid for it, but in the case of two inputs acquired any share of extra monopoly profits may be received by fringe firms. Now let us present these results formally.

Corollary of Lemmas 5-8 *(Case of inputs acquisition) When* $a = b = 0$,

(i) in the game Γ_1, as well as the games Γ_1'' and Γ_1''', *both firm get acquired at zero price in the equilibrium;*

(ii) in the game Γ_1' *there is always an equilibrium where no fringe brand is captured. Also, there is an equilibrium with one fringe acquired at a zero price, and there is a set of equilibria where fringe firms get acquired at prices* z_1 *and* z_2 *whose sum is less or equal to* 1.

From the corollary it follows that when the leading firm has a high bargaining power an increase of profits along with industry concentration is crucial for the industry concentration outcome.

In the rest of the paper we investigate the issue of how important is the number of offers from the expanding firm. The dynamic model is constructed and the following question is addressed: Are there non monopolization equilibria where the value of b is high enough? In the next section an infinitely repeated variant of the game Γ_1''' is analyzed. This game is

chosen because it provides the highest degree of bargaining power to the leading firm in the market environment.

4 A dynamic environment

One may wonder whether the non-monopolization outcome present in a short horizon industry holds in a repeated setting. To understand this better let us consider the following dynamic game denoted as Γ_2.[28]

There are infinitely many time periods $t = 0, 1, 2, ..., +\infty$. It is assumed that at the beginning of every time period t the expanding firm is allowed to select a "target" fringe firm from non-captured fringe firms. Then the leading firm makes an offer and the fringe firm of interest accepts $\{A\}$ (says Yes) or rejects $\{R\}$ (says No) it. In the end of each time period the expanding and non-captured fringe firms receive instantaneous profits according to the benchmark model described in section 2. The net profit of firms is measured as a discounted sum of the instantaneous payments with a discount factor $\delta \in (0, 1)$.[29] In the spirit of Fudenberg and Tirole (1993) we normalize the discounted sum of instantaneous payoffs by the multiplier $(1 - \delta)$. This allows us to compare payoffs of constant payment schedules with different discount factors. The continuation or net payoff of the expanding firm is given by

$$\pi^0 = (1 - \delta) \left\{ \sum_{t=0}^{+\infty} \delta^t \Pi^0_{s(t)} - \delta^{\tau_1} P^1 - \delta^{\tau_2} P^2 \right\},$$

and that of fringe firm i is

$$\pi^i = (1 - \delta) \left\{ \sum_{t=0}^{\tau_i - 1} \delta^t \Pi^i_{s(t)} + \delta^{\tau_i} P^i \right\}, (i = 1, 2),$$

[28] Our game is related to the "Division of Pie" game of Rubinstein (1982). Here, only one player makes offers. When some player accepts an offer all other agents continue to divide the rest of the pie plus additional pieces of sizes a and b which came to them as a reward.

[29] In finance the discounted stream of profits corresponds to net present value of profits with discounting $\delta = 1/(1 + r)$, where r is the interest rate.

where $s(t)$ stands for a number of fringe firms captured by the end of time period t, τ_i is timing of the capture[30] of fringe firm i, P^i is a payment to fringe firm i made by the leading firm at time τ_i, and $\Pi^i_{s(t)}$ stands for a normalized instantaneous profit of firm i which is still in business at time t given by

$$\Pi^i_s = \begin{cases} 1, & s = 2 \text{ and } i = 0 \\ a, & s = 1 \text{ and } i = 0 \\ b, & s = 1 \text{ and } i > 0 \\ 0, & \text{o/w} \end{cases}$$

By applying the Subgame Perfect Nash Equilibrium concept to the game Γ_2 we derive the central result of the paper stated in Proposition 1 below. Let us provide an intuitive description of an equilibrium behavior of the firms. An absolute control over all brands raises profits in the industry to the monopoly level. An acquisition of just one brand allows the expanding firm to collect a share of extra profits from monopolization, but this also has a positive effect on profits which are collected by the non-acquired firm. The firm has in addition b share of extra profits from monopolization. Because further industry concentration yields even higher profits, we know that $a + b < 1$. Let us turn now our attention to the dynamic game where all payoffs are expressed in terms of net discounting.

When the leading firm has full bargaining power and there is only one non-acquired firm in the industry, the fringe firm gets immediately captured with the size of compensation payment b, and the expanding firm strictly benefits from this. When the leading firm faces two fringe firms, the situation becomes not so trivial as in the case of one fringe firm. The size of compensation for the business of the first firm to acquire is not uniquely defined. It depends on the expectation of the fringe firm. If it believes that after rejecting the offer the other fringe firm will be acquired in the next period, then according to the subgame perfection paradigm this fringe firm never accepts any offer which is lower than δb (extra profits after a capture of the other firm). On the opposite, if it believes that there will be no attempts to acquire the other fringe firm unless this fringe firm agrees, then the firm of

[30] An infinite value of τ_i corresponds to ultimate survival of fringe firm i.

interest accepts any nonnegative offer. All this allows one to construct a set of equilibria where one of the fringe firms accepts some offer $z \leq \delta b$ (at any equilibrium no fringe firm can count on more than δb) in the first time period, and the other takes offer b in the next period. We call this type of situation immediate monopolization. How high the offer z can be depends on the level of benefits of the leading firm from immediate monopolization, which is equal to $[(1-\delta)/\delta]a + \delta$. The total payment $z + \delta b$ cannot be higher than this value. When b becomes large this constraint becomes binding which yields immediately monopolization equilibria with $z = [(1-\delta)/\delta]a + \delta - \delta b$ where the leading firm does not get any extra profits. The presence of such a nonprofitable equilibrium provides an equilibrium where there is no any capture. In this equilibrium the expanding firm always offers zeros and fringe firms reject such offers. This kind of behavior corresponds to the subgame perfection concept if for any nonzero offer the leading firm expects that this offer is going to be rejected with a follow up of immediate nonbenefitial monopolization, and the fringe firm which makes this rejection expects a payoff of δb. Let us stress that no firm can unilaterally affect this system of beliefs.

In a similar fashion it is possible to construct an equilibrium play where monopolization begins at an arbitrary time period. So far we have assumed that the value b is so high that there is nonbeneficial for the expanding firm monopolization. When b is low, it is still possible to put off monopolization for some time. Here the leading firm offers zero, and the fringe firms reject these offers till some time τ when monopolization starts. The expanding firm get "scared" by subequilibria where it has to pay δb to each firm, or $2\delta b$ in total. Of course, firm 0 will only wait for monopolization where it has to pay less with a minimal perceivable payment of δb. Hence, a complete capture can be delayed till time τ when the value $\delta^\tau\{[(1-\delta)/\delta]a + \delta - z - \delta b\}$ is no larger than $\{[(1-\delta)/\delta]a + \delta - 2\delta b\}$, where z is the payment to the firm which gets captured first. Note that for any positive b and any τ there is δ sufficiently high such that monopolization can be postponed till time τ.

Proposition 1 *At any subgame perfect Nash equilibrium of the game Γ_2 there are two possible outcomes:*

(i) Both fringe firms get acquired successively, first one in some time period $\tau_1 = \tau$*, and the second one in the subsequent time* $\tau_2 = \tau + 1$*. Net payoffs are*

$$\begin{aligned} \pi^0 &= [(1-\delta)a + \delta - \delta b - z]\delta^\tau, \\ \pi^1 &= z\delta^\tau \text{ and } \pi^2 = b\delta^{\tau+1}, \end{aligned}$$

where fringe firms are indexed in order of their exiting the industry. $z \in [0, \min(\delta b, (1-\delta)a + \delta - \delta b)]$ *is a payment to fringe firm* 1 *and* b *is the compensation to firm* 2*.*

The time of the first capture τ *is bounded by*

$$\tau \leq \begin{cases} \frac{\ln[(1-\delta)a+\delta-2\delta b]-\ln[(1-\delta)a+\delta-\delta b-z]}{\ln \delta}, & b < \frac{1}{2} + \frac{(1-\delta)}{2\delta}a \\ +\infty, & o/w \end{cases} \tag{7}$$

(ii) When $b \geq \frac{1}{2} + \frac{(1-\delta)}{2\delta}a \equiv \bar{b}$*, there is an equilibrium with no fringe firm acquisition.*

Remark (i) For the leading firm *the most beneficial payoff* takes place when acquisition starts in the first time period with zero initial payment. Here the expanding firm receives a strictly positive payoff of $a + \delta(1-a-b)$. (ii) *The least beneficial payoff* is equal to $max\{a + \delta(1-a-2b), 0\}$. The first expression comes for low $b < \bar{b}$ from an equilibrium where the first acquired fringe firm get captured at the price $z = [(1-\delta)a + \delta - \delta b] - \delta^{-\tau}[(1-\delta)a + \delta - 2\delta b]$ at time τ satisfying (7). (In particular, $z = \delta b$ when $\tau = 0$.) When b is high or when $b \geq \bar{b}$, the leading firm gets zero payoff because of the absence of any capture, or because it pays $(1-\delta)a + \delta - \delta b$ to the first acquired firm and δb to the other one. In this way all extra profits of the expanding firm go to the fringe firms.

PROOF: At the beginning of any time period t, due to the symmetry between the fringe firms there are three genuinely different states of the game Γ_2. These states correspond to the number of acquired fringe firms $s \in \{0, 1, 2\}$. The game starts at the state $s = 0$, then in some time period $\tau_1 \in \{0, 1, ..., +\infty\}$ it switches to the state $s = 1$ and then it switches to the state $s = 2$ in the time period $\tau_2 \in \{\tau_1 + 1, ..., +\infty\}$. Hence, any history of game states when there is eventual monopolization can be described by a sequence of two nonnegative integer

values τ_1 and τ_2, $\tau_2 > \tau_1$. A history without a complete capture can still be described by a pair of numbers that take an infinite value in the case of nonacquisition.[31] For example, the case with $\tau_1 = 0$ and $\tau_2 = +\infty$ corresponds to a capture of just one fringe firm at the first time period.

All possible subgame perfect Nash equilibria of the game Γ_2 can be found by backward induction on states. First, we find all subgame perfect Nash equilibria in a subgame which starts at the state $s = 2$. Next we move to a subgame which begins at the state $s = 1$, and then to the state $s = 0$, at which the whole game starts. To simplify the analysis we assume that in any subgame the timing starts from zero: $\tau = 0, 1, 2, ..., +\infty$, where τ is the "internal time" of a subgame of interest.

Equilibria at the state $s = 2$: When a subgame starts at the state $s = 2$, no firm has an action to choose. *Hence, there is a unique equilibrium with continuation payoffs* $(\pi^0, \pi^1, \pi^2) = (1, 0, 0)$.

Equilibria at the state $s = 1$: There are two firms in business: one is the leading firm, and the other is a fringe firm. We mark these firms 0 and 1 correspondingly. When the fringe firm is not acquired, the instantaneous payoff to the expanding firm is a, and that for the fringe firm is b.

Let us show that possible continuation payoffs of firm 1 are equal to b. First, they are no less than b. Indeed, the fringe firm is able to secure the value b by rejecting all offers all the time. And second, let $\bar{v}$ be the supremum of the subgame perfect continuation payoffs of the fringe firm. If in any time period τ the leading firm makes an offer which is higher than $(1 - \delta)b + \delta\bar{v}$ (current period payoff, given that the offer is rejected, plus the maximum possible future continuation payoff from the next time period), then by the one stage deviation principle (we will frequently use it in this proof, see Fudenberg and Tirole (1993) for details) the fringe firm must accept this offer in any equilibrium strategy. Hence, the upper bound for $\bar{v}$ is $(1 - \delta)b + \delta\bar{v}$ or $\bar{v} \leq b$. Indeed, suppose that there is an equilibrium where firm 1

[31] Note that the infinite value implies that the correspoding state never arises in the equilibrium history.

accepts an offer z in some time period τ, and $z > (1-\delta)b + \delta\bar{v}$. When firm 0 reduces z to offer z' that satisfies the condition $z > z' > (1-\delta)b + \delta\bar{v}$, the fringe firm accepts it while the expanding firm increases its payoff. Hence, in any subgame perfect equilibrium firm 1 receives a continuation payoff b.

Because the fringe firm has a continuation payoff which is equal to b, any equilibrium history should be such that in some time period τ the fringe firm accepts the offer $b/(1-\delta)$, or there is no acquisition at all ($\tau = +\infty$). Also, from the one stage deviation principle it follows that in any equilibrium strategy firm 1 accepts any offer which is strictly larger than $b/(1-\delta)$, and rejects any offer which is strictly less than $b/(1-\delta)$. Let us find out possible equilibrium values of τ. If in time period 0 the leading firm makes an offer $z > b/(1-\delta)$, the fringe firm accepts it. In this case firm 0 receives continuation value $1-z$ instead of the equilibrium payoff $(1-\delta^\tau)a + \delta^\tau(1-b)$. Hence, for the equilibrium value of τ it necessarily holds that $(1-\delta^\tau)a + \delta^\tau(1-b) \geq (1-b)$ or $(1-\delta^\tau)(a+b-1) \geq 0$. Because, according to Lemma 4(ii), the sum of a and b is smaller than 1, we have that at the equilibrium $\tau = 0$. One can check that the strategy profile where firm 0 always offers $b/(1-\delta)$ and firm 1 only rejects offers which are strictly less than $b/(1-\delta)$, yields an equilibrium. *Hence there is a unique equilibrium outcome: In the first time period the leading firm offers $b/(1-\delta)$ and the fringe firm accepts it. Firms receive continuation payoffs* $(\pi^0, \pi^1) = (1-b, b)$. In the further analysis of this equilibrium we use notation E_1 or E_2, where the subscript corresponds to the index of a fringe firm in business.

Equilibria at state $s = 0$: From the analysis above it follows that any equilibrium history has the following form: one fringe firm accepts some nonnegative offer $z/(1-\delta)$ in some time period τ, and in time period $\tau+1$ the other fringe firm accepts an offer of size $b/(1-\delta)$. In the case of non-acquisition τ takes an infinite value. Hence, in any equilibrium a continuation payoff profile is equal to $(\delta^\tau[(1-\delta)a+\delta-z-\delta b],\, \delta^\tau z,\, \delta^\tau b)$ or $(\delta^\tau[(1-\delta)a+\delta-z-\delta b],\, \delta^\tau b,\, \delta^\tau z)$. The payoffs are determined by which fringe firm is acquired first and which offer it accepts. Let us find possible equilibrium values of z. Let $\bar{z}$ be a supremum of these. When there are

two fringe firms in the industry, any fringe firm accepts any offer which is strictly greater than $\delta \max(\bar{z}, b)/(1-\delta)$, because in the case of rejection it receives a lower equilibrium continuation payoff. As a result, $\delta \max(\bar{z}, b) \geq \bar{z}$, or $\bar{z} \leq \delta b$. Because the leading firm is able to secure a zero continuation payoff by offering zero all the time it follows that at any equilibrium the expanding firm payoff must be nonnegative, or $(1-\delta)a + \delta - z - \delta b \geq 0$. All the above yields the following restriction for $\bar{z}$:

$$\bar{z} \leq Z \equiv \min(\delta b, (1-\delta)a + \delta - \delta b). \tag{8}$$

The next step of the analysis is to show that the inequality holds with equality, and any z in the segment $[0, Z]$ corresponds to some equilibrium. Let us consider the following strategy profile: firm 0 "always offers $z^*/(1-\delta)$ for some $z^* \in [0, Z]$ to fringe firm 1 until this offer gets accepted, and then always offers $b/(1-\delta)$ to firm 2", firm 1 "accepts any offer larger than or equal to $z^*/(1-\delta)$, and rejects other offers", and firm 2 "only accepts offers which are no smaller than $b/(1-\delta)$". This profile forms an equilibrium, where firm 1 gets purchased with the payment $z^*/(1-\delta)$ in time 0, and firm 2 gets captured at the price $b/(1-\delta)$ in time 1.[32] Hence, for equilibria with $\tau = 0$ possible continuation payoffs are $((1-\delta)a + \delta - z - \delta b,\ z,\ b)$ or $((1-\delta)a + \delta - z - \delta b,\ b,\ z)$, $z \in [0, Z]$. In the rest of the proof we will check for other equilibria with positive τ. There are two possibilities which differ by which value, δb or $(1-\delta)a + \delta - \delta b$, is minimal in the formula (8) for Z. We consider each situation separately.

Situation $(1-\delta)a + \delta \leq 2\delta b$. Here, $Z = (1-\delta)a + \delta - \delta b$. Let the equilibrium belonging to the ones constructed above be denoted as $E_i(z)$, $i \in \{1, 2\}$, $z \in [0, Z]$. *In the equilibrium* $E_i(z)$ *firm* i *is acquired at* $\tau = 0$ *at the price* $z/(1-\delta)$, *and the other firm gets captured at* $\tau = 1$ *with the payment* $b/(1-\delta)$. Notice that in $E_i(Z), i = 1, 2$ a continuation payoff of the expanding firm is zero. By the way of these non-beneficial equilibria for the leading firm it is possible to construct subgame perfect equilibria with continuation payoffs $(\delta^{\tau^*}[(1-\delta)a + \delta - \delta z^* - b],\ \delta^{\tau^*} z^*,\ \delta^{\tau^*} b)$ and $(\delta^{\tau^*}[(1-\delta)a + \delta - \delta z^* - b],\ \delta^{\tau^*} b,\ \delta^{\tau^*} z^*)$ for any $\tau^* \geq 0$ and any $z \leq Z$. This means that industry monopolization can be postponed to an arbitrary time period, and

[32] The optimality of these strategies can be checked by the one stage deviation principle.

the leading firm is able to buy the first fringe firm at any price which allows for nonnegative extra profits.[33] Now let us describe an equilibrium strategy profile which corresponds to the acquisition of fringe firm 1 first (for firm 2 the same strategy can be used by switching fringe firms).

At the equilibrium history the leading firm always offers zero to firm 1, which the fringe firm rejects, till time τ^*, and then all firms continue to play according to the subequilibrium $E_1(z^*)$ from time τ^* on. Now let us specify the firms' equilibrium play when the expanding firm deviates from the equilibrium strategy at time $\tau \leq \tau^*$. Namely, fringe firm i which receives an offer of $z/(1-\delta)$ does the following: it rejects it when $z < \delta b$, and accepts other offers. After rejection the game continues with the subequilibrium $E_j(Z)$, $j \neq i$, and with acceptance game moves to state $s = 1$, for which the subgame perfect equilibrium payoffs are uniquely specified. For all subgames that start from those game histories which are not equilibrium ones, or where the leading firm has just deviated, any subequilibria can be assigned, say, the subequilibrium E_1 for a subgame with state $s = 1$, and $E_1(0)$ for a subgame with state $s = 0$.

The above strategies are subgame perfect. Indeed, no fringe firm gets a positive payoff from unilateral deviation along the equilibrium path at any time $\tau < \tau^*$ (more exactly, firm 2 does not have an option to choose, and firm 1's deviation is to accept a zero offer). As for the leading firm, when it deviates at $\tau \leq \tau^*$ it receives no more than zero, because if it offers more than or equal to δb to some fringe firm, the offer gets accepted with the subsequent net payment δb to the other fringe firm in the next time period, while the net extra profits are equal to $(1-\delta)a + \delta$ (extra profits from a partial industry capture in the current period plus the continuation extra profits from complete monopolization). An offer to some fringe firm i of net value less than δb yields a zero payoff to the expanding firm because the fringe firm rejects this offer due to continuation of game into the subequilibrium $E_i(Z)$ where firm i gets net payoff δb and firm 0 receives zero.

There are no other equilibrium payoff profiles because Z is a maximum possible equilibrium payment to the fringe firm which is acquired first, and we have showed that any payment which

[33] Note that τ^* can be infinite which corresponds to everlasting operation of fringe firms in the industry.

is no greater than Z to any fringe firm can happen at *any* time period.

Situation $(1-\delta)a+\delta > 2\delta b$. Here, $Z = \delta b$. In this case a non monopolization outcome is not possible, but monopolization can be postponed to some time period τ by the same strategies constructed in the previous situation. Let us check which pair of a continuation payoff z^* of the first captured firm and time of its acquisition τ^* can be supported in equilibrium. Given a pair z^* and τ^* the leading firm obtains the continuation payoff $\pi^* = \delta^{\tau^*}[(1-\delta)a+\delta-z^*-\delta b]$. If this value is strictly smaller than $\tilde{\pi} = (1-\delta)a+\delta-2\delta b$, then the expanding firm has deviation at time $\tau = 0$ where it offers $z = \delta b + (\tilde{\pi}-\pi^*)/2$ to any fringe, this offer gets accepted (no fringe firm can count of a net payoff larger than δb), and the expanding firm receives $(\pi^*+\tilde{\pi})/2$. Because of a possibility for such deviation it follows that the condition

$$\delta^{\tau^*}[(1-\delta)a+\delta-z^*-\delta b] \geq (1-\delta)a+\delta-2\delta b \tag{9}$$

is necessary for the equilibrium z^* and τ^*. Since at any time τ any deviation of the leading firm leads to a continuation play where every fringe firm gets at least δb as a continuation payoff while the expanding firm gets net extra profits $(1-\delta)a+\delta$. This yields a set of sufficient conditions $\delta^{\tau^*}[(1-\delta)a+\delta-z^*-\delta b] \geq \delta^{\tau}[(1-\delta)a+\delta-2\delta b], \tau = 0, ..., \tau^*$, which makes the condition (9) sufficient. Q.E.D.

The positiveness of b is very crucial for the presence of a variety of equilibria. We conclude this section with a discussion of the situation with zero b. When $b = 0$, both fringe firms get captured at zero prices during first two periods. Hence, an increase in profits of a fringe firm while the other non-leading firm gets captured dramatically affects possible equilibrium payoffs. For the case of the inputs acquisition problem or Bertrand competition (zero values of a and b), let us state the following result.

Corollary of Proposition 1 *(The case of inputs acquisition or Bertrand competition ($\gamma = +\infty$)) When* $a = b = 0$, in any subgame perfect Nash equilibrium of the game Γ_2 both fringe firms get successively acquired at zero prices: the first one in time period 0, and the second one in period 1. The firms' payoffs are $\pi^0 = \delta$, $\pi^1 = 0$, and $\pi^2 = 0$.

In the next section we provide a brief discussion of how robust our results are when there are more than two fringe firms in the industry. Also, new conclusions will be derived.

5 Case of many fringe firms

Suppose now that there are n ($n \geq 2$) fringe firms in the industry. As before, the discrete variable s stands for the number of captured fringe firms, and let A be a set of brands under control of the expanding firm. Also, we denote the set of all brands by I. Under the demand given by (1) price competition yields a unique equilibrium. In this equilibrium the profit collected from brand i by a firm who controls it is given by

$$\hat{\Pi}_i^s(\gamma) = \begin{cases} \frac{n(2n+\gamma+2n\gamma)\left(2n^2+4\gamma n^2+\gamma n-2\gamma ns+2\gamma^2 n^2+\gamma^2 n-2\gamma^2 ns-\gamma^2 s\right)}{(4n^2+6\gamma n^2+2\gamma n-2\gamma ns+2\gamma^2 n^2+\gamma^2 n-\gamma^2 ns-\gamma^2 s-\gamma^2 s^2)^2}, & i \in A \\ \frac{n^2(2n+2\gamma n-\gamma s+\gamma)\left(2n+4\gamma n+\gamma-\gamma s+2\gamma^2 n+\gamma^2-\gamma^2 s\right)}{(4n^2+6\gamma n^2+2\gamma n-2\gamma ns+2\gamma^2 n^2+\gamma^2 n-\gamma^2 ns-\gamma^2 s-\gamma^2 s^2)^2}, & i \in I/A \end{cases} \tag{10}$$

where s is equal to the number of elements in the set A, or $s = |A|$.

The benchmark model can naturally be extended to a situation of more than two fringe firms by defining the normalized profit increments as follows[34]

$$a_s(\gamma) = \frac{\sum_{i\in A}\left\{\hat{\Pi}_i^s(\gamma) - \hat{\Pi}_i^0(\gamma)\right\}}{\sum_{i\in I}\left\{\hat{\Pi}_i^n(\gamma) - \hat{\Pi}_i^0(\gamma)\right\}} \text{ and } b_s(\gamma) = \frac{\sum_{i\in I/A}\left\{\hat{\Pi}_i^s(\gamma) - \hat{\Pi}_i^0(\gamma)\right\}}{|I/A|\sum_{i\in I}\left\{\hat{\Pi}_i^n(\gamma) - \hat{\Pi}_i^0(\gamma)\right\}}, 0 \leq s \leq n,$$

where $|I/A|$ is the number of non-captured brands which is equal to $n - s$.

It was shown by Deneckere and Davidson (1985) that values $a_s(\gamma)$ and $b_s(\gamma)$ are increasing in s for a positive γ. This reflects the fact that the higher industry concentration the higher profits are of the leading firm and non-captured fringe firms. Since in the case of two fringe firms we have shown existence of non-monopolization for negative values of γ, in this section we check for a possibility of having non-monopolization when γ is positive.

The games which are analyzed above can be extended to the case of many fringe firms in a natural way. In the rest of the section we describe extensions of the main games Γ_1 and Γ_2,

[34] Let us point out that $b_n(\gamma)$ is not defined, because for $s = n$ all brands are controlled by the leading firm.

which are of the central interest in this paper, and investigate under which conditions these games still have subgame perfect equilibria with non-monopolization.

n-fringe firms generalized game Γ_1 :

Stage 1: The expanding firm makes simultaneous offers $(z_1, z_2, ..., z_n)$ to fringe firms for their businesses.

Stage 2: Each fringe firm i, $i \in 1, ..., n$, observes all offers and simultaneously with the other fringe firm accepts ($\{A\}$) or rejects ($\{R\}$) its corresponding offer z_i.

Stage 3: Price competition by firms in business (the expanding firm and fringe firm(s) who rejected the offer) takes place, and the firms in business receive profits from the brands they control, according to formula (10), while the fringe firms who get captured get payments from the leading firm.

Application of the subgame perfection concept to this game implies that given an equilibrium number s^* of acquired fringe firms, firm i accepts any offer which is higher than b_{s^*-1}, and rejects offers which are smaller. Hence, the optimal behavior of the leading firm is to offer b_{s^*-1} to all fringe firms which will be captured in the equilibrium. The number of brands s^* to acquire is chosen by the leading firm on the following basis. It maximizes the difference between the extra profit $a_{s^*}(\gamma)$ and total payment to the fringe firms $s^* b_{s^*-1}(\gamma)$. Because the expanding firm has strictly positive extra profits when one fringe firm is acquired, we have that *at least one fringe firm gets captured in equilibrium.* More exactly, the equilibrium number of captured fringe firms is given by

$$s^* \in \arg \max_{s=1,...,n} \{a_s(\gamma) - s b_{s-1}(\gamma)\}.$$

When $b_{n-1}(\gamma) \geq 1/n$, it immediately follows that in equilibrium we have at least one non-captured firm. The following proposition shows this formally, and in addition it describes what happens when a number of fringe firms becomes larger.

Proposition 2 *When $b_{n-1}(\gamma) \geq 1/n$ and γ is positive, all equilibria in the n-fringe firms generalized game Γ_1 are non-monopolization. Moreover, for any $\gamma > 0$ (any degree of products*

substitutability) there is sufficiently large n^* such that for any $n \geq n^*$ the value $b_{n-1}(\gamma)$ is strictly higher than $1/n$.

PROOF: Because $a_n(\gamma) = 1$, $a_1(\gamma) > 0$ and $b_0(\gamma) = 0$, the first statement of the proposition holds because full monopolization yields non-positive extra profits, while acquisition of just one firm makes it positive. Now let us prove the second statement.

Let m denote a number of fringe firms captured. When m is equal to zero, each brand brings in the following amount of profits:

$$\hat{\Pi}_i^0(\gamma) = \frac{1+\gamma}{(2+\gamma)^2} < \frac{1}{4}, \gamma > 0, i = 0, 1, ..., n.$$

With full monopolization, each brand yields

$$\hat{\Pi}_i^n(\gamma) = \frac{1}{4}, i = 0, 1, ..., n.$$

Finally, the value which is critical for full monopolization is the profit $\hat{\Pi}_i^{n-1}(\gamma)$, $i \in A$ that a single "survived" fringe firm collects. Without loss of generality it can be assumed that this brand is indexed by n. The profit collected from this brand is equal to

$$\hat{\Pi}_n^{n-1}(\gamma, n) = \frac{(2 + 3\gamma + 2\gamma/n + \gamma^2 + 2\gamma^2/n)(2 + \gamma + 2\gamma/n)}{(4 + 4\gamma + 4\gamma/n + 3\gamma^2/n)^2}. \tag{11}$$

Given $\gamma > 0$, the function $\hat{\Pi}_n^{n-1}(\gamma, n)$ has the following limiting value[35]:

$$\lim_{n \to +\infty} \hat{\Pi}_n^{n-1}(\gamma, n) = \frac{(2+\gamma)^2}{16(1+\gamma)} > \hat{\Pi}_0^n(\gamma) = \frac{1}{4}.$$

By definition, the value of b_{n-1} is equal to

$$b_{n-1} = \frac{\hat{\Pi}_n^{n-1}(\gamma, n) - \hat{\Pi}_0^0(\gamma)}{(n+1)\left(\hat{\Pi}_0^n(\gamma) - \hat{\Pi}_0^0(\gamma)\right)},$$

and because $\hat{\Pi}_0^n(\gamma)$ does not depend upon n, it follows that

$$\lim_{n \to +\infty} n b_{n-1} = \frac{\lim\limits_{n \to +\infty} \hat{\Pi}_n^{n-1}(\gamma, n) - \hat{\Pi}_0^0(\gamma)}{\hat{\Pi}_0^n(\gamma) - \hat{\Pi}_0^0(\gamma)} = \frac{8 + 8\gamma + \gamma^2}{4(1+\gamma)} > 1.$$

[35] To take a limit it is sufficient to eliminate terms with n in equation (11).

Hence, for any $\gamma > 0$ there is such n^* that for any $n \geq n^*$ we have $b_{n-1} \geq 1/n$. Q.E.D.

Now let us briefly discuss the **n-fringe firms generalized game** Γ_2. Here in any time period the leading firm has a right to make an offer to any fringe firm. Because in this game there are more potential states[36] (to be exact, there are $n+1$ of them) than in the game Γ_2, the analysis of all equilibria becomes extremely complicated. Still, as in the previous section, when b_{n-1} is sufficiently high there are subgame perfect equilibria where the expanding firm gets zero continuation payoff, or it does not receive any additional profits from monopolization. These equilibria allow one to construct an equilibrium where no fringe firm gets ever acquired. As in the previous section a "non-beneficial monopolization" equilibrium can be used to "punish" all deviations of the leading firm from the equilibrium play. Also, monopolization can be postponed for some time for "moderate" values of b_{n-1}. The higher the value of b_{n-1} or the discount factor δ, the longer monopolization process may last. Let us state an analog of Proposition 2.

Proposition 3 *When $b_{n-1}(\gamma) > 1/n$ and γ is positive, there is such δ^* than for any discount factor δ that satisfies $\delta \geq \delta^*$, the n-fringe firms generalized game Γ_2 has a subgame perfect equilibrium with non-monopolization.*

PROOF: Here we outline the main steps of the proof. Most ideas are similar to the ones in the proof of Proposition 1. Again, all payments are expressed in terms of net or discounted values. If there is immediate monopolization (n brands get acquired during first n periods), the last fringe firm receives net payment $\delta^{n-1} b_{n-1}(\gamma)$. Given the structure of beliefs consistent with every fringe firm's expectation of being the last in the acquisition sequence, it is perceivable that the expanding firm pays up to $TP = n\delta^{n-1} b_{n-1}(\gamma)$ in total, whereas the total benefits from immediate monopolization are $TB = (1-\delta)\sum_{i=1}^{n-1} \delta^{i-1} a_i(\gamma) + \delta^{n-1}$. Let $\delta^* < 1$ be such that the total payments TP are equal to the total benefits TB (such δ^* exists because $b_{n-1}(\gamma) > 1/n$). Then for every δ such that $\delta \geq \delta^*$ it is possible to construct a

[36] Each state corresponds to the number of captured fringe firms.

subgame perfect equilibrium where the industry gets immediately monopolized with zero extra profits for the expanding firm, and the payment $\delta^{n-1}b_{n-1}(\gamma)$ for the last active fringe firm. Because of these non-beneficial equilibria there is an equilibrium where in the equilibrium play the leading firm always offers zeros, and fringe firms reject these offers. All other offers which are smaller than $\delta^{n-1}b_{n-1}(\gamma)$ are also rejected in the equilibrium, because the fringe firm under offer believes that it will receive $\delta^{n-1}b_{n-1}(\gamma)$. In turn, the expanding firm believes that any nonzero offer will bring it nonpositive extra profits. Q.E.D.

From the second part of Proposition 2 it follows that given any degree of substitutability of products the more fringe firms in an industry the more likely non-monopolization is. Let us also point out that unlike the two fringe firms situation a new type of equilibria emerges. In these equilibria only some of fringe firms get acquired eventually. Also, it can be more than one period time gap between successive captures when at least three fringe firm are left to acquire.

We conclude this section with a discussion of the inputs acquisition problem. It can be checked that as in the case of two fringe firms in the n-fringe firms generalizations of games Γ_1 and Γ_2 all fringe firms get acquired at zero prices. Moreover, in the infinite time setting all n fringe firms get captured during the first n time periods.

6 Conclusion

It has been shown that in presence of positive correlation between industry concentration and fringe firm's profit, a fringe firm can value its business higher than present value of profits it actually collects (business overvaluation). This may decrease potential gains from industry monopolization for the expanding firm, and in some cases it can make them so low that the leading firm does not get any positive extra profits which can stop the monopolization process. Other two effects of business overvaluation are delayed industry monopolization and that all the extra profit from monopolization goes to fringe firms (the leading firm stays break even). In this paper various bargaining processes of acquisition, including the ones where a leading

firm has an absolute bargaining power, are inspected. In all of them, the presence of the effect of business overvaluation was revealed.

In a price competing industry with differentiated products the possibility of non-monopolization depends on the substitutability of brands. The lower the degree of substitutability the more likely it is. This happens because for close substitutes there is a slow increase in fringe firms profits with the industry concentration.

It can be argued that a lack of bargaining rounds for the leading firm may be a cause of the absence of monopolization. Addition of extra rounds in the games of interest does not eliminate non-monopolization outcomes, even when the subgame perfect refinement concept is applied. An introduction of counteroffers from fringe firms just facilitates the main results due to a decrease in the bargaining power of the leading firm.

There is a unique subgame perfect equilibrium payoff profile in the dynamic bargaining where the leading firm has full bargaining power and there are no gains for the fringe firms from industry concentration. The fringe firms get paid their net profits, and they get captured right away. Business overvaluation effect gives rise to multiple subgame perfect equilibria with different payoff profiles and timings of capture. Because in a dynamic game every information set is a singleton, application of other refinement concepts does not reduce a set of subgame perfect equilibria. A solution of the issue of multiplicity of equilibria is a topic of future research.

7 Appendix

Analysis of Lemma 5 with mixed strategies at stage 2 of game Γ_1

Lemma 9 *If the fringe firms can play mixed strategies at stage 2, the game Γ_1 has the following outcome profile as subgame perfect equilibria: If $2b + a < 1$, both firms get captured at the price of b for each. When $2b + a = 1$, there are two equilibria: two firms are captured at the price of b for each, and only one fringe firm gets acquired at a zero price. Finally, in*

the case $2b + a > 1$ *the value*

$$\underline{\pi}(a,b) \equiv \max\left\{1-2b,\ a - \frac{\left(b+2a-\sqrt{b^2+4a(1-a-b)}\right)b}{2\left(2a+2b-1\right)}\right\} \in (0,a)$$

is the supremum of possible payoffs that the expanding firm gets at stage 2 when Nash equilibria with the lowest payoffs are selected. Let π^* *be an expected extra profit of the leading firm. There are three types of equilibria:*

(i) The expanding firm captures one fringe firm at the nonnegative price $a - \pi^*$*, given that* $\pi^* \in [\underline{\pi}(a,b), a)$*;*

(ii) One fringe firm accepts the offer z *and the other firm accepts the offer* b *with probability* $\lambda \leq z/b$*. Only a pair of* z *and* λ *such that* $\pi^* = (1-a-b)\lambda + z \in [\underline{\pi}(a,b), a)$ *yields this equilibrium;*

(iii) The leading firm submits offers $z_1, z_2 \in (0,b)$*. Firm 1 accepts this offer with probability* z_2/b*, and firm 2 takes this offer with probability* z_1/b*. This equilibrium exists when* $\pi^* = \left[(1-2a-2b)z_1z_2 + (z_1+z_2)ab\right]/b^2 \in [\underline{\pi}(a,b), a)$*.*

PROOF: At any subgame perfect equilibrium the leading firm offers (z_1^*, z_2^*), then it receives an expected payoff π^* (*equilibrium payoff*) which is an outcome of some Nash equilibrium in the subgame $\tilde{\Gamma}_1(z_1^*, z_2^*)$. Any deviation $(z_1, z_2) \neq (z_1^*, z_2^*)$ brings such an equilibrium in the subgame $\tilde{\Gamma}_1(z_1, z_2)$, where the expanding firm gets an expected payoff (*threat payoff*) that is no larger than π^*. Any game $\tilde{\Gamma}_1(z_1, z_2)$ is a 2×2 game, where each fringe firm has two strategies: accept an offer $\{A\}$ and reject an offer $\{R\}$. The normal form of the subgame $\tilde{\Gamma}_1(z_1, z_2)$ is depicted on Figure 2. Let us find out possible values of π^* and corresponding equilibria in $\tilde{\Gamma}_1(z_1^*, z_2^*)$. This will be done in a number of steps. First, we investigate all possible equilibria in mixed strategies in the subgames $\tilde{\Gamma}_1(z_1, z_2), (z_1 \geq 0, z_2 \geq 0)$, which for brevity we just call *mixed equilibria*. Second, possible threat payoffs are constructed, and finally, equilibrium payoffs are calculated.

Step 1: *Characterization of all mixed equilibria in subgames* $\tilde{\Gamma}_1(z_1, z_2)$*.*

Any fringe firm has a dominant strategy $\{A\}$ (accept an offer) when it is offered more than b. Hence, it follows that any subgame $\tilde{\Gamma}_1(z_1, z_2)$ where $z_1 > b$ and $z_2 > b$, has a unique equilibrium $\{A, A\}$ with the corresponding payoff of $1 - z_1 - z_2$ to the expanding firm. This implies that $\pi^* \geq 1 - 2b$, and a fringe firm does not "mix" strategies when it is offered more than b.

When some fringe firm i is offered $z_i = b$, it is indifferent between strategies $\{A\}$ and $\{R\}$ given that the other fringe firm j accepts its offer. If the other firm puts positive probability on the strategy $\{R\}$ then firm i's strict best response is $\{A\}$. Hence, given these facts the only possible mixed equilibrium is of the following form: firm i plays $\{\lambda A + (1-\lambda)R\}$, $\lambda \in (0,1)$, and firm j plays $\{A\}$, where firm i chooses the strategy $\{A\}$ with probability λ, and $\{R\}$ with the complementary probability. Now it is necessary for the firm j to have $\{A\}$ as the best response. This happens when $z_j \geq \lambda b$. Hence, there is a set of mixed equilibria of the form $\{\lambda A + (1-\lambda)R, A\}, \lambda \leq z_2/b$ in any subgame $\tilde{\Gamma}_1(b, z_2)$, and $\{A, \lambda A + (1-\lambda)R\}, \lambda \leq z_1/b$ in any subgame $\tilde{\Gamma}_1(z_1, b)$, where the leading firm receives payoffs $\lambda(1-b) + (1-\lambda)a - z_2$ and $\lambda(1-b) + (1-\lambda)a - z_1$, correspondingly. Note that when $z_j > b$, all equilibria are of the above type. Since $1 - b > a$ (see Lemma 4), the payoff of firm 0 is strictly increasing in λ and takes any value in segment $[a - z_j, 1 - b - z_j]$. When $z_j < b$, the payoff to firm 0 lies in the interval $[a - z_j, (1-b)z_j/b + (1 - z_j/b)a - z_j]$. Also, when $z_j < b$, there is an additional equilibrium where firm i plays $\{A\}$ and firm j plays $\{R\}$, and the expanding firm gets $a - b < 0$. Finally, in the subgame $\tilde{\Gamma}_1(b, b)$ we have a set of equilibria $\{\lambda A + (1-\lambda)R, A\}$ and $\{A, \lambda A + (1-\lambda)R\}$, $\lambda \in [0, 1]$, where the payoff of the expanding firm lies within the range $[1 - 2b, a - b]$. We will return to the subgames considered here, which we call *b-subgames*.

Now let us investigate situations where fringe firm i is offered $z_i < b$. When $z_j > b$, there are no mixed equilibria in a proper subgame. The case $z_j = b$ is studied in the previous paragraph. The last set of subgames of the form $\tilde{\Gamma}_1(z_1, z_2)$, $z_1 < b$, $z_2 < b$ is left to analyze. Any of these subgames has two equilibria in pure strategies, they are $\{A, R\}$ and $\{R, A\}$, and one mixed equilibrium $\{[z_2/b]\, A + [1 - z_2/b]\, R, [z_1/b]\, A + [1 - z_1/b]\, R\}$.[37] The expanding firm

[37] When $z_1 = z_2 = 0$, the mixed equilibrium becomes a "pure" equilibrium $\{R, R\}$.

gets $a - z_1$, $a - z_2$ and $\{(1-2a-2b)z_1z_2 + (z_1+z_2)ab\}/b^2$, correspondingly.

Step 2: *Calculation of guarantied level of threat payoffs.*

Now let us turn attention to the supremum of threat equilibrium payoffs over all possible pairs (z_1, z_2) because at any Nash equilibrium firm 0 gets at least this value. We denote by $\underline{\pi}$ the minimum level this supremum takes. As derived at Step 1, when both offers z_1 and z_2 are larger than b the corresponding subgame $\tilde{\Gamma}_1(z_1, z_2)$ has only one equilibrium yielding $1 - z_1 - z_2$ to the expanding firm. Hence, $\underline{\pi} \geq 1 - 2b$.

One can check that in any b-subgame and at any equilibrium the expanding firm has the payoff that is no larger than $a - b$. Hence, $\underline{\pi} \geq a - b$.

When one offer, say z_1, is strictly larger than b, and the other is strictly smaller than b, the corresponding subgame has a single equilibrium $\{A, R\}$, where firm 0 receives $a - z_1$. This gives the same restriction $\underline{\pi} \geq a - b$.

At last, a pair of offers that are both strictly smaller than b yields the following restriction:

$$\underline{\pi} \geq \max_{z_1 \in [0,b], z_2 \in [0,b]} \min \left\{ a - z_1, a - z_2, \frac{(1-2a-2b)z_1z_2 + (z_1+z_2)ab}{b^2} \right\} \equiv \rho(a,b). \qquad (12)$$

Given that the values of a and b belong to the open set $= \{(a,b) : 0 < a < b$ and $a + b < 1\}$ (Lemma 4 shows that that parameters a and b belong to this set), a careful analysis of (12) shows that at the maxmin value all three expressions in the curly brackets take equal values. This allows one to get a closed form expression for the function $\rho(a,b)$:

$$\rho(a,b) = a - \frac{\left(b + 2a - \sqrt{b^2 + 4a(1-a-b)}\right) b}{2\,(2a+2b-1)}. \qquad (13)$$

Next, one can show that the function $g(a,b) \equiv \rho(a,b)/a$ is continuous on the set , and is increasing in a and is decreasing in b. The map $g(a,b) : \to (0,1)$ is onto and $\lim_{(a,b)\to(0,1)} g(a,b) = 0$ and $\lim_{(a,b)\to(0,0)} g(a,b) = 1$. This shows that the function $\rho(a,b)$ takes values within the interval $(0,a)$. Finally, we have the following formula for the function $\underline{\pi}(a,b)$ depicted on Figure 5:

$$\underline{\pi}(a,b) = \max\{1-2b, \rho(a,b)\}. \qquad (14)$$

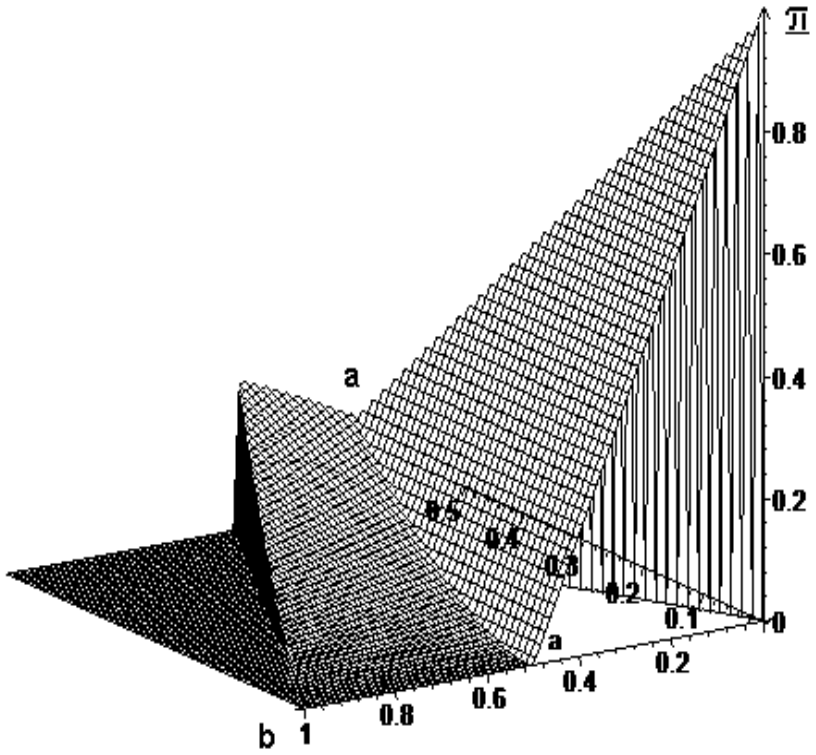

Figure 5: Graph of function $\pi(a, b)$ on set .

Step 3: *Description of equilibria.*

Because the analysis is restricted to subgame perfect equilibria, at the equilibrium play the leading firm receives payoffs that are no smaller than $\pi(a, b)$ given by (14). Because $\pi(a, b) \geq \max\{1 - 2b, 0\}$, the equilibrium subgames correspond to offers that are no larger than b. When $1 - 2b \geq a$, only equilibria of Lemma 5 arise. An interesting case is that when $1 - 2b < a$. Let us describe new equilibria that emerge in this case in addition to the ones described in Lemma 5. There are three types of equilibria:

b-equilbria (one firm accepts her offer and the other accepts offer b with some probability λ*)*: Here the expanding firm obtains any expected payoff $\pi^* \in [\pi(a, b), a)$. From the descrip-

tion of b-equilibria at Step 1 it follows that a fringe firm, say i, is offered b, and the other firm j accepts the offer z_j. The firm i accepts the offer with probability $\lambda = (\pi^* + z_j - a)/(1-a-b)$. Because $\lambda \in (0, z_j/b]$, it follows that $z_j \in [a - \pi^*, (a - \pi^*)b/(1 - a - 2b)]$.

One firm captured equilibria: The leading firm captures one fringe firm at a positive offer of $a - \pi^*$ and receives $\pi^* \in [\underline{\pi}(a, b), a)$ of extra profits. The other firm is offered no more than b.

Totally mixed equilibria: Here both fringe firms have positive offers z_1 and z_2 which are strictly smaller than b. Under these offers they play a pair of totally mixed strategies $\{[z_2/b]\, A + [1 - z_2/b]\, R,\ [z_1/b]\, A + [1 - z_1/b]\, R\}$. This kind of equilibrium takes place for the values $z_1 \in (0, b)$, $z_2 \in (0, b)$ and $\pi^* \in [\underline{\pi}(a, b), a)$ related in the following way:

$$\frac{(1 - 2a - 2b)z_1 z_2 + (z_1 + z_2)ab}{b^2} = \pi^*.$$

One can show that this type of equilibrium exists when $\rho(a, b) \geq 1 - 2b$ (see (13) for the formulas of the function $\rho(.,.)$), and π^* is sufficiently close to $\rho(a, b)$. From Figure 3 it can be seen that this happens when, given any value of a, the value of b is sufficiently high. Q.E.D.

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