Testing for Mean-Variance Spanning with Short Sales Constraints and Transaction Costs: The Case of Emerging Markets

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ABSTRACT

We propose regression-based tests for mean-variance spanning in the case where investors face market frictions such as short sales constraints and transaction costs. We test whether U.S. investors can extend their efficient set by investing in emerging markets when accounting for such frictions. For the period after the major liberalizations in the emerging markets, we find strong evidence for diversification benefits when market frictions are excluded, but this evidence disappears when investors face short sales constraints or small transaction costs. Although simulations suggest that there is a possible small-sample bias, this bias appears to be too small to affect our conclusions.

THE QUESTION OF WHETHER OR NOT an investor can extend his efficient set by including additional assets in his portfolio has recently received considerable attention in the literature. If extension of the efficient set is not possible for a specific mean-variance utility function, the mean-variance frontier of the benchmark assets and of the benchmark assets plus the additional assets intersect, that is, they have one point in common. If extension of the efficient set is not possible for any mean-variance utility function, the mean-variance frontier of the initial assets spans the frontier of the larger set of the initial assets plus the additional assets. These concepts are discussed by Huberman and Kandel (1987), who propose regression-based tests of the hypotheses of spanning and intersection for mean-variance investors. It is well known by now that a shift in the mean-variance frontier from adding assets to the investment opportunity set is tantamount to a shift in the volatility bounds of the kernels that price the assets under consideration (e.g., DeSantis (1994) and Bekaert and Urias (1996)) and that the issue is also very closely related to performance evaluation (see, e.g., Jobson and

Korkie (1988) and Chen and Knez (1996)). DeRoon, Nijman, and Werker (1997a) show how tests for spanning can be extended to allow for other utility functions, and to allow for the presence of nonmarketable risks.

Tests for intersection and spanning have been applied to numerous problems in the finance literature. A crucial assumption in almost all these applications is the absence of market frictions such as short sales restrictions and transaction costs. However, for many investors such frictions are important facts of life. The aim of this paper is to extend the tests for mean-variance spanning and intersection to take these market frictions into account.

The presence of transaction costs and short sales constraints is perhaps most predominant in the case of emerging markets. Using the Emerging Markets Data Base (EMDB) of the International Finance Corporation (IFC), both DeSantis (1994) and Harvey (1995) show that the mean-variance frontier that is based on well-developed Western markets only significantly shifts outward when the emerging markets are included. However, these results presuppose that there are no transaction costs or any other market frictions for both the developed and the emerging markets. Bekaert and Urias (1996) try to overcome this problem using returns on closed-end country funds, because the returns on these funds are attainable to investors. Based on emerging market country funds, Bekaert and Urias find only mixed evidence for the diversification benefits of emerging markets. Although the use of country funds adjusts for the effect of transaction costs and short sales constraints that investors face in emerging markets, it does not account for short sales constraints and transaction costs on the country funds themselves or on the benchmark assets.

Using industry portfolios, multinational corporation stocks, closed-end country funds, and American depository receipts, Errunza, Hogan, and Hung (1999) show that U.S. investors can create mimicking portfolios from U.S.-traded securities that are highly correlated with the IFC emerging markets indices. Their spanning tests show that for five out of the nine emerging markets that they study, direct investments in the emerging markets provide significant diversification benefits beyond diversified portfolios created from U.S.-traded securities. Errunza et al. (1999) do not consider short sales constraints or transaction costs on either the emerging markets or the U.S.-traded securities. Note that the effect of transaction costs will probably be smaller for the U.S.-traded securities than for the emerging markets, but that short sales constraints may cause the diversification benefits from direct investments in the emerging markets to be even stronger than suggested by the results of Errunza et al. (1999).

We provide direct evidence on the effect of transaction costs and short selling constraints on the diversification benefits of emerging markets by using the same IFC indices as in DeSantis (1994) and Harvey (1995), but incorporating these market frictions in our testing methodology. If frictions are ignored, we find that there are significant diversification benefits from adding emerging markets to an international stock portfolio that invests in the United States, Europe, and Japan. The evidence in favor of these diver-

sification benefits disappears when we take short sales constraints and investability restrictions into account, because in this case, for the three geographical regions, the hypothesis of spanning cannot be rejected. This effect is mainly due to the short sales constraints for the emerging markets and not to the short sales constraints on the benchmark assets. Also, with an investment horizon of six months and round trip costs of 0.5 percent on the benchmark assets, we find that significant diversification benefits from investing in the emerging markets are absent with only small transaction costs on the emerging markets. Although our simulations suggest that there is a possible small-sample bias in the asymptotic test, the magnitude of the bias appears to be small.

The plan of this paper is as follows. In Section I we, first of all, formulate the hypotheses of mean-variance spanning and intersection in the case of short sales restrictions. Regression-based tests for these hypotheses are proposed in Section II. The empirical results on investing in emerging markets are presented in Section III and in the final section we give some concluding remarks.

I. Mean-variance Spanning With Short Sales Constraints

Consider a set of K assets, whose gross returns are given by the vector $\mathbf{R}_{t+1}$. Investors can hold portfolios $\mathbf{w} \in C \subset \mathbb{R}^K$ such that $\mathbf{w}'\boldsymbol{\iota}_K = 1$, where $\boldsymbol{\iota}_K$ is a K-vector containing only ones. The set of returns available to investors is therefore given by

$$X = \{R_{t+1}^p : R_{t+1}^p = \mathbf{w}'\mathbf{R}_{t+1}, \mathbf{w} \in C, \text{ and } \mathbf{w}'\boldsymbol{\iota}_K = 1\}.$$

In case there are no market frictions, we have $C = \mathbb{R}^K$. If, in addition, the Law of One Price holds, there exists a stochastic discount factor M_{t+1} such that

$$E[M_{t+1}\mathbf{R}_{t+1}|I_t] = \boldsymbol{\iota}_K, \tag{1}$$

where I_t denotes the information set that is available to investors at time t (see, e.g., Duffie (1996)). In this paper, we will restrict ourselves to unconditional versions of equation (1) and to unconditional mean-variance spanning. Extensions of our results to the conditional case are straightforward however (see, e.g., DeSantis (1994) and DeRoon et al. (1997a)). The stochastic discount factor $m(v)_{t+1}$ that has expectation v and that corresponds to a mean-variance utility function, is a linear function of the asset returns:

$$m(v)_{t+1} = v + \boldsymbol{\alpha}'(\mathbf{R}_{t+1} - E[\mathbf{R}_{t+1}]), \tag{2a}$$

$$\boldsymbol{\alpha} = \text{Var}[\mathbf{R}_{t+1}]^{-1}(\boldsymbol{\iota}_K - vE[\mathbf{R}_{t+1}]). \tag{2b}$$

From Hansen and Jagannathan (1991) we know that the discount factor given in equation (2) has the lowest variance of all stochastic discount factors with expectation v, that price $\mathbf{R}_{t+1}$ correctly. It is also well known that $\mathbf{w} = \boldsymbol{\alpha}/(\boldsymbol{\alpha}'\boldsymbol{\iota}_K)$ is a mean-variance efficient portfolio that has a zero-beta return equal to $1/v$.

Now consider the presence of market frictions such as short sales constraints and transaction costs. These can be dealt with by letting C be a particular subset of $I\!\!R^K$ and/or by adjusting the vector of returns $\mathbf{R}_{t+1}$ to reflect the frictions. In case of short sales constraints, $C = I\!\!R^K_+$, the nonnegative part of $I\!\!R^K$.[1] When there are short sales constraints on the portfolio holdings, the condition in equation (1) must be replaced by

$$E[m(v)_{t+1}\mathbf{R}_{t+1}] \leq \boldsymbol{\iota}_K, \tag{3}$$

where the inequality sign applies componentwise.

The mean-variance efficient frontier without short sales can be found by solving the problem

$$\max_{\{\mathbf{w}\}} \mathbf{w}'E[\mathbf{R}_{t+1}] - \tfrac{1}{2}\gamma\mathbf{w}'\,\mathrm{Var}[\mathbf{R}_{t+1}]\mathbf{w},$$

$$\text{s.t. } \mathbf{w}'\boldsymbol{\iota}_K = 1 \quad \text{and} \quad w_i \geq 0, \quad \forall i, \tag{4}$$

where γ is the coefficient of risk aversion. From the Kuhn-Tucker conditions, mean-variance efficient portfolios $\mathbf{w}^*$ satisfy

$$E[\mathbf{R}_{t+1}] - \eta\boldsymbol{\iota}_K + \boldsymbol{\delta} = \gamma\,\mathrm{Var}[\mathbf{R}_{t+1}]\mathbf{w}^*,$$

$$w_i^*, \delta_i \geq 0, \quad \forall i, \tag{5}$$

$$\delta_i w_i^* = 0, \quad \forall i.$$

The vector $\boldsymbol{\delta}$ contains the Kuhn-Tucker multipliers for the restrictions that the portfolio weights are nonnegative. The Lagrange multiplier for the restriction that $\mathbf{w}'\boldsymbol{\iota}_K = 1$ is equal to η, the intercept of the line that is tangent to the mean-variance frontier in mean-standard deviation space.

Now take the mean-variance efficient portfolio for which $\eta = 1/v$, with v the expectation of a stochastic discount factor that prices $\mathbf{R}_{t+1}$ correctly subject to short sales constraints. Denote by $\mathbf{R}_{t+1}^{(v)}$ the L-dimensional subvector of $\mathbf{R}_{t+1}$ that only contains the returns of the assets for which the short sales constraints in equation (5) are not binding and let superscripts $^{(v)}$ refer to this subset. It is straightforward to show that the mean-variance efficient

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[1] More generally, if we have a vector of lower bounds $\mathbf{w}_0$ on the portfolio weights, then we have that $\mathbf{w} \geq \mathbf{w}_0$ and $\mathbf{w} - \mathbf{w}_0 \in I\!\!R^K_+$.

portfolio in equation (5) is equal to the mean-variance efficient portfolio without short sales constraints of the assets in $\mathbf{R}_{t+1}^{(v)}$ only (see, e.g., Markowitz (1991)):

$$E[\mathbf{R}_{t+1}^{(v)}] - \frac{1}{v}\,\boldsymbol{\iota}_L = \gamma^{(v)}\,\mathrm{Var}[\mathbf{R}_{t+1}^{(v)}]\mathbf{w}^{(v)} \quad \text{and}$$

$$\tag{6}$$

$$E[\mathbf{R}_{t+1}] - \frac{1}{v}\,\boldsymbol{\iota}_K + \boldsymbol{\delta} = \gamma^{(v)}\,\mathrm{Cov}[\mathbf{R}_{t+1},\mathbf{R}_{t+1}^{(v)}]\mathbf{w}^{(v)},$$

where $\mathrm{Cov}[\mathbf{R}_{t+1},\mathbf{R}_{t+1}^{(v)}]$ is the $K \times L$-dimensional covariance matrix of $\mathbf{R}_{t+1}$ and its subvector $\mathbf{R}_{t+1}^{(v)}$.

Because the mean-variance stochastic discount factor is a linear function of the mean-variance efficient portfolio, in case of short sales restrictions, the mean-variance stochastic discount factor that prices $\mathbf{R}_{t+1}$, $m_R(v)_{t+1}$, is equal to

$$m_R(v)_{t+1} = v + \boldsymbol{\alpha}^{(v)\prime}(\mathbf{R}_{t+1}^{(v)} - E[\mathbf{R}_{t+1}^{(v)}]),$$

$$\tag{7}$$

$$\boldsymbol{\alpha}^{(v)} = \mathrm{Var}[\mathbf{R}_{t+1}^{(v)}]^{-1}(\boldsymbol{\iota}_L - vE[\mathbf{R}_{t+1}^{(v)}]).$$

It is not hard to show that the stochastic discount factor as defined in equation (7) has the lowest variance of all pricing kernels that price $\mathbf{R}_{t+1}$ correctly, subject to short sales constraints. Therefore, in case of short sales constraints, the duality between mean-variance frontiers and volatility bounds still holds.

Next consider a set of N additional assets with return vector $\mathbf{r}_{t+1}$ besides the set of K benchmark assets with return vector $\mathbf{R}_{t+1}$. Mean-variance spanning of the assets $\mathbf{r}_{t+1}$ by the benchmark assets $\mathbf{R}_{t+1}$ occurs if the mean-variance stochastic discount factors that price $\mathbf{R}_{t+1}$ correctly also price $\mathbf{r}_{t+1}$, that is, if

$$E[m_R(v)_{t+1}\mathbf{r}_{t+1}] \leq \boldsymbol{\iota}_N, \tag{8}$$

holds for all values of v. Substituting equation (7) into equation (8), this is equivalent to

$$vE[\mathbf{r}_{t+1}] + \mathrm{Cov}[\mathbf{r}_{t+1},\mathbf{R}_{t+1}^{(v)}]\mathrm{Var}[\mathbf{R}_{t+1}^{(v)}]^{-1}(\boldsymbol{\iota}_L - vE[\mathbf{R}_{t+1}^{(v)}]) \leq \boldsymbol{\iota}_N. \tag{9}$$

The inequality sign in equation (8) reflects the fact that there are short sales constraints on $\mathbf{r}_{t+1}$. In the absence of short sales constraints on $\mathbf{r}_{t+1}$, the inequality becomes an equality. If there is only one value of v for which equation (8) holds, then there is intersection.

When taking transaction costs into account, it is useful to differentiate between the return on a long position in asset i, $\tau_i^\ell R_{i,t+1}$, and the return on a short position in asset i, $\tau_i^s R_{i,t+1}$ (see, e.g., Luttmer (1996)). Let $\tilde{\mathbf{R}}_{t+1}$ be a $2K$-dimensional vector, the first K elements of which are the returns on the long positions in the assets $i = 1,\ldots,K$, and the last K elements of which are the returns on the short positions in these same assets. Thus, $\tilde{R}_{i,t+1} = \tau_i^\ell R_{i,t+1}$ and $\tilde{R}_{i+K,t+1} = \tau_i^s R_{i,t+1}$. Considering $\tilde{\mathbf{R}}_{t+1}$ as the vector of returns on $2K$ different assets, transaction costs can now be handled by requiring that investors cannot go short in the first K assets ($C = I\!\!R_+^K$) and cannot go long in the last K assets ($C = I\!\!R_-^K$). Let $\tilde{m}_R(v)_{t+1}$ be the mean-variance stochastic discount factor that prices $\tilde{\mathbf{R}}_{t+1}$ correctly and let $\tilde{\mathbf{R}}_{t+1}^{(v)}$ be the L-dimensional subvector of $\tilde{\mathbf{R}}_{t+1}$ for which the constraints on the short and long positions are not binding. The notation is therefore analogous to the case of short sales constraints only. The mean-variance stochastic discount factor is now given by[2]

$$\tilde{m}_R(v)_{t+1} = v + \tilde{\boldsymbol{\alpha}}^{(v)\prime}(\tilde{\mathbf{R}}_{t+1}^{(v)} - E[\tilde{\mathbf{R}}_{t+1}^{(v)}]),$$

$$\tilde{\boldsymbol{\alpha}}^{(v)} = \text{Var}[\tilde{\mathbf{R}}_{t+1}^{(v)}]^{-1}(\boldsymbol{\iota}_L - vE[\tilde{\mathbf{R}}_{t+1}^{(v)}]). \qquad (10)$$

In a similar way, we consider long and short positions in the N additional asset as $2N$ different assets. The returns on long positions in the additional assets are given by $(\tilde{\mathbf{r}}_{t+1}^\ell)_k = \tau_k^\ell r_{k,t+1}$, $k = 1,2,\ldots,N$, and the returns on short positions are given by $(\tilde{\mathbf{r}}_{t+1}^s)_k = \tau_k^s r_{k,t+1}$, $k = 1,2,\ldots,N$. The returns on the additional assets are then spanned by the benchmark assets if

$$E[\tilde{m}_R^{[j]}(v)_{t+1}\tilde{\mathbf{r}}_{t+1}^\ell] \le \boldsymbol{\iota}_N, \quad \forall j,$$

$$E[\tilde{m}_R^{[j]}(v)_{t+1}\tilde{\mathbf{r}}_{t+1}^s] \ge \boldsymbol{\iota}_N, \quad \forall j. \qquad (11)$$

II. Testing for Intersection and Spanning

Absent short sales constraints and any other market frictions, the hypotheses of mean-variance intersection and spanning are equivalent to the condition that

$$E[m_R(v)_{t+1}\mathbf{r}_{t+1}] = \boldsymbol{\iota}_N, \qquad (12)$$

for one value of v (intersection) or for all values of v (spanning). It is well known that in this case tests for intersection and spanning can be based on the regression

$$\mathbf{r}_{t+1} = \mathbf{a} + \mathbf{B}\mathbf{R}_{t+1} + \boldsymbol{\varepsilon}_{t+1}, \qquad (13)$$

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[2] In the portfolio problem with transaction costs, agents are prevented from taking simultaneous short and long positions in one asset, which would effectively create a long position in a risk-free asset with a negative return. Therefore, $\text{Var}[\tilde{\mathbf{R}}_{t+1}^{(v)}]$ is a nonsingular matrix.

with $E[\boldsymbol{\varepsilon}_{t+1}] = \mathbf{0}$ and $E[\boldsymbol{\varepsilon}_{t+1}\mathbf{R}'_{t+1}] = \mathbf{0}$. Intersection for a given value of v implies that $\mathbf{a}v + (\mathbf{B}\boldsymbol{\iota}_K - \boldsymbol{\iota}_N) = \mathbf{0}$, and spanning implies that $\mathbf{a} = 0$ and $\mathbf{B}\boldsymbol{\iota}_K - \boldsymbol{\iota}_N = \mathbf{0}$ (Huberman and Kandel (1987), Bekaert and Urias (1996)). Alternatively, GMM tests can be used to test for intersection and spanning (DeSantis (1994), Hansen, Heaton, and Luttmer (1995), Chen and Knez (1996)). As shown in the previous section, if there are short sales restrictions on the benchmark assets $\mathbf{R}_{t+1}$, the stochastic discount factor $m_R(v)_{t+1}$ is a linear function of $\mathbf{R}^{(v)}_{t+1}$ only, and if there are short sales restrictions on the additional assets $\mathbf{r}_{t+1}$, then the equality in equation (12) becomes an inequality. For a given value of v, the restrictions implied by intersection are given in equation (9). These restrictions are equivalent to the restrictions that in the regression

$$\mathbf{r}_{t+1} = \mathbf{a}^{(v)} + \mathbf{B}^{(v)}\mathbf{R}^{(v)}_{t+1} + \boldsymbol{\varepsilon}^{(v)}_{t+1}, \qquad (14)$$

it holds true that

$$v\mathbf{a}^{(v)} + (\mathbf{B}^{(v)}\boldsymbol{\iota}_L - \boldsymbol{\iota}_N) \leq \mathbf{0}. \qquad (15)$$

Intuitively, since in case of short sales constraints, the mean-variance efficient portfolio of $\mathbf{R}_{t+1}$ for a given value of v consists of positions in only those assets for which the constraints are not binding, intersection requires that there is intersection at the unrestricted frontier of $\mathbf{R}^{(v)}_{t+1}$.

A Wald test can be used to test the inequality constraints in equation (15) (see, e.g., Kodde and Palm (1986)). Denote the left hand side of equation (15) as $v\boldsymbol{\alpha}_J(v)$, where $\boldsymbol{\alpha}_J(v)$ is the N-dimensional vector of Jensen's alphas of the assets $\mathbf{r}_{t+1}$ relative to the mean-variance efficient portfolio of $\mathbf{R}^{(v)}_{t+1}$ with zero-beta return $1/v$. The sample equivalent of $\boldsymbol{\alpha}_J(v)$ is $\hat{\boldsymbol{\alpha}}_J(v)$, and the estimated $N \times N$ covariance matrix of $\hat{\boldsymbol{\alpha}}_J(v)$, $\text{Var}[\hat{\mathbf{a}}_J(v)]$, can be obtained from the restricted covariance matrix of the OLS-estimates of equation (14), where the restrictions are given by $v\mathbf{a}^{(v)} + (\mathbf{B}^{(v)}\boldsymbol{\iota}_L - \boldsymbol{\iota}_N) = \mathbf{0}$. Following Kodde and Palm (1986), under the null hypothesis and standard regularity conditions, the test statistic

$$\xi(v) = \min_{\{\boldsymbol{\alpha}_J(v) \leq 0\}} (\hat{\boldsymbol{\alpha}}_J(v) - \boldsymbol{\alpha}_J(v))' \, \text{Var}[\hat{\boldsymbol{\alpha}}_J(v)]^{-1} (\hat{\boldsymbol{\alpha}}_J(v) - \boldsymbol{\alpha}_J(v)), \qquad (16)$$

is asymptotically distributed as a mixture of χ^2 distributions. For the case considered here, where we test whether there is intersection for the N assets $\mathbf{r}_{t+1}$, the probability of $\xi(v)$ exceeding a given value c is, under the null hypothesis, given by (see, e.g., Kodde and Palm (1986))

$$\Pr\{\xi(v) \geq c\} = \sum_{i=0}^{N} \Pr\{\chi^2_i \geq c\} w(N, i, \text{Var}[\hat{\boldsymbol{\alpha}}_J(v)]), \qquad (17)$$

where $w(N, i, \text{Var}[\hat{\boldsymbol{\alpha}}_J])$ are probability weights.[3] Given the estimated covariance matrix $\text{Var}[\hat{\boldsymbol{\alpha}}_J(v)]$, the probabilities can be determined using numerical simulation, as proposed by Gouriéroux, Holly, and Montfort (1982). Alternatively, without calculating the weights, Kodde and Palm (1986) give expressions for an upper and a lower bound on the p values of $\xi(v)$.

Of course, when implementing the intersection test in empirical applications, it is usually the case that for a particular value of v, we do not observe which assets are in $\mathbf{R}_{t+1}^{(v)}$, but have to derive this information from the asset returns in our sample. It is shown in the Appendix that this does not affect the limit distribution of the Wald test statistic for the restrictions in equation (15) however, if v corresponds to an efficient portfolio where none of the weights in $\mathbf{w}^{(v)}$ is exactly zero (i.e., $w_i^* = 0$ and $\delta_i > 0$). If this latter situation occurs, then it is easily verified that the size of the test (conditional on $\mathbf{R}_{t+1}^{(v)}$) does not depend on $\mathbf{R}_{t+1}^{(v)}$, and hence the unconditional size equals the one chosen, which shows the validity of our test.

Glen and Jorion (1993) propose a test for intersection with short sales constraints based on the difference in Sharpe ratios[4] of the benchmark assets and the total set of assets. Unlike the test statistic in equation (16) however, their test does not yield a known distribution. Apart from this, our procedure has the advantage that we avoid the assumption that one of the assets is riskless and that the test can also be used to test for spanning.

Up to now, we considered tests for intersection. Spanning implies that the restrictions in equation (15) hold for all relevant values of v. Notice that for a given set of K asset returns $\mathbf{R}_{t+1}$, there is only a finite number of subsets with $L^{(v)}$ elements, $L^{(v)} \in \{1, 2, \dots, K\}$, with $\mathbf{R}_{t+1}^{(v)}$ the $L^{(v)}$-dimensional vector containing the returns on the subset of the assets. Let $V^{[j]}$ be the set of those values of v for which the subset of assets for which the short sales constraints in the mean-variance efficient portfolios are not binding is the same, and denote the $L^{[j]}$-dimensional vector of returns for these assets as $\mathbf{R}_{t+1}^{[j]}$, that is, $\mathbf{R}_{t+1}^{[j]} = \mathbf{R}_{t+1}^{(v)}$ if and only if $v \in V^{[j]}$. Similarly, each variable or parameter that refers to the set $\mathbf{R}_{t+1}^{[j]}$ will be denoted with a superscript $[j]$. Because for $v \in V^{[j]}$ the mean-variance efficient frontier of $\mathbf{R}_{t+1}$ coincides with the mean-variance frontier of $\mathbf{R}_{t+1}^{[j]}$, the mean-variance frontier of $\mathbf{R}_{t+1}$ with short sales constraints consists of a finite number of parts of the unrestricted mean-variance frontiers of the subsets $\mathbf{R}_{t+1}^{[j]}$. It follows that the return on the additional assets $\mathbf{r}_{t+1}$ are spanned by the returns on the benchmark assets $\mathbf{R}_{t+1}$ if

$$E[m_R^{[j]}(v)_{t+1}\mathbf{r}_{t+1}] \le \boldsymbol{\iota}_N, \quad \forall j, \tag{18}$$

[3] The weights $w(N, i, \text{Var}[\hat{\boldsymbol{\alpha}}_J])$ are the probabilities that $(N - i)$ of the N elements of a vector with a $N(0, \text{Var}[\hat{\boldsymbol{\alpha}}_J])$ distribution are strictly negative.

[4] In fact, the test statistic in equation (16) can also be expressed in terms of Sharpe ratios. For details, see DeRoon, Nijman, and Werker (1997b).

where $m_R^{[j]}(v)_{t+1}$ is the mean-variance pricing kernel with expectation v that is linear in $\mathbf{R}_{t+1}^{[j]}$. If there are only short sales constraints on the benchmark assets $\mathbf{R}_{t+1}$ and not on the additional assets $\mathbf{r}_{t+1}$, the inequality in equation (18) becomes an equality. If there are only short sales constraints on $\mathbf{r}_{t+1}$ and not on $\mathbf{R}_{t+1}$, $\mathbf{R}_{t+1}^{[j]} = \mathbf{R}_{t+1}$.

If there are short sales constraints, then the mean-variance frontier of $\mathbf{R}_{t+1}$ consists of parts of the unrestricted mean-variance frontiers of the subsets of returns $\mathbf{R}_{t+1}^{[j]}$, $j = 1,2,\ldots,M$, and $\mathbf{r}_{t+1}$ can only be spanned by the returns $\mathbf{R}_{t+1}$ if it is spanned by the M subsets of $\mathbf{R}_{t+1}$. If there are also short sales restrictions on $\mathbf{r}_{t+1}$, then the conditions in equation (18) imply that there is mean-variance spanning if and only if in the M regressions

$$\mathbf{r}_{t+1} = \mathbf{a}^{[j]} + \mathbf{B}^{[j]}\mathbf{R}_{t+1}^{[j]} + \boldsymbol{\varepsilon}_{t+1}^{[j]}, \tag{19a}$$

$$\mathbf{a}^{[j]}v + \mathbf{B}^{[j]}\boldsymbol{\iota}^{[j]} \le \boldsymbol{\iota}_N, \quad \text{for all } v \in V^{[j]}. \tag{19b}$$

where $\boldsymbol{\iota}^{[j]}$ is an $L^{[j]}$-dimensional vector consisting of ones. The hypothesis that there is spanning can therefore easily be tested by using a multivariate regression of $\mathbf{r}_{t+1}$ on all $\mathbf{R}_{t+1}^{[j]}$ and using a Wald test for the restrictions in each of these regressions. Denoting $v_{\min}^{[j]} = \min_{v \in V^{[j]}} v$, and $v_{\max}^{[j]} = \max_{v \in V^{[j]}} v$, the restrictions in equation (19b) are satisfied if there is intersection for $v_{\min}^{[j]}$ and for $v_{\max}^{[j]}$, because in that case, there is also intersection for all the intermediate values of $v^{[j]}$. Therefore, testing for spanning comes down to jointly testing the restrictions:

$$\mathbf{a}^{[j]}v_{\min}^{[j]} + \mathbf{B}^{[j]}\boldsymbol{\iota}^{[j]} \le \boldsymbol{\iota}_N,$$

$$\tag{20}$$

$$\mathbf{a}^{[j]}v_{\max}^{[j]} + \mathbf{B}^{[j]}\boldsymbol{\iota}^{[j]} \le \boldsymbol{\iota}_N,$$

for $j = 1,\ldots,M$. Again, a Wald test statistic for the inequality restrictions in equation (20) is asymptotically distributed as a mixture of χ^2 distributions. With transaction costs, testing for intersection or spanning is completely analogous to the case of short sales constraints, except that we have to take into account both negativity and positivity constraints.

The total range of v can be limited beforehand. An upper bound on v is obtained if we do not impose the requirement that investors should invest *all* their wealth in the available assets, but may choose to invest only part of their wealth, that is, $0 \le \mathbf{w}'\boldsymbol{\iota} \le 1$ (see also Luttmer (1996)). In effect, this allows for the possibility of taking long positions in a risk-free asset with zero net return. This implies that the upper bound for v is 1. If we move upward along the mean-variance frontier, v decreases until $1/v$ equals the intercept of the asymptote of the lines tangent to the mean-variance frontier. This intercept is equal to the expected return on the global minimum

variance portfolio, $E[R_{t+1}^{GMV}]$, implying that the lower bound on v is given by $v = 1/E[R_{t+1}^{GMV}]$. Of course, these boundaries have to be adjusted in case there are short sales constraints and/or transaction costs on the benchmark assets $\mathbf{R}_{t+1}$.

The intersection and spanning tests presented here are closely related to the region subset test in Hansen et al. (1995), which is essentially a test for intersection. Hanson et al. (1995) estimate the minimum variance stochastic discount factor $m(v)_{t+1}$ under nonnegativity constraints (which induces a nonstandard limit distribution), in which case they end up with testing equality restrictions. On the other hand, our regression-based estimator is unrestricted with a standard asymptotic distribution, but we end up with more difficult inequality restrictions that have to be tested. This latter problem is well studied in the literature, however (see, e.g., Gouriéroux et al. (1982) and Kodde and Palm (1986)).[5]

To have an indication of the power of the spanning test with short sales constraints, Figure 1 presents the power as a function of the intercept $a^{[j]}$ in the spanning regression (19a). The figure contains the power function for the Wald spanning test statistics in the case where there are no frictions, in the case where there are short sales constraints for the new asset r_{t+1} only, and in the case where there are short sales constraints for both the new assets as well as for the benchmark assets. When there are short sales constraints on the benchmark assets, the test depends on the benchmark assets that are included in the segments, and hence the power function for the case of short sales constraints on all assets is shown when the elements in $\mathbf{R}^{[j]}$ are known as well as when they have to be estimated. The slope parameters are chosen such that in the case of short sales constraints, our test has maximum rejection probability in the null if the intercept $a^{[j]}$ is equal to zero.

For each value of $a^{[j]}$, the small-sample power is derived from a series of 1,000 simulations. In addition, for the case of no short sales, we also add the theoretical power of the spanning test, which may serve as a benchmark for the other tests.[6] For each simulation, 10 years of monthly data are generated and the empirical power for a size of 5 percent is determined. The benchmark indices are assumed to be the three stock indices used in the empirical application in Section III, where the data generating process for these indices is based on the summary statistics in Table II, assuming multivariate normality. For the new asset, the monthly standard deviation of the residual ε_{t+1} in equation (19a) is 10 percent, which is representative for the emerging markets that are analyzed in Section III. The figure plots the power for small values of $a^{[j]}$, because these appear to be the most relevant ones in the empirical application. Notice that $a^{[j]} < 0$, which is tantamount to a

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[5] A detailed comparison between our test and the region subset test proposed by Hansen et al. (1995) is provided in DeRoon et al. (1997b). This comparison can be obtained from the authors upon request.

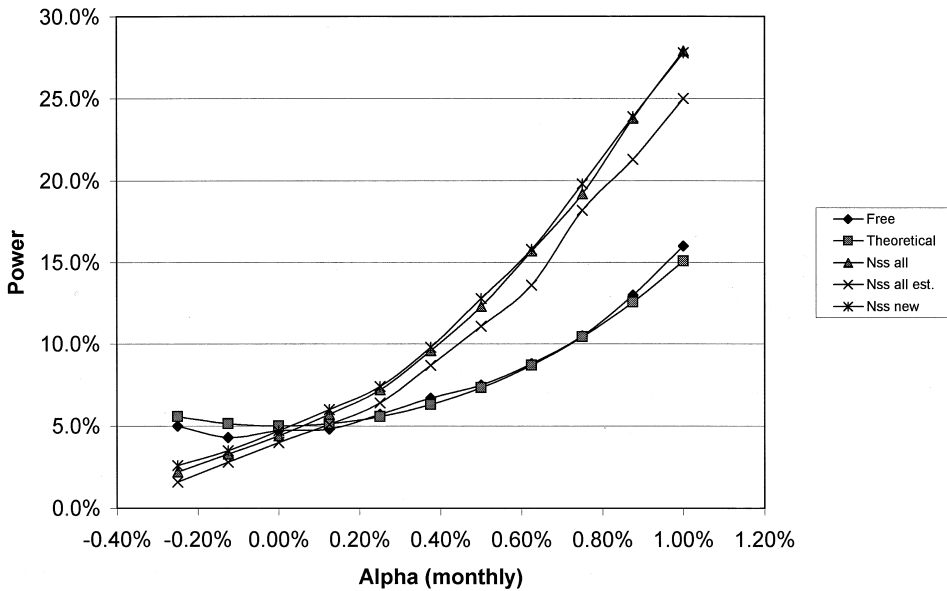[6] We thank the referee for making this suggestion.

Figure 1. Power functions. The figure presents the power function as a function of the intercept in equation (19a). For each value of the intercept (Alpha), the power is derived from a series of 1,000 simulations with 120 monthly observations each, when the rejection rate is 5 percent. The theoretical power is presented (Theoretical) as well as the power for the test with no short sales constraints (Free), with short sales constraints on the new asset only (Nss new) and with short sales constraints on all assets (Nss all) in case the benchmark assets are known, and with short sales constraints on the new assets when the benchmark assets are estimated during the simulations (Nss all est.).

negative Jensen measure $\alpha_J(v)$ in our simulations, implies that there is spanning if there are short sales constraints. This explains the low rejection probabilities for negative values of $a^{[j]}$ in Figure 1.

As Figure 1 shows, the different tests are very similar with respect to power. Notice that, for the spanning test with short sales constraints on all assets, there is only a small difference between the power functions for the case where the elements in $\mathbf{R}^{[j]}$ have to be estimated and for the case where these elements are known. This confirms the result in the Appendix that shows that the limit distribution of the spanning test is not affected by the fact that the elements in $\mathbf{R}^{[j]}$ have to be estimated. For the case where the elements in $\mathbf{R}^{[j]}$ have to be estimated, the power is somewhat smaller than in the other cases though, suggesting that in this case, there may be a minor small-sample bias, which may lead to underrejection of the null hypothesis.

Table I gives rejection rates for the asymptotic critical values of the Wald test statistics with short sales constraints in several finite samples. In this table, we use the same data generating process that underlies Figure 1, in case $a^{[j]} = 0$, with 10,000 simulations. These rejection rates will also be used in the empirical analysis in Section III to incorporate the possible effects

Table I

Simulated Rejection Rates of the Wald Test Statistics

The table contains the rejection rates in finite samples of the Wald test statistics for spanning for several significance levels. *p* values are given for the spanning test when there are short sales constraints on the new assets only (nss new) and when there are short sales constraints on both the new assets and the benchmark assets (nss all). When there are short sales on all assets, the relevant subsets from the benchmark assets where the short sales constraints are not binding are estimated during the simulation. The *p* values are derived form a series of 10,000 simulations with 5, 10, 25, or 50 years of monthly data. The simulation assumes that the new asset is spanned by the three benchmark assets which are given in Table II.

Significance	nss new			nss all		
	0.100	0.050	0.010	0.100	0.050	0.010
5 years	0.099	0.050	0.011	0.083	0.042	0.008
10 years	0.096	0.050	0.009	0.089	0.044	0.008
25 years	0.100	0.051	0.010	0.092	0.046	0.010
50 years	0.097	0.049	0.010	0.097	0.049	0.009

of a small-sample bias. The first three columns of Table I show the rejection probabilities in finite samples using the asymptotic critical values, in the case where there are only short sales constraints on the new asset. From these numbers, it appears that the rejection rates in finite samples are almost indistinguishable from the asymptotic rejection rates. Therefore, in this case there does not seem to be a small-sample bias, and we can use the asymptotic test with confidence, even in samples as small as five years of monthly data. The last three columns show similar rejection rates in the case where there are short sales constraints on both the new asset and the benchmark assets. Here the elements $\mathbf{R}^{[j]}$ are not assumed to be known, but have to be estimated. For this case, there does seem to be a possible small-sample bias. The rejection rates are always smaller than the asymptotic ones and appear to be increasing monotonically with the sample size. The bias appears to be small, though, and for sample sizes of 10 years of monthly data, the rejection rates are close enough to the asymptotic ones for the test to be used with confidence. For the empirical analysis in Section III the large sample properties of the tests appear to be sufficiently informative, as the small-sample adjustment does not alter any of the conclusions.

III. Empirical Results for Emerging Markets

A. Data

We use 17 indices from the Emerging Markets Data Base of the International Finance Corporation. Monthly observations on the Global Indices are available over the period of January 1985 until June 1996, for six Latin

American countries, seven Asian countries, one European, one Middle Eastern, and two African countries. The Morgan Stanley Capital International (MSCI) Indices for the United States, Europe, and Japan serve as the benchmark assets. The dataset is therefore very similar to the ones used by DeSantis (1994) and Harvey (1995). For all these indices we use (unhedged) monthly holding returns in U.S. dollars. The indices for both the emerging markets and for the developed markets are calculated with dividends reinvested. All data are obtained from Datastream.

Bekaert et al. (1998) provide ample indications that the behavior of the emerging market returns might have been changing over time. One important reason for this is the many liberalizations that have taken place in the emerging markets (see, e.g., Bekaert (1995)), causing the emerging markets to become more integrated with the developed markets. Therefore, we focus on the diversification benefits that are offered by the emerging markets for the postliberalization periods only. Starting dates for the emerging markets data that we used (if different from January 1985) are given in Panel B of Table II. Results for the regions Latin America, Asia, and "Other" are based on data since the last liberalization in that region.

Some basic summary statistics for net monthly holding returns are given in Table II. Table II provides summary data on the three benchmark indices as well as for the emerging markets. In this table and the following, the emerging markets are organized according to their geographical region: Latin America, Asia, and Other. A quick look at the data reveals that the emerging markets indices are usually much more variable than the benchmark indices, but also have somewhat higher average returns. For the monthly holding returns we use, the average standard deviation of the emerging markets indices is 10.43 percent and the average expected return is 1.82 percent, compared with 5.54 percent and 1.46 percent for the benchmark indices. Also notice that there is substantial cross-sectional variation in the average returns of the emerging markets. The average correlation of the emerging markets with the developed markets (not reported in Table II) is only 0.09 whereas the correlation between the developed markets themselves is as high as 0.33.

B. Results for Spanning Tests With Short Sales Constraints

Based on the summary statistics, it may be conjectured that, in the absence of market frictions, many emerging markets yield diversification benefits relative to the benchmark indices for the United States, Europe, and Japan. Table III shows Wald test statistics for the hypothesis that the returns on these three indices span the returns for each emerging market. For each group, the first line shows the spanning test statistic and the associated p value in case there are no short sales restrictions on any asset. The joint tests for spanning for all the emerging markets within a geographical group show that spanning is always rejected at any confidence level. These results merely confirm the findings of, for instance, DeSantis (1994) and

Table II
Summary Statistics

Panel A provides summary statistics for monthly dollar returns on the MSCI Indices that serve as the benchmark assets. Panel B provides summary statistics for the IFC Emerging Markets Data Base. The overall sample period is January 1985 until June 1996, giving a total of 138 observations. For the emerging markets, if major liberalizations occurred in this period, only the postliberalization periods as indicated in the last column of Panel B are used.

Panel A: Benchmark Assets (January 1985–June 1996)		
	Avg. (%)	St. Dev. (%)
United States	1.38	4.16
Europe	1.58	4.91
Japan	1.43	7.55

Panel B: Emerging Markets (Postliberalization Periods)			
		Avg. (%)	St. dev. (%)	Liberalization
Latin America				
Argentina	Arg	3.16	17.42	Dec 1989
Brazil	Bra	2.91	14.12	Jul 1991
Chile	Chi	2.59	7.80	Apr 1990
Colombia	Col	3.18	11.33	Feb 1991
Mexico	Mex	1.96	9.92	May 1989
Venezuela	Ven	0.29	13.39	Dec 1990
Asia				
India	Ind	0.58	8.66	Nov 1992
Korea	Kor	0.49	7.68	Jan 1992
Malaysia	Mal	1.37	7.64	—
Pakistan	Pak	1.93	9.93	Feb 1991
Philippines	Phi	2.44	8.39	Nov 1991
Taiwan	Tai	1.47	11.22	Jan 1991
Thailand	Tha	2.39	8.69	—
Other				
Greece	Gre	2.11	11.81	—
Jordan	Jor	0.64	4.86	—
Nigeria	Nig	1.69	15.27	—
Zimbabwe	Zim	2.51	9.21	—

Harvey (1995), as well as those of Errunza et al. (1999). As noted before, however, the diversification benefits suggested by the first line in Table III may not be attainable to investors, because they may require short selling of the emerging markets indices, the benchmark indices, or both.

If we do not allow investors to go short in the emerging markets, while still retaining the possibility to sell the benchmark indices short, the conclusions are very different. The joint tests for spanning for Latin America and Asia no longer reject the null hypothesis in this case. It is only for the group Other that we still find significant diversification benefits from emerging markets. Notice that for the individual countries, the *p* values are often smaller than in the no-frictions case, because now a one-sided null distribution

Table III

Spanning Tests with Short Sales Constraints

The table presents test results for the hypothesis that there is mean-variance spanning of emerging markets by three benchmark assets, which are the MSCI indices for the United States, Europe, and Japan, after liberalizations in the emerging markets have taken place. For each emerging market, results are shown for the period after liberalization of the stock market has taken place, as reported in Table II. If there is no liberalization during the sample period, the entire sample from January 1985 until June 1996 is used. The numbers in the table are Wald test statistics. The numbers in parentheses are p values associated with the asymptotic distribution of the Wald test statistics.

Panel A: Latin America							
	Arg	Bra	Chi	Col	Mex	Ven	All
No restrictions							
Wald	2.12	2.28	4.38	5.14	1.02	1.85	18.12
(p)	(0.347)	(0.320)	(0.112)	(0.077)	(0.599)	(0.397)	(0.000)
No short sales of emerging markets							
Wald	0.76	0.85	3.52	4.01	0.93	0.01	5.60
(p)	(0.221)	(0.242)	(0.036)	(0.031)	(0.201)	(0.524)	(0.163)
			[**]	[**]			
No short sales							
Wald	0.27	0.40	3.51	4.01	0.85	0.01	5.03
(p)	(0.606)	(0.298)	(0.061)	(0.026)	(0.194)	(0.480)	(0.148)
			[**]	[**]			
Investable indices, no short sales							
Wald	0.27	0.38	3.94	4.43	0.95	3.78	5.57
(p)	(0.603)	(0.301)	(0.047)	(0.023)	(0.170)	(0.026)	(0.127)
			[**]	[**]			

Panel B: Asia								
	Ind	Kor	Mal	Pak	Phi	Tai	Tha	All
No restrictions								
Wald	3.16	3.33	0.74	4.42	3.19	1.31	4.60	16.50
(p)	(0.206)	(0.189)	(0.689)	(0.110)	(0.203)	(0.520)	(0.100)	(0.000)
No short sales of emerging markets								
Wald	0.12	0.12	0.05	1.95	2.72	0.70	3.28	1.33
(p)	(0.433)	(0.436)	(0.481)	(0.112)	(0.065)	(0.249)	(0.044)	(0.633)
					[*]		[**]	
No short sales								
Wald	0.12	0.13	0.05	1.67	2.67	0.55	3.25	1.30
(p)	(0.451)	(0.404)	(0.680)	(0.094)	(0.068)	(0.226)	(0.082)	(0.655)
					[**]		[**]	
Investable indices, no short sales								
Wald	0.21	0.18	0.97	2.03	1.63	0.53	0.61	3.11
(p)	(0.403)	(0.369)	(0.329)	(0.088)	(0.114)	(0.236)	(0.429)	(0.362)
				[*]				

Panel C: Other					
	Gre	Jor	Nig	Zim	All
No restrictions					
Wald	4.28	76.37	4.29	24.29	105.07
(p)	(0.118)	(0.000)	(0.117)	(0.000)	(0.000)
No short sales of emerging markets					
Wald	1.59	1.09	0.83	8.38	11.16
(p)	(0.125)	(0.186)	(0.227)	(0.001)	(0.013)
				[***]	[**]
No short sales					
Wald	1.58	1.08	0.83	8.31	11.08
(p)	(0.227)	(0.312)	(0.387)	(0.006)	(0.036)
				[***]	[**]
Investable indices, no short sales					
Wald	1.59	1.02	NA	3.21	3.25
(p)	(0.240)	(0.334)	NA	(0.105)	(0.391)
				[**]	

Significance levels based on simulated test statistics are given in square brackets: * = 10 percent, ** = 5 percent, *** = 1 percent. NA = Not available.

is the relevant one. For some individual countries, like Chile and Colombia, the outward shift of the estimated mean-variance frontier is big enough to reject the null hypothesis of spanning. For individual countries, the 5 percent critical value of the test statistic is determined by the χ_1^2 distribution. Even though the estimated outward shift that results from adding all six Latin American countries is even bigger, the joint test shows that this shift is not big enough for the resulting test statistic to exceed the 5 percent critical value, which is now determined by a mixture of χ_i^2 distributions, with $i = 1, 2, \ldots, 6$. A similar situation occurs for the Asian markets.[7] For both Latin America and Asia, the estimated diversification benefits mainly result from one or two countries only. There is too much sampling error in the data for the diversification benefits to show up in the joint tests. This also shows up in the estimated mean-variance efficient portfolio weights, which show that in case of short sales constraints, investors would mainly like to invest in Chile and Colombia (for Latin America) or in the Philippines (for Asia). These results can be obtained from the authors upon request. Finally, the use of the p values that are adjusted for small-sample bias as reported in Table I (and from similar simulation results for the joint tests) does not alter any of the conclusions.

It may be the case though, that the diversification benefits offered by the emerging markets depend on whether or not the portfolio of the benchmark assets contains short positions. To account for short sales restrictions on the benchmark assets as well, Table III also presents spanning tests in the case where there are short sales restrictions on both the emerging markets and the benchmark assets. These results are presented in the third line for each geographical group in Table III. Adding short sales constraints on the benchmark assets does not change the results. The Wald test statistics are similar to the case when there are short sales constraints on the emerging markets only, implying that there are no diversification benefits for the emerging markets of Latin America and Asia. For the group Other, the joint test still rejects the null hypothesis of spanning. Again, the conclusions do not change when we use the adjusted p values from Table I, even though in this case there may be a small-sample bias.

To shed some light on these results, Figure 2 shows the mean-variance frontiers for the benchmark assets and the Asian markets for the cases discussed above. From this figure, it is obvious that there is a big shift in the unrestricted frontier if the Asian markets are added to the three benchmark assets. Adding short sales constraints for the Asian markets causes the diversification benefits to be much less pronounced, as can be seen from the inward shift of the frontier, which is now segmented. According to the tests in Table III, this inward shift makes the difference with the frontier of the

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[7] In addition to this, because the postliberalization periods are different for different countries, the joint test for the Asian markets is based on a smaller number of observations than the test for Thailand, for instance, which may result in a joint test statistic that is numerically smaller than the individual ones.
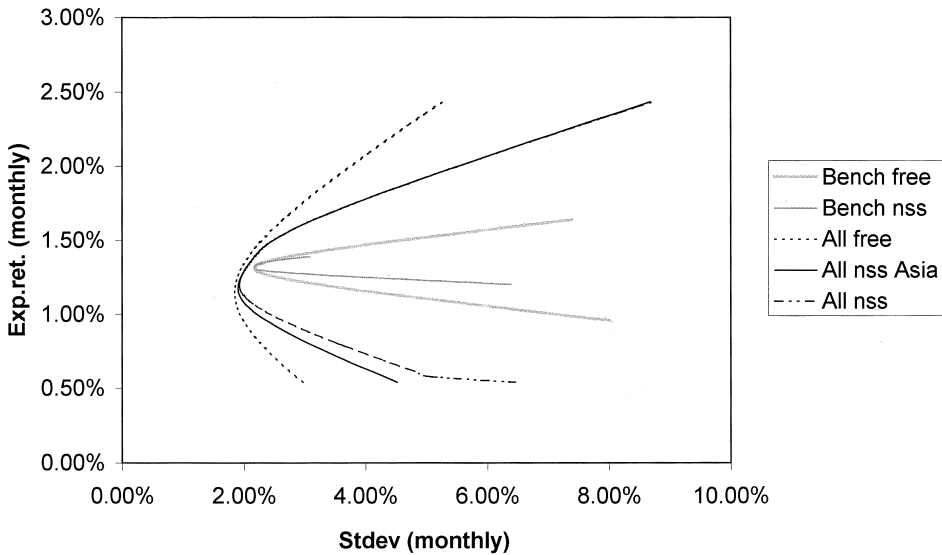
Figure 2. Mean-variance frontiers for the benchmark assets and Asia. The figure presents the unconstrained and segmented frontiers for the benchmark assets (Bench) and for the benchmarks plus the Asian markets (All) with (free) or with no (nss) short sales allowed.

benchmark assets insignificant. Finally, adding short sales constraints on the benchmark assets as well only makes a small difference in the frontiers. For the frontier of the benchmark assets, short sales constraints mainly make a difference for the inefficient part of the frontier and they put an upper limit on the efficient part. For the frontier of all markets together, adding short sales constraints on the benchmarks hardly makes a difference relative to the case where there are short sales constraints on the Asian markets only. Therefore, this figure illustrates that the main effect of short sales constraints operates through the emerging markets, as we also concluded from the results in Table III.

The results above show that adding short sales constraints considerably weakens the evidence in favor of emerging markets, except for the group Other. This latter result must be interpreted with some caution, because even for the emerging markets for which the null hypothesis is rejected, the diversification benefits may not be attainable, because of foreign ownership restrictions. Bekaert (1995) discusses several measures of the extent of foreign ownership restrictions in emerging markets. These measures suggest that these restrictions may be important for the countries for which the hypothesis of spanning is rejected (in particular for Zimbabwe). Thus, the diversification benefits suggested by Table III may be difficult or impossible to obtain.

To address this issue, the last line for each geographical group in Table III gives the results for the spanning tests in case the IFC Investable Indices are used instead of the Global Indices. The null hypothesis is again whether

the emerging market indices are spanned by the benchmark assets in case there are short sales constraints on both the emerging markets and the benchmark assets. Joint tests for all emerging markets within a geographical group now never reject the hypothesis of spanning, including the group Other. This suggests that ownership restrictions for these countries are indeed binding.

C. Results for Spanning Tests With Transaction Costs

In this section, we consider the effects of transaction costs on the hypothesis that the mean-variance frontier of the benchmark indices spans the frontiers of the benchmark indices plus the individual emerging markets. We assume that investors face a round trip cost of 0.5 percent when trading the benchmark assets, and that they have a holding period of six months.[8] With a holding period of six months and a round trip cost of 0.5 percent on the benchmark assets, efficient portfolios consist of long positions in the MSCI Indices for the U.S., Europe, and Japan index or in the U.S. and Europe index only.

Our analysis focuses on the transaction costs that are needed to keep investors out of the emerging markets. In addition to paying transaction costs, investors are also not allowed to sell the emerging markets short and we use the IFC Investable Indices to incorporate possible ownership restrictions. Table IV presents levels of transaction costs in the emerging markets above which the hypothesis of spanning cannot be rejected at the 5 percent and 10 percent level, respectively. For instance, in the case of Argentina, a round-trip cost below 0.10 percent is needed to reject spanning by the benchmark assets at the 10 percent level. For Venezuela, for instance, spanning can never be rejected at the 10 percent level, no matter how low the transaction costs are, which is indicated as "NR." The estimates in Table IV suggest that with 0.5 percent round-trip costs on the benchmark assets and short selling restrictions on the emerging markets, transaction costs for the emerging markets need not be particularly high to keep investors out of these markets. Especially in the Asian markets, we often find that even in the absence of transaction costs on the emerging markets, spanning is not rejected. It is only in a few cases that a transaction cost of at least two times the level in the benchmark assets is needed to keep investors out of the market.

To get some further intuition about the importance of these transaction costs, the third line for each geographic group in Table IV gives an estimate of the actual round-trip costs in the emerging markets. These estimates are from Barings Securities and reported by Bekaert et al. (1998). The reported transaction costs are calculated from the percentage spread, which is the difference between the offer and bid price divided by the average of the offer and bid price for a security. For each country, the percentage spreads of

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[8] Results for holding periods of 3 and 12 months are similar to those for 6 months and can be obtained from the authors upon request.

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Table IV

Transaction Cost Bounds

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The table presents estimated transaction cost bounds for the emerging markets in order to reject spanning of each emerging market by three benchmark assets at the 5 percent and 10 percent significance levels. The three benchmark assets are the MSCI indices for the United States, Europe, and Japan. The table assumes that there is a 0.5 percent round-trip cost on the benchmark assets and that there is a holding period of one month. The estimated transaction costs are in percentages. All results are based on the IFC Investable Indices for the postliberalization periods. The actual transaction costs are from Baring Securities as reported by Bekaert et al. (1996).

Panel A: Latin America						
	Arg	Bra	Chi	Col	Mex	Ven
10% bound	0.10	0.05	1.00	1.40	0.50	NR
5% bound	NR	NR	0.65	0.80	NR	NR
Actual tr. cost	1.55	0.85	3.93	1.00	0.93	NA

Panel B: Asia							
	Ind	Kor	Mal	Pak	Phi	Tai	Tha
10% bound	NR	NR	NR	NR	0.10	NR	NR
5% bound	NR	NR	NR	NR	NR	NR	NR
Actual tr. cost	1.50	NA	0.69	0.38	0.94	0.47	0.70

Panel C: Other				
	Gre	Jor	Nig	Zim
10% bound	NR	0.25	NA	1.20
5% bound	NR	0.10	NA	0.50
Actual tr. cost	0.48	0.58	NA	NA

NR = never rejected in the absence of transaction costs. NA = not available.

individual stocks are weighted by the capitalization of each stock within each country (see Bekaert et al. (1998)). Interestingly, except for Colombia, the actual transaction costs are always higher than the calculated 10 percent bounds in Table IV, and the actual transaction costs are in every case higher than the 5 percent bounds. This shows that, at least for a holding period of six months, the joint effects of transaction costs in the emerging markets, short sales constraints, a round-trip cost of 0.5 percent on the benchmark assets, and possible ownership restrictions make it hard to reject the hypothesis of mean-variance spanning.

<div align="center">

IV. Concluding Remarks

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In this paper, we show how regression techniques can be used to test for mean-variance spanning and intersection in the case where there are short sales constraints and/or transaction costs. When there are short sales

constraints on the benchmark assets, the mean-variance frontier consists of parts of the mean-variance frontiers of subsets of the set of benchmark assets. If the benchmark assets are to span a new set of assets, there has to be spanning for each subset of the benchmark assets. This can be incorporated in regression-based tests for spanning, by using a multivariate regression in which the returns on the new assets are regressed on the returns of the relevant subsets of the benchmark assets. Short sales restrictions on the new assets require testing for inequality restrictions rather than equalities. Following the ideas presented in, for example, Luttmer (1996), transaction costs can be handled by looking at short and long positions in an asset as two different securities. Transaction costs can then be dealt with in the same way as short sales constraints.

There is substantial evidence available in the literature that suggests that, in the absence of market frictions, U.S. investors can benefit from including emerging markets assets in their well-diversified international portfolio of developed market assets. We try to shed some further light on this issue by testing whether emerging market indices are spanned by developed market indices when investors face short sales constraints and/or transaction costs. We find that, when accounting for short sales constraints and investability restrictions, the evidence in favor of diversification benefits of the emerging markets disappears, that is, for the three geographical regions, we can no longer reject the hypothesis of spanning. This is mainly due to the short sales constraints on the emerging markets. Although our simulations suggest that there is a possible small-sample bias in the asymptotic test, the size of this bias is small and does not affect our conclusions. Also, with a round-trip cost of 0.5 percent in the developed markets and a holding period of six months, only small transaction costs in the emerging markets are needed for possible diversification benefits to be insignificant.

Appendix: Proof of the Validity of the Test

In this appendix, we prove a simple but useful lemma. This lemma shows that the fact that we possibly use the incorrect regressions in our spanning and intersection tests (due to sample variation) is asymptotically negligible. Short sales restrictions on the benchmark assets are handled by testing for spanning and intersection on subsets of the available assets, where there is only a finite number of such subsets. The probability of choosing the right subsets tends to one and this turns out to be a sufficient condition for the validity of the tests. Suppose that we are given a finite number of Wald test statistics, $\xi(v^{[1]})_T, \ldots, \xi(v^{[M]})_T$, as defined in equation (16), where T is the sample size. Let the space of all possible values of v be partitioned in $V^{[1]}, \ldots, V^{[M]}$, with the interpretation that, depending on the value of the parameter v, one of the test statistics $\xi(v^{[1]})_T$ has desirable properties. Let j indicate the set $V^{[j]}$ to which $v^{[j]}$ belongs. If v_0 denotes the true value of v,

one would like to use the test $\xi(v_0)_T$, of course, but this is not possible, because v_0 is unknown. Assume, however, that we are given a parameter estimate $\hat{v}_T$, such that, under v_0

$$\Pr\{\hat{v}_T \in V^{[j]}\} \to 1,\, T \to \infty.$$

Now we have the following result.

LEMMA 1: *For each $c \in I\!R$, we have*

$$\lim_{T \to \infty} \Pr\{\xi(\hat{v}_T)_T \leq c\} - \Pr\{\xi(v_0)_T \leq c\} = 0.$$

Proof: The proof is very straightforward, using

$$\Pr\{\xi(\hat{v}_T)_T \leq c\} = \sum_{j=1}^{M} \Pr\{\xi(v^{[j]})_T \leq c \text{ and } v^{[j]} = \hat{v}_T\}$$

$$= \Pr\{\xi(v_0)_T \leq c\} - \Pr\{\xi(v_0)_T \leq c \text{ and } v_0 \neq \hat{v}_T\}$$

$$+ \sum_{v \neq v_0} \Pr\{\xi(v)_T \leq c \text{ and } v = \hat{v}_T\},$$

and that the latter two terms converge to zero.   Q.E.D.

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