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Journal of Empirical Finance 11 (2004) 29–53
Journal of EMPIRICAL FINANCE
www.elsevier.com/locate/econbase

Evaluating style analysis

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Accepted 3 December 2002

Abstract

In this paper we analyze the use and implications of (return-based) style analysis. First, style analysis may be used to estimate the relevant factor exposures of a fund. We use a simple simulation experiment to show that imposing portfolio and positivity constraints in style analysis leads to significant efficiency gains if the factor loadings are indeed positively weighted portfolios, in particular when the factors have low cross-correlations. If this is not the case though, imposing the constraints can lead to biased exposure estimates. Second, style analysis may be used in performance measurement. If the actual factor exposures are a positively weighted portfolio and if the risk-free rate is one of the benchmarks, then the intercept coincides with the Jensen measure. In general, the intercept in the style regression can only be interpreted as a special case of the familiar Jensen measure. Third, style estimates may be compared with actual portfolio holdings. We show that the actual portfolio holdings will in general not reveal the actual investment style of a fund because of cross exposures between the asset classes and because fund managers may hold securities that on average do not have a beta of one relative to their own asset class. Although return-based style analysis is less suitable to predict future portfolio holdings, our empirical analysis suggests that it performs better than holding-based style analysis in predicting future fund returns.

JEL classification: G11; G23; C52
Keywords: Style analysis; Mutual funds

1. Introduction

In recent years, return-based style analysis, as introduced by Sharpe (1992), has become a very popular tool for analyzing mutual fund returns. Essentially, in return-based style

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doi:10.1016/j.jempfin.2002.12.003

analysis, a factor model is used to explain fund returns (see e.g. Brown and Goetzmann, 1997). The factors are taken to be the returns on several factor or benchmark portfolios, such as value, growth, small cap, momentum, country, or sector portfolios. Standard style analysis imposes that the factor loadings are positive and that they sum to one. These factor loadings therefore constitute a positively weighted portfolio, and mutual fund returns can be decomposed in the return on the style portfolio and an idiosyncratic fund return.

In this paper, we analyze the use and implications of return-based style analysis. First, style analysis may be used to determine the factor exposures. Return-based style analysis determines the mimicking portfolio of mutual funds or other investment opportunities with positive portfolio weights, i.e., the positively weighted style portfolio that is closest to the mutual fund in a least squares sense. When no constraints are imposed on the factor loadings, we will refer to this as *weak* style analysis.[1] The case where only a portfolio constraint is imposed will be referred to as *semi-strong* style analysis, and the case where both the portfolio and the positivity constraints are imposed will be referred to as *strong* style analysis, or style analysis as proposed by Sharpe (1992). If the actual factor exposures constitute a positively weighted portfolio, a simple simulation experiment shows that we obtain significant efficiency gains in style estimates when imposing the constraints in the estimation process. Our simulation experiment suggests that using strong style analysis rather than weak style analysis can lead to a reduction in size of the confidence intervals of the style coefficients up to almost 90%. The highest efficiency gains occur when the actual coefficient is on or close to the boundary of zero. When using return-based style analysis to determine the relevant factor exposures, biased estimates may occur if the factor exposures are in fact not a positively weighted portfolio.

Second, style analysis may be used in performance measurement (see e.g. Lucas and Reipe, 1996). One possible application of the mimicking (style) portfolio is as a benchmark in evaluating the performance of the mutual fund. We discuss this application in some detail and show how it is related to the more traditional Jensen measure. In general, the intercept in the style regression can only be interpreted as a special case of the Jensen measure. We also derive some general conditions under which an investment in the mutual fund is more or less attractive than an investment in the mimicking portfolio.

Third, the mimicking portfolio obtained in style analysis may be compared with the actual portfolio holdings of the mutual fund. We show that the actual mutual fund portfolio holdings in general will not reveal the investment style of the fund. Therefore, holding-based style analysis does not necessarily yield the actual style because of cross-correlations between the asset classes or because the fund manager selects assets that have relatively high or low betas relative to their own index. In such cases, return-based style analysis can still be expected to yield the actual investment style.

In our empirical analysis, we focus on the difference between portfolio holdings and estimated style exposures. As suggested by the theoretical analysis, we find that estimated style exposures indeed differ substantially from actual portfolio holdings. Because of these differences, return-based style analysis is less suitable for predicting future portfolio

[1] Agarwal and Naik (2000) refer to this as *generalized* style analysis.

holdings than holding-based style analysis.[2] However, if the aim is to predict future fund returns, factor exposures seem to be more relevant than actual portfolio holdings, and return-based style analysis performs better than holding-based style analysis. In Section 2, we discuss the relation between unrestricted factor loadings and (positively weighted) mimicking portfolios, i.e., between weak, semi-strong, and strong style analysis, and illustrate the efficiency gains from strong style analysis. Section 3 considers the relationship between return-based style analysis and the actual mutual fund portfolio holdings. In Section 4, we consider the relation between style analysis and performance measurement. Section 5 illustrates the application of style analysis using data for US-based internationally diversified mutual funds. Section 6 concludes.

2. Style analysis and factor exposures

We start by evaluating the effects of the portfolio and positivity constraints in style analysis. Suppose that K factor (mimicking) portfolios with return vector R_t drive the asset returns. In addition, there are N mutual funds with return vector r_t, for which we have the linear factor model

$$r_t = a + BR_t + \varepsilon_t, \tag{1}$$

where $E[\varepsilon_t]=E[\varepsilon_t R_{i,t}]=0$ for $i=1, \ldots, K$. In this case $B=\Sigma_{rR}\Sigma_{RR}^{-1}$, and $a=\mu_r - B\mu_R$, where Σ is a covariance matrix, and μ is an expected return vector. When using Eq. (1) as a factor model, we do not impose any constraint on a and B. In particular, the rows of B do not necessarily constitute positively weighted portfolios. On the other hand, in style analysis, it is common to refer to the regression in Eq. (1) as the style regression, where we impose the constraints that the rows of B are positively weighted portfolios. In the sequel, if there are no restrictions on B, we refer to this as weak style analysis and to $a+\varepsilon_t$ as the weak idiosyncratic returns. If we define a_i as the ith element of a and b_i as the ith row of B, then a_i and b_i are the solutions to the problem

$$\min_{\alpha,\beta} E[(r_{i,t} - \alpha - \beta' R_t)^2]. \tag{2}$$

The vector b_i reflects the fund mimicking positions or the minimum variance hedge positions for the mutual fund. To see the effect of the portfolio constraint $\Sigma_j \beta_j = 1$, let $\tilde{a}_i$ and $\tilde{b}_i$ be the solutions of the problem

$$\begin{aligned} &\min_{\alpha,\beta} E[(r_{i,t} - \alpha - \beta' R_t)^2], \\ &\text{s.t. } \beta'\iota_K = 1 \end{aligned} \tag{3}$$

where ι_K is a K-dimensional vector of ones. Thus, $\tilde{b}_i$ are the factor exposures which are constrained to sum to one, i.e., they characterize a portfolio. The case where only the portfolio constraint is imposed will be referred to as semi-strong style analysis. Using

[2] This finding is similar to Rekenthaler et al. (2002).

standard least squares results, it is straightforward to show that the coefficients $\tilde{b}_i$ can be written as

$$\tilde{b}_i = b_i + (1 - b_i'\iota_K)\Sigma_{RR}^{-1}\iota_K\left(\iota_K'\Sigma_{RR}^{-1}\iota_K\right)^{-1}. \tag{4}$$

Note that the last part of this expression equals the Global Minimum Variance (GMV) Portfolio of the factor portfolios: $w_{\mathrm{GMV}} = \Sigma_{RR}^{-1}\iota_K(\iota_K'\Sigma_{RR}^{-1}\iota_K)^{-1}$. Defining $c_i = b_i'\iota_K$, the ith row of $\tilde{b}_i$ reads

$$\tilde{b}_i = c_i\left(\frac{b_i}{b_i'\iota_K}\right) + (1 - c_i)w_{\mathrm{GMV}}. \tag{5}$$

Thus, for each mutual fund, the semi-strong style coefficients, or portfolio restricted exposures $\tilde{b}_i$, are equal to a weighted average of the GMV portfolio and a hedge portfolio $b_i/b_i'\iota_K$. It follows immediately from Eq. (5) that $\tilde{b}_i$ only coincides with the unrestricted exposures b_i if $c_i = 1$, which is the case if the weak style coefficients already are a portfolio. In a similar fashion, it is straightforward to show that the portfolio constraint implies that the intercept $\tilde{a}_i$ equals

$$\tilde{a}_i = a_i + (b_i'\iota_K - 1)E[R_t^{\mathrm{GMV}}]. \tag{6}$$

The semi-strong style coefficients in Eq. (5) yield the style portfolio that is closest to the mutual fund in a least squares sense, i.e., it is the best mimicking portfolio. Since the difference between the mutual fund return $r_{i,t}$ and the return on the mimicking portfolio $\tilde{b}_i' R_t$ is simply the tracking error, $e_{i,t} = r_{i,t} - \tilde{b}_i' R_t$, the mimicking portfolio is the portfolio that yields the lowest tracking error variance. Eqs. (4) and (6) moreover imply that if the portfolio restriction is not valid, these mimicking portfolio weights and the resulting intercept may give biased estimates of the actual factor loadings B, and the associated intercept a, where the bias in B is linear in the GMV portfolio, w_{GMV}. If the factor exposures would in fact constitute a portfolio, then it is well known that imposing the portfolio constraint in style estimation leads to more efficient results than unconstrained estimation, i.e., semi-strong style analysis would be more precise than weak style analysis. If the portfolio constraint in style analysis actually reflects the portfolio constraints faced by the fund manager, imposing the portfolio constraint will in general yield better style estimates. On the other hand, if the fund manager is allowed to take leveraged positions as is the case for hedge funds e.g., the use of semi-strong style analysis would bias the estimates (see also Fung and Hsieh, 1997).

In addition to the portfolio constraint, it is common in style analysis to impose positivity constraints on the estimated factor exposures. The style portfolios $\hat{b}_i$ and the associated intercepts $\hat{a}_i$ are then the solution to the problem

$$\begin{aligned}&\min_{\alpha,\beta} E[(r_{i,t} - \alpha - \beta' R_t)^2],\\ &\qquad \text{s.t. } \beta'\iota_K = 1 \\ &\qquad\quad \beta \geq 0,\end{aligned} \tag{7}$$

where the inequality sign applies componentwise. We refer to this case as strong style analysis. If we order the benchmarks as $R_t' = (R_{1t}' R_{2t}')$ such that the positivity constraints are not binding for R_{1t} and binding for R_{2t} [implying that $\beta' = (\beta_1' \ 0_2')$, where 0_2 is a vector of zeros with the same dimension as R_{2t}], then the coefficients $\hat{b}_{1i}$ coincide with the portfolio constrained coefficients in a regression of the mutual fund return on the benchmarks R_{1t} only. It follows that the coefficients $\hat{b}_{1i}$ can be written as

$$\hat{b}_{1i} = c_i^{(1)} \left(\frac{b_i^{(1)}}{b_i^{(1)\prime} \iota_1} \right) + (1 - c_i^{(1)}) w_{\mathrm{GMV}}^{(1)}, \tag{8}$$

where

$$c_i^{(1)} = b_i^{(1)\prime} \iota_1,$$

$$w_{\mathrm{GMV}}^{(1)} = \frac{\Sigma_{11}^{-1} \iota_1}{\iota_1' \Sigma_{11}^{-1} \iota_1},$$

and the coefficients $b_i^{(1)}$ result from the regression

$$r_{i,t} = a_i^{(1)} + b_i^{(1)\prime} R_{1t} + \varepsilon_{i,t}^{(1)}. \tag{9}$$

The coefficients $b_i^{(1)}$ can be expressed in terms of the (unrestricted) weak style exposures b_i as

$$b_i^{(1)} = b_{1i} + \Sigma_{11}^{-1} \Sigma_{12} b_{2i}. \tag{10}$$

Similarly, the intercept $\hat{a}_i$ can be written as

$$\hat{a}_1 = a_i^{(1)} + (b_i^{(1)\prime} \iota_1 - 1) E[R_{1t}^{\mathrm{GMV}}]. \tag{11}$$

Again, we find that the strong style portfolio is a weighted average of the GMV portfolio $w_{\mathrm{GMV}}^{(1)}$ and a hedge portfolio $b_i^{(1)}/b_i^{(1)\prime} \iota_1$, but now, these portfolios are based on the subset of benchmarks, R_{1t}, for which the positivity constraints are not binding.

The strong style coefficients as given in Eq. (8) reflect the positively weighted portfolio of the factors that mimics the mutual fund. Although it is the best positively weighted mimicking portfolio, there is an additional potential bias in the estimated coefficients relative to the actual factor exposures in Eq. (1), because of the positivity constraints. Similarly to semi-strong style analysis, however, to the extent that the portfolio and the positivity constraints hold for the individual assets, the constraints imposed by strong style analysis reflect the constraints faced by the fund manager—and these constraints should not be imposed if the fund manager can take leveraged and short positions in the various asset classes. If the portfolio and positivity constraints are valid, the use of strong style analysis leads to more efficient estimates than either semi-strong or weak style analysis, although it is not straightforward to obtain analytical expressions for the gain in efficiency that results from imposing the constraints.

Table 1
Efficiency gains from the constraints in style estimation

Panel I

	$\beta_1 = 0.00$	$\beta_2 = 0.50$	$\beta_3 = 0.50$
ρ = 0.00			
Weak	[− 0.22; 0.22]	[0.28; 0.73]	[0.28; 0.72]
Semi-strong	[− 0.18; 0.17]	[0.33; 0.68]	[0.33; 0.68]
Strong, 5%	[0.00; 0.05]	[0.34; 0.66]	[0.34; 0.66]
Strong, 10%	[0.00; 0.06]	[0.34; 0.66]	[0.34; 0.66]
Strong, 50%	[0.00; 0.12]	[0.33; 0.66]	[0.32; 0.66]
ρ = 0.33			
Weak	[− 0.23; 0.24]	[0.25; 0.74]	[0.26; 0.74]
Semi-strong	[− 0.21; 0.22]	[0.28; 0.71]	[0.29; 0.71]
Strong, 5%	[0.00; 0.06]	[0.31; 0.69]	[0.31; 0.69]
Strong, 10%	[0.00; 0.07]	[0.30; 0.69]	[0.31; 0.69]
Strong, 50%	[0.00; 0.15]	[0.28; 0.69]	[0.29; 0.69]
ρ = 0.67			
Weak	[− 0.32; 0.33]	[0.17; 0.82]	[0.17; 0.82]
Semi-strong	[− 0.30; 0.30]	[0.20; 0.80]	[0.20; 0.80]
Strong, 5%	[0.00; 0.08]	[0.25; 0.74]	[0.25; 0.75]
Strong, 10%	[0.00; 0.019]	[0.24; 0.76]	[0.24; 0.76]
Strong, 50%	[0.00; 0.22]	[0.20; 0.77]	[0.20; 0.77]

Panel II

	$\beta_1 = 0.05$	$\beta_2 = 0.45$	$\beta_3 = 0.50$
ρ = 0.00			
Weak	[− 0.17; 0.27]	[0.23; 0.68]	[0.28; 0.72]
Semi-strong	[− 0.13; 0.22]	[0.28; 0.63]	[0.32; 0.68]
Strong, 5%	[0.00; 0.09]	[0.30; 0.62]	[0.35; 0.67]
Strong, 10%	[0.00; 0.11]	[0.30; 0.62]	[0.34; 0.67]
Strong, 50%	[0.00; 0.19]	[0.28; 0.62]	[0.33; 0.67]
ρ = 0.33			
Weak	[− 0.18; 0.29]	[0.20; 0.68]	[0.26; 0.72]
Semi-strong	[− 0.16; 0.27]	[0.23; 0.66]	[0.29; 0.71]
Strong, 5%	[0.00; 0.11]	[0.26; 0.65]	[0.32; 0.70]
Strong, 10%	[0.00; 0.13]	[0.26; 0.65]	[0.31; 0.70]
Strong, 50%	[0.00; 0.22]	[0.24; 0.65]	[0.29; 0.70]
ρ = 0.67			
Weak	[− 0.27; 0.38]	[0.12; 0.77]	[0.17; 0.82]
Semi-strong	[− 0.25; 0.35]	[0.15; 0.75]	[0.20; 0.80]
Strong, 5%	[0.01; 0.12]	[0.21; 0.69]	[0.27; 0.76]
Strong, 10%	[0.00; 0.14]	[0.20; 0.71]	[0.25; 0.77]
Strong, 50%	[0.00; 0.29]	[0.15; 0.73]	[0.20; 0.78]

Table 1 (*continued*)

Panel III			
	$\beta_1 = 0.25$	$\beta_2 = 0.25$	$\beta_3 = 0.50$
ρ = 0.00			
Weak	[0.03; 0.47]	[0.03; 0.47]	[0.28; 0.72]
Semi-strong	[0.07; 0.42]	[0.08; 0.43]	[0.32; 0.68]
Strong, 5%	[0.09; 0.39]	[0.09; 0.41]	[0.34; 0.68]
Strong, 10%	[0.08; 0.41]	[0.09; 0.42]	[0.33; 0.67]
Strong, 50%	[0.07; 0.42]	[0.08; 0.43]	[0.32; 0.68]
ρ = 0.33			
Weak	[0.02; 0.49]	[0.00; 0.49]	[0.26; 0.74]
Semi-strong	[0.04; 0.47]	[0.03; 0.46]	[0.29; 0.71]
Strong, 5%	[0.08; 0.41]	[0.07; 0.40]	[0.32; 0.71]
Strong, 10%	[0.07; 0.43]	[0.07; 0.42]	[0.31; 0.71]
Strong, 50%	[0.05; 0.46]	[0.04; 0.45]	[0.29; 0.71]
ρ = 0.67			
Weak	[− 0.07; 0.58]	[− 0.09; 0.57]	[0.17; 0.82]
Semi-strong	[− 0.05; 0.55]	[− 0.05; 0.55]	[0.20; 0.80]
Strong, 5%	[0.07; 0.40]	[0.07; 0.39]	[0.30; 0.77]
Strong, 10%	[0.05; 0.44]	[0.05; 0.44]	[0.26; 0.78]
Strong, 50%	[0.00; 0.54]	[0.00; 0.53]	[0.20; 0.79]

The table shows the results from a simulation experiment where a mutual fund is simulated from a set of three factors:

$$r_t = \alpha + \beta_1 R_{1,t} + \beta_2 R_{2,t} + \beta_3 R_{3,t} + \varepsilon_t,$$

where various choices for β are used and where $\alpha = 0$, stdev($R_{i,t}$) = 3%, stdev(ε_t) = 2.5%, and the correlation between each of the benchmarks is always 0.00, 0.33, or 0.67. The table shows the average of the 95% confidence intervals for the estimated coefficients b_i, based on 1000 simulations of 60 months of returns. For the strong style estimates, the reported intervals are based on Kim–Stone–White confidence intervals with pretest levels of either 5%, 10%, or 50%.

To illustrate the efficiency gains that result from imposing the portfolio and positivity constraints, Table 1 shows the results of a simple simulation experiment. The table shows 95% confidence intervals for the estimated style parameters, based on 60 monthly simulated returns on three benchmark portfolios as well as a mutual fund, using

$$r_t = \alpha + \beta_1 R_{1,t} + \beta_2 R_{2,t} + \beta_3 R_{3,t} + \varepsilon_t,$$

where we use $\alpha = 0$, stdev ($R_{i,t}$) = 3.0%, and stdev (ε_t) = 2.5%. The correlation between all the benchmark portfolios is 0.00, 0.33, or 0.67, as is shown in the first column. The three panels of Table 1 show simulation results for different values of the coefficients β_i. The choice of the coefficients β_i is always such that the portfolio and positivity constraints are satisfied, but the actual coefficient β_1 may be on or close to the boundary of zero. For the different choices of the coefficients and the different correlations between the benchmarks, the table shows the average confidence intervals of the estimated coefficients, based on 1000 simulations.

For the weak and semi-strong style analysis, the confidence intervals are based on the estimated standard errors, assuming that the estimated style coefficients are normally distributed. In case of strong style analysis, this assumption is no longer valid, because the distribution is truncated at zero. Therefore, the reported confidence intervals are the confidence intervals suggested by Kim et al. (2000), which we refer to as KSW confidence intervals. The procedure suggested by Kim, Stone, and White implies that a pretest is used in which the hypothesis $\beta_i = 0$ is tested. The pretest level used in Table 1 is either 5%, 10%, or 50%. Details on the KSW confidence intervals can be found in the Appendix. In all cases, the table reports the average confidence bounds over the 1000 simulations.

As the table shows, the confidence intervals for the strong style estimates are always smaller than the confidence levels for the weak and semi-strong style estimates. It is only in Panel III, where all three coefficients β_i are well within the parameter space [0;1], that the strong style confidence intervals are close to the semi-strong style confidence intervals. From the first two panels, it is obvious that if one of the parameters is on or close to the boundary, the gains in efficiency from using strong style analysis can be significant. For instance, in the first column of Panel I, we see that the size of the confidence interval of $b_1(\beta_1 = 0)$ in case of strong style analysis is close to 10% of the size of the confidence interval of the weak estimate and less than 15% of the size of the interval in case of semi-strong style analysis. The table also shows that the confidence levels for strong style analysis depend on the pretest level used when constructing the KSW confidence intervals. When the pretest level is increased from 5% to 10%, the confidence intervals tend to become somewhat wider, although the differences are very small. As the pretest level is further increased to 50%, the differences become more apparent, especially when β_1 is on or close to boundary. In that case, the confidence interval about doubles when the pretest level is increased from 5% to 50%.

The efficiency gains from using strong style analysis are not only the result of a lower bound of zero, as is implied by the KSW intervals, but also by a much tighter upper bound. In the Panel II, where $\beta_1 = 0.05$, the size of the confidence interval for b_1 in case of strong style analysis is still only 25% or less of the size of the intervals in case of weak or semi-strong style analysis. These gains in efficiency that occur when β_1 is on or close to the boundary are also present in the estimates of the other style coefficients, where the confidence intervals are always smallest in case of strong style analysis. Thus, while using strong style analysis will result in biased estimates when the portfolio and positivity constraints are not true, strong style analysis leads to much more efficient style estimates when the constraints do hold, especially when one of the parameters is close to or on the boundary of zero and when there is a relatively high correlation between the benchmarks.

3. Style analysis and mutual fund portfolio holdings

One obvious point of interest in return-based style analysis is the relation between the estimated style and the actual portfolio holdings of the mutual fund. After all, one might claim that there is no need to use return data to determine the style of a fund when the actual portfolio holdings are known.[3] However, as we will show below, the individual

[3] Applications of style analysis based on portfolio holdings rather than returns can be found, e.g., in Wermers (2000, 2001).

portfolio holdings may not yield the actual portfolio style, because the portfolio holdings do not necessarily coincide with the factor exposures that are created by these holdings. In this section, we analyze the use of style analysis given the fund's portfolio holdings, in order to see if style analysis corresponds to the actual portfolio holdings. Notice that R_t contains the returns on K benchmark or factor portfolios which themselves consist of individual assets. Most fund managers typically invest in a subset of the assets underlying an index only and, moreover, give the assets in their portfolio different weights than the index. Denote the vector of the stock returns that are present in benchmark index i as $R_t^{(i)}$, where $R_t^{(i)}$ has $K^{(i)}$ elements. The index return $R_{i,t}$ itself is defined by a particular index portfolio $x^{(i)}$, i.e.,

$$R_{i,t} = x^{(i)\prime} R_t^{(i)}.$$

The fund manager chooses a portfolio $v^{(i)}$ from $R_t^{(i)}$ for which in general $v^{(i)} \neq x^{(i)}$. Assuming that the manager chooses portfolios $v^{(i)}$, from K asset classes, he also has to determine the weights w_i assigned to each asset class. Thus, we have that $\Sigma_j v_j^{(i)} = 1$, $\forall i$, $\Sigma_i w_i = 1$, and $v_j^{(i)} \geq 0$ and $w_i \geq 0$, $\forall i, j$. The manager's return on asset class i is equal to

$$r_t^{(i)} = v^{(i)\prime} R_t^{(i)}. \tag{12}$$

The return on the fund is therefore equal to

$$r_t = \Sigma_{i=1}^K w_i r_t^{(i)} = \Sigma_{i=1}^K w_i \Sigma_{j=1}^{K^{(i)}} v_j^{(i)} R_{j,t}^{(i)}. \tag{13}$$

Next, note that we can also use the factor model in Eq. (1) for the individual asset returns $R_{j,t}^{(i)}$:

$$R_{j,t}^{(i)} = \alpha_j^{(i)} + \beta_{1,j}^{(i)} R_{1,t} + \ldots + \beta_{K,j}^{(i)} R_{K,t} + \varepsilon_{j,t}^{(i)}, \tag{14}$$

If we use the correct factor portfolios, then standard asset pricing models imply the portfolio constraint $\sum_i \beta_{i,j}^{(i)} = 1$ should hold, because all individual assets should be spanned by the factor portfolios. If the betas $\beta_{i,j}^{(i)}$ sum to one for each individual asset, in combining Eq. (14) with Eq. (13), we should also have

$$\Sigma_{i=1}^K w_i \Sigma_{j=1}^{K^{(i)}} v_j^{(i)} \beta_{i,j}^{(i)} = 1,$$

implying that the coefficients in the style regression should indeed sum to one and that the portfolio constraint is a valid constraint if the fund manager faces such a constraint as well (see also Huberman et al., 1987, for example). However, also note that if the net weight in asset $R_j^{(i)}$ is $z_j^{(i)} = w_i v_j^{(i)}$, then the expected style coefficient b_k for factor k in the style regression equals

$$b_k = \sum_{i=1}^{K} \sum_{j=1}^{K^{(i)}} z_j^{(i)} \beta_{k,j}^{(i)},$$

since in principle every asset can have an exposure with respect to index k. Therefore, this style coefficient b_k will in general not coincide with the actual portfolio holdings in index

k, which is $w_k \Sigma_{j=1}^{K^{(k)}} v_j^{(k)} = w_k$. The reason for this difference arises because the fund manager does not necessarily hold assets that have $\beta_{k,j}^{(i)} = 1$ with respect to their own index ($k = i$) and can also have a factor loading on other factor indices ($k \neq i$).

Thus, if the fund manager is restricted to hold (positively weighted) portfolios, the portfolio constraint should also hold in the style regression, implying that semi-strong style analysis should yield better results than weak style analysis. To the extent that the positivity constraints hold for the individual assets in Eq. (14), the positivity constraints should also hold for mutual fund returns if the fund manager is not allowed to take short positions. Although it may be reasonable to assume that positivity constraints (Eq. (14)) will hold for most assets, this is mainly an empirical question. However, even though some individual assets may have a negative loading on some factor portfolios, these negative weights are in most cases not likely to show up in the factor loadings of the mutual funds, as the fund will typically be a broad portfolio of individual assets, giving the negative factor loadings of some individual assets only a small weight. Strong style analysis might then be preferable to weak style analysis because of the efficiency gains. The analysis in this section shows that in determining the actual style, return-based (strong) style analysis will in general also be preferable to holding-based style analysis, because these holdings do not yield the actual style of the fund, unless the factor loadings of the individual assets are equal to one.

4. Style analysis and performance measurement

One way in which the style portfolio $\hat{b}_i$ can be used is to provide a benchmark to evaluate the performance of the mutual fund. Since $\hat{b}_i$ reflects the best positively weighted mimicking portfolio, it seems natural to compare the mutual fund returns r_t with the returns on the mimicking portfolio $\hat{b}_i' R_t$. The intercept $\hat{a}_i$ in the style regression

$$r_{i,t} = \hat{a}_i + \hat{b}_i' R_t + e_{i,t}, \tag{15}$$

gives the expected excess return of the mutual fund relative to the mimicking portfolio. If it is possible to find a perfect mimicking portfolio $\hat{b}_i$, implying that $\mathrm{Var}[e_{i,t}] = 0$, then a positive value of $\hat{a}_i$ implies that the fund return can only be obtained at higher cost when using the benchmarks, and that investors will strictly prefer the mutual fund over the mimicking portfolio. If $\mathrm{Var}[e_{i,t}] > 0$, a positive value of $\hat{a}_i$ does not necessarily mean that the fund outperforms the mimicking portfolio though, since the mutual fund may also be riskier than the mimicking portfolio. If the choice is to invest either in the mimicking portfolio or in the mutual fund, the performance can therefore best be measured by the Sharpe ratio, which gives the excess expected return of the portfolio (or fund) relative to its standard deviation:

$$\mathrm{Sh}_i = \frac{E[r_{i,t}] - R^f}{\sigma(r_{i,t})}.$$

Since the difference in expected returns between the mutual fund and the mimicking portfolio is the style intercept, $\hat{a}_i$, a positive value of $\hat{a}_i$ will induce a higher Sharpe ratio, unless this is offset by a higher standard deviation of the mutual fund, $\sigma(r_{i,t})$. The variance of the mutual fund return can be written as

$$\begin{aligned}\mathrm{Var}[r_{i,t}] &= \mathrm{Var}[\hat{b}_i' R_t + e_{i,t}] = \mathrm{Var}[\hat{b}_i' R_t] + \mathrm{Var}[e_{i,t}] + 2\mathrm{Cov}[\hat{b}_i' R_t, e_{i,t}] \\ &= \mathrm{Var}[\hat{b}_i' R_t] + \mathrm{Var}[e_{i,t}] + 2 \times \frac{1 - b_i' \iota_K}{\iota_K' \Sigma_{RR}^{-1} \iota_K},\end{aligned}$$

where the last term arises because the error term, $e_{i,t}$ may be correlated with $\hat{b}_i' R_t$ due to the portfolio constraint (see Eq. (5)). Thus, the variance of the mutual fund return will exceed that of the mimicking portfolio return if

$$\frac{1 - b_i' \iota_K}{\iota_K' \Sigma_{RR}^{-1} \iota_K} > -\frac{1}{2} \times \mathrm{Var}[e_{i,t}] \Leftrightarrow 1 - b_i' \iota_K > -\frac{1}{2} \frac{\mathrm{Var}[e_{i,t}]}{\mathrm{Var}[R_t^{\mathrm{GMV}}]}. \tag{16}$$

Similarly, the variance of the mutual fund return is smaller if the inequality is reversed. Notice that a necessary condition for a smaller variance of the mutual fund return is that $b_i' \iota_K > 1$, implying that—without the portfolio constraint—the mimicking portfolio would require a bigger investment than the mutual fund. In addition to this, it follows from Eq. (16) that in terms of variance, the mutual fund becomes more attractive than the mimicking portfolio if $\mathrm{Var}[R_t^{\mathrm{GMV}}]$ increases and if $\mathrm{Var}[e_{i,t}]$ decreases.

Evaluating the fund using the intercept a_i in the style regression is reminiscent of the Jensen measure for the fund, using the same asset classes as the benchmark assets. Therefore, an alternative way of analyzing the mutual fund performance is by using the Jensen measure, which is the intercept in a regression of the mutual fund excess returns on the benchmark excess returns:

$$r_{i,t} - \eta = \alpha_{J,i} + B(R_t - \eta \iota_K) + \varepsilon_{i,t}. \tag{17}$$

Here, η is the zero-beta rate associated with a mean-variance efficient portfolio, which can be replaced by the risk-free rate if the risk-free deposit is one of the benchmark assets. A high value of the Jensen measure indicates that the maximum obtainable Sharpe ratio from the benchmark assets R_t only can be improved upon if the investor also includes the mutual fund in his investment portfolio. Thus, whereas the Sharpe ratio can be used to make a choice between two investment alternatives, the mutual fund and the benchmark portfolio, the Jensen measure gives the improvement in the Sharpe ratio that can be obtained if the mutual fund is added to the benchmark assets (see, e.g., Jobson and Korkie, 1984 and 1989). From Eq. (6), it follows that the portfolio restricted intercept $\tilde{a}_i$ equals a special case of the generalized Jensen measure, since $\tilde{a}_i$ equals the intercept $\alpha_{J,i}$ in the regression

$$r_{i,t} - E[R_t^{\mathrm{GMV}}] = \alpha_{J,i} + \beta\left(R_t - E[R_t^{\mathrm{GMV}}]\iota_K\right) + u_{i,t}. \tag{18}$$

Thus, for investors with a zero-beta rate equal to the expected return on the GMV portfolio, we obtain the Jensen measure as the portfolio restricted intercept in a regression of the fund returns on the benchmark returns. In a similar fashion, the intercept $\hat{a}_i$ in the style analysis, which includes both the portfolio and the positivity constraints is also a special case of the Jensen measure as in Eq. (18), but based on the subset R_{1t} only, for which the positivity constraints are not binding. It should be noted at this point that if the actual factor exposures are in fact positively weighted portfolios and if one of the factors or benchmarks is the risk-free deposit (for which we do not necessarily have to impose the positivity constraint), the intercept in the strong style regression will actually coincide with the Jensen measure for any mean-variance investor and not only for investors with a very low risk aversion.

At this point, it is also worthwhile to note that Eq. (17) has a clear interpretation in terms of optimal portfolio weights. Given that the investor holds a mean-variance efficient portfolio of the benchmark assets R_t, the Jensen measure, together with the (co)variances of the residuals ε_t, yield the optimal weights in the mean-variance portfolio that invests in both the benchmarks (R_t) and the mutual funds (r_t) (see, e.g., Treynor and Black, 1973; DeRoon and Nijman, 2001). If the benchmarks or factor portfolios used in style analysis are the relevant factors in explaining asset returns, mean-variance investment portfolios will also based on those factor portfolios. Therefore, if the portfolio and positivity constraints imposed in strong style analysis are valid, then the question whether or not to include an individual mutual fund in an investment portfolio can be answered by analyzing $\hat{a}_i/\sigma(\varepsilon_i)$, i.e., the ratio of the average tracking error over the standard deviation of the tracking error. This ratio is also known as the information or appraisal ratio. When the covariance matrix of the tracking errors $\varepsilon_{i,t}$ is diagonal, this ratio can also be used when considering different mutual funds simultaneously.

In summary, the performance measurement of the mutual fund relative to the mimicking portfolio should not be based on the intercept $\hat{a}_i$ only, since the mutual fund may be also be riskier than the mimicking portfolio which may actually result in a lower Sharpe ratio even though the intercept $\hat{a}_i$ is positive. In addition, although the $\hat{a}_i$ may be interpreted in terms of the Jensen measure, it should be noted that, in general, $\hat{a}_i$ is the Jensen measure for investors with a very low risk aversion. However, if the portfolio and positivity constraints are valid and if one of the assets is the risk-free deposit, $\hat{a}_i$ actually coincides with the Jensen measure irrespective of the risk aversion of the investor.

5. Data and empirical analysis

In the previous sections, we showed that the portfolio and positivity constraints in return-based style analysis will in general lead to efficiency gains, although they will also yield biased style estimates if the constraints are in fact not true. In addition, we showed that return-based style analysis will in general give different insights than the actual portfolio holdings. In order to illustrate the potential consequences, we use style analysis in a sample of 18 US-based internationally investing mutual funds over the period January 1989 through April 1999. The mutual fund data are obtained from Morningstar's Principia Pro database and have as reported investment style 'foreign' or 'world', where the distinction between the two styles is that in case of 'foreign', it is not allowed to hold US stocks. The fund returns are in dollars and net of expenses. Our sample is comparable to the sample of Cumby and Glenn (1990) and DeRoon et al. (1998), studying the performance of, respectively, 15 funds over the period January 1982 through June 1988 and 18 funds over the period January 1982 through December 1994.

In Table 2, we present some summary statistics for the sample of funds that we employ. Overall, the funds have similar levels of risk as measured by their standard deviations, and average returns varying from 0.57% to 1.32%. It appears that New Perspective realized the highest average return with the lowest standard deviation. The fund charges an initial load fee of 5.75%, and is by far the largest fund in size. The worldwide diversified fund First Invest Global charges the highest load fee of 6.25%, while six funds in the sample do not charge an initial load fee. The Vanguard International Growth fund can be characterized as a passively managed fund, while the other funds in the sample follow an active selection strategy.

5.1. Style analysis and fund performance

As mentioned in the analysis of Section 4, style analysis is often used to provide a benchmark in order to evaluate the performance of mutual funds (see, e.g. Sharpe, 1992 and Fung and Hsieh, 1997). A question that receives considerable attention in the performance evaluation literature is why people invest in actively managed mutual funds (see, e.g. Gruber, 1996). Actively managed mutual funds are characterized by active stock selection strategies and market timing strategies in order to beat the return on a benchmark. In contrast, passively managed mutual funds mainly follow buy and hold strategies, where the investment objective is to replicate as close as possible a certain benchmark or market

Table 2
Summary statistics

Mutual fund	Style	Average return (%)	Standard deviation (%)	Net assets (mln $)	Front load (%)
Alliance Global Sm	W	0.79	4.76	74.4	4.25
Alliance Intl	F	0.61	4.42	76.7	4.25
Bailard, Biehl Intl	F	0.48	4.46	113.4	0.00
Evergreen Intl Gr	F	0.57	3.97	66.0	0.00
First Invest Global	W	0.98	4.44	312.4	6.25
Kemper Intl	F	0.78	4.04	398.4	5.75
Nations Intl Gr	F	0.77	4.27	22.4	0.00
New Perspective	W	1.32	3.56	23061.1	5.75
Oppenheimer Global	W	1.25	4.34	3580.5	5.75
Phoenix-Aberdeen	W	1.01	4.20	185.3	4.75
Putnam Global Gr	W	1.11	3.93	3518.3	5.75
Scudder Intl	F	0.97	4.02	3103.7	0.00
T. Rowe Price Intl	F	0.93	4.23	10006.7	0.00
Templeton Global Sm	W	0.84	3.86	1095.8	5.75
Templeton Gr	W	1.19	3.81	12319.5	5.75
Templeton World	W	1.19	3.93	8589.9	5.75
United Intl Gr	F	1.03	4.55	1236.4	5.75
Vanguard Intl Gr	F	0.88	4.35	7601.6	0.00

The table reports the average monthly fund return over the period January 1989 through April 1999, and the corresponding standard deviation of the fund return. The column labeled 'Style' presents the reported investment foreign (F) or world (W). The column 'Net assets' reports the size of the fund as measured at the end of 1998, while the column 'Front load' reports the load fee that the fund charges for a position in the fund.

Table 3
Intercepts and slope coefficients strong style analysis

Mutual fund	Style	α	North America		Europe	
			Growth	Value	Growth	Value
Alliance Global Sm	W	– 0.62 [– 1.10; 0.01]	0.29 [0.00; 0.29]	0.54 [0.48; 0.97]	0.09 [0.00; 0.09]	0.00 [0.00; 0.00]
Alliance Intl	F	– 0.59 [– 0.89; – 0.20]	0.06 [0.00; 0.06]	0.08 [0.02; 0.08]	0.41 [0.37; 0.61]	0.16 [0.00; 0.16]
Bailard, Biehl Intl	F	– 0.67 [– 0.92; – 0.25]	0.00 [0.00; 0.00]	0.08 [0.01; 0.08]	0.46 [0.42; 0.67]	0.17 [0.01; 0.17]
Evergreen Intl Gr	F	– 0.78 [– 1.27; – 0.35]	0.00 [0.00; 0.00]	0.18 [0.09; 0.18]	0.20 [0.03; 0.44]	0.31 [0.10; 0.49]
First Invest Global	W	– 0.11 [– 0.34; 0.27]	0.00 [0.00; 0.00]	0.26 [0.18; 0.50]	0.17 [0.03; 0.36]	0.37 [0.18; 0.54]
Kemper Intl	F	– 0.76 [– 1.27; – 0.39]	0.09 [0.00; 0.09]	0.06 [0.00; 0.06]	0.34 [0.17; 0.55]	0.28 [0.14; 0.46]
Nations Intl Gr	F	– 0.48 [– 0.73; – 0.13]	0.00 [0.00; 0.00]	0.14 [0.07; 0.14]	0.28 [0.12; 0.47]	0.31 [0.16; 0.50]
New Perspective	W	– 0.20 [– 0.49; 0.08]	0.10 [0.00; 0.10]	0.44 [0.39; 0.62]	0.10 [0.00; 0.10]	0.25 [0.20; 0.35]
Oppenheimer Global	W	– 0.03 [– 0.38; 0.43]	0.00 [0.00; 0.00]	0.32 [0.24; 0.67]	0.29 [0.12; 0.51]	0.27 [0.09; 0.43]
Phoenix-Aberdeen	W	– 0.40 [– 0.84; 0.12]	0.00 [0.00; 0.00]	0.63 [0.53; 1.00]	0.29 [0.17; 0.59]	0.00 [0.00; 0.00]
Putnam Global Gr	W	– 0.14 [– 0.40; 0.28]	0.00 [0.00; 0.00]	0.43 [0.38; 0.68]	0.15 [0.00; 0.36]	0.30 [0.11; 0.50]
Scudder Intl	F	– 0.46 [– 0.85; – 0.16]	0.07 [0.00; 0.07]	0.07 [0.00; 0.07]	0.30 [0.16; 0.46]	0.26 [0.12; 0.41]
T. Rowe Price Intl	F	– 0.32 [– 0.51; – 0.04]	0.00 [0.00; 0.00]	0.10 [0.04; 0.10]	0.34 [0.26; 0.45]	0.28 [0.20; 0.37]
Templeton Global Sm	W	– 0.91 [– 1.52; – 0.33]	0.65 [0.56; 1.00]	0.00 [0.00; 0.00]	0.22 [0.08; 0.22]	0.00 [0.00; 0.00]
Templeton Gr	W	– 0.22 [– 0.48; 0.14]	0.63 [0.56; 0.98]	0.00 [0.00; 0.00]	0.27 [0.19; 0.52]	0.00 [0.00; 0.00]
Templeton World	W	– 0.13 [– 0.38; 0.19]	0.61 [0.53; 0.83]	0.04 [0.00; 0.04]	0.27 [0.19; 0.47]	0.00 [0.00; 0.00]
United Intl Gr	F	– 0.22 [– 0.63; 0.28]	0.00 [0.00; 0.00]	0.16 [0.07; 0.16]	0.47 [0.21; 0.78]	0.29 [0.04; 0.61]
Vanguard Intl Gr	F	– 0.31 [– 0.53; 0.00]	0.06 [0.00; 0.06]	0.00 [0.00; 0.00]	0.29 [0.14; 0.45]	0.34 [0.20; 0.52]

Mutual fund	Pacific		Cash	Sharpe ratio		Jensen α (S.E.)
	Growth	Value		Fund	Mim. ptf.	
Alliance Global Sm	0.08 [0.03; 0.32]	0.00 [0.00; 0.00]	0.00 [0.00; 0.00]	0.166	0.388	– 0.82 (0.26)
Alliance Intl	0.26 [0.20; 0.42]	0.00 [0.00; 0.00]	0.03 [0.00; 0.03]	0.138	0.295	– 0.44 (0.17)
Bailard, Biehl Intl	0.09 [0.00; 0.09]	0.17 [0.13; 0.30]	0.04 [0.00; 0.04]	0.108	0.276	– 0.48 (0.16)
Evergreen Intl Gr	0.02 [0.00; 0.02]	0.20 [0.16; 0.38]	0.08 [0.02; 0.18]	0.144	0.355	– 0.33 (0.17)
First Invest Global	0.08 [0.00; 0.08]	0.12 [0.08; 0.24]	0.00 [0.00; 0.00]	0.221	0.274	– 0.17 (0.15)
Kemper Intl	0.09 [0.04; 0.09]	0.05 [0.00; 0.05]	0.10 [0.03; 0.19]	0.193	0.418	– 0.27 (0.18)
Nations Intl Gr	0.17 [0.12; 0.28]	0.06 [0.00; 0.06]	0.04 [0.00; 0.04]	0.180	0.314	– 0.27 (0.16)
New Perspective	0.07 [0.04; 0.14]	0.00 [0.00; 0.00]	0.04 [0.00; 0.09]	0.371	0.439	0.01 (0.10)
Oppenheimer Global	0.08 [0.03; 0.08]	0.04 [0.00; 0.04]	0.00 [0.00; 0.00]	0.287	0.339	– 0.04 (0.19)
Phoenix-Aberdeen	0.08 [0.02; 0.08]	0.00 [0.00; 0.00]	0.00 [0.00; 0.00]	0.241	0.387	– 0.39 (0.24)
Putnam Global Gr	0.05 [0.01; 0.05]	0.07 [0.05; 0.07]	0.00 [0.00; 0.00]	0.282	0.332	– 0.15 (0.13)
Scudder Intl	0.17 [0.13; 0.28]	0.04 [0.00; 0.04]	0.08 [0.02; 0.16]	0.241	0.380	– 0.05 (0.14)
T. Rowe Price Intl	0.23 [0.20; 0.32]	0.01 [0.00; 0.01]	0.04 [0.00; 0.04]	0.221	0.310	– 0.10 (0.12)
Templeton Global Sm	0.04 [0.00; 0.26]	0.00 [0.00; 0.00]	0.10 [0.00; 0.22]	0.217	0.538	– 0.45 (0.22)
Templeton Gr	0.07 [0.04; 0.18]	0.00 [0.00; 0.00]	0.02 [0.00; 0.02]	0.312	0.396	– 0.14 (0.14)
Templeton World	0.08 [0.05; 0.20]	0.00 [0.00; 0.00]	0.00 [0.00; 0.00]	0.304	0.363	– 0.17 (0.14)
United Intl Gr	0.07 [0.00; 0.07]	0.02 [0.00; 0.02]	0.00 [0.00; 0.00]	0.227	0.308	– 0.22 (0.22)
Vanguard Intl Gr	0.21 [0.17; 0.34]	0.06 [0.00; 0.06]	0.05 [0.00; 0.05]	0.201	0.281	– 0.06 (0.13)

The table reports the intercepts and slope coefficients from a strong style analysis of 18 international investing mutual funds on 6 international MSCI Growth and Value indices and a Cash return, based on the period January 1989 until April 1999. The columns 'Sharpe ratio' report the Sharpe ratio of the each fund and of its mimicking portfolio. The last column reports the Jensen measure of each fund relative to the six MSCI indices, using excess returns.

index. Consequently, due to the higher trading activity, actively managed mutual funds usually have much higher operating expenses than passively managed funds, i.e., on average respectively 1.0% vs. 0.2% per year. Since these operating expenses are deducted from a mutual fund's gross income, investors might be interested in a potentially cheaper alternative. Most studies report that actively managed funds provide lower net returns than the passively managed funds (see, e.g. Wermers, 2000).

In order to examine whether it is more attractive to invest in a combination of passively managed funds or in one of the 17 actively managed funds in our sample, we report in Table 3 the estimation results of the following strong style analysis:

$$\begin{aligned} r_{i,t} = \hat{a}_i &+ \hat{b}_1 R_t^{(G\_NA)} + \hat{b}_2 R_t^{(V\_\mathrm{NA})} + \hat{b}_3 R_t^{(G\_\mathrm{EUR})} + \hat{b}_4 R_t^{(V\_\mathrm{EUR})} \\ &+ \hat{b}_5 R_t^{(G\_\mathrm{PAC})} + \hat{b}_6 R_t^{(V\_\mathrm{PAC})} + \hat{b}_7 R_t^{(\mathrm{Cash})} + e_{i,t} \end{aligned} \tag{19}$$

where $R_t^{(G\_)}$ and $R_t^{(V\_)}$ denote the returns in period t on the regional MSCI Growth and Value indices of North America (NA), Europe (EUR), and Pacific (PAC) and a benchmark reflecting the returns on a risk-free deposit $R_t^{(\mathrm{Cash})}$. All benchmark data are obtained from Datastream International. The table also reports the average tracking error $\hat{a}_i$ of the strong style analysis. This tracking error can be interpreted as the average relative under- or outperformance of the mutual fund with respect to the passive benchmarks.

It appears from Table 3 that the actively managed funds in the sample relatively underperform their corresponding mimicking portfolio that is a combination of the MSCI indices. The underperformance varies between 0.91% (i.e., 10.9% annually) for Templeton Global Small Companies fund and 0.03% (i.e., 0.4% annually) for Oppenheimer Global. However, as discussed in Section 4, a negative average tracking error does not necessarily indicate that it is optimal for investors to invest in the mimicking portfolio if the choice is restricted to invest in either the mutual fund or the mimicking portfolio, since all funds contain some residual risk relative to the mimicking portfolio, which may or may not be correlated with the factor returns. In order to answer the question whether the funds or the mimicking portfolios are more attractive investments, we report in two columns of Table 3 the Sharpe ratios of the mutual funds and the corresponding mimicking portfolios. Since from the strong style analysis it followed that the style intercept $\hat{a}_i$ is negative for all the actively managed funds in the sample, a higher Sharpe ratio of the fund can only be caused by a lower standard deviation of the mutual fund compared to the mimicking portfolio. However, the Sharpe ratios of the mimicking portfolios are almost uniformly higher than the Sharpe ratios of the funds, and at least in economic terms significantly so.

As shown by, e.g., Jobson and Korkie (1989), the Jensen measure is the relevant one if the investor wants to analyze the benefits of adding the fund to an efficient portfolio of the benchmark assets only, i.e., whether the Sharpe ratio of the benchmark assets can be improved by adding mutual funds to the portfolio. Therefore, we propose to use the Jensen measure, as given in Eq. (17), as an alternative performance measure that answers the question whether an investor can improve the maximum obtainable Sharpe ratio of his initial portfolio by also investing in an actively managed internationally investing mutual fund. To this end, Table 3 also reports the Jensen measure as obtained from the regression (19), leaving out Cash and using excess returns instead. In this case, we naturally do not impose portfolio or positivity constraints.

The Jensen measures are negative for all but one of the funds, although in most cases, they are not significantly different from zero. For most funds, the Jensen measures are also lower than the alphas from the style regression. The negative signs of the Jensen measures imply that starting from an investment in the MSCI benchmark indices, investors can only improve the Sharpe ratio of their portfolio if they add a short position in one of the mutual funds. Thus, whereas the style analysis implied that when choosing between the benchmark indices and the mutual funds, investors would prefer an investment in the benchmark indices, the Jensen measures imply that there are also no benefits from adding the mutual funds to a portfolio of the benchmark indices.

5.2. Style analysis, portfolio weights, and fund exposure

It was argued in Section 3 that, in general, there is no need for the style estimates to coincide with the actual portfolio holdings of a mutual fund. Even though the portfolio and positivity constraints may be valid in order to reflect the restrictions faced by the fund manager, the style estimates will differ from the actual portfolio weights because the manager may select stocks with relatively high or low betas, or because there are cross-correlations between the benchmarks. Nevertheless, the style estimates will reflect the sensitivity of the fund for certain factor or benchmark portfolios, i.e., the fund exposures.

In order to illustrate the differences between the style estimates and the actual portfolio holdings, in this subsection, we will apply style analysis on the sample of 18 internationally investing mutual funds and compare it with the actual portfolio holdings over the sample period January 1991 through April 1999. Note that the sample period is slightly different from the previous analysis, which is due to the fact that from the mutual funds in the sample, we observe the reported holdings (at an annual frequency) for the investment regions North America, Europe, and Pacific only over this shorter sample period.

In order to illustrate that style analysis does not necessarily accurately estimate the portfolio holdings of fund managers, we first apply strong style analysis using four asset classes, i.e., regional indices of North America, Europe, and Pacific, and a benchmark reflecting cash positions. Table 4 reports the estimated exposures for these style indices over the period January 1991 through April 1999, and subsequently compares them with the average reported holdings over the same period.

The bottom rows of Table 4 give an indication of the difference between the estimated strong style exposures and the reported actual holdings. On average, the estimated style exposures exceed the reported holdings for North America (11.0%) and Europe (12.0%), whereas the style exposures are lower than the reported holdings for the Pacific index (− 7.0%). For all three indices, we find that the estimated exposures and reported holdings are highly correlated (approximately 0.90).

In Section 3, it was explained that differences between the estimated style exposures and the reported holdings can be due to high and low beta stocks that are held by the fund or by correlations between the indices. Table 5 reports some results of strong style analysis using benchmarks at a more disaggregated level that indicate which factor is most important in explaining the difference between estimated style and reported holdings. For instance, in case of North America, we now use four different indices: US Growth and Value indices and Canadian Growth and Value indices. If the betas of these subindices

Table 4
Estimated exposures and reported holdings

Mutual fund	Style	Estimated exposures (average reported holdings)			
		North America	Europe	Pacific	Other
Alliance Global Sm	W	0.59 (0.53)	0.28 (0.20)	0.13 (0.10)	0.00 (0.02)
Alliance Intl	F	0.05 (0.02)	0.58 (0.47)	0.34 (0.38)	0.03 (0.02)
Bailard, Biehl Intl	F	0.14 (0.03)	0.63 (0.53)	0.20 (0.30)	0.03 (0.03)
Evergreen Intl Gr	F	0.18 (0.05)	0.50 (0.39)	0.15 (0.25)	0.17 (0.06)
First Invest Global	W	0.40 (0.29)	0.44 (0.35)	0.16 (0.25)	0.00 (0.03)
Kemper Intl	F	0.14 (0.04)	0.60 (0.52)	0.16 (0.30)	0.10 (0.05)
Nations Intl Gr	F	0.14 (0.01)	0.56 (0.48)	0.21 (0.33)	0.09 (0.06)
New Perspective	W	0.52 (0.33)	0.39 (0.30)	0.08 (0.11)	0.01 (0.03)
Oppenheimer Global	W	0.41 (0.24)	0.46 (0.35)	0.13 (0.15)	0.00 (0.06)
Phoenix-Aberdeen	W	0.39 (0.31)	0.49 (0.34)	0.12 (0.18)	0.00 (0.05)
Putnam Global Gr	W	0.43 (0.26)	0.43 (0.35)	0.13 (0.24)	0.00 (0.03)
Scudder Intl	F	0.12 (0.03)	0.55 (0.47)	0.24 (0.34)	0.08 (0.02)
T. Rowe Price Intl	F	0.11 (0.02)	0.60 (0.49)	0.25 (0.33)	0.04 (0.05)
Templeton Global Sm	W	0.38 (0.32)	0.38 (0.29)	0.07 (0.13)	0.17 (0.06)
Templeton Gr	W	0.46 (0.30)	0.33 (0.27)	0.13 (0.15)	0.08 (0.05)
Templeton World	W	0.47 (0.33)	0.39 (0.28)	0.13 (0.15)	0.02 (0.05)
United Intl Gr	F	0.11 (0.03)	0.83 (0.58)	0.06 (0.14)	0.00 (0.08)
Vanguard Intl Gr	F	0.08 (0.01)	0.65 (0.50)	0.25 (0.39)	0.02 (0.02)
Mean difference		0.11	0.12	− 0.07	0.00
Stdev difference		0.04	0.05	0.05	0.05
Correlation		0.98	0.94	0.88	0.32
GMV-portfolio		− 0.02	0.02	0.05	0.95

The table reports the estimated exposures to regional indices based on return-based style analysis over the period January 1991 through April 1999 and the average reported holdings over the corresponding sample period. The table also reports the mean difference between the estimated exposures and the reported holdings, the standard deviation of this difference, and the correlation between the estimated exposures and the reported holdings. The bottom row reports the composition of the GMV portfolio of the four regional indices.

relative to the aggregate North America index are not equal to one, and if the weights of these subindices in the aggregate index differ from the weights assigned to them by the fund manager, then this will cause a difference between the estimated style exposures and the reported holdings, as follows from Section 3. In a similar way, we split each regional index in Value and Growth indices for the underlying countries. For Europe, these are France, Germany, Italy, and the UK, and for the Pacific area, these are Australia, Hong Kong, and Japan.[4]

For each fund, Table 5 first of all reports the sum of the estimated strong style exposures, $\Sigma_i \hat{b}_i$. If style analysis provides consistent estimates of the actual portfolio holdings, then these summed exposures should be close to the estimated exposures to the aggregate indices in Table 4. For North America, although the summed exposures have the same order of magnitude as the aggregate exposures in Table 4, they are certainly not equal. In addition, the difference between the summed exposures and the reported holdings is not smaller than the difference between the aggregate exposures and the reported

[4] Summary statistics and betas for these subindices can be obtained from the authors upon request.

Table 5
Aggregated estimated exposures

Mutual fund	Style	Estimated exposure							
		North America		Europe		Pacific		Other	
		$\Sigma\hat{b}_i$	$\sum \frac{\hat{b}_i}{\beta^{(i)}}$	$\Sigma\hat{b}_i$	$\sum \frac{\hat{b}_i}{\beta^{(i)}}$	$\Sigma\hat{b}_i$	$\sum \frac{\hat{b}_i}{\beta^{(i)}}$	$\Sigma\hat{b}_i$	$\sum \frac{\hat{b}_i}{\beta^{(i)}}$
Alliance Global Sm	W	0.656	0.647	0.193	0.194	0.152	0.235	0.000	0.000
Alliance Intl	F	0.102	0.111	0.485	0.484	0.400	0.628	0.013	0.013
Bailard, Biehl Intl	F	0.145	0.157	0.535	0.530	0.270	0.478	0.051	0.051
Evergreen Intl Gr	F	0.213	0.213	0.426	0.427	0.203	0.372	0.158	0.158
First Invest Global	W	0.392	0.389	0.378	0.377	0.230	0.415	0.000	0.000
Kemper Intl	F	0.164	0.172	0.476	0.476	0.281	0.631	0.078	0.078
Nations Intl Gr	F	0.152	0.156	0.426	0.420	0.344	0.768	0.077	0.077
New Perspective	W	0.460	0.445	0.343	0.345	0.187	0.446	0.010	0.010
Oppenheimer Global	W	0.400	0.399	0.392	0.399	0.207	0.387	0.000	0.000
Phoenix-Aberdeen	W	0.360	0.348	0.402	0.413	0.238	0.582	0.000	0.000
Putnam Global Gr	W	0.370	0.354	0.394	0.404	0.236	0.499	0.000	0.000
Scudder Intl	F	0.137	0.142	0.450	0.448	0.346	0.615	0.067	0.067
T. Rowe Price Intl	F	0.090	0.090	0.498	0.490	0.373	0.723	0.039	0.039
Templeton Global Sm	W	0.415	0.409	0.244	0.243	0.155	0.384	0.186	0.186
Templeton Gr	W	0.415	0.413	0.237	0.236	0.261	0.580	0.086	0.087
Templeton World	W	0.427	0.428	0.273	0.273	0.261	0.574	0.040	0.040
United Intl Gr	F	0.168	0.183	0.626	0.639	0.207	0.591	0.000	0.000
Vanguard Intl Gr	F	0.092	0.095	0.534	0.536	0.374	0.707	0.000	0.000
Mean difference		0.11	0.11	0.02	0.02	0.03	0.30	0.00	0.00
Stdev difference		0.03	0.03	0.05	0.05	0.05	0.10	0.05	0.05
Correlation		0.98	0.98	0.91	0.91	0.89	0.67	0.36	0.36

The columns '$\Sigma\hat{b}_i$' in the table reports the estimated exposures aggregated at a regional level of a strong style analysis, using as asset classes the growth and value indices of each country underlying a regional index. The columns '$\sum \frac{\hat{b}_i}{\beta^{(i)}}$' report a weighted estimated exposure. The table also reports the mean difference between the estimated exposures and the reported holdings, the standard deviation of this difference, and the correlation between the estimated exposures and the reported holdings.

holdings as can be found in Table 4. The bottom three rows of the table summarize the relation between the summed style exposures and the actual reported weights. Comparing the mean and standard deviation of the difference with the ones reported in Table 4, it can be seen that the use of subindices does not give any improvement for the North American case. In addition, the correlation between the summed style exposures and the actual reported weights in Table 5 is almost identical to the one reported in Table 4, which is based on the aggregate index.

This picture changes if we focus on the European and Pacific indices. For the European indices, the summed exposures in Table 5 are much closer to the actual reported holdings than the estimated exposures in Table 4. The average difference decreases from 12.0% in Table 4 to 2.0% in Table 5. For the Pacific region, a similar story holds, i.e., the average difference changes from -7.0% in Table 4 to $+3.0\%$ in Table 5.

From Section 5, the summed exposures are likely to differ from the reported holdings if the betas of the subindices relative to the aggregate indices are different from one. To correct for this, Table 5 also reports the sum of the estimated style exposures for each region, weighted by the $\beta_j^{(i)}$ of each subindex j relative to the aggregate regional index i. If the style exposure for the disaggregated indices reflect the actual portfolio weights assigned by the fund manager, then this weighted sum should be closer to the reported holdings, assuming that the betas of the individual stocks with respect to the subindex are relatively close to one. Comparing the two columns for each region in Table 5, we see that the two summed exposures are very close in case of North America and Europe, but not for the Pacific case. This reflects the fact that the β's of the subindices relative to the aggregate index are close to one in case of North America and Europe, whereas in the Pacific case, they can be as low as 0.36.[5] However, even though for the Pacific case, the β's are clearly different from one, the weighted summed exposures do not explain the difference between the estimated aggregate exposure and the reported holdings in Table 4. On the contrary, the average difference between the summed exposure and the actual reported holdings increase from 3.0% to 30.0% and the correlation between the summed style exposures and the reported holdings even decreases.

Although the analysis is limited by the availability of the data, Table 5 indicates that the differences between estimated exposures and reported holdings is not likely to be explained by the fact that fund managers hold on average high or low beta stocks relative to the index. It follows then that the difference between reported holdings and estimated exposures is more likely to be caused by the correlations between the different indices.

5.3. Return-based style analysis versus holding-based style analysis

Although the previous section showed that there are clearly differences between actual portfolio holdings and estimated style exposures, this does not imply that holding-based style analysis is not useful. A recent practitioner's article by Rekenthaler et al. (2002) has argued that portfolio holdings provide a more accurate prediction of style. In order to address the questions which method best predicts the future 'style' and which method best describes the future return behavior of the mutual fund (out-of-sample), we report in Tables 6 and 7 the Mean Absolute Deviations (MAD) of the predicted holdings versus the actual holdings and the MAD of the predicted returns versus the realized returns over the period 1992–1998. The benchmarks used are the aggregated regional indices. The predicted holdings and returns are based either on a return-based style analysis (RBSA) or on holding-based style analysis (HBSA), in which case the last reported holdings are used. In case of return-based style analysis, we base our prediction on a moving window style regression of 36 months, while in the case of the holdings-based prediction, the last reported actual holding is used. For instance, the first reported actual holding we observe is the portfolio holding at the end of 1991. These portfolio holdings are used to predict the holdings at the end of 1992, or to predict the monthly fund returns during 1992. In case of return-based style analysis, we use the monthly returns over the period December 1989–November 1992 to predict the style, or the returns in a moving window of 36 months over the period January 1989–December 1991 to predict the monthly fund returns during 1992. Notice that this procedure may result in a relative advantage for holding-based style analysis, since a change in style would result in an immediate change in the holdings, whereas the return-based exposures only adjust gradually over a 3-year period.

[5] The betas of the subindices relative to the aggregate indices can be obtained from the authors upon request.

Table 6
Mean absolute deviations predicted holdings

Mutual fund	MAD predicted holdings							
	RBSA	HBSA	RBSA	HBSA	RBSA	HBSA	RBSA	HBSA
	North America		Europe		Pacific		Other	
Alliance Global Sm	0.12	0.07	0.10	0.08	0.05	0.05	0.15	0.07
Alliance Intl	0.05	0.01	0.12	0.06	0.06	0.08	0.08	0.03
Bailard, Biehl Intl	0.06	0.02	0.28	0.12	0.20	0.09	0.32	0.18
Evergreen Intl Gr	0.23	0.03	0.09	0.06	0.08	0.05	0.21	0.07
First Invest Global	0.11	0.05	0.14	0.05	0.09	0.05	0.11	0.02
Kemper Intl	0.14	0.03	0.12	0.09	0.13	0.05	0.11	0.04
Nations Intl Gr	0.12	0.00	0.13	0.06	0.13	0.06	0.13	0.03
New Perspective	0.16	0.04	0.14	0.05	0.07	0.02	0.24	0.05
Oppenheimer Global	0.15	0.06	0.15	0.07	0.07	0.06	0.23	0.07
Phoenix-Aberdeen	0.14	0.11	0.18	0.12	0.07	0.06	0.18	0.08
Putnam Global Gr	0.13	0.05	0.11	0.03	0.12	0.05	0.12	0.03
Scudder Intl	0.09	0.01	0.15	0.07	0.13	0.05	0.11	0.04
T. Rowe Price Intl	0.08	0.02	0.18	0.07	0.11	0.04	0.14	0.05
Templeton Global Sm	0.15	0.05	0.10	0.05	0.08	0.06	0.16	0.05
Templeton Gr	0.19	0.04	0.12	0.02	0.07	0.03	0.23	0.05
Templeton World	0.17	0.04	0.12	0.03	0.07	0.04	0.22	0.05
United Intl Gr	0.09	0.03	0.22	0.08	0.05	0.07	0.26	0.08
Vanguard Intl Gr	0.06	0.01	0.17	0.05	0.15	0.04	0.07	0.03

The table reports the mean absolute deviations of the predicted holdings versus the reported holdings. The predictions are based either on a rolling window strong style analysis of 36 months (labeled 'RBSA') or on the reported holdings of the previous year (labeled 'HBSA').

It is obvious from Table 6 that last year's holdings give a better prediction of the current holdings than the rolling style estimates. The MAD for the holding-based style analysis is usually about 0.05, whereas for return-based style analysis, it is about 0.15. Thus, if the aim of the analysis is to predict future portfolio holdings, holding-based style analysis performs better than return-based style analysis. This could be expected beforehand, since we already knew from the theoretical analysis in Section 3 and the empirical analysis in the previous section that estimated style exposures can deviate from actual portfolio holdings.

The return-based style exposures may be more useful though in terms of predicting or explaining fund returns. If the interest is in predicting fund returns conditional on the factor returns, which is the case for instance in performance measurement and asset allocation studies, the factor exposures may be more relevant than the actual portfolio holdings. To this end, Table 7 shows the MAD of the actual versus the predicted return, where the predicted return is conditional on the realized factor returns, based on either last year's actual portfolio holdings or on the 3-year estimated strong style exposure. Thus, each month in year t we predict the fund return as

$$\hat{r}_t = \sum_{j=1}^{K} w_{j,t-1} R_{j,t},$$

where $w_{j,t-1}$ are either the actual portfolio holdings at time $t-1$ or the estimated rolling window exposures based on the last 3 years until time $t-1$.

Table 7
Mean absolute deviations predicted returns

Mutual fund	MAD predicted returns	
	RBSA	HBSA
Alliance Global Sm	2.12	2.22
Alliance Intl	1.15	1.12
Bailard, Biehl Intl	1.25	2.01
Evergreen Intl Gr	1.46	1.65
First Invest Global	1.01	1.07
Kemper Intl	1.47	1.62
Nations Intl Gr	1.20	1.34
New Perspective	0.85	1.32
Oppenheimer Global	1.49	1.82
Phoenix-Aberdeen	1.64	1.67
Putnam Global Gr	1.01	1.24
Scudder Intl	1.10	1.07
T. Rowe Price Intl	0.88	1.14
Templeton Global Sm	2.15	2.10
Templeton Gr	1.44	1.66
Templeton World	1.23	1.47
United Intl Gr	1.93	2.21
Vanguard Intl Gr	1.04	1.23

The table reports the mean absolute deviations of the predicted return and the actual return. The predicted returns are based either on a rolling window strong style analysis of 36 months (RBSA) or on the last reported information regarding the holdings (HBSA).

From Table 7, we see that in all but three cases, the MAD between the actual and the predicted returns is smaller for the return-based style analysis than for the holding-based style analysis. The average MAD for the holding-based style analysis is 1.55% per month, whereas for the return-based style analysis, it is only 1.36% per month. Assuming that the MADs for the various mutual funds are uncorrelated a t-test for the difference between these two means would give a t-value of 4.3, suggesting a significant difference between the two.

Thus, although holding-based style analysis may be preferred to return-based style analysis, if the aim is to predict future portfolio holdings, return-based style analysis may be more attractive if we want to predict future fund returns. Return-based style analysis seems to be more suitable to identify the actual factor exposures that are relevant for predicting future returns and identifying the risk exposures of the fund, which do not follow immediately from the actual portfolio holdings of the fund.

6. Summary and conclusions

The portfolio and positivity constraints that are usually imposed in return-based style analysis may lead to biased estimates if the actual factor exposures of the mutual fund are not a positively weighted portfolio, but can be expected to lead to significant efficiency gains if these constraints are in fact true. Return-based style analysis will in general give a better estimate of the actual investment style than the fund's portfolio holdings, because of cross-correlations between asset classes and because the fund manager may select assets with relatively high or low betas relative to their own index.

In relative performance evaluation, the aim of style analysis is to determine a benchmark portfolio that mimics the fund under consideration. In this case, the portfolio and positivity constraints are required since in weak style analysis, the factor exposures do not necessarily sum to one nor are they positive. Although the intercept in the strong style regression indicates whether the fund under- or outperforms the mimicking portfolio on a relative basis, it may only be interpreted as the Jensen measure for a very specific group of investors, unless the portfolio and positivity constraints are valid and one of the benchmark assets is the risk-free asset.

Both from the theoretical and from the empirical analysis, we find that estimated style exposures may deviate from portfolio holdings. Actual portfolio holdings are better predictors of future portfolio holdings than estimated style exposures are. However, if the aim is to predict future fund returns, factor exposures seem to be more relevant than actual portfolio holdings and return-based style analysis performs better than holding-based style analysis.

Acknowledgements

Financial support by the Institute for Quantitative Research Europe (INQUIRE) is gratefully acknowledged by the authors. Furthermore, we are grateful to Bas Werker, Thierry Chauveau, Geert Bekaert (the editor), an anonymous referee, seminar participants from the Norwegian School of Management, and participants of the 2001 German Finance Association meetings and the 2001 European Financial Management Association meetings for helpful comments and suggestions.

Appendix A

This appendix shows the main steps in deriving the asymptotic confidence intervals for strong style analysis. To simplify notation, define θ=(α β')′, $\hat{\theta}$=($\hat{a}$ $\hat{b}'$), e=(0ι')′, and X_t=(1R_t'). From Andrews (1999), Kim et al. (2000) derive that with inequality constraints on the style coefficients in Eq. (1), we get

$$\sqrt{T}\left(\hat{\theta}-\theta_0\right) \rightarrow \hat{\lambda},$$

where the limiting random variable $\hat{\lambda}$ is the solution to the problem

$$\min_{\lambda}(\lambda - E[X_tX_t'\,]^{-1}G)'\,E[X_tX_t'\,](\lambda - E[X_tX_t'\,]^{-1}G)$$

$$\text{s.t. } e'\,\lambda = 0,$$

$$Q\lambda \leq 0,$$

where G is a normal random vector with mean zero and covariance matrix $E[\varepsilon_t^2 X_t X_t']$, and Q is a $L \times K$ matrix of zeros, except $q_{ij} = -1$ if θ_j is the ith element of θ_0 that is zero. To determine which elements of θ_0 are zero, a pretest is done using a semi-strong style regression of the fund returns on the factor returns and setting those elements of θ_0 equal to zero for which we cannot reject the hypothesis that the corresponding semi-strong style coefficients are zero at the chosen pretest significance level. In the paper, we use as pretest levels 5%, 10%, and 50%. Using Monte Carlo simulations, the 95% confidence bounds for $\hat{\lambda}$ can be determined, which can be denoted as z_L and z_U:

$$1 - \text{pretest level} = \Pr\{z_L \le \hat{\lambda} \le z_U\}.$$

From this confidence interval, we can derive a confidence interval for θ_0 using

$$\Pr\{z_L \le \hat{\lambda} \le z_U\} = \Pr\{z_L \le \sqrt{T}(\hat{\theta} - \theta_0) \le z_U\},$$

implying that the confidence interval is given by

$$\hat{\theta} - z_U \sqrt{T}, \hat{\theta} - z_L \sqrt{T}.$$

When the simulations result in a lower bound on β in the strong style analysis that is smaller than zero, this lower bound is set to zero (see Kim et al., 2000).

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